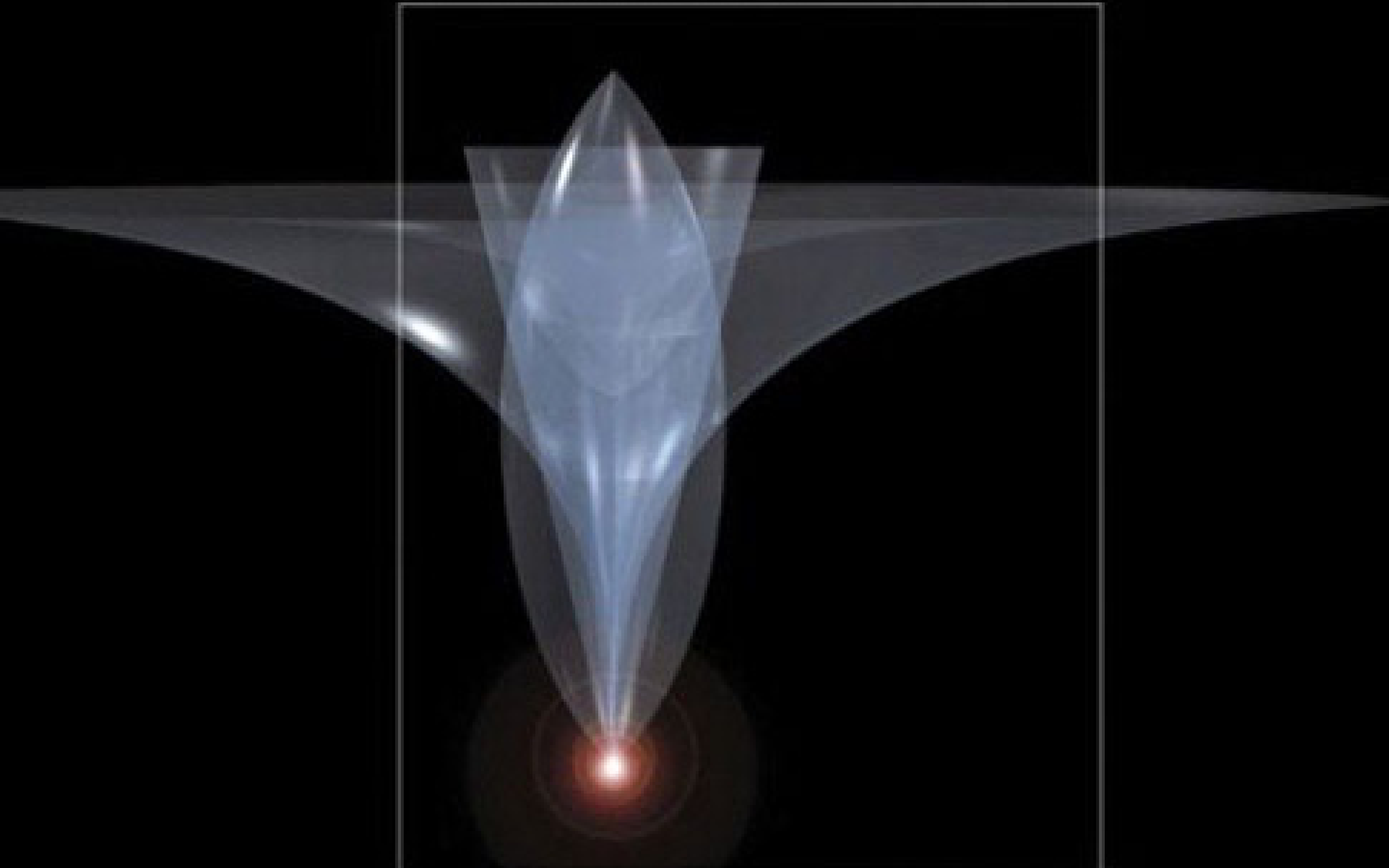
INTRODUCTION TO

COSMOLOGY



BARBARA RYDEN

Today: The road to the Big Bang.

Wednesday: The road to ΛCDM.

Thursday: The road to the future.

Warning:
Astronomers at Work

The gigayear (Gyr) is a unit of time:

1 Gyr = 10^9 years = 3.16×10^{16} s

The megaparsec (Mpc) is a unit of length:

1 Mpc = 10^6 parsecs = 3.09×10^{22} m

≈ 0.003 light-Gyr

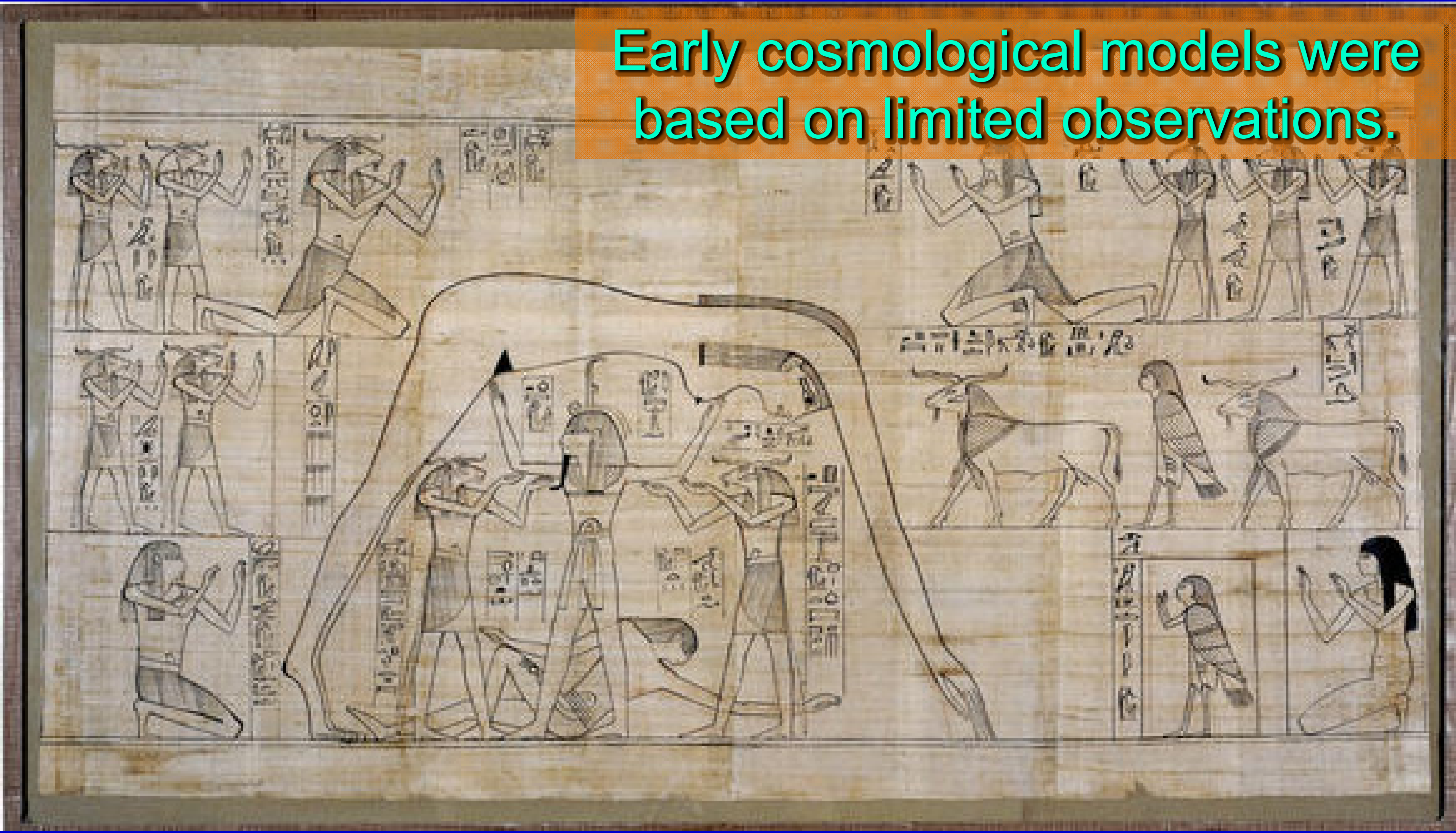
Apparent magnitude (m) is a measure of flux:

