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Quantum Chromodynamics (II)

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Contents:

1. Electron-positron annihilation into hadrons.

2. Deep Inelastic Scattering.

3. QCD in hadronic collisions: factorization.

4. QCD radiation.

5. Jets.

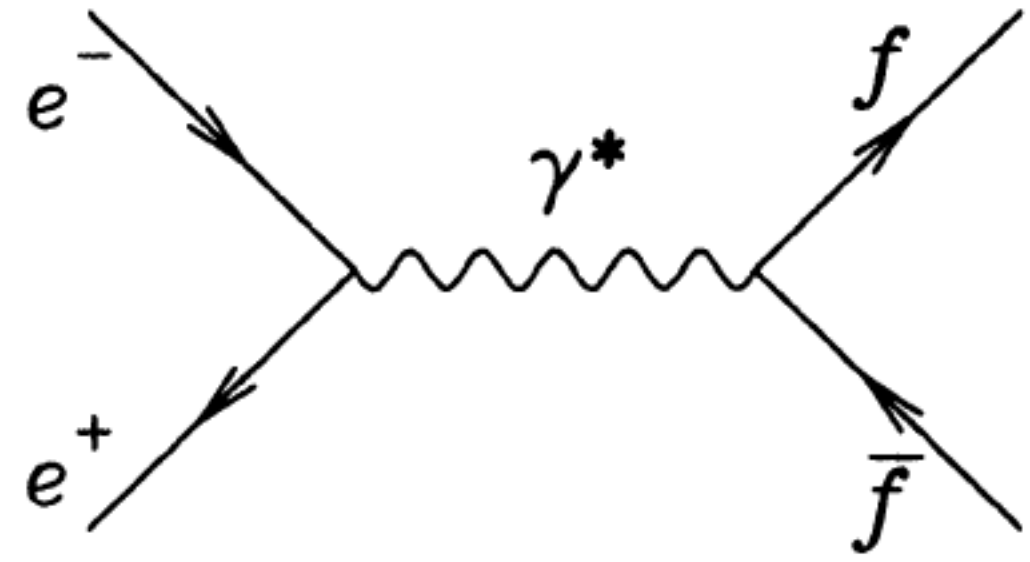
Bibliography:

➜ G. P. Salam, *Elements of QCD for hadron colliders*, CERN Yellow Report CERN-2010-002, arXiv:1011.5131.

➜ R. K. Ellis et al., *QCD and collider physics*, Cambridge University Press 1996.

$\sigma(e^+e^- \to \text{hadrons})$: tree level

➜ Take the Feynman diagram neglecting EW contributions (i.e. Z exchange), and consider the cms (as done in LEP), $s=(p_{e^+}+p_{e^-})^2=E_{cm}^2$. Hadronization happens much later ($1/\Lambda_{QCD}$) than qqbar production ($1/E_{cm}$).



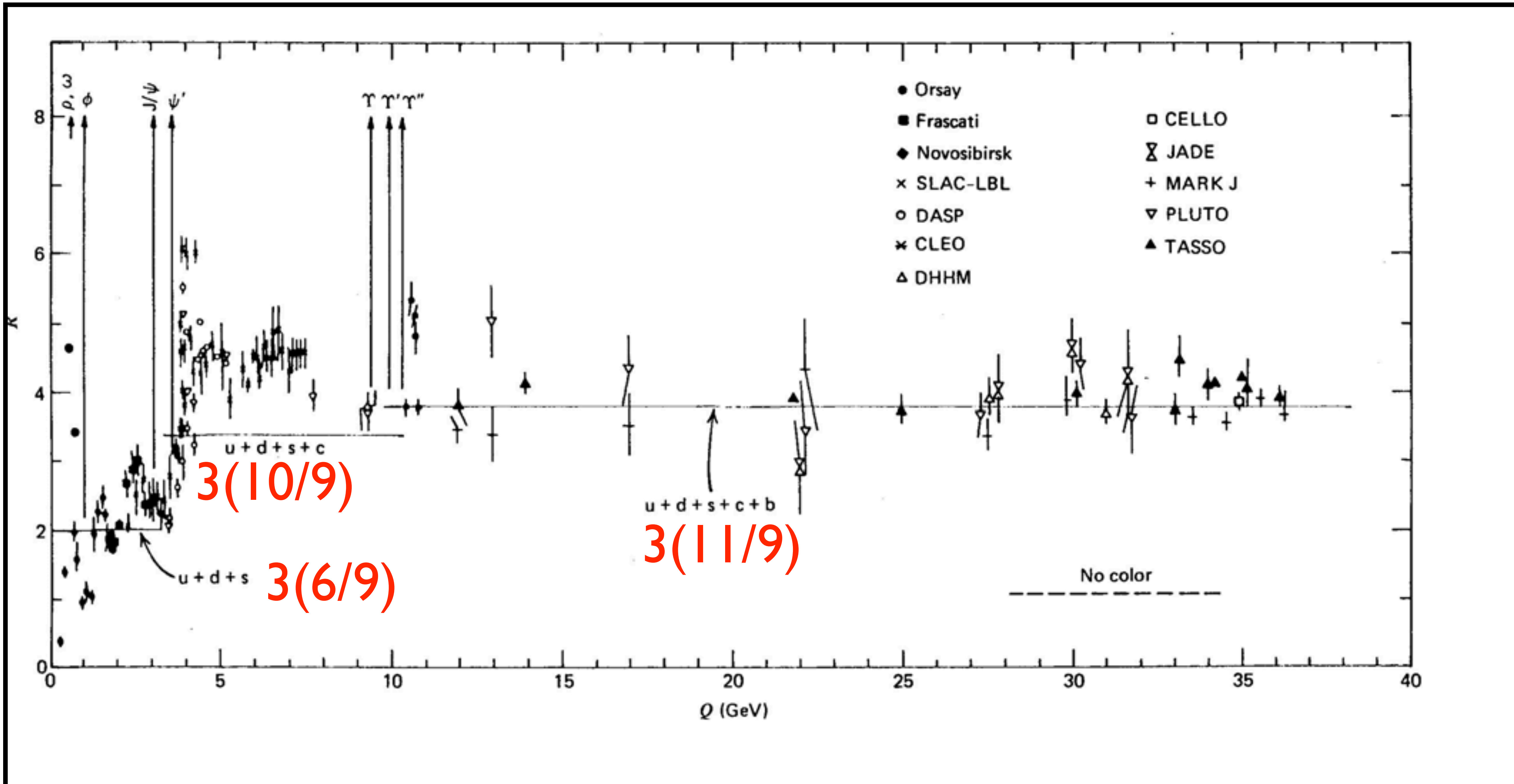
$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2 Q_f^2}{2s}(1+\cos^2\theta) \Rightarrow \sigma_0 = \frac{4\pi\alpha^2}{3s}Q_f^2$$

➜ The total cross section at tree level (i.e. $\mathcal{O}(\alpha_s^0)$) gives:

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\sum_q \sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q Q_q^2$$

Evidence for 3 colours!!!

σ(e^+e^-→hadrons): tree level

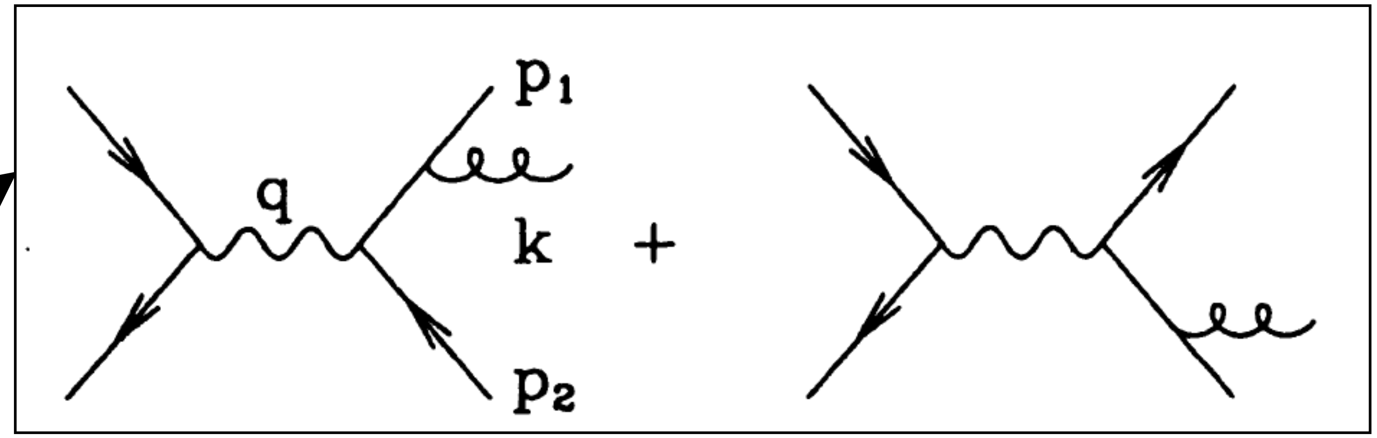


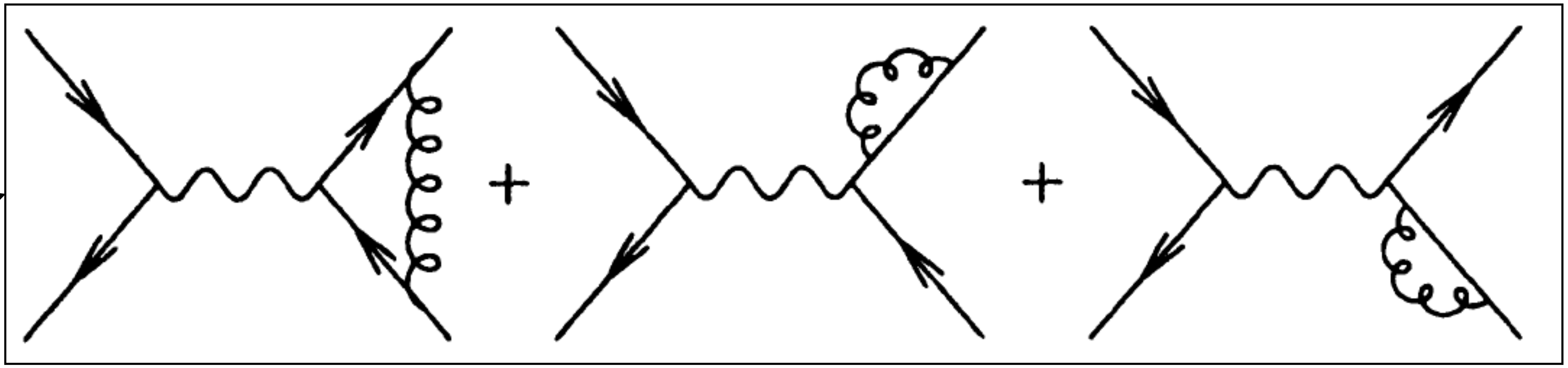
$$R = \frac{\cdots}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\sum_q \cdots}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q Q_q^2$$

Evidence for 3 colours!!!

σ(e⁺e⁻→hadrons): HO corrections

➜ Now consider the $\mathcal{O}(\alpha_s)$ corrections: they can be real and virtual.





➜ The virtual corrections contain UV divergencies which are dealt with by the usual renormalization procedure: fields and running of the coupling constant. But they also contain other divergencies: IR divergencies for massless quarks.

$$I \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2+i\varepsilon)((p_1+k)^2-m^2+i\varepsilon)((p_2-k)^2-m^2+i\varepsilon)} \sim \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(2p_1\cdot k)(-2p_2\cdot k)}$$

logarithmic divergence

$\sigma(e^+e^- \to$ hadrons): IRC safety

➜ The real corrections (gluon emission probability) contain two types of divergencies: IR or soft ($\omega \to 0$); collinear or mass (see later) ($\theta \to 0$).

colour 'charge'

$$dP_i \propto \frac{\alpha_s C_R}{\pi} \frac{dx_i}{x_i} \frac{dk_{T,i}^2}{k_{T,i}^2} \propto \frac{d\omega_i}{\omega_i} \frac{d\theta_i}{\theta_i}, \quad \omega_i = x_i E, \quad \theta_i \simeq k_{T,i}/\omega_i$$

real

$$\sigma^{q\bar{q}g}(\epsilon) = \sigma_0\, 3 \sum_q Q_q^2 \frac{\alpha_S}{2\pi} H(\epsilon) \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + O(\epsilon)\right] C_F$$

$$H(\epsilon) = \frac{3(1-\epsilon)^2}{(3-2\epsilon)\Gamma(2-2\epsilon)} = 1 + O(\epsilon)\text{, d=4-2}\varepsilon$$

virtual

$$\sigma^{q\bar{q}(g)}(\epsilon) = \sigma_0\, 3 \sum_q Q_q^2 \frac{C_F \alpha_S}{2\pi} H(\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + O(\epsilon)\right]$$

$$\Longrightarrow R = 3 \sum_q Q_q^2 \left\{1 + \frac{\alpha_S}{\pi} + O(\alpha_S^2)\right\}$$

➜ Soft divergencies cancel between virtual and real contributions.

➜ Collinear divergencies cancel for sufficiently inclusive quantities (sum over initial and final states, we cannot distinguish a q from a q +collinear g): KLN theorems.

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colour 'charge'

$$dP_i \propto \frac{\alpha_s C_R}{\pi} \frac{dx_i}{x_i} \frac{dk^2_{T,i}}{k^2_{T,i}}$$

This observable is IR and collinear (IRC) safe, can be computed reliably in perturbation theory.

real

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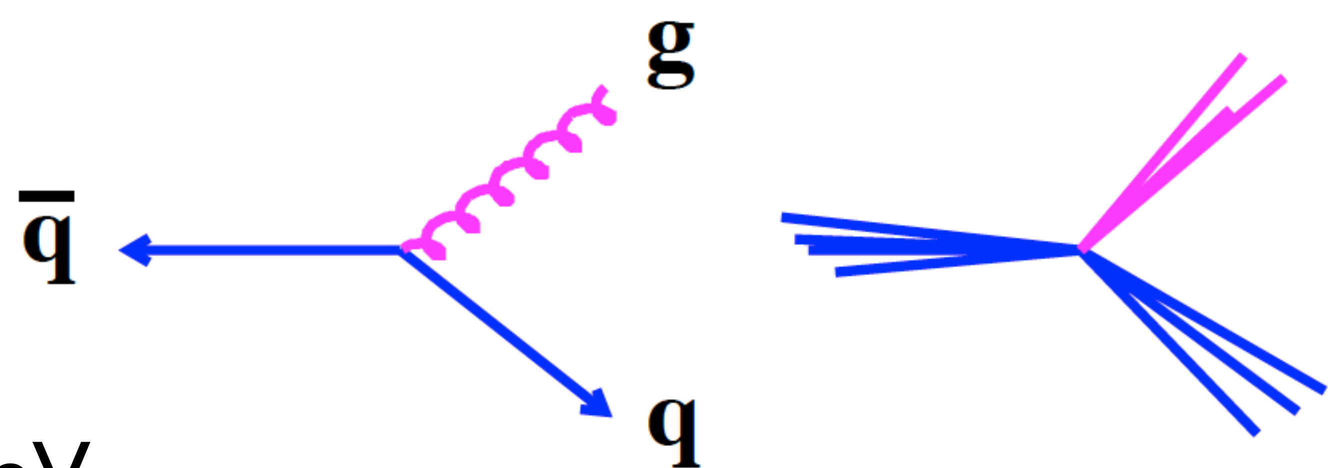
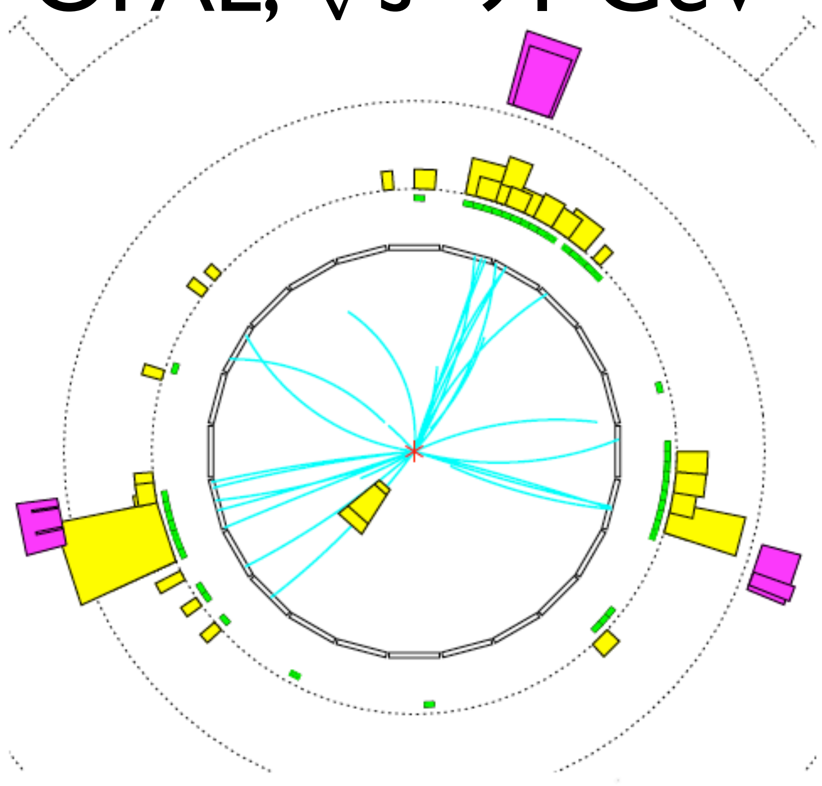
σ(e⁺e⁻→hadrons): spin-1 gluon

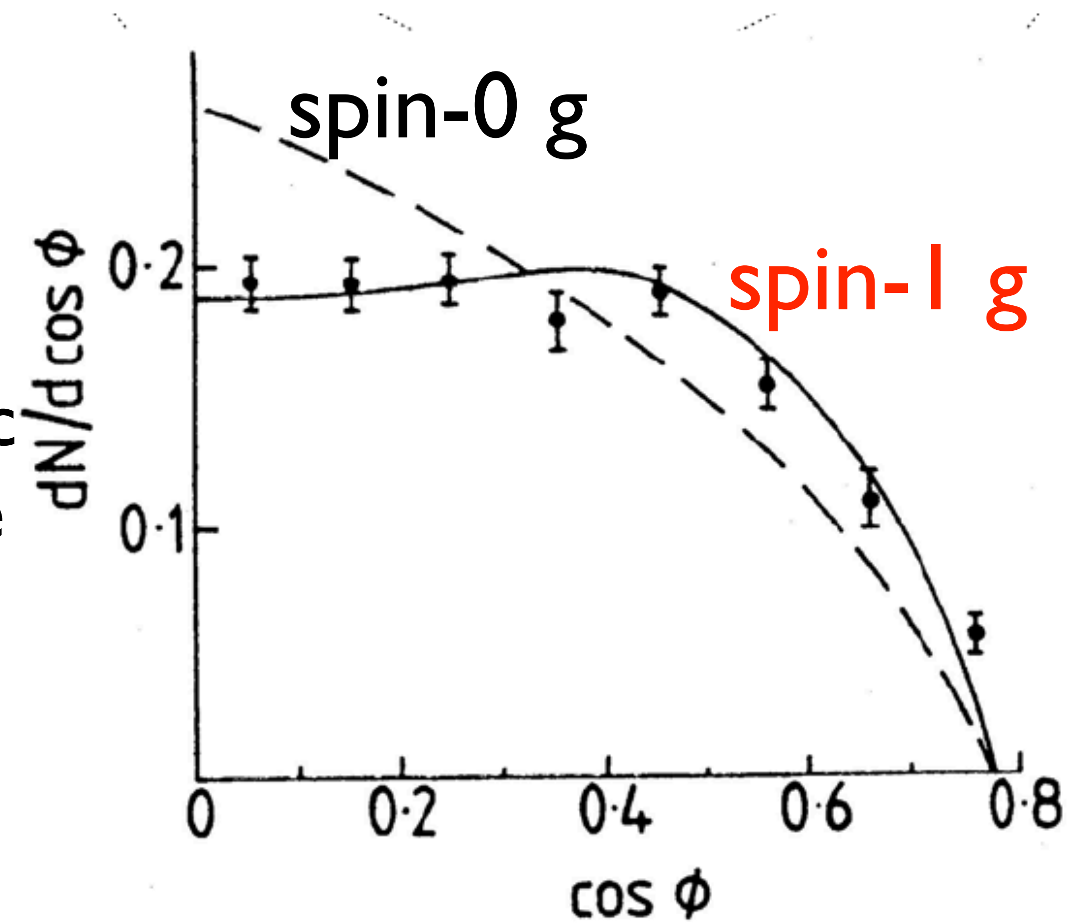
➜ $e^+e^- \to$ qqbarg with the gluon emission at large angle produces three collimated showers of hadrons (i.e. three jets, see later), evidence of the existence of the gluons and of its spin through its angular distribution!



OPAL, $\sqrt{s}$=91 GeV

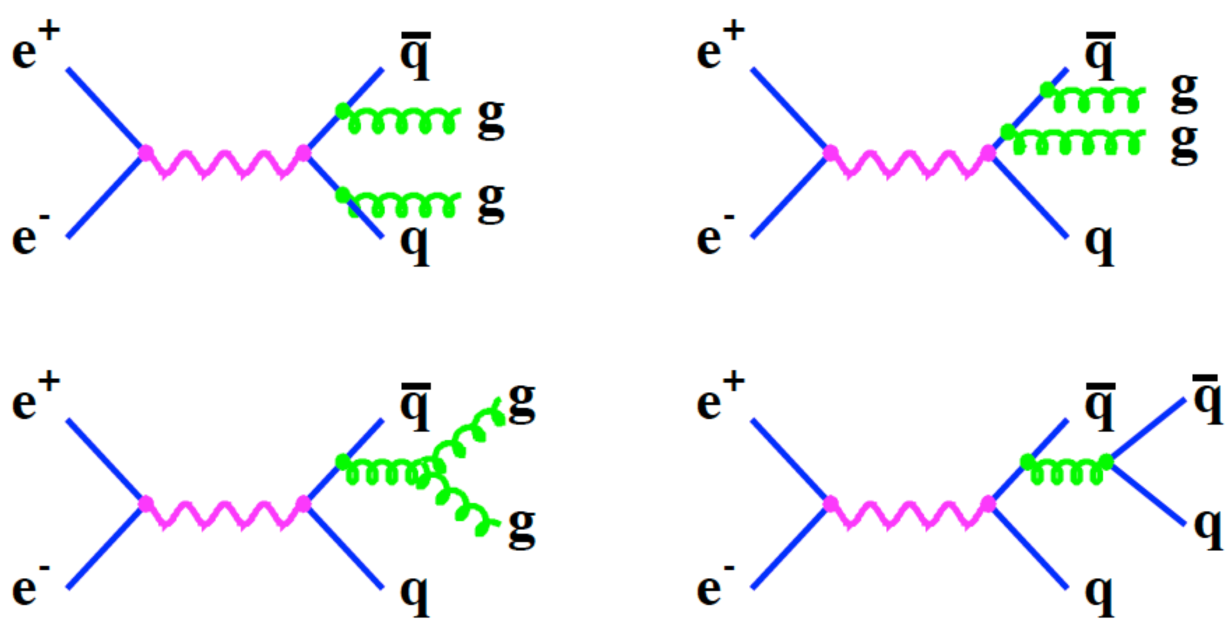


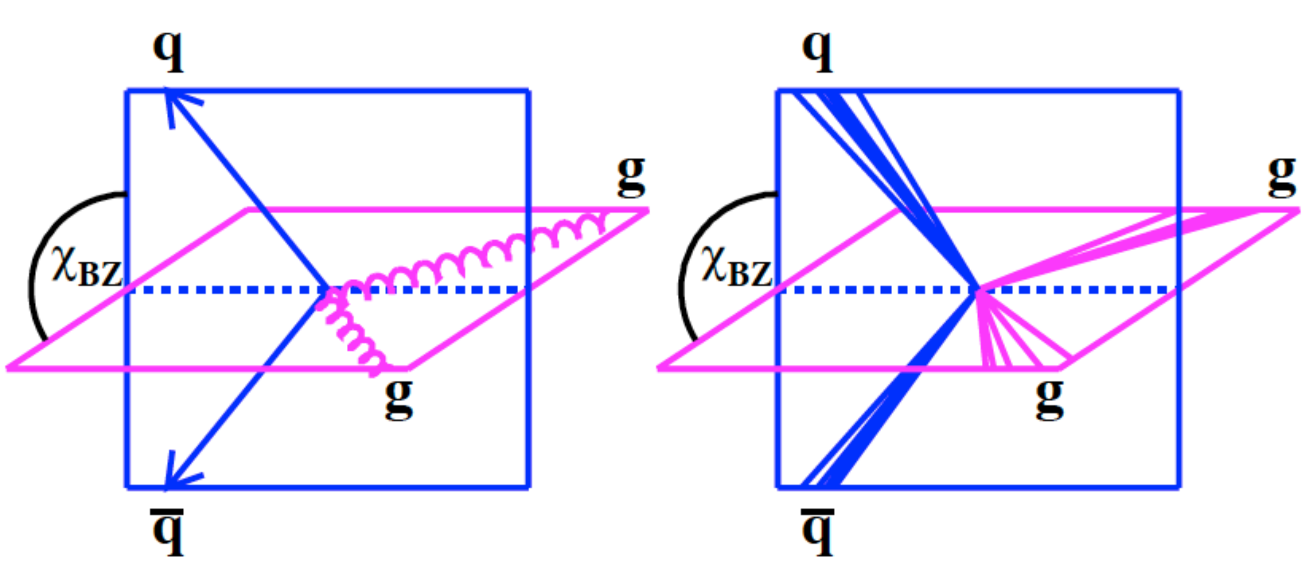
φ: angle between the most energetic (q) jet and the other two.

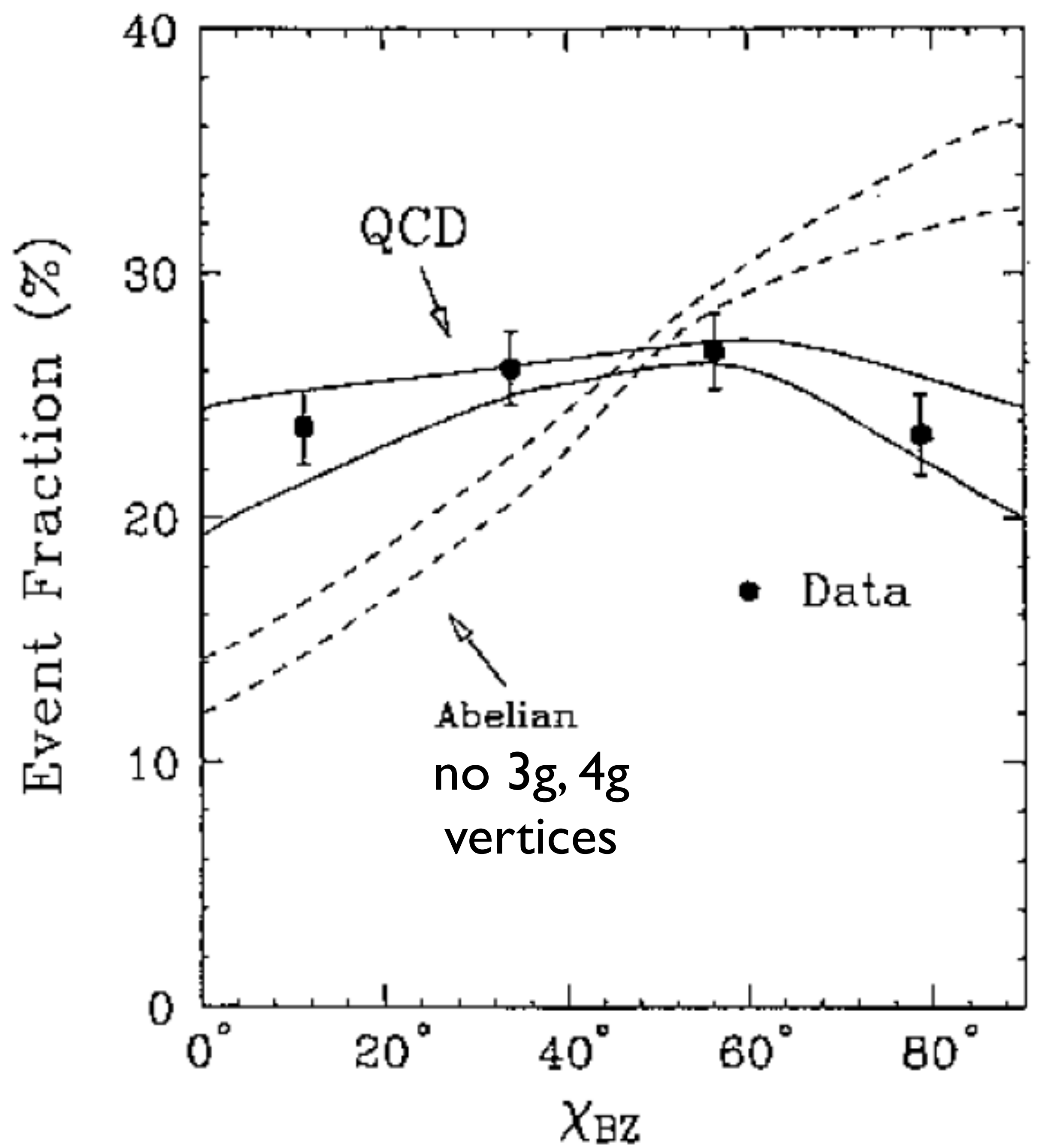


σ(e^+e^-→hadrons): ggg vertex

➜ ggg vertex (3 spin-1 particles) produces a different angular structure than qqbarg vertex (2 spin-1/2, 1 spin-1 particle).







