

QCD

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Third IDPASC school

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---Introduction.QED and QCD differences

---Asymptotic freedom(Callan-Symanzyk equation,
deep inelastic,jets)

---Confinement

---Symmetries.Isospin,Quiral symmetry,
(spontaneus symmetry breaking,Goldtone bosons
Scale symmetry(trace anomaly,vacuum structure,
instantons,topological charge

---QCD at high temperature.(Phase transitions,
Confinement and quiral symmetry transitions
Elliptical flow.Little bang versus big-bang)

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$$

$$\psi ~\rightarrow~ \psi' = \exp\left[iq\alpha(x)\right]\psi,$$

$$\bar{\psi} ~\rightarrow~ \bar{\psi}' = \exp\left[-iq\alpha(x)\right]\bar{\psi},$$

$$A_n \rightarrow A'_n = A_n - \partial_n a(\vec{x}),$$

$$D_\mu=\partial_\mu+iqA_\mu,$$

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi - q\bar{\psi}\gamma^\mu A_\mu\psi$$

$$\mathcal{L}_{QED} = \mathcal{L}_0 - j^\mu A_\mu, ~~ j^\mu = q\bar{\psi}\gamma^\mu\psi.$$

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu},~~ F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu.$$

$$\mathcal{L}=\sum_q\bar{\psi}_q^J i\gamma^\mu \partial_\mu \psi_q^k-\sum_qm_q\bar{\psi}_q^J \psi_q^J,$$

$$el~color,~j,k=1,2,3,~y~q=d,u,s,c,b,t$$

$$\psi_q(x)\longrightarrow \psi'_q(x)=\exp\left[ig_s\alpha_a(x)T_a\right]\psi_q(x),$$

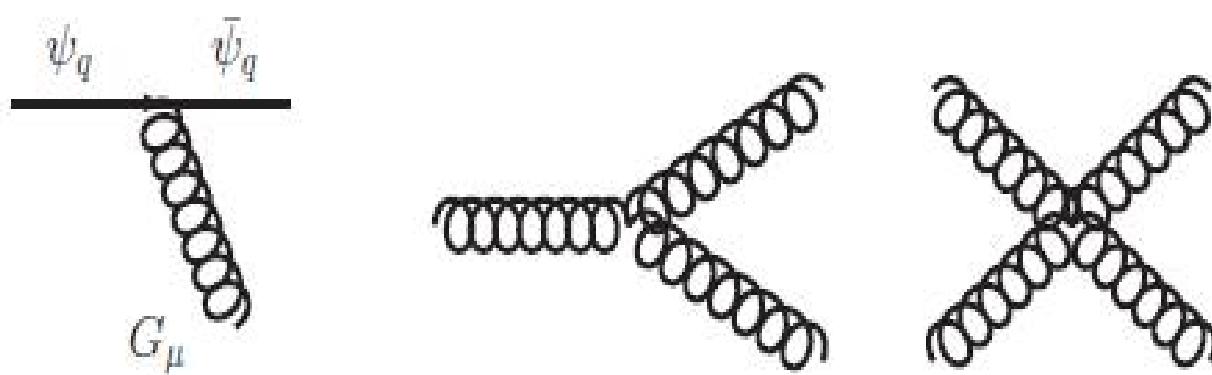
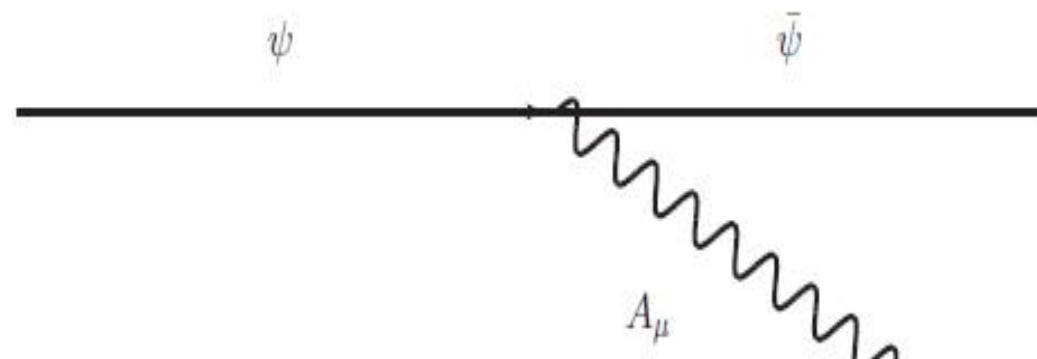
$$[T_a,T_b]=if_{abc}T_c,$$

$$G^a_\mu\longrightarrow G^a_\mu=\bar{G}^a_\mu-\partial_\mu\alpha_a(x)-g_sf_{abc}\alpha_b(x)G^c_\mu,$$

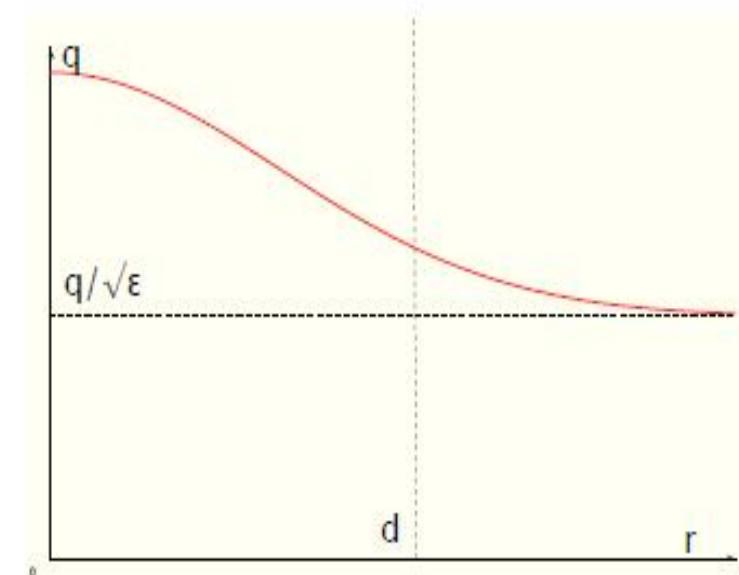
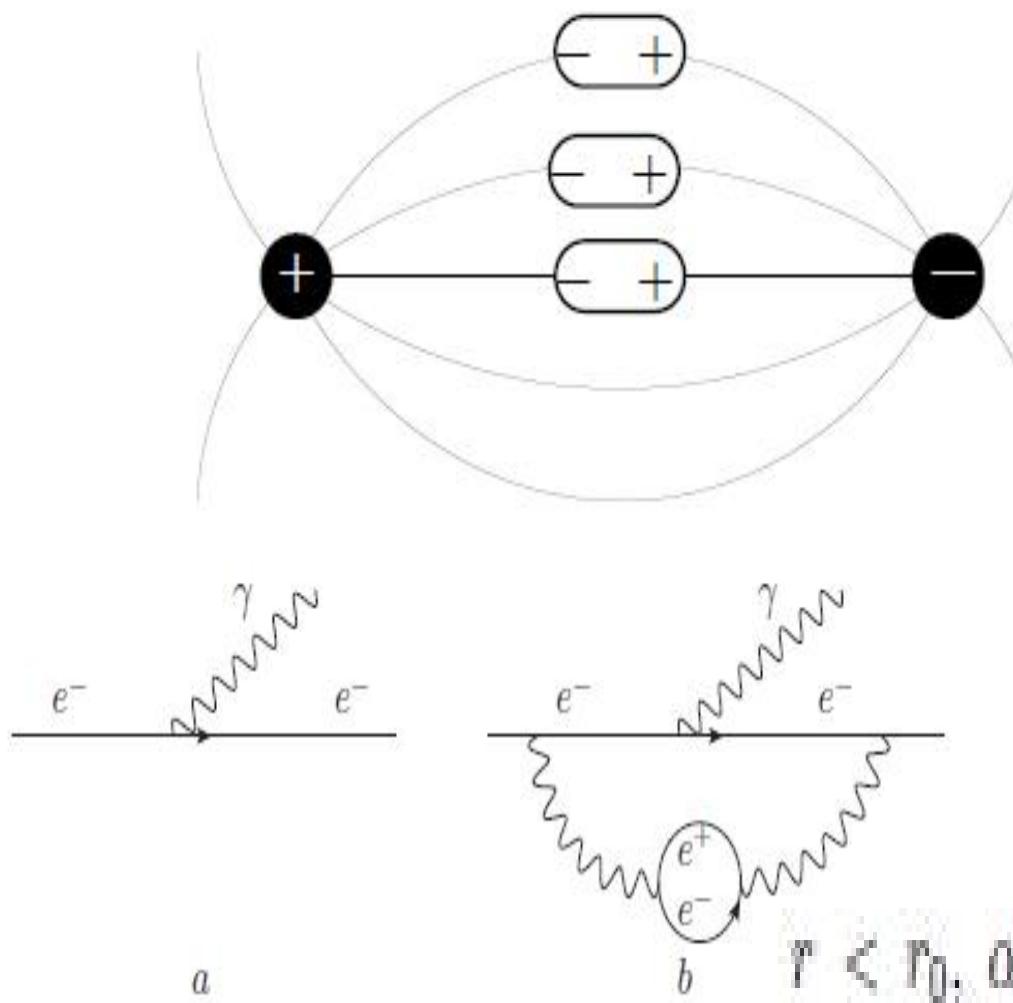
$$D_\mu = \partial_\mu + ig_s T_a G^a_\mu$$

$$\mathcal{L}_{QCD}=\bar{\psi}_q i\gamma^\mu D_\mu \psi_q-m_q\bar{\psi}_q \psi_q=\bar{\psi}_q i\gamma^\mu \partial_\mu \psi_q-g_s\bar{\psi}_q i\gamma^\mu \psi_q T_a G^a_\mu-m_q\bar{\psi}_q \psi_q,$$

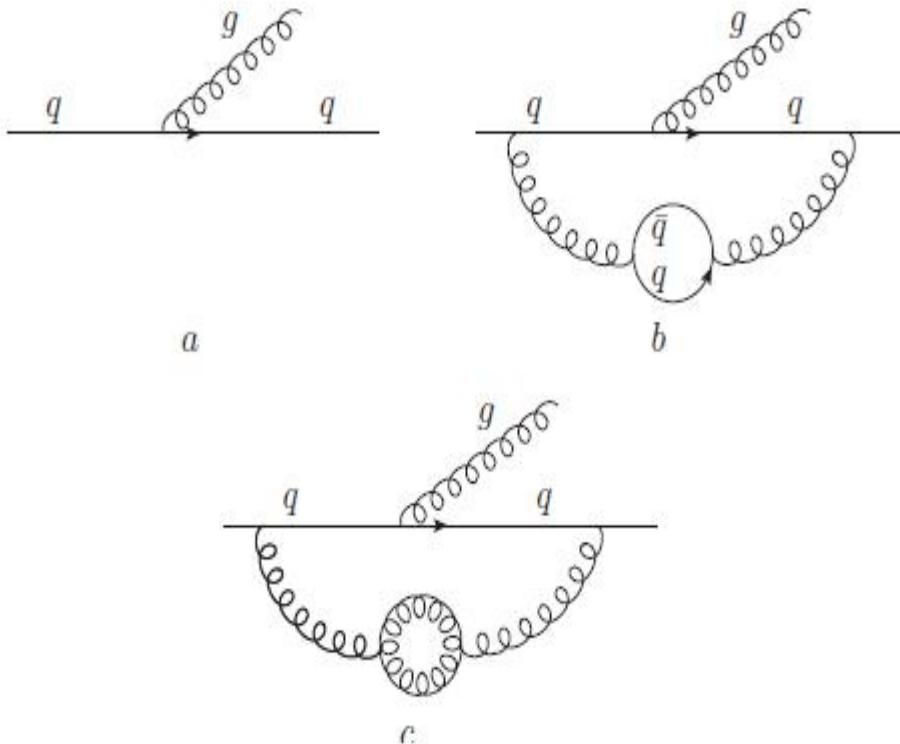
$$-\frac{1}{4}F^{\mu\nu}_aF^a_{\mu\nu},~~F^a_{\mu\nu}=\partial_\mu G^a_\nu-\partial_\nu G^a_\mu-g_sf_{abc}G^b_\mu G^c_\nu.$$



Charge screening in QED and in QCD. Asymptotic Freedom



$$\alpha_{EM}(r) = \frac{\alpha_{EM}(r_0)}{1 + \frac{\alpha_{EM}(r_0)}{3\pi} \ln(r^2/r_0^2)}$$



$$\alpha_s(r) = \frac{\alpha_s(r_0)}{1 - \frac{\alpha_s(r_0)}{12\pi}(11N_c - 2n_f)\ln(r^2/r_0^2)} \quad r \rightarrow 0, \alpha_s(r) \rightarrow 0,$$

$$\alpha_s = g_s^2/(4\pi)$$

$$\Gamma_B(s, \alpha_0, \Lambda^2) = Z(\mu^2)^{-1} \Gamma(s, \alpha(\mu^2), \mu^2)$$

$$d\Gamma_B/d\mu^2 = 0$$

$$\mu^2 \frac{\partial \Gamma}{\partial \mu^2} + \mu^2 \frac{\partial \alpha}{\partial \mu^2} \frac{\partial \Gamma}{\partial \alpha} - \mu^2 \frac{\Gamma}{Z} \frac{\partial Z}{\partial \mu^2} = 0.$$

Defining $\beta(\alpha) = \mu^2 \frac{\partial \alpha}{\partial \mu^2}$ and $\gamma(\alpha) = -\frac{\mu^2}{Z} \frac{\partial Z}{\partial \mu^2}$

group equation (RGÉ),

$$\left[\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha) \frac{\partial}{\partial \alpha} + \gamma(\alpha) \right] \Gamma(s, \alpha(\mu^2), \mu^2) = 0.$$

Callan-Symanzyk Equation

$$\ln\left(\frac{\mu^2}{\mu_0^2}\right) = \int_{\alpha(\mu_0^2)}^{\alpha(\mu^2)} \frac{dx}{\beta(x)}$$

