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IDPASC School of Flavour Physics

History of CP and T Violation in Kaon Physics

K^0 Mesons, $K^0 \leftrightarrow \overline{K}^0$ Transitions

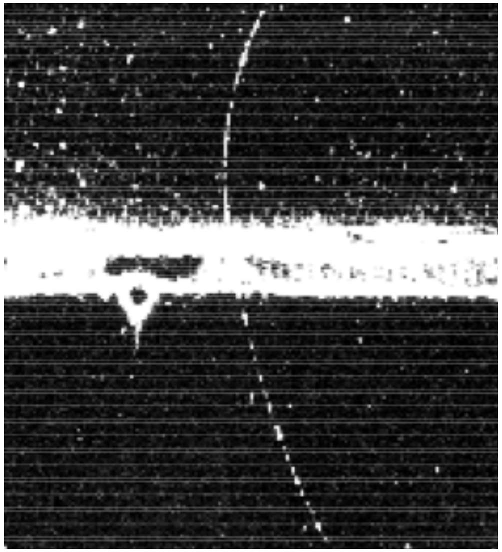
Four Types of CP Symmetry Breaking („Violation“)

Motion Reversal, „Time Reversal“, T Symmetry, CPT Symmetry

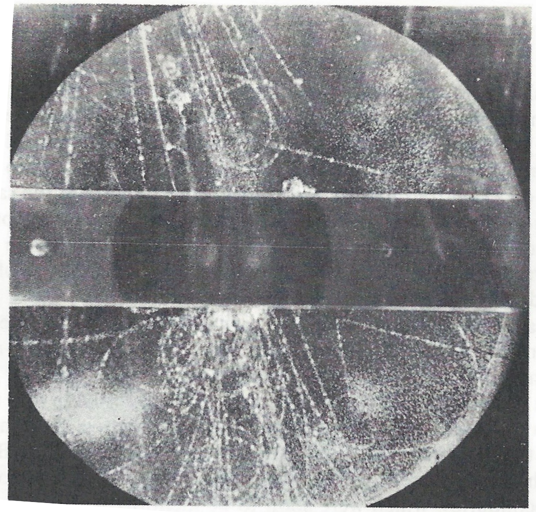
How to distinguish violations of CP & T from violations of CP & CPT

Particles, Antiparticles, Strangeness

The first antiparticle 1932: discovered by Carl Anderson (1905 - 1991) in his cloud chamber, predicted 1928 by Paul Dirac (1902 - 1984)



The first neutral Kaon 1947: discovered by Rochester and Butler, again with cosmic rays in a cloud chamber. Gell-Mann 1953 called Kaons strange particles; they are produced with large rate and decaying with small rate.



Pais, Gell-Mann, Nishijima:

Strangeness $S(p,n,\pi^+,\pi^-)=0$, $S(K^+,K^0)=+1$,

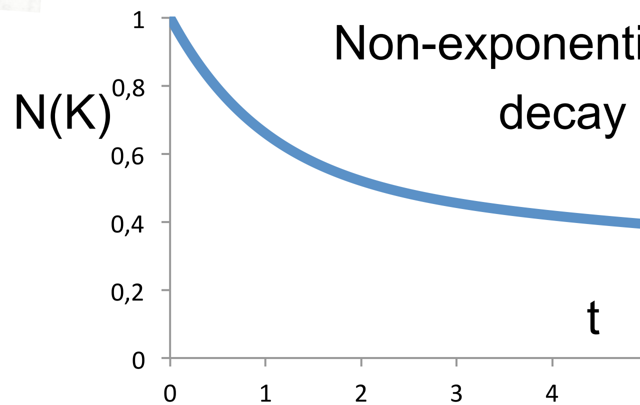
$S(\Lambda,K^-,\overline{K}^0)=-1$ conserved in production:

$\pi^- + p \to \Lambda + K^0$, $\pi^+ + p \to p + K^+ + \overline{K}^0$, σ strong!

