Flavour Physics, CP violation in the S.M. and Beyond

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Lectures given at the 3rd IDPASC School

Santiago de Compostela

January 2013

Brief Review of the Flavour Structure of the Standard Model (SM)

Gauge group: $SU(3)_C \times SU(2)_L \times U(1) \equiv G_{SM}$

Gauge fields: G_{SM} has 12 generators $\Rightarrow$ 12 gauge fields:

$$SU(2)_L \rightarrow W_j^\mu$$

$$SU(3)_C \rightarrow G_k^\mu \rightarrow$$ 8 gluons which mediate strong interactions

$$U(1) \rightarrow B^\mu$$

$B^\mu, W_j^\mu \rightarrow W_\mu^\pm, Z_\mu, A_\mu \rightarrow$ mediate electroweak interactions

Fermion Fields

		Representation
Left-handed quarks :	$Q_L = \begin{bmatrix} (u^0)^\alpha_L \\ (d^0)_L \end{bmatrix}_j$	$(3, 2, 1/6)$
Left-handed leptons :	$L_L \begin{bmatrix} \nu^0_L \\ l^0_L \end{bmatrix}_j$	$(1, 2, -1/2)$
Right-handed up quarks :	$(u^0)^\alpha_{R_j}$	$(3, 1, 2/3)$
Right-handed down quarks :	$(d^0)^\alpha_{R_j}$	$(3, 1, -1/3)$
Right-handed charged leptons :	$(l^0_R)_j$	$(1, 1, -1)$

$\alpha \rightarrow$ colour index ; $j \rightarrow$ family index

No $\nu^0_{R_j}$ are introduced! Why?

Scalar fields

$$\Phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix}$$

Representation

$(1, 2, 1/2)$

Gauge interactions are **determined** by the **covariant derivative** which is dictated by the transformation properties of the various fields, under the gauge group. In general:

$$D^\mu = \partial^\mu - i \underbrace{g_s}_{SU(3)_c} L_k G_k^\mu - i \underbrace{g}_{SU(2)_L} T_j W_j^\mu - i \underbrace{g'}_{U(1)_Y} Y B^\mu$$

$L_k \to$ 8 generators of $SU(3)_c$; $T_j \to$ 3 generators of $SU(2)_L$

$Y \to$ 1 generator of $U(1)$

The specific form of the generators depends, of course on the gauge group representations to which the various fields belong.

For example in the **SM**:

$Q_L \quad : \quad L_k = \frac{1}{2}\lambda_k \quad ; \quad T_j = \frac{1}{2}\tau_j \quad ; \quad Y = 1/6$

$u^{o\alpha}_R \quad : \quad L_k = \frac{1}{2}\lambda_k \quad ; \quad T_j = 0 \quad ; \quad Y = 2/3$

$d^{o\alpha}_R \quad : \quad L_k = \frac{1}{2}\lambda_k \quad ; \quad T_j = 0 \quad ; \quad Y = -1/3$

$L_L \quad\quad L_k = 0 \quad ; \quad T_j = \frac{1}{2}\tau_j \quad ; \quad Y = -1/2$

$\Phi \quad\quad L_k = 0 \quad ; \quad T_j = \frac{1}{2}\tau_j \quad ; \quad Y = 1/2$

$\lambda_k \; (k = 1, \dots 8)$ Gell-Mann matrices

$\tau_j \; (j = 1, 2 3)$ Pauli matrices

In order to describe physics at low energy the SM gauge symmetry has to be spontaneously broken.

$$SU(3)_C \times SU(2)_L \times U(1) \longrightarrow SU(3)_C \times U(1)_{em}$$

Spontaneous gauge symmetry breaking, rather than explicit breaking is essential in order to achieve renormalizability

G. 't Hooft, T. Veltman

mass terms like $m_W^2 W_\mu^+ W^{\mu -}$ are **not invariant under gauge transformations!!**

Gauge symmetry breaking in the SM.

The most general gauge invariant, renormalizable scalar potential is:

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

where $\Phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix}$; $\boxed{\lambda > 0}$

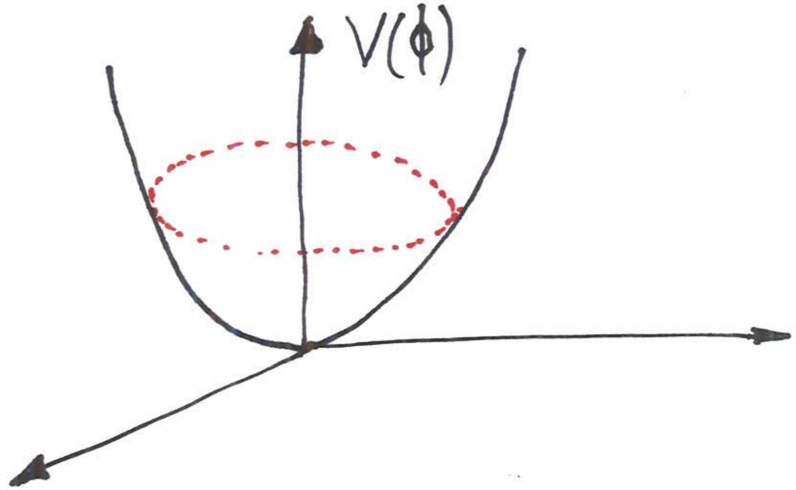
Two possibilities:

$\mu^2 > 0 \rightarrow$ minimum at $\langle 0 | \Phi | 0 \rangle = 0$

$\mu^2 < 0 \rightarrow$ minimum at $\langle 0 | \Phi | 0 \rangle = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{bmatrix}$

The two possibilities :

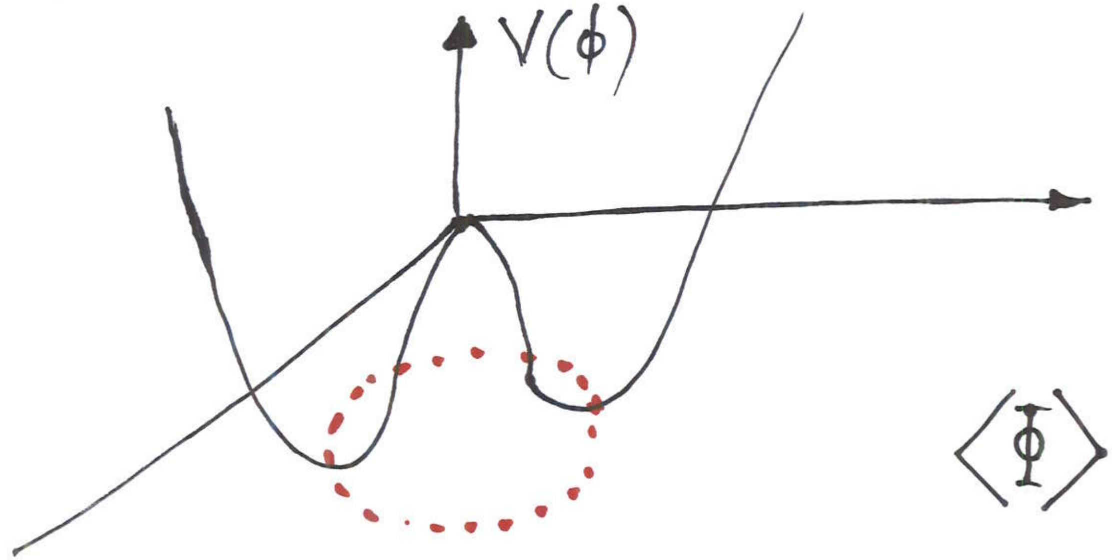
1) $\mu^2 > 0$ $\lambda > 0$



Minimum of the potential at

$$\langle 0|\phi|0\rangle = 0$$

2)



$$\langle \Phi \rangle = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{bmatrix}$$

Electric charge is always conserved!!

For $\mu^2 < 0$, $\lambda > 0$, there is **Spontaneous gauge sym. breaking**

$$SU(3)_C \times SU(2) \times U(1) \longrightarrow SU(3)_C \times U(1)_{em}$$

Check that $U(1)_{em}$ is not broken:

$$Q = T_3 + Y$$

For the Higgs doublet:

$$T_3 = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \; ; \; Y = \begin{bmatrix} 1/2 & \\ & 1/2 \end{bmatrix} \; ; \; Q = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Therefore:

$$Q \begin{bmatrix} 0 \\ 1/\sqrt{2}\, v \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1/\sqrt{2}\, v \end{bmatrix} = 0$$

and thus

$$e^{i\alpha Q} \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} = \left[1 + i\alpha Q + \dots \right] \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix}$$

Some important points:

(i) Electric charge is automatically conserved in the SM. This is no longer true in extensions of the SM with two Higgs doublets which includes the case of supersymmetric extensions of the SM. In the general two Higgs doublets model (2HDM), without loss of generality, one has: **T.D. Lee**

$$\langle 0|\Phi_1|0\rangle = \begin{bmatrix} 0 \\ v_1/\sqrt{2} \end{bmatrix} \; ; \; \langle 0|\phi_2|0\rangle = \begin{bmatrix} y \\ \frac{1}{\sqrt{2}} v_2 e^{i\theta} \end{bmatrix}$$

In order to preserve charge conservation in 2HDM one has to choose a region of the parameter space where the minimum is at: $y = 0$

(ii) The SM does not provide an explanation for the charges of elementary fermions.

The values of Y are chosen in such a way that one obtains the correct charges.

Example:

$$Q_{u_L^0} = \frac{1}{2} + Y_{u_L^0} = 2/3 \Rightarrow Y_{u_L^0} = 1/6$$

$$Q_{d_L^0} = -1/2 + Y_{d_L^0} = -1/3$$

$$\Rightarrow Y_{d_L^0} = 1/6$$

Therefore:

$$\boxed{Y_{Q_L} = 1/6}$$

Why do we have :

$$Q_p = -Q_e \qquad ?$$

One can find a solution to this fundamental question in the framework of grand-unification, for example

$$SU(5)$$

A convenient parametrization :

$$\Phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} = \begin{bmatrix} G^+ \\ 1/\sqrt{2}\,(v + H^0 + i G^0) \end{bmatrix}$$

Abers and B. Lee

Through the Brout-Englert-Higgs-mechanism BEH

a "miracle" happens :

$G^\pm$, G^0 → would-be Goldstone Bosons

$G^\pm$ → are absorbed as longitudinal components of $W^\pm$ which acquire a mass :

$$M_W = \frac{g v}{2}$$

G^0 → is absorbed as the longitudinal component of $Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu$

$\tan\theta_W \equiv g'/g$

Through the BEH mechanism, Z_μ **acquires a mass:**

$$\boxed{m_Z = \sqrt{g^2 + g'^2}\ \frac{v}{2} = \frac{M_W}{\cos\theta_W}}$$

The orthogonal combination :

$$A_\mu = \cos\theta_W\, B_\mu + \sin\theta_W\, W_\mu^3$$

remains massless and is identified with the photon:

$$\begin{bmatrix} A_\mu \\ Z_\mu \end{bmatrix} = \begin{bmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{bmatrix} \begin{bmatrix} B_\mu \\ W_\mu^3 \end{bmatrix}$$

Fermion masses in the SM

In the Standard Model, no fermion mass terms are gauge invariant. Since we want to have a gauge invariant Lagrangian, no mass terms are introduced, they are generated through Yukawa interactions, upon gauge symmetry breaking

Consider, for example the electron mass:

$$m_e \bar{e} e = m_e (\bar{e}_L e_R + \bar{e}_R e_L)$$

belongs to a doublet   singlet

$$e_L = \frac{1}{2}(1-\gamma_5)e$$

$$e_R = \frac{1}{2}(1+\gamma_5)e$$

Within the SM, all fermion masses are protected by gauge symmetry. This is a very important feature of the SM.

