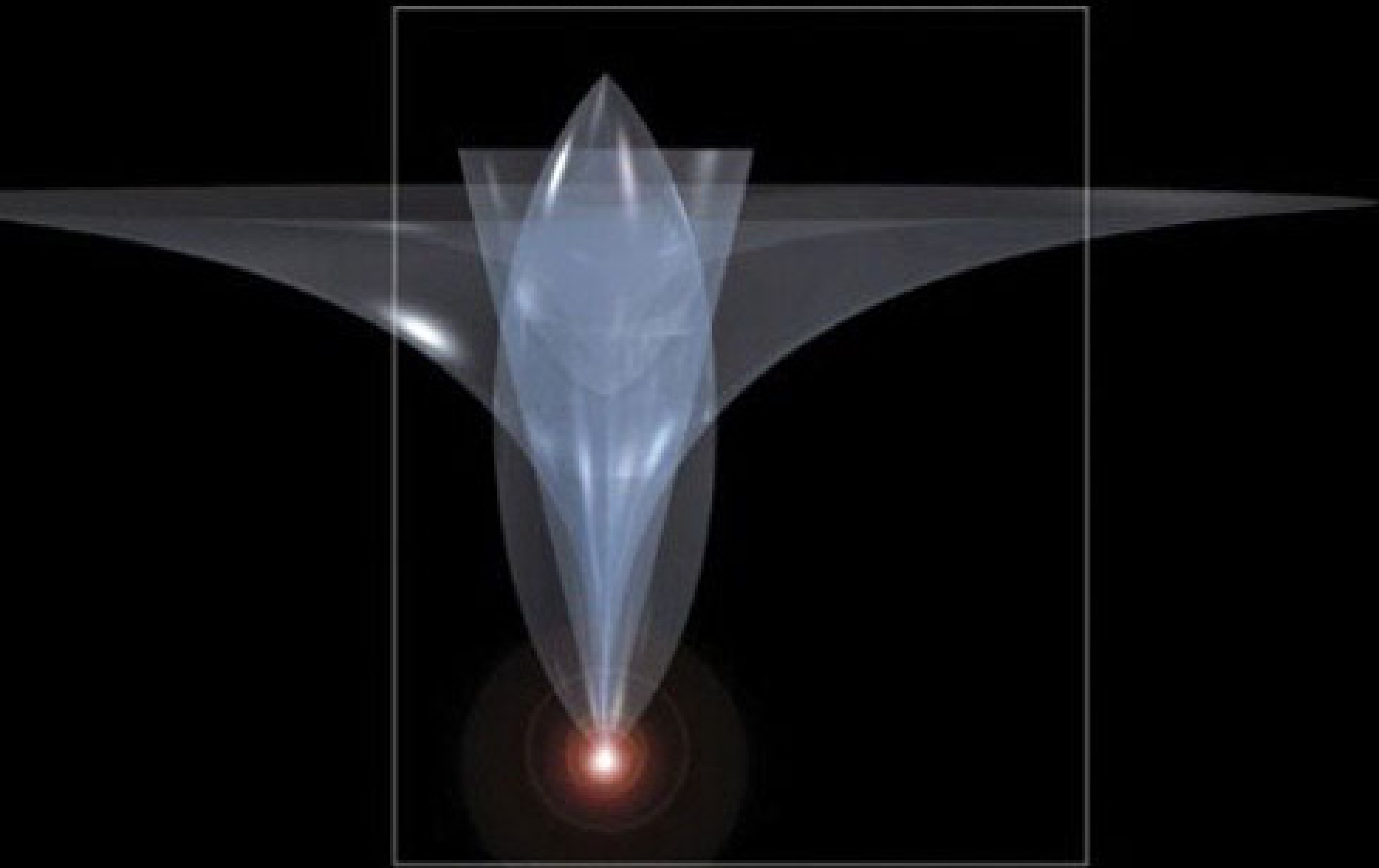
INTRODUCTION TO COSMOLOGY II



BARBARA RYDEN

A summary of the cosmological principle (thanks to Sydney Harris):

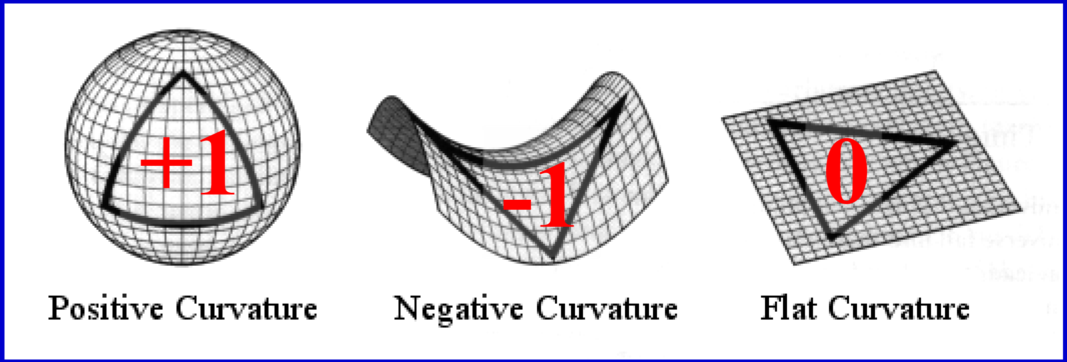
"I'VE SEEN OUT TO THE LIMIT OF THE OBSERVABLE UNIVERSE, AND BELIEVE ME, IT'S NO BETTER OUT THERE THAN IT IS HERE."

In a universe obeying the cosmological principle, spacetime curvature is described by:

Curvature constant $\kappa = -1, 0, +1$

Radius of curvature R_0 (if $\kappa \neq 0$)

Scale factor $a(t)$



In a universe where GR holds true, spacetime curvature is determined by:

Energy density $\varepsilon(t)$

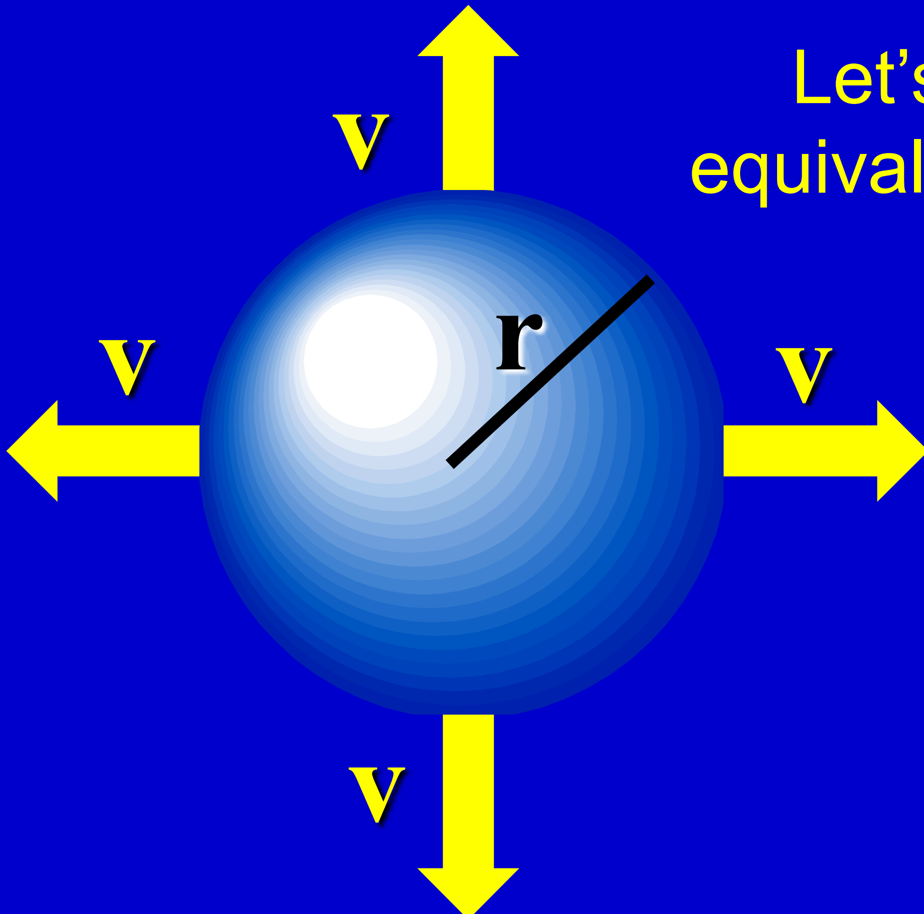
Pressure $P(t)$

The equation that links the curvature of space to its energy density is the **Friedman equation**.