$$m = C - 2.5 \log_{10} f$$

Absolute magnitude (M) is a measure of luminosity:

$$M = K - 2.5 \log_{10} L$$

Cosmology is based on **observations** of the universe around us.

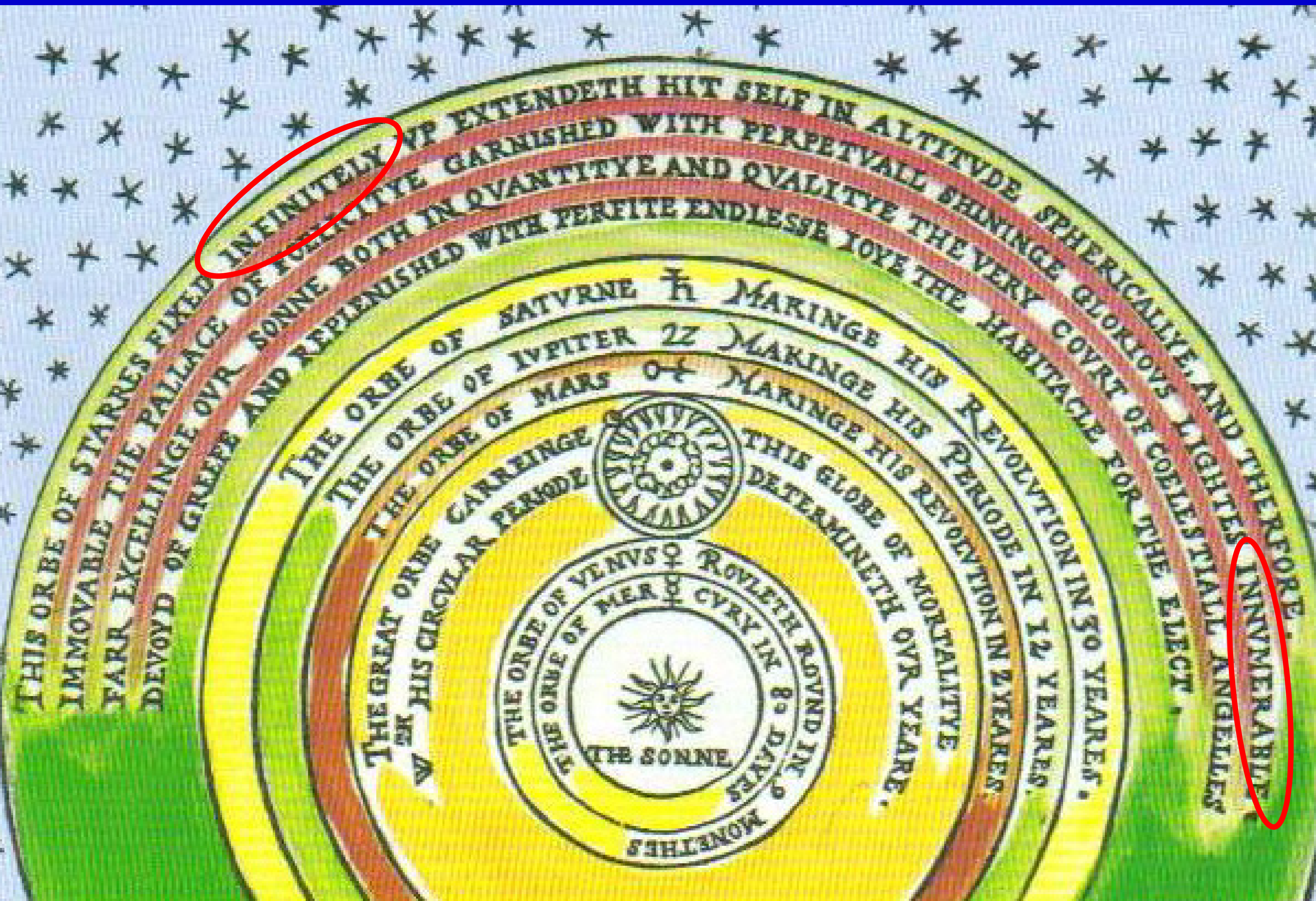


Observation 1: The night sky is dark.