Consider that the SM is grand-unified through a gauge group $G \supset G_{SM}$, where G could be, for example $SU(5)$ or $SO(10)$

Since fermion masses are not invariant under G_{SM}, they do not acquire a mass (it would be very heavy!!) when the breaking:

$$G \rightarrow G_{SM}$$

takes place.

scale of breaking

$$G \xrightarrow{\quad V \quad} G_{SM}$$

(SU(5), SO(10), etc) $\qquad\qquad$ $SU(3)_c \times SU(2) \times U(1)$

This breaking gives mass to the gauge particles corresponding to the generators of the broken symmetries. Of the 24 gauge bosons of SU(5), there are 12 which acquire a mass $\approx gV$ at this stage. The scale of breaking is $V \gg v$. Since the fermion masses are not invariant under G_{SM}, they are protected from acquiring a mass of order $g_y V$.

In the SM, fermions acquire mass through **Yukawa couplings.**

Quark masses :

$$\left(Y_d\right)_{jk} \left(\overline{u^0}\ \overline{d^0}\right)_{jL} \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} \left(d^0_k\right)_R + \left(Y_u\right)_{jk} \left(\overline{u^0}\,\overline{d^0}\right)_{jL} \begin{bmatrix} \phi^0 \\ \phi^- \end{bmatrix} \left(u^0_k\right)_R + \text{h.c.}$$

$$\phi^0 = \frac{v}{\sqrt{2}} + H \rightarrow \quad \left(m_d\right)_{jk} = \frac{1}{\sqrt{2}} v \left(Y_d\right)_{jk}$$

$$\left(m_u\right)_{jk} = \frac{1}{\sqrt{2}} v \left(Y_u\right)_{jk}$$

Gauge invariance does not constrain the flavour structure of Yukawa couplings

m_d, m_u arbitrary complex matrices.

Similarly for <u>charged lepton masses</u>

$$Y_\ell^{jk} \left(\overline{\nu}\ \overline{\ell}\right)_{jL} \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} \ell_{kR} + h.c.$$

After gauge symmetry breaking :

$$\phi^0 = \frac{v}{\sqrt{2}} + H^0$$

Charged leptons acquire a mass :

$$(m_\ell)_{jk} = Y^{jk} \frac{v}{\sqrt{2}}$$

In the SM, **Yukawa couplings are the only couplings which can be complex !!**

What about neutrino masses?

In the SM, <u>neutrinos are massless</u>

no Dirac mass → since ν_R is not introduced

no Majorana mass at tree level ($\nu_L^T C \nu_L$) → due to the absence of Higgs triplets

no Majorana mass in higher orders → due to exact B-L conservation

Fermion mass matrices can be diagonalized by bi-unitary transformations

$$u_L^0 = U_L^u\, u_L \quad ; \quad u_R^0 = U_R^u\, u_R$$

$$d_L^0 = U_L^d\, d_L \quad ; \quad d_R^0 = U_R^d\, d_R$$

$$l_L^0 = U_L^l\, l_L \quad ; \quad l_R^0 = U_R^l\, l_R$$

$$M_u \rightarrow U_L^{u\dagger} M_u U_R^u = d_u \equiv \text{diag.}(m_u, m_c, m_t)$$

$$M_d \rightarrow U_L^{d\dagger} M_d U_R^d = d_d \equiv \text{diag.}(m_d, m_s, m_b)$$

$$M_l \rightarrow U_L^{l\dagger} M_l U_R^l = d_l = \text{diag.}(m_e, m_\mu, m_\tau)$$

In terms of the mass eigenstates u,d,l how do the gauge currents look?

Charged currents – The charged current interactions are written in the weak-basis as:

$$\mathcal{L}_W = \frac{g}{\sqrt{2}}\left[\overline{u}^{0}_{iL}\,\gamma^{\mu}\,d^{0}_{iL} + \overline{\nu}^{0}_{iL}\,\gamma^{\mu}\,l^{0}_{L}\right]W^{+}_{\mu} + h.c.$$

In the mass eigenstate basis:

$$\mathcal{L}_W = \frac{g}{\sqrt{2}}\left[\overline{u}_L\,\gamma^{\mu}\,U^{u\dagger}_L U^{d}_L\,d_L + \overline{\nu}^{0}_L\,\gamma^{\mu}\,U^{l}_L\,l_L\right]W^{+}_{\mu} + h.c.$$

$$V_{CKM} \equiv U^{u\dagger}_L U^{d}_L$$

Is the matrix U_L^l (i.e. leptonic mixing) physically meaningful in the context of the SM? **Answer**: No!

Note that since neutrinos are **massless in the SM**, one can always redefine **neutrino fields**:

$$\overline{\nu}_L^0 \gamma^\mu U_L^l l_L \; ; \; \overline{\nu}_L^0 U_L^l = \overline{\nu}_L$$

and the charged current becomes: $\overline{\nu}_L \gamma^\mu l_L$

$\Rightarrow$ **In the SM, there is no leptonic mixing! therefore no neutrino oscillations in the SM!**

Electromagnetic current

$$J^\mu_{em} = \frac{2}{3}\left[\bar{u}^0_L \gamma^\mu u^0_L + u^0_R \gamma^\mu u^0_R\right] - \frac{1}{3}\left[\bar{d}^0_L \gamma^\mu d^0_L + \bar{d}^0_R \gamma^\mu d^0_R\right]$$

$$- \left[\bar{l}^0_L \gamma^\mu l^0_L\right] - \left[l^0_R \gamma^\mu l^0_R\right]$$

In the mass eigenstate basis:

$$J^\mu_{em} = \frac{2}{3}\left[\bar{u}_L U^u_L U^u_L \gamma^\mu u_L + \bar{u}_R U^{u\dagger}_R U^u_R \gamma^\mu u_R\right]$$

$$- \frac{1}{3}\left[\bar{d}_L U^{d\dagger}_L U^d_L \gamma^\mu d_L + \bar{d}_R U^{d\dagger}_R U^d_R \gamma^\mu d_R\right]$$

$$- \left[\bar{l}_L \gamma^\mu U^{l\dagger}_L U^l_L l_L\right] - \left[\bar{l}_R \gamma^\mu U^{l\dagger}_R U^l_R l_R\right]$$

J^μ_{em} keeps its form in the mass eigenstate basis.

Neutral current interactions in the mass eigenstate basis

In terms of quark mass eigenstates, one obtains:

$$\mathcal{L}_Z = \frac{g}{\cos\theta_W}\Big[\bar{u}_L \gamma^\mu \overbrace{U_L^{u\dagger} U_L^u}^{\mathbb{1}} u_L - \bar{d}_L \gamma^\mu \overbrace{(U_L^{d\dagger} U^d)}^{\mathbb{1}} d_L - 2\sin^2\theta_W J^\mu_{em}\Big]$$

Flavour changing neutral currents are naturally absent at tree-level in the Standard Model, due to the GIM mechanism

"Charm" was invented in order to achieve this cancelation of FCNC

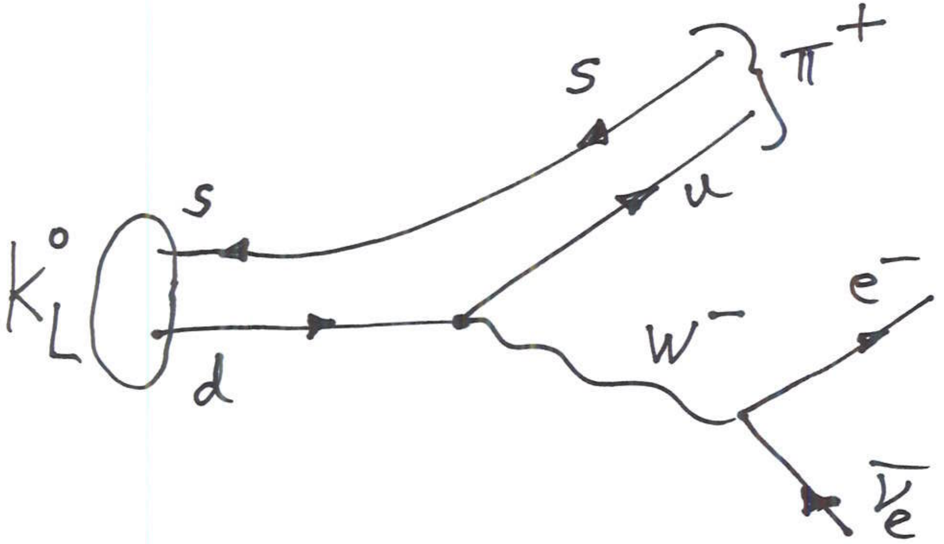
Exercise : Suppose that "charm" did not exist, so that one would have

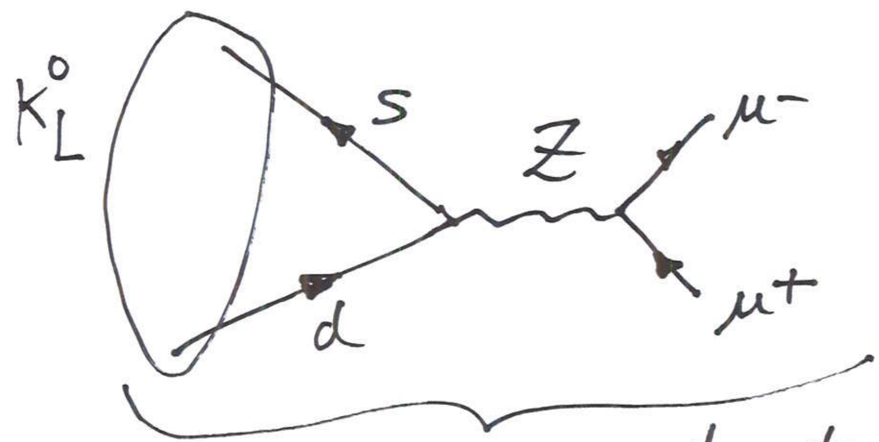
$$\begin{bmatrix} u_1^0 \\ d_1^0 \end{bmatrix} \qquad d_{2L}^0 \,,\; u_{1R}^0 \,,\; d_{1R}^0 \;\; d_{2R}^0$$

Show that in this model FCNC automatically arise.

Historical note : Prior to the arise of renormalizable gauge interactions, physicists considered the possibility that weak neutral currents could exist. However there was a **strong prejudice** against neutral currents due to the stringent limits on the strength of FCNC.