Alexander Friedman(n)

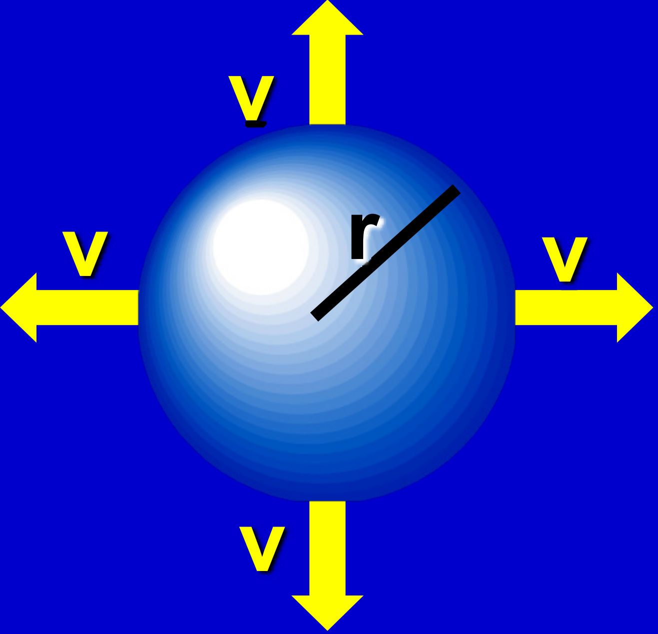
The Einstein field equation is a set of 10 nonlinear 2nd-order differential equations; hard to solve in general.

Friedman (1922) looked at the special case of a universe that is spatially homogeneous & isotropic.



Let’s look at the **Newtonian** equivalent of Friedman’s problem.

A sphere of radius $r(t)$ and density $\rho(t)$ is expanding uniformly; the speed at its surface is $v(t) = dr/dt$.



Newton says: gravitational acceleration at the surface of the sphere is

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2}$$

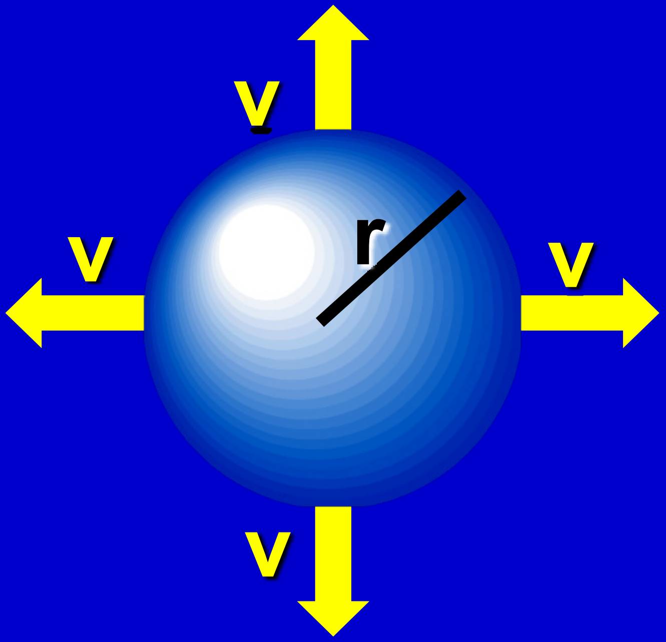
where M is the (constant) mass of the sphere.
Multiply by dr/dt and integrate to find

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{GM}{r} + U$$

constant of integration

Energy is conserved.

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 - \frac{GM}{r} = U$$



Mass is conserved.

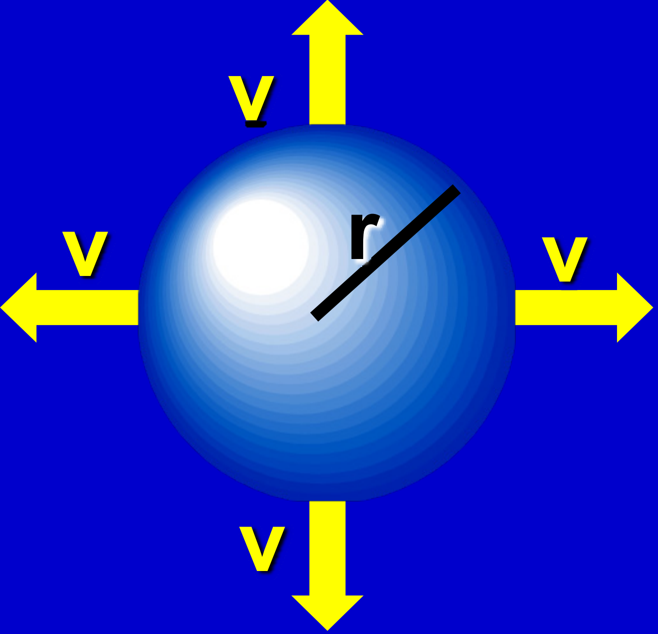
$$M = \frac{4\pi}{3}\rho(t)\, r(t)^3$$

Expansion is isotropic about the sphere's center.

$$r(t) = a(t)\, r_0$$

Put it all together and get

$$\frac{1}{2} r_0^2 \dot{a}^2 = \frac{4\pi}{3} G r_0^2 \rho(t)\, a(t)^2 + U$$



As I was saying,

$$\frac{1}{2} r_0^2 \dot{a}^2 = \frac{4\pi}{3} G r_0^2 \rho(t) a(t)^2 + U$$

Divide by $r_0^2 a^2/2$ and find

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) + \frac{2U}{r_0^2}\frac{1}{a(t)^2}$$

This is the **Friedman equation** in its Newtonian form.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) + \frac{2U}{r_0^2}\frac{1}{a(t)^2}$$

Three possible fates for the expanding sphere:

1) **U>0** Right hand side of equation is always positive; the sphere continuously expands.

2) **U<0** Right hand side vanishes at some maximum scale factor; the sphere then recollapses.

3) **U=0** Critical case where expansion speed exactly equals escape speed.

The critical case U=0: boundary between eternal expansion and eventual recollapse.

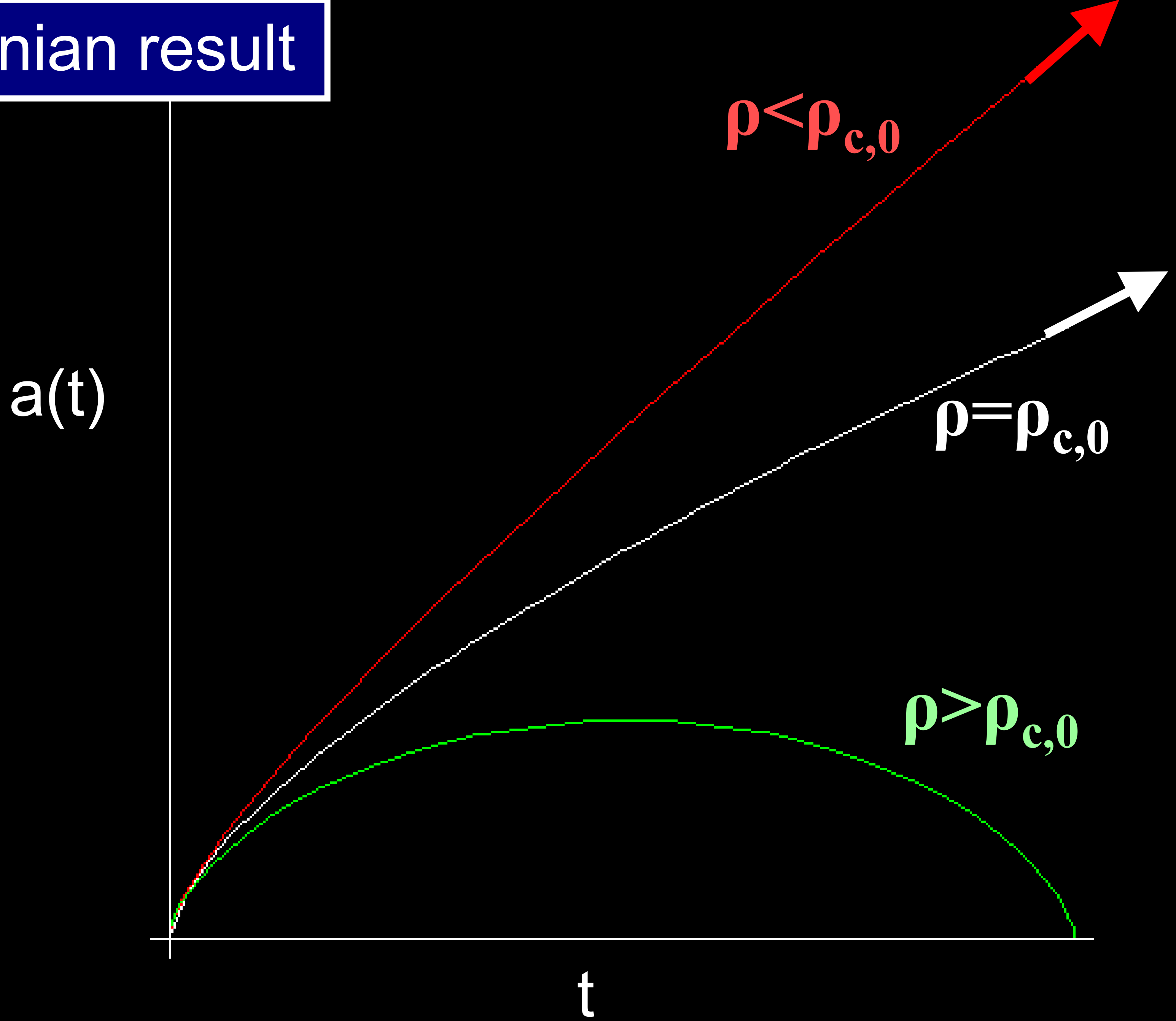
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) + \cancel{\frac{2U}{r_0^2}\frac{1}{a(t)^2}}$$