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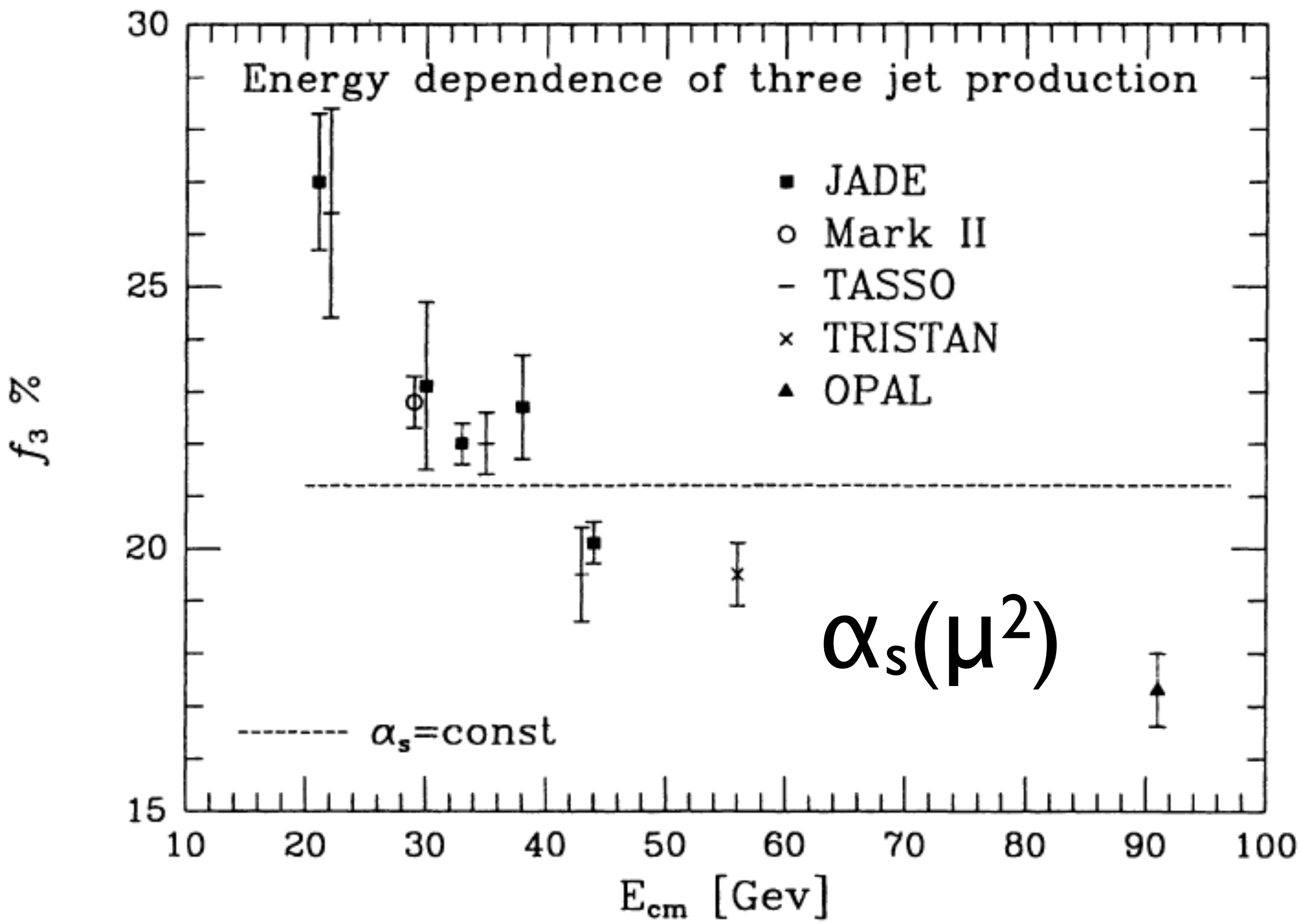

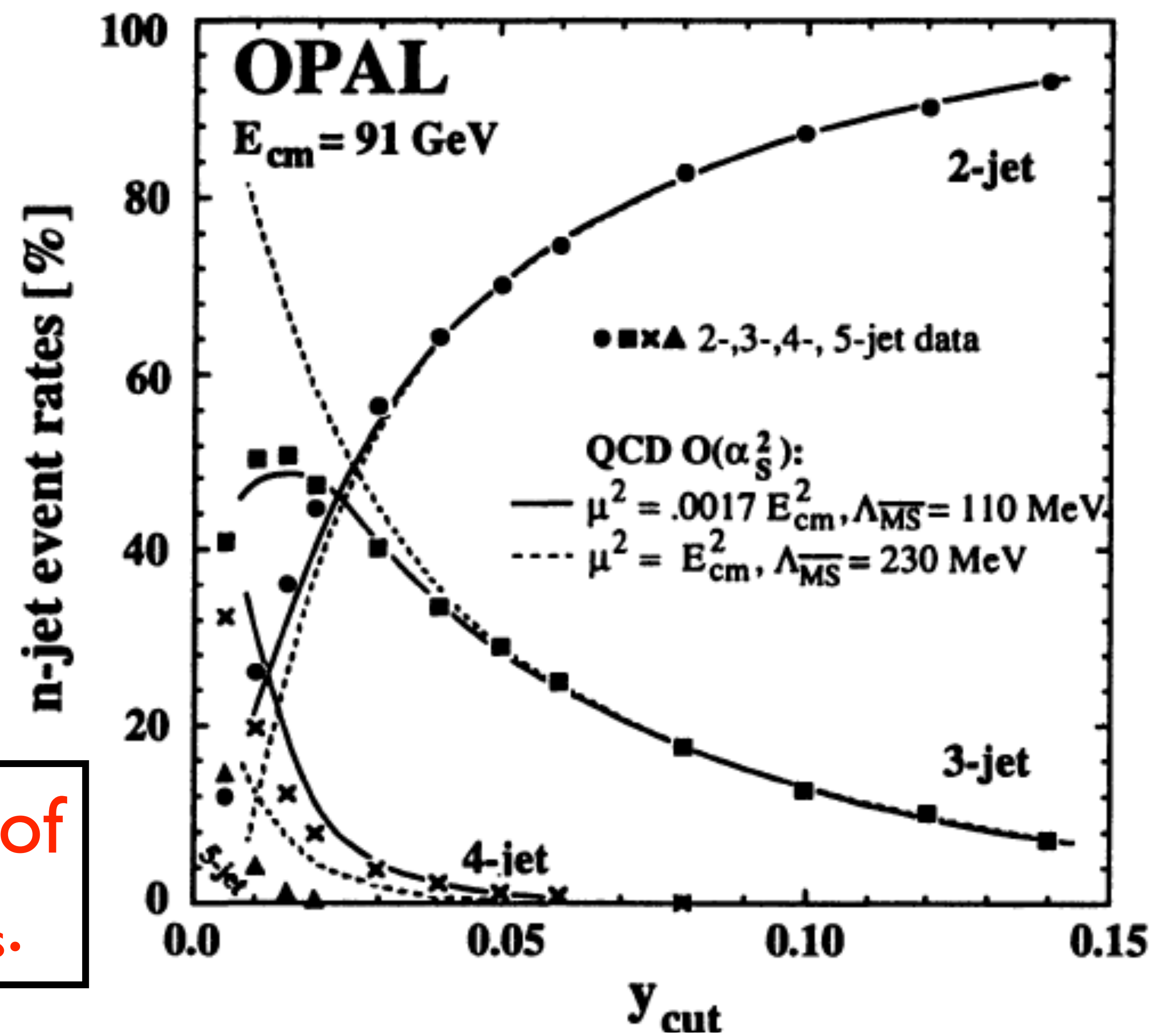

Sensitive check to the structure of QCD and a measurement of α_s.

$\bar{q}$ $\bar{q}$

cut of invariant mass of 2-jet system

σ(e⁺e⁻→hadrons): ggg vertex

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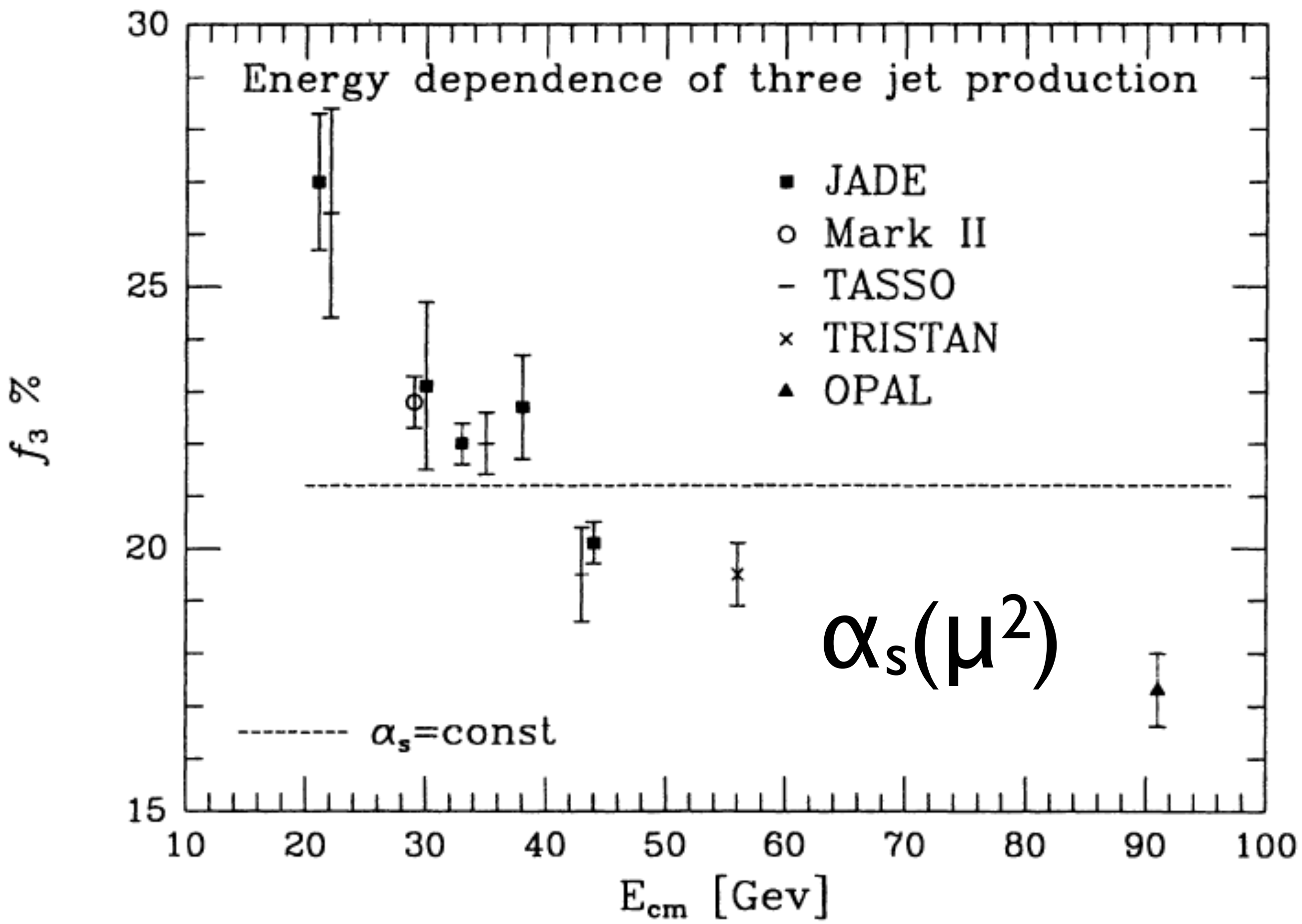
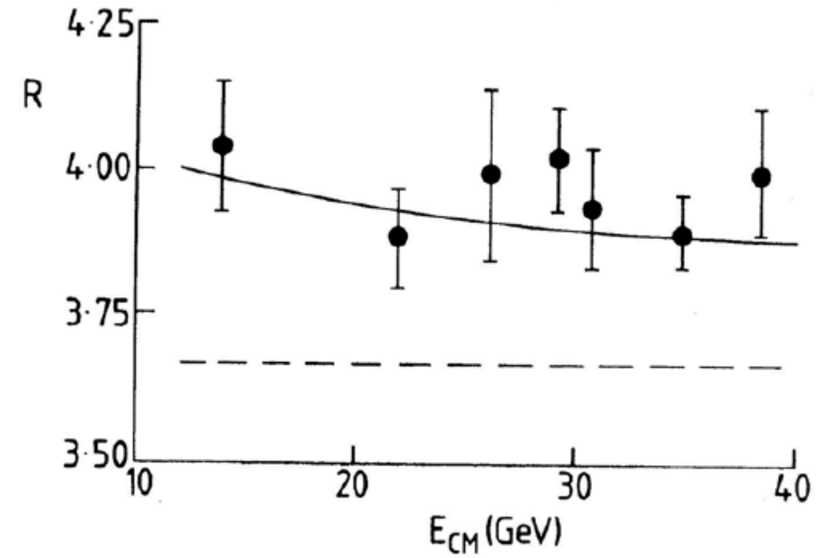


$$R_\mu = 3\sum Q_q^2\left(1+\frac{\alpha_S}{\pi}\right)$$

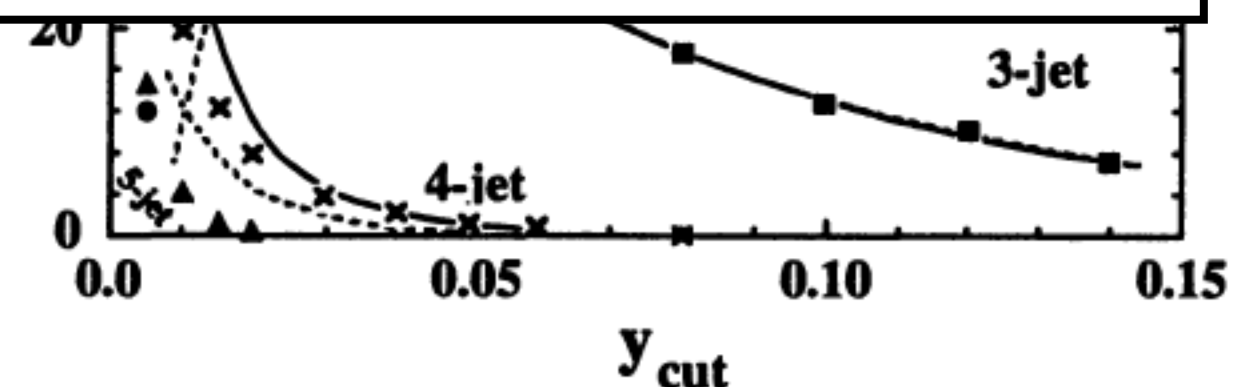


$$\sum_q Q_q^2 = 3\tfrac{2}{3}$$

Therefore $(1+\frac{\alpha_S}{\pi}) \approx \frac{3.9}{3.66}$

giving $\alpha_S(q^2 = 25^2) \approx 0.20$

Sensitive check to the structure of QCD and a measurement of α_s.



cut of invariant mass of 2-jet system

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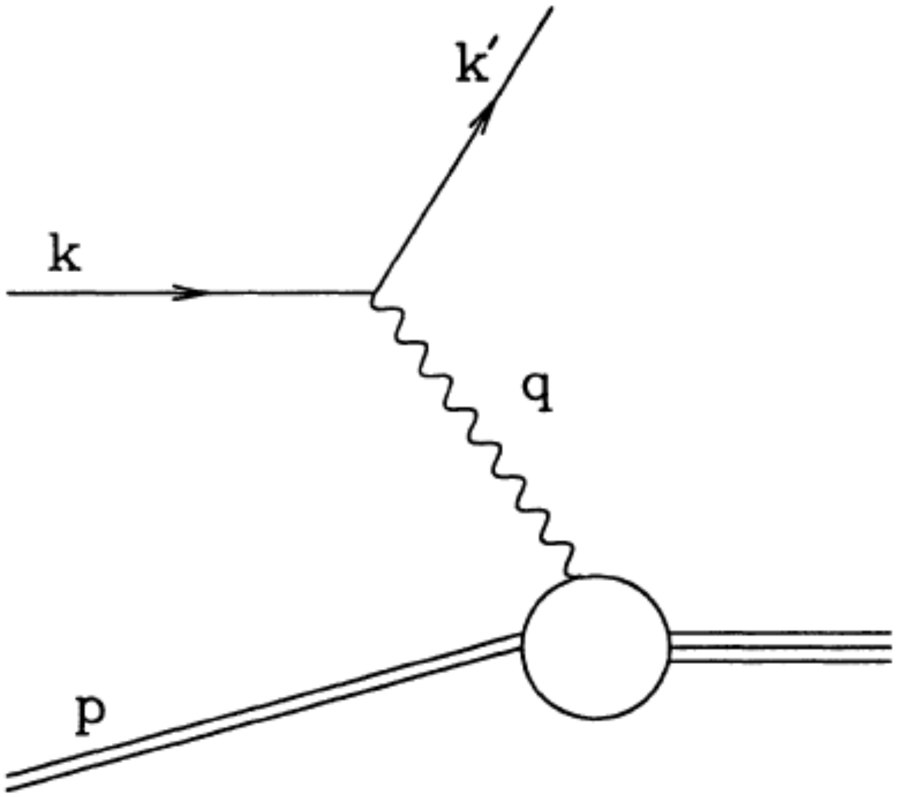
DIS: basics

➜ Consider the process of lepton (e, μ, ν) scattering on a proton (or neutron or nucleus): equivalent to the Rutherford experiment.



$$
\begin{aligned}
Q^2 &= -q^2 \\
M^2 &= p^2 \\
\nu &= p \cdot q = M(E' - E) \\
x &= \frac{Q^2}{2\nu} = \frac{Q^2}{2M(E - E')} \\
y &= \frac{q \cdot p}{k \cdot p} = 1 - E'/E \,,
\end{aligned}
$$

proton rest frame

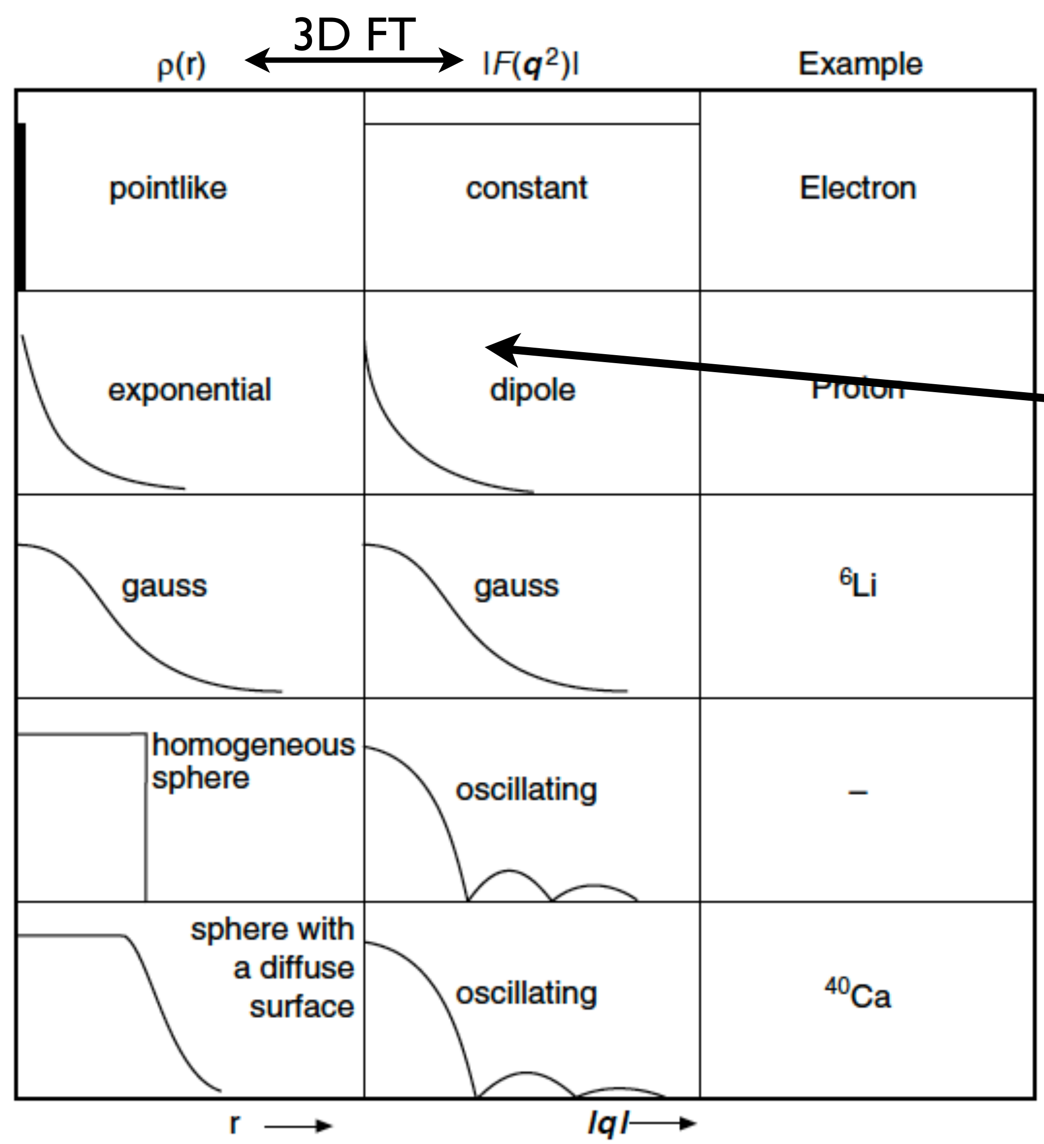
➜ For charged lepton scattering and neglecting Z exchange,

$$
\frac{d^2\sigma^{em}}{dx\,dy} = \frac{8\pi\alpha^2 ME}{Q^4}\Big[\Big(\frac{1+(1-y)^2}{2}\Big)2xF_1^{em} + (1-y)(F_2^{em} - 2xF_1^{em}) - (M/2E)xyF_2^{em}\Big]
$$

F_1, F_2: structure functions of the hadron

DIS: proton substructure

➜ Let us compare elastic scattering (x=1) on a pointlike s=1/2 particle with that on a proton and the inelastic one (for x~$\mathcal{O}(1)$):



$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{point}\\\text{spin }1/2}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[1 + 2\tau \tan^2\frac{\theta}{2}\right]$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\theta}{2}\right]$$

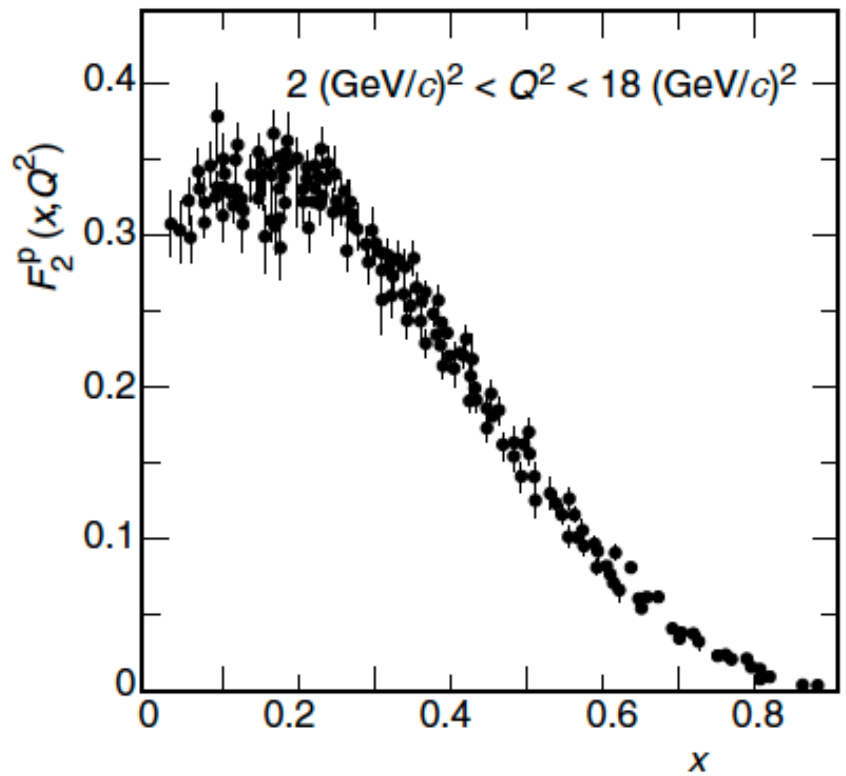
$$\frac{d^2\sigma}{d\Omega\, dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2\frac{\theta}{2}\right]$$

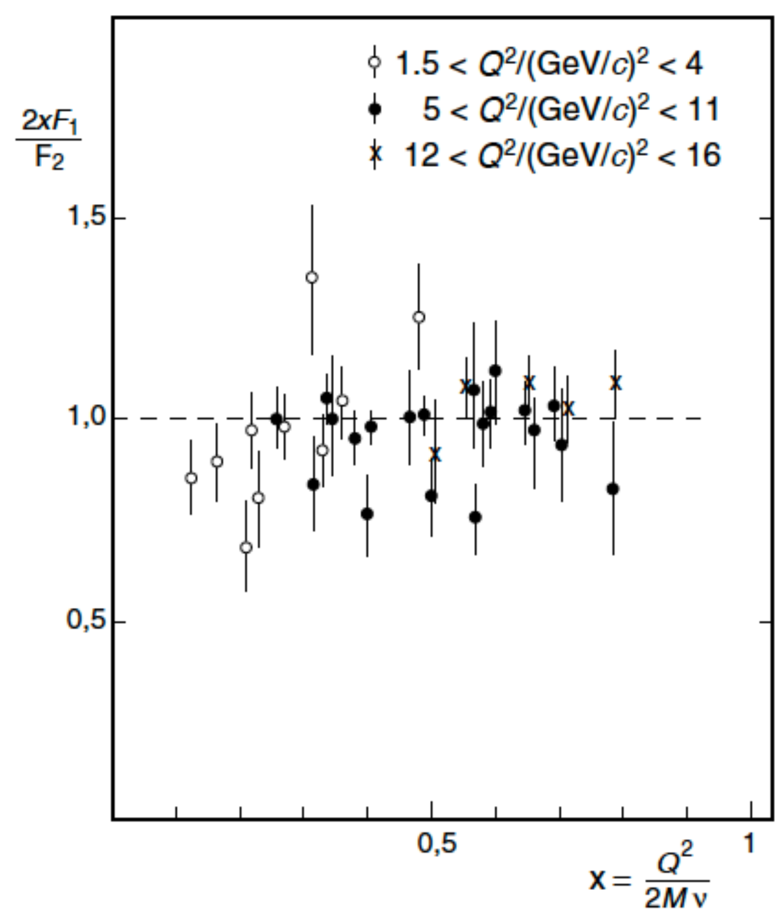
$$F_1(x,Q^2) = Mc^2\, W_1(Q^2,\nu)$$
$$F_2(x,Q^2) = \nu\, W_2(Q^2,\nu)\,.$$

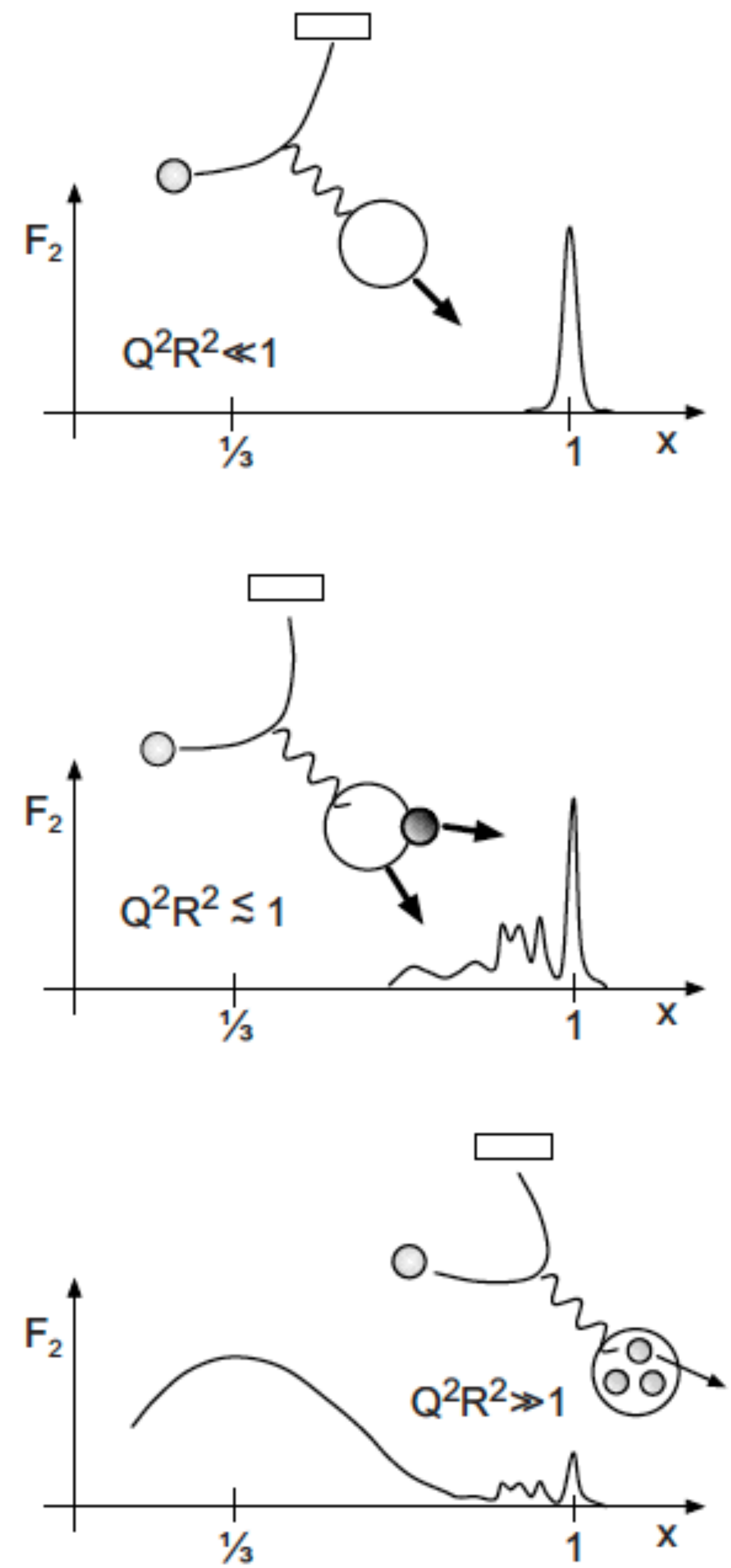
➜ For fixed x, $F_{1,2}$ roughly independent of Q (note $1/Q^4$ behaviour of proton form factors): Bjorken scaling, pointlike scatterers.

➜ $2xF_1=F_2$: Callan-Gross relation, spin-1/2 scatterers.