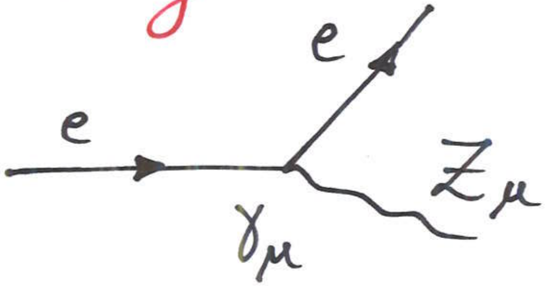
Example : The decay $K_L \to \mu^+\mu^-$ has a branching ratio **extremely suppressed**, with respect to the decay $K_L^0 \to \pi^+ e^- \bar{\nu}_e$. If FCNC existed they would have branching ratios of the same order of magnitude :

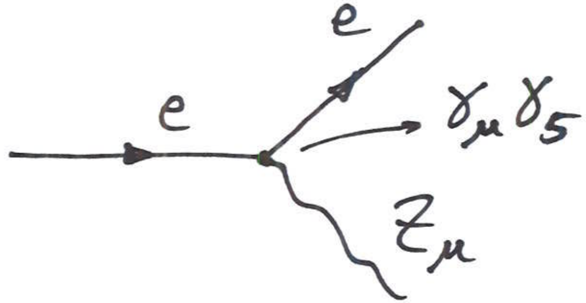




Does not exist at tree-level in the SM

Neutral-current interactions violate Parity





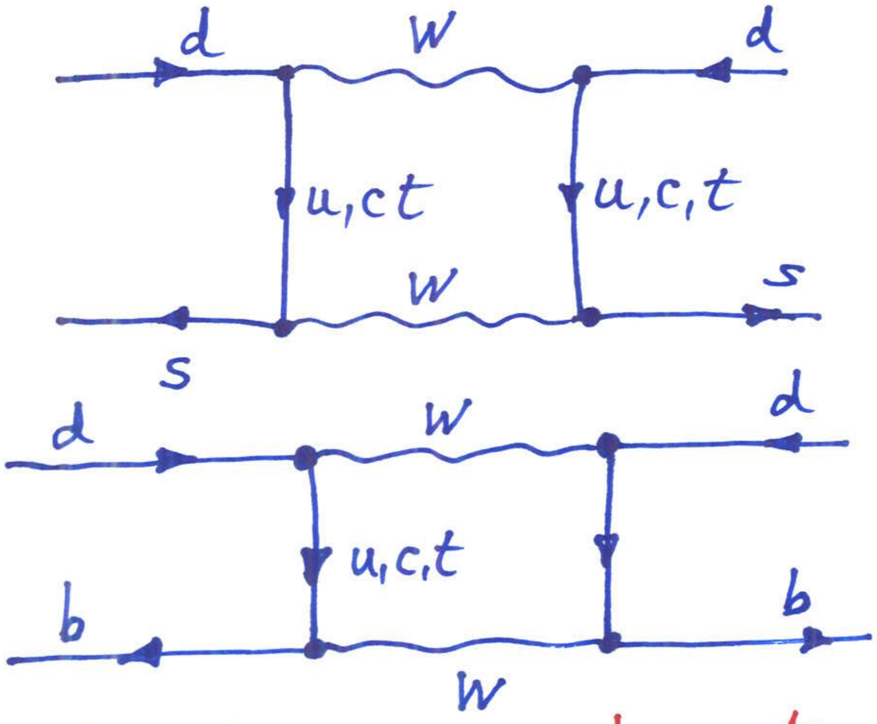
The simultaneous presence of both couplings leads to Parity Violation

$\Downarrow$

Parity violation in atomic physics

They were discovered, after some misleading experimental data

As a result of the **GIM mechanism** there are no tree-level contributions to $K^0-\overline{K^0}$, $B_d-\overline{B_d}$, $B_s-\overline{B}_s$, $D^0-\overline{D^0}$ mixings. However in the SM there are higher order corrections which are **calculable**



$K^0-\overline{K^0}$ mixing

$B_d^0-\overline{B_d^0}$ mixing

- Led to the correct estimate of the charm quark mass by B. Lee and M.K. Gaillard
- Provided the first indirect evidence of a large top mass

Exercise: Consider a simple extension of the SM which consists of the **addition of an isosinglet quark D to the Standard Model**

$$D_L\,,\ D_R\,,\ Q=-1/3 \qquad \text{isosinglet under } SU(2)$$

(i) Write down the most general quark mass terms which are obtained in the framework of this model

(ii) Derive the structure of charged currents

(iii) Derive the structure of neutral currents, showing that there are **FCNC** in this model

(iv) Show that although **non-vanishing FCNC are naturally suppressed in this model**

- Neutral Currents have played a crucial rôle in the construction of the SM and its experimental tests.
- The discovery of Neutral weak currents was the first great success of the SM
- An important feature of Flavour-Changing-Neutral Currents (FCNC):

  They are forbidden at tree level, both in the SM and in most of its extensions
- EPS prize to Gargamelle collaboration in 2009
- EPS prize to GIM in 2011.

At loop level FCNC are generated and have played a crucial rôle in testing the SM and in putting bounds on New Physics beyond the SM: $K^0-\bar{K}^0$, $D^0-\bar{D}^0$, $B^0_d-\bar{B}^0_d$, $B^0_s-\bar{B}^0_s$

rare kaon decays, rare b-meson decays

CP violation

SM contributes to these processes at loop level

$\Downarrow$

New Physics has a chance to give significant contributions

The need to suppress FCNC led to two dogmas:

- No Z-mediated FCNC at tree level
- No FCNC in the scalar sector, at tree level.

Glashow and Weinberg (PRD 1977)
E. A. Paschos (PRD 1977)
derived necessary and sufficient conditions

(i) All quarks of fixed charge and helicity must transform according to the same irreducible representation of SU(2) and correspond to the same eigenvalue of T_3

(ii) All quarks should receive their contributions to the quark mass matrix from a single neutral scalar vev

Can one violate these two dogmas in reasonable extensions of the SM? Yes!

"Reasonable" means that FCNC should be naturally suppressed, without fine-tuning.

- In the gauge sector, the Dogma can be violated through the introduction of a $Q = 1/3$ and/or $Q = 2/3$ vector-like quark.

$\Downarrow$

Naturally small violations of 3×3 unitarity of V^{CKM}

$\Downarrow$

Z-mediated, Naturally suppressed FCNC at tree level

Fundamental Properties of the CKM matrix

$$\left(\bar{u}\ \bar{c}\ \bar{t}\right)_L \gamma^\mu \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu$$

The CKM matrix is complex, but some of its phases have no physical meaning.

This is due to the fact that one has the freedom to **rephase the quark fields** u_α, d_α (mass eigenstates):

$$u_\alpha = e^{i\varphi_\alpha} u'_\alpha$$

$$d_k = e^{i\varphi_k} d'_k$$

under this rephasing $\quad V'_{\alpha k} = \exp i(\varphi_k - \varphi_\alpha) V_{\alpha k}$

Solution : Look for **rephasing invariants** :

moduli : $|V_{\alpha i}|$

Quartets : $Q_{\alpha i \beta j} \equiv V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i}$

One requires $\alpha \neq \beta$, $i \neq j$, otherwise the quartet becomes the product of two square moduli.

Invariants of higher order may in general be written as functions of the **quartets and the moduli**. Exercise : Show that :

$$\boxed{V_{\alpha i} V_{\beta j} V_{\gamma k} V^*_{\alpha j} V^*_{\beta k} V^*_{\gamma i} = \frac{Q_{\alpha i \beta j}\, Q_{\beta i \gamma k}}{|V_{\beta i}|^2}}$$

Examples of rephasing invariant quartets

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{us} V_{cb} V_{ub}^* V_{cs}^* \equiv Q_{uscb}$$

$$V_{cd} V_{ts} V_{td}^* V_{cs}^* \equiv Q_{cdts}$$

etc

The Flavour Sector of the SM

Charged current interactions

$$\sum_j \left(\overline{u}^0 \quad \overline{c}^0 \quad \overline{t}^0\right)_j \gamma_\mu \begin{pmatrix} d^0 \\ s^0 \\ b^0 \end{pmatrix}_j W^\mu$$

flavour diagonal in a Weak-Basis (WB)

$$M_u = Y_u \frac{v}{\sqrt{2}} \quad ; \quad M_d = Y_d \frac{v}{\sqrt{2}}$$

Gauge invariance does not constrain the flavour structure of Y_u, Y_d.

$M_u, M_d \rightarrow$ arbitrary, complex 3×3 matrices

The two matrices M_u, M_d contain $(18+18)$ parameters, but most of them are not physical. Due to the fermion families replication the gauge interaction part of $\mathcal{L}_{SM}$ has a very large flavour symmetry. One can make weak-basis transformations which change M_u, M_d but do not change the physical content of M_u, M_d.

Large redundancy in M_u, M_d.

By making a WB transformation:

$$u_L^0 = W_L u_L^{0\prime} \quad ; \quad u_R^0 = W_R^u u_R^{0\prime}$$

$$d_L^0 = W_L d_L^{0\prime} \quad ; \quad d_R^0 = W_R d_R^{0\prime}$$

gauge currents remain flavour diagonal, but M_u, M_d change:

$$M_u \longrightarrow M_u' = W_L^\dagger M_u W_R^u$$

$$M_d' = W_L^\dagger M_d W_R^d$$

But the physical content does not change!

without loss of generality, one can make a WB transformation so that :

$$M_u = \mathrm{diag.}\,(m_u, m_c, m_t)$$

$$M_d = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{12}^* & M_{22} & M_{23} \\ M_{13}^* & M_{23}^* & M_{33} \end{bmatrix} \longrightarrow \text{Hermitian}$$

The only rephasing invariant phase is

$$\varphi \equiv \arg\,(M_{12} M_{23} M_{13}^*)$$

Counting parameters :

$$\begin{aligned} m_u, m_c, m_t &\longrightarrow 3 \\ |M_{d\,ij}| &\longrightarrow 6 \\ \varphi &\longrightarrow \underline{1} \\ & 10 \end{aligned}$$

Difficulty in following a bottom-up approach in the search for a solution to the Flavour Puzzle: Even if there is a Flavour symmetry behind the spectrum of fermion masses and mixings, in what Weak-basis will the symmetry be transparent?

Texture zeros are Weak-Basis dependent.

CP violation

Important point: In the study of CP violation, one should allow for the most general CP transformation. Typically, $\mathcal{L}_{CP}$ leaves a large freedom of choice in the **definition of CP transformation**. CP is violated if and only if there is **no possible choice of CP transformation which leaves the Lagrangian invariant.** For a review, see

G.C. Branco, L. Lavoura, J. P. Silva
Oxford University press

CP violation can be investigated in the fermion mass eigenstate basis or in a weak-basis. We shall consider both cases.

Important point : A pure gauge Lagrangian is necessarily CP invariant (**Grimus, Rebelo, 1997**)

Let us study the CP properties of the SM, after spontaneous gauge symmetry breaking, and after diagonalization of the quark mass matrices

$$m_d = \mathrm{diag}.\,(m_d, m_s, m_b)$$

$$m_u = \mathrm{diag}.\,(m_u, m_c, m_t)$$

non-degenerate

In the mass eigenstate basis, the most general CP transformation is:

$$CP\, W^{+\mu}(t,\vec{r})(CP)^{\dagger} = -e^{i\xi_W}\, W^{-}_{\mu}(t,-\vec{r})$$

$$CP\, W^{-\mu}(t,\vec{r})(CP)^{\dagger} = -e^{-i\xi_W}\, W^{+}_{\mu}(t,-\vec{r})$$

$$CP\left[u_\alpha(t,\vec{r})\right](CP)^{\dagger} = e^{i\xi_\alpha}\gamma^0 C\, \bar{u}_\alpha^{T}(t,-\vec{r})$$

$$CP\left[d_k(t,\vec{r})\right]CP^{\dagger} = e^{i\xi_k}\gamma^0 C\, \bar{d}_k^{T}(t,-\vec{r})$$

Invariance of charged current weak interactions under CP constrains $V_{\alpha k}$ to satisfy the following condition:

$$V_{\alpha k}^{*} = e^{i(\xi_W+\xi_k-\xi_\alpha)}\, V_{\alpha k}$$

If one considers a single element of V_{CKM} the previous condition can always be satisfied by using the freedom to choose ξ_W, ξ_K, ξ_α.