But not conserved in decay:

$K^0 \to \pi^+\pi^-$, $\overline{K}^0 \to \pi^+\pi^-$. Γ weak!

2nd very strange property:



Non-exponential decay law.

Transitions $K^0 \leftrightarrow \bar{K}^0$

As long as it lives, an unstable particle has always one alternative:

To decay or not to decay.

The decay probability per time interval is a constant Γ.

This leads to the exponential decay law $N(t) = N(0)\, e^{-\Gamma t}$.

The unstable particle K^0 is an exception with always two alternatives:

To decay, to transform itself into a $\bar{K}^0$ or to remain a K^0.

This multiple choice leads to the non-exponential decay law.

The transition $K^0 \rightarrow \bar{K}^0$ has been predicted by Gell-Mann and Pais 1955:

PHYSICAL REVIEW VOLUME 97, NUMBER 5 MARCH 1, 1955

Behavior of Neutral Particles under Charge Conjugation

M. GELL-MANN,* *Department of Physics, Columbia University, New York, New York*

AND

A. PAIS, *Institute for Advanced Study, Princeton, New Jersey*

(Received November 1, 1954)

Some properties are discussed of the θ^0, a heavy boson that is known to decay by the process $\theta^0 \rightarrow \pi^+ + \pi^-$. According to certain schemes proposed for the interpretation of hyperons and K particles, the θ^0 possesses an antiparticle $\bar{\theta}^0$ distinct from itself. Some theoretical implications of this situation are discussed with special reference to charge conjugation invariance. The application of such invariance in familiar instances is surveyed in Sec. I. It is then shown in Sec. II that, within the framework of the tentative schemes under consideration, the θ^0 must be considered as a "particle mixture" exhibiting two distinct lifetimes, that each lifetime is associated with a different set of decay modes, and that no more than half of all θ^0's undergo the familiar decay into two pions. Some experimental consequences of this picture are mentioned.

Read it,

it's like an

objet d'art!

more on Transitions $K^0 \leftrightarrow \bar{K}^0$

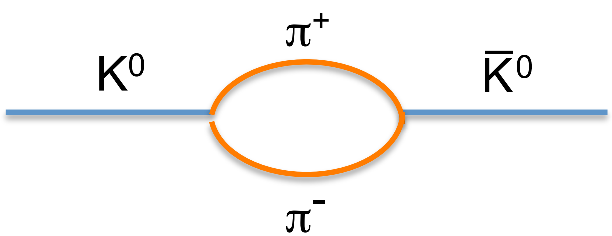
Gell-Mann & Pais 1955: Charge Conjugation = Operator C, $C(\mu^-) = \mu^+$, $C(p) = \bar{p}$,

Charge conjugation symmetry: $m(\bar{p}) = m(p)$ ...

$C(\gamma) = -\gamma$, $C(\pi^0) = +\pi^0$. But: $C(n) = \bar{n} \neq n$ (B-number), $C(K^0) = \bar{K}^0 \neq K^0$ (S-number).

Experimentally known: decay $K^0(\Theta^0) \rightarrow \pi^+ \pi^-$ and its rate Γ.

Postulated by C symmetry: existence of $\bar{K}^0$, and decay $\bar{K}^0 \rightarrow \pi^+\pi^-$ with the same rate

And: 

with a rate $\Gamma(K^0 \rightarrow \bar{B}^0) \approx \Gamma$.

Any state $\Psi = \psi_1 K^0 + \psi_2 \bar{K}^0$ is possible; only the state K_1 with $\psi_1 = +\psi_2$ decays into 2π;

the orthogonal state K_2 with $\psi_1 = -\psi_2$ does not, and lives much longer than $1/\Gamma$.

