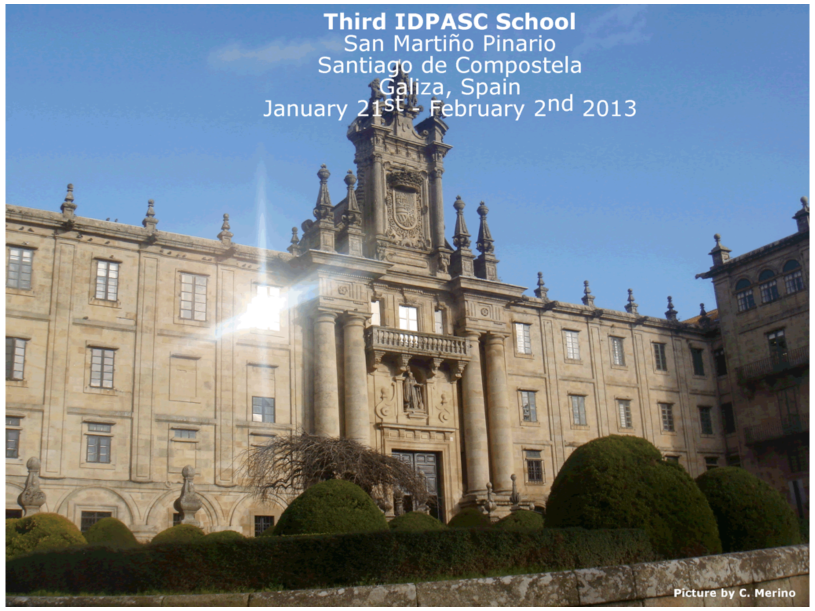

Introduction to AdS/CFT

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Correlation functions

Euclidean correlators → $\langle \mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\rangle$

Perturb the lagrangian by a source → $\mathcal{L}\to\mathcal{L}+J(x)\,\mathcal{O}(x)\equiv\mathcal{L}+\mathcal{L}_J$

Generating function

$$Z_{QFT}[J] = \Big\langle \ \exp[\int \mathcal{L}_J] \ \Big\rangle_{QFT} \quad \longrightarrow \quad \Big\langle \prod_i \mathcal{O}(x_i) \Big\rangle = \prod_i \frac{\delta}{\delta J(x_i)} \log Z_{QFT}[J]\Big|_{J=0}$$

AdS/CFT prescription

$$Z_{QFT}[\phi_0] = \Big\langle \exp\Big[\int \phi_0\,\mathcal{O}\Big] \Big\rangle_{QFT} = Z_{gravity}[\phi\to\phi_0]$$

$$\phi_0(x) = \phi(z=0,x) = \phi|_{\partial AdS}(x)$$

$$Z_{gravity}[\phi\to\phi_0] = \sum_{\{\phi\to\phi_0\}} e^{S_{gravity}}$$

When classical gravity dominates

$$\boxed{Z_{QFT}[\phi_0] \;=\; e^{S^{on-shell}_{gravity}[\phi\rightarrow\phi_0]}} \;\Longrightarrow$$

typically divergent

After renormalizing of the action

$$\log Z_{QFT} \;=\; S^{ren}_{grav}[\phi \rightarrow \phi_0] \;\Longrightarrow\; \boxed{\langle\mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\rangle \;=\; \frac{\delta^{(n)} S^{ren}_{grav}[\phi]}{\delta\varphi(x_1)\,\cdots\varphi(x_n)}\bigg|_{\varphi=0}}$$

one-point function ➡

$$\langle\mathcal{O}(x)\rangle_\varphi \;=\; \frac{\delta S^{ren}_{grav}[\phi]}{\delta\varphi(x)} \;\Longrightarrow\; \langle\mathcal{O}(x)\rangle_\varphi \;=\; \lim_{z\rightarrow 0}\; z^{d-\Delta}\;\frac{\delta S^{ren}_{grav}[\phi]}{\delta\phi(z,x)}$$

variation of the action

$$S_{grav} = \int_{\mathcal{M}}\int dz\, d^dx \mathcal{L}[\phi,\partial\phi] \;\Longrightarrow\; \delta S_{grav} = \int_{\mathcal{M}}\int dz\, d^dx\Big[\frac{\partial\mathcal{L}}{\partial\phi}\,\delta\phi \;+\; \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\,\delta(\partial_\mu\phi)\Big]$$

$$\Longrightarrow\; \delta S_{grav} = \int_{\mathcal{M}}\int dz\, d^dx\Big[\Big(\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\Big(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\Big)\Big)\delta\phi \;+\; \partial_\mu\Big(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi\Big)\Big]$$

on-shell action ➡

$$\delta S^{on-shell}_{grav} = \int_\epsilon^\infty\int d^dx\;\partial_z\Big(\frac{\partial\mathcal{L}}{\partial(\partial_z\phi)}\delta\phi\Big) \;=\; -\int_{\partial M} d^dx\;\frac{\partial\mathcal{L}}{\partial(\partial_z\phi)}\delta\phi\Big|_{z=\epsilon}$$

canonical momentum → $\boxed{\Pi = -\frac{\partial \mathcal{L}}{\partial(\partial_z \phi)}}$ → $\delta S_{grav}^{on-shell} = \int_{\partial M} d^d x \, \Pi(\epsilon, x) \, \delta\phi(\epsilon, x)$

→ $\frac{\delta S_{grav}^{on-shell}}{\delta \phi(\epsilon, x)} = \Pi(\epsilon, x) = -\frac{\partial \mathcal{L}}{\partial(\partial_z \phi)}$

renormalized action → $S^{ren} = S_{grav}^{on-shell} + S_{ct}$

renormalized momentum

$$\Pi^{ren}(z, x) = \frac{\delta S^{ren}}{\delta \phi(z, x)} \quad \longrightarrow \quad \Pi^{ren}(\epsilon, x) = -\frac{\partial \mathcal{L}}{\partial(\partial_z \phi(\epsilon, x))} + \frac{\delta S_{ct}}{\delta \phi(\epsilon, x)}$$

one-point function in the presence of a source

$$\boxed{\langle \mathcal{O}(x) \rangle_{\varphi} = \lim_{z \to 0} z^{d-\Delta} \, \Pi^{ren}(z, x)}$$