However, it can be readily shown that the condition constrains all quartets and all rephasing invariant functions of V to be real. Therefore:

There is CP violation in the SM if and only if any of the rephasing invariant functions of V_{CKM} is not real.

Parameter Counting

In the SM with n_g generations, V_{CKM} is a $n_g \times n_g$ unitarity matrix.

$n_g^2 \longrightarrow$ parameters of a n_g-dimensional unitary matrix.

Therefore one obtains:

$$\underbrace{N_{parameters}}_{\text{total number of physical parameters}} = n_g^2 - \underbrace{(2n_g - 1)}_{\text{number of phases which be removed by rephasing}} = (n_g - 1)^2$$

for $n_g = 3$

$N_{par.} = 4$

$$N_{\substack{\text{physical}\\\text{phases}}} = (n_g-1)^2 - \underbrace{\frac{1}{2}(n_g-1)}_{\text{number of "angles"}} = \frac{1}{2}(n_g-1)(n_g-2)$$

Therefore :

For $n_g = 2 \Rightarrow$ no CP violating phase

For $n_g = 3 \Rightarrow$ one CP violating phase

Another way of conforming this. For 2 generations, there is only one rephasing invariant quartet :

$Q_{udcs} \equiv V_{ud} V_{cs} V_{us}^* V_{cd}^*$; However the orthogonality relation : $V_{ud} V_{cd}^* + V_{us} V_{cs}^* \Rightarrow Q_{udcs} = -|V_{us}|^2 |V_{cs}|^2$ which is real!

Consider now the case of three generations.

Orthogonality of the first two rows of V leads to :

$$\boxed{V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0}$$

Multiplying by $V_{us}^* V_{cs}$ and taking imaginary parts one obtains:

$$\mathrm{Im}\, Q_{udcs} = -\mathrm{Im}\, Q_{ubcs}$$

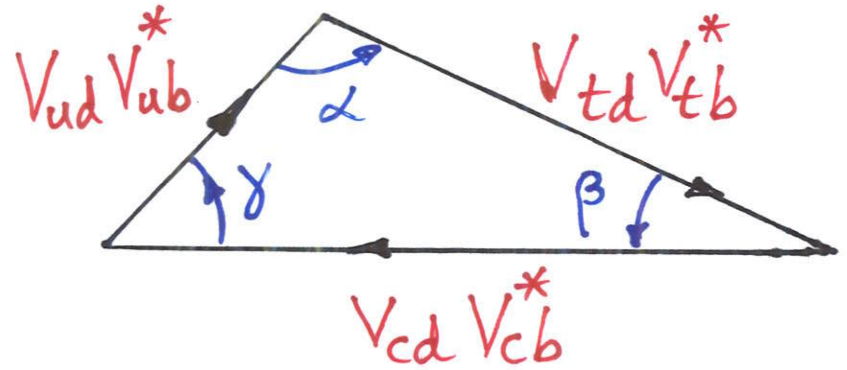
In an analogous way, one can show that for $n_g = 3$ the imaginary parts of all quartets are equal, up to a sign.

Geometrical Interpretation of Im Q and the Unitarity triangles

In the SM with 3 generations $|\mathrm{Im}\, Q|$ gives the strength of CP violation. Consider the orthogonality between the first and 3^{rd} columns of V:

$$\boxed{V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0}$$

This equation may be interpreted as a "triangle":



Under rephasing, the triangle rotates.

Orientation of the triangle has no physical meaning

Obviously, the internal angles of the triangles are rephasing invariant.

$$\alpha \equiv \arg\left[-V_{td}V_{ub}V_{ud}^{*}V_{tb}^{*}\right] = \arg\left(-Q_{ubtd}\right)$$

$$\beta \equiv \arg\left[-V_{cd}V_{tb}V_{cb}^{*}V_{td}^{*}\right] = \arg\left(-Q_{tbcd}\right)$$

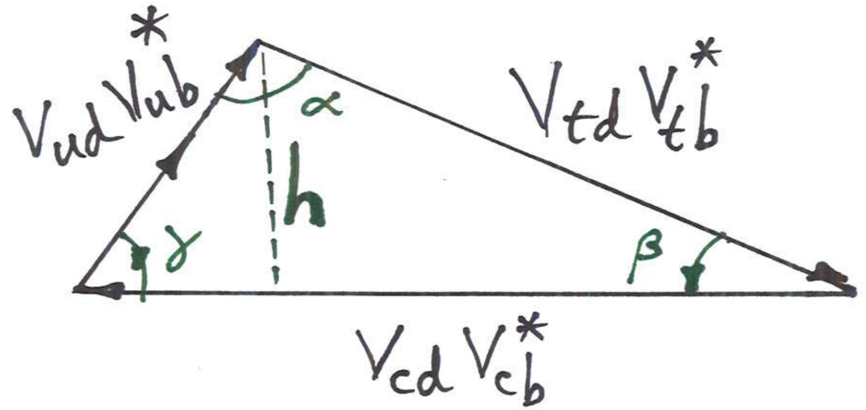
$$\gamma \equiv \arg\left[-V_{ud}V_{cb}V_{ub}^{*}V_{cd}^{*}\right] = \arg\left(-Q_{cbud}\right)$$

$$\alpha + \beta + \gamma = \arg(-1) = \pi \; (\mathrm{mod.}\,\pi)$$

This is true "**by definition**", <u>no test of unitarity</u> !!

Geometrical Interpretation of Im Q

The quantity Im Q has a simple geometrical interpretation



Area of the triangle: $A = |V_{cd}V_{cb}^*|\frac{h}{2}$;

Since $h = |V_{ud}V_{ub}^*|\sin\gamma$ and $\gamma = \arg(-Q_{cbud})$

one obtains

$$\text{Area} = \frac{1}{2}|\text{Im}\, Q_{udcb}|$$

Since all $|\text{Im}\, Q|$ are equal, all triangles have same area

Experimentally we know that:

$$|V_{CKM}| \cong \begin{bmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} \qquad \lambda \approx .22$$

The six unitarity triangles:

$$\overset{\lambda}{V_{ud}V_{us}^*} + \overset{\lambda}{V_{cd}V_{cs}^*} + \overset{\lambda^5}{V_{td}V_{ts}^*} = 0 \qquad T_{ds}$$

$$\overset{\lambda^3}{V_{ud}V_{ub}^*} + \overset{\lambda^3}{V_{cd}V_{cb}^*} + \overset{\lambda^3}{V_{td}V_{tb}^*} = 0 \qquad T_{db}$$

$$\overset{\lambda^4}{V_{us}V_{ub}^*} + \overset{\lambda^2}{V_{cs}V_{cb}^*} + \overset{\lambda^2}{V_{ts}V_{tb}^*} = 0 \qquad T_{sb}$$

$$\overset{\lambda}{V_{ud}V_{cd}^*} + \overset{\lambda}{V_{us}V_{cs}^*} + \overset{\lambda^5}{V_{ub}V_{cb}^*} = 0 \qquad T_{uc}$$

$$\overset{\lambda^3}{V_{ud}V_{td}^*} + \overset{\lambda^3}{V_{us}V_{ts}^*} + \overset{\lambda^3}{V_{ub}V_{tb}^*} = 0 \qquad T_{ut}$$

$$\overset{\lambda^4}{V_{cd}V_{td}^*} + \overset{\lambda^2}{V_{cs}V_{ts}^*} + \overset{\lambda^2}{V_{cb}V_{tb}^*} = 0 \qquad T_{ct}$$

Strength of CP Violation in the SM

$$|\mathrm{Im}\, Q| = |\underset{\lambda^0}{V_{ud}}\, \underset{\lambda^3}{V_{ub}}\, \underset{\lambda}{V_{cd}}\, \underset{\lambda^2}{V_{cb}}| \sin\gamma$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

In order to account for CP violation in the Kaon sector, $\sin\gamma$ should be of order 1. So $|\mathrm{Im}\, Q| \simeq \lambda^6$

The strength of CP violation (measured by $\mathrm{Im}\, Q$) is small in the SM, due to the smallness of some $|V_{ij}|$, like $|V_{ub}|$, $|V_{cb}|$. What would be the maximal possible value of $\mathrm{Im}\, Q$?

$$V = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^* \\ 1 & \omega^* & \omega \end{bmatrix} \qquad \omega = \exp(i\, 2\pi/3)$$

$$\mathrm{Im}\, Q = \frac{1}{6\sqrt{3}} \simeq .096$$

Standard Parametrization of V_{CKM}

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

One of the advantages of the **Standard Parametrization** is that the s_{ij} are simply related to directly measured quantities:

$$s_{13} = |V_{ub}| \; ; \quad s_{12} = \frac{|V_{us}|}{(1-|V_{ub}|)^{1/2}} \; ; \quad s_{23} = \frac{|V_{cb}|}{\sqrt{1-|V_{ub}|^2}}$$

Once s_{ij} are fixed all data has to be fit by a single parameter: δ_{13}

Invariant Approach to CP Violation

J. Bernabéu, M. Gronau, G.C.B. 1986

Consider a Lagrangian written as:

$$\mathcal{L} = \mathcal{L}_{CP} + \mathcal{L}_{remaining}$$

where $\mathcal{L}_{CP}$ is the part of the Lagrangian which conserves CP, typically the **gauge interactions.** In order to analyse whether the whole Lagrangian violates CP, one has to check whether the CP transformation under which $\mathcal{L}_{CP}$ is invariant implies non-trivial restrictions i.e. restrictions which are not satisfied – on the rest of the Lagrangian, i.e. $\mathcal{L}_{remain.}$

In the case of the SM, the most general CP transformations which leave $\mathcal{L}_{CP}$ invariant are :

$$(CP)\,(u^0_L)\,(CP)^\dagger = e^{i\xi_W}\, K_L\, \gamma^0 C\, \bar{u}^{0\,T}_L$$

$$(CP)\,(d^0_L)\,(CP)^\dagger = K_L\, \gamma^0 C\, \bar{d}^{0\,T}_L$$

$$(CP)\,(u^0_R)(CP)^\dagger = K^u_R\, \gamma^0 C\, \bar{u}^{0\,T}_R$$

$$(CP)\; d^0_R\;(CP)^\dagger = K^d_R\, \gamma^0 C\, \bar{d}^{0\,T}_R$$

where K_L , K^u_R , K^d_R are unitary matrices acting in flavour space.