$$H(t)^2 = \frac{8\pi G}{3}\rho(t)$$

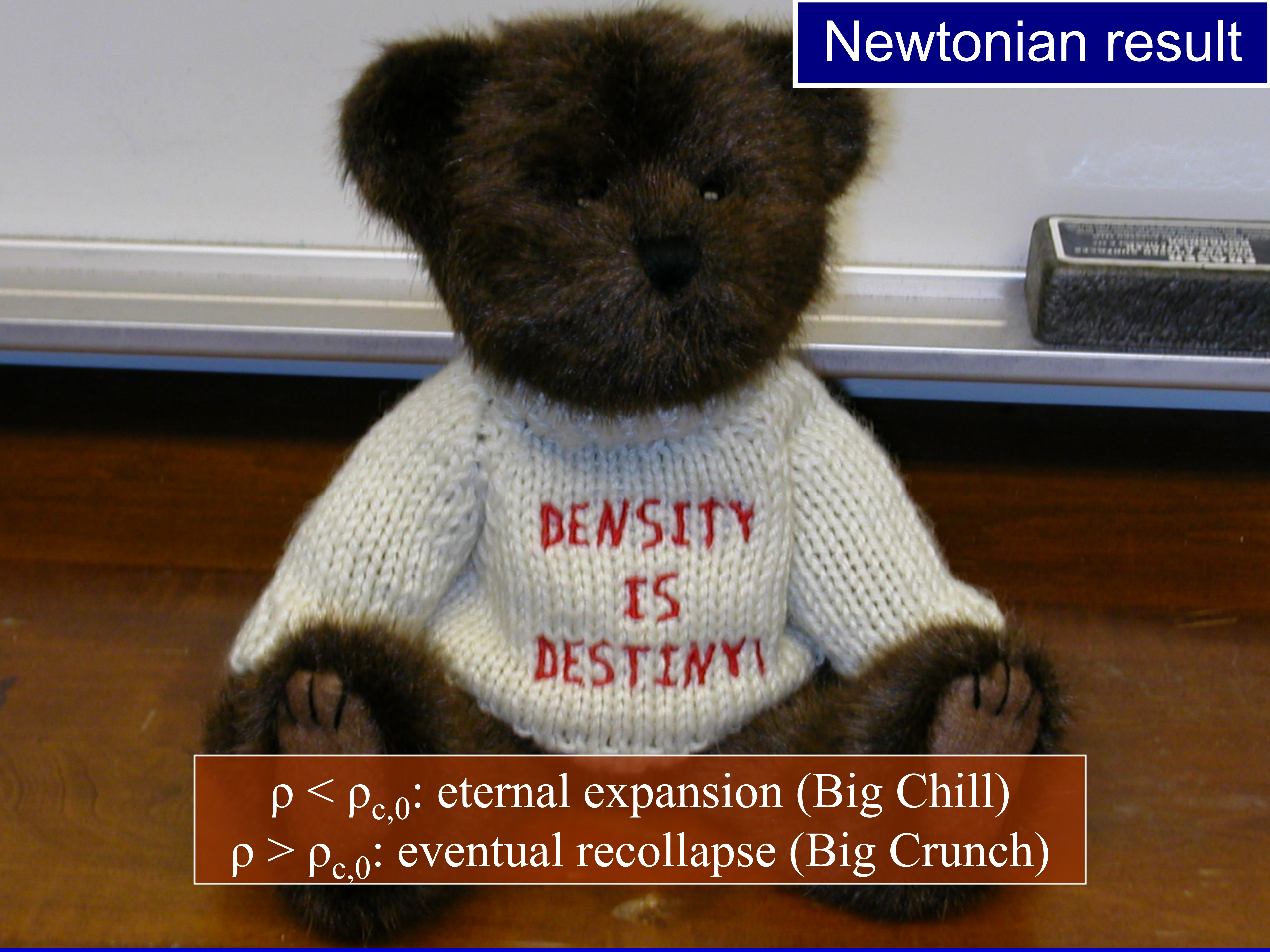
For a given value of H_0, there's a **critical density** $\rho_{c,0}$.

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} = 9.5\times10^{-30}\,\mathrm{g\,cm^{-3}}$$

Newtonian result



Newtonian result

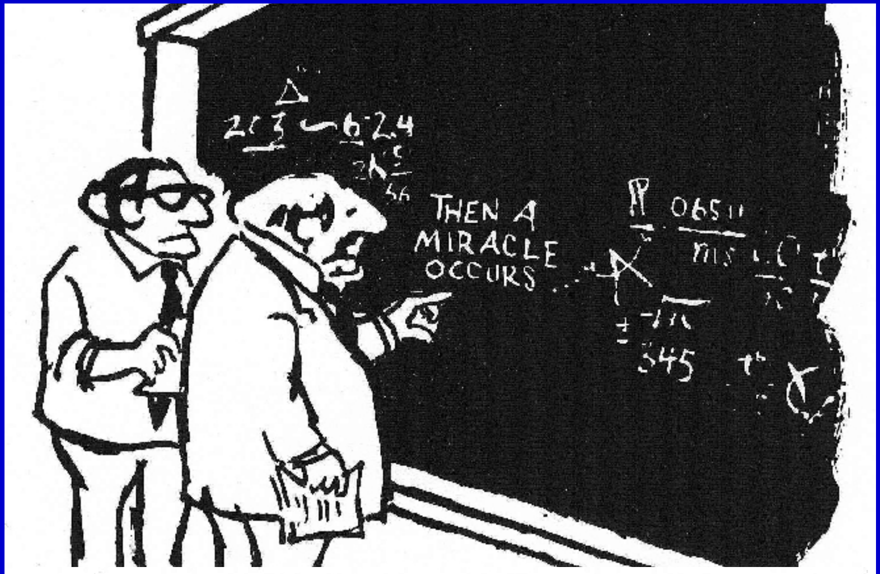


$\rho < \rho_{c,0}$: eternal expansion (Big Chill)

$\rho > \rho_{c,0}$: eventual recollapse (Big Crunch)

Newtonian Friedman equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) + \frac{2U}{r_0^2}\frac{1}{a(t)^2}$$



"I think you should be more explicit here in step two."

GR Friedman equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c}{R_0^2 a(t)}$$

What do we gain from adopting GR?

1) For mass density ρ, we substitute energy density ε. (Massless particles, such as photons, are influenced by gravity & contribute to gravity.)

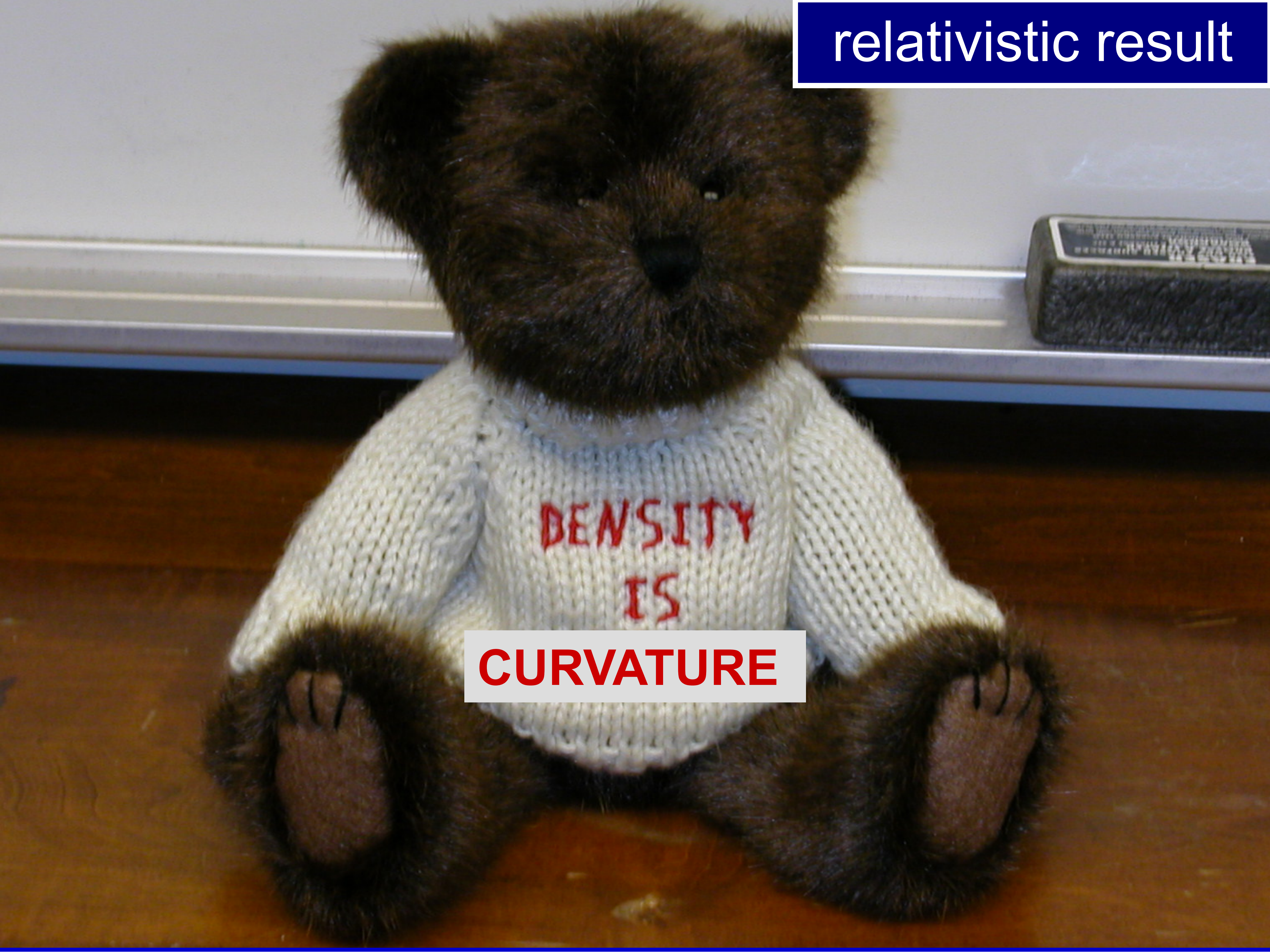
2) The critical density $\varepsilon_{c,0} = \rho_{c,0}c^2$ now divides **positively** curved universes from **negatively** curved universes.