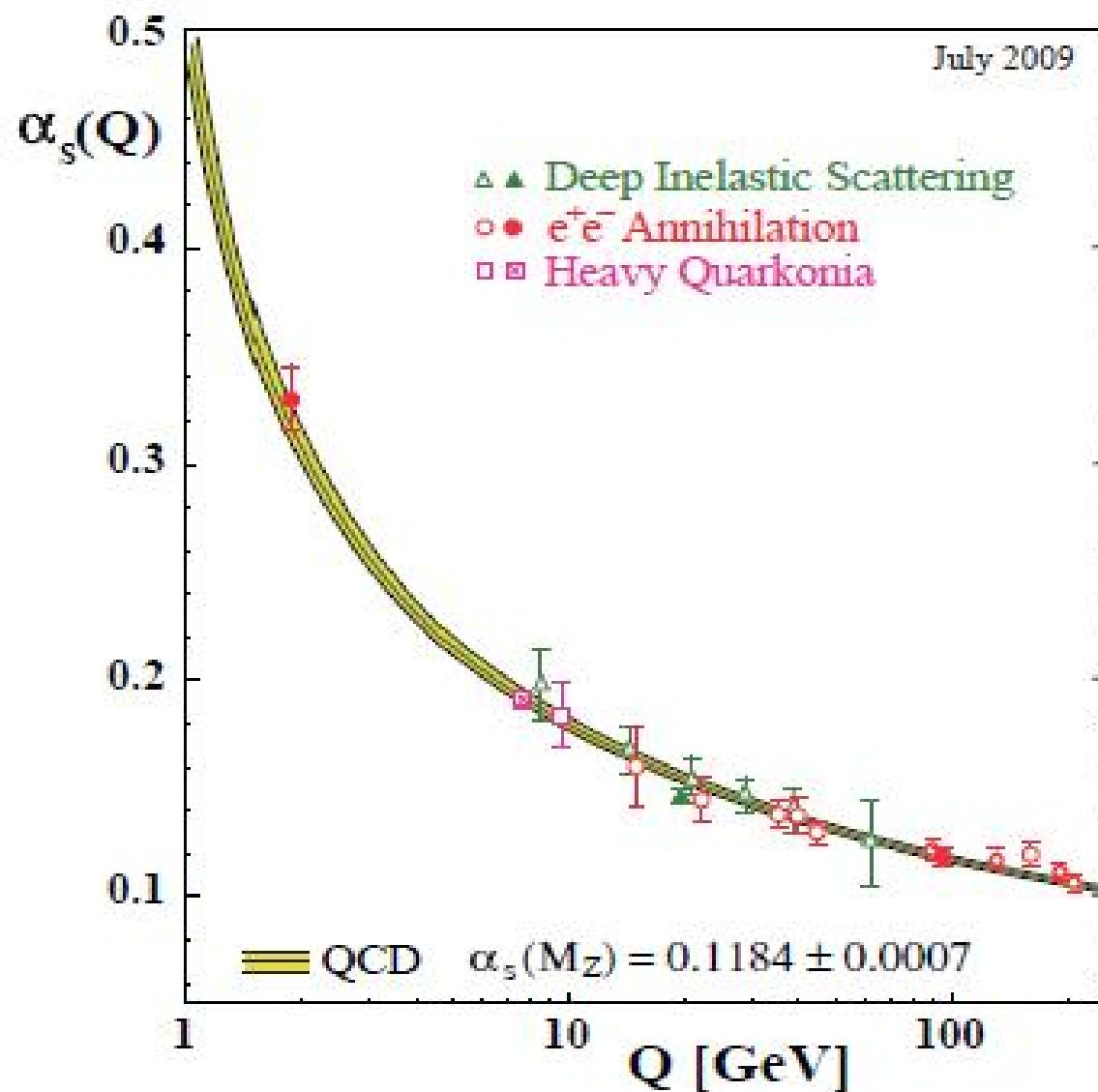
$$\beta(\alpha_s) = -b_0\alpha_s^2 - b_1\alpha_s^3 - b_2\alpha_s^4 + O(\alpha_s^5)$$

$$b_0 = \frac{33 - 2n_f}{12\pi},$$

$$b_1 = \frac{153 - 19n_f}{24\pi^2},$$

$$b_2 = \frac{77139 - 15099n_f + 325n_f^2}{3456\pi^3}$$

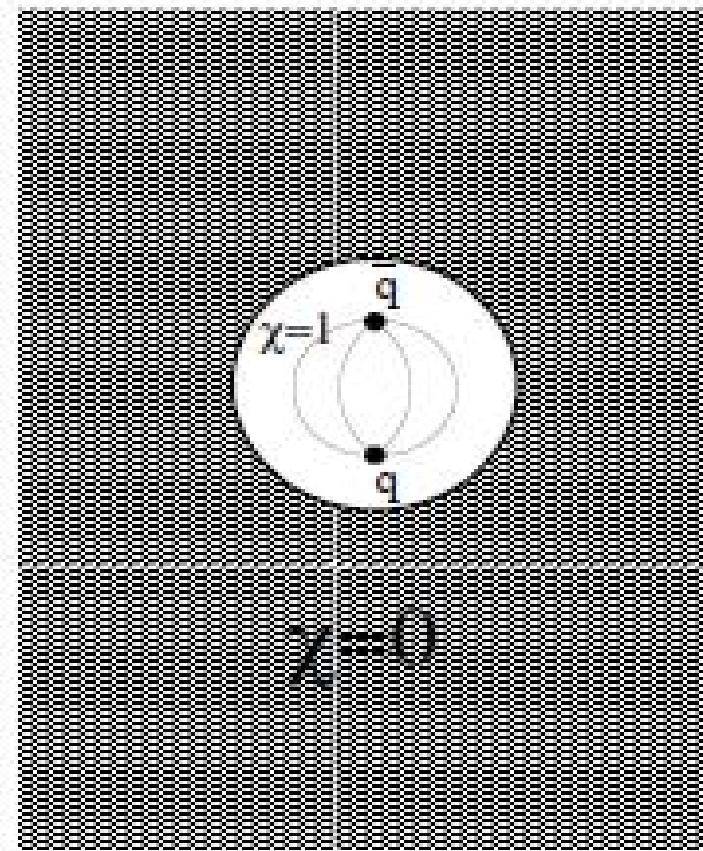
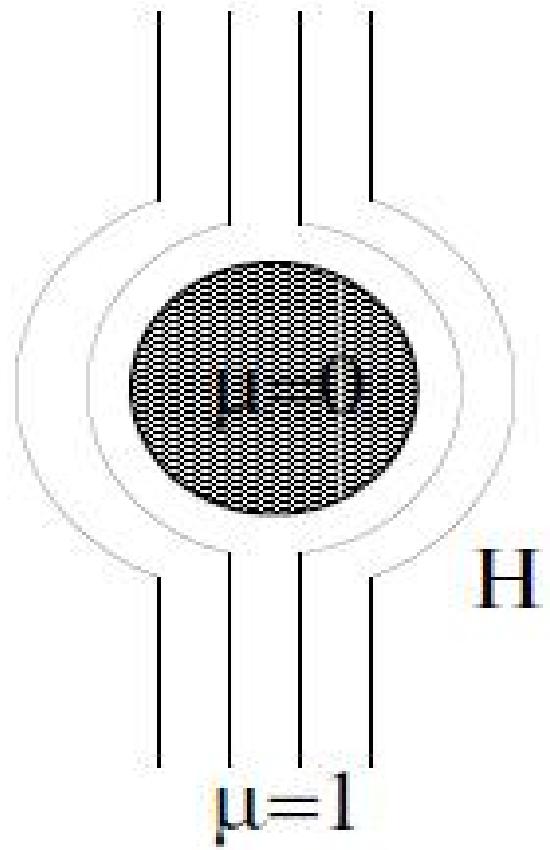
$$\alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + \alpha_s(\mu_0^2)b_0 \ln(\mu^2/\mu_0^2)}$$

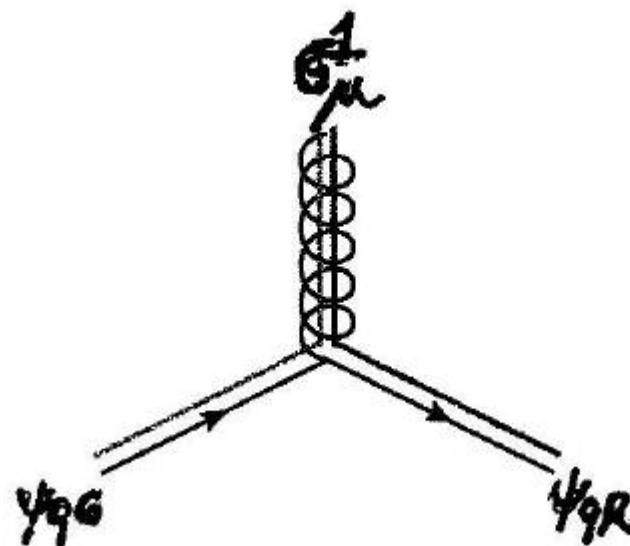


CONFINEMENT

Analogy superconductor QED-Vacuum QCD
perfect color dielectric conductor

- Superconductor is perfect diamagnet with zero magnetic susceptibility(electron pairs in BCS).Repels the magnetic field
- Vacuum in QCD is a perfect color dielectric, zero dielectric constant,done by condensate qqbar and gluons.Repels the cromoelectric field, keeping it inside hadrons



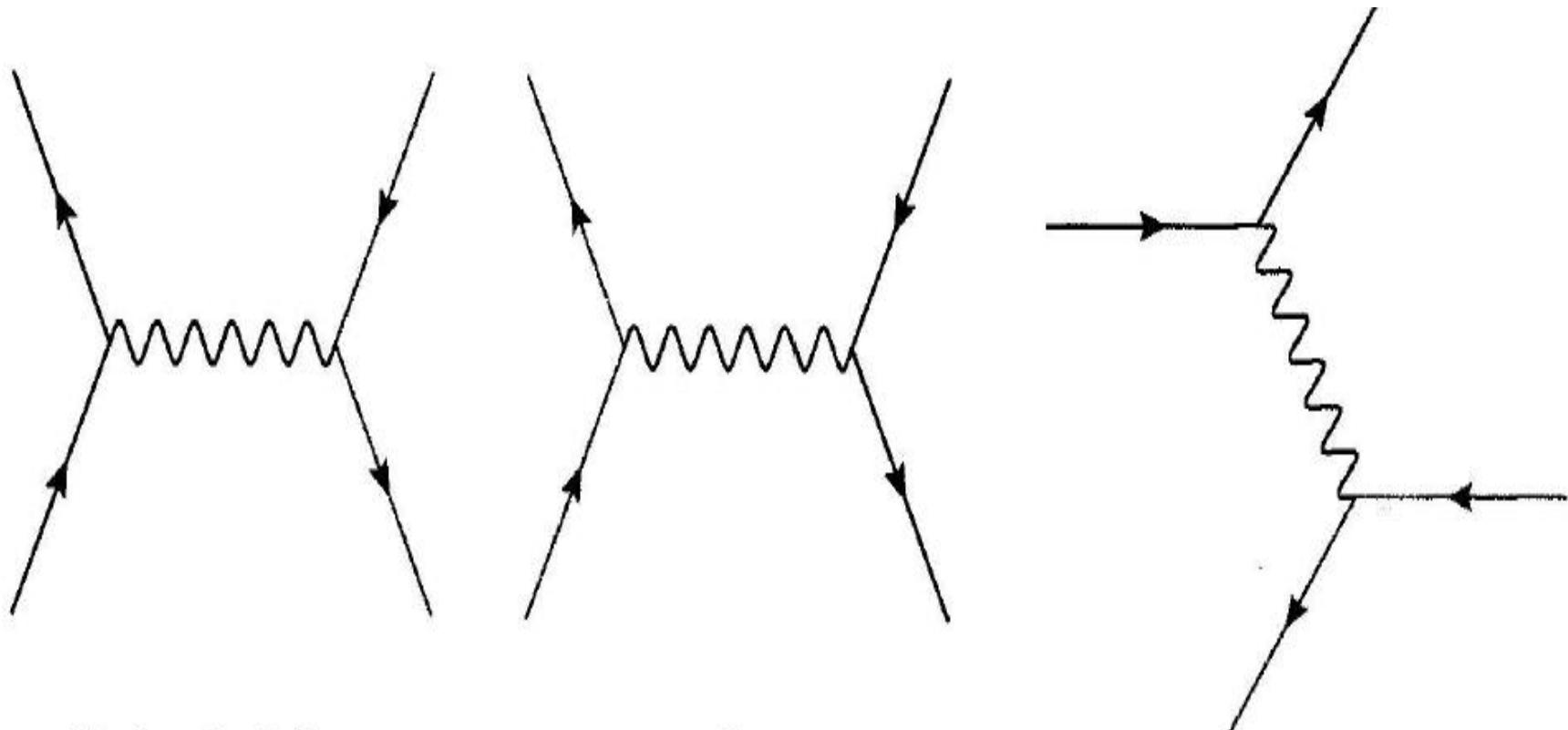


$$\begin{aligned} & \propto -\frac{i}{2}g_s \quad \bar{\psi}_{qR} \quad \lambda^1 \quad \psi_{qL} \\ & = -\frac{i}{2}g_s \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \end{aligned}$$

$$e^+ e^- \rightarrow Z \rightarrow q\bar{q} : \sum_{\text{colors}} |M|^2 \propto \delta_{ij}\delta_{ji}^* = \text{Tr}\{\delta\} = N_C = 3$$