It can be shown that in order for $\mathcal{L}_{Yukawa}$ (or equivalently M_u, M_d) to be **CP invariant**, the following relations have to be satisfied

$$K_L^\dagger M_u K_R^u = M_u^*$$
$$K_L^\dagger M_d K_R^d = M_d^*$$

$$\boxed{\text{Exercise: Prove the above result.}}$$

The existence of the matrices K_L, K_R^u, K_R^d is a necessary and sufficient condition for **CP invariance in the SM**

$$\underbrace{K_L^\dagger M_u K_R^u = M_u^* \quad ; \quad K_L^\dagger M_d K_R^d = M_d^*}$$

$$\Downarrow$$

$$\underbrace{K_L^\dagger M_u M_u^\dagger K_L = (M_u M_u^\dagger)^* = H_u^T \; ; \; K_L^\dagger M_d M_d^\dagger K_L = (M_d M_d^\dagger)^* = H_d^T}$$

$$\Downarrow$$

$$\underbrace{K_L^\dagger \left[H_u, H_d\right] K_L = \left[H_u^T, H_d^T\right] = -\left[H_u, H_d\right]^T}$$

$$K_L^\dagger \left[H_u, H_d\right]^r K_L = -\left\{\left[H_u, H_d\right]^r\right\}^T \quad r \text{ odd}$$

One concludes :

$$\boxed{\operatorname{tr}\left[H_u, H_d\right]^r = 0}$$

Therefore, one concludes that in the SM **CP invariance** implies : $\mathrm{tr}\,[H_u, H_d]^r = 0$

For $r = 1 \longrightarrow$ trivially satisfied

For $r = 3$:

$$\mathrm{tr}\,[H_u, H_d]^3 = 6i\,(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \times$$
$$\times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\,\mathrm{Im}\,Q$$

$$\boxed{\text{Exercise – Proove the above result. Hint : choose to work in a Weak-basis where either } H_u \text{ or } H_d \text{ is diagonal.}}$$

In the Standard Model neutrinos are strictly massless

- No Dirac mass since ν_R is not introduced
- No Majorana mass at tree level, since no scalar triplet is introduced.
- No Majorana mass at higher orders, due to exact B-L conservation

$\nu_L^T m C \nu_L$ violates (B-L) by 2 units

Same applies to SU(5) GUT, where B − L is an accidental symmetry

No leptonic mixing in the SM:

$$\bar{\nu}^0_{L_i} \gamma_\mu l^0_{L_i} W^\mu \qquad \nu^0_{L_i}, l^0_{L_i} \text{ weak eigenstates}$$

After diagonalization of the charged lepton mass matrix:

$$\bar{\nu}^0_L \gamma^\mu V l_L \qquad l_L \text{ mass eigenstates}$$

But V can be eliminated through a redefinition of ν^0_L: $\bar{\nu}_L \gamma_\mu l_L W^\mu$ flavour diagonal

Observation of neutrino oscillations provides clear evidence for New Physics beyond the SM.

Minimal extension of the SM which allows for non-vanishing neutrino masses:

A "strange" feature of the SM:

No ν_{R_i} are introduced.

What happens if we introduce ν_{R_i}?

Dirac masses for neutrinos are generated:

$$(Y_\nu)_{ij}\, \bar{L}_i\, \tilde{\phi}\, \nu_{R_j} \longrightarrow m_D = (Y_\nu)_{ij} \frac{v}{\sqrt{2}}$$

If one writes the most general Lagrangian consistent with gauge invariance and renormalizability, one has to include the mass term:

$$(M_R)_{ij}\, \nu_{R_i}^T C\, \nu_{R_j}$$

One may have $M_R \gg v$, since the mass term is gauge invariant.

This leads to the seesaw mechanism, with:

$$(m_\nu)_{light} \approx \frac{v^2}{M}$$

$$(M)_{heavy} \approx M_R$$

Note that this minimal extension of the SM, sometimes denoted SMν, is actually "simpler" and more "natural" than the SM, providing a simple and plausible explanation for the smallness of neutrino masses.

For the moment let us consider the low energy limit of the SMU.

Neutrino masses and mixing at low energies:

$$\mathcal{L}_{mass} = -\overline{l}_L m_\ell l_R - \frac{1}{2} \nu_L^T C m_\nu \nu_L + h.c.$$

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \overline{l}_L \gamma^\mu \nu_L W_\mu + h.c.$$

m_ℓ, m_ν encode all information about lepton masses and mixing in general

m_ℓ – arbitrary complex matrix

m_ν – symmetric complex matrix

There is a great redundancy in m_ℓ, m_ν so not all of its physical parameters are physical. This redundancy stems from the freedom to make weak-basis (WB) transformations:

$$\nu_L = W_L \nu_L' \quad ; \quad \ell_L = W_L \ell_L' \, ; \, \ell_R = W_R \ell_R'$$

m_ℓ, m_ν transform as: W_L, W_R unitary matrices

$$m_\ell' = W_L^\dagger m_\ell W_R \quad ; \quad m_\nu' = W_L^T m_\nu W_L$$

One can use the freedom to make WB transformations to go to a basis where

$$m_\ell = d_\ell \rightarrow \text{diagonal and real}$$

In this basis, one can still make a rephasing:

$$\ell''_{L,R} = K_L \, \ell'_{L,R} \quad ; \quad \nu''_L = K_L \nu'_L$$

with $K_L = \text{diag}(e^{-i\varphi_1}\ e^{-i\varphi_2}\ e^{-i\varphi_3})$. Under this rephasing d_ℓ remains invariant, but m_ν transforms as:

$$(m''_\nu)_{ij} = e^{i(\varphi_i + \varphi_j)} (m'_\nu)_{ij}$$

One can eliminate n phases from m_ν

So the number of physical phases in m_ν is:

$$\boxed{N_\phi = \frac{1}{2}\, n(n+1) - n = \frac{1}{2}\, n(n-1)}$$

For $n = 3$ one has $N_\phi = 3$. So altogether one has in m_ν :

$$|(m_\nu)_{ij}| \longrightarrow 6 \text{ real parameters}$$

$$N_\phi \longrightarrow 3 \text{ phases}$$

The individual phases of $(m_\nu)_{ij}$ have no physical meaning because they are not rephasing invariant. But one can construct polynomials of $(m_\nu)_{ij}$ which are rephasing invariant

Examples of rephasing invariant polynomials:

$$P_1 \equiv (m_\nu^*)_{11} (m_\nu^*)_{22} (m_\nu)_{12}^2 \; ; \quad P_2 \equiv (m_\nu^*)_{11} (m_\nu^*)_{33} (m_\nu)_{13}^2$$

$$P_3 \equiv (m_\nu^*)_{33} (m_\nu^*)_{12} (m_\nu)_{13} (m_\nu)_{23}$$

Generation of leptonic mixing in the charged current

$$U_L^{\ell\dagger} m_\ell U_R^\ell = d_\ell \quad ; \quad U^{\nu T} m_\nu U^\nu = d_\nu$$

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \bar{\ell}_L \gamma_\mu U \nu_L W^\mu + h.c.$$

$$U = U_L^{\ell\dagger} U^\nu \rightarrow \text{PMNS matrix}$$

In this basis, there is still freedom to rephase the charged lepton fields : $\ell_j \rightarrow \ell_j' = \exp(i\phi_j)\ell_j$

Due to the Majorana nature of neutrinos the rephasing:

$$\nu_K \to \nu'_K = \exp(-i\psi_K)\,\nu_K \;; \quad \psi_K \text{ arbitrary}$$

is not allowed, since it would not leave the Majorana mass terms invariant: $\nu_{LK}^T C\, m_K\, \nu_{LK}$

In the mass eigenstate basis:

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\left(\bar{e}\ \bar{\mu}\ \bar{\tau}\right)_L \gamma_\mu \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}_L W^\mu + \text{h.c.}$$

For the moment we do not introduce the contraints of 3×3 unitarity. Note that in the contest of type-one seesaw the PMNS matrix is not unitary.

Rephasing invariant quantities.

Recall the situation in the quark sector:

$$(\bar{u}\ \bar{c}\ \bar{t})_L \gamma_\mu \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}_L W^\mu + h.c.$$

Rephasing invariant quantities:

9 moduli

4 rephasing invariant phases

$$\beta \equiv \arg(-V_{cd} V_{tb} V_{cb}^* V_{td}^*)$$

$$\gamma \equiv \arg(-V_{ud} V_{cb} V_{ub}^* V_{cd}^*)$$

$$\chi \equiv \beta_s = \arg(-V_{cb} V_{ts} V_{cs}^* V_{tb}^*)$$

$$\chi' = \arg(-V_{us} V_{cd} V_{ud}^* V_{cs}^*)$$

Novel feature in the leptonic sector with Majorana neutrinos:

rephasing invariant bilinears of the type:

$$\arg\left(U_{\ell\alpha}\, U^{*}_{\ell\beta}\right)$$ (no summation of repeated indices)

" Majorana-type phases "

There are six independent Majorana-type phases. This is true even when unitarity is not imposed on U_{PMNS}. It applies to a general framework with an arbitrary number of right-handed neutrinos.

A possible choice for the six independent Majorana-type phases:

$$\beta_1 \equiv \arg\left(U_{e1} U_{e2}^*\right) \qquad \gamma_1 \equiv \arg\left(U_{e1} U_{e3}^*\right)$$

$$\beta_2 \equiv \arg\left(U_{\mu 1} U_{\mu 2}^*\right) \qquad \gamma_2 \equiv \arg\left(U_{\mu 1} U_{\mu 3}^*\right)$$

$$\beta_3 \equiv \arg\left(U_{\tau 1} U_{\tau 2}^*\right) \qquad \gamma_3 \equiv \arg\left(U_{\tau 1} U_{\tau 3}^*\right)$$

One can choose the following four independent Dirac-type invariant phases:

$$\sigma_{e\mu}^{12} \equiv \arg\left(U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^*\right) = \beta_1 - \beta_2$$
$$\sigma_{e\tau}^{12} \equiv \arg\left(U_{e1} U_{\tau 2} U_{e2}^* U_{\tau 1}^*\right) = \beta_1 - \beta_3$$
$$\sigma_{e\mu}^{13} \equiv \arg\left(U_{e1} U_{\mu 3} U_{e3}^* U_{\mu 1}^*\right) = \gamma_1 - \gamma_2$$
$$\sigma_{e\tau}^{13} \equiv \arg\left(U_{e1} U_{\tau 3} U_{e3}^* U_{\tau 1}^*\right) = \gamma_1 - \gamma_3$$

A "Surprise":

If one assumes 3×3 unitarity of U_{PMNS}, the full leptonic mixing matrix can be obtained from the six independent Majorana phases.

- Normalization of rows and columns plays an important rôle. Prevents "blowing up" of unitarity triangles.

For three generations and assuming 3×3 unitarity the U_{PMNS} matrix can be parametrized by:

$$U = VK \qquad K = \mathrm{diag.}\left(1,\, e^{i\alpha_1/2},\, e^{i\alpha_2/2}\right)$$

$$V = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ \cdot & \cdot & s_{23}c_{13} \\ \cdot & \cdot & c_{23}c_{13} \end{bmatrix}$$

One can eliminate the phase δ from the first row by writing:

$$U = VK' \;;\quad K' = \mathrm{diag}\left(1\ 1\ e^{i\delta}\right)K$$

convenient for the analysis of $0\nu\beta\beta$.