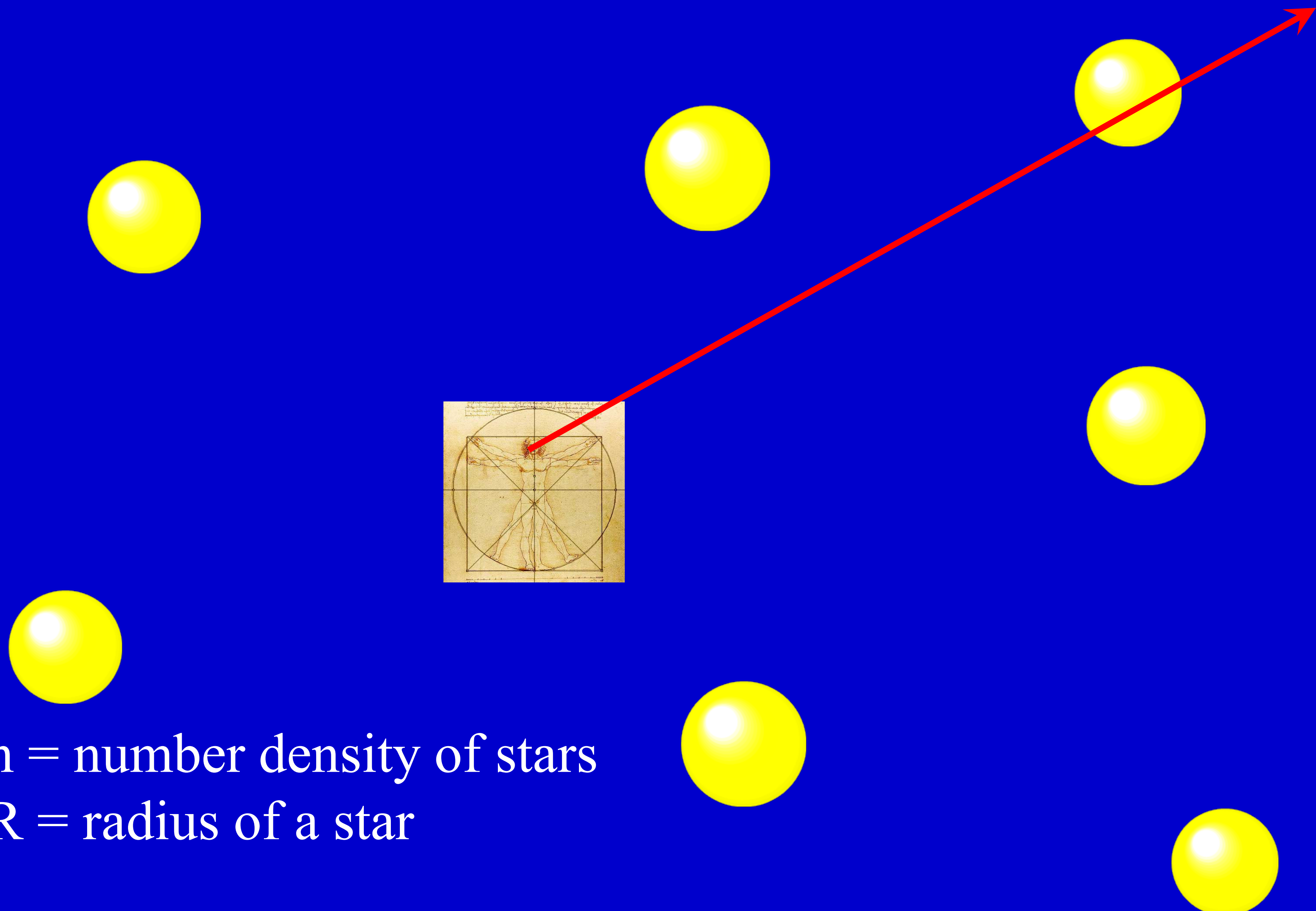
(This statement is known as “Olbers’ Paradox”.)

Why is the darkness of the night sky paradoxical?

We can blame Thomas Digges (English: 1546-1595)



How bright do we expect the sky to be in an infinite universe filled with stars?



n = number density of stars

R = radius of a star

Mean free path before your line of sight meets the surface of a star:

$$\lambda = \frac{1}{n\pi R^2}$$

$n \sim 10^9\ \mathrm{Mpc}^{-3}$, $R \sim 1R_\odot \sim 700{,}000\ \mathrm{km} \sim 2\times10^{-14}\ \mathrm{Mpc}$

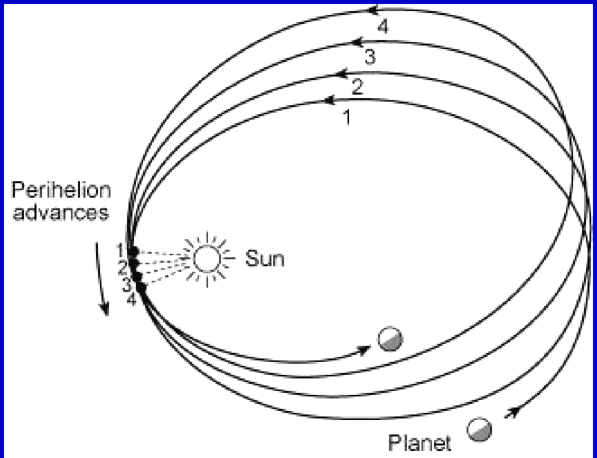
$$\lambda \sim 10^{18}\ \mathrm{Mpc}$$

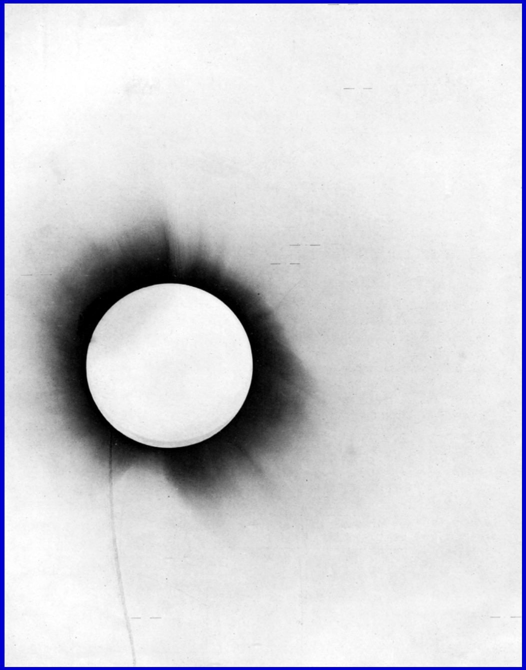
Implication: The universe can’t be infinitely large and infinitely old and filled with immortal stars.

If it were, every line of sight would end at a star’s surface, and the sky would be uniformly bright.

More precisely, the universe can’t be more than $\sim 10^{18}$ Mpc across and $\sim 3\times10^{15}$ Gyr old, and have had stars during all that time.

Observation 2: General Relativity accurately describes gravity inside the solar system.