Linear response theory

path integral representation →

$$\boxed{\langle \mathcal{O}(x)\rangle_\varphi \;=\; \int [D\psi]\, \mathcal{O}(x)\, e^{S_E[\psi]+\int d^d y\, \varphi(y)\mathcal{O}(y)}}$$

At linear order

$$\langle \mathcal{O}(x)\rangle_\varphi \;=\; \langle \mathcal{O}(x)\rangle_{\varphi=0} \,+\, \int d^d y\, G_E(x-y)\, \varphi(y) \qquad\qquad G_E(x-y) \;=\; \langle \mathcal{O}(x)\, \mathcal{O}(y)\rangle$$

For normal-ordered operators

$$\langle \mathcal{O}(x)\rangle_{\varphi=0} = 0 \quad \longrightarrow \quad \boxed{\langle \mathcal{O}(x)\rangle_\varphi \;=\; \int d^d y\, G_E(x-y)\, \varphi(y)}$$

In momentum space

$$\boxed{G_E(k) \;=\; \frac{\langle \mathcal{O}(k)\rangle_\varphi}{\varphi(k)} \quad \longrightarrow \quad G_E(k) \;=\; \lim_{z\to 0} z^{2(d-\Delta)}\, \frac{\Pi^{ren}(z,k)}{\phi(z,k)}}$$

Scalar field in Euclidean AdS_{p+1}

$$S = -\frac{\eta}{2}\int dz\, d^dx\sqrt{g}\left[g^{MN}\partial_M\,\phi\,\partial_N\,\phi + m^2\,\phi^2\right] \qquad \eta \to \text{constant} \quad \Longrightarrow$$

$$S = -\frac{\eta}{2}\int dz\, d^dx\partial_M\left[\sqrt{g}\,\phi\,g^{MN}\partial_N\phi\right] + \frac{\eta}{2}\int dz\, d^dx\,\phi\,\sqrt{g}\left[\frac{1}{\sqrt{g}}\,\partial_M\left(\sqrt{g}g^{MN}\partial_N\phi\right) - m^2\,\phi\right] \quad \Longrightarrow$$

$$S^{on-shell} = -\frac{\eta}{2}\int dz\, d^dx\partial_M\left[\sqrt{g}\,\phi\,g^{MN}\partial_N\phi\right] = \frac{\eta}{2}\int d^dx\left(\sqrt{g}\,\phi\,g^{zz}\partial_z\phi\right)_{z=\epsilon} \quad \Longrightarrow$$

$$\boxed{\Pi \equiv \eta\,\sqrt{g}\,g^{zz}\partial_z\phi = -\frac{\partial\mathcal{L}}{\partial(\partial_z\phi)}} \quad \Longrightarrow \quad \boxed{S^{on-shell} = \frac{1}{2}\int_{z=\epsilon} d^dx\;\Pi(z,x)\,\phi(z,x)}$$

In momentum space

$$\left.\begin{array}{l} \phi(z,x) = \displaystyle\int \frac{d^dk}{(2\pi)^d}e^{ik\cdot x}\,f_k(z) \\[2em] \Pi(z,x) = \displaystyle\int \frac{d^dk}{(2\pi)^d}e^{ik\cdot x}\,\Pi_k(z) \end{array}\right] \quad \Longrightarrow \quad \boxed{S^{on-shell} = \frac{1}{2}\int \frac{d^dk}{(2\pi)^d}\,\Pi_{-k}(z=\epsilon)\,f_k(z=\epsilon)}$$

Near $z = 0$

$$\phi(z,x) \approx A(x)\, z^{d-\Delta} + B(x)\, z^{\Delta} \qquad \longrightarrow \qquad \boxed{f_k(z) \approx A(k)\, z^{d-\Delta} + B(k)\, z^{\Delta}}$$

expansion of the momentum → $\Pi(z,x) \approx \eta L^{d-1}\Big[(d-\Delta)\, A(x)\, z^{-\Delta} + \Delta\, B(x)\, z^{\Delta-d}\Big]$ →

$$\boxed{\Pi_{-k}(z) = \eta L^{d-1}\Big[(d-\Delta)\, A(-k)\, z^{-\Delta} + \Delta\, B(-k)\, z^{\Delta-d}\Big]}$$

on-shell action →

$$\boxed{S^{on-shell} = \frac{\eta}{2} L^{d-1} \int \frac{d^dk}{(2\pi)^d}\Big[\underset{\substack{\big\downarrow \\ \text{divergent}}}{\epsilon^{-2\nu}\,(d-\Delta)\, A(-k)\, A(k)} + \underset{\substack{\big\downarrow \\ \text{finite}}}{d\, A(-k)\, B(k)}\Big]}$$

local boundary counterterm

$$S_{ct} \sim \int_{\partial AdS} d^dx\, \sqrt{\gamma}\, \phi^2 \qquad \longleftarrow \qquad ds^2_{z=\epsilon} = \gamma_{\mu\nu}\, dx^\mu\, dx^\nu = \frac{L^2}{\epsilon^2}\, \delta_{\mu\nu} dx^\mu\, dx^\nu$$

$$\Big\downarrow$$

$$\int_{\partial AdS} d^dx \sqrt{\gamma}\, \phi^2(\epsilon, x) = L^d \int \frac{d^dk}{(2\pi)^d}\Big[\epsilon^{-2\nu}\, A(-k)\, A(k) + 2\, A(-k)\, B(k)\Big]$$