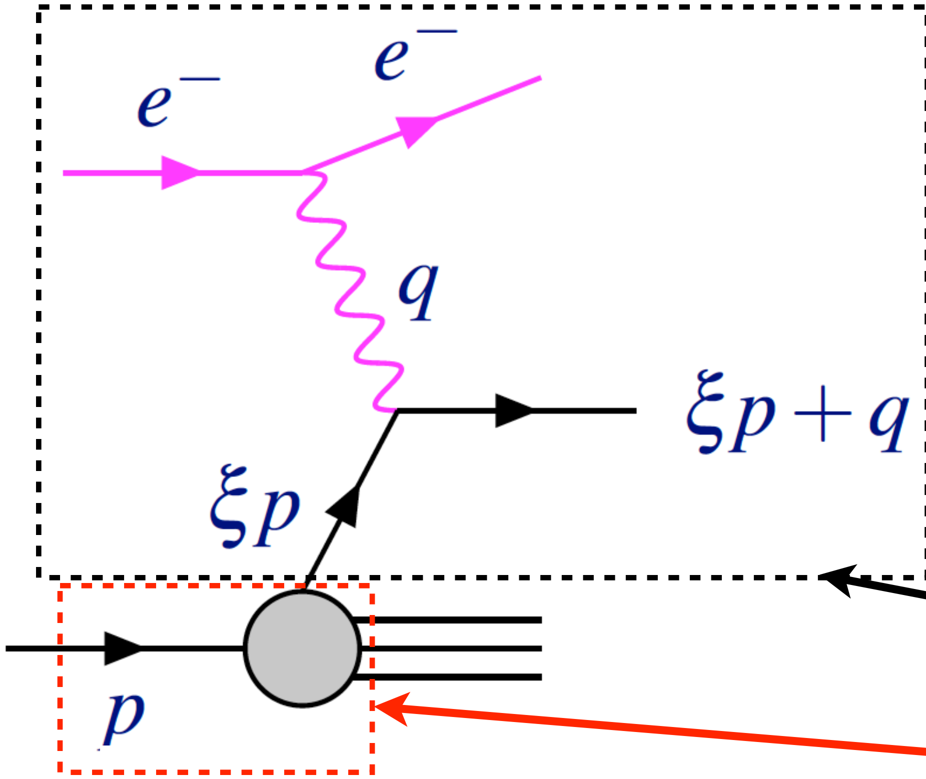
DIS: proton substructure







DIS: parton model



➜ For very large p (IMF), the hadron can be considered an incoherent superposition of quanta (partons) during the interaction ($Q \gg \Lambda_{QCD}$): parton model (Feynman, Bjorken, Gribov).

$$\sigma(e^-(k)p(p) \to e^-(k')X) = \int_0^1 d\xi \sum_f f_{q_f}(\xi)\,\sigma(e^-(k)q_f(\xi p) \to e^-(k')\,q_f(\xi p+q))$$

$$\begin{aligned} F_2(x) = 2xF_1(x) &= \sum_{q,\bar{q}} \int_0^1 d\xi\; q(\xi)\; x e_q^2 \delta(x-\xi) \\ &= \sum_{q,\bar{q}} e_q^2\, x q(x)\,. \end{aligned}$$

➜ Relation between PDFs for valence and sea quarks and gluons:

$$F_2^{eN}(x) = \frac{5}{18} F_2^{\nu N}(x)$$

electric charges

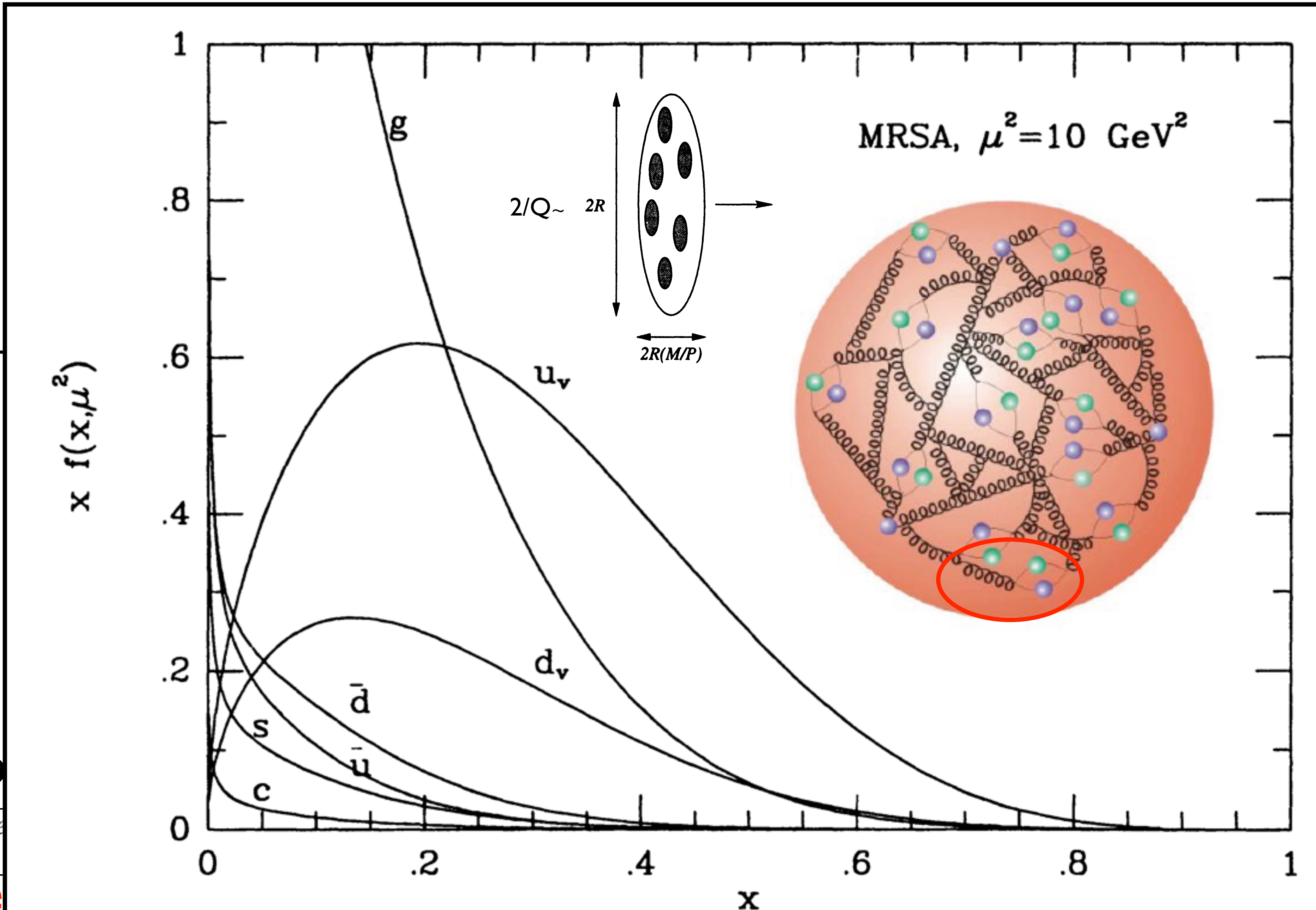
$$F_2^{ep} - F_2^{en} = \frac{1}{3} x(u_v(x) - d_v(x))$$

valence

$$\int_0^1 F_2^{\nu N}(x)dx = \int_0^1 x\left(q(x) + \bar{q}(x)\right)dx = 0.44$$

gluons

DIS: parton model

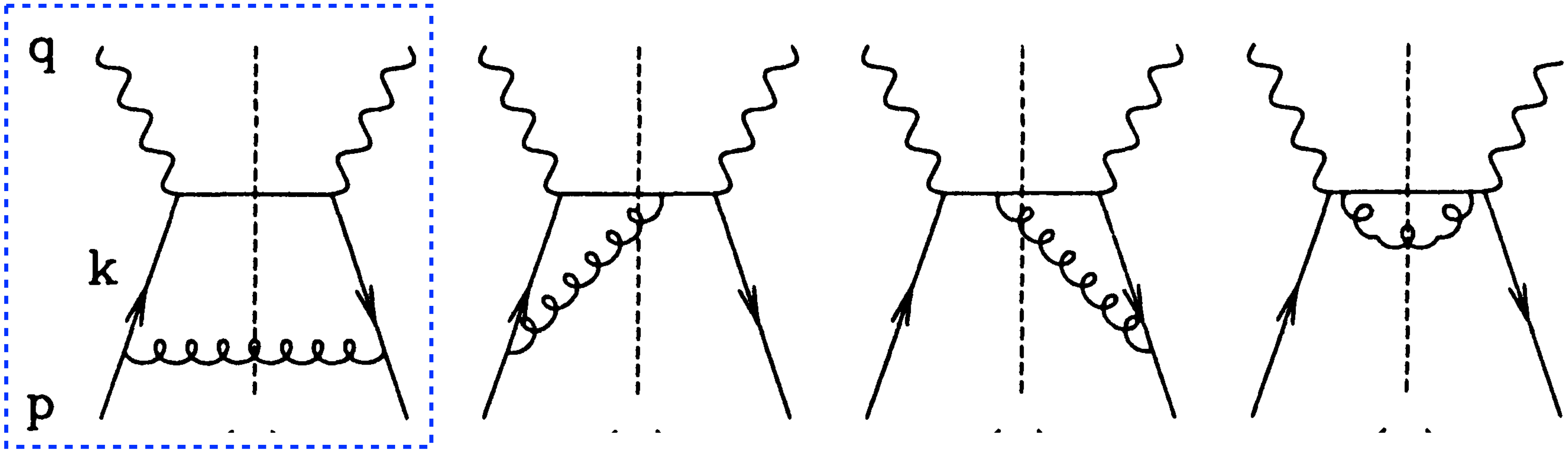


gluons

DIS: QCD corrections

➜ The parton model receives corrections from the fact that partons radiate: PDFs evolve with scale Q, DGLAP evolution equations.

only diagram that gives (logarithmic) divergencies (in LC gauge)



$$Q^2 \partial_{Q^2} \begin{pmatrix} q_i(x,Q^2) \\ \bar{q}_i(x,Q^2) \\ g(x,Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{d\xi}{\xi} \begin{pmatrix} P_{q_i q_j}\left(\frac{x}{\xi}\right) & 0 & P_{q_i g}\left(\frac{x}{\xi}\right) \\ 0 & P_{q_i q_j}\left(\frac{x}{\xi}\right) & P_{q_i g}\left(\frac{x}{\xi}\right) \\ P_{gq}\left(\frac{x}{\xi}\right) & P_{gq}\left(\frac{x}{\xi}\right) & P_{gg}\left(\frac{x}{\xi}\right) \end{pmatrix} \begin{pmatrix} q_j(x,Q^2) \\ \bar{q}_j(x,Q^2) \\ g(x,Q^2) \end{pmatrix}$$

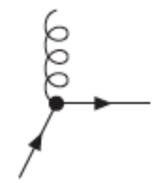
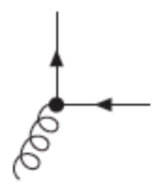
DGLAP@LO

➜ PDFs are unknown, non-perturbative quantities but we know its perturbative evolution (at leading logarithmic accuracy). They have to be extracted from data.

$$q(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P \middle| \bar{\psi}(0) \not{n} \psi(\lambda n) \middle| P \right\rangle$$

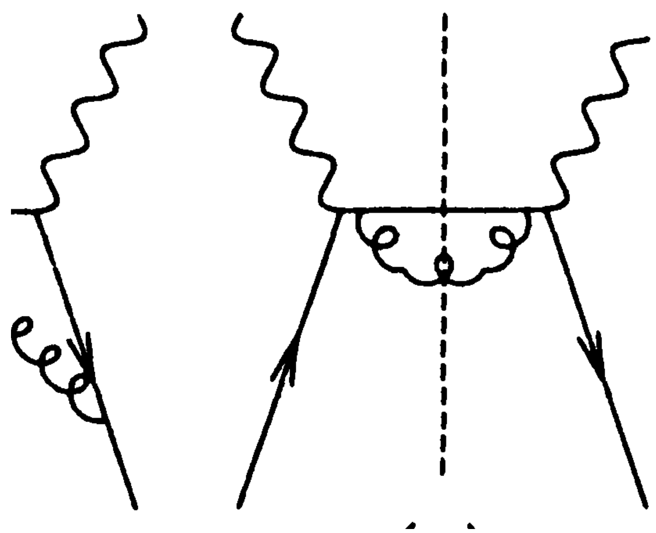
DIS: QCD corrections

➜ The parton m act that partons radiate: PDFs ev equations.

Diagram	Splitting
	$P_{qq}=C_F\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)\right]$
	$P_{gq}=C_F\left[\frac{1+(1-x)^2}{x}\right]$
	$P_{qg}=T_R\left[x^2+(1-x)^2\right]$
	$P_{gg}=2C_A\left[\frac{x}{(1-x)_+}+(1-x)\left(x+\frac{1}{x}\right)\right]+\frac{11C_A-4n_fT_R}{6}\delta(1-x)$

$T_R=1/2$
$C_F=(N^2-1)/(2N)=4/3$
$C_A=N=3$

only diagram that gives (logarithmic) divergencies (in LC gauge)



$$Q^2\partial_{Q^2}\begin{pmatrix} q_i(x,Q^2)\\ \bar{q}_i(x,Q^2)\\ g(x,Q^2)\end{pmatrix}=\frac{\alpha_s(Q^2)}{2\pi}\int_x^1\frac{d\xi}{\xi}\begin{pmatrix} P_{q_iq_j}\left(\frac{x}{\xi}\right) & 0 & P_{q_ig}\left(\frac{x}{\xi}\right)\\ 0 & P_{q_iq_j}\left(\frac{x}{\xi}\right) & P_{q_ig}\left(\frac{x}{\xi}\right)\\ P_{gq}\left(\frac{x}{\xi}\right) & P_{gq}\left(\frac{x}{\xi}\right) & P_{gg}\left(\frac{x}{\xi}\right)\end{pmatrix}\begin{pmatrix} q_j(x,Q^2)\\ \bar{q}_j(x,Q^2)\\ g(x,Q^2)\end{pmatrix}$$

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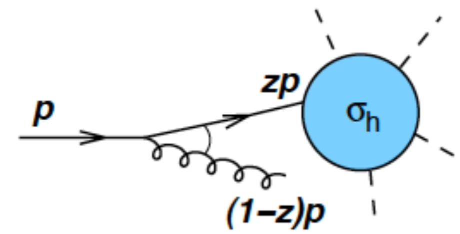
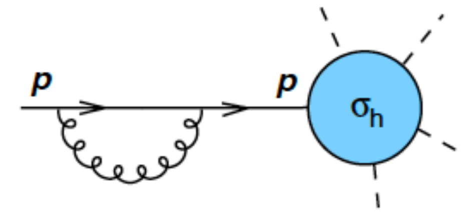
$$q(x)=\int\frac{d\lambda}{2\pi}e^{i\lambda x}\left\langle P\left|\bar{\psi}(0)\not{n}\psi(\lambda n)\right|P\right\rangle$$

DIS: virtual plus real

➜ When we consider radiation from initial state (before a hard scattering σ_h), both real and virtual correction appear:



$$\sigma_{g+h}(p) \simeq \sigma_h(zp)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$

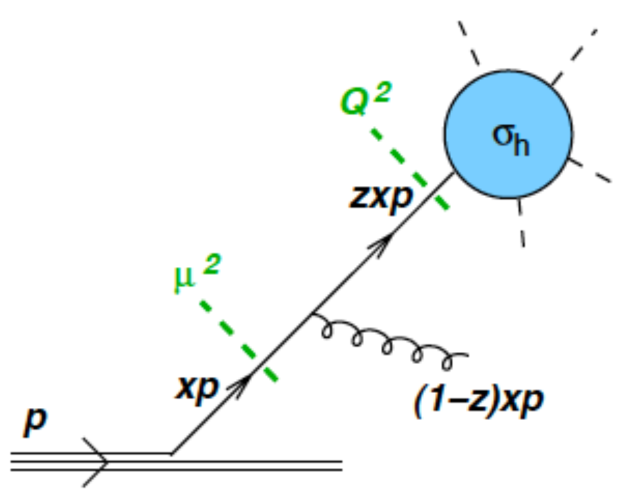


$$\sigma_{g+h}(p) \simeq -\sigma_h(p)\frac{\alpha_s C_F}{\pi}\frac{dz}{1-z}\frac{dk_t^2}{k_t^2}$$

➜ They combine into a IR finite but collinerly divergent cross section:

$$\sigma_{g+h} + \sigma_{V+h} \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_0^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{infinite}} \underbrace{\int_0^1 \frac{dz}{1-z}[\sigma_h(zp) - \sigma_h(p)]}_{\text{finite}}$$

➜ The collinear divergence is absorbed in a redefinition of the PDFs putting a cut-off: the independence of its choice leads to DGLAP.

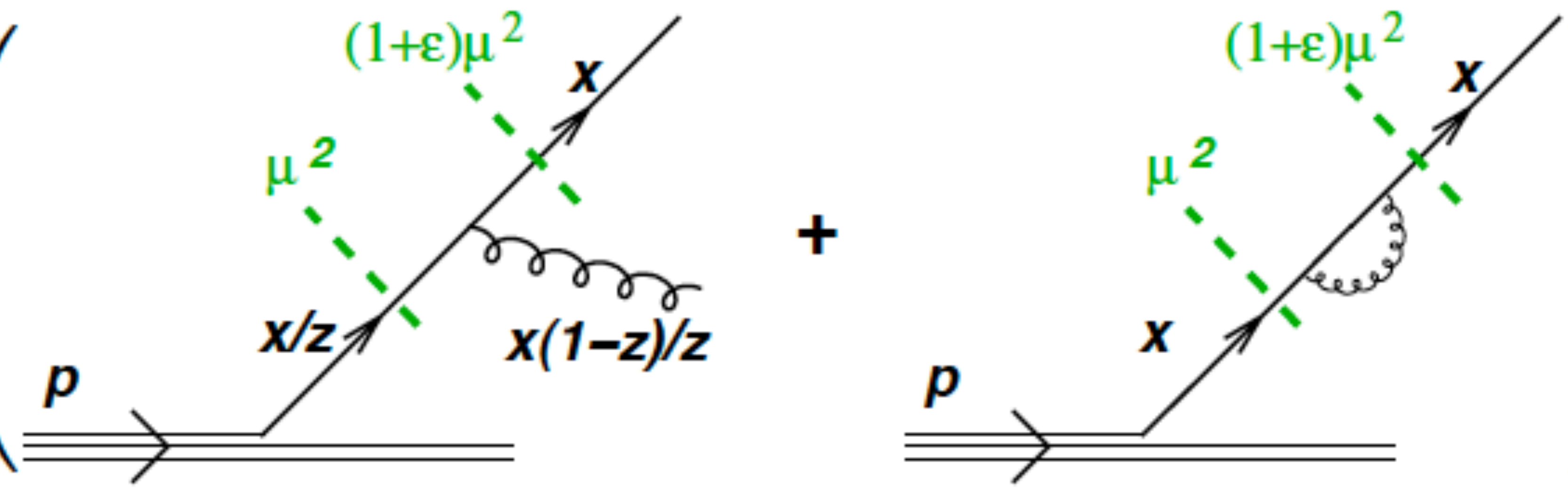


$$\sigma_0 = \int dx\ \sigma_h(xp)\ q(x, \mu_F^2)\,,$$

$$\sigma_1 \simeq \frac{\alpha_s C_F}{\pi} \underbrace{\int_{\mu_F^2}^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{finite (large?)}} \underbrace{\int \frac{dx\, dz}{1-z}\, [\sigma_h(zxp) - \sigma_h(xp)]\ q(x, \mu_F^2)}_{\text{finite}}$$

DIS: virtual plus real

$$\frac{dq(x,\mu_{\mathrm{F}}^2)}{d\ln\mu_{\mathrm{F}}^2} = \frac{1}{\epsilon}\left(\; \ldots \; \right)$$



$$= \frac{\alpha_{\mathrm{s}}}{2\pi}\int_x^1 dz\, p_{qq}(z)\,\frac{q(x/z,\mu_{\mathrm{F}}^2)}{z} - \frac{\alpha_{\mathrm{s}}}{2\pi}\int_0^1 dz\, p_{qq}(z)\, q(x,\mu_{\mathrm{F}}^2)$$

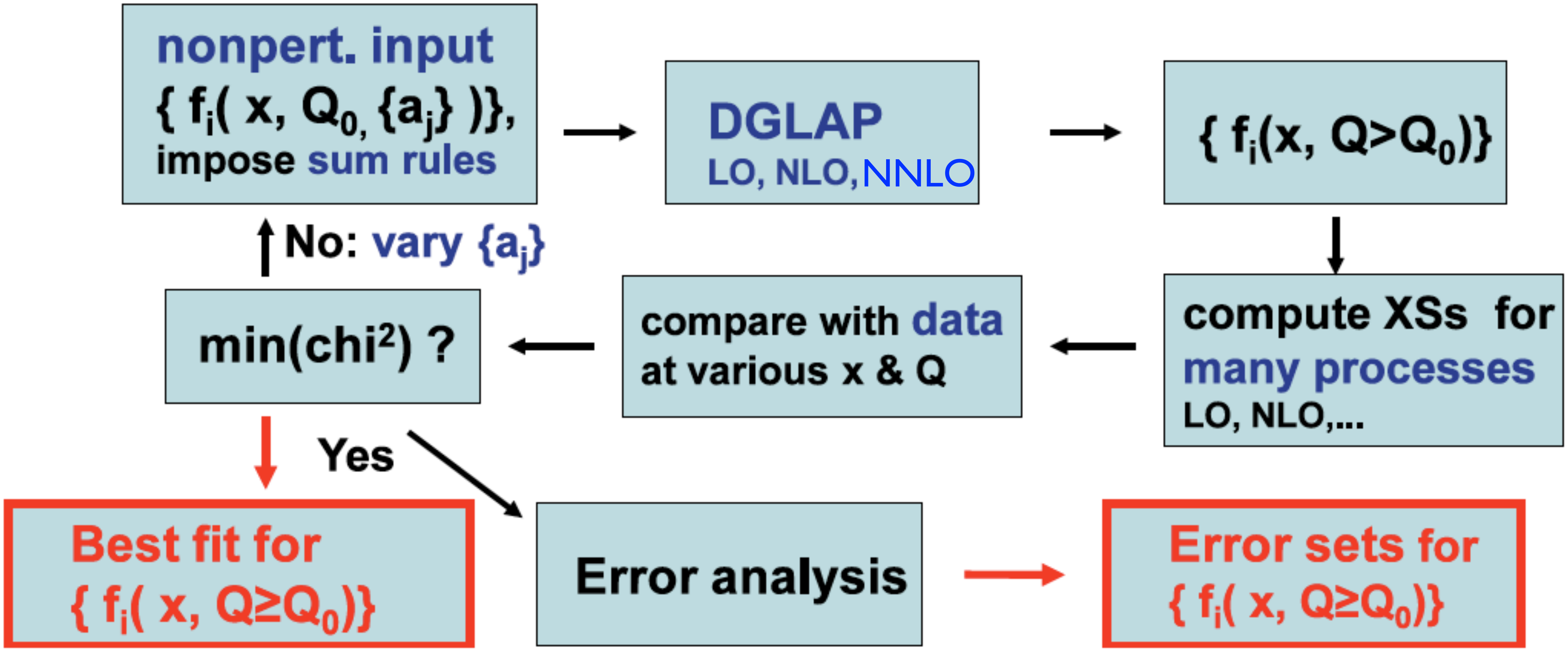
$$= \underbrace{\frac{\alpha_{\mathrm{s}}}{2\pi}\int_x^1 dz\, P_{qq}(z)\,\frac{q(x/z,\mu_{\mathrm{F}}^2)}{z}}_{P_{qq}\otimes q}, \qquad P_{qq} = C_F\left(\frac{1+z^2}{1-z}\right)_+$$

$$\int_x^1 dz\,[g(z)]_+\, f(z) = \int_x^1 dz\, g(z)\, f(z) - \int_0^1 dz\, g(z)\, f(1)$$
$$= \int_x^1 dz\, g(z)\,(f(z)-f(1)) - \int_0^x dz\, g(z)\, f(1)$$

DIS: DGLAP global analysis

➜ Fits to as many data as possible: DIS charged lepton and neutrino data, Drell-Yan, jets, photons,...

➜ Present accuracy: NNLO for evolution, NLO for all cross sections. Several groups: CT, MSTW, NNPDF, ABJM, HERAPDF,...



DIS: DGLAP global analysis

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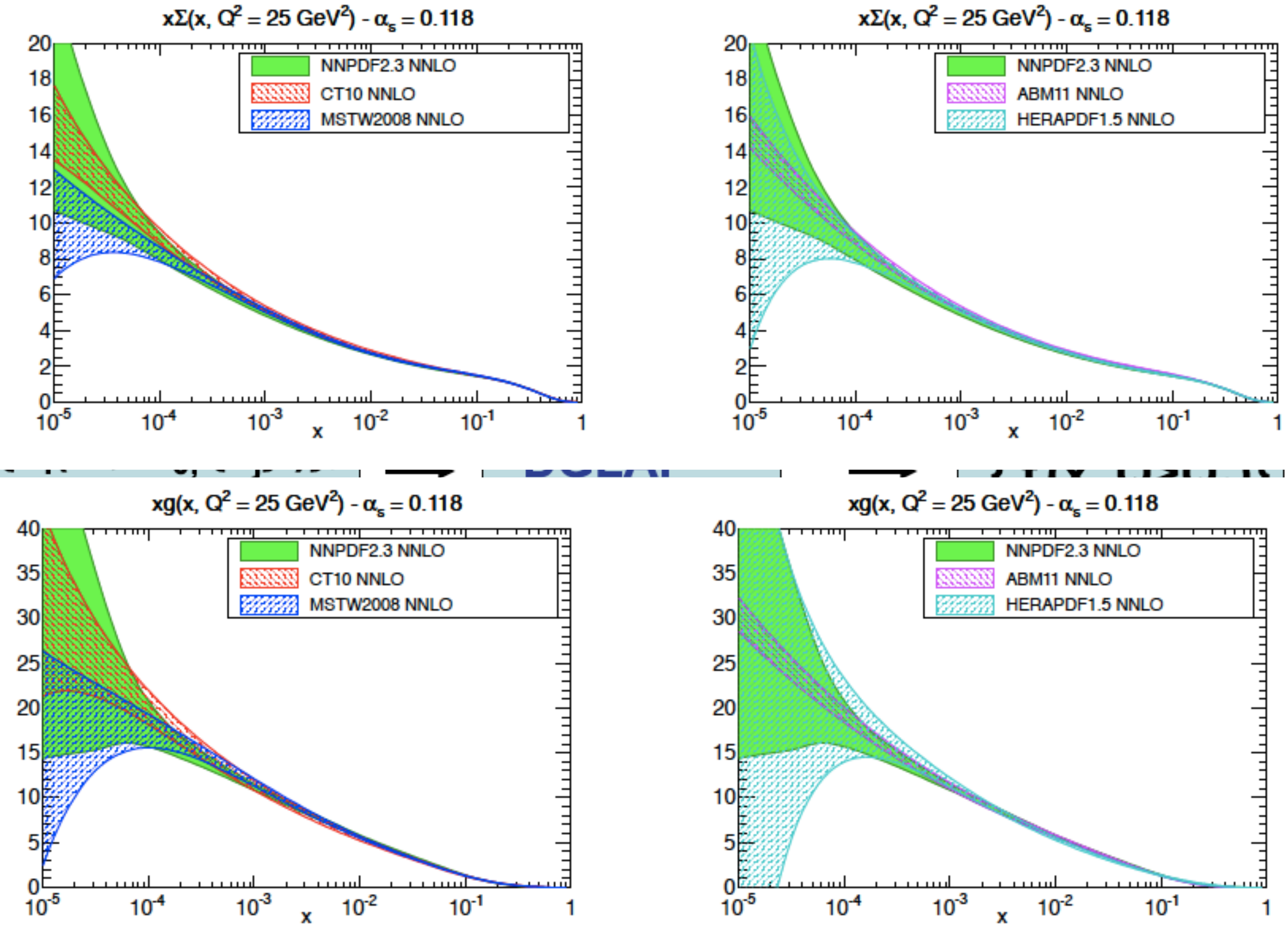

Be
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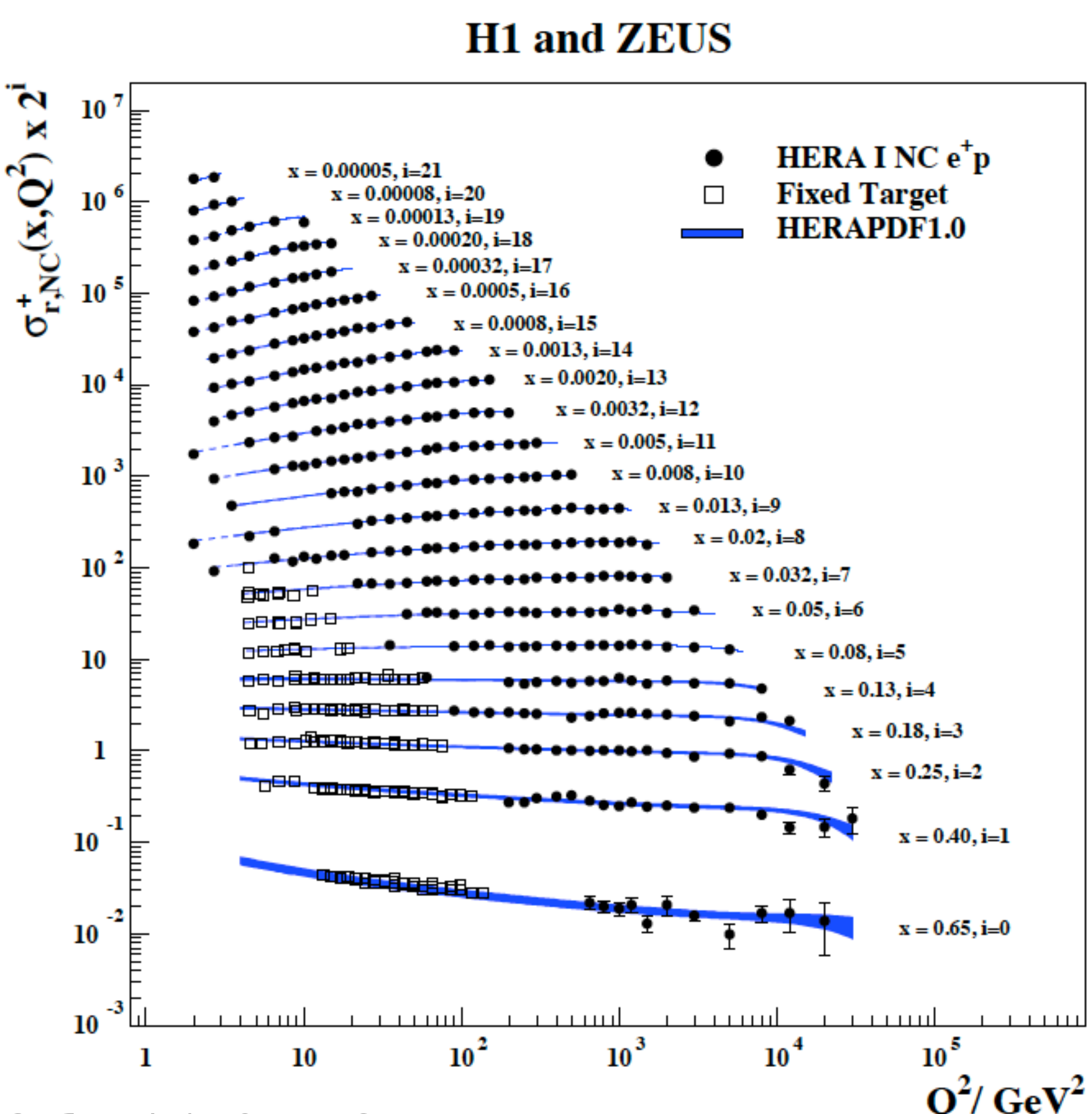
DIS: legacy from HERA

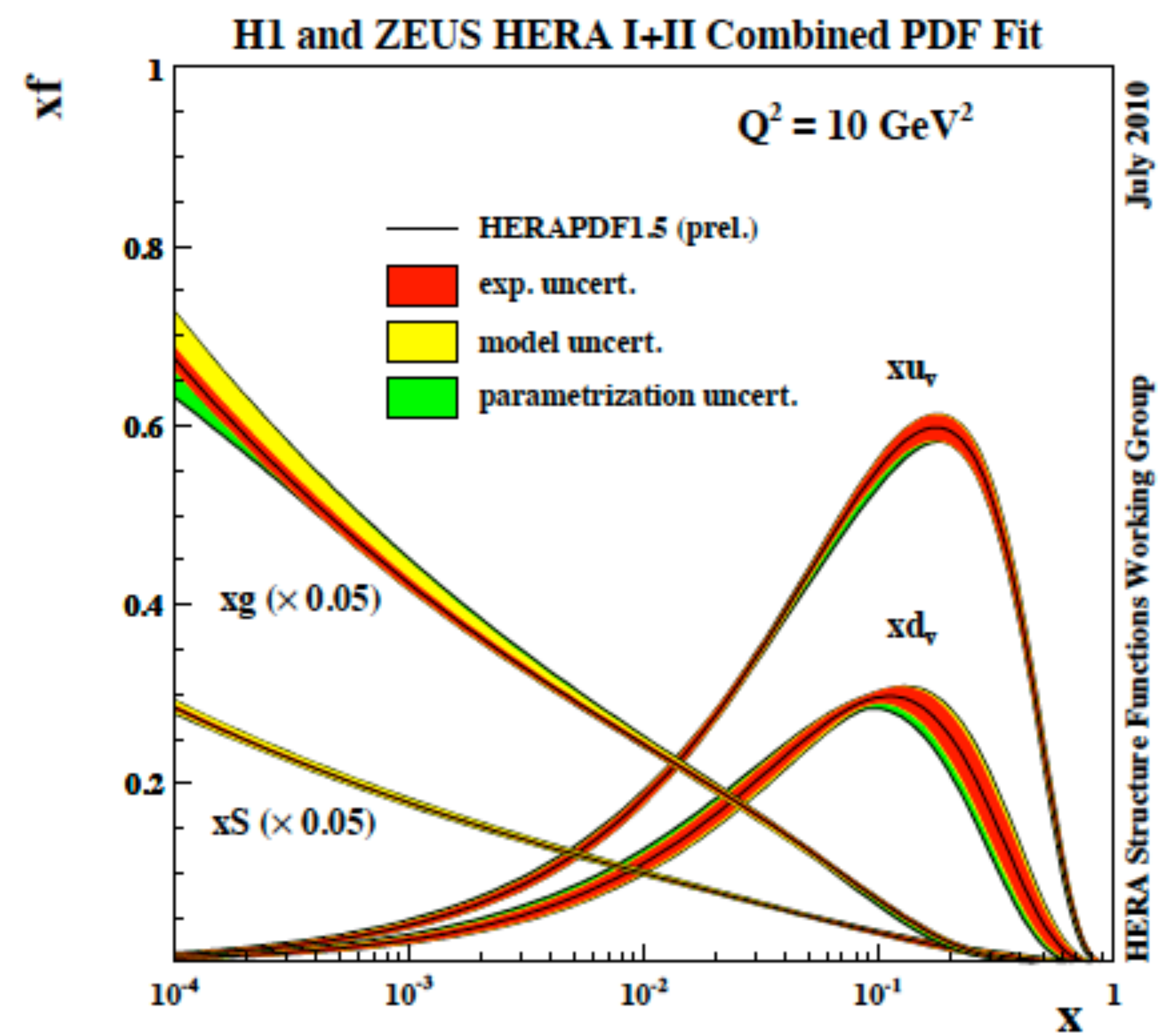
➜ Three pQCD-based alternatives to describe small-x ep and eA data (differences at moderate $Q^2(>\Lambda^2_{QCD})$ and small x):

➢ DGLAP evolution (fixed order pQCD).