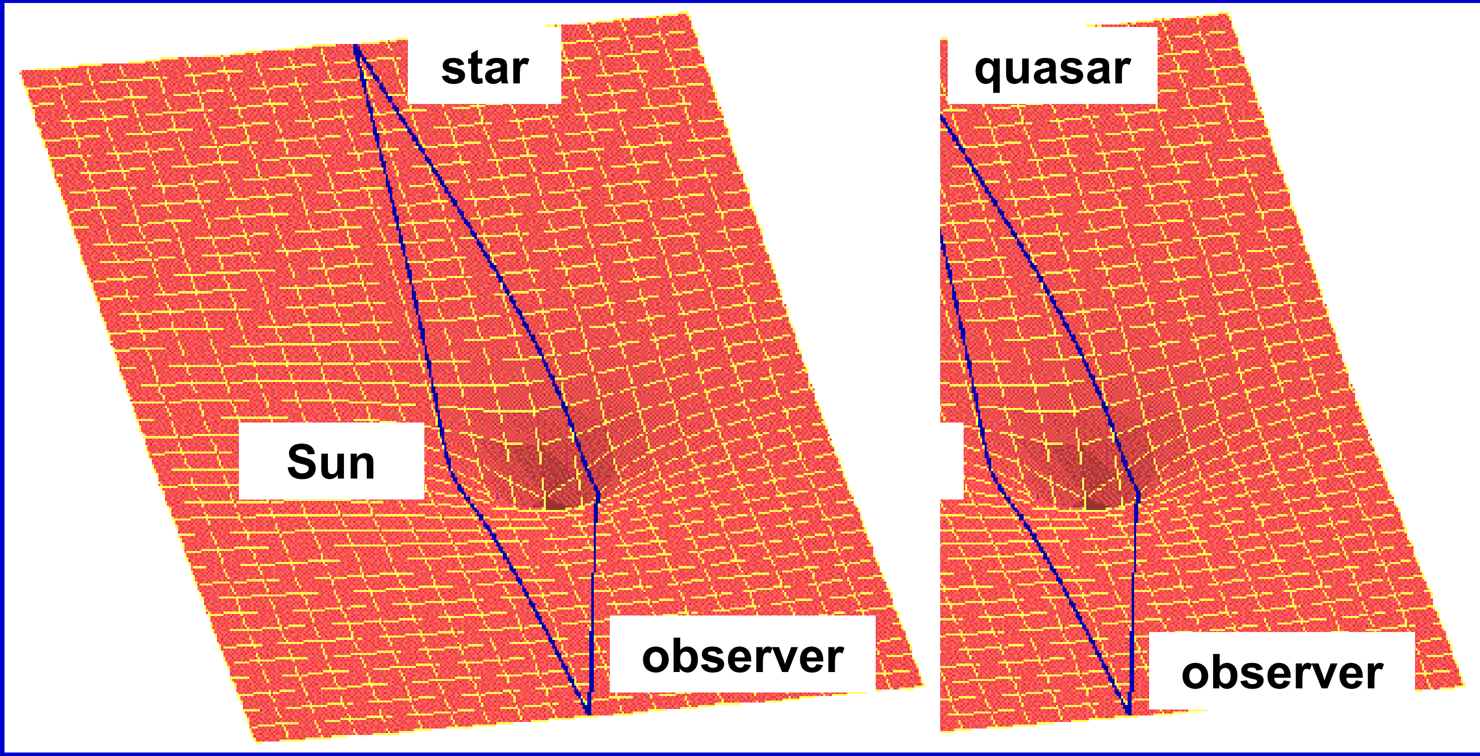


Precession of the perihelion of Mercury.

Gravitational lensing by the Sun during a total eclipse.

Gravitational redshift of light.

Implication: It's not crazy to assume that General Relativity accurately describes gravity on larger scales.



Einstein field equation:

Einstein tensor $$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$ stress-energy tensor

In a dilute perfect gas, $T_{\mu\nu}$ depends on the energy density ε and pressure P.

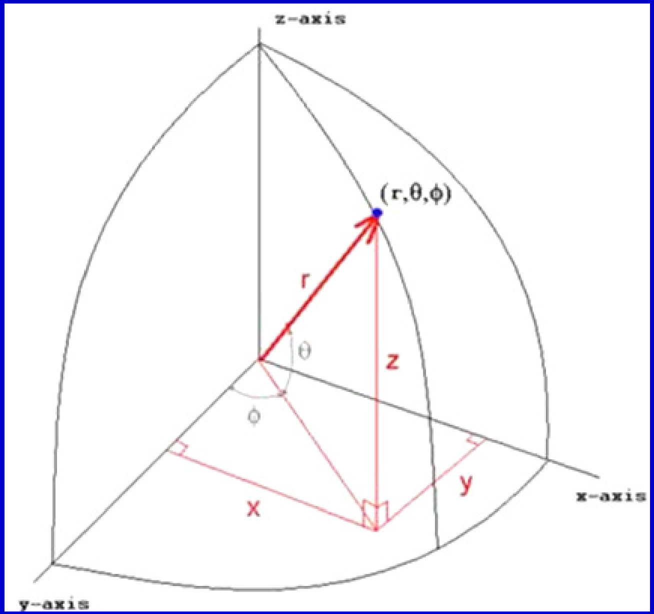
$G_{\mu\nu}$ depends on the metric $g_{\mu\nu}$ of 4-dimensional spacetime, which is a measure of local curvature.

$$ds^2 = \sum_{\mu,\nu} g_{\mu\nu} dx^{\mu} dx^{\nu}$$

Euclidean (3-dimensional) space:

$(x^1, x^2, x^3) = (x,y,z)$
$ds^2 = dx^2 + dy^2 + dz^2$

$(x^1, x^2, x^3) = (r,\theta,\varphi)$
$ds^2 = dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\varphi^2$

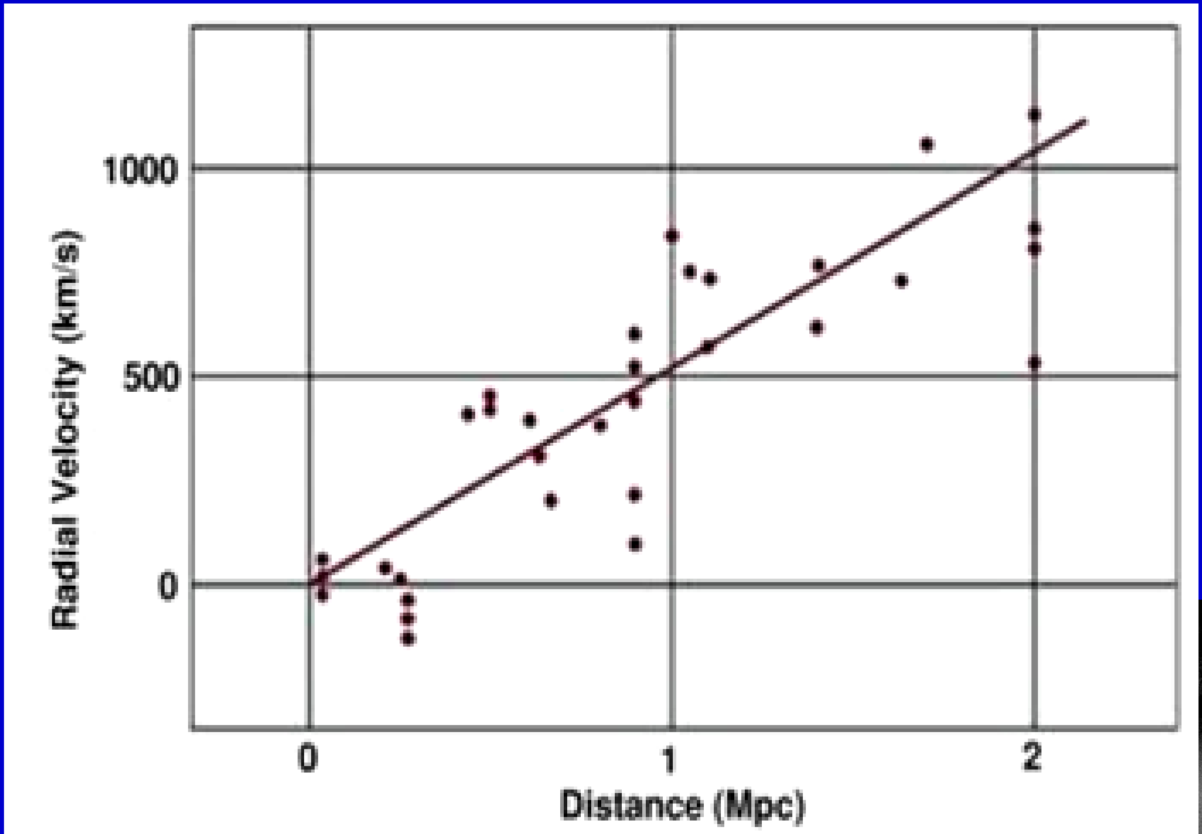


Minkowski (4-dimensional) spacetime:

$(x^0, x^1, x^2, x^3) = (ct,x,y,z)$
$ds^2 = - c^2dt^2 + dx^2 + dy^2 + dz^2$

$(x^0, x^1, x^2, x^3) = (ct,r,\theta,\varphi)$
$ds^2 = -c^2dt^2 + dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\varphi^2$

Observation 3: Galaxies have a **redshift** proportional to their distance.