Drell-Yan process $q\bar{q} \rightarrow Z \rightarrow e^+ e^- : \frac{1}{9} \sum_{\text{colors}} |M|^2 \propto \frac{1}{9} \delta_{ij}\delta_{ji}^* = \frac{1}{9} \text{Tr}\{\delta\} = \frac{1}{3}$

$$1/N_C$$



Hadronic Z decay

$$e^- e^+ \rightarrow \gamma^*/Z^0 \rightarrow q\bar{q}$$

$$\propto N_C$$

Drell-Yan

$$q\bar{q} \rightarrow \gamma^*/Z^0 \rightarrow \ell^+ \ell^-$$

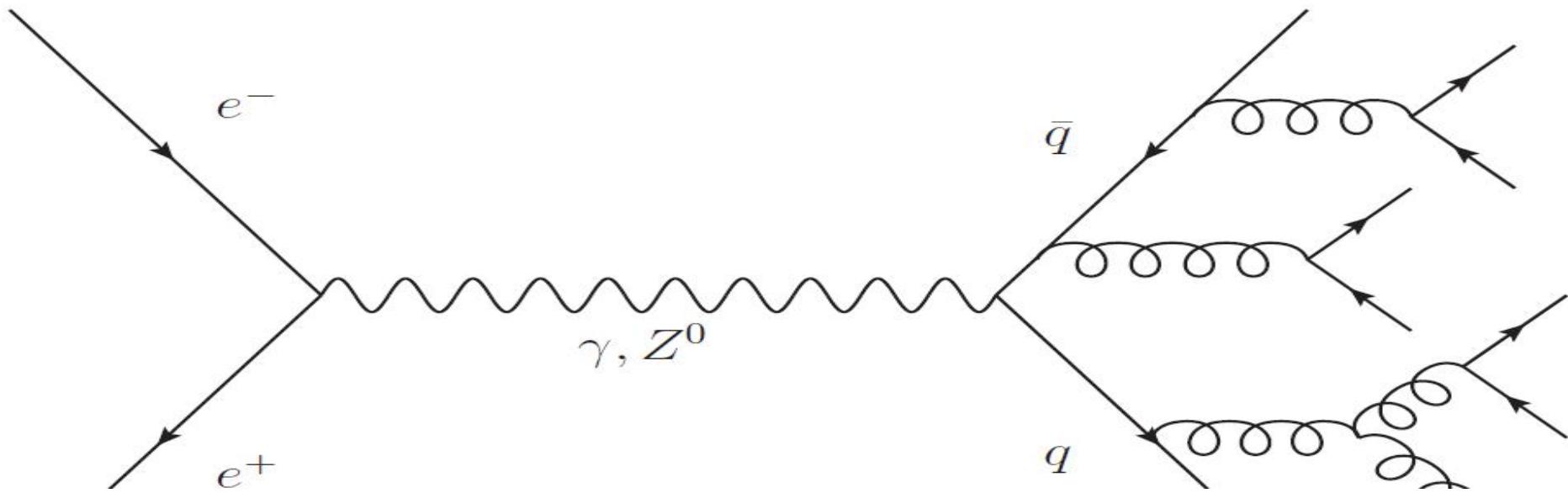
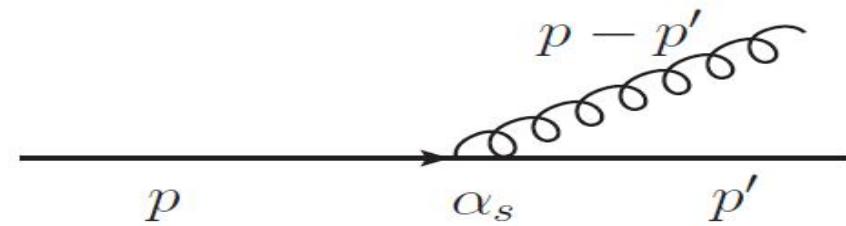
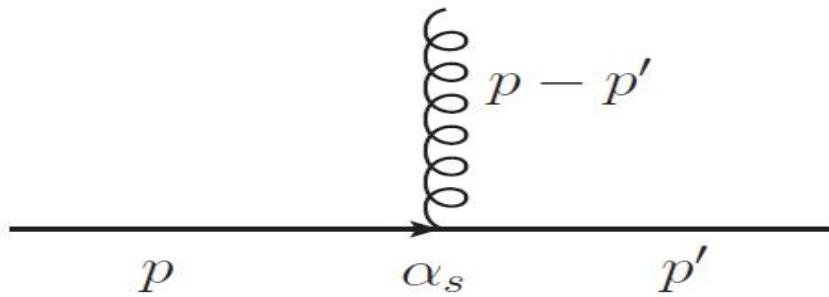
$$\propto 1/N_C$$

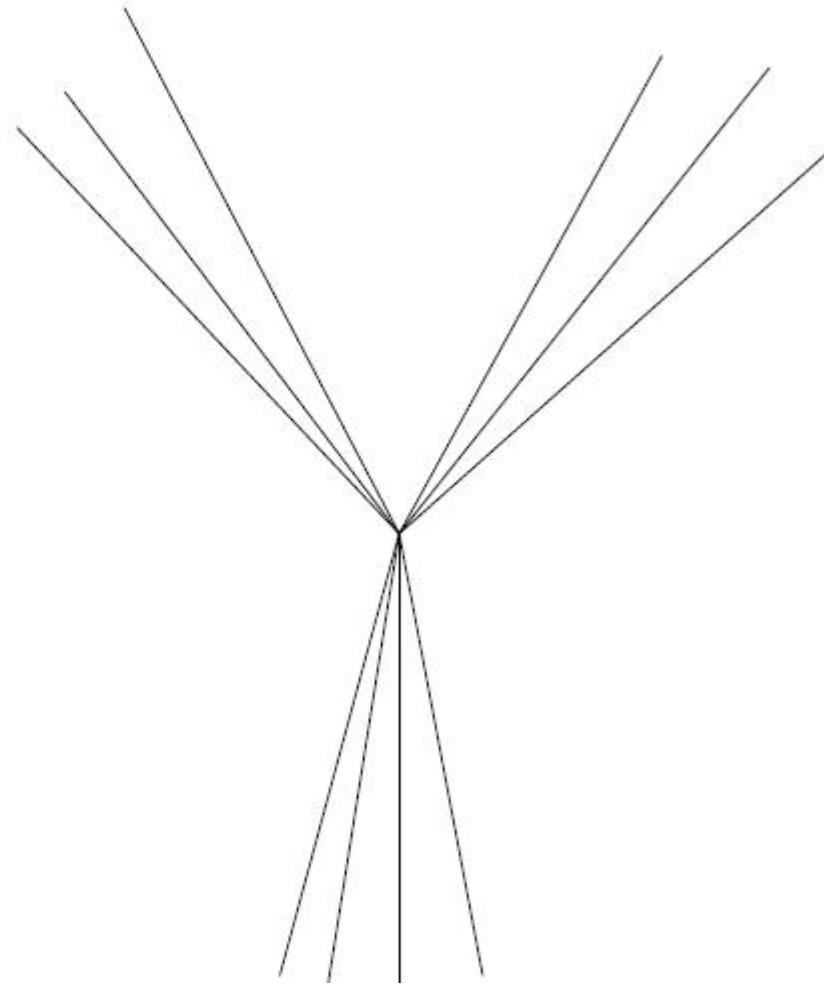
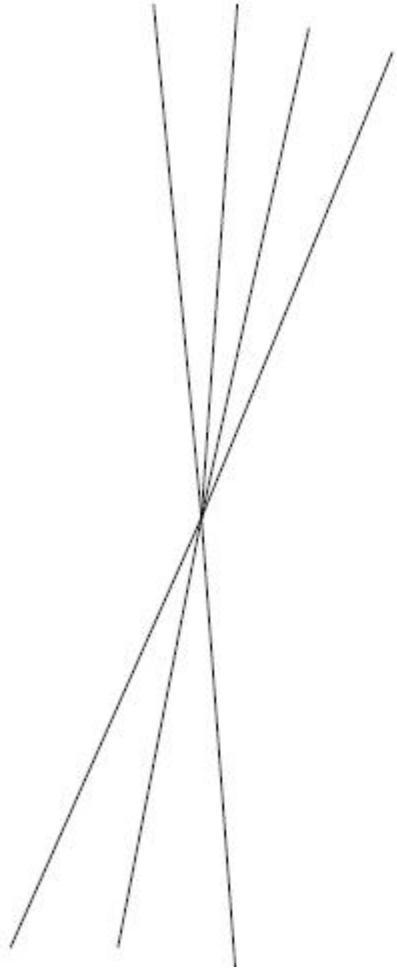
DIS

$$\ell\bar{\ell} \xrightarrow{\gamma^*/Z^*} \ell\bar{\ell}$$

$$\propto 1$$

Jets





$Z \rightarrow qg\bar{q}$:

$$\sum_{\text{colors}} |M|^2 \propto \delta_{ij} t_{jk}^a (t_{\ell k}^a \delta_{i\ell}^*)^*$$

$$= \text{Tr}\{t^a t^a\}$$

$$= \frac{1}{2} \text{Tr}\{\delta\} = 4,$$

$$\text{Tr}\{t^a t^b\} = T_R \delta^{ab}$$

$$a, b \in [1, \dots, 8]$$

$$\begin{aligned} \sum_a t_{ij}^a t_{jk}^a &= C_F \delta_{ik} \\ a &\in [1, \dots, 8] \\ i, j, k &\in [1, \dots, 3] \end{aligned}$$

$$\sum_{c,d} f^{acd} f^{bcd} = C_A \delta^{ab}$$

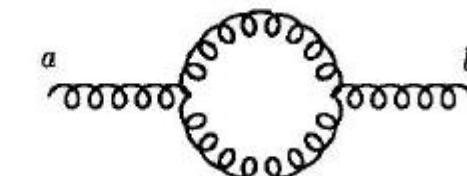
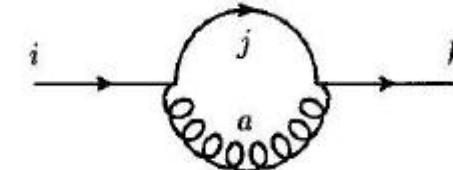
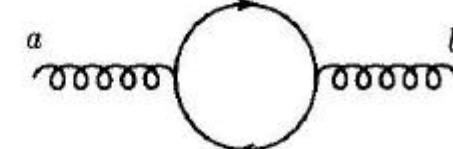
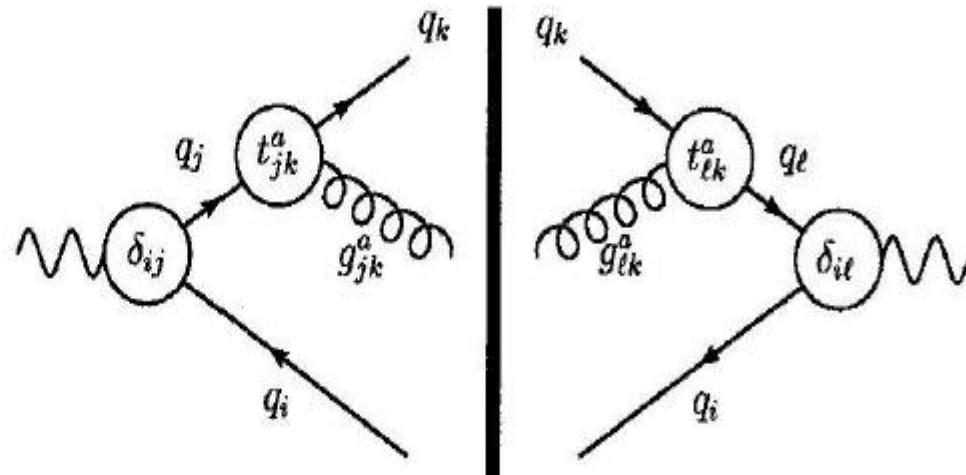
$$a, b, c, d \in [1, \dots, 8]$$

$$t_{ij}^a t_{k\ell}^a = T_R \left(\delta_{jk} \delta_{i\ell} - \frac{1}{N_C} \delta_{ij} \delta_{k\ell} \right) \quad i, j, k, \ell \in [1, \dots, 3]$$

$$T_R = \frac{1}{2}$$

$$C_F = \frac{4}{3}$$

$$C_A = N_C = 3$$



$$\begin{array}{c} j \\ \swarrow \searrow \\ k \end{array} \quad \begin{array}{c} i \\ \swarrow \searrow \\ \ell \end{array} \quad \alpha \quad \left[\begin{array}{c} \nearrow \searrow \\ \nearrow \end{array} \right] \quad \left[\begin{array}{c} \nearrow \searrow \\ \nearrow \end{array} \right] \quad \frac{-1}{N_C} \quad (\text{Fierz})$$

Symmetries

- Isospin $m_u = m_d$

$$\begin{aligned} u &\rightarrow \alpha u + \beta d, & V = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}, & \det V = 1 & V \in SU(2) \\ d &\rightarrow \gamma u + \delta d, \end{aligned}$$

$$m_u < m_d$$

$$m_u \bar{u}u + m_d \bar{d}d = \frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) + \frac{1}{2}(m_d - m_u)(\bar{d}d - \bar{u}u)$$

Quiral

$$m_u=m_d=0.$$

$$\left(\begin{array}{c} u_R \\ d_R \end{array}\right) \longrightarrow V_R \left(\begin{array}{c} u_R \\ d_R \end{array}\right), \quad \left(\begin{array}{c} u_L \\ d_L \end{array}\right) \longrightarrow V_L \left(\begin{array}{c} u_L \\ d_L \end{array}\right), \quad V_R,V_L \in SU(2)_{SU(2)_L\times SU(2)_R}$$

$$\vec{I}(I^+,I^-,\boldsymbol{I}^3)\quad\vec{I}_5(I_5^+,I_5^-,\boldsymbol{I}_5^3)$$

$$I^+ ~=~ \int d\vec{x}\,\bar{u}\gamma^0d = \int d\vec{x}\,u^+d,$$

$$I_5^+ ~=~ \int d\vec{x}\,\bar{u}\gamma^0\gamma_5d = \int d\vec{x}\,u^+\gamma_5d$$

$$\partial_\mu (\bar{u}\gamma^\mu d)=i(m_u-m_d)\bar{u}d,\qquad \partial_\mu (\bar{u}\gamma^\mu\gamma_5 d)=i(m_u+m_d)\bar{u}\gamma_5d$$

Spontaneous symmetry breaking (fundamental state is not symmetrical)

- No particle multiplets with opposite parity

If the vacuum is invariant under any S of a group G $S|\Omega\rangle = |\Omega\rangle$

$$S|\Omega\rangle = e^{ieQ}|\Omega\rangle \simeq (1 + ieQ)|\Omega\rangle = |\Omega\rangle \quad Q|\Omega\rangle = 0.$$

$$\tilde{I}|\Omega\rangle = 0 \quad \tilde{I}_5|\Omega\rangle \neq 0.$$

- The three states $\vec{I}_5|\Omega\rangle$ have the same vacuum energy, and the same spin, therefore we have three massless bosons (Golstone bosons) negative parity and $I=1$. It is natural the identification:

$$I_5^+|\Omega\rangle = |\pi^+\rangle, \quad I_5^0|\Omega\rangle = |\pi^0\rangle, \quad I_5^-|\Omega\rangle = |\pi^-\rangle.$$