Cancel the divergence $\longrightarrow$ $S_{ct} = -\frac{\eta}{2}\,\frac{d-\Delta}{L}\int_{\partial AdS} d^dx\,\sqrt{\gamma}\,\phi^2$ $\longrightarrow$

$$\boxed{S_{ct} = -\frac{\eta}{2}\,(d-\Delta)\,L^{d-1}\int\frac{d^dk}{(2\pi)^d}\left[\epsilon^{-2\nu}\,A(-k)\,A(k)\;+\;2\,A(-k)\,B(k)\right]}$$

$S^{ren} = S^{on-shell} + S_{ct}$ $\longrightarrow$ $\boxed{S^{ren} = \frac{\eta}{2}\,L^{d-1}\,(2\Delta-d)\int\frac{d^dk}{(2\pi)^d}\,A(-k)\,B(k)}$

Represent f_k as $f_k(z) = A(k)\,\phi_1(z,k) + B(k)\,\phi_2(z,k)$ $\quad\phi_1(z,k)\approx z^{d-\Delta}$ $\quad\phi_2(z,k)\approx z^{\Delta}$

Regularity of f_k at $z\to\infty$ fixes $\chi = \frac{B}{A}$ $\longrightarrow$

$\varphi(k) = A(k)$ $\longrightarrow$ $S^{ren} = \frac{\eta}{2}L^{d-1}\,(2\Delta-d)\int\frac{d^dk}{(2\pi)^d}\,\chi(k)\varphi(k)\,\varphi(-k)$

$\langle\mathcal{O}(k)\rangle_\varphi = (2\pi)^d\,\frac{\delta S^{ren}}{\delta\varphi(-k)} = \eta\,L^{d-1}\,(2\Delta-d)\,\chi(k)\varphi(k)$ $\longrightarrow$ $\boxed{\langle\mathcal{O}(k)\rangle_\varphi = 2\nu\,\eta\,L^{d-1}B(k)}$

The subleading contribution is proportional to the VEV

2-point function $\longrightarrow$ $\boxed{G_E(k) = 2\nu\,\eta\,L^{d-1}\,\frac{B(k)}{A(k)}}$

Determination of B/A

Define $f_k(z) = z^{\frac{d}{2}}\, g_k(z) \longrightarrow z^2\,\partial_z^2\, g_k \,+\, z\,\partial_z\, g_k - (\nu^2 + k^2\, z^2) g_k = 0$ **modified Bessel eq.**

two ind. sols. for $f_k \longrightarrow \boxed{z^{\frac{d}{2}}\, I_{\pm\nu}(kz)}$ $\quad I_{\pm\nu}(z) \approx \frac{1}{\Gamma(1\pm\nu)}\left(\frac{z}{2}\right)^{\pm\nu}, \qquad (z\to 0)$

Then $\longrightarrow$ $\phi_1(z,k) \,=\, \Gamma(1-\nu)\left(\frac{k}{2}\right)^{\nu} z^{\frac{d}{2}}\, I_{-\nu}(kz) \qquad \phi_2(z,k) \,=\, \Gamma(1+\nu)\left(\frac{k}{2}\right)^{-\nu} z^{\frac{d}{2}}\, I_{\nu}(kz)$

$$\boxed{f_k(z) = z^{\frac{d}{2}}\left[\Gamma(1-\nu)\left(\frac{k}{2}\right)^{\nu} A(k)\, I_{-\nu}(kz) \,+\, \Gamma(1+\nu)\left(\frac{k}{2}\right)^{-\nu} B(k)\, I_{\nu}(kz)\right]}$$

As $I_{\pm\nu}(z) \approx \frac{e^z}{\sqrt{2\pi z}}, \qquad (z\to\infty) \longrightarrow$

$$f_k(z) \approx \frac{z^{\frac{d}{2}}\, e^{kz}}{\sqrt{2\pi k z}}\left[\Gamma(1-\nu)\left(\frac{k}{2}\right)^{\nu} A(k) \,+\, \Gamma(1+\nu)\left(\frac{k}{2}\right)^{-\nu} B(k)\right] \qquad (z\to\infty)$$

Regularity at the IR →

$$\frac{B(k)}{A(k)} = -\frac{\Gamma(1-\nu)}{\Gamma(1+\nu)}\left(\frac{k}{2}\right)^{2\nu} = \frac{\Gamma(-\nu)}{\Gamma(\nu)}\left(\frac{k}{2}\right)^{2\nu} \rightarrow$$

$$\boxed{G_E(k) \;=\; 2\nu\,\eta\,L^{d-1}\,\frac{\Gamma(-\nu)}{\Gamma(\nu)}\left(\frac{k}{2}\right)^{2\nu}}$$

position space →

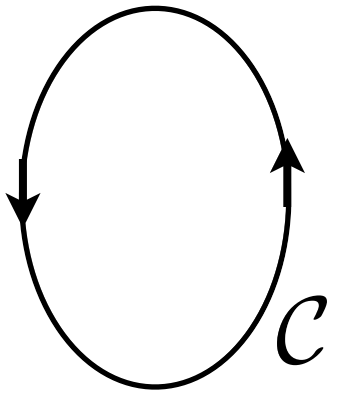
$$G_E(x) = \int \frac{d^dk}{(2\pi)^d}\, e^{ikx}\, G_E(k)$$

$$\int \frac{d^dk}{(2\pi)^d}\, e^{ikx}\, k^n = \frac{2^n}{\pi^{\frac{d}{2}}}\,\frac{\Gamma\left(\frac{d+n}{2}\right)}{\Gamma\left(-\frac{n}{2}\right)}\,\frac{1}{|x|^{d+n}} \quad \rightarrow \quad \boxed{\langle\mathcal{O}(x)\mathcal{O}(0)\rangle = \frac{2\nu\eta L^{d-1}}{\pi^{\frac{d}{2}}}\,\frac{\Gamma\left(\frac{d}{2}+\nu\right)}{\Gamma\left(-\nu\right)}\,\frac{1}{|x|^{2\Delta}}}$$

$$\langle\mathcal{O}(x)\mathcal{O}(0)\rangle \sim |x|^{-2\Delta} \rightarrow \Delta \text{ scaling dimension of } \mathcal{O}(x)$$