This relation is known as "Hubble's Law"
(Edwin Hubble, 1929)

Hubble's law in mathematical form:

$$c\,z = H_0\,d$$

c = speed of light

z = redshift

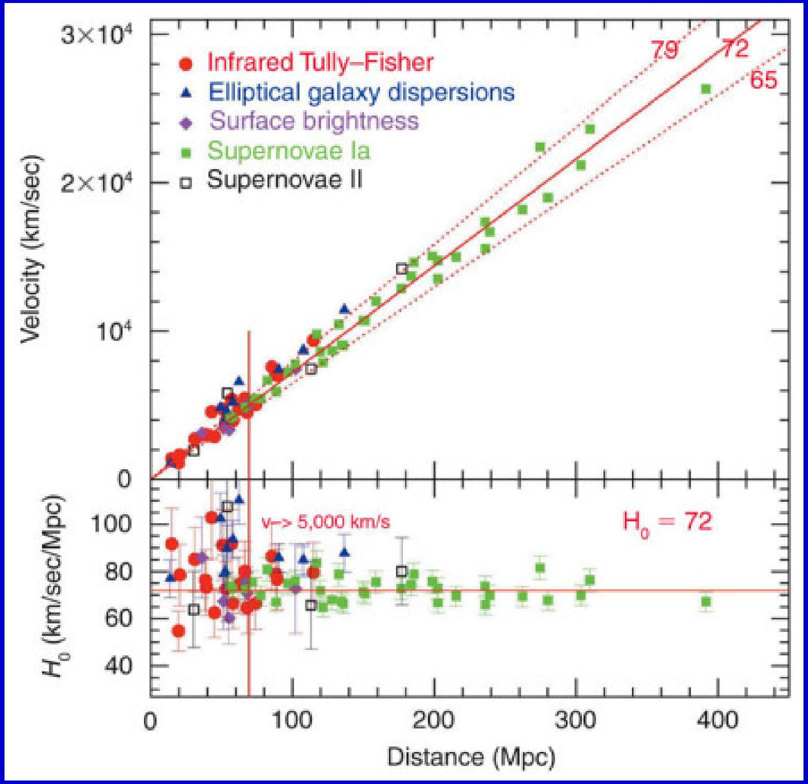
d = distance

H_0 = Hubble constant

This linear relation holds for $z \ll 1$.

This relation has scatter.

Implication: Space is expanding.
(Note: the Newtonian explanation would be "galaxies are moving *through* space".)



$$H_0 = cz/d \approx 71 \text{ km/sec/Mpc} \approx 2.3 \times 10^{-18} \text{ /sec}$$

How long has space been expanding?

Travel time = distance / speed

$t = d / v = d / (H_0 d) = 1/H_0$, independent of d

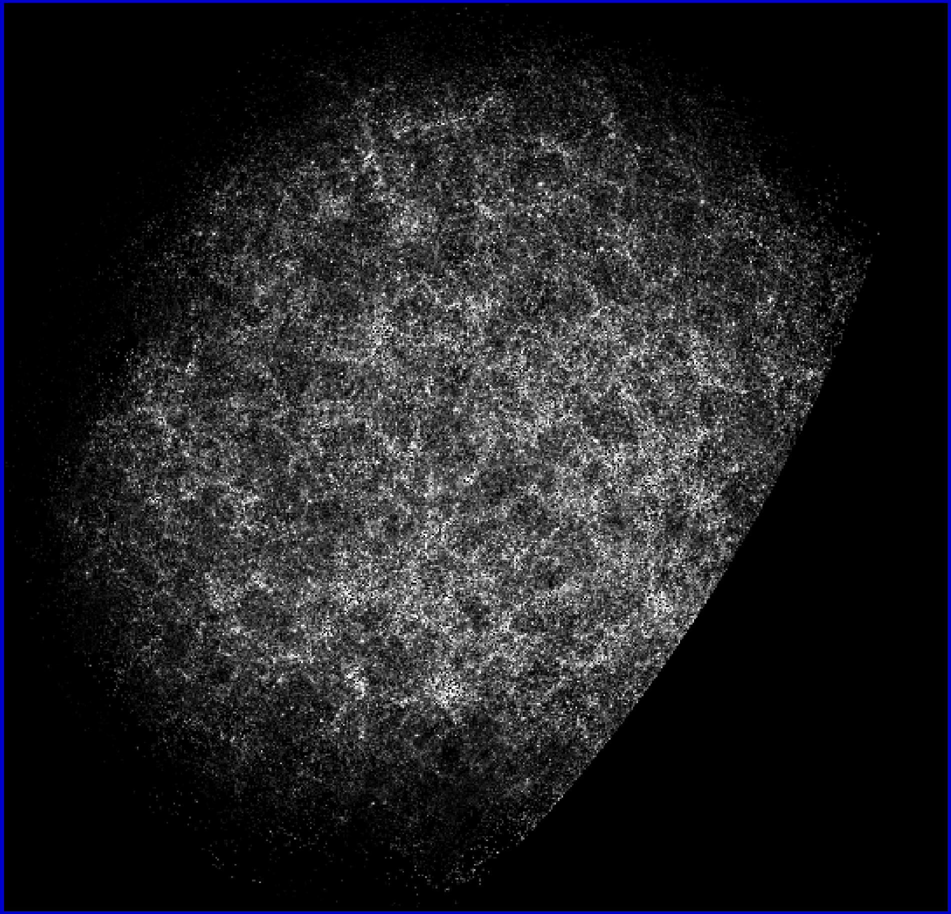
$1/H_0 \approx 4.3 \times 10^{17}$ sec ≈ 13.8 Gyr = "Hubble time"

$c/H_0 \approx 1.3 \times 10^{26}$ m ≈ 4200 Mpc = "Hubble distance"