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{sb}, \quad \mathcal{H}_{sb} = \int d\vec{x} (m_u \bar{u} u + m_d \bar{d} d),$$

$$[\mathcal{H}_0, \vec{I}] = 0 = [\mathcal{H}_0, \vec{I}_5].$$

Scale Symmetry and Trace Anomaly

- For massless, QCD is invariant under

$$x \rightarrow \lambda x, \quad \psi_q \rightarrow \lambda^{3/2} \psi_q, \quad G_\mu^a \rightarrow \lambda G_\mu^a,$$

$$s^\mu = T^{\mu\nu} x_\nu,$$

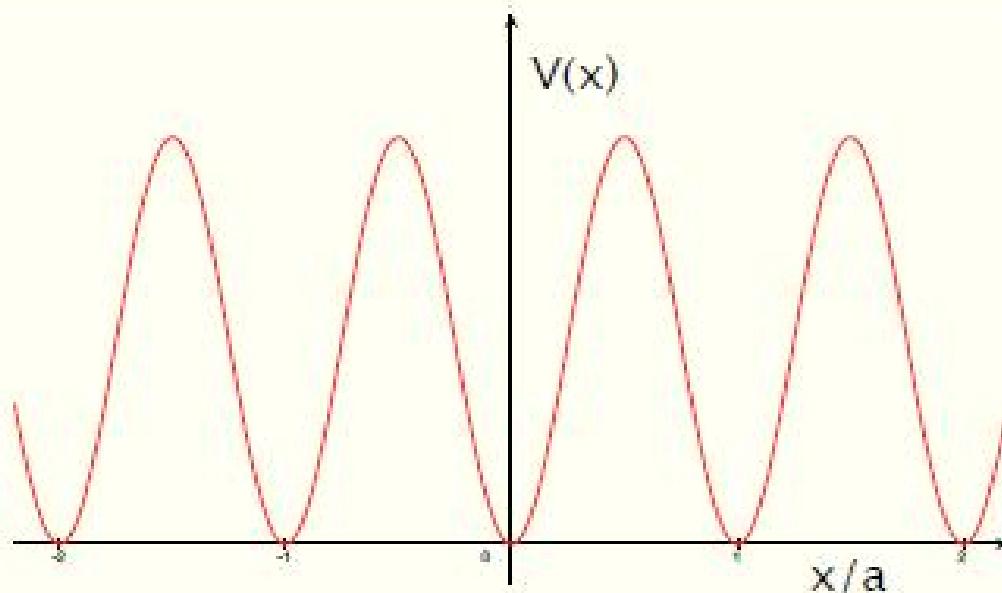
$$\partial_\mu s^\mu = T_\mu^\mu = \frac{\beta(g_s)}{2g_s} F_{\mu\nu}^a F^{\mu\nu a} + [1 + \gamma(g_s)] m_q \bar{\psi}_q \psi_q,$$

$$T_\mu^\mu = \epsilon - 3P \quad \beta(g_s) = -\frac{g_s^3}{3(4\pi)^2} (11N_c - 2n_f)$$

Vacuum Structure, Instantons, Topological Charge

$$\mathcal{L}_\theta = g_s^2 \frac{\theta}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a ,$$

$$\partial_\mu K^\mu = \text{traza}\left(\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a\right), \quad K^\mu \equiv 2\epsilon^{\mu\nu\rho\sigma} \text{traza}\left(F_\nu^a \partial_\rho F_\sigma^a - \frac{1}{3}g_s f_{abc} F_\nu^a F_\rho^b F_\sigma^c\right).$$



$$\psi(x) = \sum_n c_n \psi(x - x_n) \quad x_n = n\hat{a}, \quad c_n = e^{inx\theta}$$

$$|\theta\rangle = \sum_n e^{inx\theta} |n\rangle$$

- The classical theory has degenerate vacua labelled by the winding number n (topological charge)

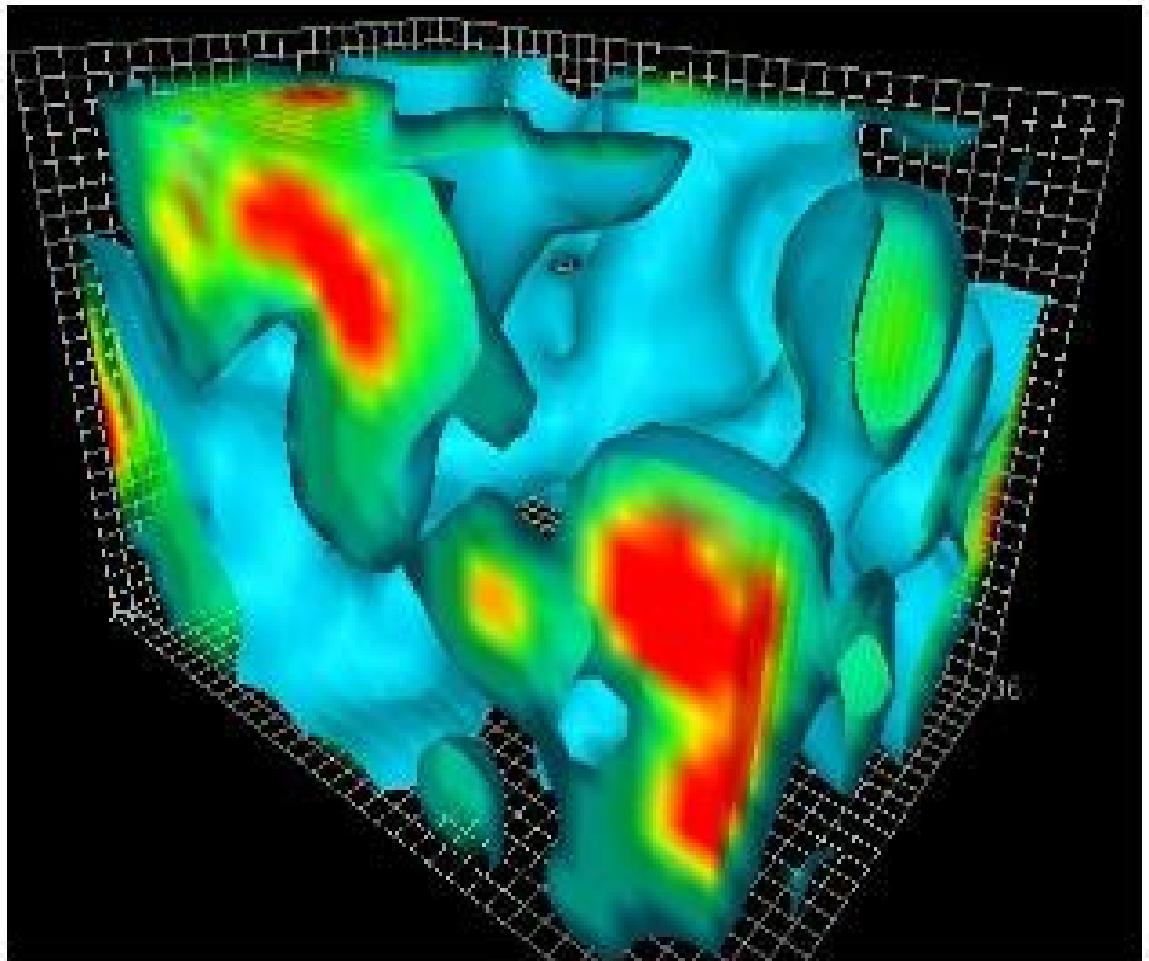
In the quantized theory

$$|\theta\rangle = \sum_n e^{in\theta} |\underline{A}_n\rangle$$

where the state $|\underline{A}_n\rangle$ is an eigenstate of \underline{A}_n
which satisfies

- Finite euclidean action
- Solutions of the classical field equations with zero Euclidean energy
- Evolves from \underline{A}_n at it=-Infinite to $\underline{A}_{n+\nu}$ at it=infinite

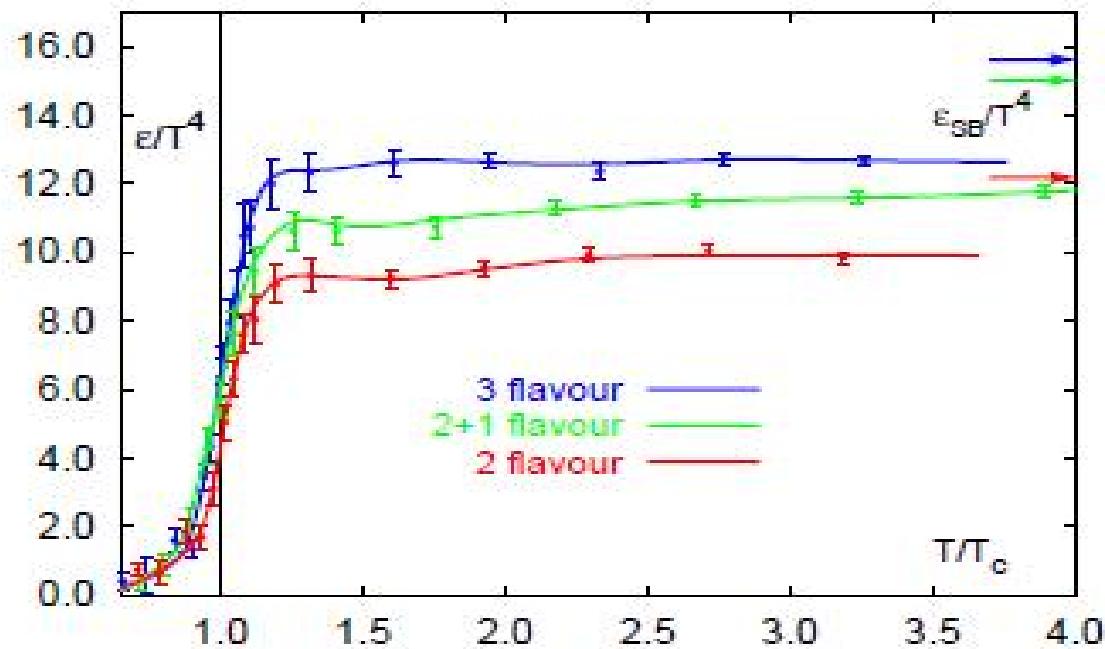
$$\begin{aligned}\nu &= -\frac{g^2}{16\pi^2} \int d\tau d^3r \partial_\alpha^E K_\alpha^E \quad (\alpha = x, y, z, \tau) \\ &= -\frac{g^2}{16\pi^2} \int_{r=\infty} dS_\alpha K_\alpha^E,\end{aligned}$$