Wilson loops and quark-antiquark potentials

→ External charge moving along a $\mathcal{C}$ in QED



Action → $S_{\mathcal{C}} = \oint_{\mathcal{C}} A_\mu \, dx^\mu$

equivalent to insert in the path integral → $e^{iS_{\mathcal{C}}} = e^{i \oint_{\mathcal{C}} A_\mu \, dx^\mu} \equiv W(\mathcal{C})$

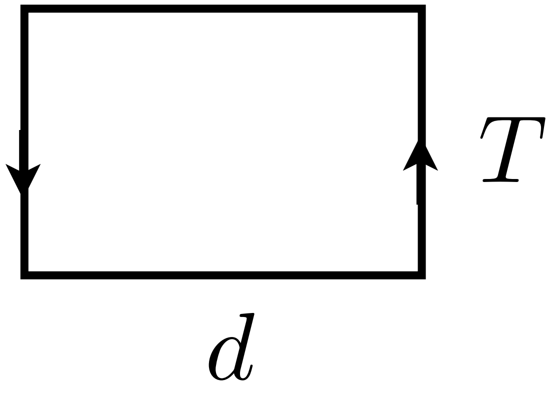
$\boxed{W(\mathcal{C}) \rightarrow \text{Wilson loop}}$ $\quad$ Math. → holonomy of A_μ along $\mathcal{C}$

Phase factor due to the propagation of a quark along the closed curve $\mathcal{C}$

$$\boxed{W(\mathcal{C}) = \mathrm{Tr}\, P \exp \left[i \oint_{\mathcal{C}} A_\mu \, dx^\mu \right]} \qquad A_\mu = A_\mu^a \, T^a$$

$W(\mathcal{C}) \rightarrow$ amplitude for a creation of a $q\bar{q}$ pair

Rectangular Wilson loop in Euclidean space



Amplitude for the propagation of a $q\bar{q}$ pair separated a distance d

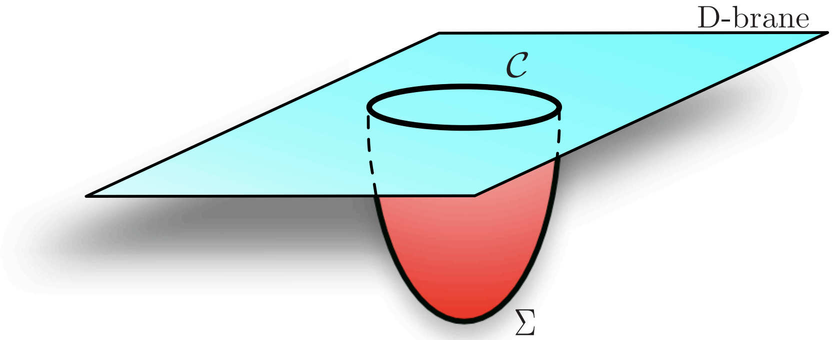
$$\lim_{T\to\infty} \langle W(\mathcal{C})\rangle \sim e^{-T\,E(d)} \quad \Longrightarrow \quad E(d) \to \text{the energy of a } q\bar{q} \text{ pair}$$

In a confining theory → $\boxed{E(d) \approx \sigma\, d}$ $\qquad \sigma \to \text{constant}$

$$\lim_{T\to\infty} \langle W(\mathcal{C})\rangle \sim e^{-\sigma\,T\,d} \sim e^{-\sigma\,(\text{area enclosed by the loop})} \quad \Longrightarrow$$

area law

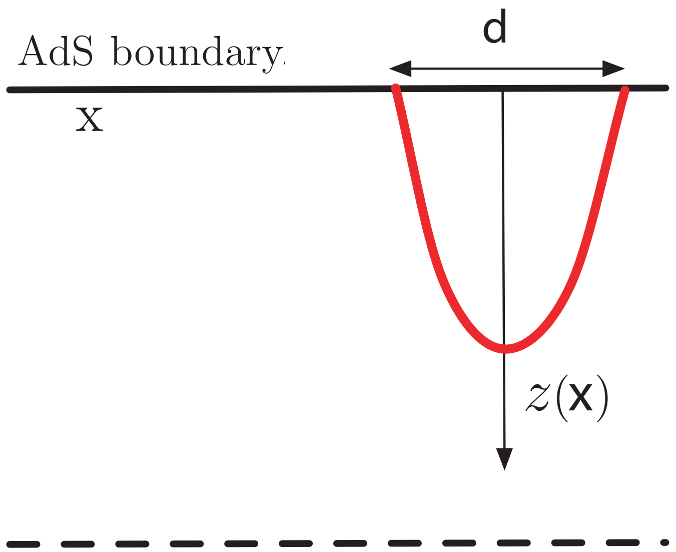
In AdS/CFT → Realized as open string ending on a D-brane →



$$\langle W(\mathcal{C})\rangle = Z_{string}(\partial\Sigma = \mathcal{C}) \quad \xrightarrow[\text{large } \lambda]{} \quad \langle W(\mathcal{C})\rangle = e^{-S(\mathcal{C})}$$

$S(\mathcal{C}) \to$ extremal Nambu-Goto action

hanging string extended in x in AdS_5



induced metric

↓

$$ds^2 = \frac{L^2}{z^2}\left[\, dt^2 + (1 + z'^{\,2})\, dx^2 \,\right]$$

↓

$$S = \frac{1}{2\pi\alpha'} \int dt \int dx \,\sqrt{g} = \frac{TL^2}{2\pi\alpha'} \int dx \frac{\sqrt{1+z'^{\,2}}}{z^2}$$