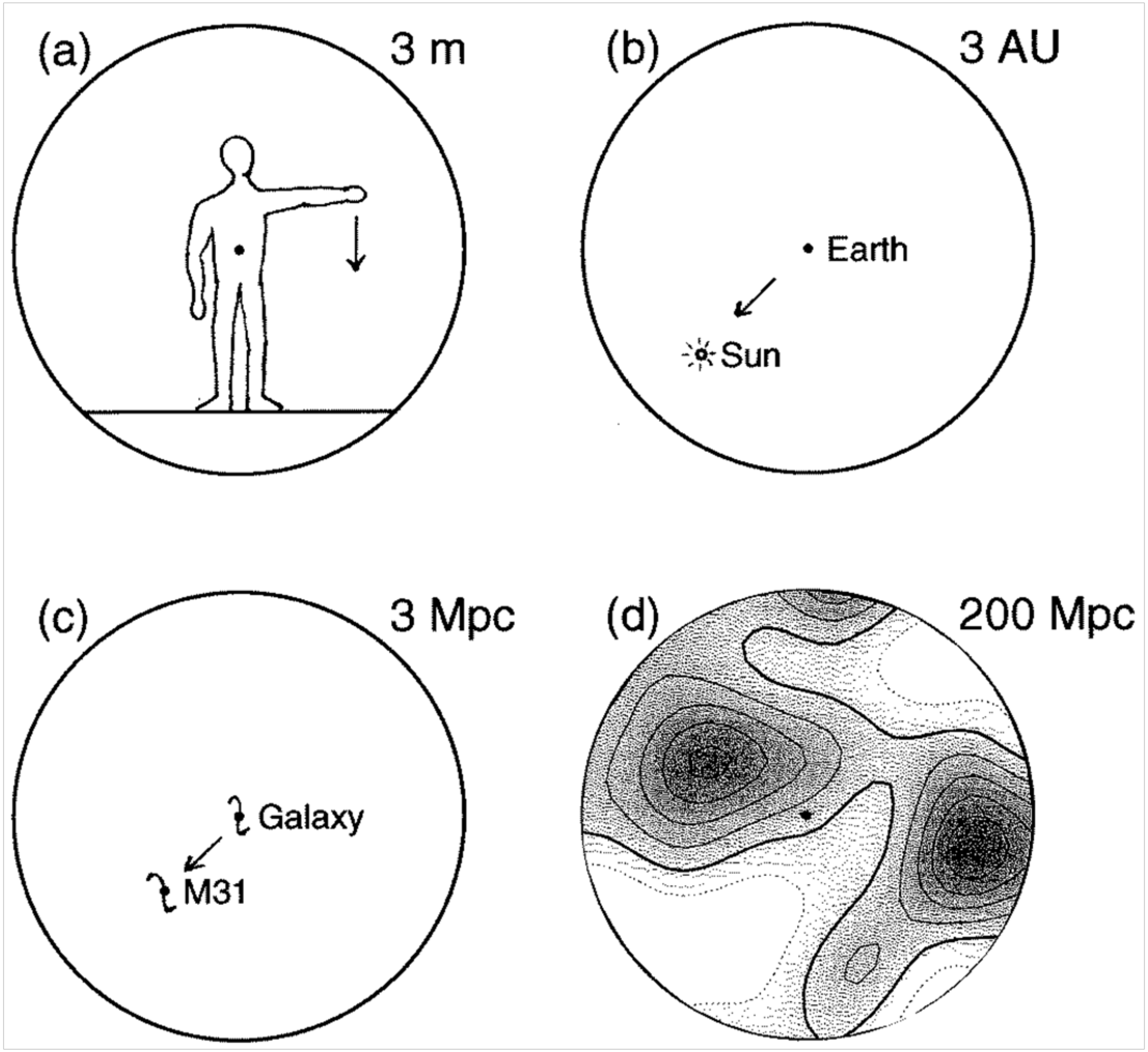
$\ll 10^{18}$ Mpc

Observation 4: The universe is **isotropic** on large scales.

distribution of galaxies on the sky →



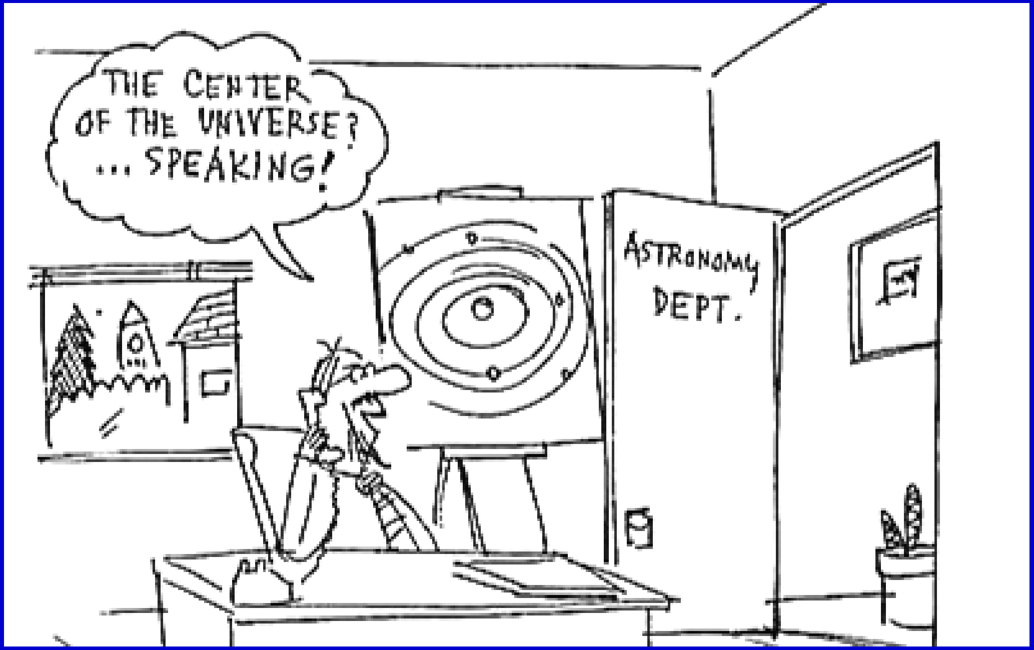
Isotropic = the same in all directions.
"Large scales" = larger than ~100 Mpc.



(a)
3 m
(b)
3 AU
Earth
Sun
(c)
3 Mpc
Galaxy
M31
(d)
200 Mpc

Assumption: There's nothing special about our location in the universe (on large scales).

(This statement is known as the "Copernican Principle".)