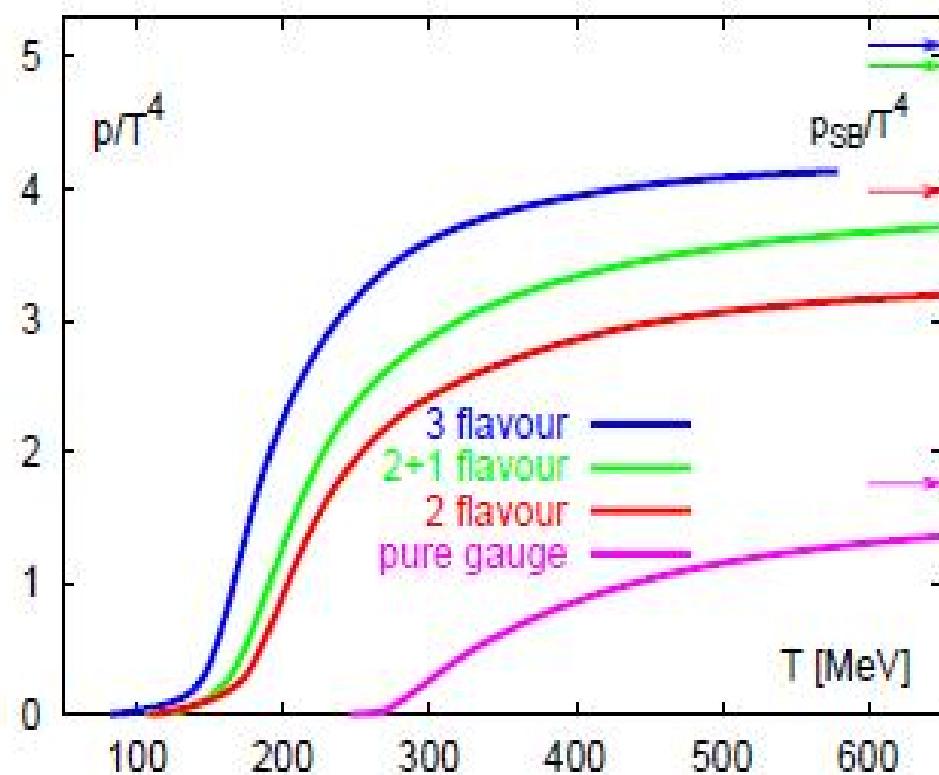
QCD at high temperature

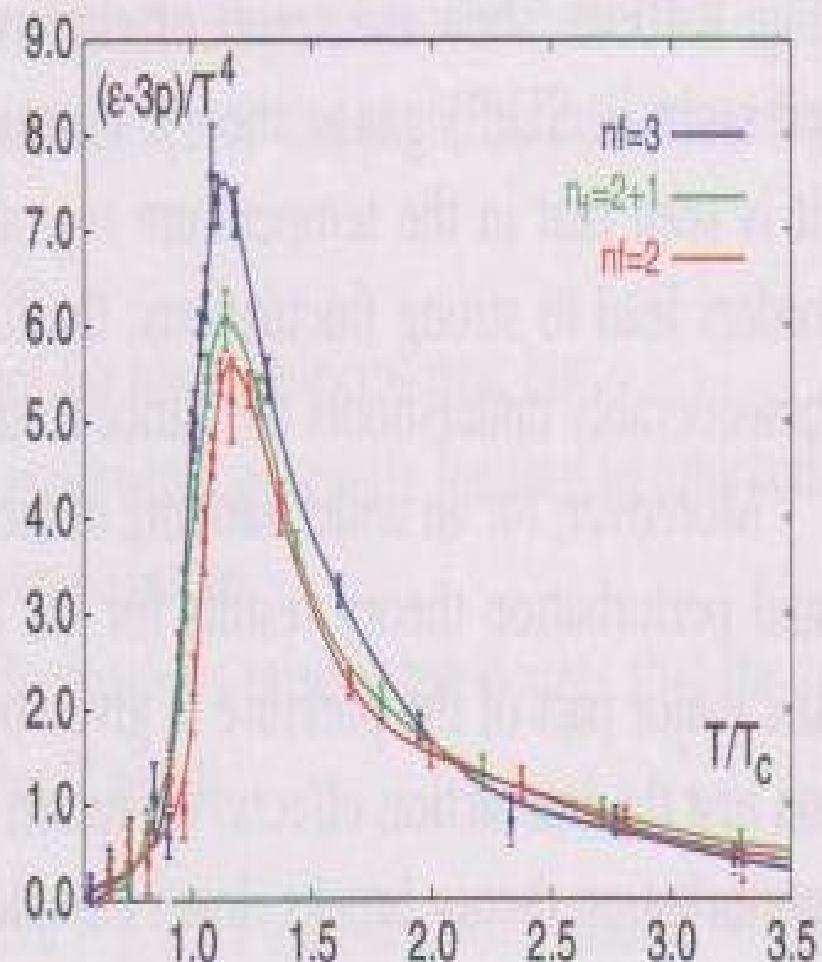
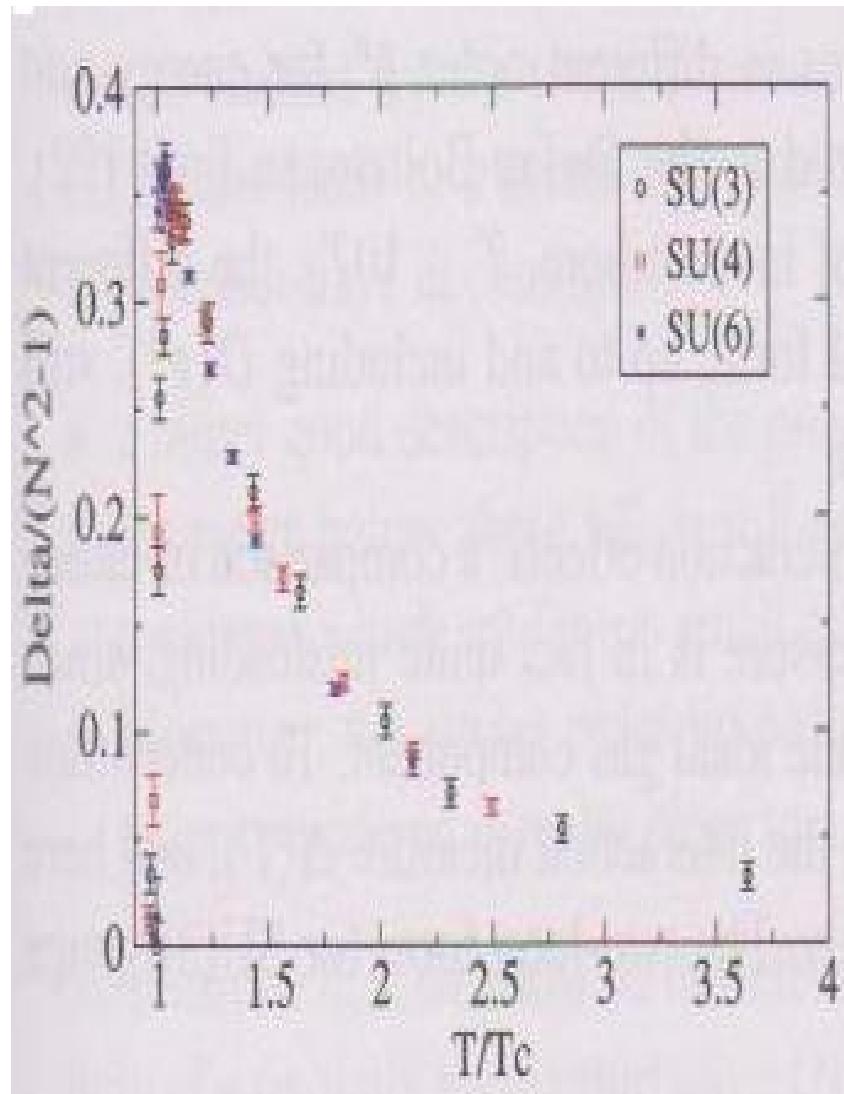
$$\epsilon_{\text{HG}} = \frac{\pi^2}{30} 3 T^4 \simeq T^4,$$

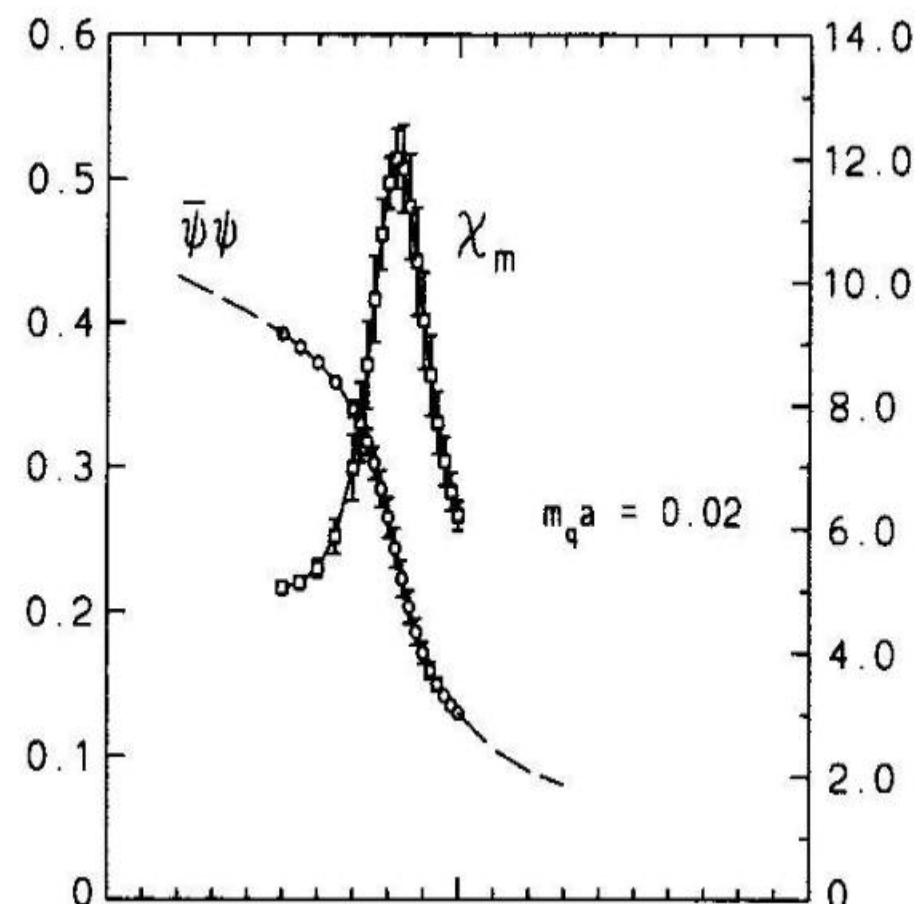
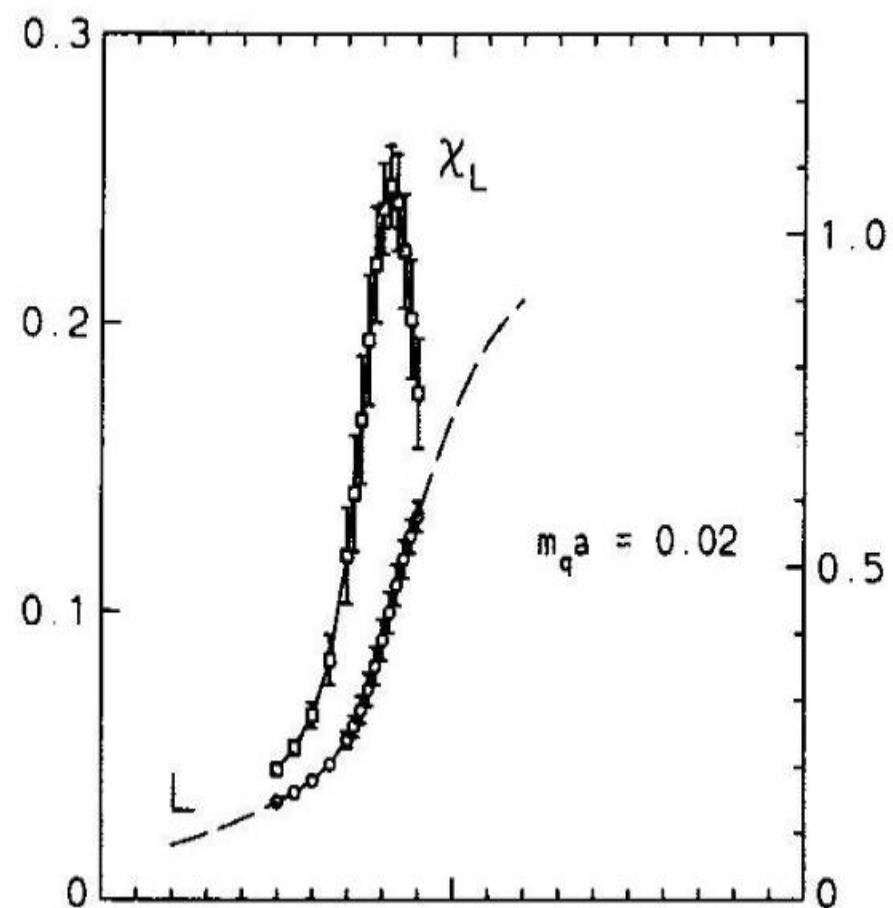
$$\epsilon_{\text{QGP}} = \frac{\pi^2}{30} \left[2 \times 8 + \frac{7}{8} \times 2(3) \times 2 \times 2 \times 3 \right] T^4 = \frac{\pi^2}{30} [16 + 21(31.5)] T^4,$$

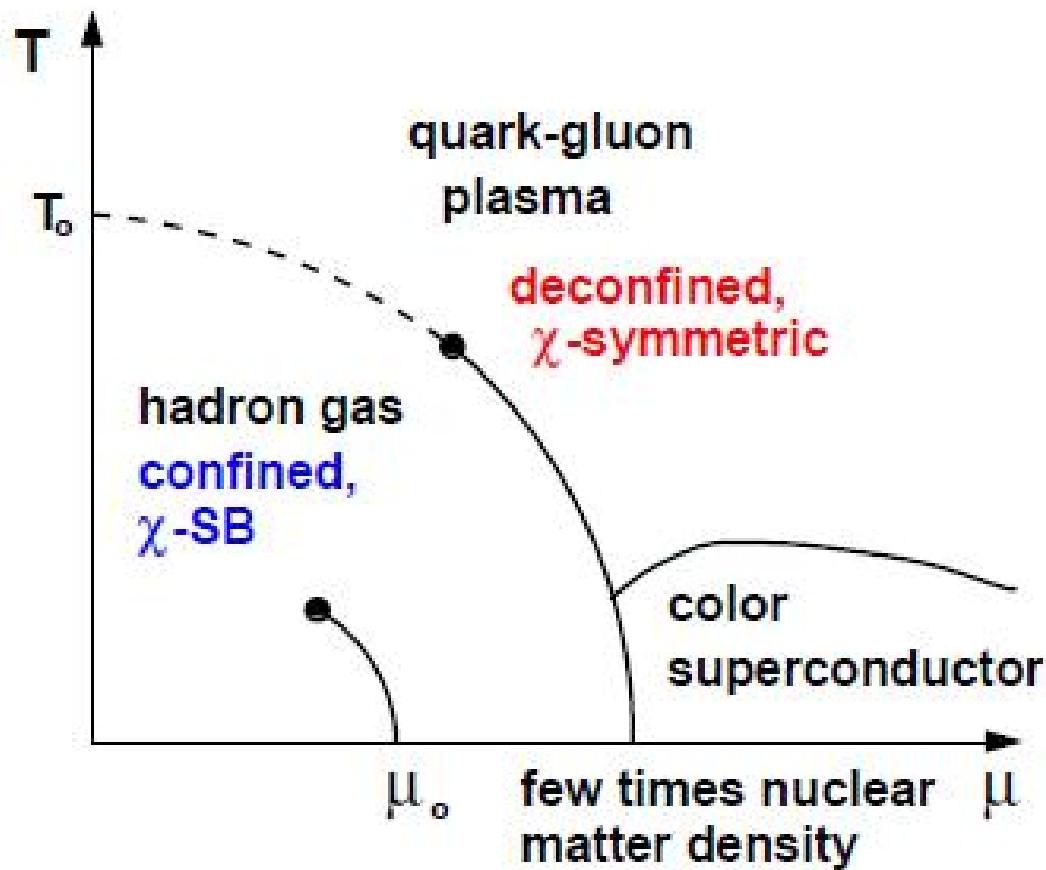


$$\epsilon = \frac{\langle p_T \rangle \left. \frac{dN}{dy} \right|_{y=0}}{\tau_0 \pi R_p^2 A^{2/3}}$$











Big Bang

Little Bang

Global Characteristics

Mass density Ω , Age

Expansion (galaxies)

Hubble Flow

Nucleosynthesis (H, He, Li)

Thermodynamics at $t \sim 100$ s

Large Scale Structure

Density Fluctuations

Microwave BG

T at decoupling

Temperature Fluctuations

signal from the earliest phase

Global Characteristics

Expansion (Hadrons)

Particle Ratios (π, K, p, \dots)

Thermodynamics at $t \sim 3 \times 10^{-5}$ s

Event Structures (e.g. in eN/dy)

Fluctuations at Phase Transition

Thermal Radiation ($\gamma, \ell\ell$)

Temp. evolution $\int T dt$

Colour Screening ($J/\Psi, Y$)



Characterizing the Little Bang (1)



● Particle Production and Energy density ϵ :

Produced Particles: $dN_{ch}/d\eta = 1600 \pm 76$ (stat)

- ~ 30,000 particles in total, ~ 400 times
- somewhat on high side of expectations
- growth with energy faster in AA than pp

Energy density $\epsilon > 3 \times$ RHIC (fixed t_0)

Temperature + 30%

- lower limit, likely t_0

Particle production
at RHIC (BNL)

1666: First documented use of the word
'Annus Mirabilis'

(HS = 200 GeV, Au+Au, $t^* = 1000 \mu\text{m}$)

HICMID = RHIC, 1000
HICMID+ZPC+ART

Data

Matter under extreme conditions:

$$\epsilon > 15 \text{ GeV/fm}^3$$

~ 50 times core of a neutron star
(40 billion tons/cm³)

50 protons packed into the volume of one p !