$$\varepsilon > \varepsilon_{c,0} \quad \kappa = +1$$

$$\varepsilon = \varepsilon_{c,0} \quad \kappa = 0$$

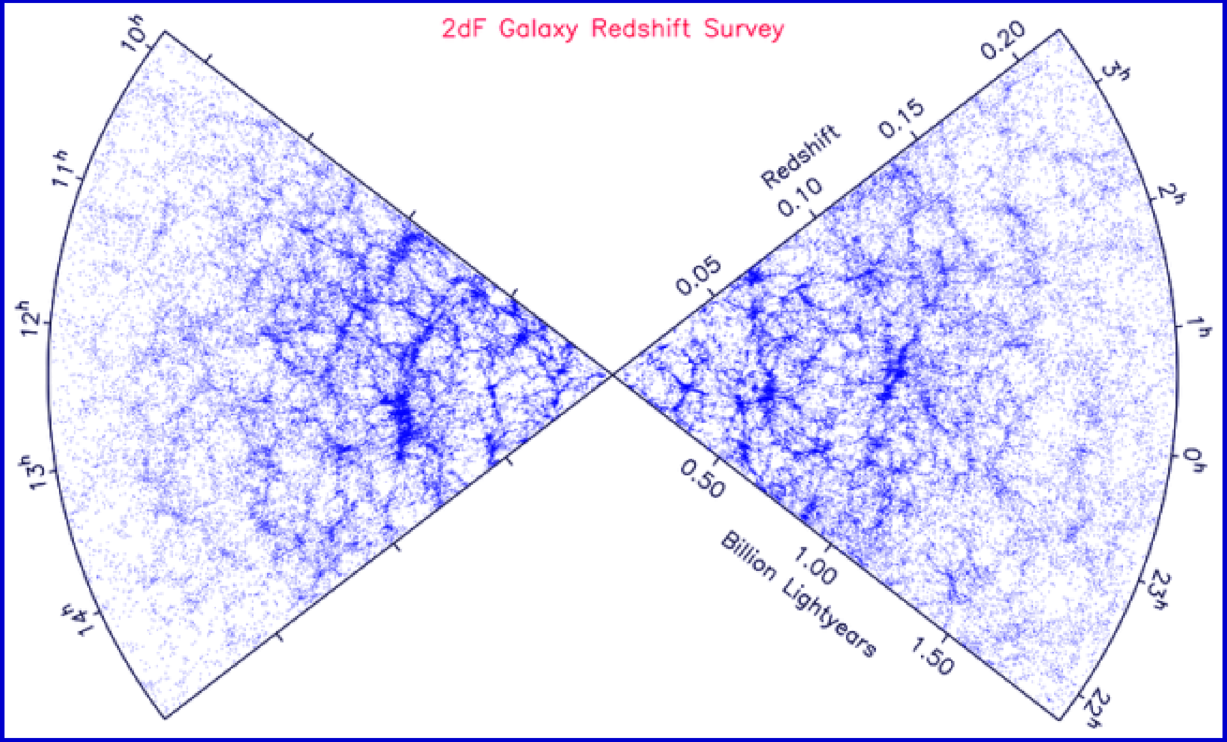
$$\varepsilon < \varepsilon_{c,0} \quad \kappa = -1$$

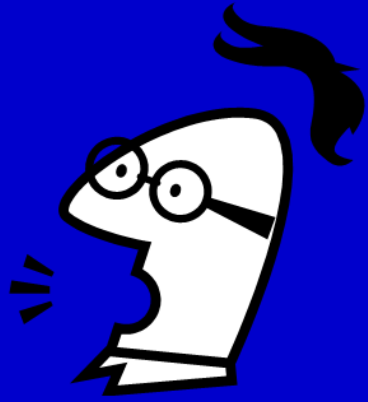
relativistic result



$$\varepsilon_{c,0} = \frac{3H_0^2c^2}{8\pi G} = 5.3\,\mathrm{GeV\,m^{-3}}$$

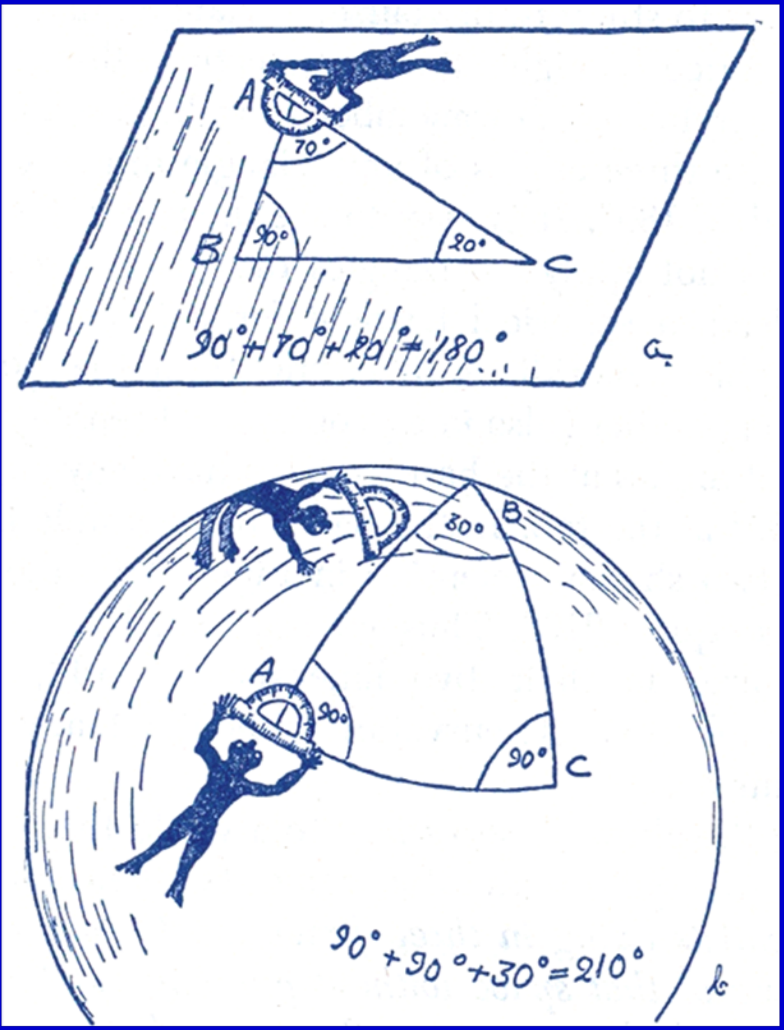
This density is very low by galactic standards. However, most of space consists of intergalactic voids; it's not immediately obvious whether $\varepsilon_0 > \varepsilon_{c,0}$.





Startling realization!

We can determine whether the present energy density is greater than or less than $\varepsilon_{c,0}$ by **measuring the curvature of space**.