Lagrangian →
$$\boxed{\mathcal{L} = \frac{TL^2}{2\pi\alpha'} \frac{\sqrt{1+z'^{\,2}}}{z^2}}$$

$$T = \int dt$$

b. c. → $z(x=-d/2) = z(x=d/2) = 0$

$z_* \rightarrow$ maximal value of z

$z_* = z(x=0)$

First integral → $z' \dfrac{\partial \mathcal{L}}{\partial z'} - \mathcal{L} = \text{constant}$ →

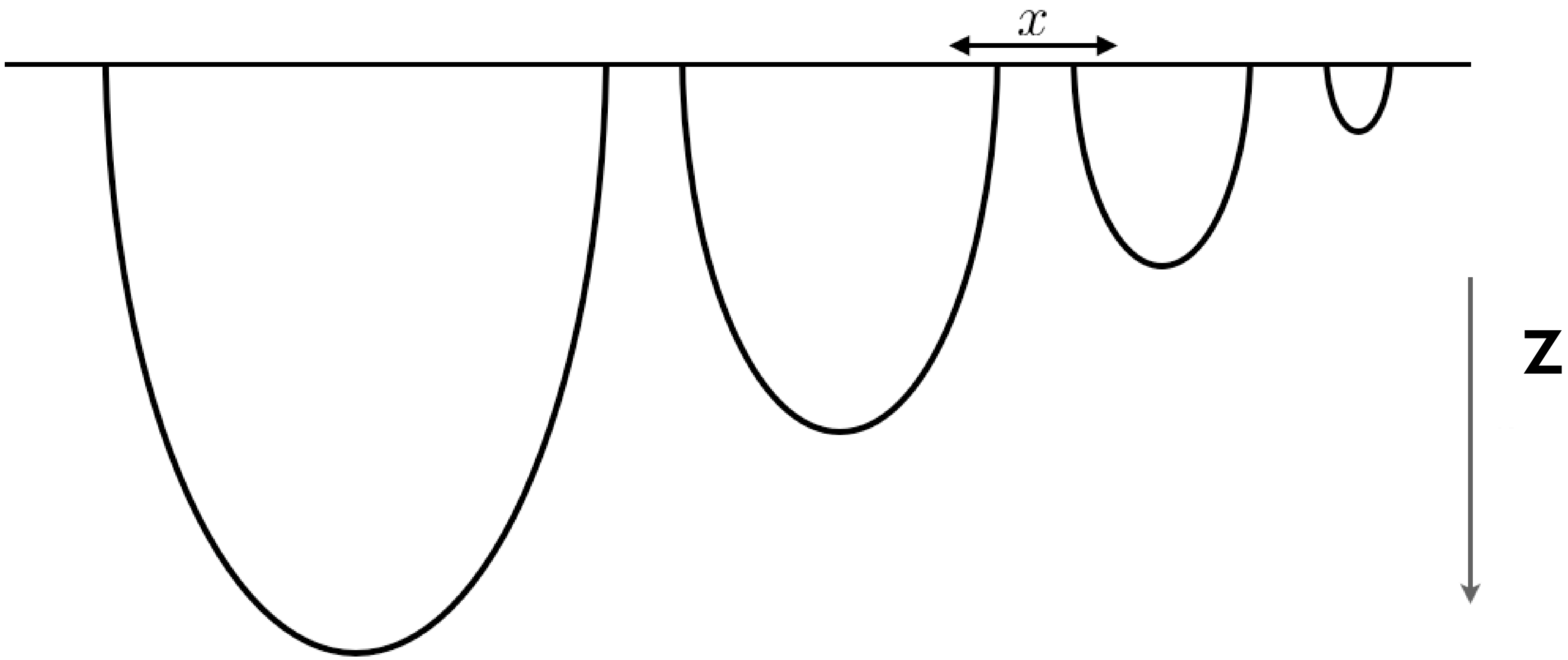
$$z^2 \sqrt{1+z'^{\,2}} = \text{constant}$$

→

$$\boxed{x = \pm z_* \int_1^{\frac{z}{z_*}} \frac{y^2}{\sqrt{1-y^4}}\, dy}$$

$$\frac{d}{2} = z_* \int_0^1 \frac{y^2}{\sqrt{1-y^4}}\,dy \quad \Longrightarrow \quad \boxed{z_* = \frac{d}{2\sqrt{2}\,\pi^{\frac{3}{2}}}\left(\Gamma\left(\frac{1}{4}\right)\right)^2}$$

long distances in $x \rightarrow$ deeper into the AdS bulk



on-shell action → $S = \frac{T\,L^2 z_*^2}{2\pi\alpha'} \int \frac{dx}{z^4}$

Change variables from x to z → $\frac{dx}{dz} = \frac{1}{z'} = \frac{z^2}{\sqrt{z_*^4 - z^4}}$ →

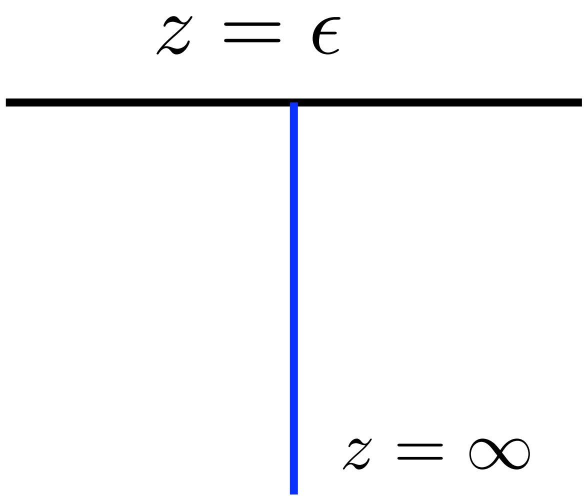
→ $S = 2 \times \frac{T L^2 z_*^2}{2\pi\alpha'} \int_\epsilon^{z_*} \frac{dz}{z^2\sqrt{z_*^4 - z^4}}$ → $S = \frac{T L^2}{\pi\alpha' z_*} I_\epsilon$ →