Temperature $> 4 \times 10^{12}$ eK

$> 200,000$ times center of Sun !

Bakola et al. [12]

Bozek et al. [13]

Sarkaryan et al. [14]

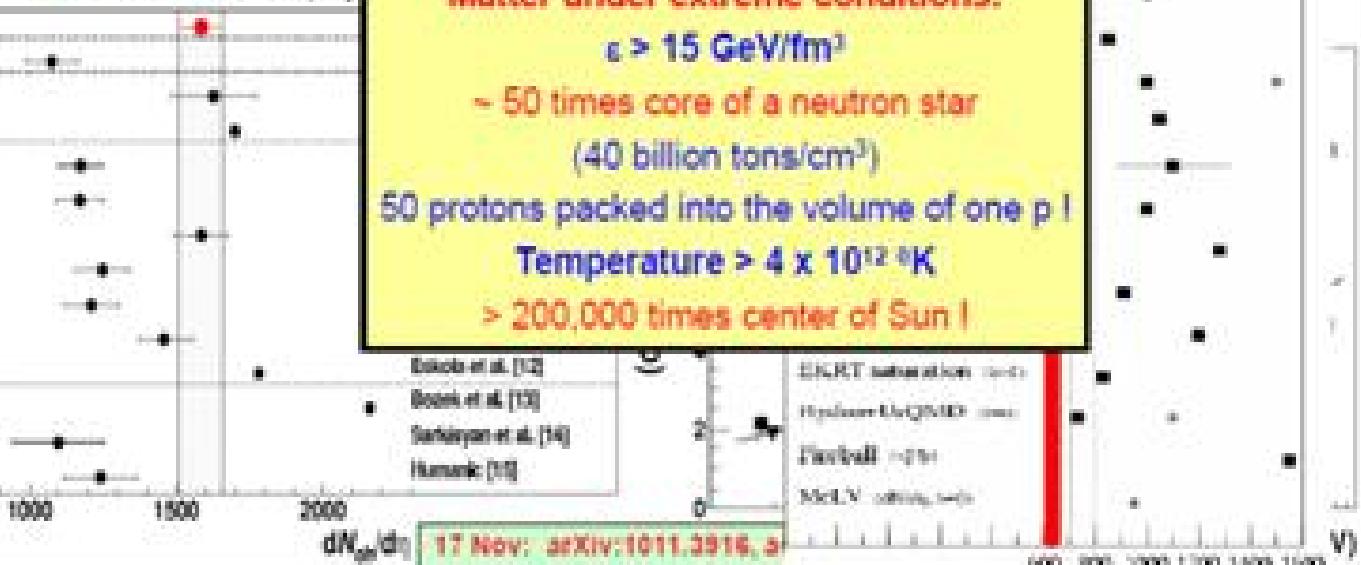
Nahmici [15]

EKRT saturation

Hydrokinetic QGP [16]

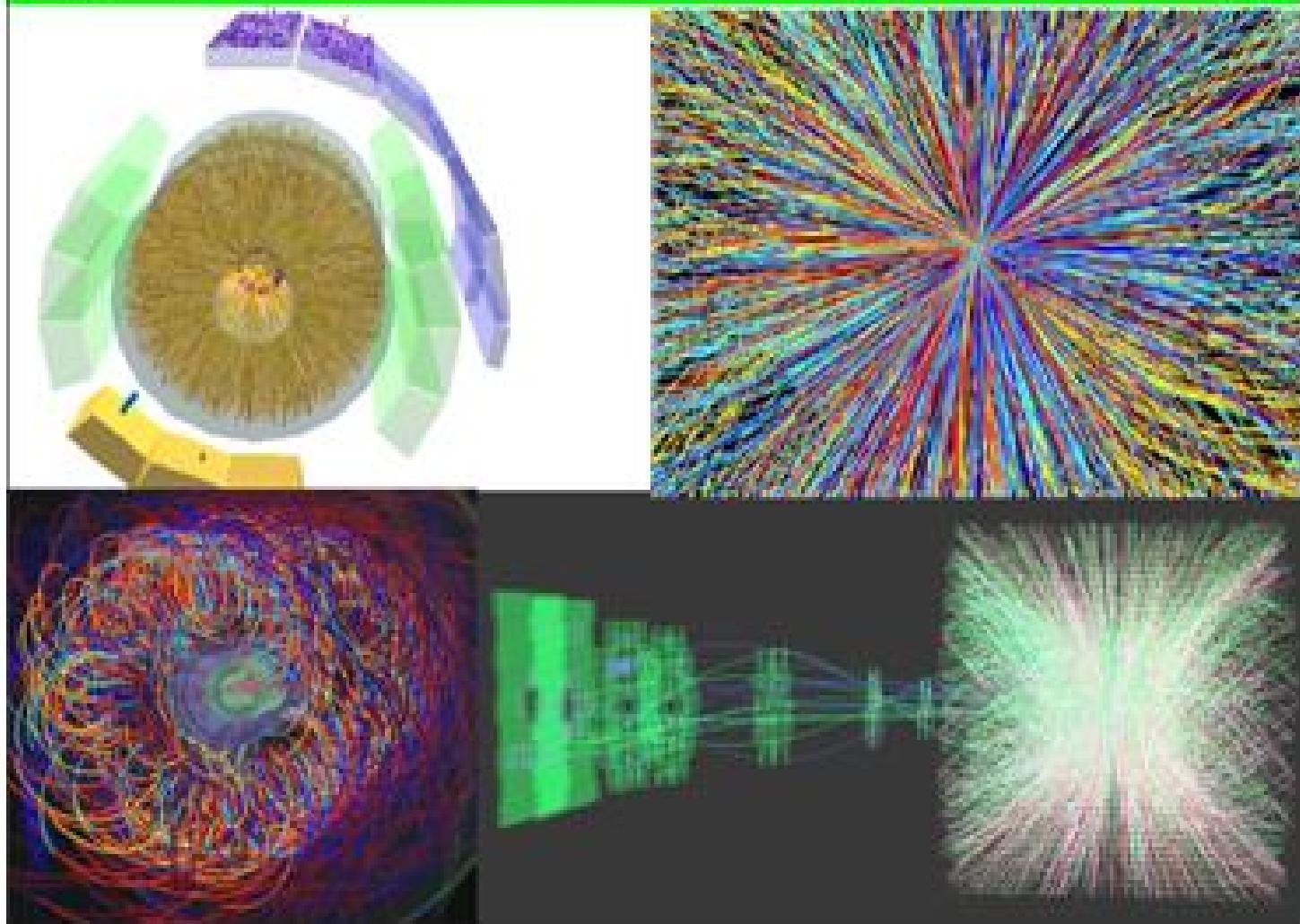
Florkowski [17]

Melkman [18]





Results from the Heavy Ion Run





Characterizing the Little Bang (2)

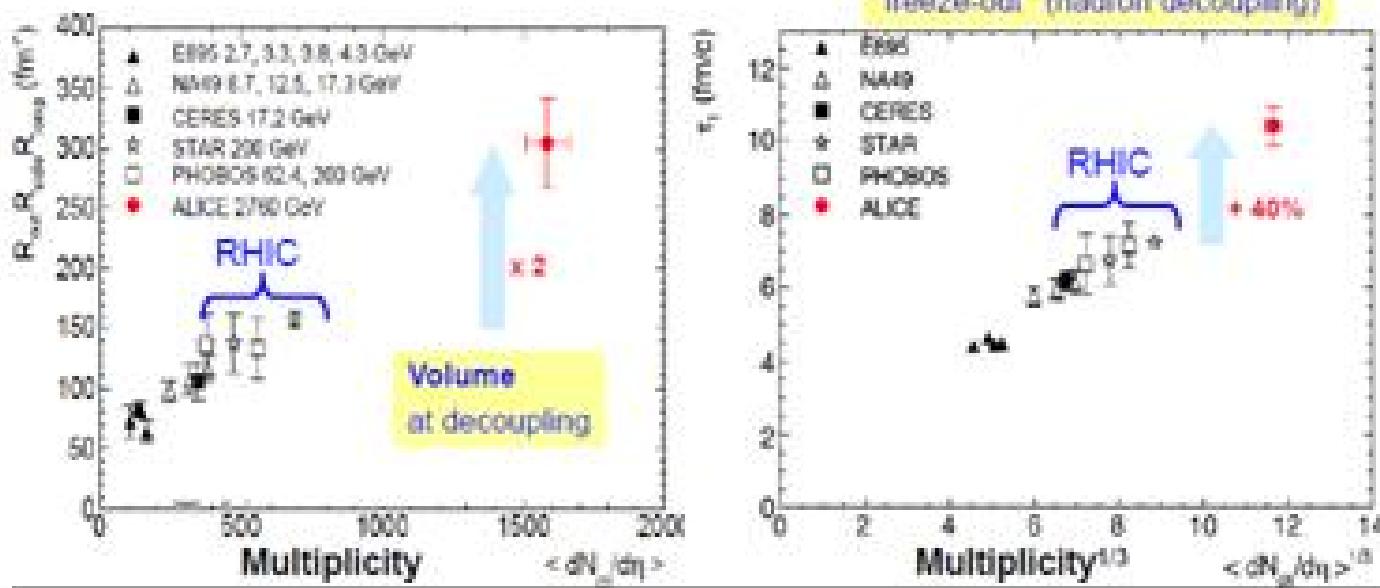


- Volume and Lifetime:

- ⇒ Identical particle interferometry
 - Quantum effect, leading to Bose Einstein Condensate at zero temperature
- ⇒ Volume $\approx 2 \times$ RHIC ($\approx 200 \text{ fm}^3$)
 - observable 'comoving' volume !
- ⇒ Lifetime $\approx +30\text{-}40\%$ ($> 10 \text{ fm/c} \sim 3 \times 10^{-23} \text{ s}$)

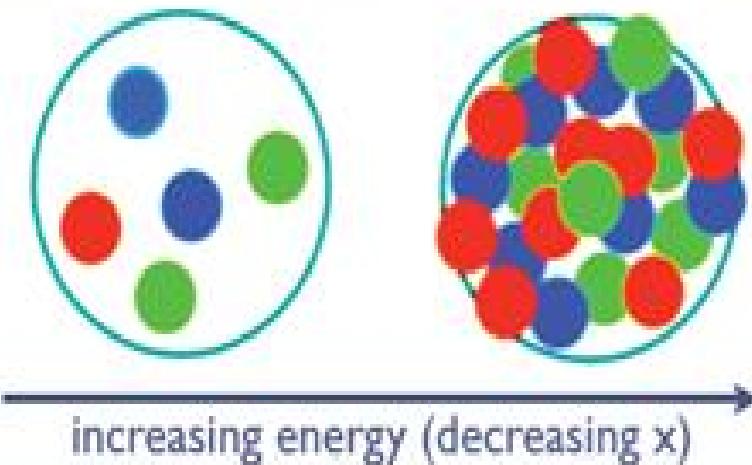
'Little Bang' lives some 10^{40} less than current age of Universe..

Lifetime: from collision to 'freeze-out' (hadron decoupling)



Color screening in partonic densities: picture

Saturation scale when interaction probability becomes $\mathcal{O}(1)$



$$N_g \sim \frac{1}{x^\lambda} \implies Q_{\text{sat}}^2 \sim \frac{A^{1/3}}{x^\lambda}$$

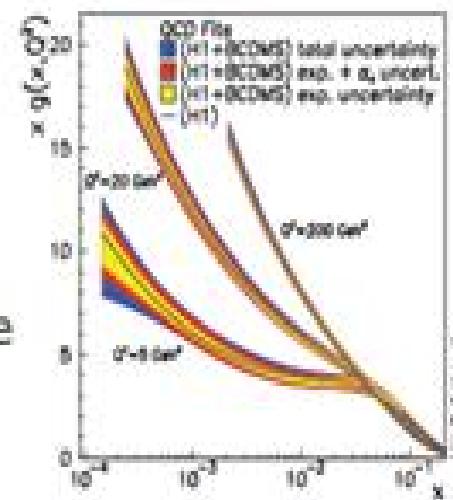
Strong fields and large occupation numbers.
Semiclassical approach: Color Glass Condensate
Geometric scaling

transverse area of the gluon

$$\alpha_s \frac{1}{Q_{\text{sat}}^2} A N_g(x, Q_{\text{sat}}^2) \sim \pi R_A^2$$