George Gamow,
One Two Three Infinity

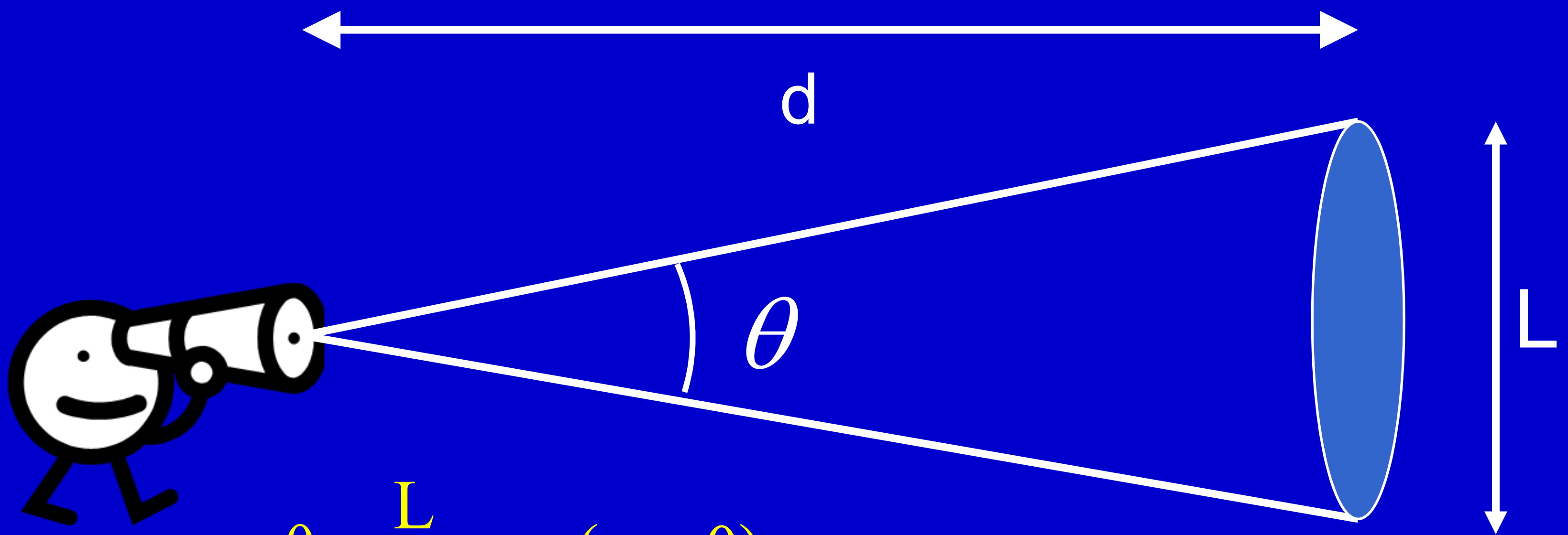
In principle, measuring the curvature of space is simple: draw big triangles. (Connect 3 points with **geodesics)**.

If the angles at the vertices sum to π radians, space is flat ($\kappa = 0$).

If they sum to $> \pi$, space is positively curved ($\kappa = +1$).

If they sum to $< \pi$, space is negatively curved ($\kappa = -1$).

Since light travels along geodesics, we can use light to trace out two sides of the necessary triangle.



$$\theta = \frac{L}{d} \qquad (\kappa = 0)$$

$$\theta = \frac{L}{R_0 \sin(d/R_0)} > \frac{L}{d} \qquad (\kappa = +1)$$

$$\theta = \frac{L}{R_0 \sinh(d/R_0)} < \frac{L}{d} \qquad (\kappa = -1)$$

If space is curved, then its radius of curvature R_0 can't be much smaller than the Hubble distance, $c/H_0 \approx 4200$ Mpc. **Why?**

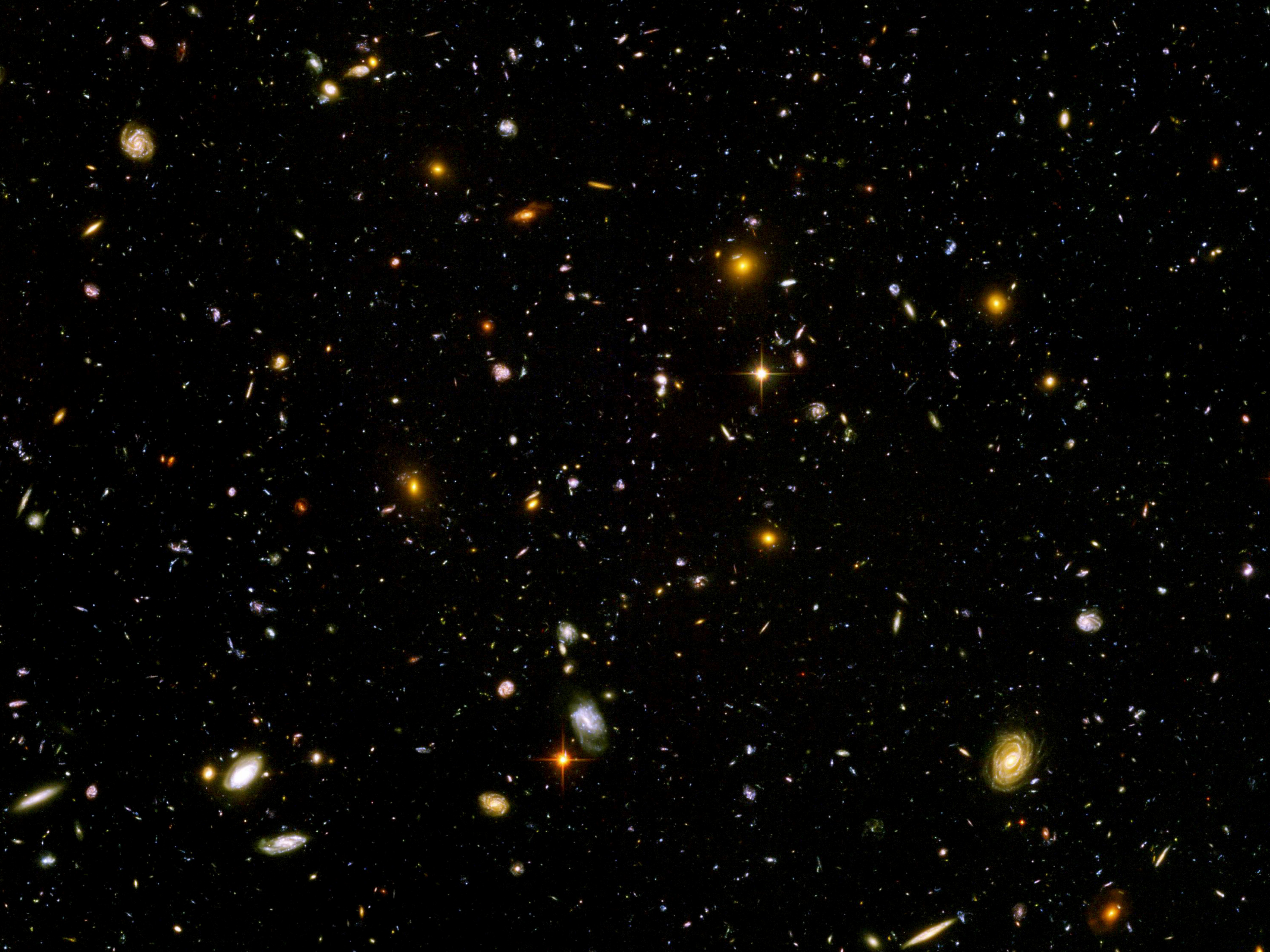
For **positively** curved space, $\theta = \dfrac{L}{R_0 \sin(d/R_0)}$

The angular size of galaxies blows up as $d \to \pi R_0$.

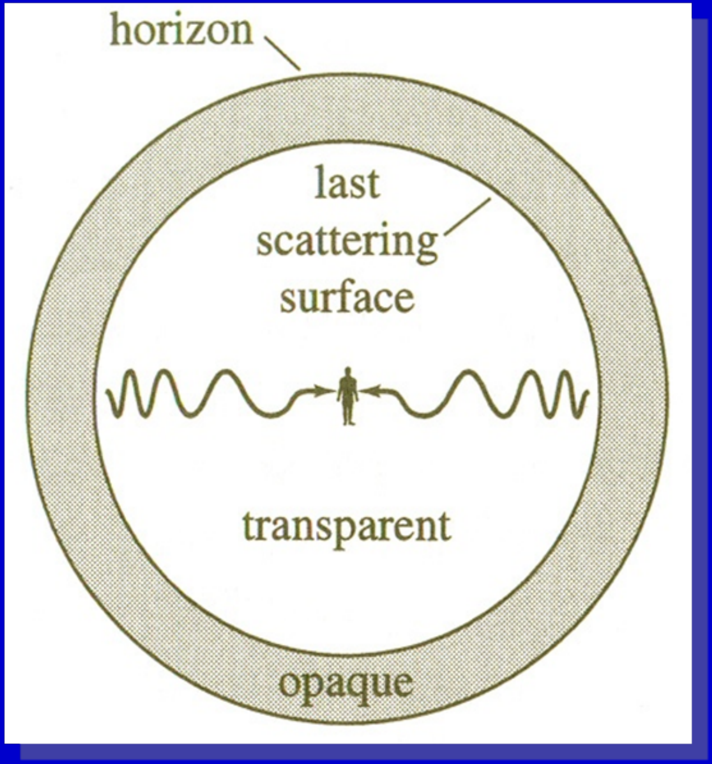
For **negatively** curved space, $\theta = \dfrac{L}{R_0 \sinh(d/R_0)}$

The angular size dwindles exponentially for $d \gtrsim R_0$.

We can see galaxies at $d \sim c/H_0$; they are neither absurdly large in angle nor unresolvably small.



We want to look at objects whose distance d is as big as possible: beyond the most distant galaxies.