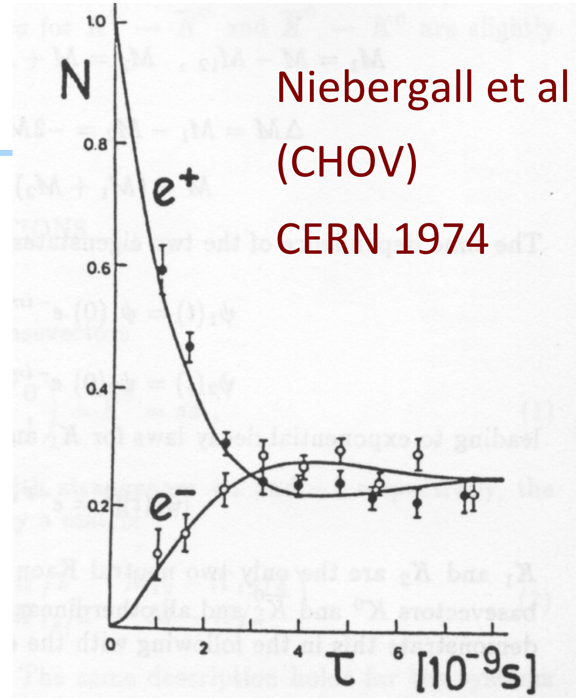
1956 Landé et al: ~ 20 events compatible with $K^0 \not\rightarrow \pi^+\pi^-$ and with $\tau_2 / \tau_1 > 10$.

1957 Weak decays violate P and C, but CP remains a conserved symmetry.

1961 R. H. Good et al: Passage of a neutral K beam through a set of „regenerators“

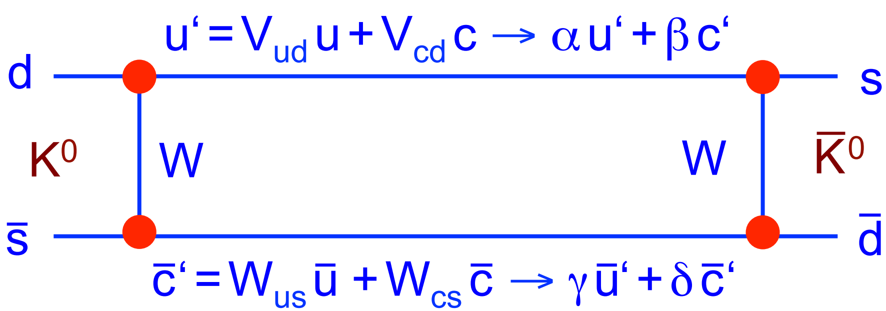
produces an interference pattern yielding „$\Gamma(K^0 \rightarrow \bar{B}^0)$“ $\approx \Delta m = (0.85 \pm 0.25)\,\Gamma$.

and more on $K^0 \leftrightarrow \bar{K}^0$



Beautiful demonstration 1974, K^0 mesons at t=0 (produced by $K^+ p \to K^0 p \pi^+$) develop into $\psi_1 K^0 + \psi_2 \bar{K}^0$ identified by decays $K^0 \to \pi^- e^+ \nu$ and $\bar{K}^0 \to \pi^+ e^- \nu$:

Quarks were known since 10 years, and the St. Model was advanced; it explains $K^0 \leftrightarrow \bar{K}^0$ by a box graph:

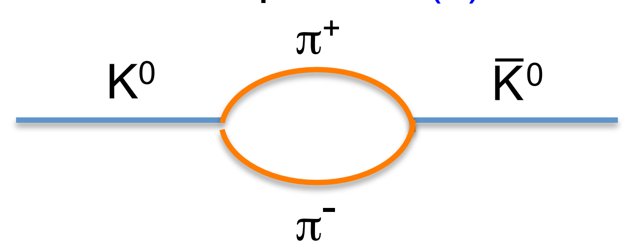


Quark oscillations $u' \to c'$ in hadron-size distances require $m(u) \neq m(c)$ like neutrino oscillations in astronomical distances.