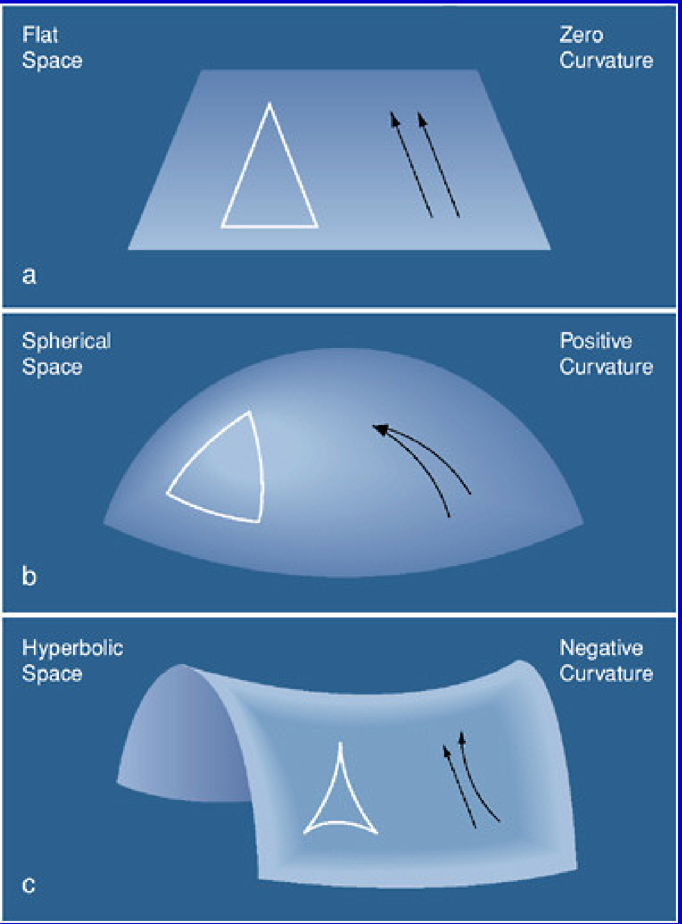
The Earth is not at the center of the universe.
The Sun is not at the center.
The Galaxy is not at the center. (There is no center.)

Implication: The universe is **homogeneous**
& isotropic on large scales.
(This is known as the "**Cosmological Principle**".)

If the universe is isotropic around you,
and if your location isn't special,
then the universe is isotropic around **every** observer.

A universe isotropic around every observer must be
homogeneous (the same in all locations).

There are **three** possible metrics for a homogeneous & isotropic 3-dimensional space.



Flat (or Euclidean): $\kappa = 0$

$$ds^2 = dr^2 + r^2\, d\Omega^2$$

(where $d\Omega^2 \equiv d\theta^2 + \sin^2\theta\, d\varphi^2$)

Positively curved: $\kappa = +1$

$$ds^2 = dr^2 + R^2 \sin^2 (r/R)\, d\Omega^2$$

(where R = radius of curvature)

Negatively curved: $\kappa = -1$

$$ds^2 = dr^2 + R^2 \sinh^2 (r/R)\, d\Omega^2$$

The **Robertson-Walker metric** gives the spacetime curvature of an expanding, homogeneous, isotropic universe.

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[dr^2 + S_\kappa(r)^2 \, d\Omega^2 \right]$$

$a(t)$ = scale factor

$a(t_0)$ = scale factor today = 1

$$S_\kappa(r) = \begin{cases} R_0 \sin(r / R_0) & \kappa = +1 \\ r & \kappa = 0 \\ R_0 \sinh(r / R_0) & \kappa = -1 \end{cases}$$

The cosmological principle is a powerful concept.

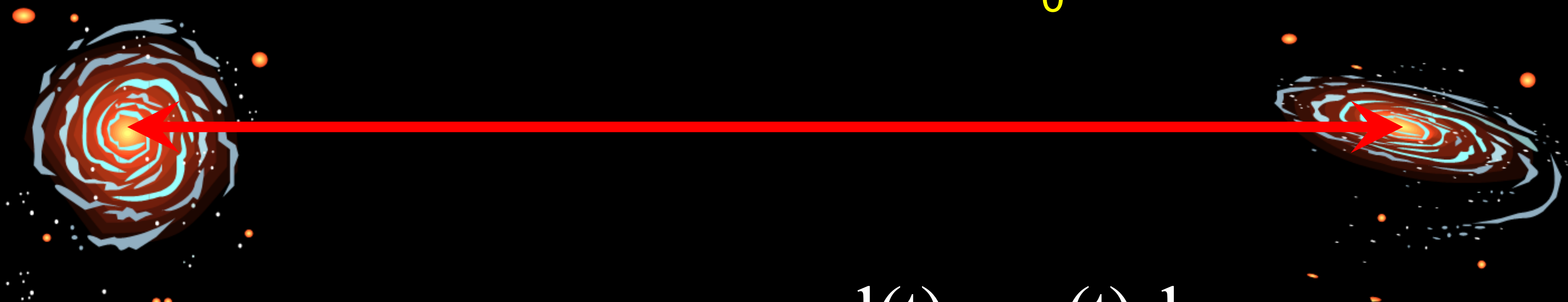
If we assume isotropy & homogeneity,
then spacetime curvature is described by
a number, a length, & a scalar function:

Curvature constant: $\kappa = -1, 0, \text{ or } +1$

Present-day radius of curvature: R_0

Scale factor: $a(t)$

The scale factor a(t) is related to the observed Hubble constant H_0.



Distance between two galaxies: $$d(t) = a(t)\,d_0$$

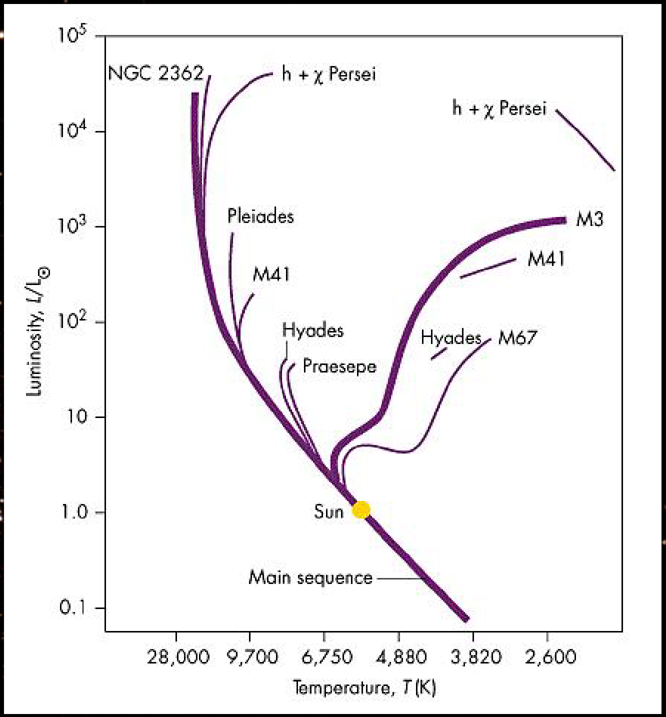
Relative speed of two galaxies: $$v(t) = \dot{d} = \dot{a}\,d_0$$

Hubble's law for the two galaxies: $$v(t) = \left(\frac{\dot{a}}{a}\right) d(t)$$

$$H(t) = \frac{\dot{a}}{a}$$

$$\boxed{H(t_0) = H_0 = \left.\frac{\dot{a}}{a}\right|_{t=t_0} = 71\,\mathrm{km\,s^{-1}\,Mpc^{-1}}}$$

Observation 5: The stars in globular clusters have ages of ~ 14 Gyr.