Dirac and Majorana unitarity triangles

Dirac unitarity triangles

$$T_{e\mu} : U_{e1} U^{*}_{\mu 1} + U_{e2} U^{*}_{\mu 2} + U_{e3} U^{*}_{\mu 3} = 0$$

$$T_{e\tau} : U_{e1} U^{*}_{\tau 1} + U_{e2} U^{*}_{\tau 2} + U_{e3} U^{*}_{\tau 3} = 0$$

$$T_{\mu\tau} : U_{\mu 1} U^{*}_{\tau 1} + U_{\mu 2} U^{*}_{\tau 2} + U_{\mu 3} U^{*}_{\tau 3} = 0$$

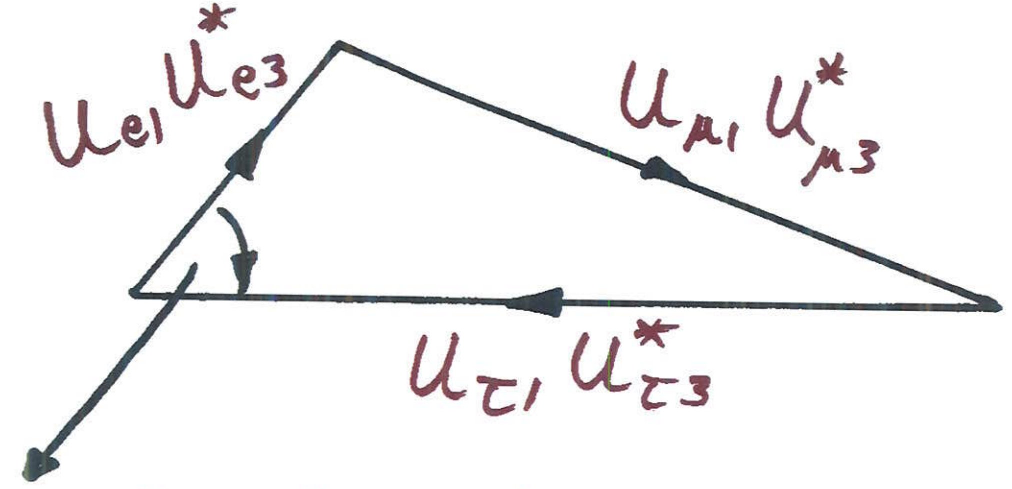
Majorana unitarity triangles

$$T_{12} : U_{e1} U^{*}_{e2} + U_{\mu 1} U^{*}_{\mu 2} + U_{\tau 1} U^{*}_{\tau 2} = 0$$

$$T_{13} : U_{e1} U^{*}_{e3} + U_{\mu 1} U^{*}_{\mu 3} + U_{\tau 1} U^{*}_{\tau 3} = 0$$

$$T_{23} : U_{e2} U^{*}_{e3} + U_{\mu 2} U^{*}_{\mu 3} + U_{\tau 2} U^{*}_{\tau 3} = 0$$

Example of a Majorana Triangle



$\arg\left(-U_{e1}U_{e3}^*U_{\tau 1}^*U_{\tau 3}\right) = \pi - (\delta_3 - \delta_1) \rightarrow$ Dirac-type phase

Majorana phases – they give the directions of the sides of the Majorana unitary triangles. "Arrows have no meaning! They can be reversed if one makes, for example the rephasing: $\nu_3 \rightarrow \nu_3' = -\nu_3$.

The limit of CP invariance

Majorana triangles provide necessary and sufficient conditions for having CP invariance with Majorana neutrinos.

- Vanishing of their common area:

$$A = \frac{1}{2} |\mathrm{Im}\, Q|$$

- Orientation of all the "collapsed" triangles along the real axis or the imaginary axis. If one of these triangles T_{ik} is parallel to the imaginary axis, that means that the neutrinos i, k have opposite CP parities.

Neutrinoless double-beta decay ($0\nu\beta\beta$)

$0\nu\beta\beta$ is sensitive to Majorana-type phases:

$$|m_{ee}|^2 = m_1^2 |U_{e1}|^4 + m_2^2 |U_{e2}|^4 + m_3^2 |U_{e3}|^4 +$$
$$+ 2 m_1 m_2 |U_{e1}|^2 |U_{e2}|^2 \cos(2\beta_1) + 2 m_1 m_3 |U_{e1}|^2 |U_{e3}|^2 \cos(2\gamma_1)$$
$$+ 2 m_2 m_3 |U_{e2}|^2 |U_{e3}|^2 \cos\left[2(\beta_1 - \gamma_1)\right]$$

Note that the angle $(\gamma_1 - \beta_1)$ is the argument of $(U_{e1}^* U_{e2} U_{e1} U_{e3}^*)$ which is not a rephasing invariant Dirac-type quartet.

If one adopts an explicit parametrization:

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ \times & \times & s_{23}c_{13}e^{i\delta} \\ \times & \times & c_{23}c_{13}e^{i\delta} \end{bmatrix}\begin{bmatrix} 1 & & \\ & e^{i\alpha_1/2} & \\ & & e^{i\alpha_2/2} \end{bmatrix}$$

$$m_{ee} = \left| \left[c_{13}^2 \left(m_1 c_{12}^2 + m_2 e^{-i\alpha_1} s_{12}^2 \right) + m_3 e^{-i\alpha_2} s_{13}^2 \right] \right|$$

This is the reason why this parametrization is useful for the analysis of $0\nu\beta\beta$.

Determining the neutrino mass matrix from experiment

We have seen that in the WB where the charged lepton mass matrix is diagonal, real one has:

$$m_\ell = \mathrm{diag.}(m_e, m_\mu, m_\tau) \,; \qquad M_\nu = \begin{bmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ \cdot & m_{\mu\mu} & m_{\mu\tau} \\ \cdot & \cdot & m_{\tau\tau} \end{bmatrix}$$

Can use rephasing freedom to make $m_{ee}, m_{\mu\mu}, m_{\tau\tau}$ real, but $m_{e\mu}, m_{e\tau}, m_{\mu\tau}$ complex.

Altogether: 6 real parameters + 3 phases = 9

How many physical quantities in M_ν can be obtained through feasible experiments?

$$\left.\begin{array}{l} \Delta m^2_{12}\,,\ \Delta m^2_{23} \\ \theta_{12}\,,\ \theta_{23}\,,\ \theta_{13} \\ \mathrm{Im}\,Q \rightarrow \text{CP violation in } \nu \text{ oscillations} \\ 0\nu\beta\beta \rightarrow m_{ee} \end{array}\right\} \text{7 measurable quantities (Glashow's counting)}$$

$$\underbrace{7 < 9}$$

S. Glashow, Frampton, Mafartia

... We arrive at the dreadful conclusion that no presently conceivable set of feasible experiments can fully determine the neutrino mass matrix!

Some of the ways out:

- Postulate "texture zeros" in M_ν → S. Glashow et al
  various sets of zero allowed by experiment
- Postulate det. $M_\nu = 0$ → G.C.B, R.G. Felipe, F. Joaquim, T. Yanagida
  7 parameters in M_ν!

CP-odd Weak-basis invariants in the leptonic sector with Majorana neutrinos

The relevant part of the Lagrangian is:

$$\mathcal{L} = -\bar{l}_L m_l l_R - \frac{1}{2} \nu_L^T C m_\nu \nu_L + \frac{g}{\sqrt{2}} \bar{l}_L \gamma_\mu \nu_L W^\mu + h.c.$$

- The CP transformation properties of the various fields are dictated by the part of the Lagrangian which conserves CP, namely the gauge interactions

- One has to keep in mind the fact that the gauge sector of the SM does not distinguish the various families of fermions. The most general CP transformations which leave $\mathcal{L}_{gauge}$ invariant is:

$$CP\, l_L (CP)^\dagger = W_L \gamma^0 C \bar{l}_L^T$$

$$CP\, \nu_L (CP)^\dagger = W_L \gamma^0 C \bar{\nu}_L^T$$

$$CP\, l_R (CP)^\dagger = W_R \gamma^0 C \bar{l}_R^T$$

where W_L, W_R are unitary matrices acting in generation space.

The Lagrangian of the leptonic sector conserves CP if and only if the leptonic mass matrices m_ν, m_ℓ satisfy:

$$W_L^T\, m_\nu\, W_L = -m_\nu^* \quad ; \quad W_L^\dagger\, m_\ell\, W_R = m_\ell^*$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$W_L^\dagger\, \tilde{h}_\nu\, W_L = (\tilde{h}_\nu)^* \qquad\qquad W_L^\dagger\, h_\ell\, W_L = h_\ell^*$$

$\tilde{h}_\nu \equiv h_\nu^*$

$$W_L^\dagger [\tilde{h}_\nu, h_\ell] W_L = [\tilde{h}_\nu^*, h_\ell^*] = [\tilde{h}_\nu^T, h_\ell^T] = -[\tilde{h}_\nu, h_\ell]^T$$

$$W_L^\dagger [\tilde{h}_\nu, h_\ell]^3 W_L = -\left\{[\tilde{h}_\nu, h_\ell]^3\right\}^T$$

CP invariance implies $\longrightarrow$ $\mathrm{tr}[\tilde{h}_\nu, h_\ell]^3 = 0$!!

CP invariance $\Rightarrow \operatorname{tr}\left[\tilde{h}_\nu, h_\ell\right]^3 = 0$

Valid for an arbitrary number of generations!!

First derived for the quark sector by G.C.B, J. Bernabeu, M. Gronau

$$\operatorname{tr}\left[\tilde{h}_\nu, h_\ell\right]^3 = -6i\left(m_\mu^2 - m_e^2\right)\left(m_\tau^2 - m_\mu^2\right)\left(m_\tau^2 - m_e^2\right) \times \Delta m_{21}^2\, \Delta m_{31}^2\, \Delta m_{32}^2 \times \operatorname{Im} Q$$

$$\operatorname{Im} Q = \frac{1}{8} \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \sin\delta$$

This invariant is sensitive to Dirac type CP violation

For 3 generations, the vanishing of this invariant is the necessary and sufficient for the absence of Dirac-type CP violation.

What about Majorana-type CP violation?

Using the previous method, one can derive (G.C.B., L. Lavoura, M.N. Rebelo) that the following invariant is sensitive to Majorana type CP violation:

$$\boxed{I^{CP}_{Majorana} = \mathrm{Im}\,\mathrm{tr}\left(m_\ell m_\ell^\dagger m_\nu^* m_\nu m_\nu^* m_\ell^T m_\ell^* m_\nu\right)}$$

The simplest way to check that $I^{CP}_{Maj.}$ is sensitive to Majorana-type CP violation is by evaluating in the case of 2 generations of Majorana neutrinos:

$$I^{CP}_{Maj.} = \frac{1}{4} m_1 m_2 \Delta m^2_{21} \left(m_\mu^2 - m_e^2\right) \sin^2(2\theta) \sin 2\gamma$$

$$U = \begin{bmatrix} \cos\theta & -\sin\theta\, e^{i\gamma} \\ \sin\theta\, e^{-i\gamma} & \cos\theta \end{bmatrix}$$

The invariant is very smart!!

Violating 3×3 Unitarity

Suppose that one drops the requirement of 3×3 unitarity. How many independent parameters are there in 3×3 V^{CKM}?

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} & \cdots \\ V_{cd} & V_{cs} & V_{cb} & \cdots \\ V_{td} & V_{ts} & V_{tb} & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$9 \text{ moduli} + \underset{(9-5)}{\underset{\downarrow}{4}} \text{ rephasing invariant phases} = \\ = 13 \text{ parameters}$$

A convenient choice for the 4 independent rephasing invariant phases is:

$$\beta \equiv \arg\left(-V_{cd} V_{tb} V_{cb}^{*} V_{td}^{*}\right)$$

$$\gamma \equiv \arg\left(-V_{ud} V_{cb} V_{ub}^{*} V_{cd}^{*}\right)$$

$$\chi \equiv \arg\left(-V_{cb} V_{ts} V_{cs}^{*} V_{tb}^{*}\right) \equiv \beta_s$$

$$\chi' \equiv \arg\left(-V_{us} V_{cd} V_{ud}^{*} V_{cs}^{*}\right)$$

The SM with 3 generations predicts a series of exact relations among the 13 measurable (in principle) quantities.