$I_\epsilon = \int_{\epsilon/z_*}^{1} \frac{dy}{y^2\sqrt{1 - y^4}}$ → diverges when $\epsilon \to 0$

$I_\epsilon = -\frac{\pi^{\frac{3}{2}}\sqrt{2}}{\left(\Gamma\left(\frac{1}{4}\right)\right)^2} + \frac{z_*}{\epsilon}$ → $E = -\frac{4\pi^2 L^2}{\left(\Gamma\left(\frac{1}{4}\right)\right)^4 \alpha'} \frac{1}{d} + \frac{L^2}{\pi\alpha'} \frac{1}{\epsilon}$

The divergent term corresponds to the quark and antiquark masses

String hanging from the boundary $z = \epsilon$ to $z = \infty$ at fixed x



$$ds^2 = \frac{L^2}{z^2}\left(dt^2 + dz^2\right)$$

$$\Big\downarrow$$

$$S_| = \frac{T}{2\pi\alpha'}\int_\epsilon^\infty \frac{L^2}{z^2}dz^2 = \frac{TL^2}{2\pi\alpha'\,\epsilon} \longrightarrow E_{||} = 2\times\frac{L^2}{2\pi\alpha'\,\epsilon}$$

$q\bar{q}$ potential $\longrightarrow$ $V_{q\bar{q}} = E - E_{||}$ $\longrightarrow$ $V_{q\bar{q}} = -\frac{4\pi^2 L^2}{\left(\Gamma\left(\frac{1}{4}\right)\right)^4\alpha'}\frac{1}{d}$

$L^2 = \sqrt{N^2 g_{YM}}\,\alpha' = \sqrt{\lambda}\,\alpha'$ $\longrightarrow$ $\boxed{V_{q\bar{q}} = -\frac{4\pi^2\sqrt{\lambda}}{\left(\Gamma\left(\frac{1}{4}\right)\right)^4}\frac{1}{d}}$

$\longrightarrow$ Coulombic (conformal invariant)

$\longrightarrow$ non-perturbative in λ

Perturbative result $\longrightarrow$ $V^{(per)}_{q\bar{q}} = -\frac{\pi\lambda}{d}$

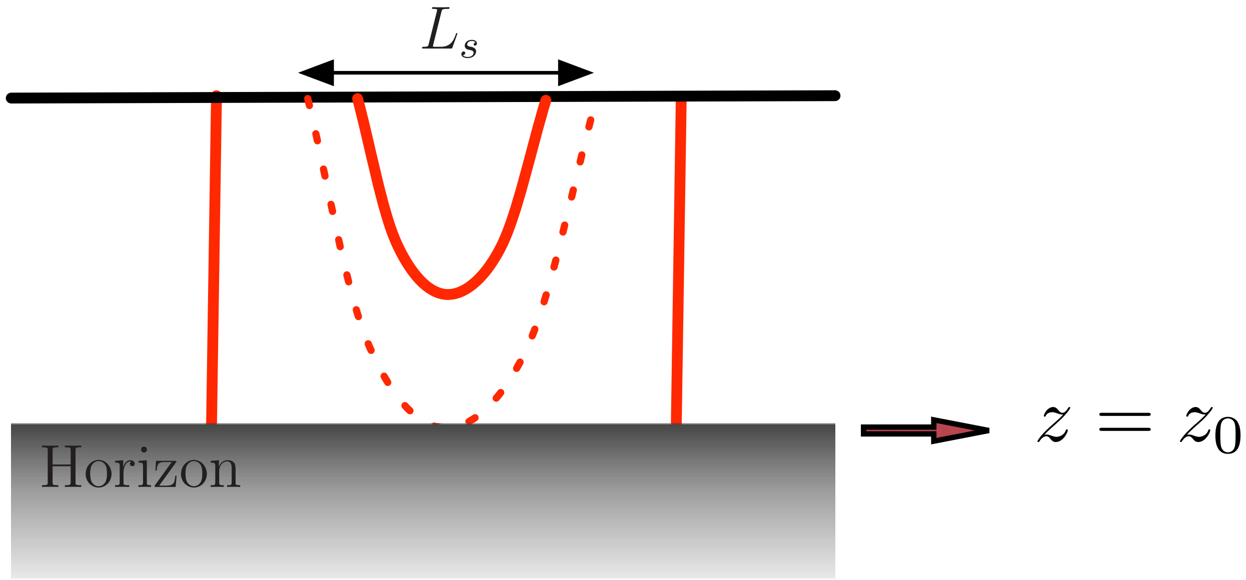
Non-perturbative result from a classical mechanics calculation!!