$$14 \text{ Gyr} \approx 1/H_0$$

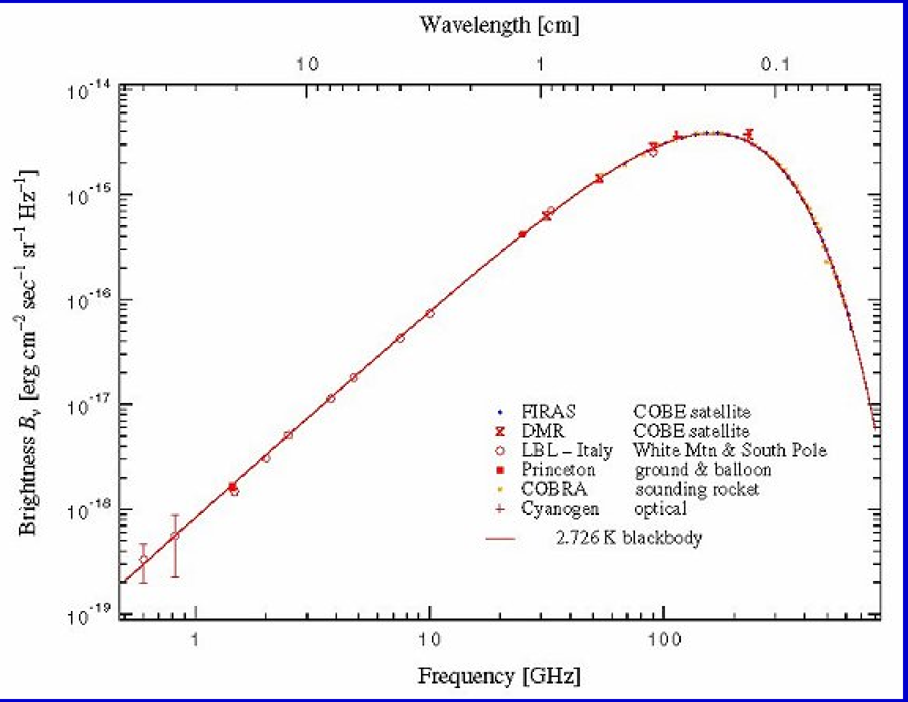
Implication: The universe is at least as old as the stars that it contains.

"I am as old as my tongue,
and slightly older than my teeth."

Observations are consistent with a universe that began expanding ~ 14 Gyr ago, with globular clusters forming soon afterwards. (Soon = within a billion years.)

Observation 6: There's a nearly isotropic microwave background with a blackbody spectrum.

Wilson & Penzias, New Jersey, 1965



Penzias & Wilson, Stockholm, 1978

The "Cosmic Microwave Background" (CMB) has a temperature T = 2.725 K.

The CMB today:

Energy density:

$$\varepsilon = \alpha T^4 = 4.17 \times 10^{-14}\,\mathrm{J\,m^{-3}} = 0.26\,\mathrm{MeV\,m^{-3}}$$

This is 10 times the average energy density of starlight.

Number density of photons:

$$n = \beta T^3 = 4.11 \times 10^8\,\mathrm{m^{-3}}$$

This is **2 billion** times the average number density of baryons (protons & neutrons).

The past and future CMB:

Photon wavelength **increases** as the universe expands.

$$\lambda \propto a(t)$$

Photon energy **decreases**.

$$E = hc/\lambda \propto a(t)^{-1}$$

Number density & energy density of photons **decrease**.

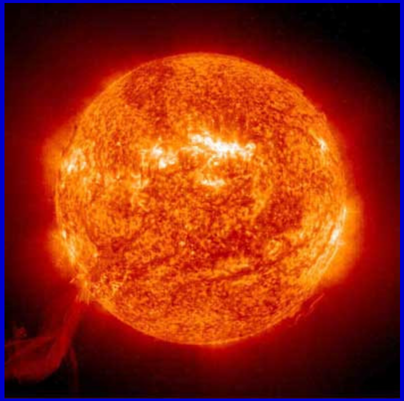
$$n \propto a(t)^{-3},\ \varepsilon \propto \langle E \rangle n \propto a(t)^{-4}$$

Blackbody temperature **decreases**.

$$kT \propto \langle E \rangle \propto a(t)^{-1}$$

Implication: Blackbody radiation is produced by opaque objects; thus, the universe was once **opaque**, although it's now mostly transparent.

The CMB was hotter in the past: above some critical temperature it would have photoionized hydrogen, providing opacity by electron scattering.



(Today, **stars** are opaque because they're made mostly of ionized hydrogen.)

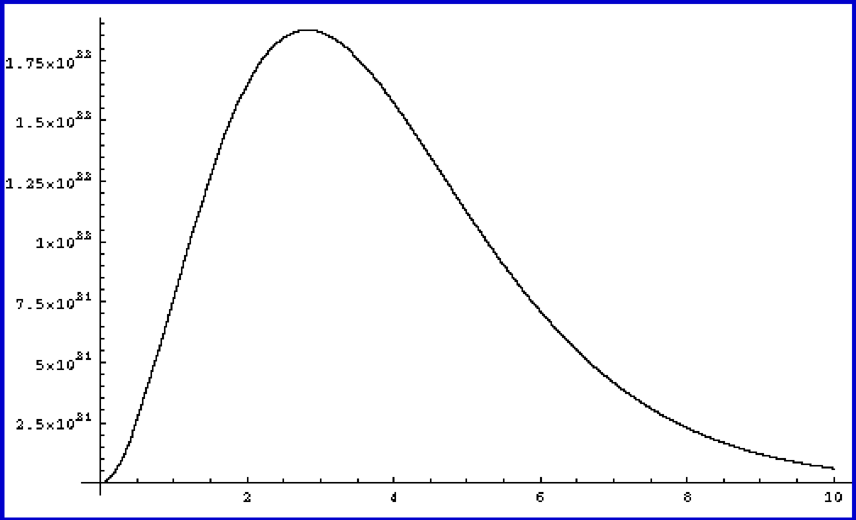
At what temperature T is blackbody radiation energetic enough to ionize hydrogen?

$$H + \gamma \leftrightarrow p + e^{-}$$

Since the ionization energy of hydrogen is $\chi = 13.6$ eV, we might guess $T = \chi / k = 160{,}000$ K.

Applied to the early universe, this guess is an **overestimate**.

(1) Protons are badly outnumbered by photons.



(2) Blackbody spectra have exponential tails to high photon energy.

Mean photon energy = 2.7 kT.
One photon in 500 will have E > 10 kT.
One photon in 30 billion will have E > 30 kT.

Because of the high-energy tail in the spectrum, recombination takes place **not** when $\chi / kT \approx 1$, but when $\chi / kT \approx -\ln \eta$, where $\eta = n_{baryon}/n_{photon} \approx 5\times10^{-10}$.

Detailed calculation shows that recombination occurs when $kT_{rec} = 0.024\chi = 0.32$ eV.

This corresponds to a temperature $T_{rec} = 3700$K, and a scale factor $a(t_{rec}) = 2.725\text{K} / 3700\text{K} = 0.00074$.

The observations are consistent with the

Big Bang Theory.

Definition of “Big Bang Theory”:

The universe has expanded from an initial dense, hot state that existed at some finite time in the past.