➢ Resummation schemes (of $[\alpha_s \ln(1/x)]^n$ terms).

➢ Non linear effects: saturation.





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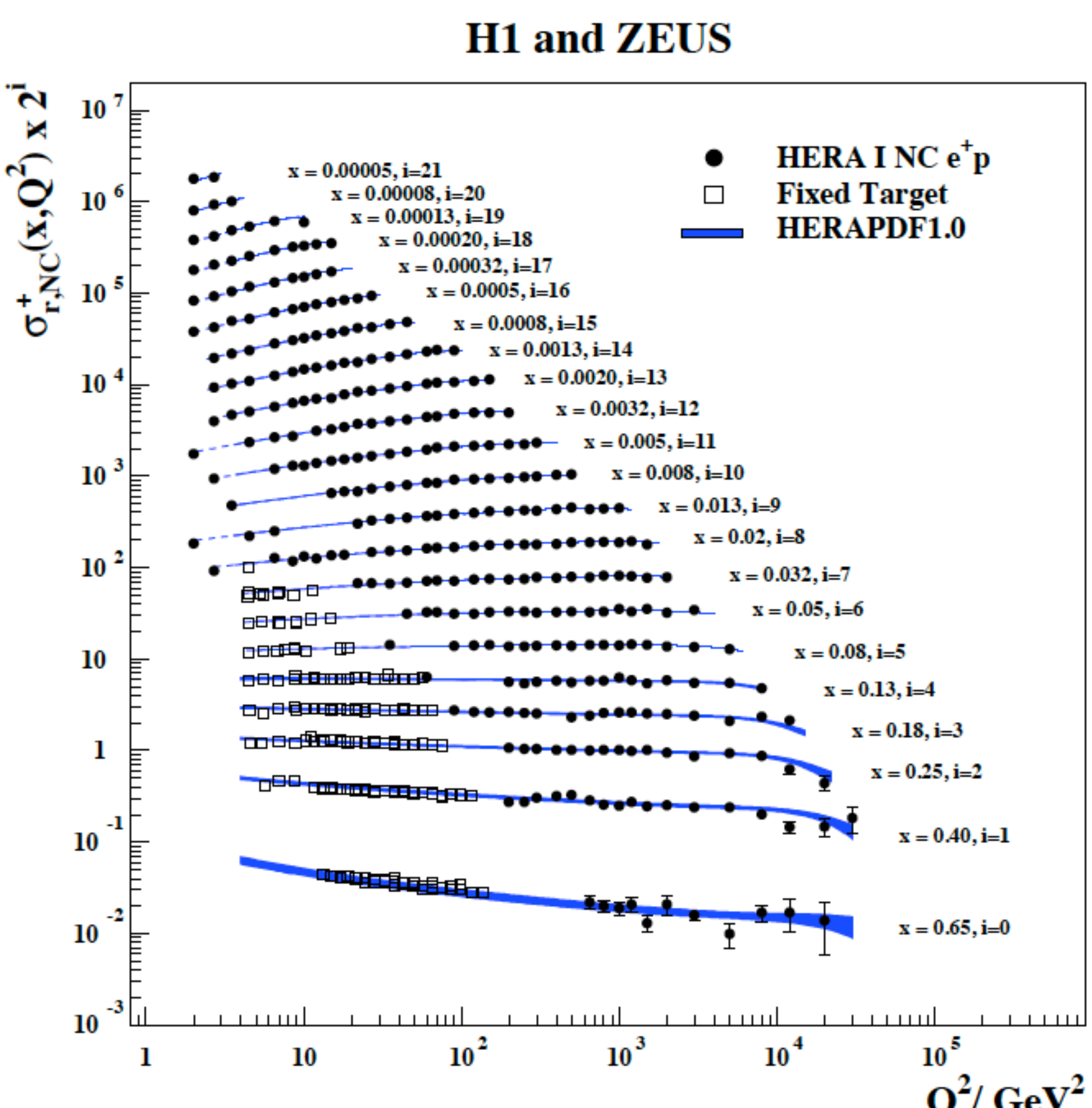
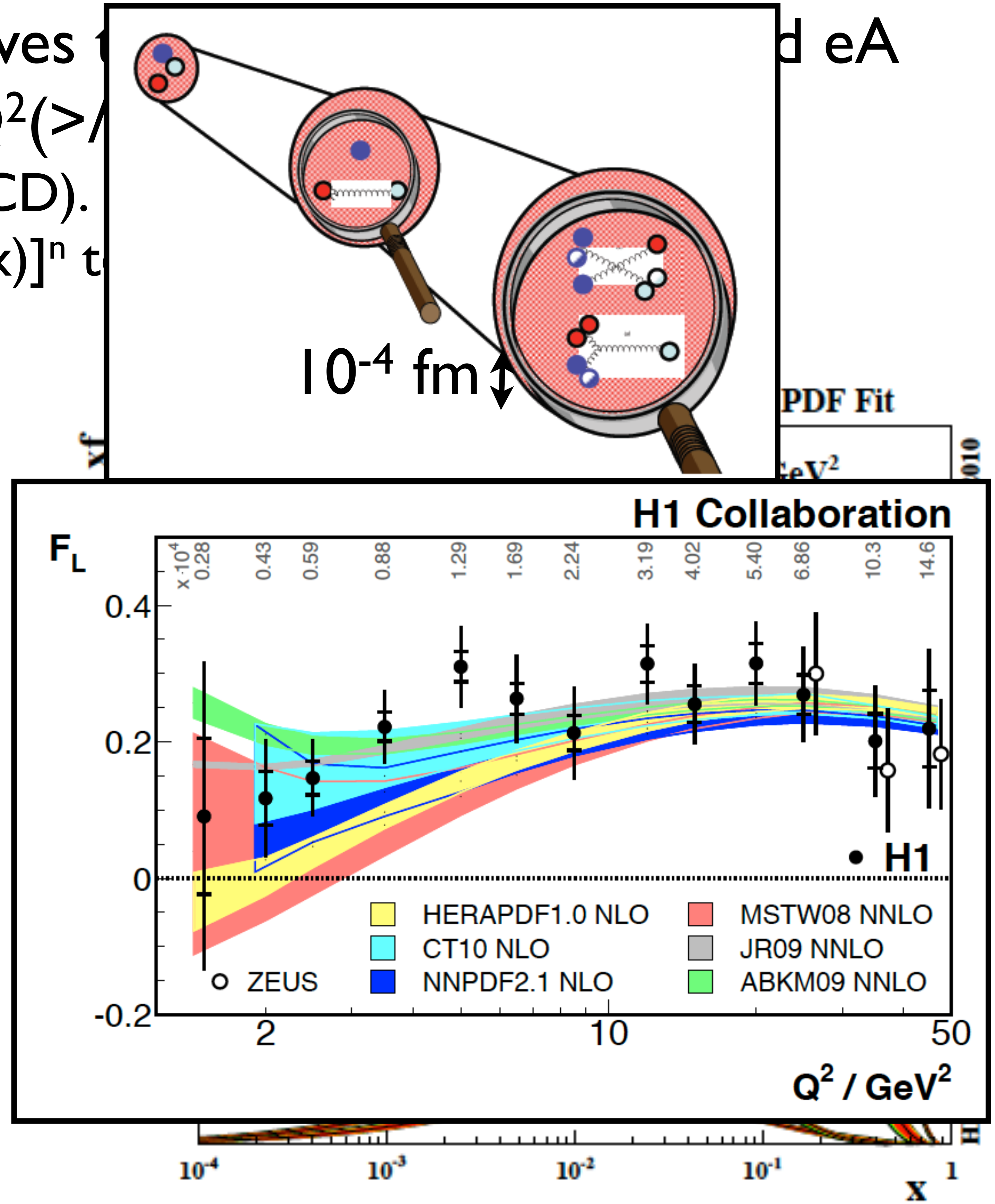




DIS: nuclei

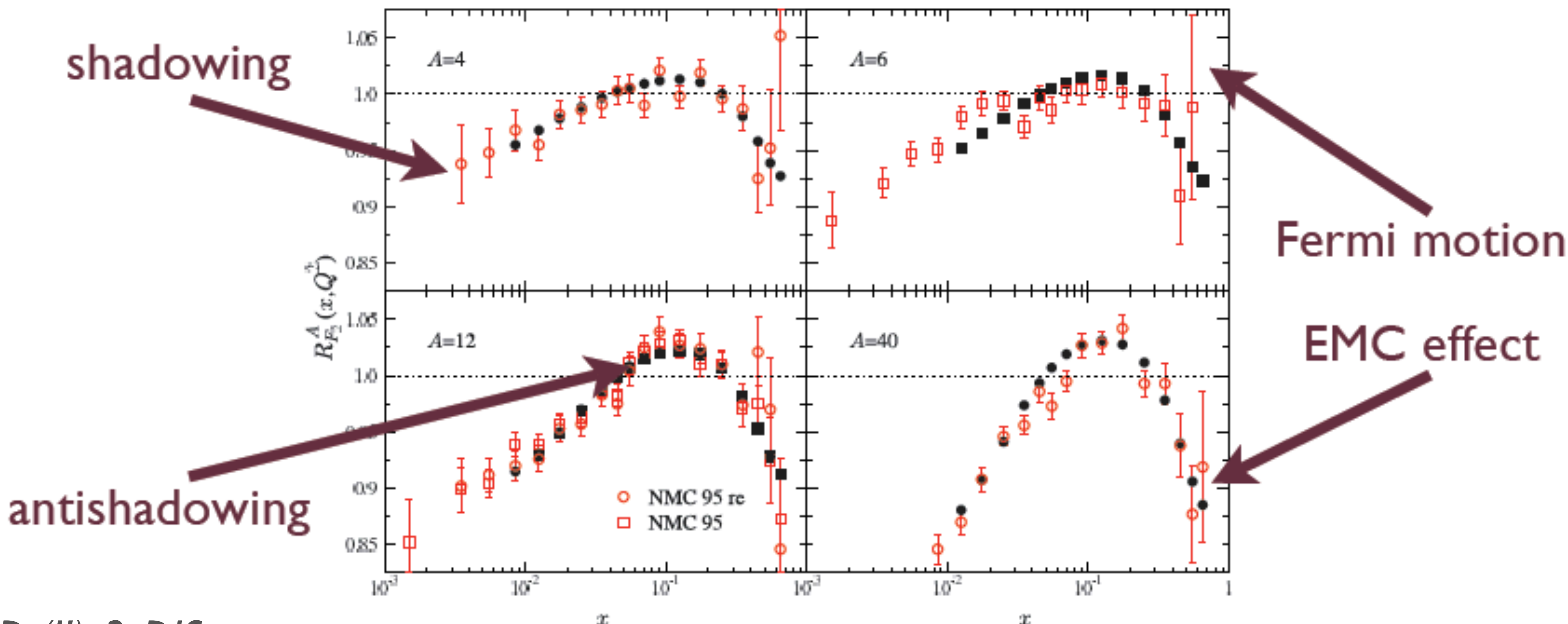
➜ Nuclear PDFs are not merely a superposition of proton and neutron ones.

➜ The same DGLAP approach is currently used.

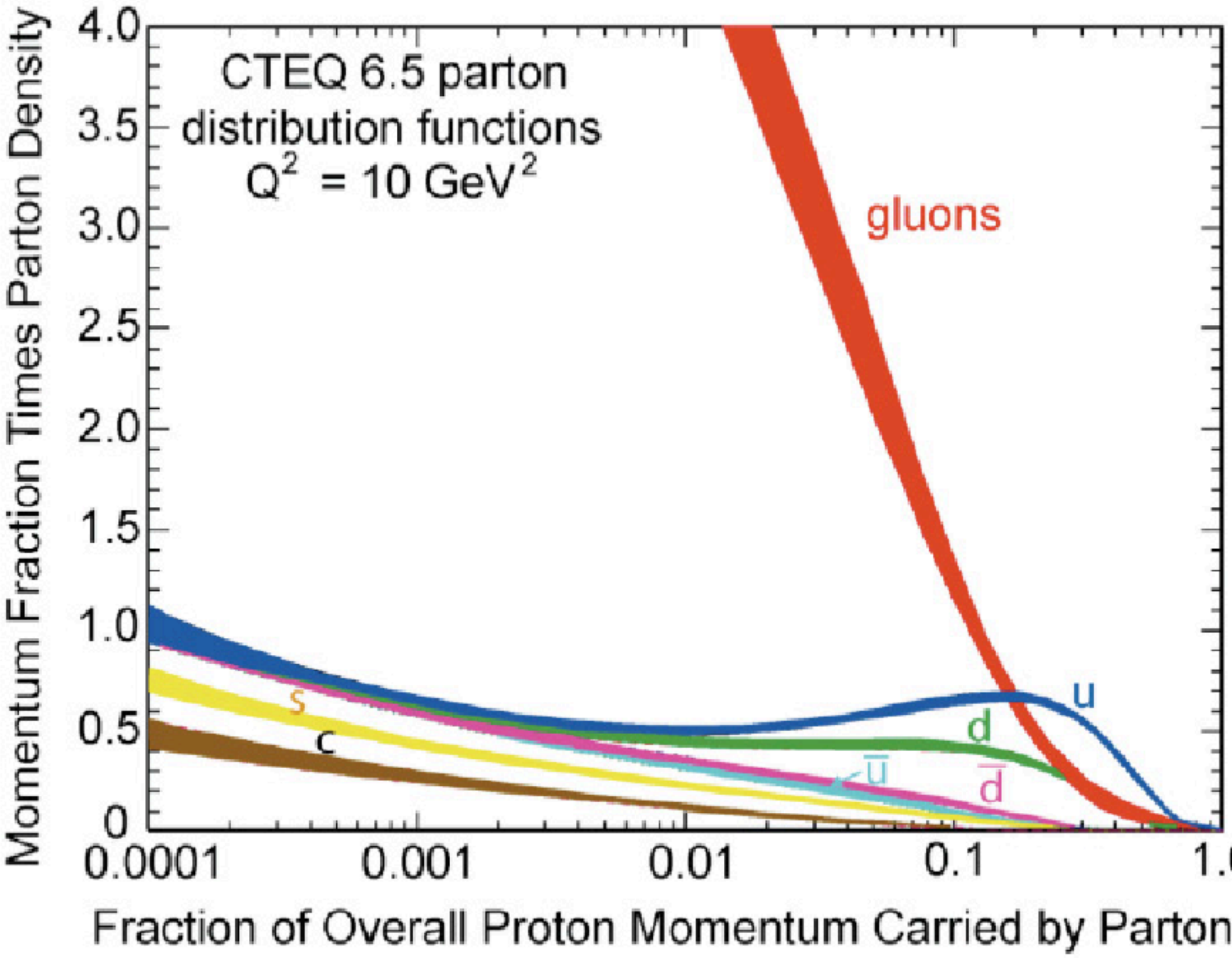
➜ They are needed to use several sets of data e.g. with neutrinos.

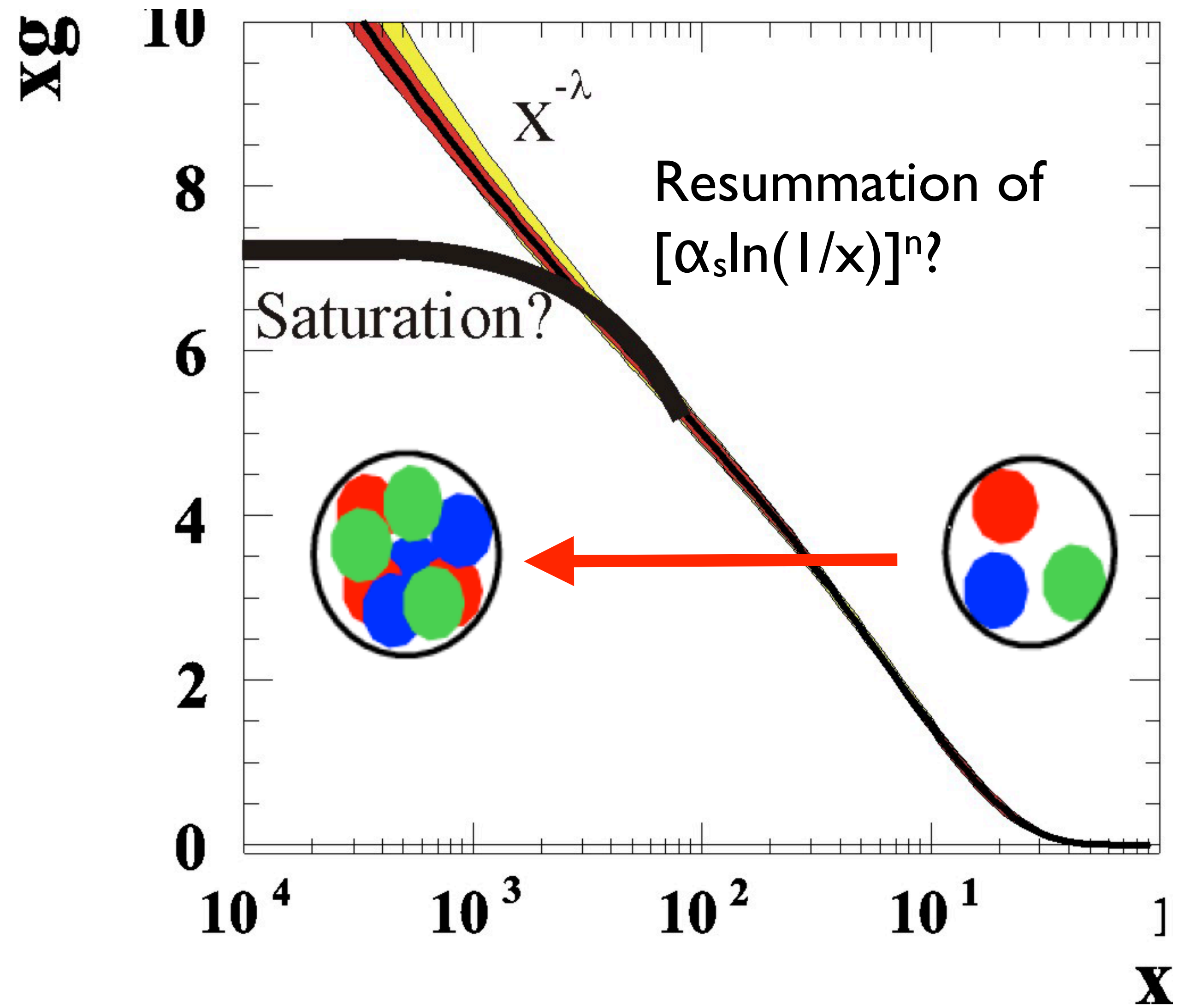
$$R^A_{F_2}(x,Q^2) = \frac{F_2^A(x,Q^2)}{AF_2^p(x,Q^2)}$$

Origin: multiple scattering?; evolution: linear or non-linear?



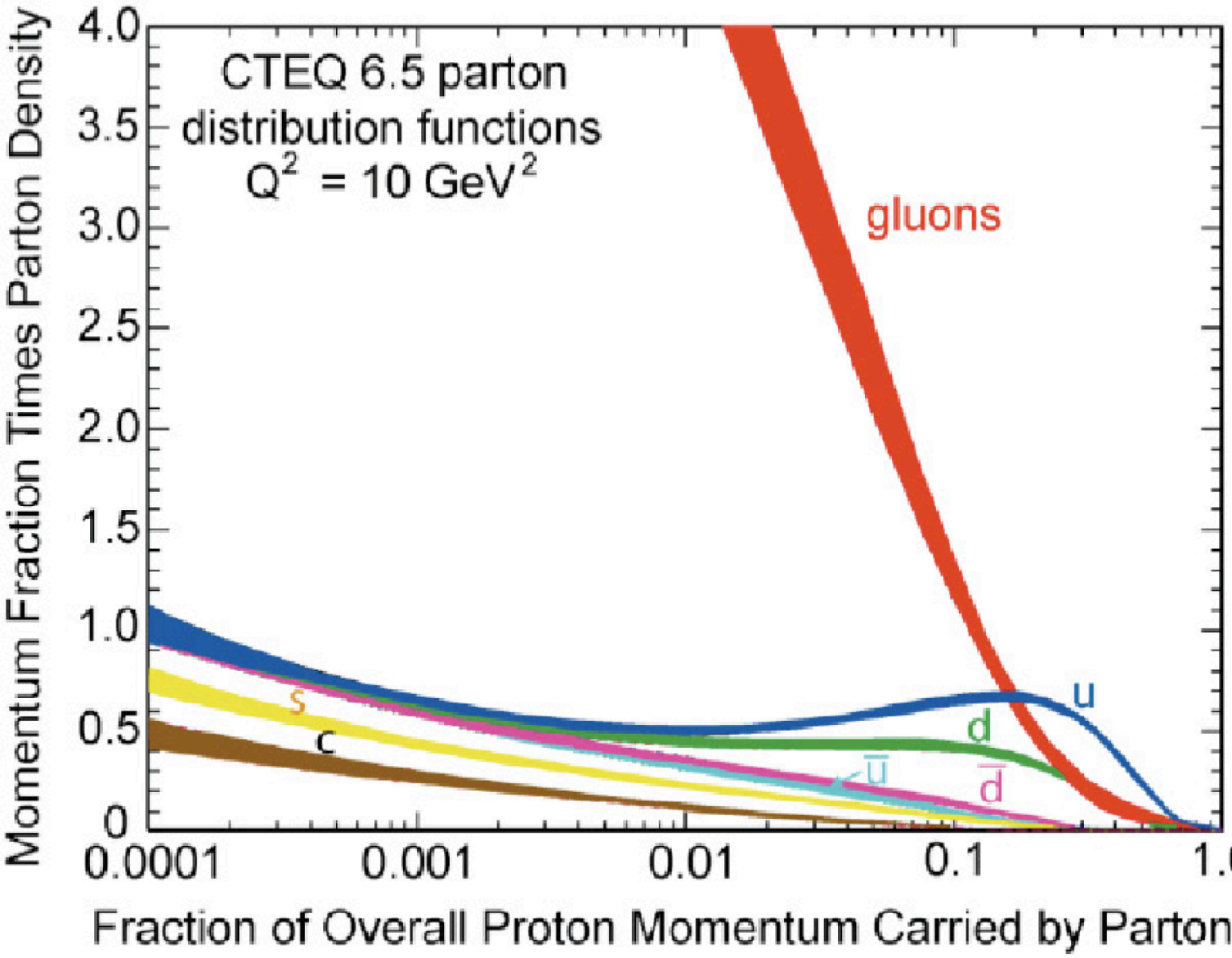
DIS: beyond DGLAP

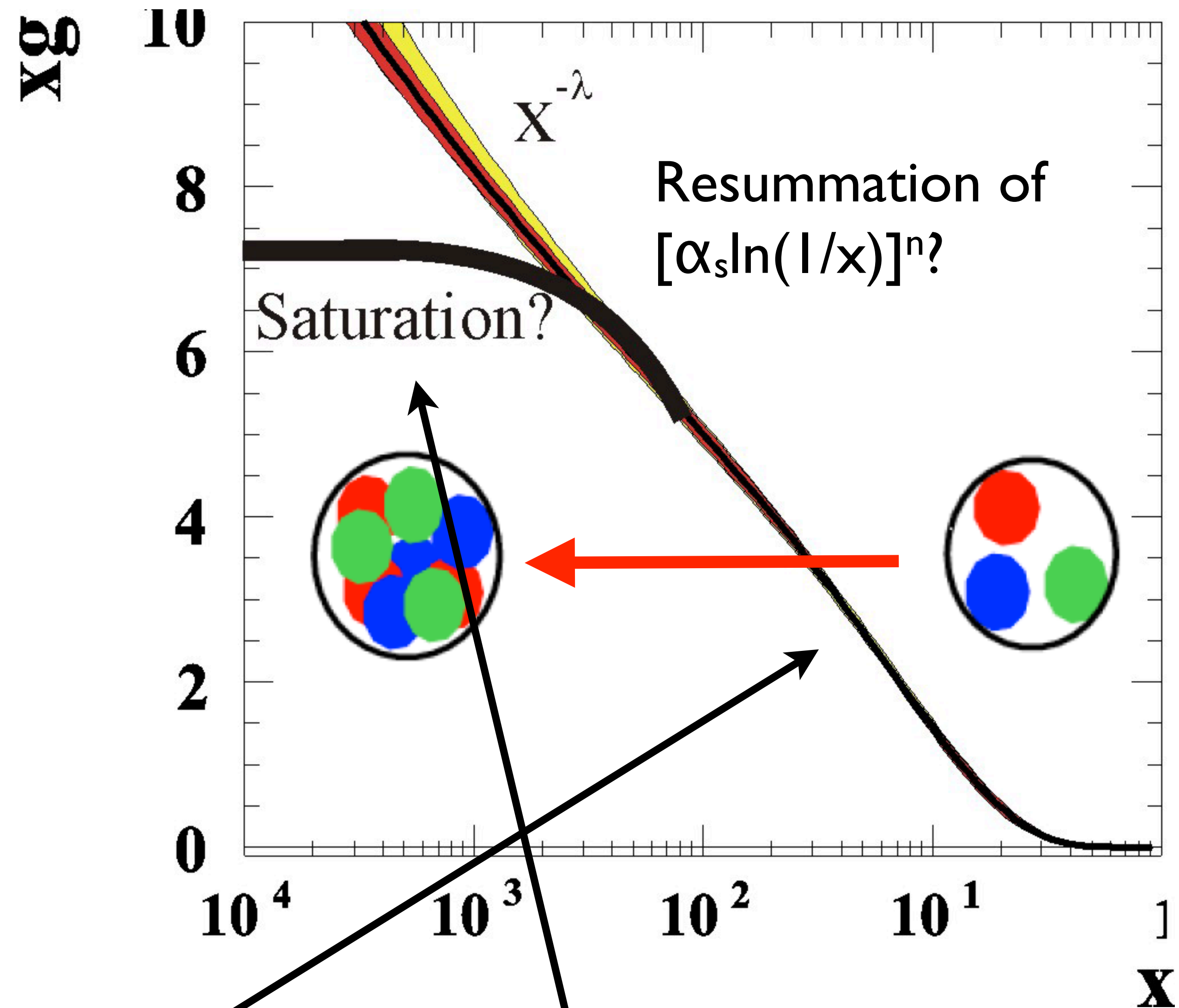




- QCD radiation of partons when x decreases leads to a large number of partons (gluons), provided each parton evolves independently (linearly, $\Delta[xg] \propto xg$).
- This independent evolution breaks at high densities (<u>small x or high mass number A</u>): non-linear effects ($g \leftrightarrow gg$, $\Delta[xg] \propto xg - k(xg)^2$).