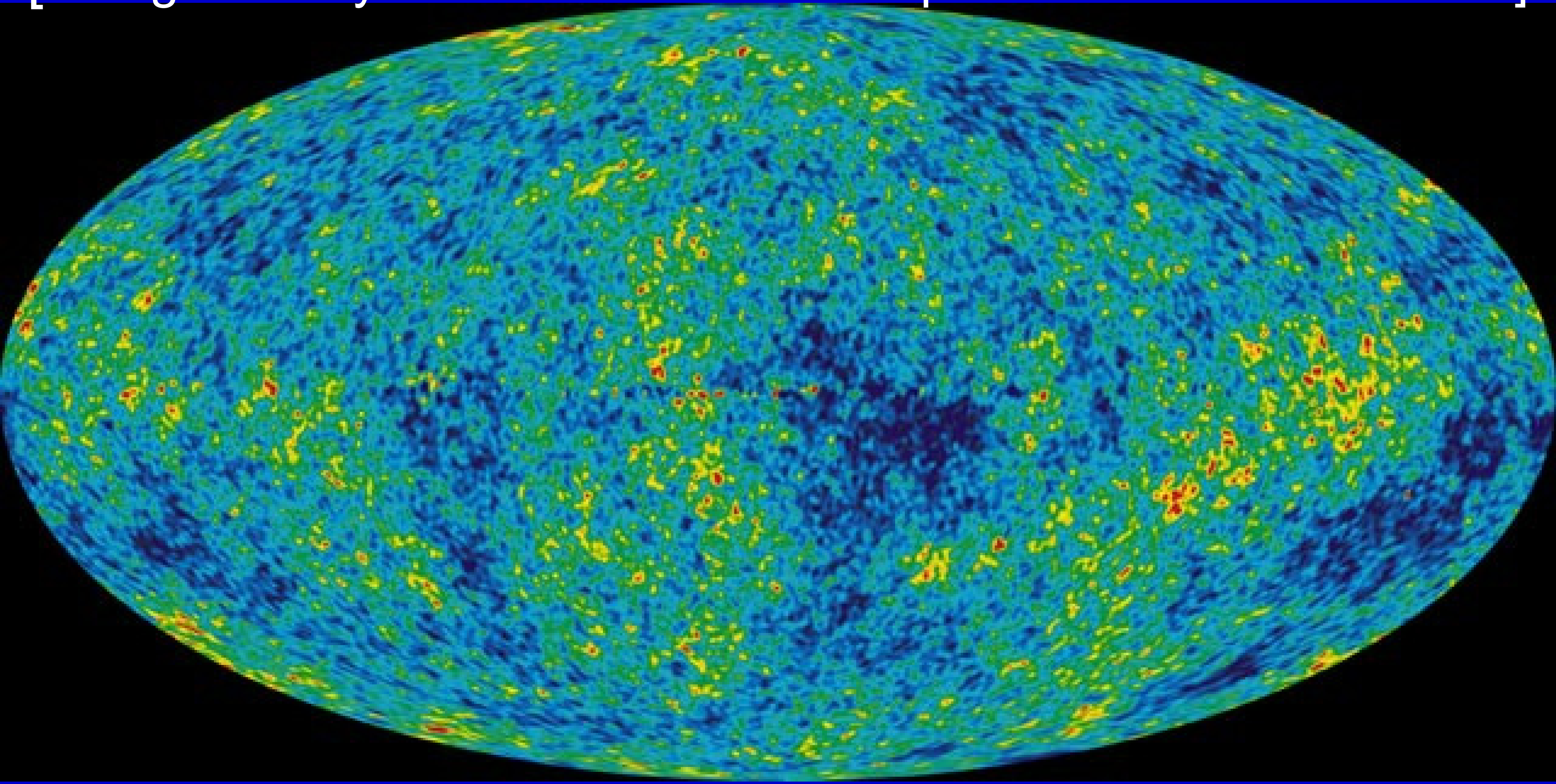


"A telescope is a time machine". Outward in space = backward in time.

When we observe the Cosmic Microwave Background, we're looking at the **last scattering surface**, the surface of the opaque ionized gas filling the early universe.

Map of the last scattering surface, from the Wilkinson Microwave Anisotropy Probe (WMAP).

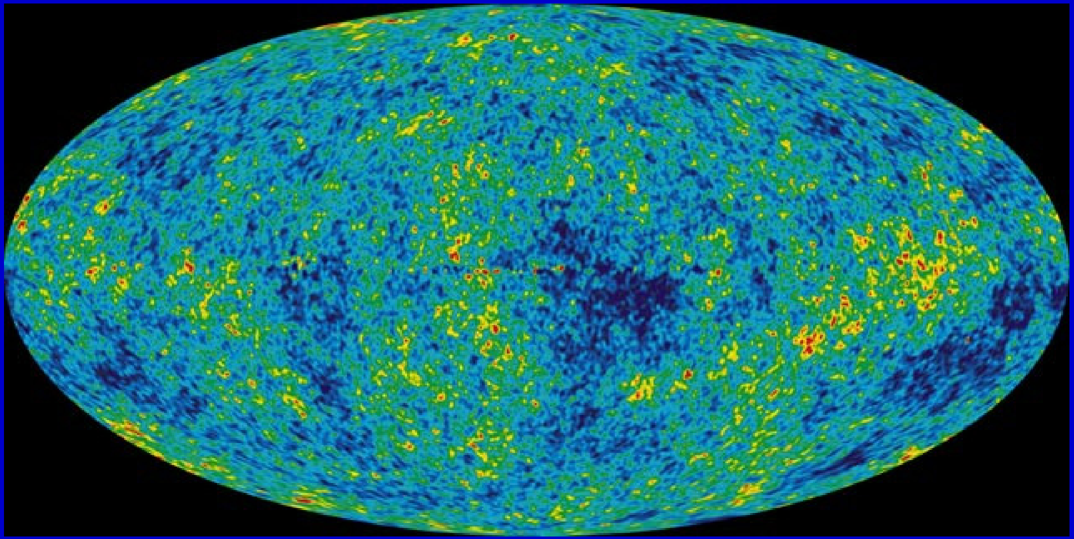
[Foreground synchrotron emission & dipole distortion subtracted.]



False color = temperature.

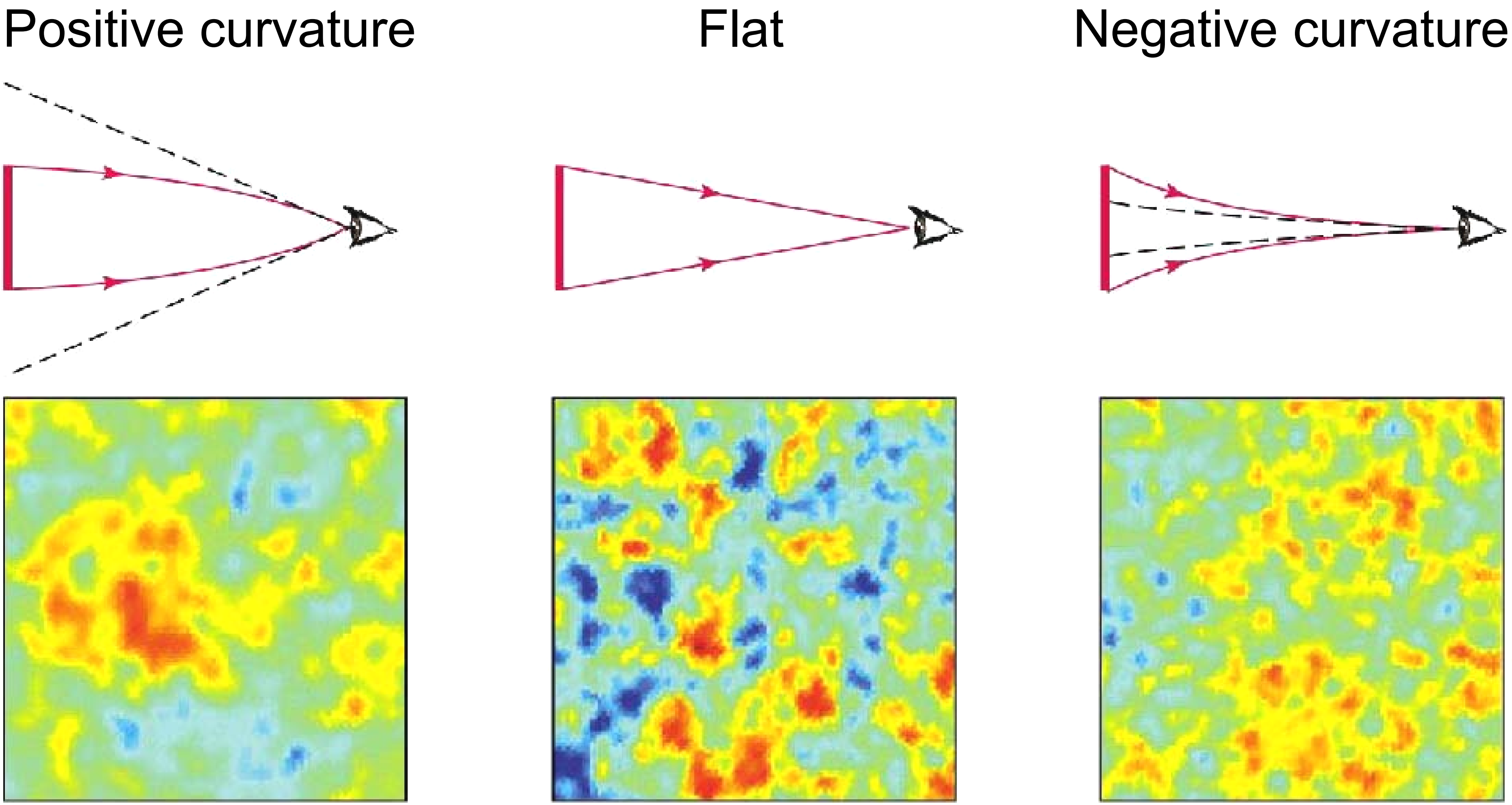
$(dT/T)_{rms} = 1.1 \times 10^{-5}$

“Hot spots” = regions of high-density photon-baryon gas.
“Cold spots” = regions of low density.