Warning :

$$\alpha + \beta + \gamma = \pi$$

is not a test of unitarity! It is true, by definition!

$$\alpha \equiv \arg\left(-V_{td} V_{ub} V_{ud}^* V_{tb}^*\right)$$

$$\beta \equiv \arg\left(-V_{cd} V_{tb} V_{cb}^* V_{td}^*\right)$$

$$\gamma \equiv \arg\left(-V_{ud} V_{cb} V_{ub}^* V_{cd}^*\right)$$

In the derivation of the unitary relations, it is useful to adopt a convenient phase convention. One can choose :

$$\arg(V) = \begin{pmatrix} 0 & \chi' & -\gamma \\ \pi & 0 & 0 \\ -\beta & \pi+\chi & 0 \end{pmatrix}$$

We have used the 5 rephasing degrees of freedom to fix 5 of the nine phases. We are left with 4

Some exact relations predicted by the SM

$$|V_{ub}| = \frac{|V_{cd}||V_{cb}|}{|V_{ud}|} \frac{\sin\beta}{\sin(\beta+\gamma)}$$

$$\sin\chi = \frac{|V_{td}|}{|V_{ts}|} \frac{|V_{cd}|}{|V_{cs}|} \sin\beta$$

$$\frac{|V_{ub}|}{|V_{td}|} = \frac{\sin\beta}{\sin\gamma} \frac{|V_{tb}|}{|V_{ud}|}$$

$$\sin\chi = \frac{|V_{us}||V_{ub}|}{|V_{cs}||V_{cb}|} \sin(-\chi + \chi' + \gamma)$$

Violations of any of these exact relations signals the presence of New Physics which may involve deviations of 3×3 unitarity or not.

The presence of New Physics contributions to $B_d-\bar{B}_d$ and $B_s-\bar{B}_s$ mixings affects the extraction of $|V_{td}|$, $|V_{ts}|$ from the data, even in the framework of New Physics which respects 3×3 unitarity.

Example: SUSY extensions of the SM

In many of the extensions of the SM the dominant effect of New Physics arises from new contributions to B_d-$\bar{B}_d$ and B_s-$\bar{B}_s$ mixings, which is convenient to parametrize as:

$$M_{12}^q = (M_{12}^q)^{SM} r_q^2 e^{2i\theta_q} \qquad q = d, s$$

$\Delta M_{B_d} = r_d^2 (\Delta M_{B_d})^{SM} \longrightarrow$ affects the extraction of $|V_{td}|$ from ΔM_{B_d}

$\Delta M_{B_s} = r_s^2 (\Delta M_{B_s})^{SM} \longrightarrow$ affects the extraction of $|V_{ts}|$ from ΔM_{B_s}.

$$S_{J/\psi K_S} = \sin(2\beta + 2\theta_d) = \sin(2\bar{\beta})$$

$$S_{\rho^+\rho^-} = \sin(2\alpha - 2\theta) = \sin(2\bar{\alpha})$$

How to detect the Presence of New Physics?

Answer: One can use the exact relations predicted by the SM.

$$(db) \longrightarrow \quad |V_{ub}| = \frac{V_{cd}V_{cb}}{V_{ud}} \frac{\sin\beta}{\sin(\gamma+\beta)} \rightarrow \text{extraction of } \theta_d$$

$$(sb) \longrightarrow \quad \sin\chi = \frac{|V_{us}||V_{ub}|}{|V_{cs}||V_{cb}|} \sin(\gamma - \chi + \chi') \rightarrow \text{extraction of } \theta_s$$

Extraction of θ_d :

$$\tan\theta_d = \frac{R_u \sin(\gamma+\bar{\beta}) - \sin\bar{\beta}}{\cos\bar{\beta} - R_u\cos(\gamma+\bar{\beta})} \; ; \qquad R_u = \frac{|V_{ud}||V_{ub}|}{|V_{cd}||V_{cb}|}$$

Extraction of θ_s :

$$\tan\theta_s = \frac{\sin\bar{\chi} - C\sin(\gamma-\bar{\chi})}{C\cos(\gamma-\bar{\chi}) + \cos\bar{\chi}} \; ; \qquad C = \frac{|V_{us}||V_{ub}|}{|V_{cs}||V_{cb}|}$$

To an excellent approximation one has:

$$\sin\chi \simeq \frac{|V_{us}|^2}{|V_{ud}|^2}\,\frac{\sin\beta\,\sin\gamma}{\sin(\gamma+\beta)} \rightarrow \text{Silva, Wolfenstein}$$

$$\sin\chi = \frac{|V_{td}|}{|V_{ts}|}\,\frac{|V_{cd}|}{|V_{cs}|}\,\sin\beta \rightarrow \text{Botella, Nebot, Rebelo, GCB}$$

$\Rightarrow$ If either (γ, χ) or $\left(\frac{\Delta M_{B_d}}{\Delta M_{B_s}}, \chi\right)$ are measured with some precision, one has novel stringent tests of the SM, where contribution of New Physics can be significant.

Since we are considering experimental tests of 3×3 unitarity of the V^{CKM} matrix one should ask the following question:

- Can one have self-consistent extensions of the SM, where deviations of (3×3) unitarity of V^{CKM} may occur?
- Can these deviations be naturally small?

The answer to both questions is:

Yes!!

We shall consider extensions of the SM with Vector-like isosinglet quarks of $Q = -1/3$ or $Q = 2/3$.

Question : Why consider these extensions ?

what can vector-like quarks do for us ?

Seven reasons to consider vector-like quarks

1. They provide a self-consistent framework with naturally small violations of 3×3 unitarity of V^{CKM}

$$\Downarrow$$

2. Lead to naturally small Flavour-Changing Neutral Currents (FCNC) (mediated by Z_μ)

3. Provide the simplest framework to have Spontaneous CP violation, with a Vacuum Phase generating a non-trivial CKM phase.

Important requirement:

There is experimental evidence of a Complex V^{CKM} even if one allows for the presence of New Physics.

4. Provide New Physics contributions to $B_d - \bar{B}_d$ mixing and $B_s - \bar{B}_s$ mixing.

5. Provide a simple solution to the Strong CP problem, which does not require Axions.

6. May contribute to the understanding of the observed pattern of fermion masses and mixing.

7. Provide a framework where there is a common origin of all CP Violations:

(i) CP Violation in the Quark Sector

(ii) CP Violation in the Lepton Sector detectable through neutrino oscillations

$U_{e3} \neq 0$ and "relatively large": Great News!!

(iii) CP violation needed to generate the Baryon Asymmetry of the Universe (BAU) through Leptogenesis.

Comment:

There is nothing strange in having deviations of 3×3 unitarity. The PMNS matrix in the leptonic sector in the context of the type-one seesaw (SMν) is not 3×3 unitary!

A Minimal Model

Consider an extension of the SM, where the following new fields are introduced:

- A vectorial quark D^0, with both D^0_L and D^0_R are $SU(2)_L$ singlets with change $Q = -1/3$ (or $Q = 2/3$)
- 3 right-handed neutrinos $\nu^0_{R_j}$
- A neutral complex singlet S

- Since we want to have Spontaneous CP violation, we impose CP invariance at the Lagrangian level: All couplings real.
- Introduce a Z_4 symmetry on the Lagrangian, under which:

$$\psi_\ell^0 \to i\,\psi_\ell^0 \;;\quad e_{Rj}^0 \to i\, e_{Rj}^0 \;;\quad \nu_{Rj}^0 \to i\,\nu_{Rj}^0$$

$$D^0 \to -D^0 \;;\quad S \to -S$$

The Z_4 symmetry is crucial to obtain a solution of the Strong CP problem and Leptogenesis

Scalar Potential

The Scalar potential contains various terms which do not have phase dependence, but there are terms with phase dependence.

$$V_{phase} = \left[\mu^2 + \lambda_1 S^* S + \lambda_2 \phi^\dagger \phi\right]\left(S^2 + S^{*2}\right) + \lambda_3 \left(S^4 + S^{*4}\right)$$

There is a range of the parameters of the Higgs potential, where the minimum is at:

$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}} \quad ; \quad \langle S \rangle = \frac{V}{\sqrt{2}} e^{i\theta}$$

Most general $SU(2)_L \times U(1) \times SU(3)_c \times Z_4$ invariant Yukawa couplings in the quark sector :

$$\mathcal{L}_Y = -(\bar{u}^0 \; \bar{d}^0)_{L_i} \left[g_{ij}\, d^0_{R_j} + h_{ij}\, \tilde{\phi}\, u^0_{R_j} \right] - \bar{M}\left(\bar{D}^0_L D^0_R\right)$$

$$- \left(f_i S + f' S^* \right) \bar{D}^0 d^0_{R_i} + \text{h.c.}$$

Quark mass matrix for down-type quarks :

$$\left(\bar{d}^0_{1L} \;\; \bar{d}^0_{2L} \;\; \bar{d}^0_{3L} \;\; \bar{D}^0_L\right) \left[\begin{array}{ccc:c} & \overset{3\times 3,\ \text{real}}{m_d} & & 0 \\ \hdashline M_1 & M_2 & M_3 & \bar{M} \end{array} \right] \begin{bmatrix} d^0_{1R} \\ d^0_{2R} \\ d^0_{3R} \\ D^0_R \end{bmatrix}$$

"zero" due to Z_4 symmetry

$$M_j = f_j V e^{i\theta} + f_j' V e^{-i\theta}$$

$$\mathcal{M} \equiv \left[\begin{array}{c|c} m_d & 0 \\ \hline M_1, M_2, M_3 & \overline{M} \end{array}\right] ;$$

$$U_L^{\dagger} \left(\mathcal{M} \mathcal{M}^{\dagger} \right) U_L = \text{diag.} \left(d^2, D^2 \right)$$

$$\mathcal{U} = \begin{bmatrix} K & R \\ S & T \end{bmatrix}$$; One can easily derive :

$$K^{-1} \left[m_d m_d^{\dagger} - \frac{m_d \overbrace{M^{\dagger} M}^{\text{complex}} m_d^{\dagger}}{\left(M M^{\dagger} + \overline{M}^2 \right)} \right] K = d^2$$

A remarkable feature of the Model:

The phase θ arising from $\langle S \rangle$, generates a non-trivial CKM phase, provided $|M_j|$ and $\bar{M}$ are of the same order of magnitude (This is "natural")

$$K^{-1} \, m_{eff} \, m_{eff}^{\dagger} \, K = \text{diag.}\left(m_d^2, m_s^2, m_b^2\right)$$

$$m_{eff} \, m_{eff}^{\dagger} = m_d m_d^{\dagger} - \frac{m_d M^{\dagger} M m_d^{\dagger}}{M M^{\dagger} + \bar{M}^2}$$

$$M_j = \left(f_j V e^{i\theta} + f_j' V e^{-i\theta}\right)$$

Naturally small deviations of 3×3 unitarity

Naturally Small Flavour-Changing Neutral Currents

For definiteness, consider the case of one isosinglet $Q=-1/3$ quark

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\left(\bar{u}\ \bar{c}\ \bar{t}\right)_L \gamma^\mu \left[\underset{3\times 3\ V_{CKM}}{K}\ \ R\right]\begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix} W_\mu^+$$

$$\mathcal{L}_Z = -\frac{g}{2\cos\theta_W}\left\{\left(\bar{u}\ \bar{c}\ \bar{t}\right)_L \gamma^\mu \begin{bmatrix} u \\ c \\ t \end{bmatrix}_L - \left[\bar{d}\ \bar{s}\ \bar{b}\ \bar{D}\right]\begin{bmatrix} K^+K & K^+R \\ R^+K & R^+R \end{bmatrix}\gamma^\mu \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix}\right.$$

$$\left. - \sin^2\theta_W\, J_{em}^\mu \right\} Z_\mu$$

Why deviations of 3×3 unitarity are naturally small :

$$\mathcal{U}_L^\dagger \mathcal{M}\mathcal{M}^\dagger \mathcal{U}_L = \mathrm{diag.}(m_d^2, m_s^2, m_b^2, M_D^2)$$

$$\mathcal{U}_L = \begin{bmatrix} K & R \\ S & T \end{bmatrix} \quad ; \quad K^\dagger K + S^\dagger S = 1$$

but $\quad S \approx -\dfrac{M\, m_d^\dagger K}{\overline{M}^2} \to O(m/M)$;

$K^\dagger K = 1 - O(m^2/M^2)$. Note that there is nothing strange about violations of 3×3 unitarity.