DIS: beyond DGLAP





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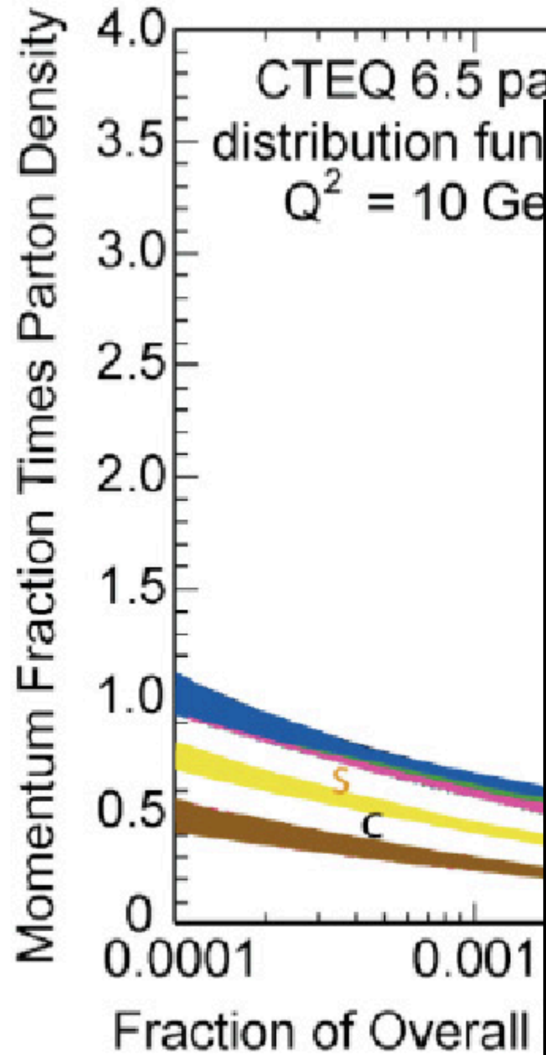

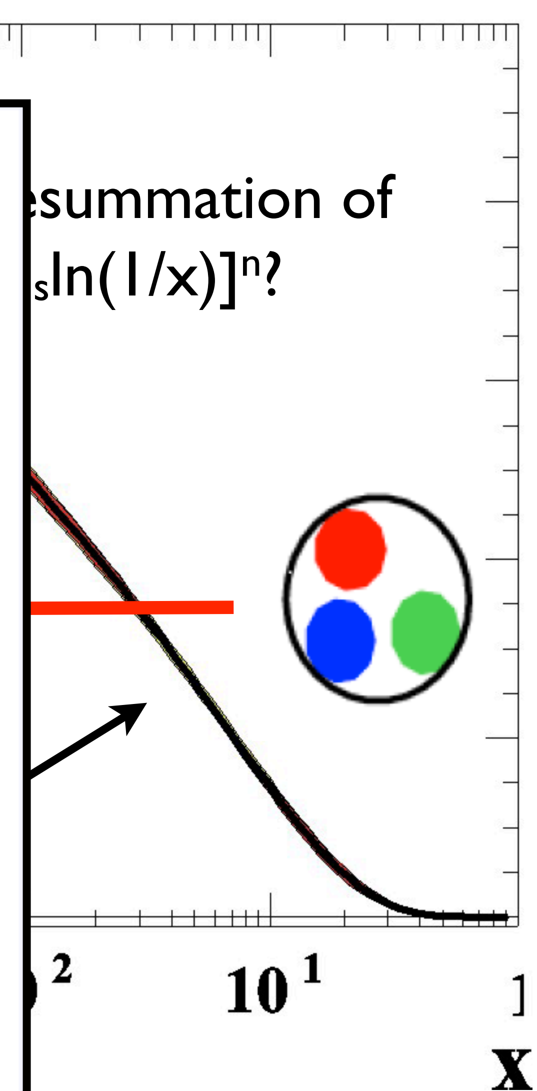

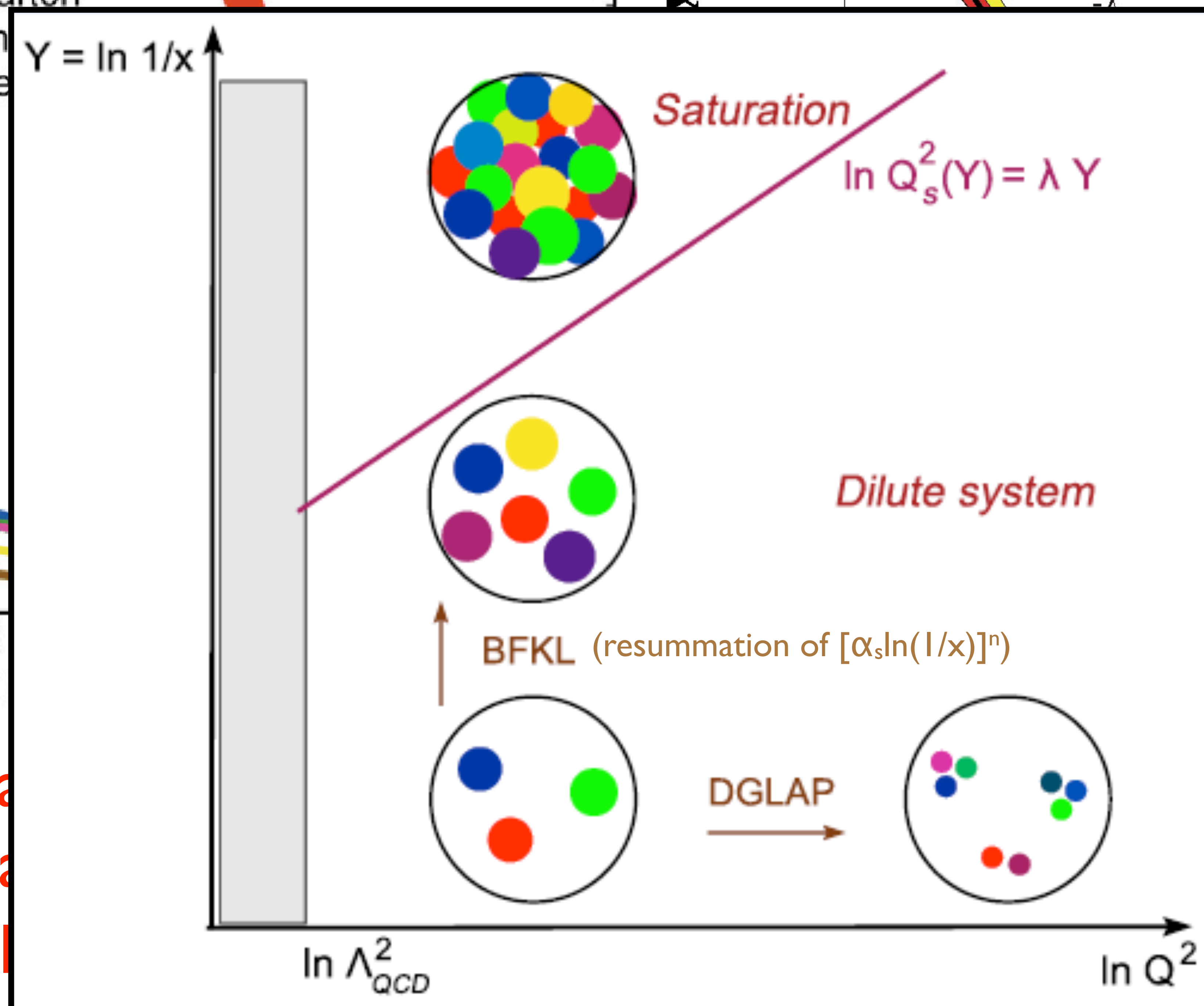

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Bibliography:

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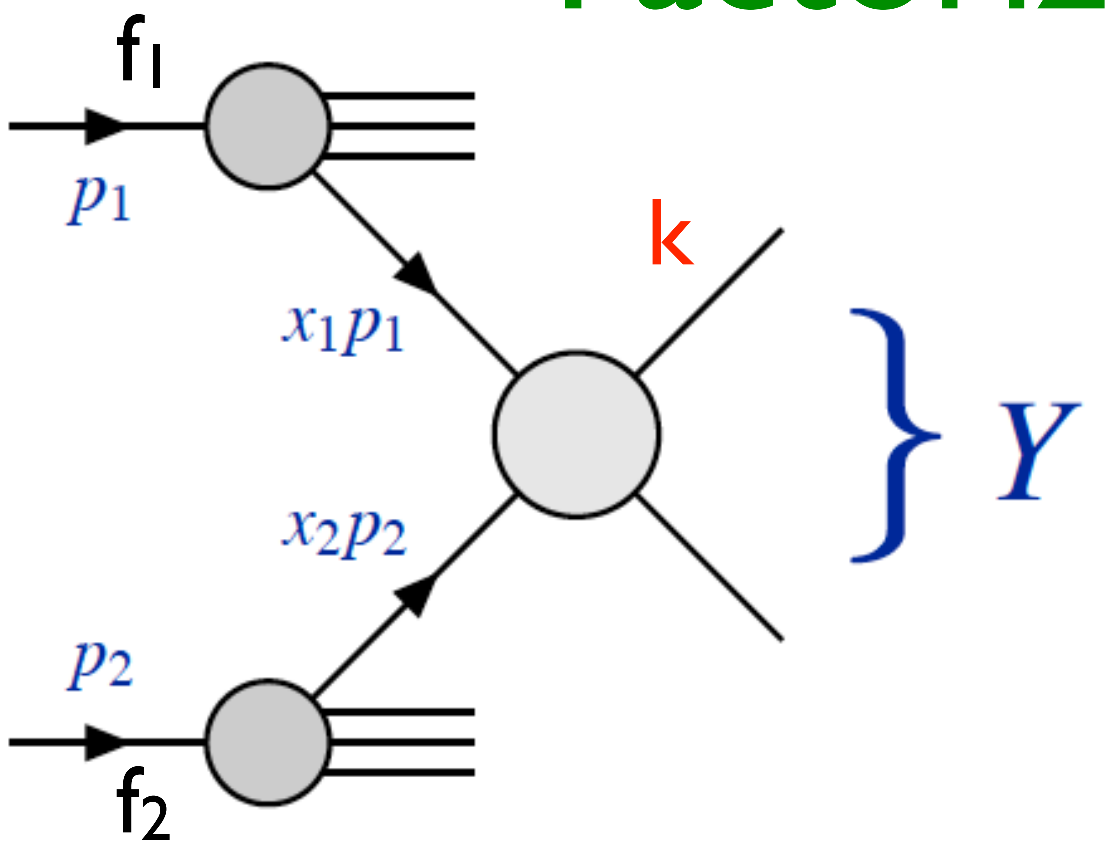
➜ R. K. Ellis et al., *QCD and collider physics*, Cambridge University Press 1996.

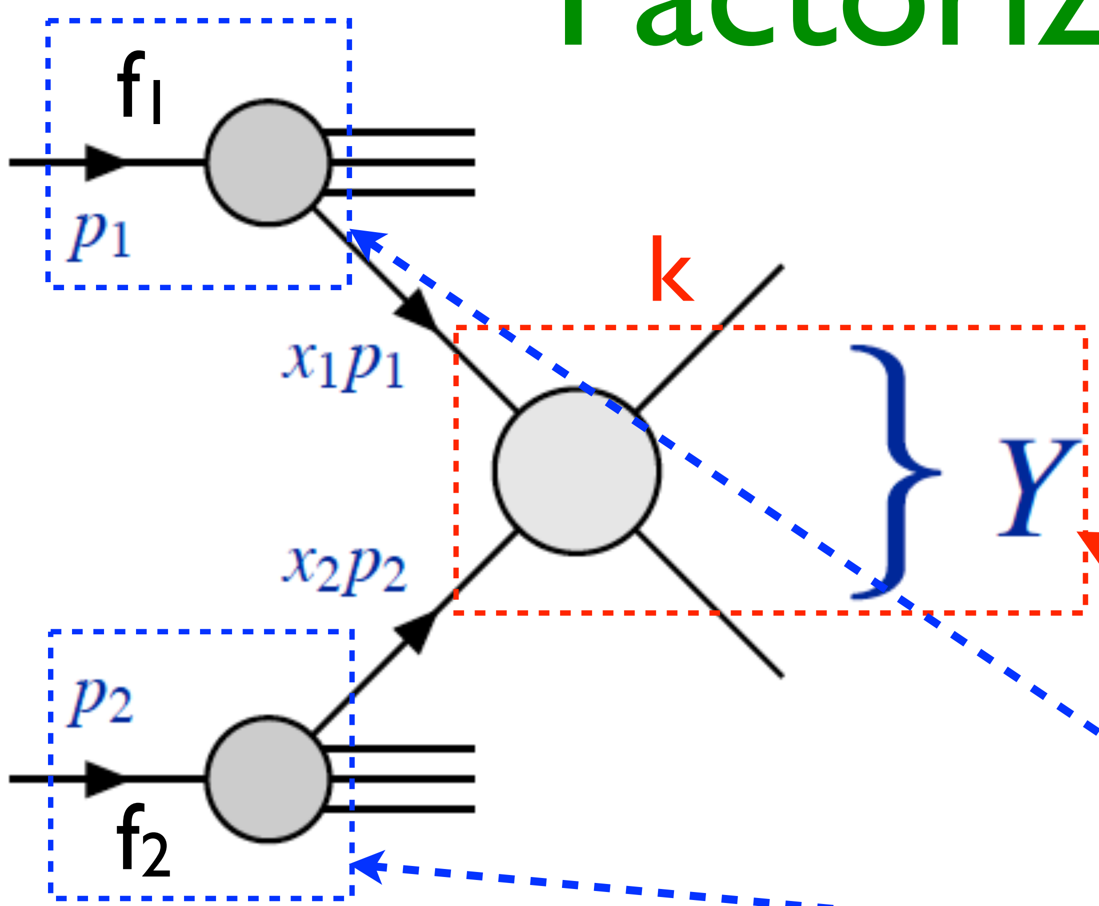
Factorization: basics



$$\sigma(h_1(p_1)+h_2(p_2)\to Y+X)=\int_0^1 dx_1\int_0^1 dx_2\sum_{f_1,f_2} f_{f_1}(x_1)f_{f_2}(x_2)\sigma(f_1+f_2\to Y)$$

- ➜ For $E_{cm}\sim S$=mass, energy, p_T,...>>Λ_{QCD}: collinear factorization.
- ➜ x_i: momentum fraction of hadron N (in hadron/nuclei) taken by parton i.
- ➜ z: momentum fraction of parton i taken by hadron h.
- ➜ Scales: Q, μ_F for factorization, μ_R for renormalization.
- ➜ f's (PDFs) and D's (fragmentation functions) evolved according to DGLAP.
- ➜ Partonic σ computed at (N)NLO (order $\alpha_s^{2(3)}$,...) for all observables (h, H, γ, DY, jets).
- ➜ Need of resummation of large logs (e.g. $\log(M_Q/p_T)$).

Factorization: basics



$$\sigma(h_1(p_1)+h_2(p_2)\to Y+X)=\int_0^1 dx_1\int_0^1 dx_2\sum_{f_1,f_2} f_{f_1}(x_1)f_{f_2}(x_2)\sigma(f_1+f_2\to Y)$$

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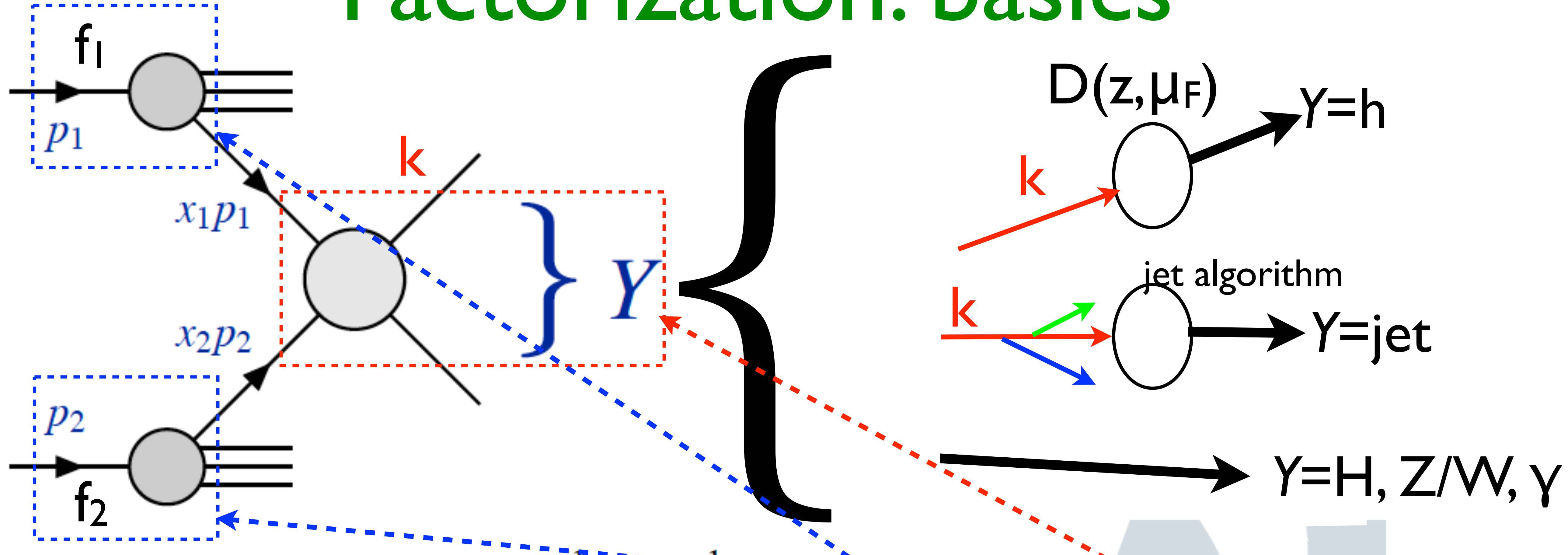

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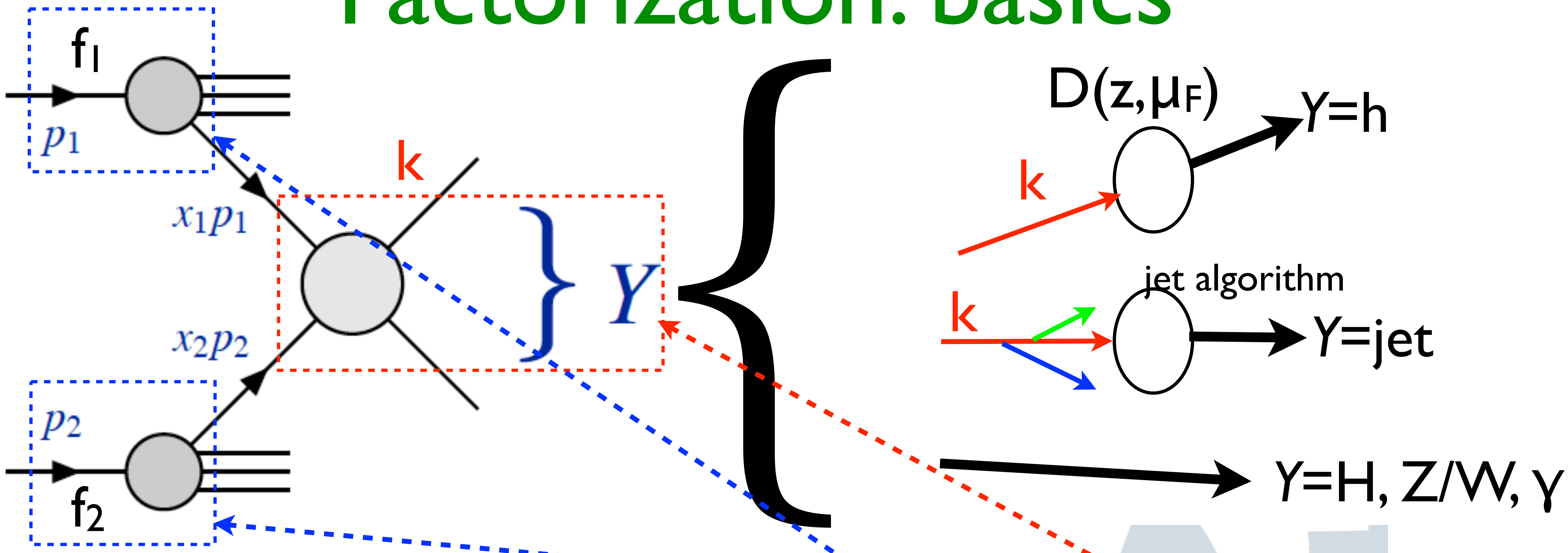

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Q μ_R

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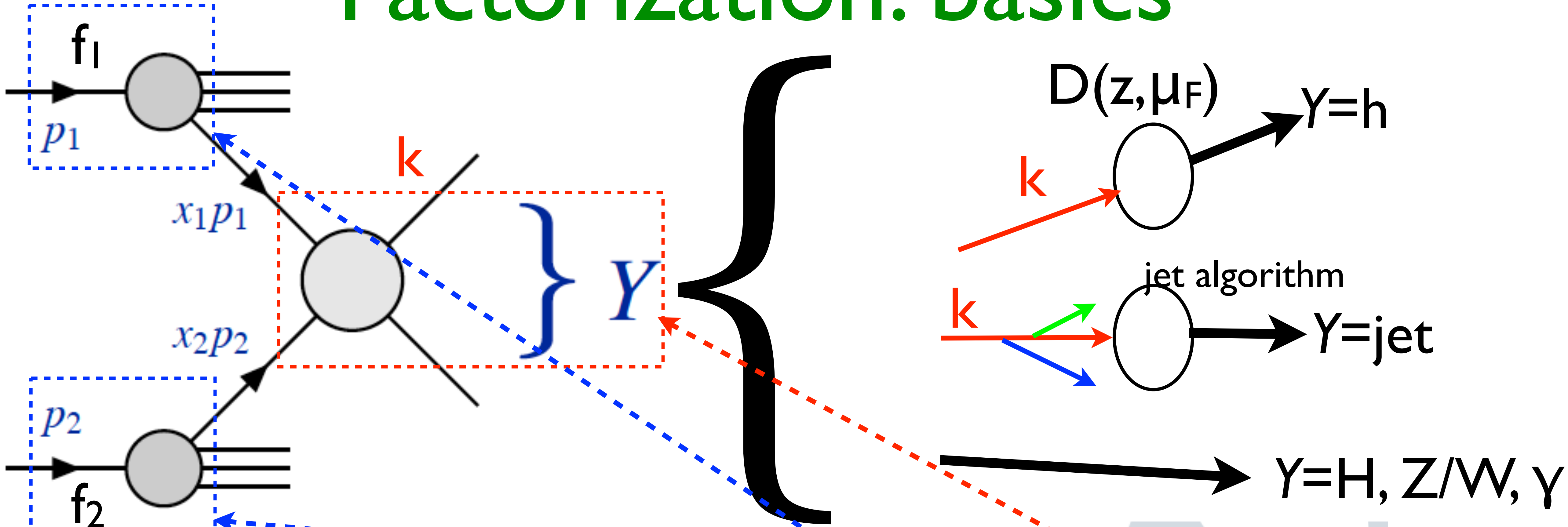

$$\sigma(h_1(p_1)+h_2(p_2)\to Y+X)=\int_0^1 dx_1\int_0^1 dx_2 \sum_{f_1,f_2} f_{f_1}(x_1) f_{f_2}(x_2)\,\sigma(f_1+f_2\to Y) + \mathcal{O}(1/S^n)$$

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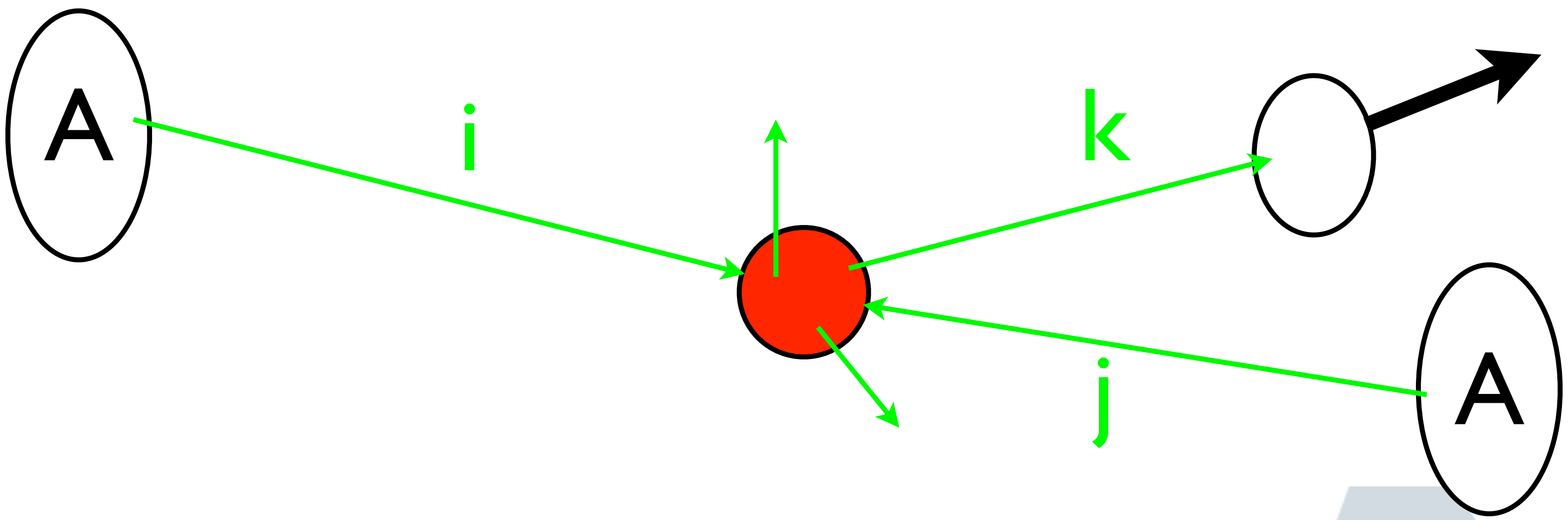
Factorization: fundamentals



$$\sigma(h_1(p_1)+h_2(p_2)\rightarrow Y+X)=\int_0^1 dx_1 \int_0^1 dx_2 \sum_{f_1,f_2} f_{f_1}(x_1) f_{f_2}(x_2)\,\sigma(f_1+f_2\rightarrow Y)+O(1/S^n)$$

➜ Origin of collinear factorization: separation of scales between short distances pieces (hard scattering at a parton level, pQCD at fixed-order) and large distances pieces (PDFs, FFs).

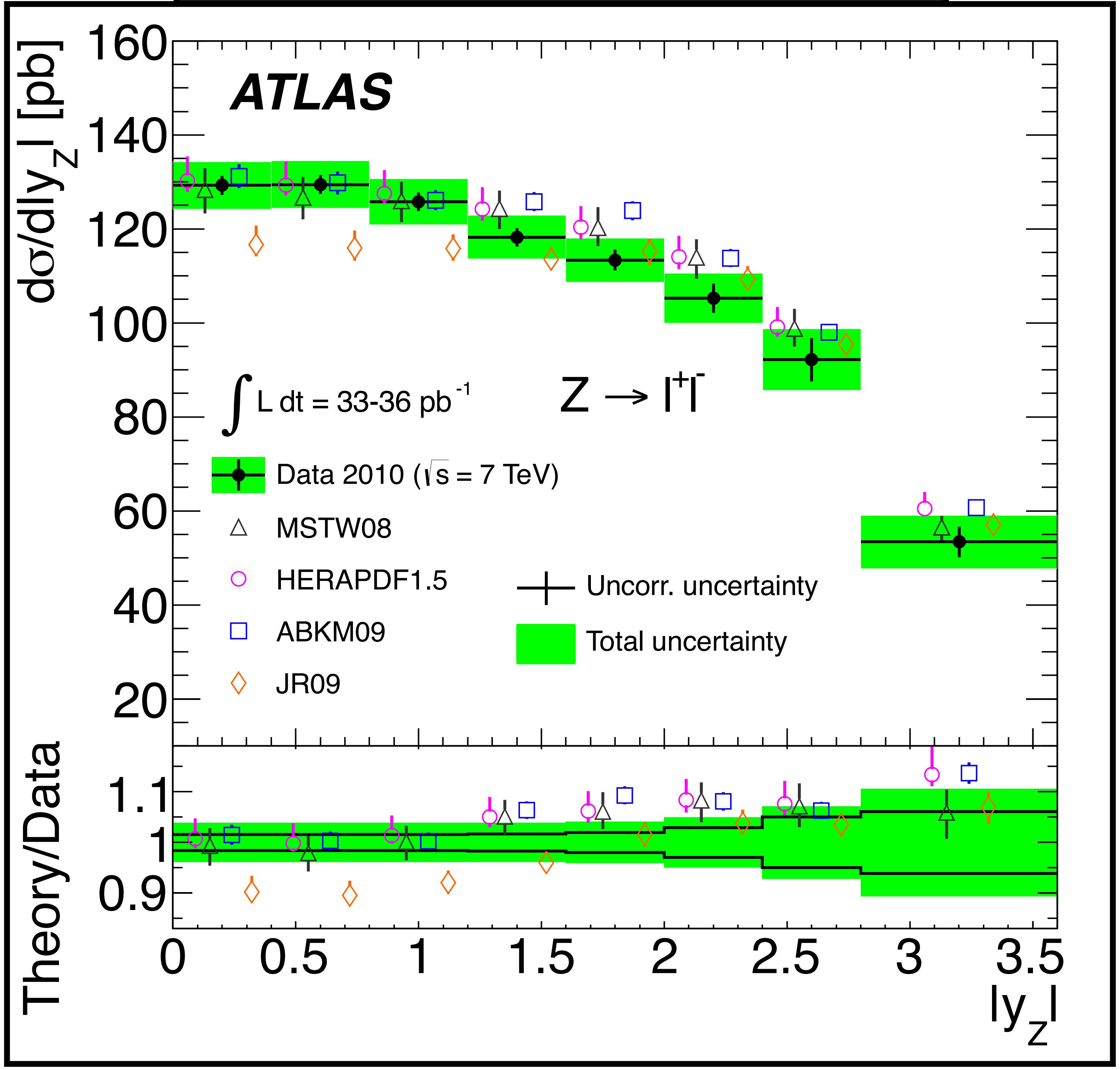
➜ Strictly proven only for e^+e^-, DIS and Drell-Yan ($hh'\rightarrow l^+l^-+X$), for sufficiently inclusive quantities (1 hadron, two jets,...) and for given kinematical regions and observables.

➜ Assumed for hh' and for less inclusive observables.

➜ Other factorizations proposed e.g. k_T-factorization for $E_{cm} >> S >> \Lambda_{QCD}$.

Factorization: PDFs for discoveries

LHC starts to be sensitive to differences in pdfs!!!



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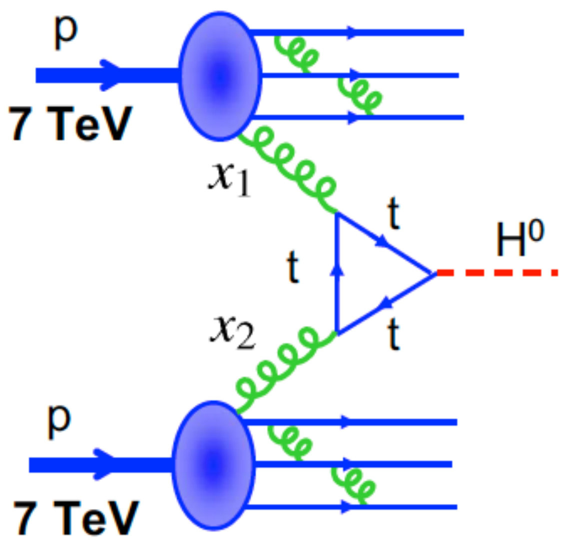
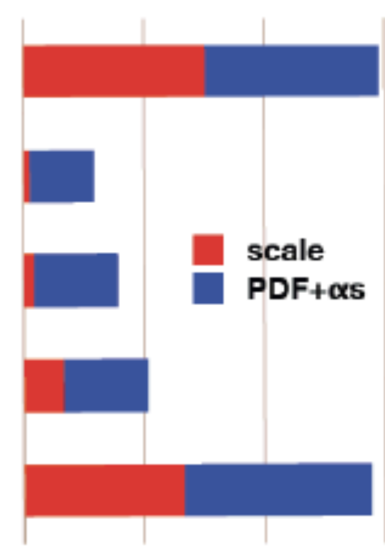


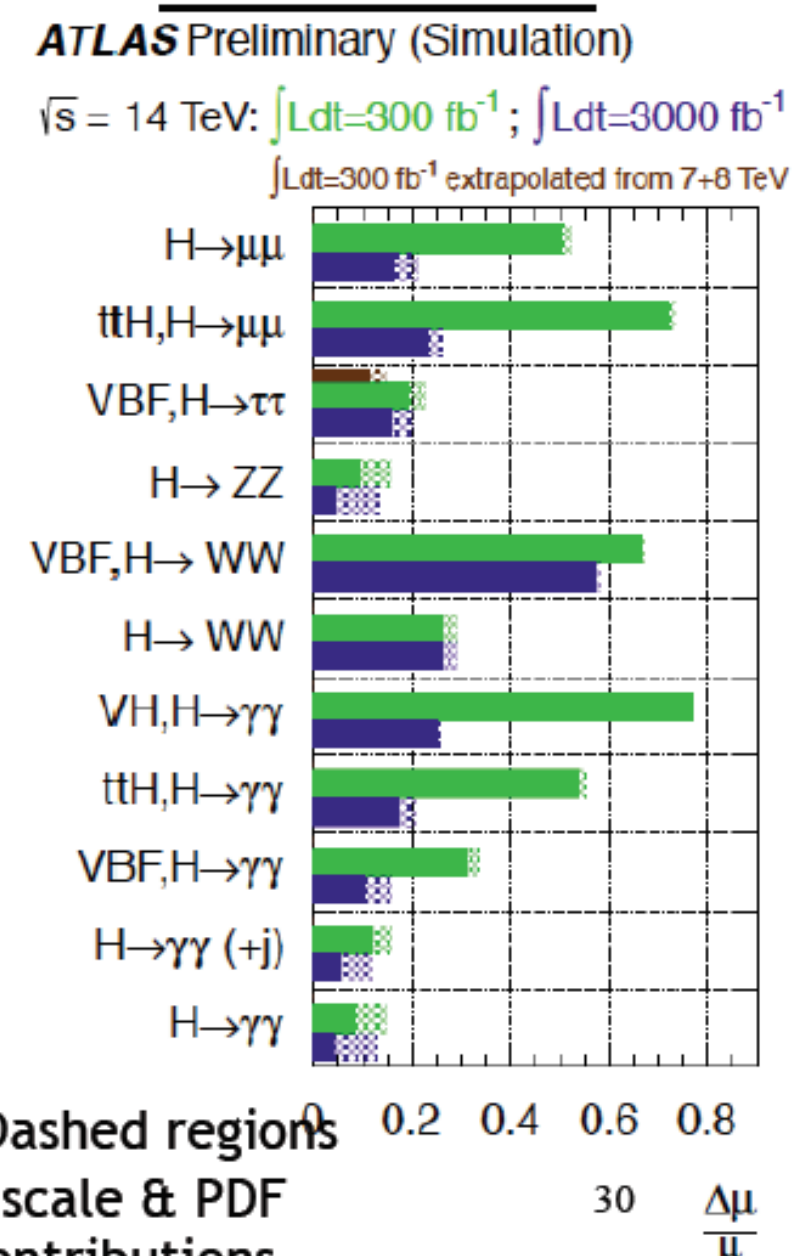
PDF Uncertainties for Higgs Physics

Theory Cross Section Uncertainties
(125 GeV Higgs
J Campbell, ICHEP'12)

		σ (8 TeV)	uncertainty
NNLL QCD +NLO EW	gg→H	19.5 pb	14.7%
	VBF	1.56 pb	2.9%
NNLO QCD +NLO EW	WH	0.70 pb	3.9%
	ZH	0.39 pb	5.1%
NLO QCD	ttH	0.13 pb	14.4%



Projected Experimental Uncertainties

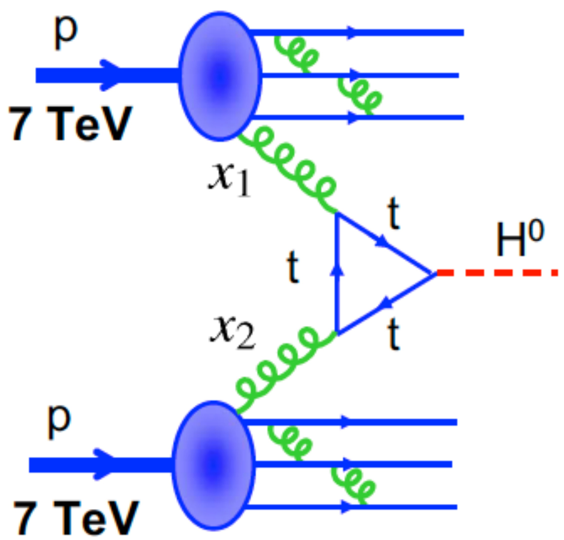


Similarly fermionic modes (bbbar, ccbar)

... tests of Standard Model in Higgs sector may become limited by knowledge of PDFs in HL-LHC era

[Dashed regions = scale & PDF contributions]

Factorization: PDFs for discoveries



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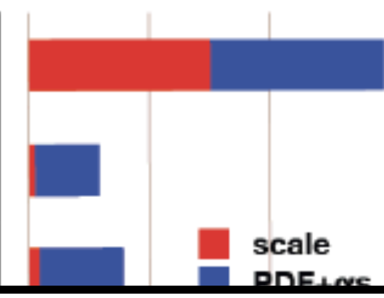

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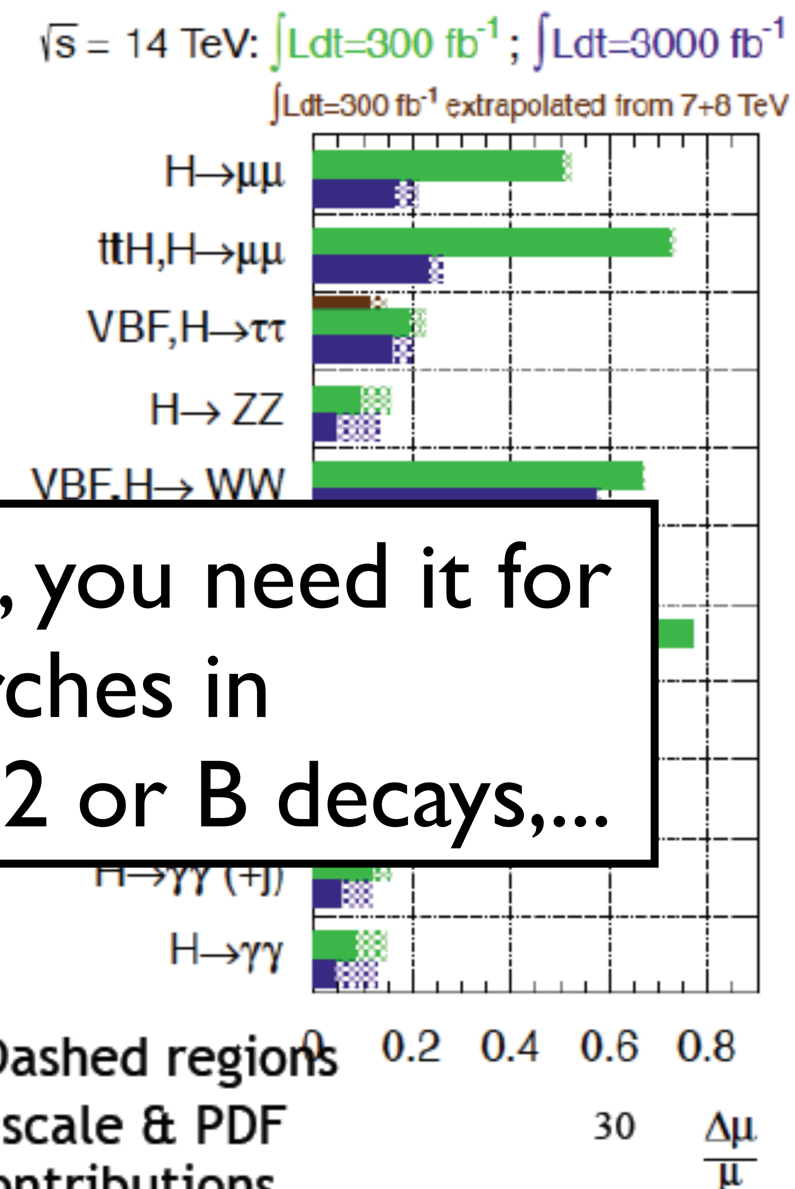

→ Even if you are not a QCD person, you need it for EW/BSM physics: background for searches in colliders, hadronic contributions to g-2 or B decays,...