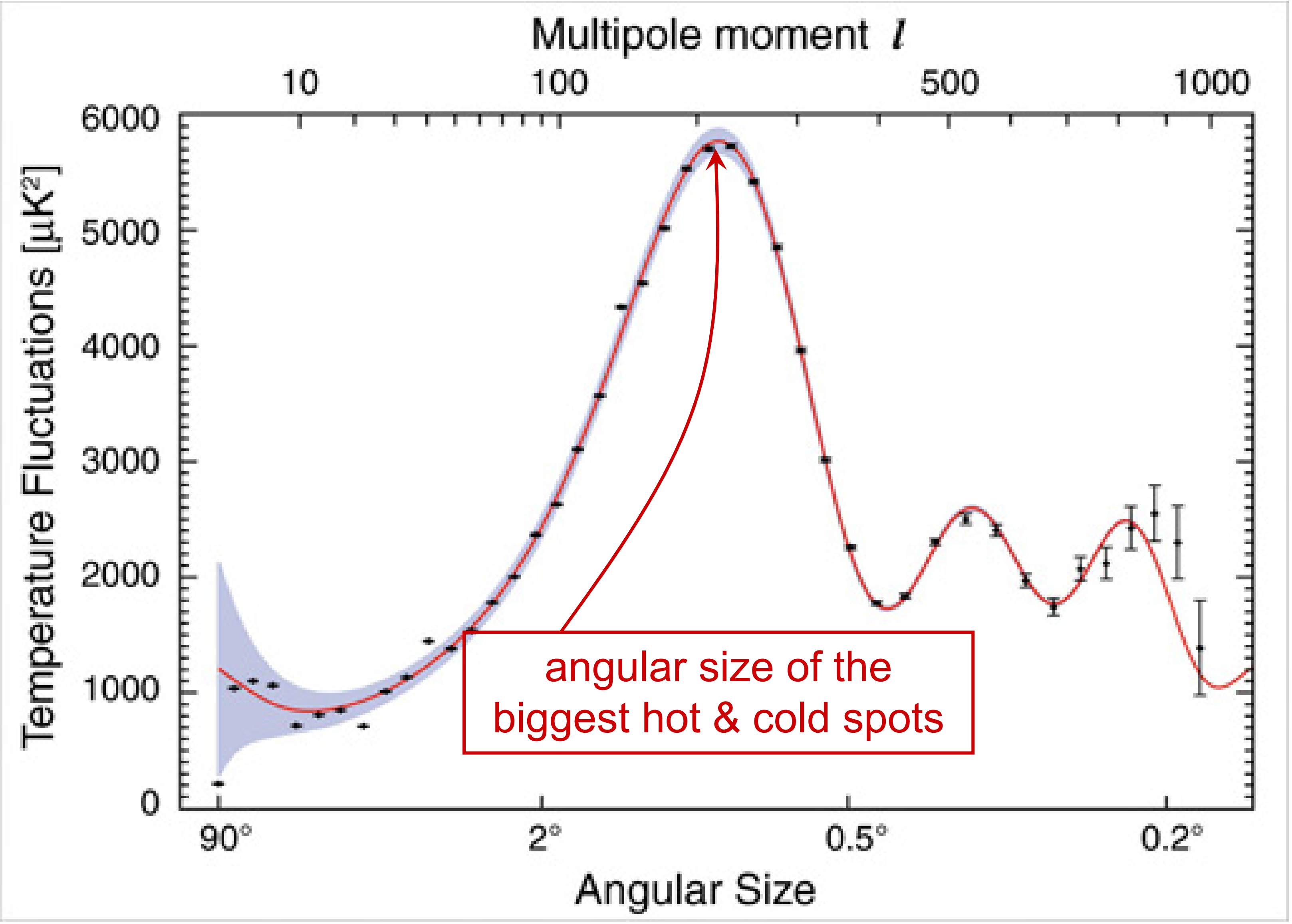


The physical size of the biggest “spots” is dictated by t_{ls}, the age of the universe when a typical CMB photon last scattered from a free electron.

Largest spots are ~ct_{ls} in radius: ~250,000 light-years.


Positive curvature
Flat
Negative curvature

Amplitude of CMB temperature fluctuations, expressed as a function of the multipole moment l.



Because there exists a critical density $\varepsilon_c(t)$, cosmologists often express density in terms of the **density parameter** $\Omega(t)$, where

$$\Omega(t) = \frac{\varepsilon(t)}{\varepsilon_c(t)}$$

That is, Ω is the actual mean density of the universe at a given time, divided by the critical density at that time.

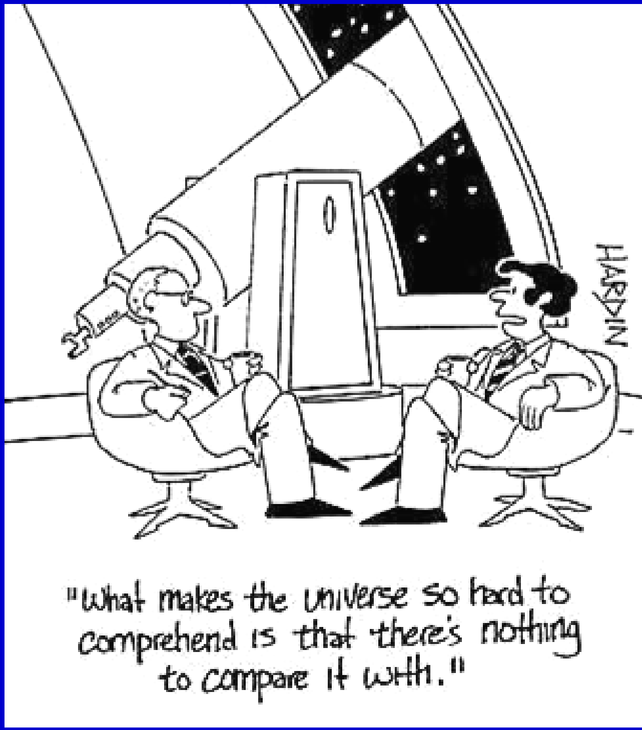
$$\Omega > 1 \qquad \kappa = +1$$

$$\Omega = 1 \qquad \kappa = 0$$

$$\Omega < 1 \qquad \kappa = -1$$

Results from the angular size of temperature fluctuations in the CMB:

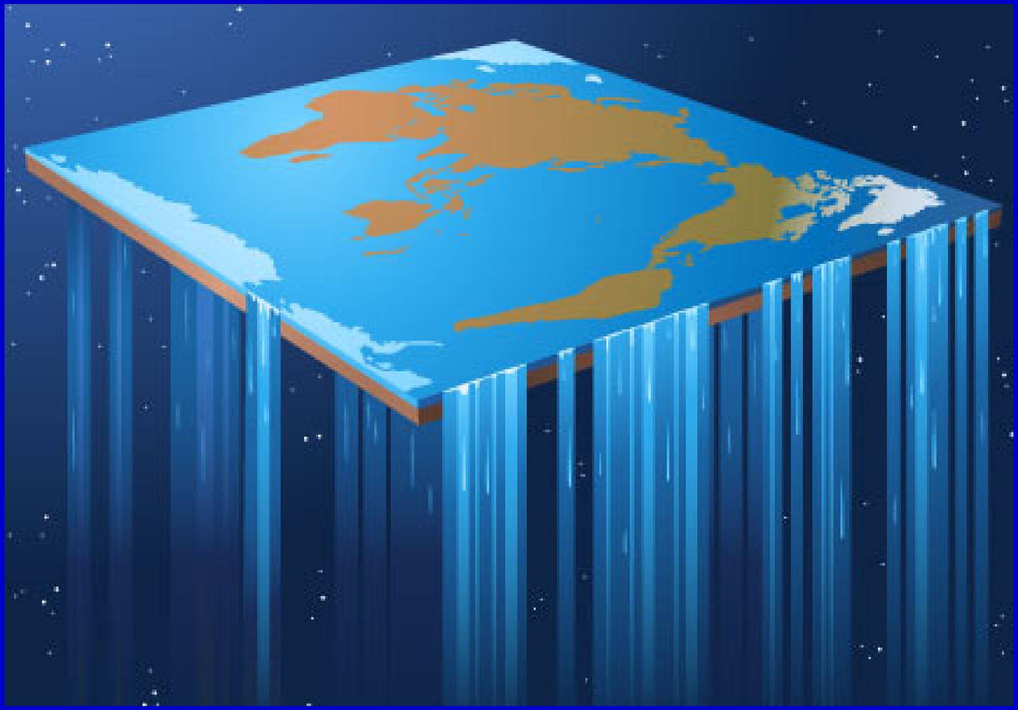
$$\Omega_0 = 1.080^{+0.093}_{-0.071}$$



Small print: data from WMAP 7-year data release only; 68% confidence interval.