Quark-antiquark potential at finite temperature

Consider an AdS black hole

$$ds^2 = \frac{L^2}{z^2}\left[\, f(z)\, dt^2 \,+\, d\vec{x}^{\,2} \,+\, \frac{dz^2}{f(z)}\,\right] \qquad f(z) = 1 - \frac{z^4}{z_0^4} \qquad T = \frac{1}{\pi z_0}$$

→ Temperature



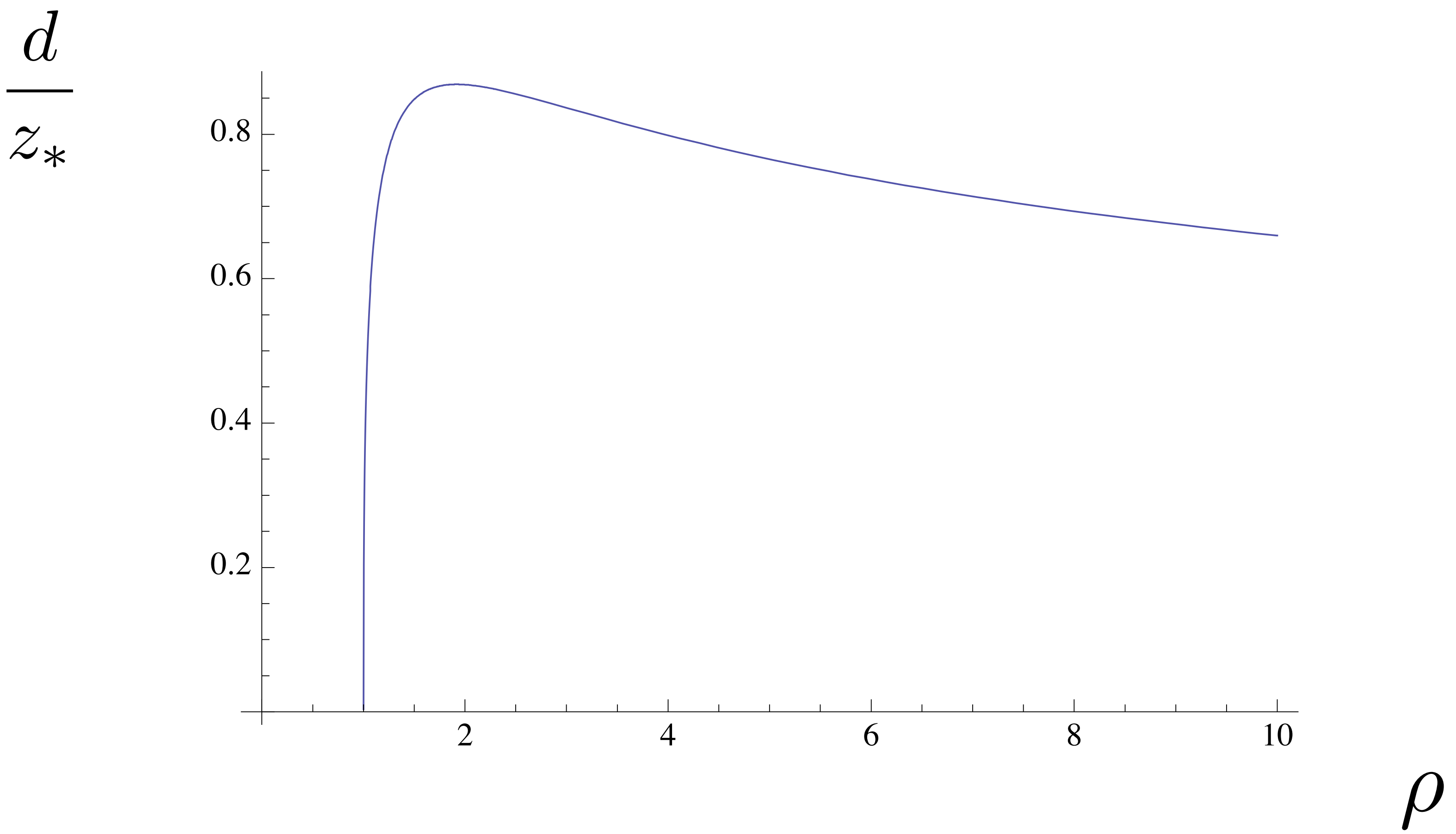
Action → $$S = \frac{\tau L^2}{2\pi\alpha'}\int dx \frac{\sqrt{f(z) + z'^{\,2}}}{z^2} \qquad \tau = \int dt$$

First integral → $$\frac{z^2\,\sqrt{f(z) + z'^{\,2}}}{f(z)} = \text{constant} = \frac{z_*^2}{\sqrt{f(z_*)}}$$

$q\bar{q}$ distance $\Rightarrow$

$$\boxed{d = 2z_* \sqrt{\rho - 1} \int_0^1 \frac{y^2 dy}{\sqrt{(1-y^4)(\rho - y^4)}}}$$

$$\rho \equiv \left(\frac{z_0}{z_*}\right)^4$$



- $z_* \to z_0 \Rightarrow \rho \to 1 \Rightarrow d \to 0$
- There is a maximal value of d $(d_{max} \sim z_0)$
- At high T → disconnected configuration energetically favored

Models thermal screening in a plasma!

$q\bar{q}$ potential in a confining background

Generated by analytic continuation of the AdS black hole

$$ds^2 = \frac{L^2}{z^2}\left[-dt^2 + dx_1^2 + dx_2^2 + f(z)du^2 + \frac{dz^2}{f(z)}\right]$$ → space ends at $z = z_0$