... tests of Standard Model in Higgs sector may become limited by knowledge of PDFs in HL-LHC era

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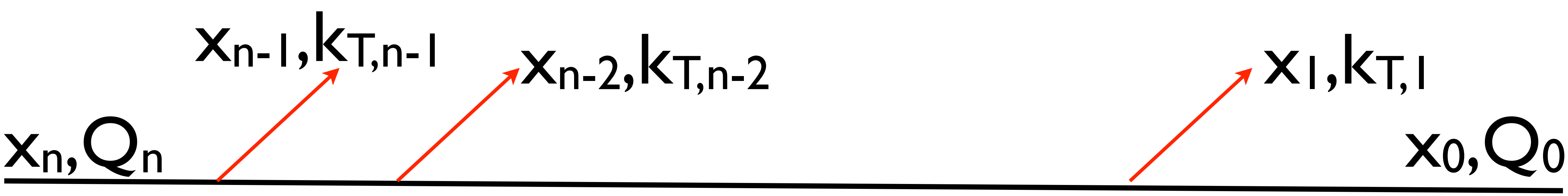
➜ R. K. Ellis et al., *QCD and collider physics*, Cambridge University Press 1996.

QCD radiation: one emitter

➜ A massless on-shell particle cannot radiate. Some virtuality of order $k_T^2/[x(1-x)]$ has to be allowed, either for initial or final state radiation.

➜ The bulk of radiation is determined by the divergencies in the emission kernel: infrared and collinear for massless emitters.



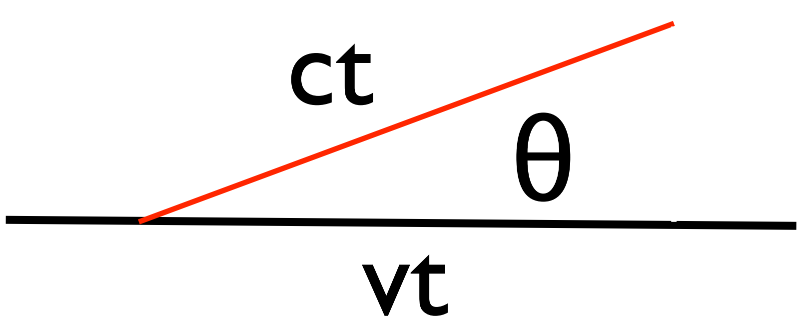
$$dP_i = \frac{dx_i}{x_i}\frac{dk_{T,i}^2}{k_{T,i}^2}, \quad \omega_i = x_i E, \quad \theta_i^2 \simeq \frac{k_{T,i}^2}{\omega_i^2}$$

$$Q_n^2 \gg k_{T,n-1}^2 \gg k_{T,n-2}^2 \gg \ldots \gg k_{T,1}^2 \gg Q_0^2$$

DLL

$$x_n \ll x_{n-1} \ll x_{n-2} \ll \ldots \ll x_1 \ll x_0$$

➜ A massive particle cannot emit collinearly: dead cone.



$$vt \simeq ct(1-\theta_0^2) \Rightarrow \theta_0^2 = m^2/E^2, \quad \theta^2 \to \theta^2 + \theta_0^2$$

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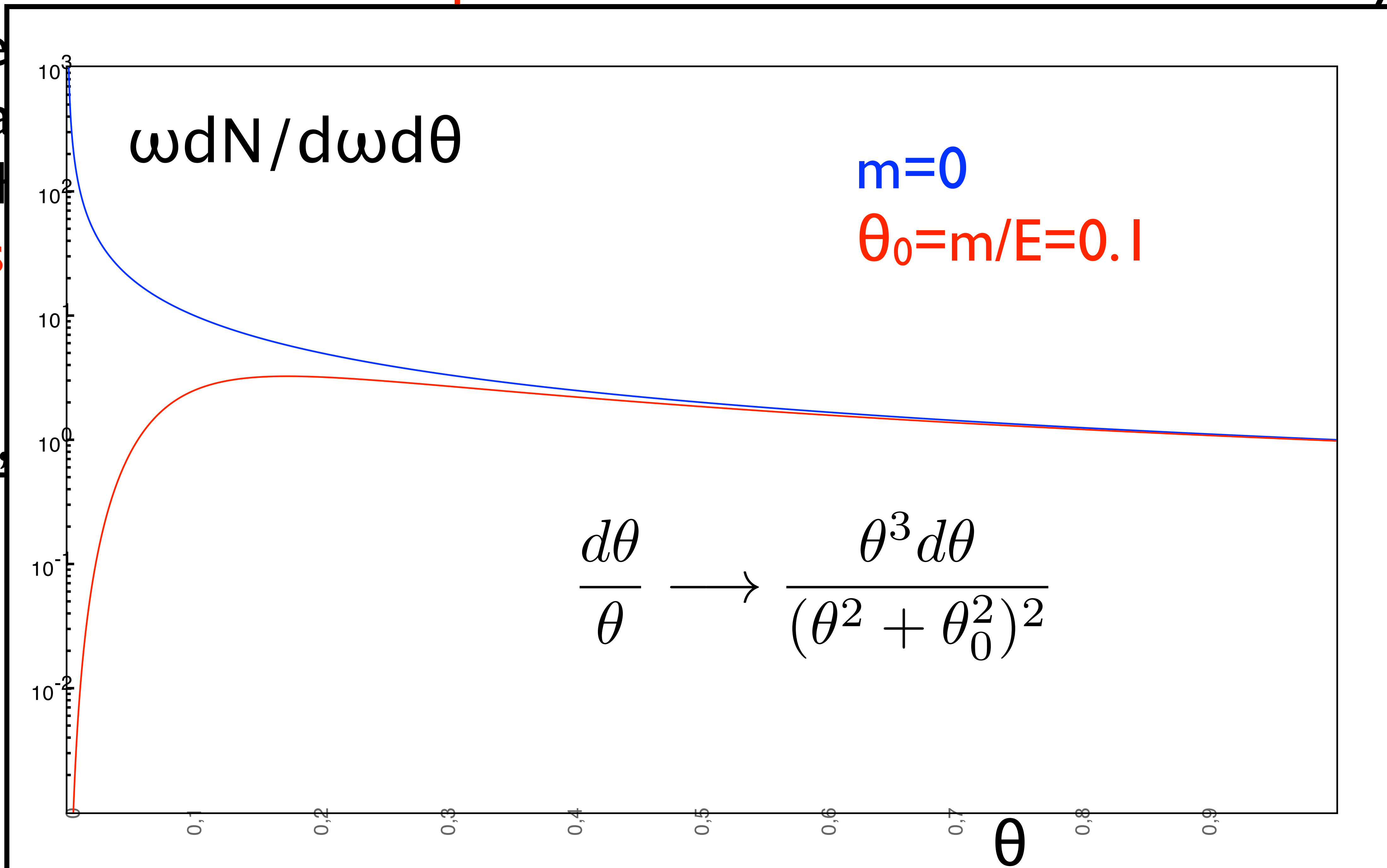


$x_{n,}$ ... Q_0

dP_i ... $\gg Q_0^2$

$\ll x_0$

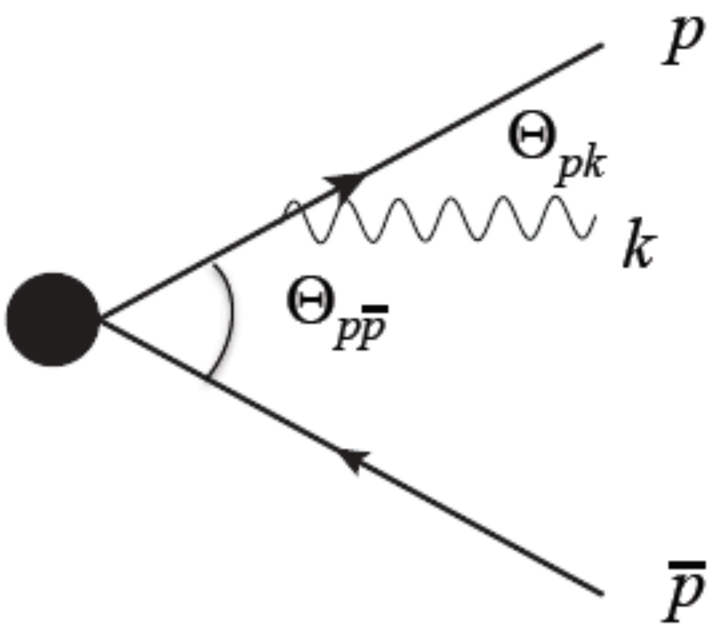
➜ A

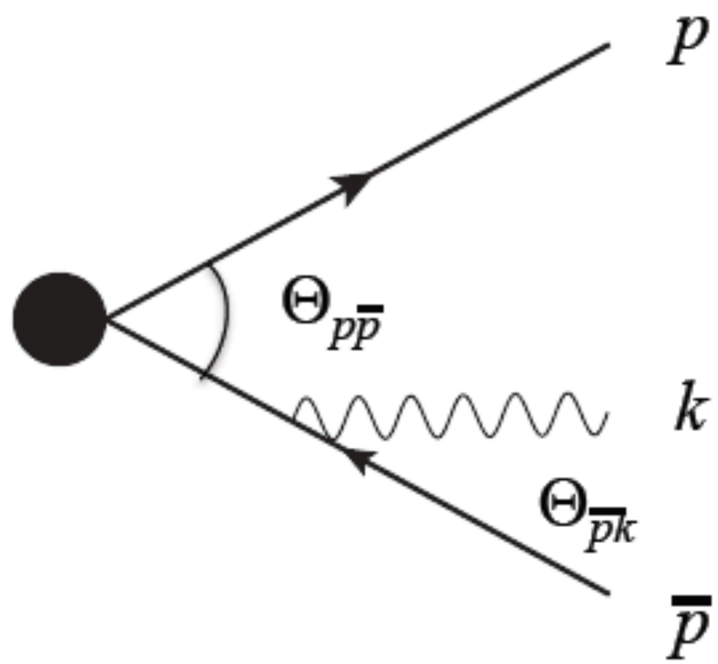
θ

vt

$$vt \simeq ct(1-\theta_0^2) \Rightarrow \theta_0^2 = m^2/E^2, \quad \theta^2 \to \theta^2 + \theta_0^2$$

QCD radiation: antenna

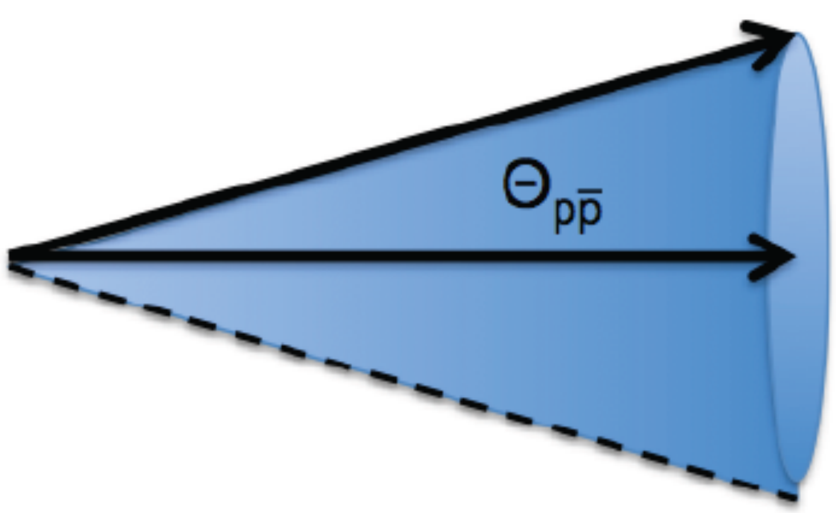




$$(2\pi)^2 E \frac{dN}{d^3k} = \alpha_s C_F \frac{p \cdot \bar{p}}{(p \cdot k)(\bar{p} \cdot k)}$$

➜ If we make a probabilistic interpretation: $dN = dN_q + dN_{qbar}$,

$$dN_q \propto \alpha_s \frac{d\omega}{\omega} \frac{d\theta_{pk}}{\theta_{pk}} \Theta(\theta_{p\bar{p}} - \theta_{pk})$$



➜ Quantum interference leads to a probabilistic picture!

QCD radiation: antenna

➜ Qualitatively: |qqbarg> → |qqbar>+|g>

$$D_{q\bar{q}} = \theta_{q\bar{q}} t_{coh}, \quad t_{coh} \sim \omega/k_T^2, \quad D_g \sim 1/k_T$$

$$D_{q\bar{q}} = \frac{\theta_{q\bar{q}}}{k_T \theta_g} > D_g \Rightarrow \theta_g < \theta_{q\bar{q}}$$

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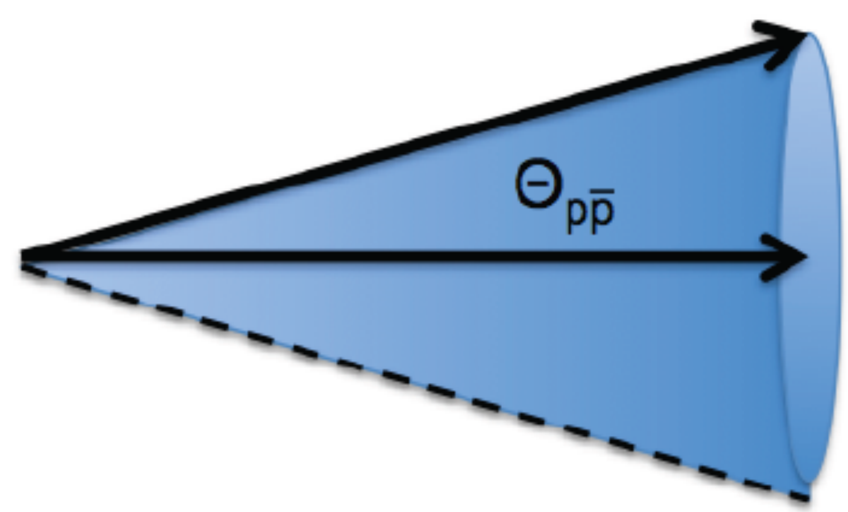

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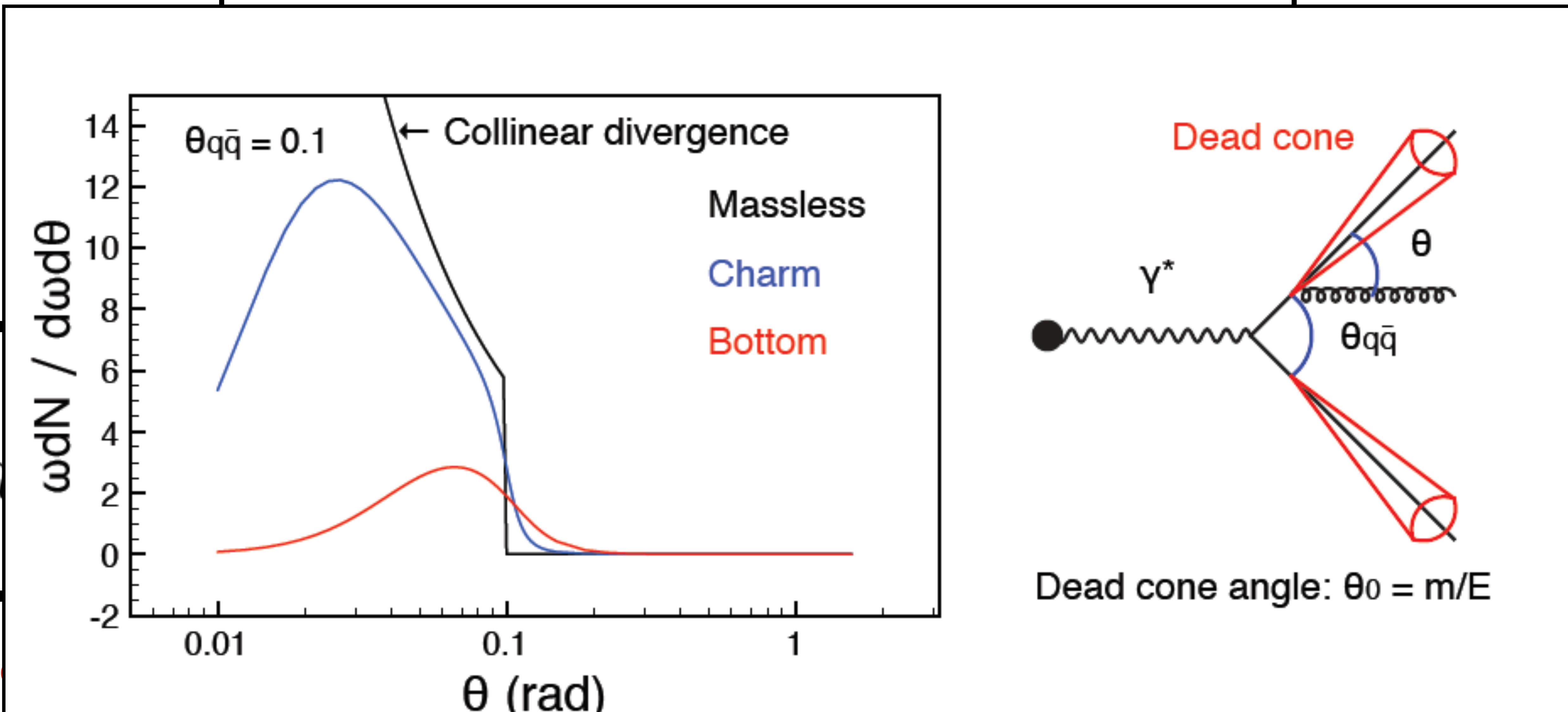


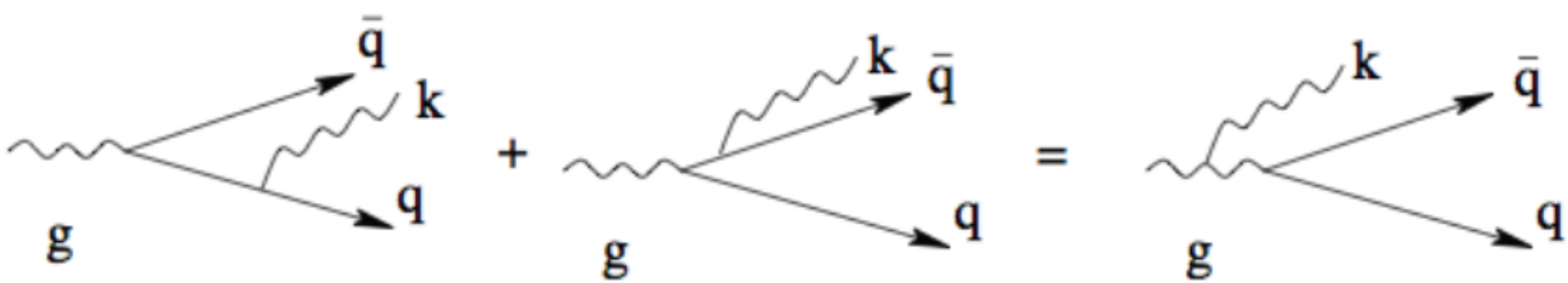
QCD radiation: antenna

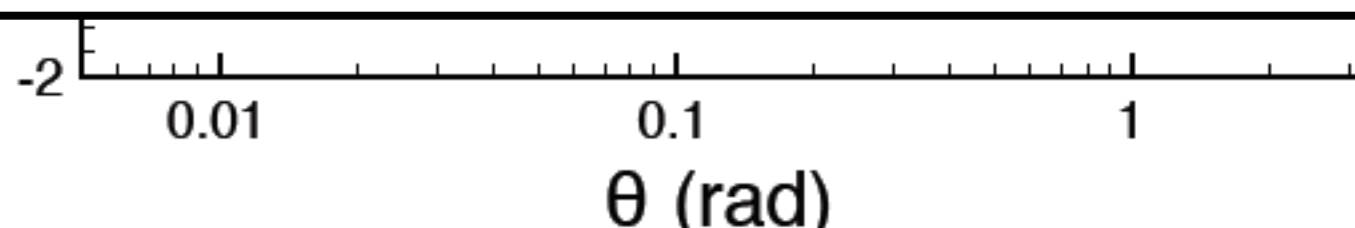
➜ Qualitatively: |qqbarg> → |qqbar>+|g>

$$D_{q\bar{q}} = \theta_{q\bar{q}} t_{coh}, \quad t_{coh} \sim \omega/k_T^2, \quad D_g \sim 1/k_T$$

$$D_{q\bar{q}} = \frac{\theta_{q\bar{q}}}{k_T \theta_g} > D_g \Rightarrow \theta_g < \theta_{q\bar{q}}$$

➜ In the soft limit, the same happens irrespective of the total colour charge of the pair:

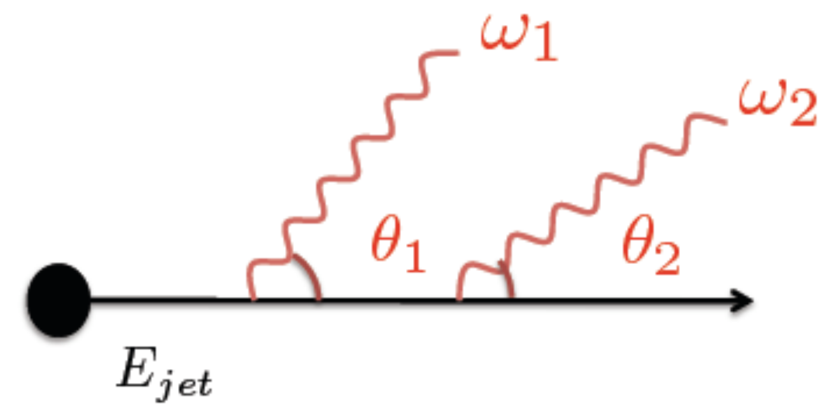




Dead cone angle: $\theta_0 = m/E$

QCD radiation: consequences

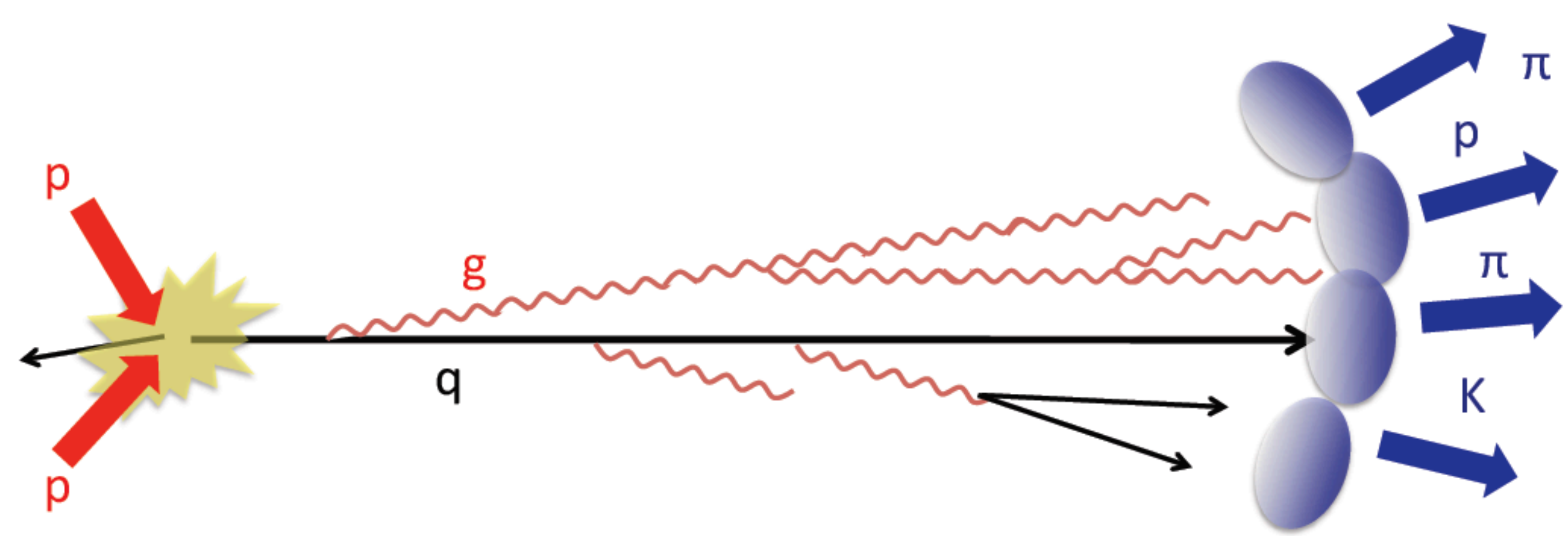
➜ Coherence changes strongly the multiplicity in QCD parton cascades!!!



$$N^{coh} = \int_0^{E_{jet}} \frac{d\omega_1}{\omega_1} \int_{\mu/\omega_1}^{1} \frac{d\theta_1}{\theta_1} \int_0^{\omega_1} \frac{d\omega_2}{\omega_2} \int_{\mu/\omega_2}^{\theta_1} \frac{d\theta_2}{\theta_2} = \frac{1}{4!} \ln^4 E_{jet}/\mu$$

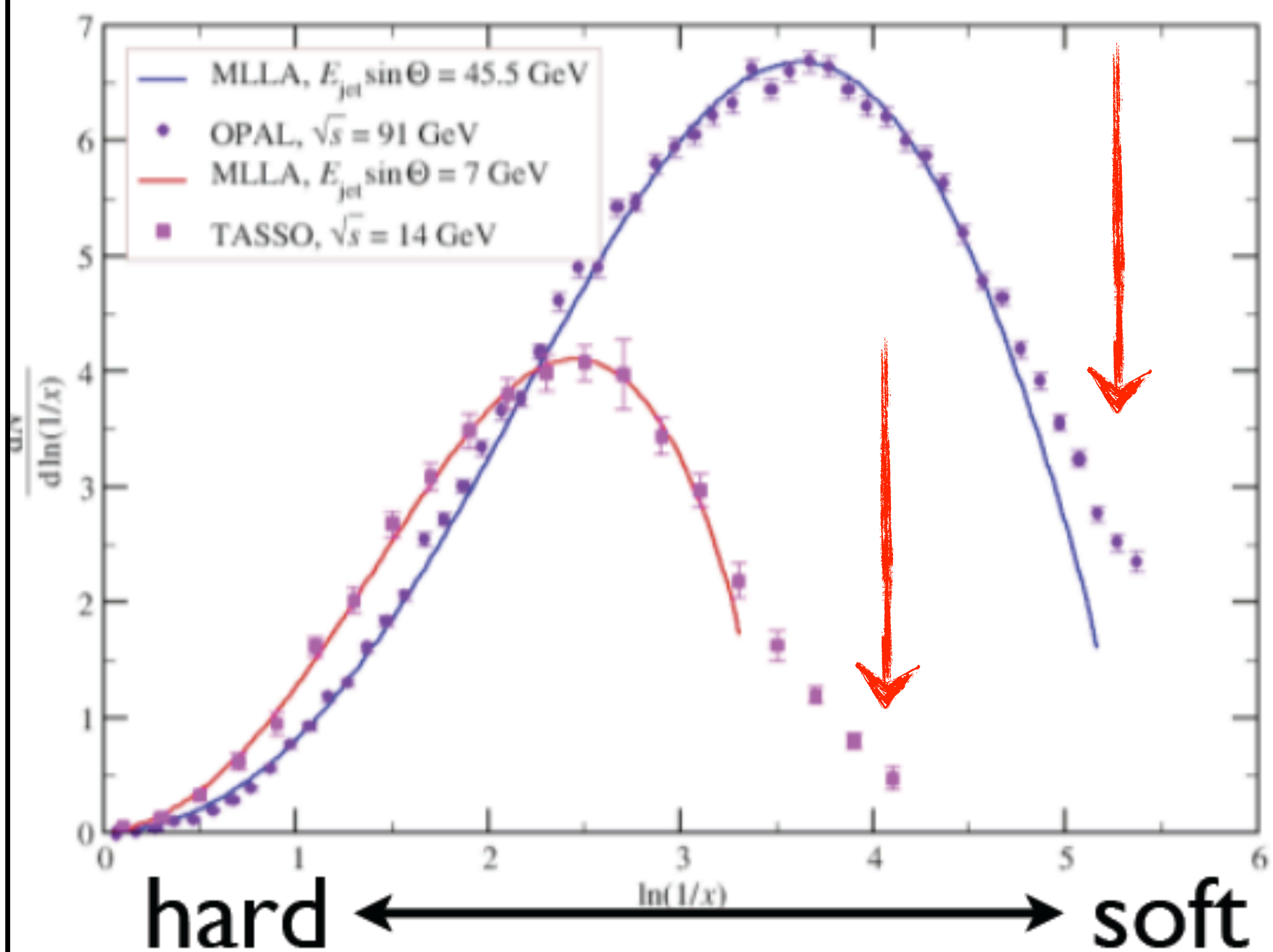
$$N^{incoh} = \int_0^{E_{jet}} \frac{d\omega_1}{\omega_1} \int_{\mu/\omega_1}^{1} \frac{d\theta_1}{\theta_1} \int_0^{\omega_1} \frac{d\omega_2}{\omega_2} \int_{\mu/\omega_2}^{1} \frac{d\theta_2}{\theta_2} > N^{coh}$$

➜ It provides the probabilistic variable for Monte Carlos for QCD branching (PYTHIA, HERWIG, SHERPA,...): ordering variables m^2, θ, k_T (equivalent at high energies).