The density parameter Ω is close to one:
energy density of the universe is close to 5.3 GeV m^{-3} :
the universe is close to flat.

Believing in a flat Earth is not socially respectable,



but believing in a flat Universe is acceptable.

In fact, for the rest of this lecture, I will join the "Flat Universe Society", and assume $\Omega 0=1$.

Accounting: what contributes to the energy density of the universe?

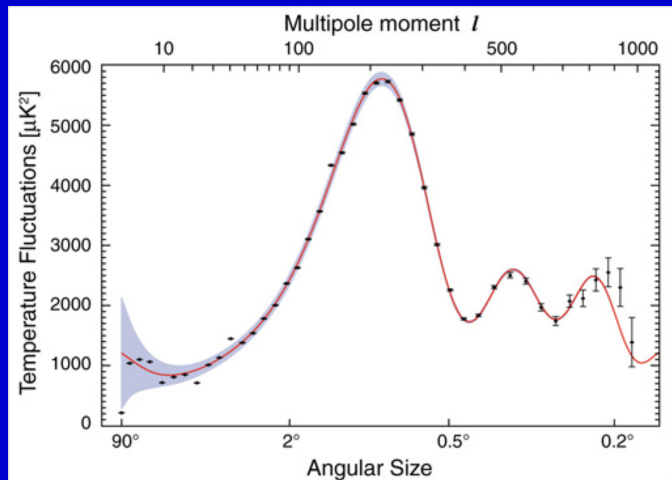
Energy density of **photons** = αT^4 = 0.26 MeV m^{-3}.

Density parameter of photons:

$\Omega_{phot,0} \approx 0.26$ MeV m^{-3} / 5300 MeV $m^{-3} \approx 5 \times 10^{-5}$.

Photons contribute only 1 part in 20,000 of the energy density of the universe today.

Accounting: what about the contribution of baryonic matter?



Location of the peaks in the CMB temperature fluctuation spectrum depends on the **sound speed** of the photon-baryon gas.

The sound speed in turn depends on the baryon-to-photon ratio. More baryons → slower sound speed.

Result of fitting the CMB temperature fluctuation spectrum:

$$\Omega_{\text{bary},0} = 0.0449 \pm 0.0028$$

Small print: data from WMAP 7-year data release only; 68% confidence interval.

Everyday baryonic matter (protons, neutrons, & enough electrons to keep everything neutral) contributes less than 5% of the density.

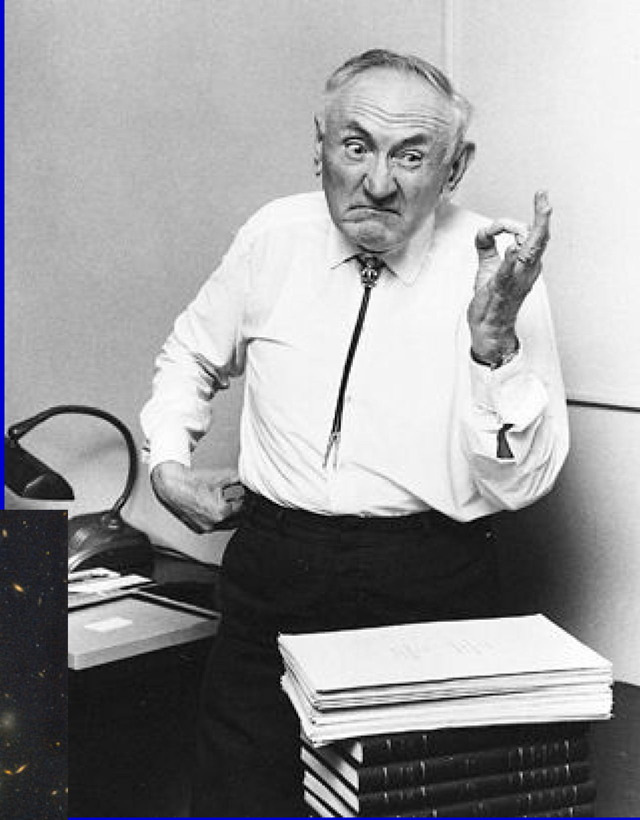
Accounting: what provides the rest of the energy density of the universe?

Photons ~0.005%, baryonic matter ~4%.
What's the other ~96%???

One possibility is **dark matter**: massive particles that don't interact with photons (except through gravity).

Dark matter doesn't absorb photons,
it doesn't emit photons,
it doesn't scatter photons.

The phrase “dark matter” (originally “dunkle Materie”) was popularized by Fritz Zwicky, in his study of the Coma cluster of galaxies (1933).



Speed of an orbiting galaxy within a cluster is $v \sim (GM/r)^{1/2}$: more massive cluster $\rightarrow$ faster galaxies.

We can find the total mass of the Coma cluster from the velocity dispersion of its galaxies:

$$M_{total} = 2 \times 10^{15} M_{sun}$$

Mass of hot dilute gas in the Coma cluster:

$$M_{gas} = 2 \times 10^{14} M_{sun}$$

Mass of stars in the Coma cluster:

$$M_{stars} = 3 \times 10^{13} M_{sun}$$

Best estimate for the density parameter of dark matter:

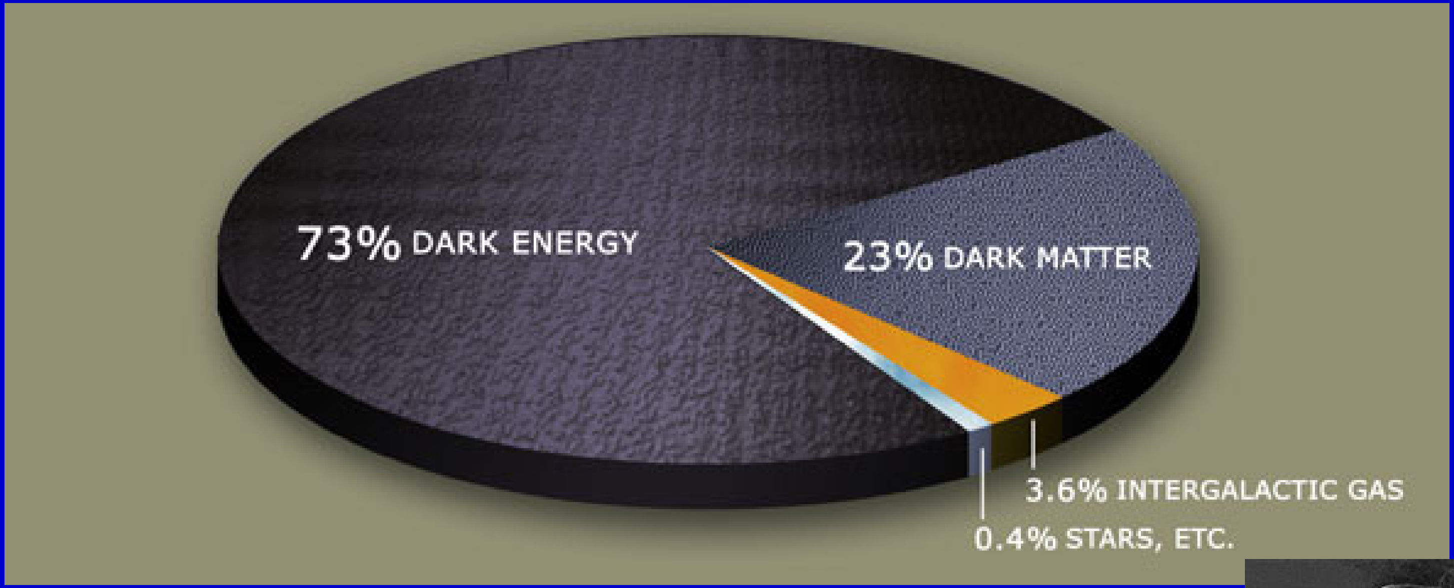
$$\Omega_{dm,0} = 0.23$$

Matter (non-relativistic massive particles) contributes only 27% of the energy density of the universe today.

Radiation (massless or highly relativistic particles) contributes an utterly negligible amount.

What's the other ~73%?

Various lines of evidence indicate that it's **dark energy** (a component of the universe with negative pressure which causes the expansion to accelerate).



The dark energy could be Λ, Einstein's notorious **cosmological constant**.