The PMNS matrix is not unitary in the framework of seesaw mechanism, type I.

Confronting experiment

Can extensions of the SM, with vector-like quarks "solve" some of the tensions between SM and experiment?

Answer: Yes! In the framework of an extension of the SM, with one $Q = 2/3$ vector-like quark, it has been shown that the tensions can be solved and various correlations are predicted.

F. Botella, M. Nebot, GCB arXiv:1207.4440 (2012)

But the important point is for experiment/theory to confirm that deviations are really there

From Cecilia Tarantino talk at ICHEP2012

From a closer look

UTfit

From the UTA (excluding its exp. constraint)

	Prediction	Measurement	Pull
$\sin 2\beta$	0.81±0.05	0.680±0.023	2.4 ←
γ	68°±3°	76°±11°	<1
α	88°±4°	91°±6°	<1
$\lvert V_{cb}\rvert \cdot 10^3$	42.3±0.9	41.0±1.0	<1
$\lvert V_{ub}\rvert \cdot 10^3$	3.62±0.14	3.82±0.56	<1
$\varepsilon_K \cdot 10^3$	1.96±0.20	2.23±0.01	1.4 ←
$BR(B \to \tau\nu) \cdot 10^4$	0.82±0.08	1.67±0.30	-2.7 ←

Leptonic Sector

Recall that the leptonic fields transform under Z_4 as: $\psi_L^0 \to i\psi_L^0$; $e_R^0 \to i e_R^0$; $\nu_R^0 \to i\nu_R^0$

Leptonic Yukawa terms:

$$\mathcal{L}_\ell = \overline{\psi}_L^0 G_\ell \phi e_R^0 + \overline{\psi}_L^0 G_\nu \tilde{\phi} e_R + \frac{1}{2} \nu_R^{0T} C (f_\nu S + f_\nu' S^*) \nu_R^0 + h.c.$$

Leptonic mass matrices:

$$M_\nu = \begin{bmatrix} 0 & m \\ m^T & M \end{bmatrix} \qquad m_\ell = \frac{v}{\sqrt{2}} G_\ell \quad ; \quad m = \frac{v}{\sqrt{2}} G_\nu$$

$$M = \frac{V}{\sqrt{2}} f_\nu^+ \cos\alpha + i f_\nu^- \sin\alpha$$

$$f^\pm \equiv f_\nu \pm f_\nu'$$

Leptonic Mixing

In the weak-basis where m_l is diagonal, real, the light neutrino masses and low energy leptonic mixing are obtained from

$$K^{\dagger}\left[m \frac{1}{M} m^{T}\right] K = d_{\nu}$$

m is real, but since M is a generic complex matrix, m_{eff} is also a generic complex matrix.

$K \rightarrow$ 3 complex phases one Dirac type, two Majorana-type

PMNS

Conclusions

- Vector like quarks provide a very interesting scenario for New Physics
- They are a crucial ingredient in the simplest model of spontaneous CP violation where a complex V^{CKM} is generated from a vacuum phase
- They provide a consistent framework where there are naturally small violations of 3x3 unitarity in V^{CKM}, leading to naturally small Z-FCNC.
- They provide a simple solution to the Strong CP problem, without axions

- The Standard Model and its CKM mechanism for mixing and CP violation is in good agreement with experiment. This is a remarkable fact in view of the large amount of data:

$|V_{us}|, |V_{ub}|, |V_{cb}|, \gamma \longrightarrow$ completely fix V^{CKM}

Other measurable quantities include:

$$\underbrace{\varepsilon_K,\ B_d - \bar{B}_d \text{ mixing},\ B_s - \bar{B}_s \text{ mixing},\ \beta,\ \beta_s,\ \text{rare } B\text{-decays, rare Kaon decays, etc., etc.}}_{}$$

Have to be accomodated without free parameters

Unfortunately there are "Hadronic uncertainties."

- There is room for New Physics which could be detected in LHCb and future super-B factories.
- The spectrum of Fermion Masses and the Pattern of quark and lepton mixing remains one of the Fundamental questions in Particle Physics. It is very likely that detectable New Physics be involved in the solution of the Flavour Puzzle
- Remember Neutrinos!!

Generation of the Baryon Asymmetry of the Universe (BAU)

The ingredients to dynamically generate BAU from an initial state with zero B.A., were formulated by Sakharov (1967)

(i) Baryon number violation

(ii) C and CP Violation

(iii) Departure from thermal equilibrium

All these ingredients exist in the SM, but it has been established that in the SM, one cannot generate the observed BAU:

$$\boxed{\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.20 \pm .15) \times 10^{-10}}$$

n_B, $n_{\bar{B}}$, n_γ number densities of baryons, antibaryons and photons at present time.

Reasons why the SM cannot generate sufficient BAU:

(i) CP violation in the SM is too small

$$\boxed{\frac{\mathrm{tr}\,[H_u, H_d]^3}{T_{ew}^{12}} \approx 10^{-20}}$$

(ii) Successful baryogenesis requires a strongly first order phase transition which would require a light Higgs mass

$$m_H \lesssim 70\,\mathrm{GeV}$$

One concludes that an explanation of the observed BAU requires New Physics beyond the SM. Leptogenesis, suggested by Fukugita and Yanagida is one the simplest and most attractive mechanisms to generate BAU:

Out of equilibrium decays of right-handed neutrinos create a lepton asymmetry which is in turn converted into a baryon asymmetry by $(B+L)$ violating but $(B-L)$ conserving sphaleron interactions.

The SMν, i.e. the extension of the SM consisting of adding 3 right-handed neutrinos has all the ingredients to have Leptogenesis. For an excellent review, see Sacha Davidson, E. Nardi, Y. Nir.

$$\mathcal{L}_m = -\left[\overline{\nu_L^0}\, m_D\, \nu_R^0 + \frac{1}{2}\, \nu_R^{0T} C\, M_R\, \nu_R^0 + \overline{l_L^0}\, m_l\, l_R^0\right] + h.c.$$

$$= -\left[\frac{1}{2}\, n_L^T C\, \mathcal{M}^*\, n_L + \overline{l_L^0}\, m_l\, l_R^0\right] + h.c.$$

with $n_L = \begin{bmatrix} \nu_L^0 \\ (\nu_R^0)^c \end{bmatrix}$

The full neutrino mass matrix is a 6×6 matrix :

$$\mathcal{M} = \begin{bmatrix} 0 & m \\ m^T & M \end{bmatrix}$$

diagonalized by :

$$V^T \mathcal{M}^* V = \mathcal{D} \; ; \quad \mathcal{D} = \mathrm{diag.}(m_1 m_2 m_3 M_1 M_2 M_3)$$

$$V = \begin{bmatrix} K & G \\ S & T \end{bmatrix} \qquad \mathcal{D} = \begin{bmatrix} d & \\ & D \end{bmatrix}$$

unitary 6×6 matrix

One can show that :

$$S^{\dagger} \simeq K^{\dagger} m M^{-1} \quad ; \quad G = m T^{*} D^{-1} \simeq m D^{-1}$$

U_{PMNS}

$$\underbrace{-K^{\dagger} m \frac{1}{M} m^{T} K^{*} = d}_{\text{usual seesaw formula}}$$

The leptonic charged current interactions are :

$$-\frac{g}{\sqrt{2}} \left(\bar{l}_{iL} \gamma_{\mu} K_{ij} \nu_{jL} + \bar{l}_{iL} \gamma_{\mu} G_{ij} N_{jL} \right) W^{\mu}$$

$$+ \text{h.c.}$$

Counting parameters in the leptonic sector:

Without loss of generality, one can choose a weak basis where the charged lepton mass matrix is diagonal, and also the righthanded neutrino mass matrix is diagonal. In this basis, the Yukawa coupling matrix Y_D entering in the Dirac neutrino mass matrix is an arbitrary complex matrix. 3 of the 9 phases in Y_D can be eliminated by rephasing. So altogether:

$$\underbrace{3}_{(m_\ell)_i} + \underbrace{3}_{(M_R)_i} + \underbrace{9}_{\text{real par. in } Y_D} + \underbrace{6}_{\text{phases in } Y_D} = 21!$$

Lepton - Asymmetry generated through CP violating decays of the Heavy neutrinos:

$$N \to l + H$$

Unflavoured Leptogenesis:

$$A^j = \frac{\sum_i N_i^j - \bar{N}_i^j}{\sum_i N_i^j + \bar{N}_i^j} \propto \sum_{k \neq j} c_k \, \mathrm{Im}\left[\left(m_D^\dagger m_D\right)_{jk}^2\right]$$

Casas and Ibarra parametrization:

$$m_D = i\, U_\nu \sqrt{d}\; R \sqrt{D}$$

$R \to$ complex orthogonal matrix.

$$m_D^\dagger m_D = -\sqrt{D_R}\, R^\dagger \sqrt{d}\, \sqrt{d}\, R \sqrt{D_R}$$

Leptogenesis independent of U_ν

- In general, it is not possible to establish a connection between CP asymmetries needed for leptogenesis and CP violation detectable in neutrino oscillations
  One may have leptogenesis even if U is real.

- The connection may be established with further theoretical assumptions.

Can one have a WB invariant which is sensitive to the CP violating phases entering in unflavoured Leptogenesis?

Yes! G.C.B., T. Morozumi, B.M. Nobre, M.N. Rebelo
Nucl. Physics B (2001)

$$I \equiv \mathrm{Im}\,\mathrm{tr}\left[h H M^* h^* M\right]$$

$$= M_1 M_2 (M_2^2 - M_1^2)\, \mathrm{Im}\,(h_{12}^2) + M_1 M_3 (M_3^2 - M_1^2)\, \mathrm{Im}(h_{13}^2) + M_2 M_3 (M_3^2 - M_2^2)\, \mathrm{Im}(h_{23})^2$$

$h \equiv m_D^\dagger m_D \; ; \; H \equiv M^\dagger M$

Conclusions

- Neutrino Oscillations provide clear evidence for Physics beyond the SM and the discovery of $U_{e3} \neq 0$ opens up the exciting possibility of detecting leptonic Dirac-type CP violation through neutrino oscillations.
- Leptogenesis is an attractive framework to generate BAU which can occur in the framework of SMν. Difficult to test experimentally, but ...

- It is urgent to conceive "feasible experiments" which can measure physical quantities in M_ν beyond the seven quantities mentioned by Glashow et al.

  Difficult but what looks impossible today, may be possible tomorrow!

- It would be very nice if some years from now, we have a workshop with a title like:

  "The leptonic unitarity triangle fit"