QCD radiation: consequences

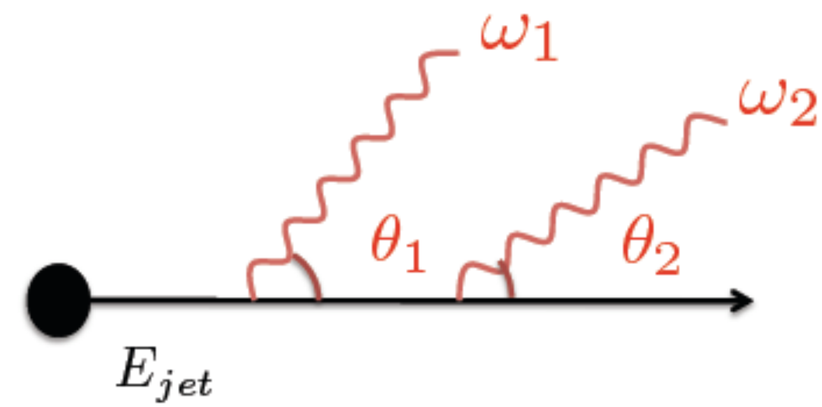
$$\langle dN_q\rangle_\phi \propto \frac{d\omega}{\omega}\frac{d\theta_{pk}}{\theta_{pk}}\Theta\left(\theta_{p\bar{p}}-\theta_{pk}\right)$$



TASSO Collaboration, Z. Phys. C 47 (1990)

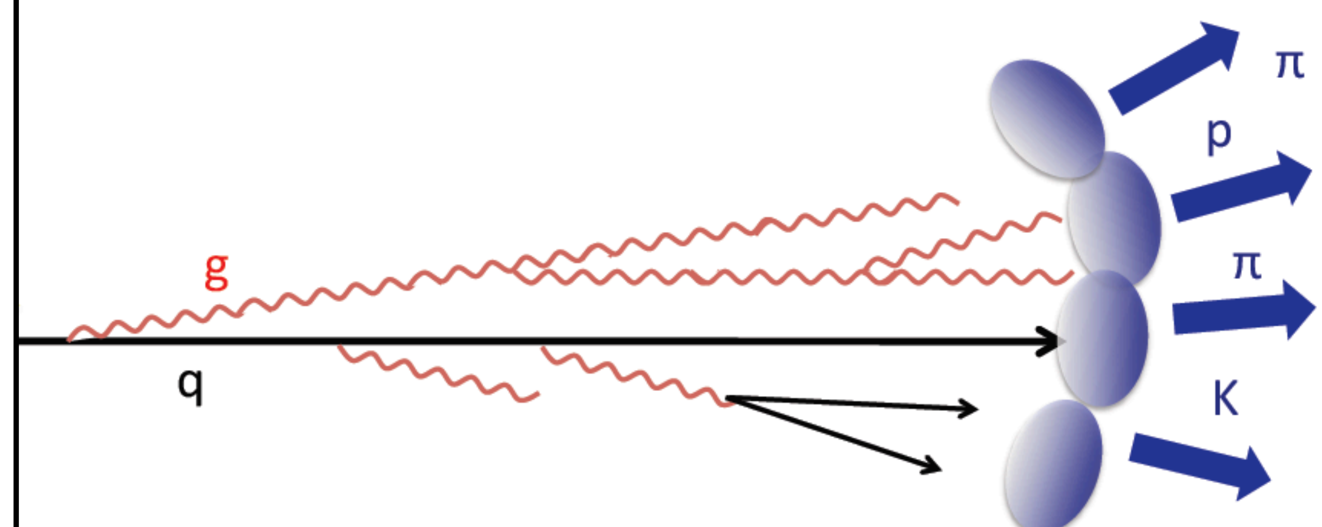
OPAL Collaboration, Phys. Lett. B 247 (1990)

!!!



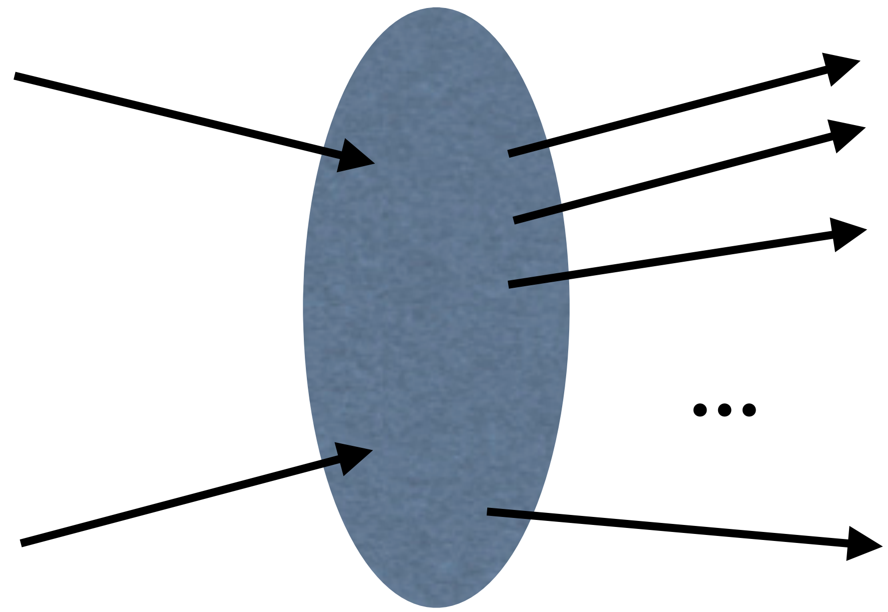
$$\int_{\mu/\omega_2}^{\theta_1}\frac{d\theta_2}{\theta_2}=\frac{1}{4!}\ln^4 E_{jet}/\mu$$

$$\int_{\mu/\omega_2}^{1}\frac{d\theta_2}{\theta_2} > N^{coh}$$

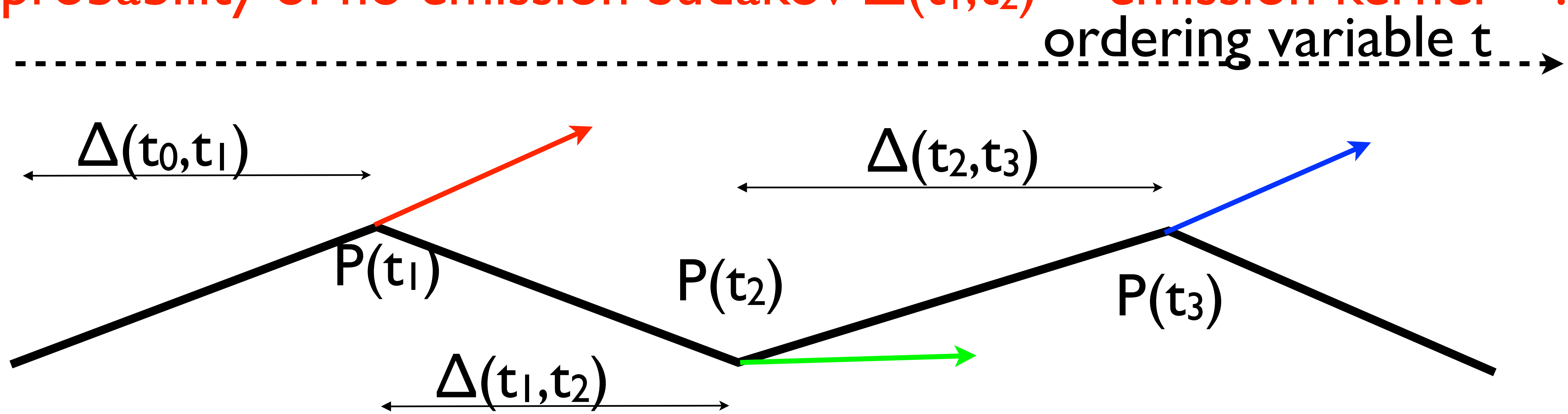


QCD radiation: branching

➜ In QFT, we would like to compute diagrams with any number of external legs that would give the cross section for production of any number of partons. This is not possible for a large number even a tree level - a subject on its own.



➜ The structure of coherent QCD radiation provides a probabilistic picture that allows a sequential treatment of the branching process: iteration of emission kernel at t_1 $P(t_1)$ × probability of no emission Sudakov $\Delta(t_1,t_2)$ × emission kernel × ....



Contents:

1. Electron-positron annihilation into hadrons.

2. Deep Inelastic Scattering.

3. QCD in hadronic collisions: factorization.

4. QCD radiation.

5. Jets.

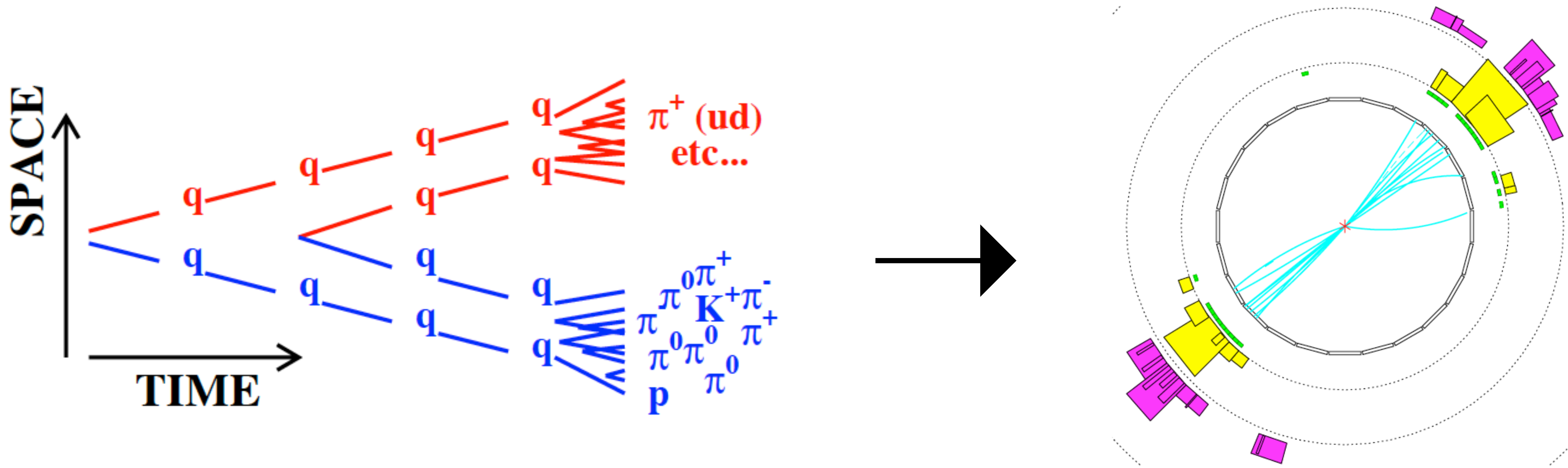
Bibliography:

➜ G. P. Salam, *Elements of QCD for hadron colliders*, CERN Yellow Report CERN-2010-002, arXiv:1011.5131.

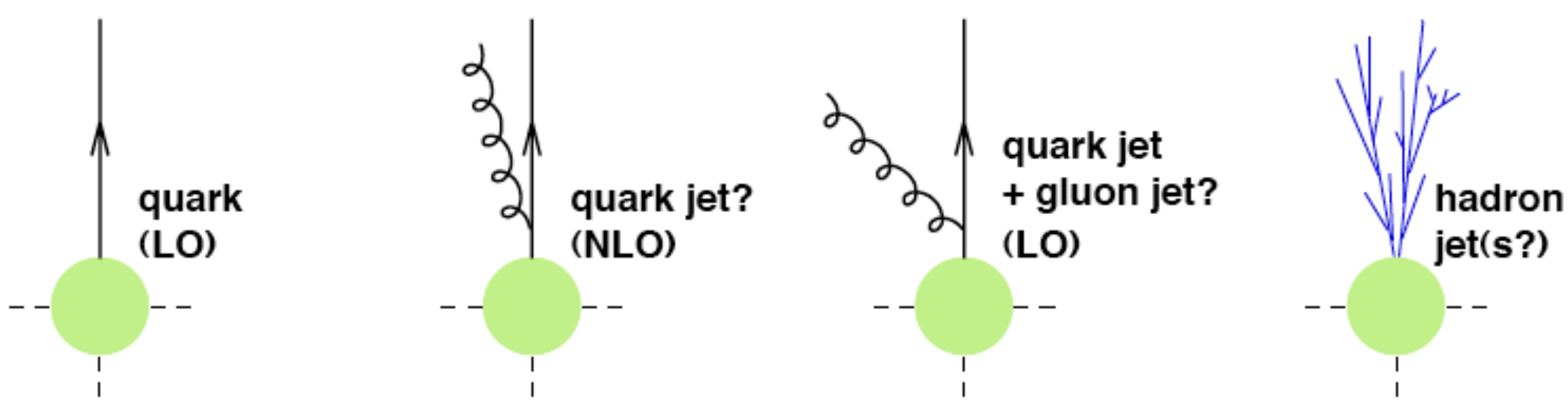
➜ R. K. Ellis et al., *QCD and collider physics*, Cambridge University Press 1996.

Jets: definition

➜ QCD at high energies tends to produce collimated showers of hadrons - jets, due to the collinear singularities of emission kernels, of gluon self interaction, of coherence and of confinement (string).



➜ Partons are ill-defined beyond LO, jets are the closest object to them which can be well defined.



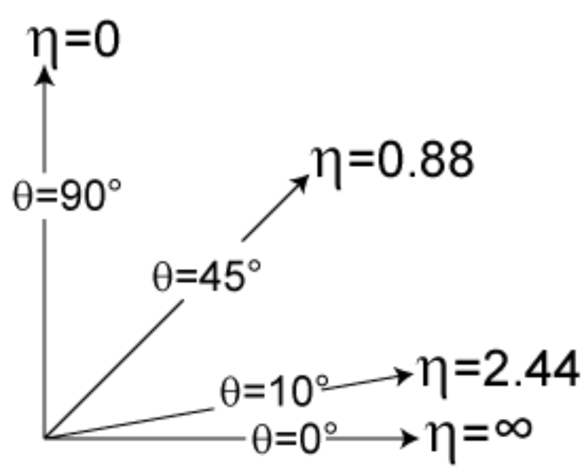
Jets: definition

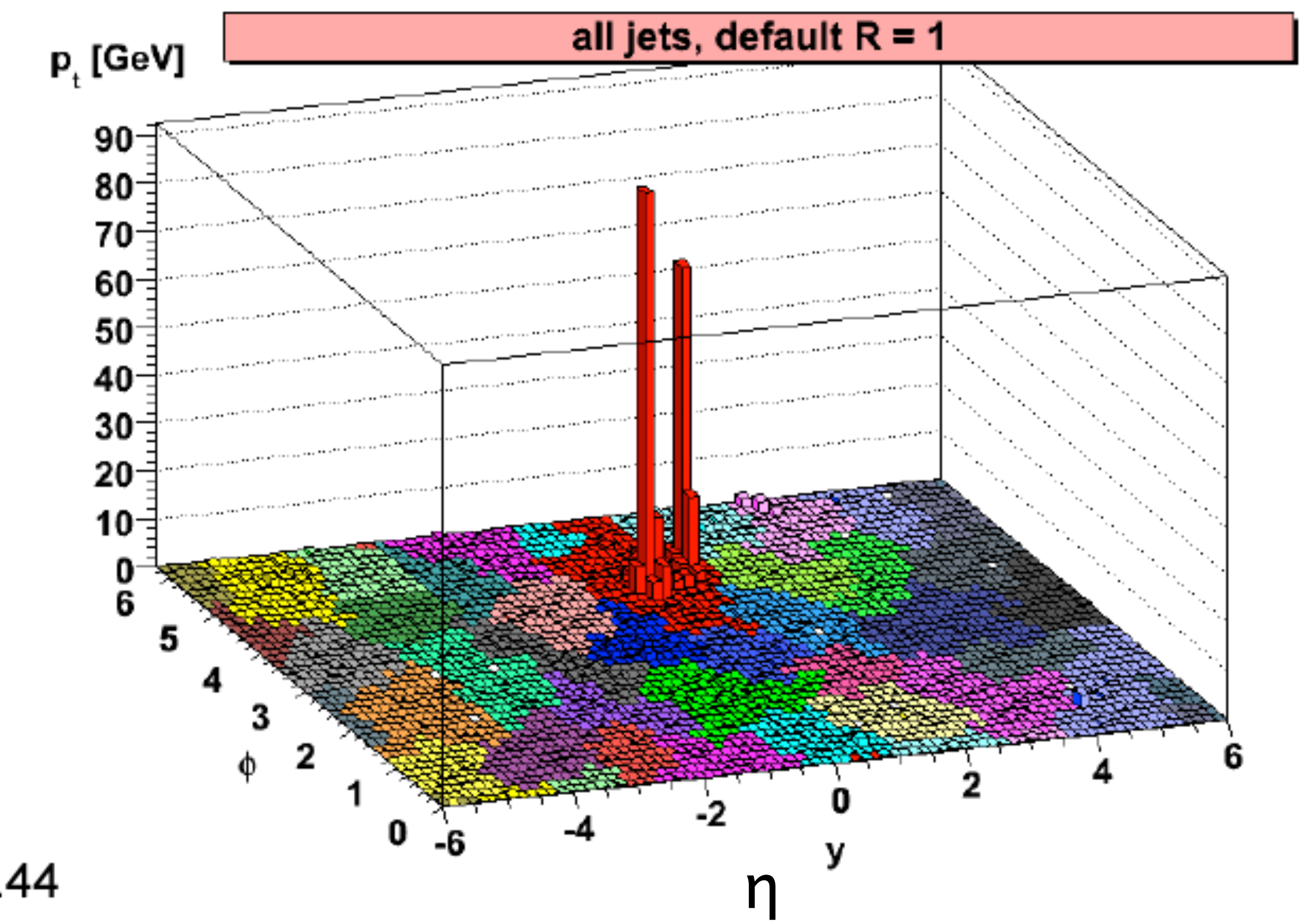
➜ Jets have to be defined: definition must be

➢ Stable at all order of perturbation theory (i.e. $p_i \to p_j + p_k$, $p_j || p_k$ or $p_j \to 0$, $p_k \to 0$: IRC safe).

➢ Insensitive to parton → hadron transition: flow of energy, calorimetric measurements.

Jets and their constituents are located in a plane defined by the azimuthal angle ϕ times the pseudorapidity $\eta = -\ln \mathrm{tg}(\theta/2)$ (= y for $m=0$), with θ the polar angle.



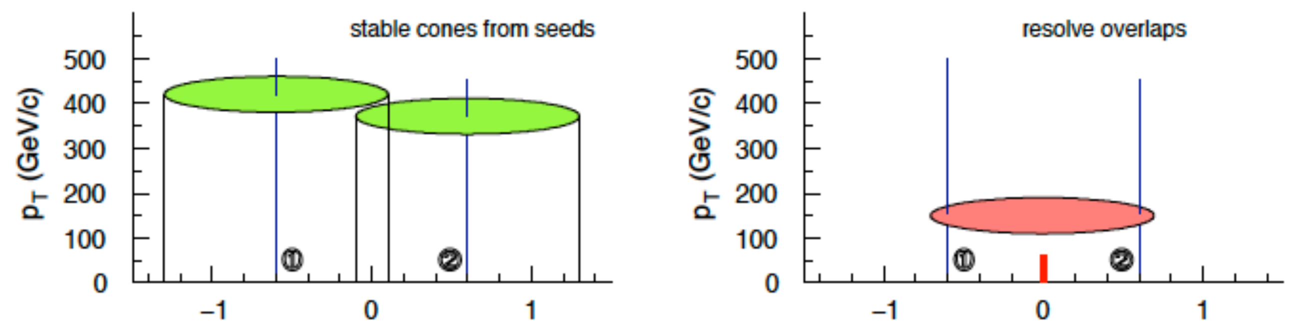


Jets: IRC safety

➜ Algorithms that privilege a particle (use a seed, 'cones') present problems (of comparison experiment/theory) with IRC safety.

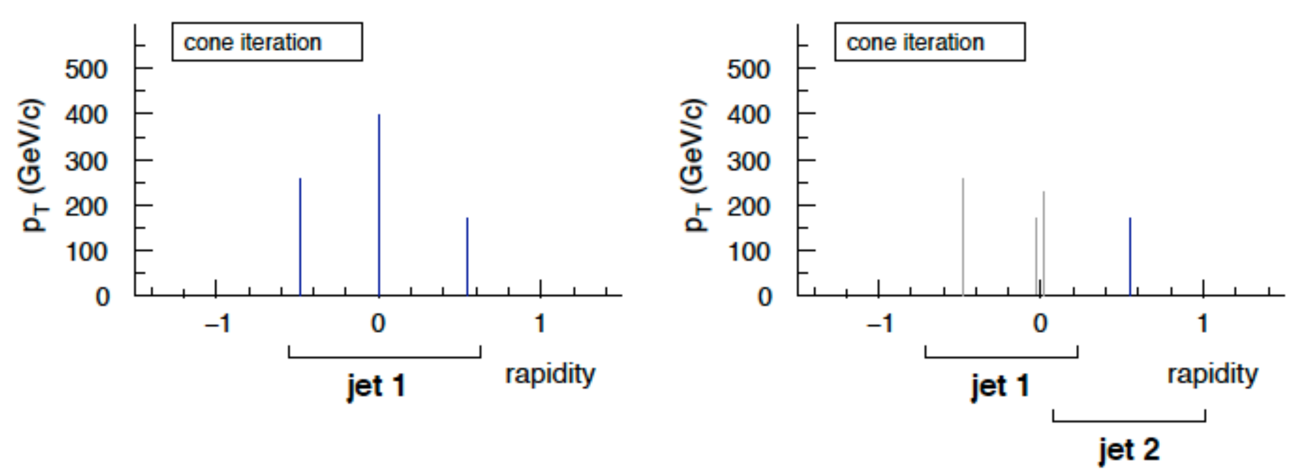
➜ This can be traced back from an incomplete cancellation of divergencies between real and virtual corrections.

infrared unsafety



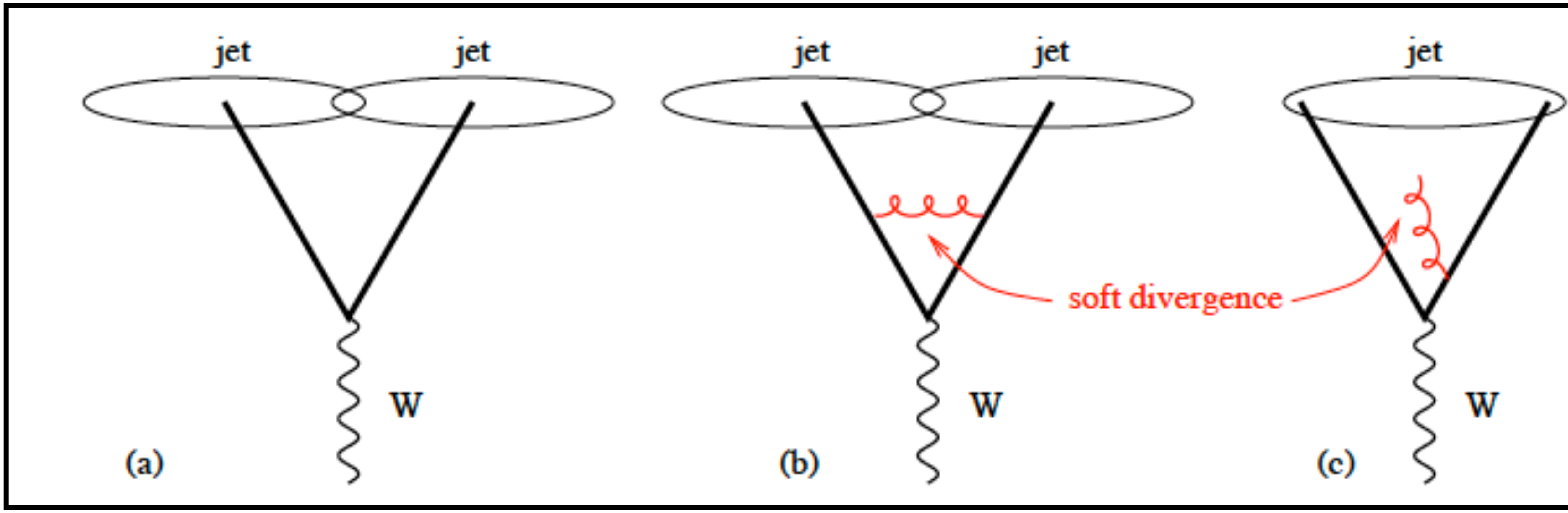
When a soft particle changes the jets

collinear unsafety



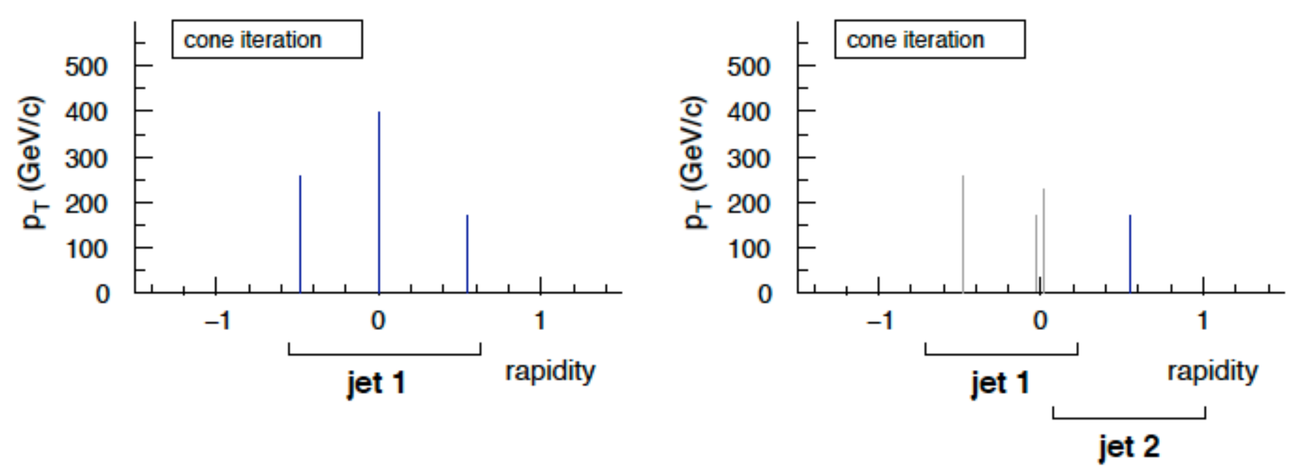
When a collinear splitting changes the jets

Jets: IRC safety



➜ This can be traced back from an incomplete cancellation of divergencies between real and virtual corrections.

collinear unsafety



When a collinear splitting changes the jets

Jets: sequential algorithms

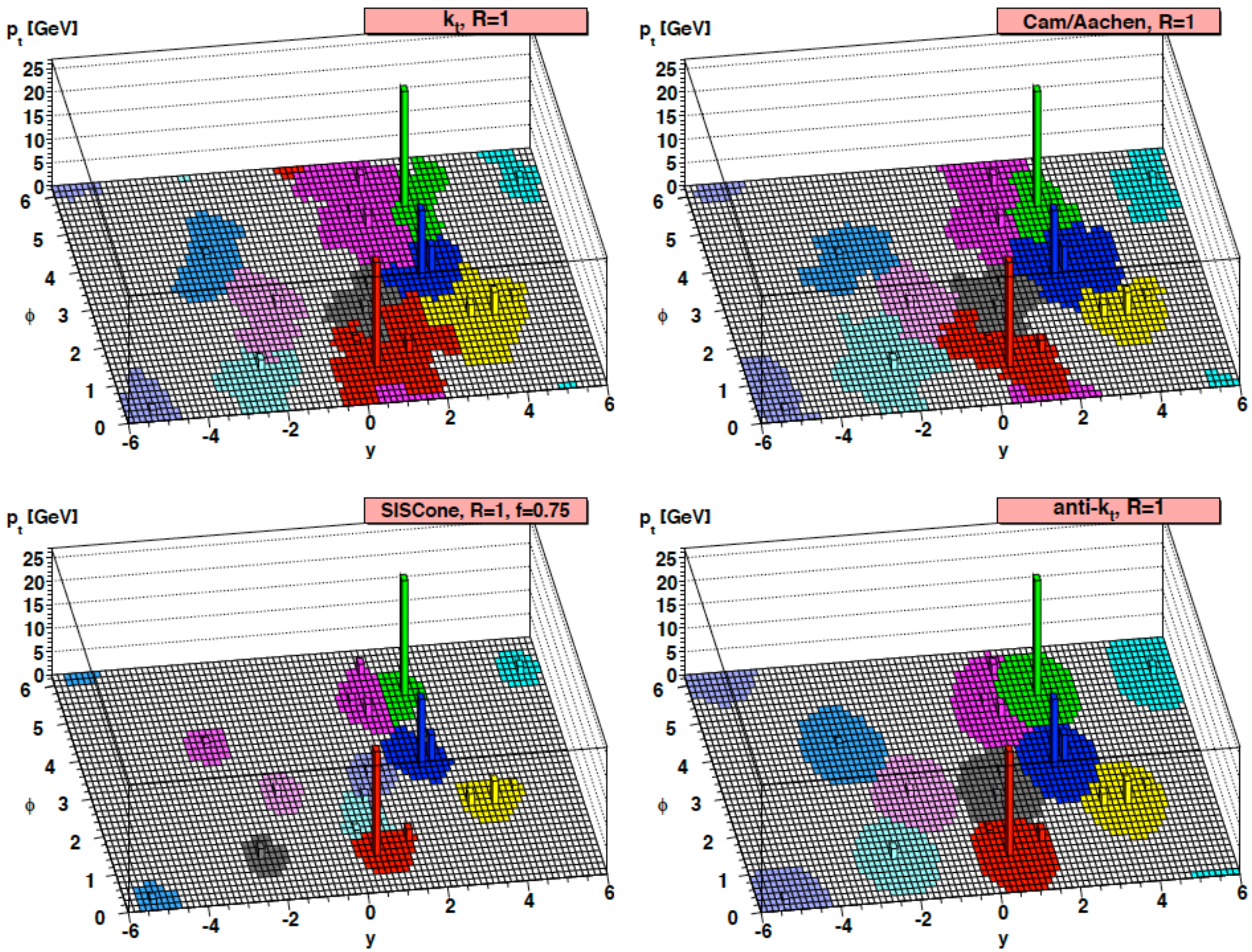
➜ Seedless or sequential recombination algorithms are IRC safe.