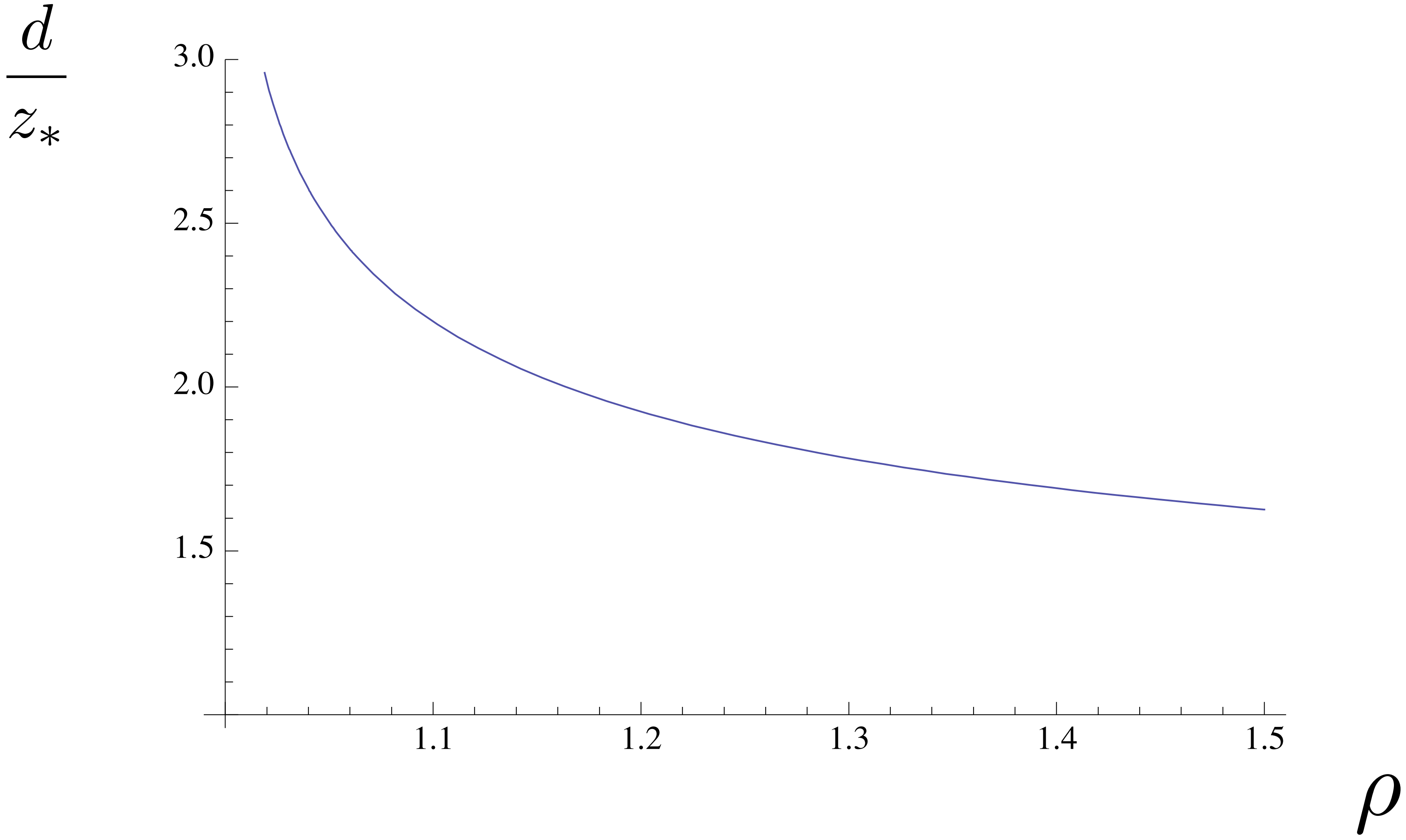
Induced metric in Euclidean signature → $$ds^2 = \frac{L^2}{z^2}\left[dt^2 + \left(1 + \frac{z'^{\,2}}{f}\right)dx^2\right]$$

Action → $$S = \frac{\tau L^2}{2\pi\alpha'}\int \frac{dx}{z^2}\sqrt{1 + \frac{z'^{\,2}}{f(z)}}$$

First integral → $$\frac{z^2}{\sqrt{f(z)}}\sqrt{f(z) + z'^{\,2}} = z_*^2$$ →

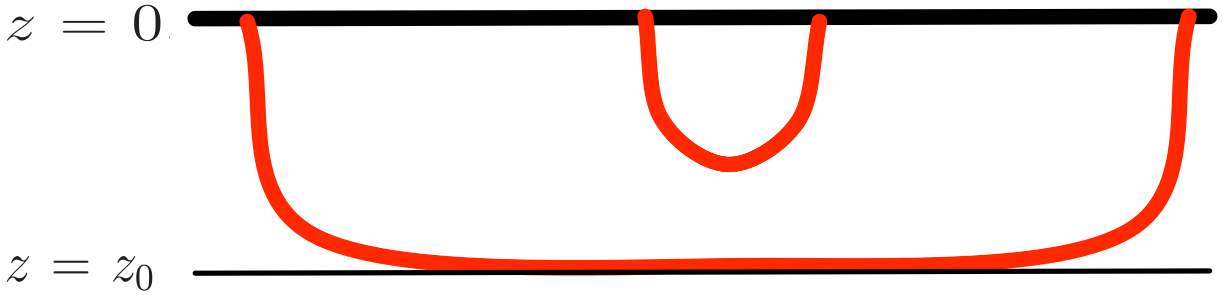
$$\boxed{x = \pm z_*\sqrt{\rho}\int_1^{\frac{z}{z_*}} \frac{y^2 dy}{\sqrt{(1-y^4)(\rho - y^4)}}}$$

$$d = 2z_* \sqrt{\rho} \int_0^1 \frac{y^2 dy}{\sqrt{(1-y^4)(\rho - y^4)}} \qquad \rho \equiv \left(\frac{z_0}{z_*}\right)^4$$



- No maximal distance
- The distance diverges as we approach the end of the space

Qualitative picture →



When $d \to \infty$ the profile is almost rectangular

→ Vertical parts → masses of the static quarks

→ Horizontal part → $q\bar{q}$ potential

$$S_{horizontal} = \frac{\tau L^2}{2\pi\alpha'} \frac{d}{z_0^2}$$ → Area law → Confinement

$$\boxed{V = \sigma_s\, d} \rightarrow \sigma_s = \frac{L^2}{2\pi\alpha'}\frac{1}{z_0^2} \rightarrow \boxed{\sigma_s = \frac{\sqrt{\lambda}}{2\pi z_0^2}}$$

effective string tension

mass gap → $M \sim \dfrac{1}{z_0}$

$z_0 \to$ glueball size

$$\rightarrow \boxed{\sigma_s \sim \sqrt{\lambda}\, M^2}$$