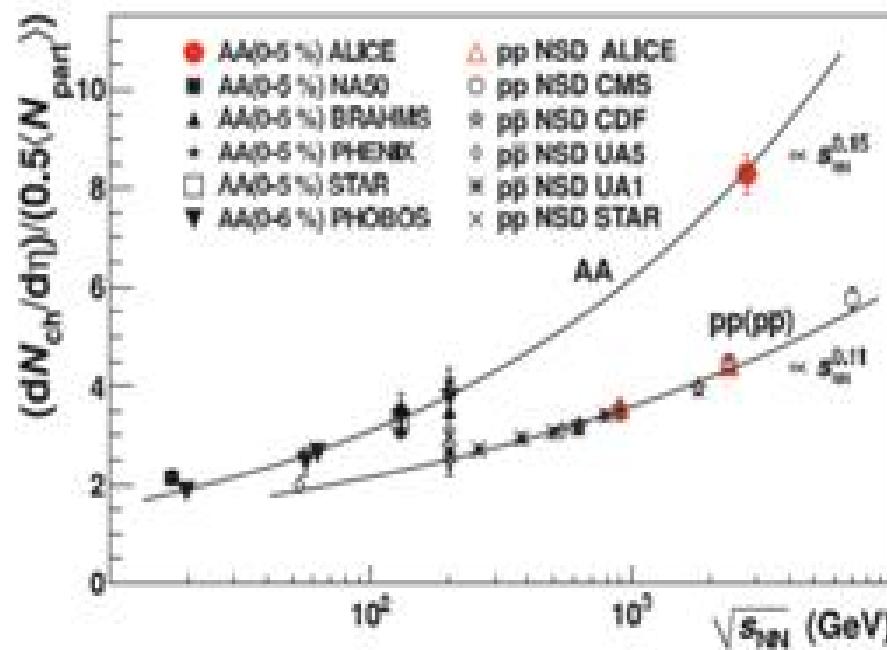
transverse area of the nucleus

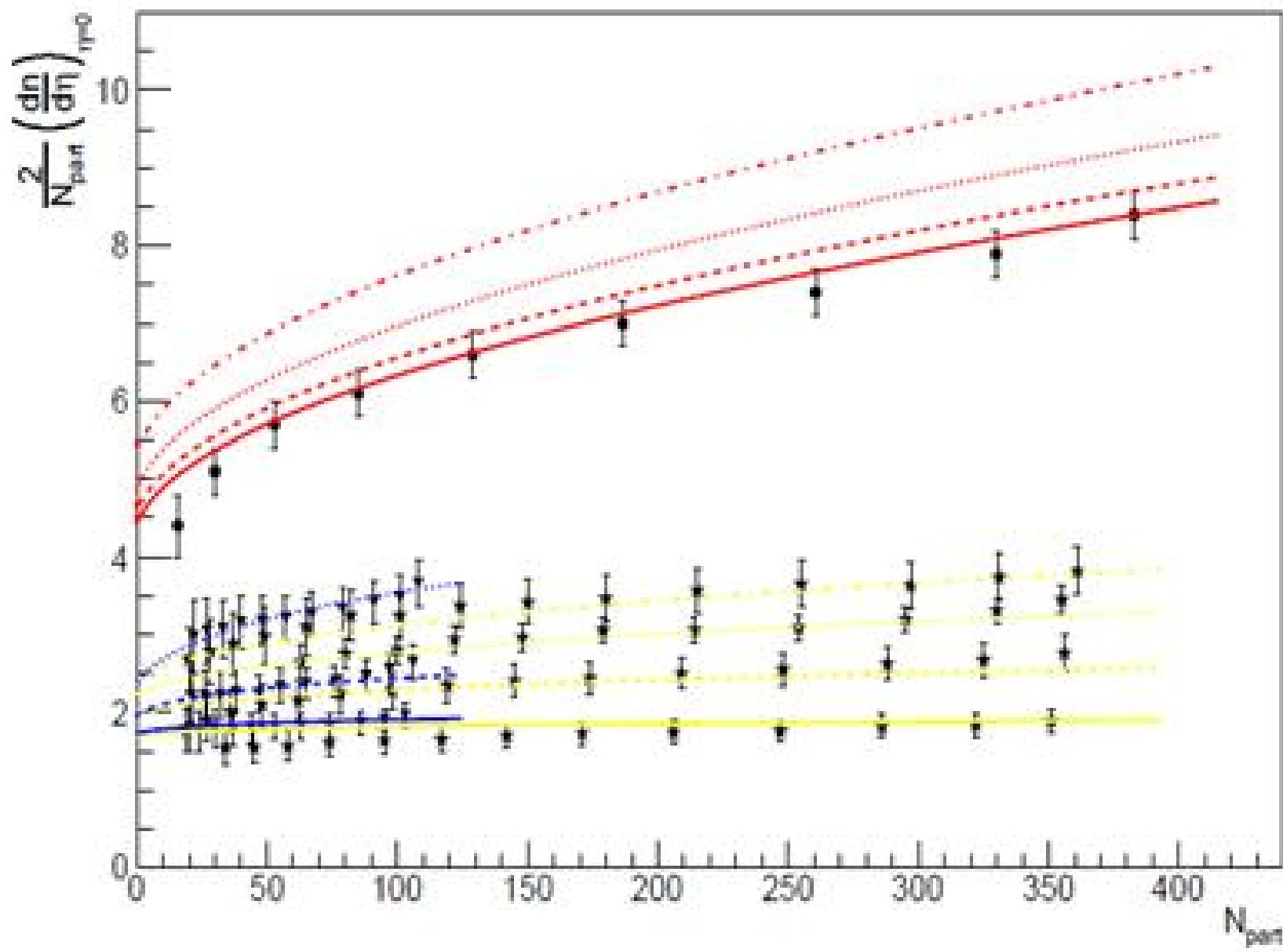
$$R_A \sim A^{1/3}$$



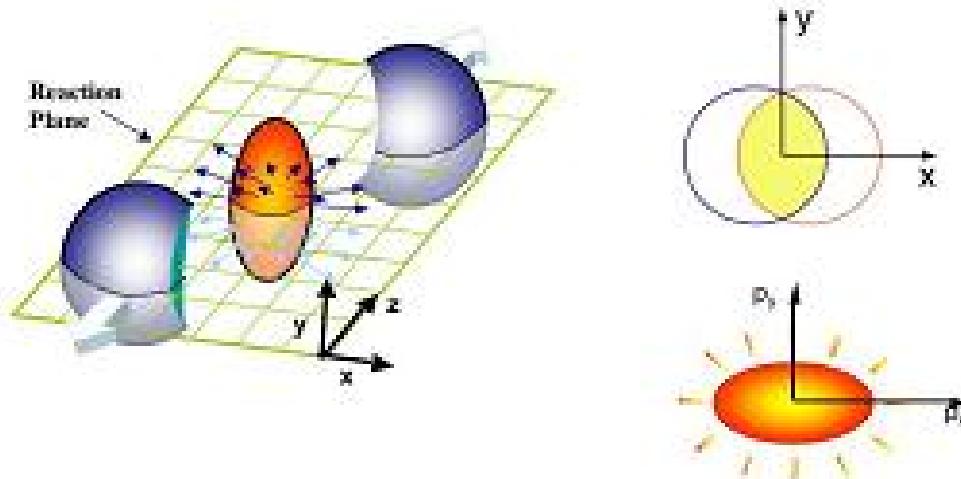
New data: multiplicities

[ALICE arxiv:1011.3916]





ELLIPTIC FLOW



$$\frac{E d^2 N}{d\phi^2} = \frac{d^2 N}{p_t dp_t d\phi^2} = \frac{d^2 N}{2\pi p_t dp_t d\phi} [1 + 2V_2 \cos(\phi - \Phi_2) + 2V_3 \cos 2(\phi - \Phi_3) + \dots]$$

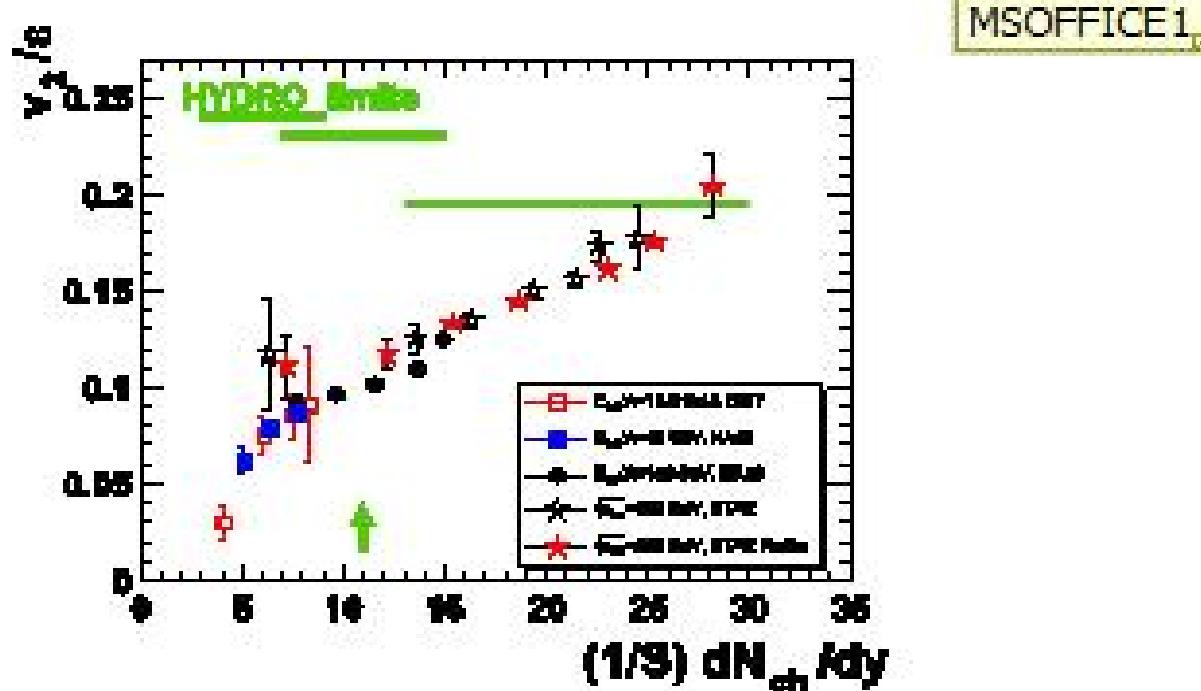
eccentricity $\epsilon = \left\langle \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2} \right\rangle$ spatial anisotropy of the almond

$$V_2 = \left\langle \frac{p_y^2 - p_x^2}{p_y^2 + p_x^2} \right\rangle \text{ momentum anisotropy}$$

pressure gradient is mainly along the direction of impact parameter (x axis)
 in fluid, the p_\perp distribution will reflect the fluid profile. Spatial distribution is carried over the momentum distribution (partons or hadrons must interact each other)

We expect

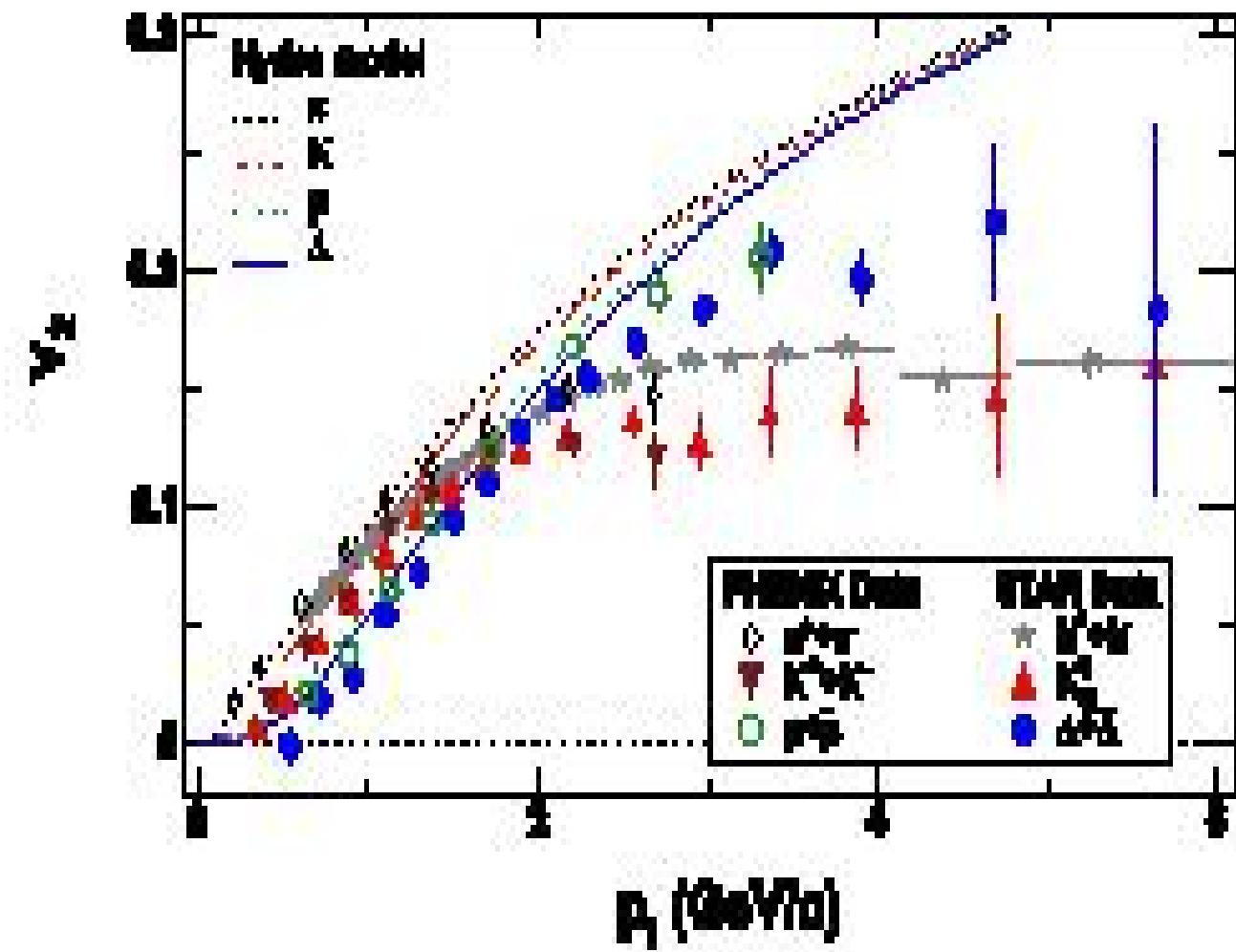
$$v_s \square \varepsilon \text{ density of scatterings} = \varepsilon \frac{1}{\pi S} \frac{dn}{d\nu}$$



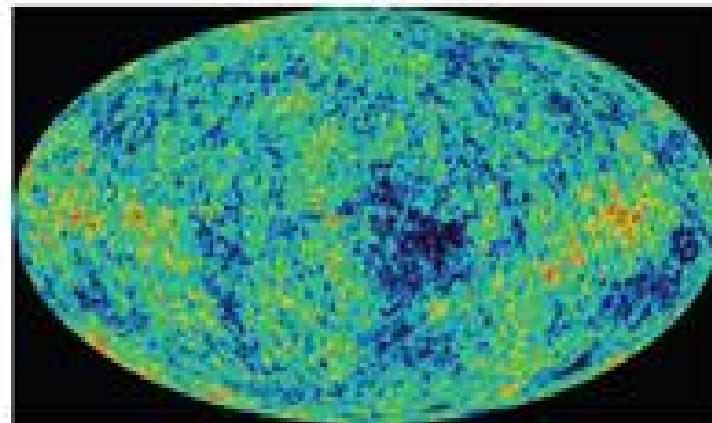
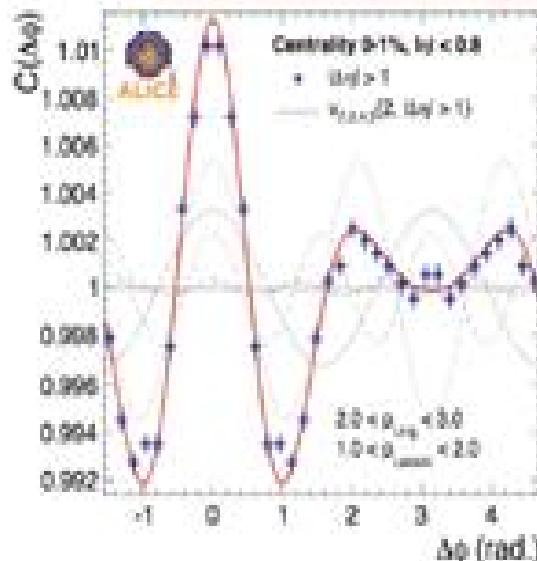
Hydrodynamics perfect fluid result is for full thermalized system
and a soft EoS (e energy density cs speed sound)