➜ Problems of speed of computation solved: mathematical tools imported from other fields.

$d_{iB}=p_{Ti}^{2n}$
$d_{ij}=\min(p_{Ti}^{2n},p_{Tj}^{2n})\Delta R_{ij}^2/R^2$
$\Delta R_{ij}^2=\eta_{ij}^2+\phi_{ij}^2$

Find $\min(d_{iB},d_{ij})$
If d_{iB}, remove i
If d_{ij}, merge i and j
Go on until exhausting the list.

n=1: k_T
n=0: C/A
n=-1: antik_T



Jets: contributions and problems

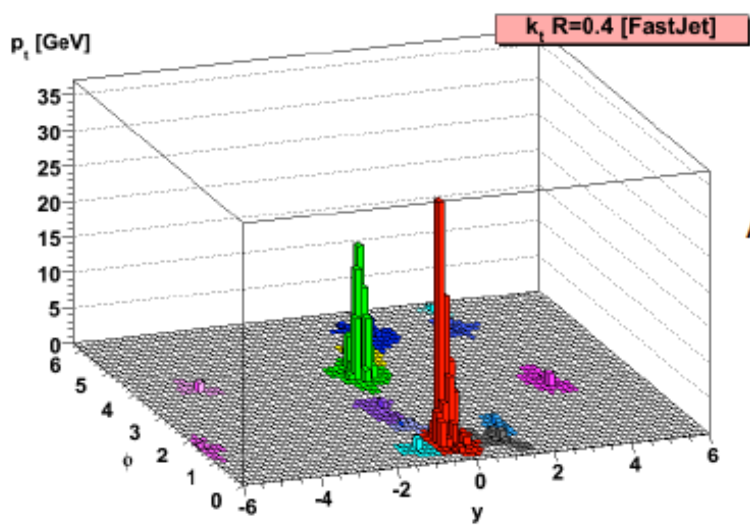
➜ Take a jet of radius R:

$$\langle p_{T\,jet} - p_{T\,parton} \rangle = \langle \delta p_T \rangle = a p_T \alpha_s \ln R - b\frac{1}{R} + cR^2$$

perturbative radiation

non-perturbative contribution

underlying event, pileup⇒subtraction



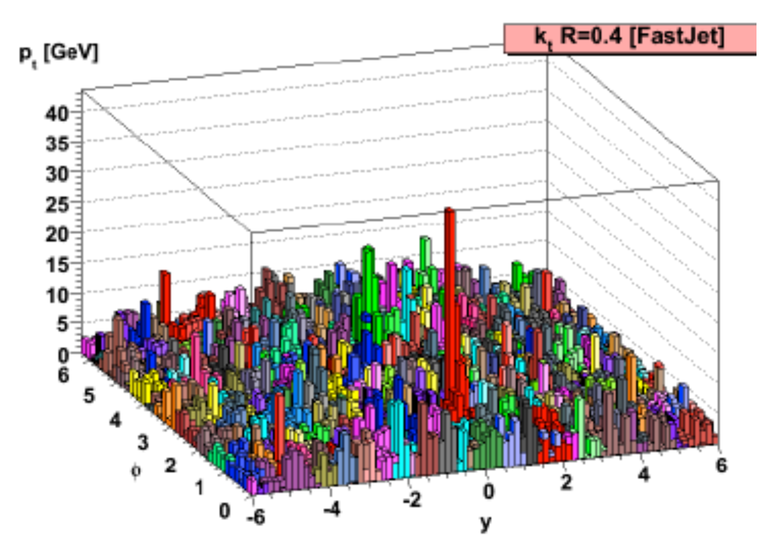
An example hard event

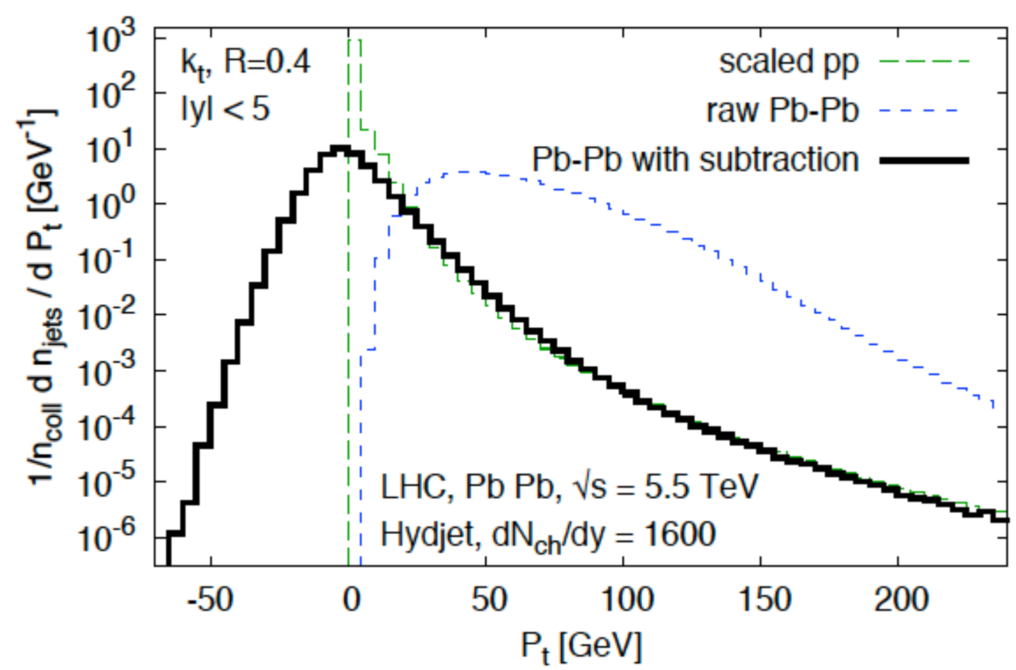
$p_t \sim 100$ GeV

Generated with Pythia

Mixed into LHC HI environment

HydJet, $dN_{ch}/dy \simeq 1600$





Ongoing effort to deal with this both in pp and in PbPb at LHC.

Jets: contributions and problems

➜ Take a jet of radius R:

$$\langle \delta p_t \rangle_{\text{pert}} = \int \frac{d\theta^2}{\theta^2} \int dz \underbrace{p_t\big(\max[z, 1-z] - 1\big)}_{\delta p_t} \frac{\alpha_s\big(\theta\,(1-z)\,p_t\big)}{2\pi} P_{qq}(z)\,\Theta\big(\theta - f_{\text{alg}}(z)R\big)$$

perturbative radiation

non-perturbative contribution

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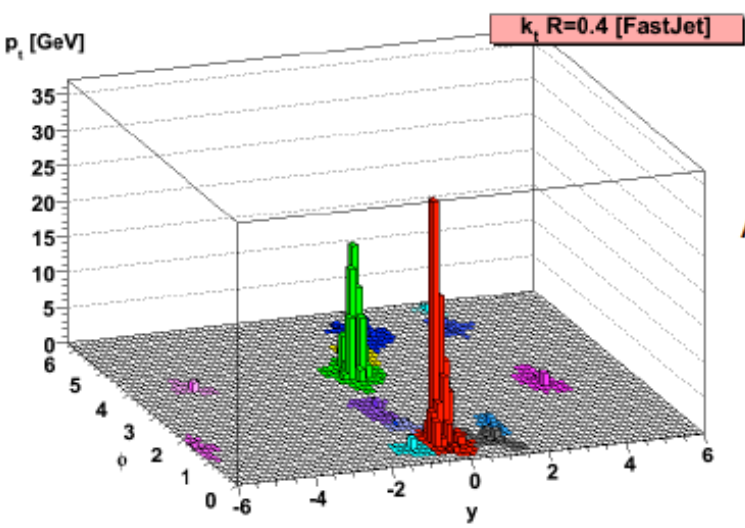

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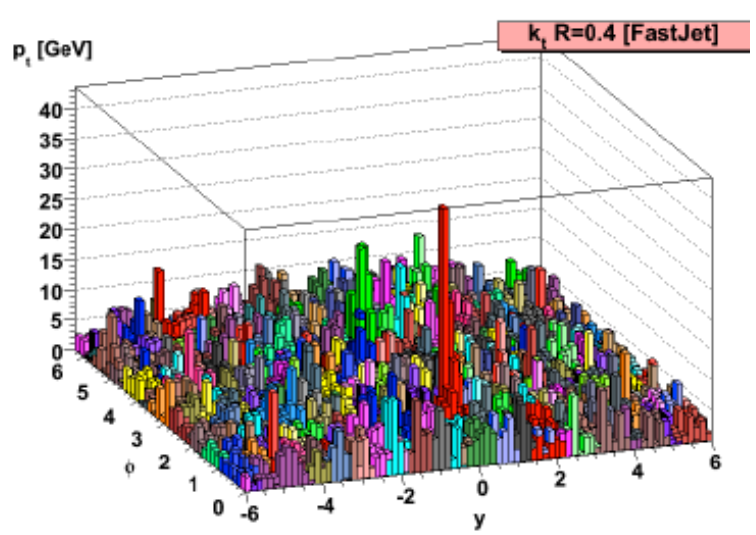

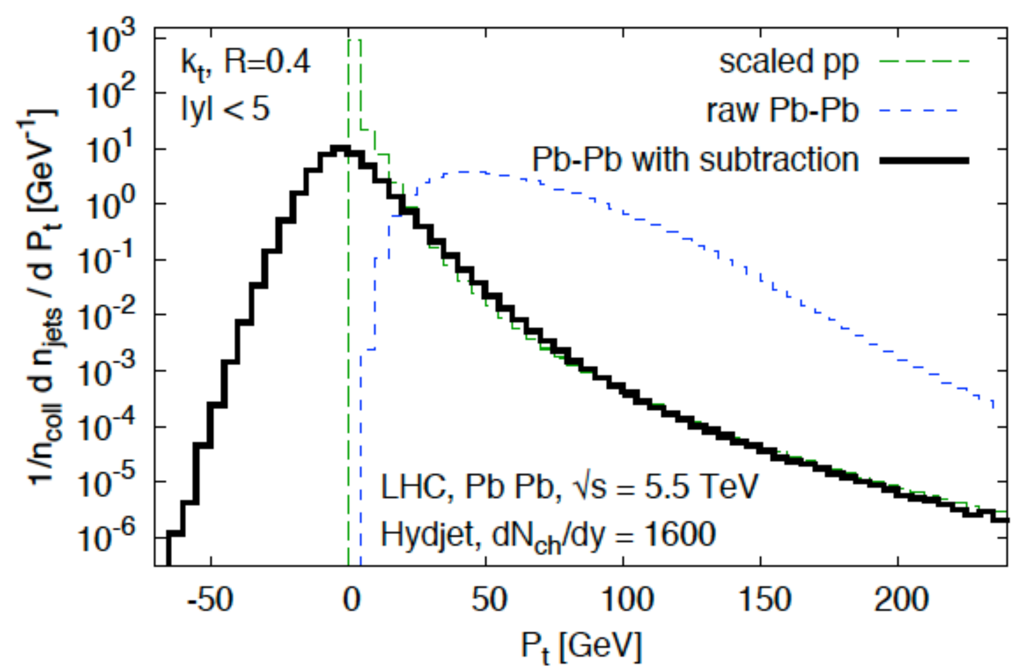

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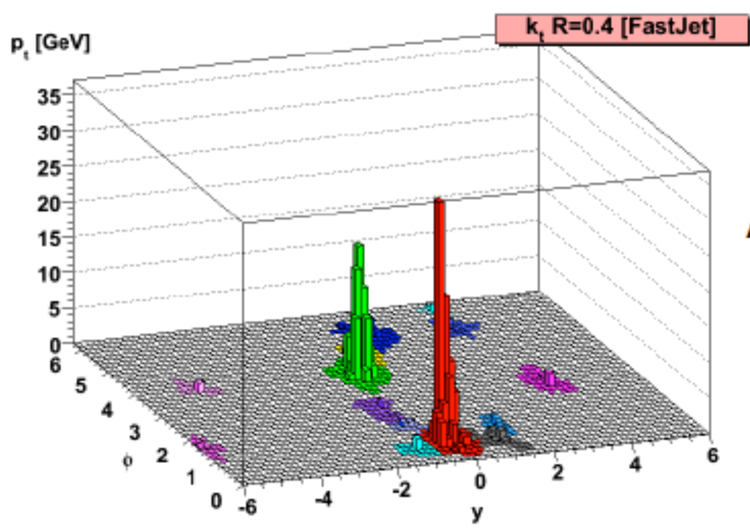

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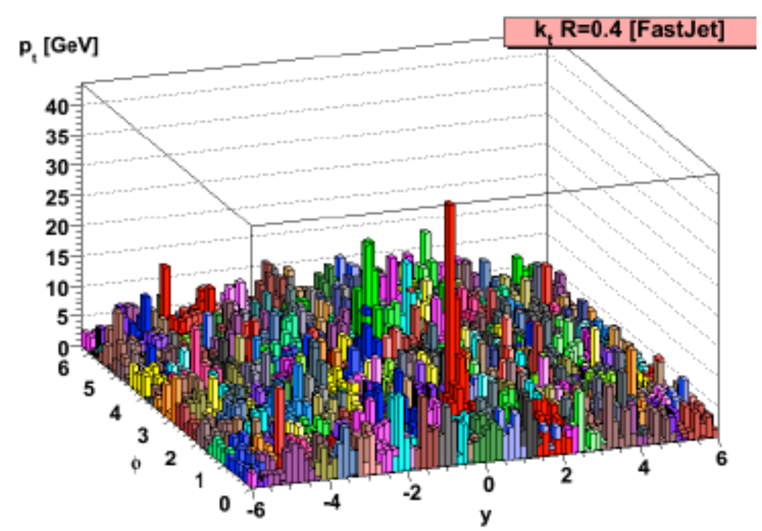

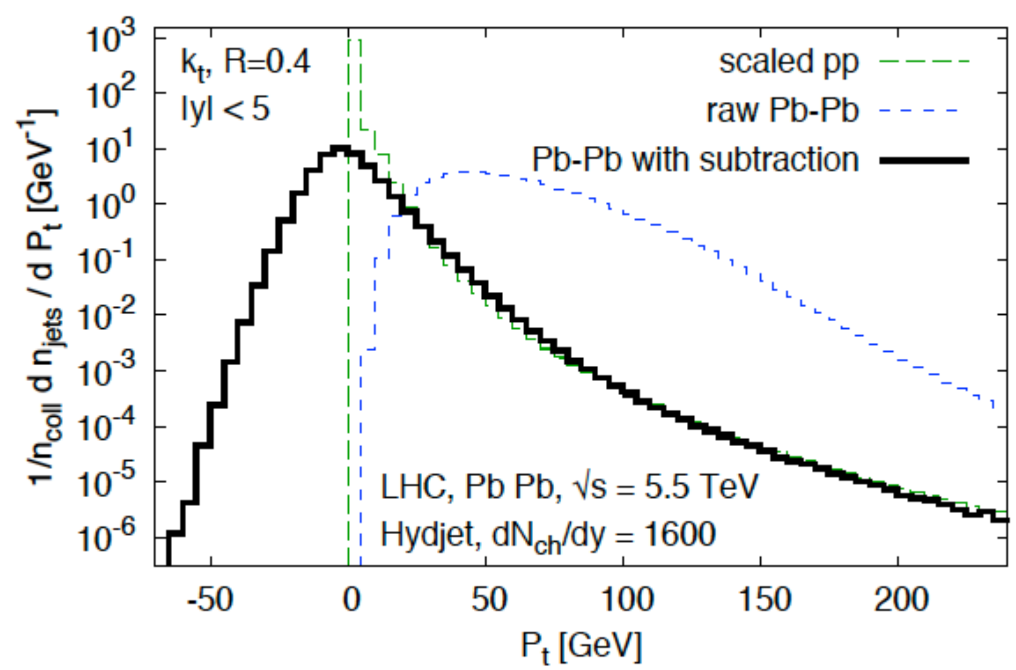

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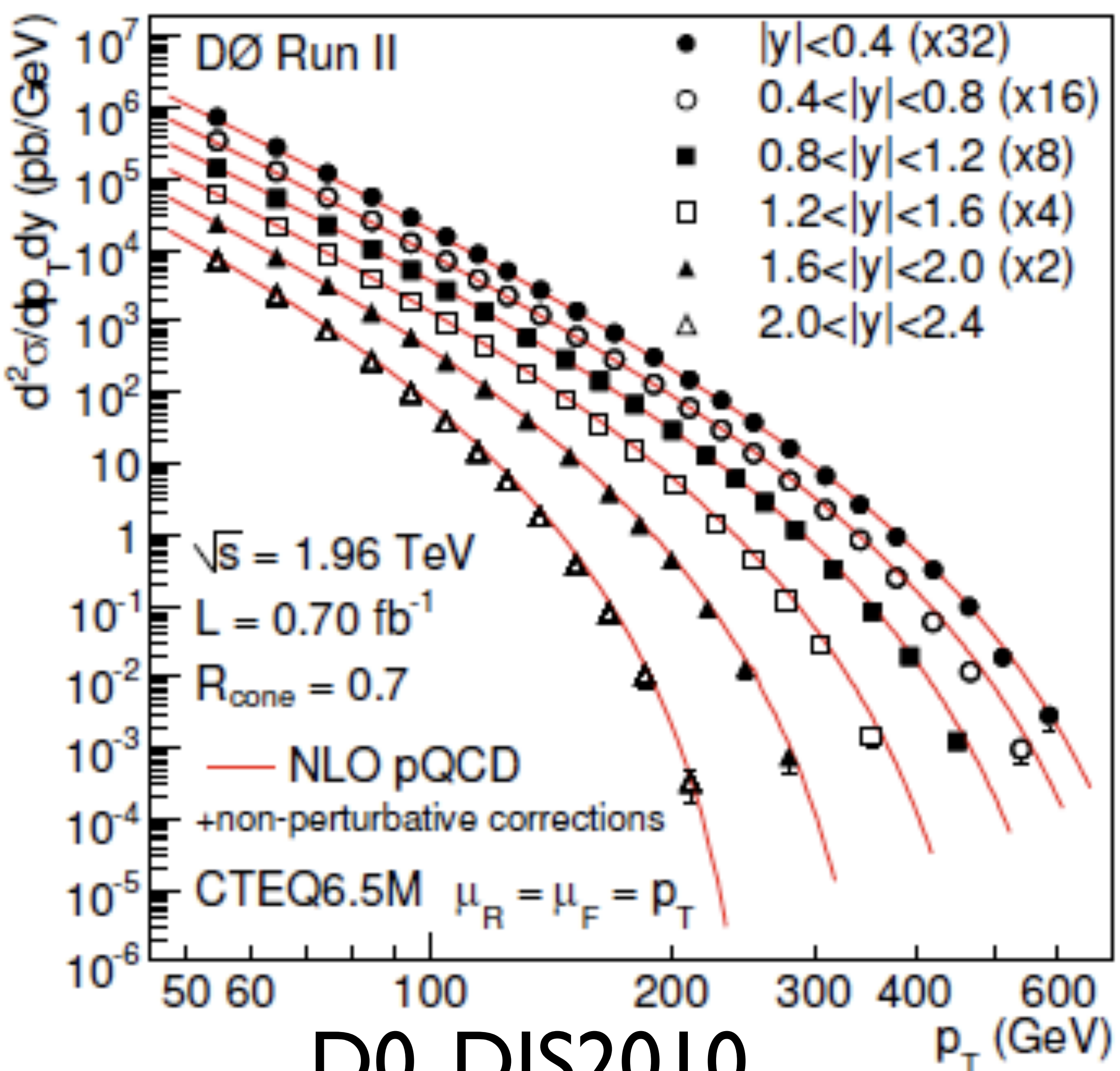
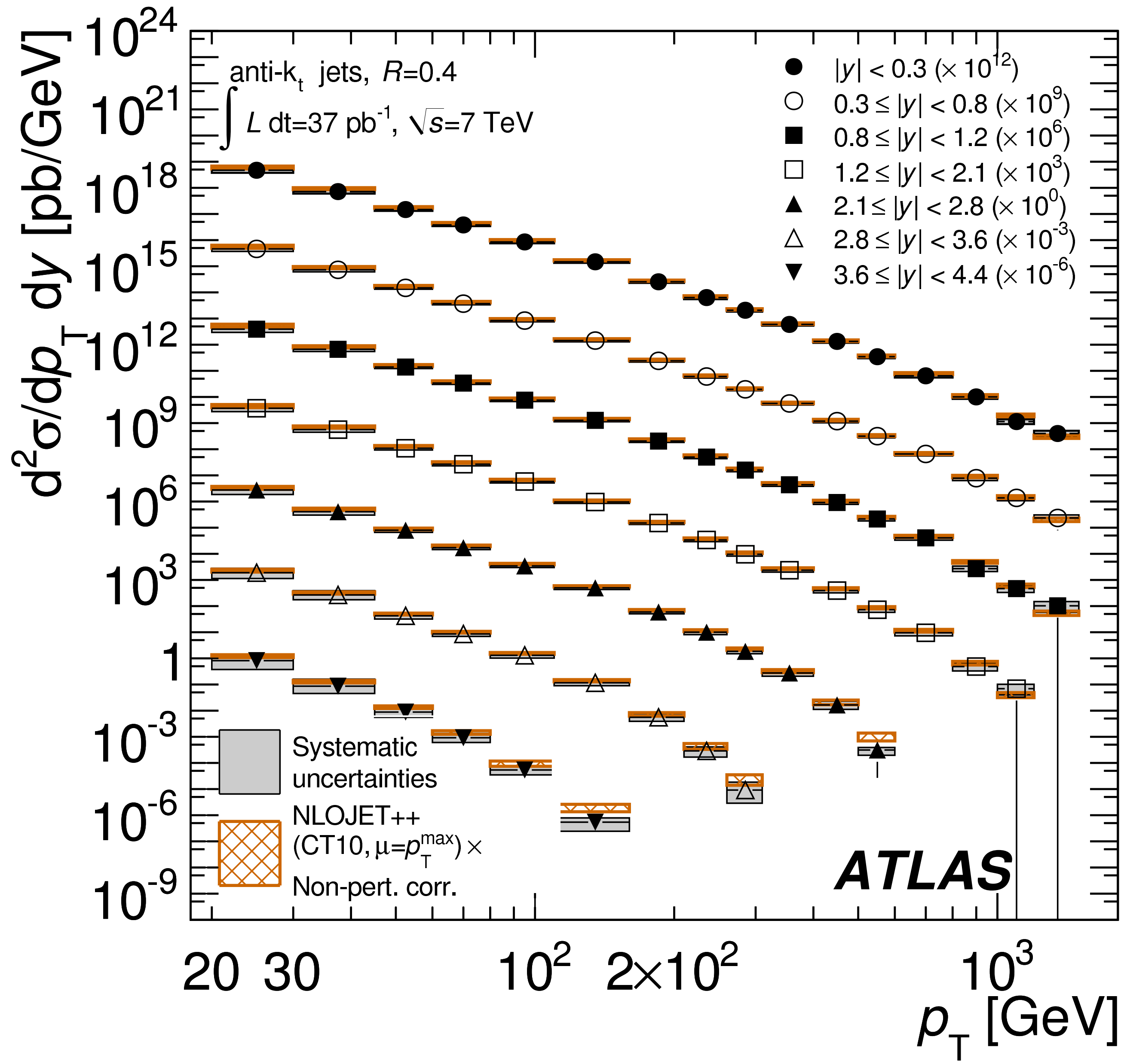


D0, DIS2010



Mixed into LHC HI environment

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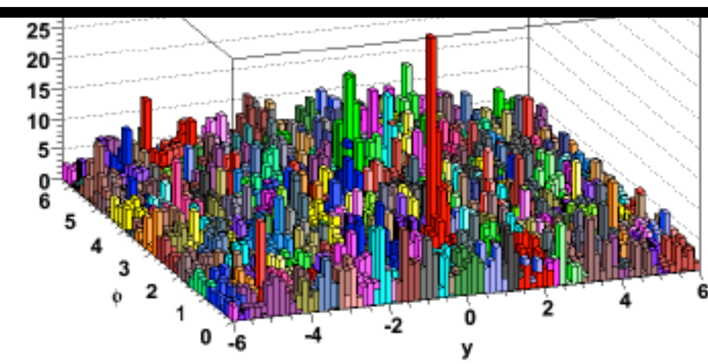

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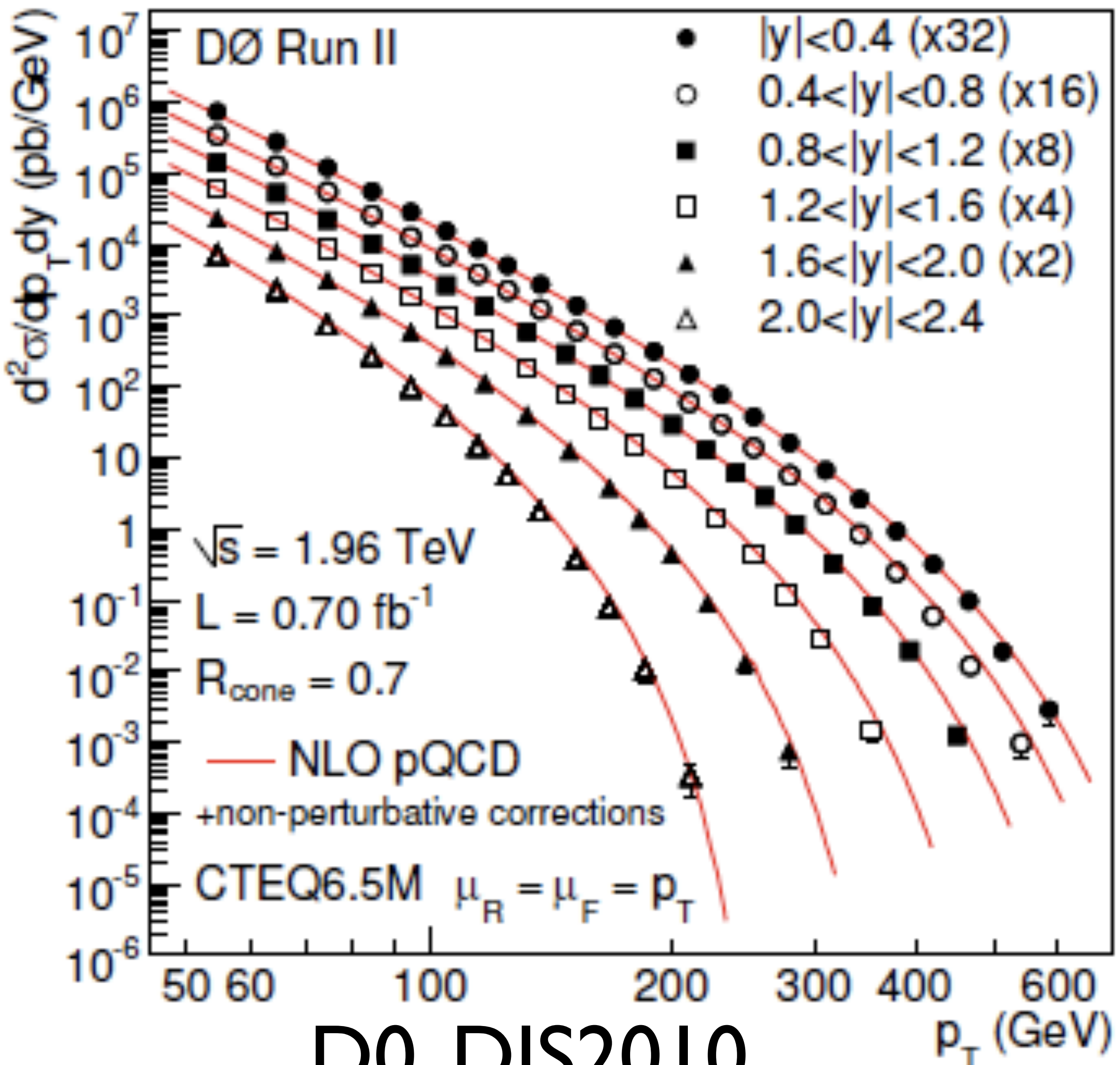

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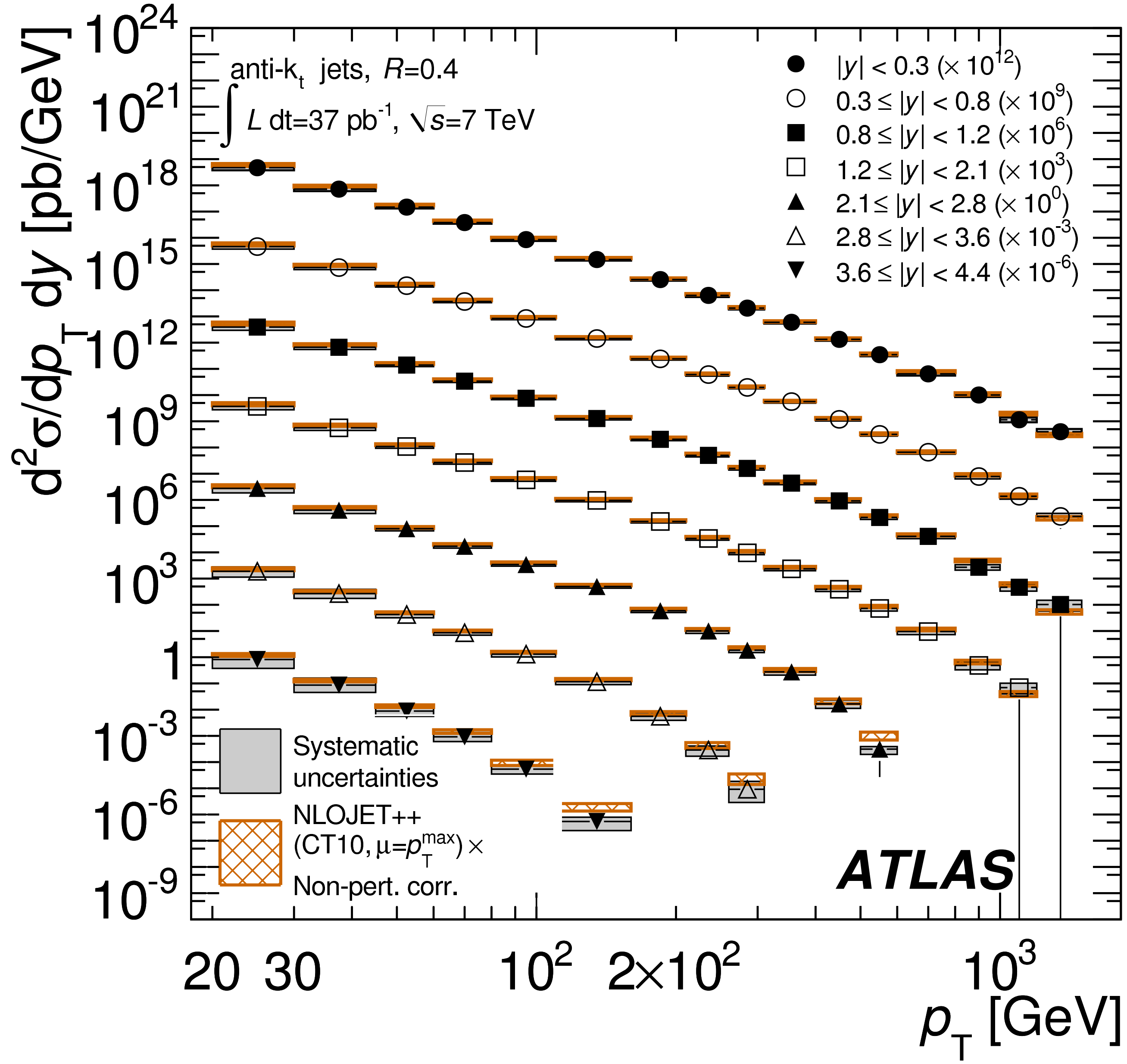

Mixed int

➜ Even if you are not a QCD person, you need it for EW/BSM physics: all non-purely leptonic signals involve jets e.g. H→bbar, H→Z's, W's decaying into q's,...

deal

pp

and in PbPb at LHC.