Reynolds number $Re = \rho L v / \text{viscosity}$ Mach number $Ma = v / cs$,
comptesibility: Knudsen number $Kn = \lambda / L$

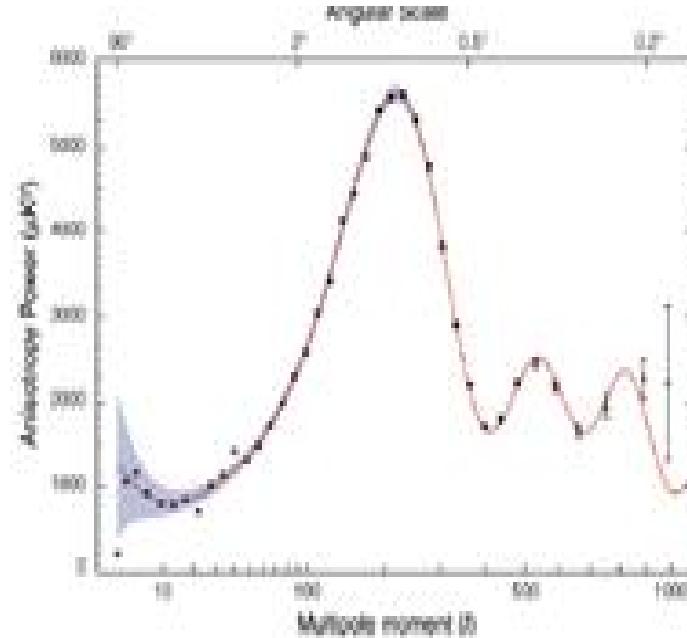
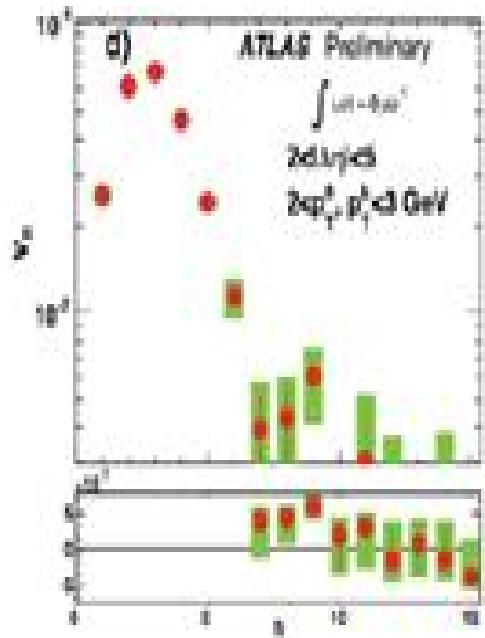
Viscosity = $\rho c s \lambda$ $Ma = Re \times Kn$ $Ma = 1; Re = 1/Kn$



Little Bangs versus Big Bang



- Fluctuation analyzed since 2009
~ 100 % uncertainty on η/s
- Improved measurements upcoming
(future RHIC & LHC running)
- After more than a decade of analysis
and measurements,
~1 % accuracy on key parameter
- Improved measurements upcoming
(PLANCK, ...)



- Uncertainties in initial conditions
- Many observables

$$v_n(p_T), n=1,\dots,10$$

- Many small events

- Uncertainties in initial condition
- Many observables

$$c_l, l=1,\dots,1000$$

- One big event
(additional uncertainty: cosmic variance)

Characterizing the Matter (1)

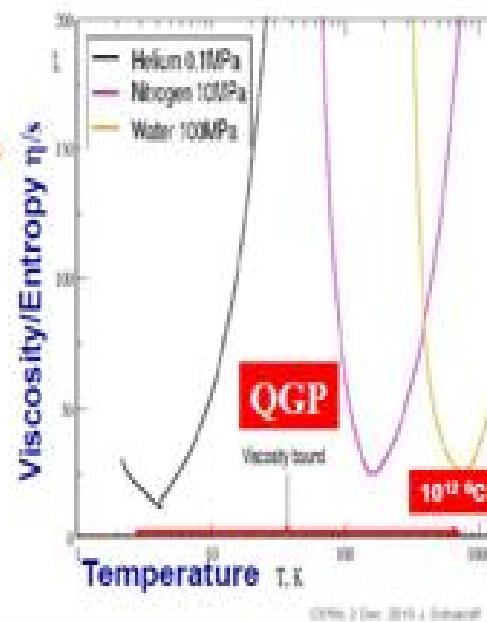
- RHIC discovery in 2005: The QGP is a (almost) perfect liquid

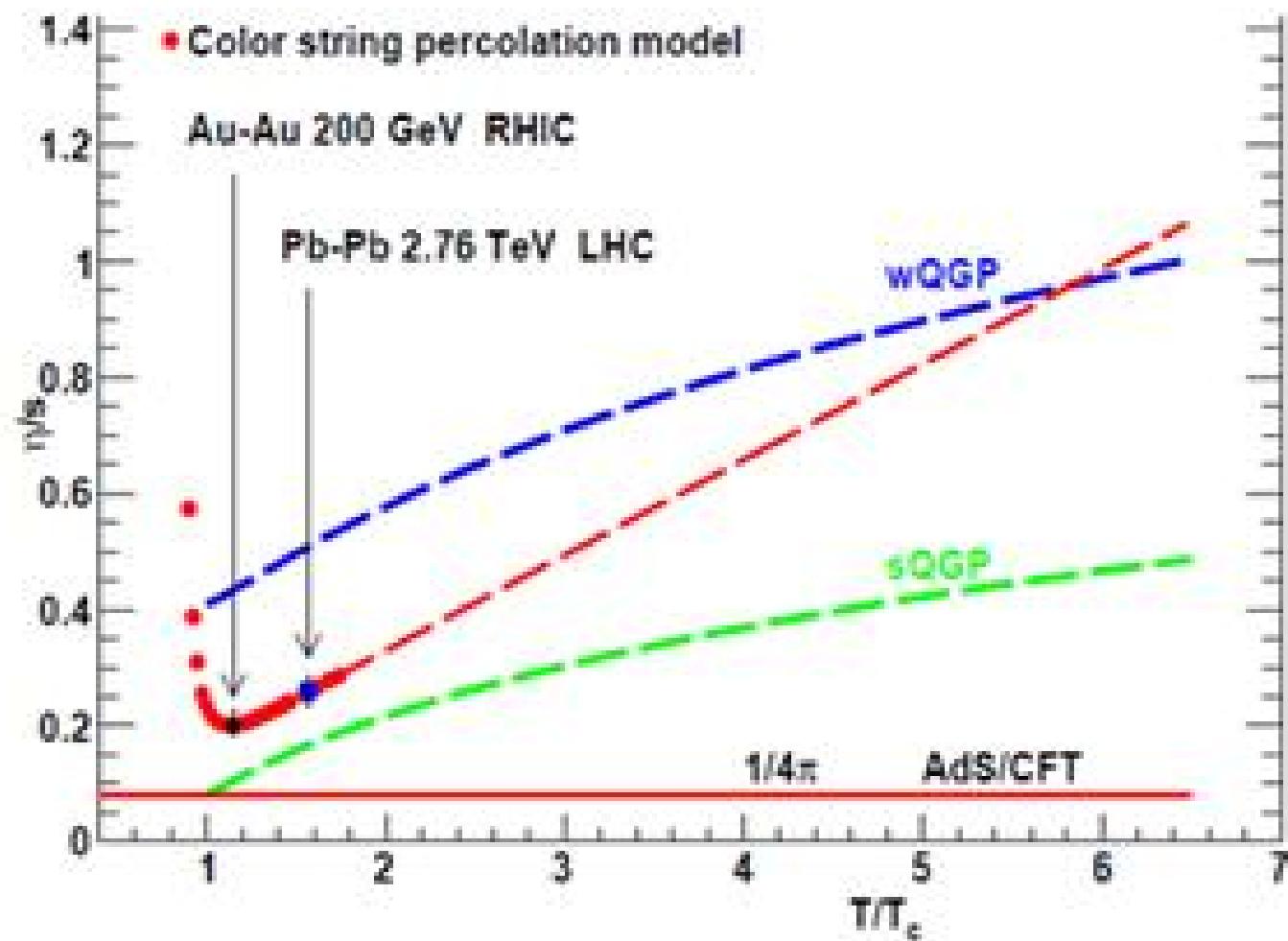
- ⇒ perfect liquid → Viscosity $\eta = 0$ ('response to pressure gradients')
→ strong interactions in the liquid)
 - ⇒ QGP almost ideal fluid, $\eta/S < 0.2 - 0.5$
 - usually use Viscosity/Entropy (η/S dimensionless number)

- unexpected result

- ⇒ QGP thought to behave like a gas
(weakly interacting)
 - ⇒ closest Theory prediction $\eta/S > 1/4\pi \approx 0.08$
 - AdS/CFT:
 - SUSY string theory in 5 dimensions !

BNL Press release, April 18, 2005:
QGP = "Perfect" Liquid
New state of matter more remarkable than predicted -
raising many new questions







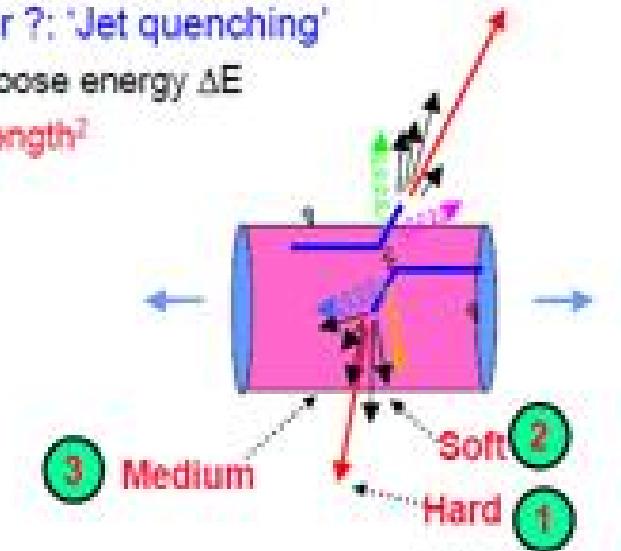
Characterizing the Matter (2)



- How strongly interacting is the matter ?: 'Jet quenching'

- ⇒ quarks/gluons traveling through QGP loose energy ΔE

- some unusual properties, e.g. $\Delta E \sim \text{Length}^2$
(not $\Delta E \sim L$, as in normal matter !)



- ⇒ how much energy is lost ? (measures 'interaction strength' of QGP)

- look at high momentum ('hard') part of jets

- ⇒ how is it lost ?

- many soft or few hard scatterings

- look at low momentum ('soft') part

- ⇒ 'response of QGP'

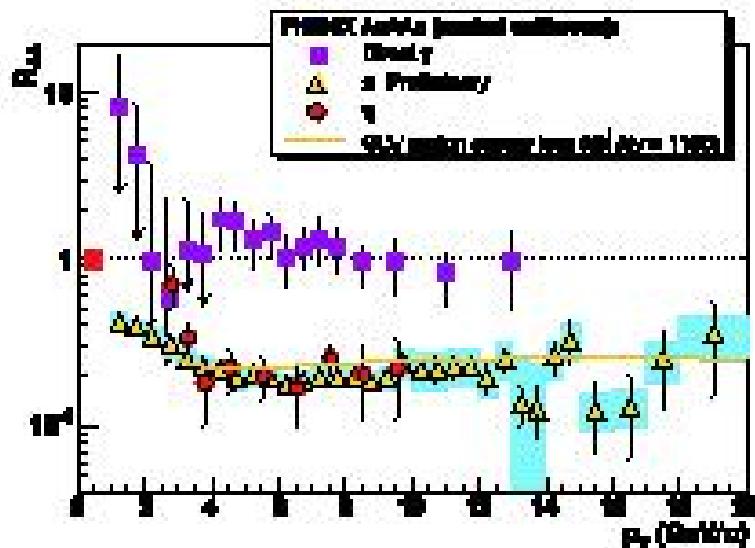
- shock waves, Mach cones ??

- look at average ('very soft') particles of the medium

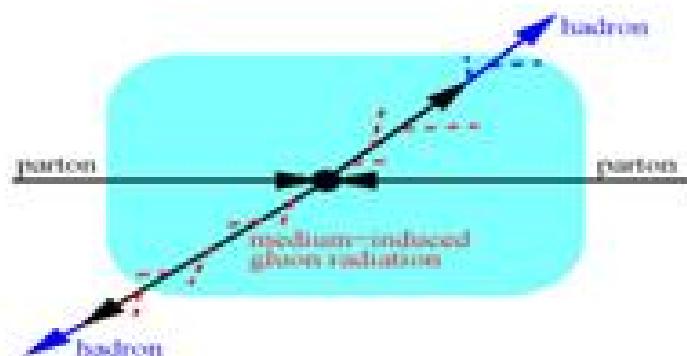


Transverse Momentum Suppression

$$R_{AA}(p_T) = \frac{d^2N_{AA}/dp_T d\eta}{T_{AA}(b)d^2\sigma_{NN}/dp_T d\eta'}$$



Photons in agreement
with perturbative QCD



Energy loss by
medium induced gluon
radiation



Charged Jets

- Jets in ALICE (TPC)
 - we see qualitatively a similar effect to Atlas/CMS