2nd remark: The St. Model is CPT-symmetric, therefore antiquarks mix with $W_{ij} = V_{ij}^*$.

M K Gaillard and B W Lee 1974: The observed $K^0 \leftrightarrow \bar{K}^0$ rate requires m(c) < 5 GeV.

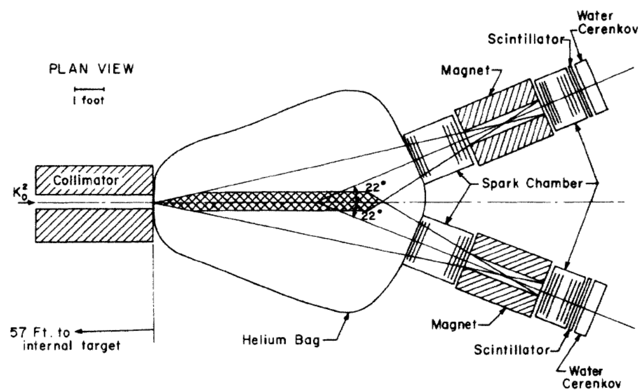
November 1974: Discovery of J and ψ with $m(c) \approx 1.5$ GeV. Half of the $K^0 \leftrightarrow \bar{K}^0$ rate is box graph and half is „long range interaction“:

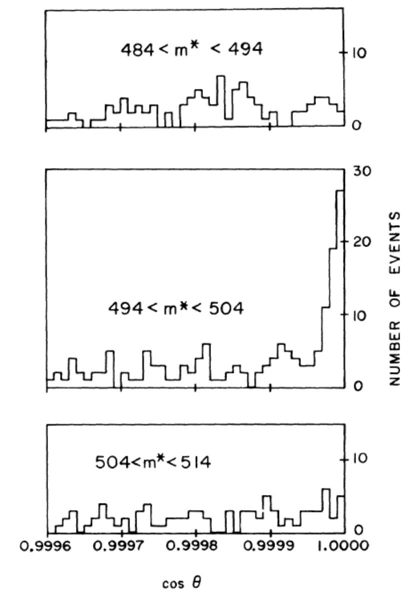


Back to History: Discovery of CP Violation

1963 Adair et al: „Anomalous Regeneration“, more $\pi^+\pi^-$ decays observed in a K_2 beam passing through a liquid-hydrogen regenerator than expected.

1964 Christenson Cronin Fitch Turlay: The same happens in He gas ≈ vacuum, i. e. CP symmetry is broken, either in the decay $K_2(CP=-1) \rightarrow \pi^+\pi^-$ $(CP=+1)$ or because the long- living state K_L in $K^0\,\overline{K}^0$ oscillations is not the CP eigenstate K_2.





1967 Steinberger et al: $$\Delta_e = \frac{N\left(K_L \rightarrow \pi^- e^+ \nu\right) - N\left(K_L \rightarrow \pi^+ e^- \nu\right)}{N\left(K_L \rightarrow \pi^- e^+ \nu\right) + N\left(K_L \rightarrow \pi^+ e^- \nu\right)} = \left(2.24 \pm 0.36\right)\cdot 10^{-3},$$

1967 Schwartz et al: $$\Delta_\mu = \frac{N\left(K_L \rightarrow \pi^- \mu^+ \nu\right) - N\left(K_L \rightarrow \pi^+ \mu^- \nu\right)}{N\left(K_L \rightarrow \pi^- \mu^+ \nu\right) + N\left(K_L \rightarrow \pi^+ \mu^- \nu\right)} = \left(4.0 \pm 1.3\right)\cdot 10^{-3},$$

$\Delta S = \Delta Q$, i.e. $K^0 \rightarrow \ell^+$ and $\overline{K}^0 \rightarrow \ell^-$ ➔ CP symmetry is broken in $K^0\,\overline{K}^0$ oscillations.

Four Types of CP Violation

(1) CP violation in $K^0\bar{K}^0$ transitions: Long time after the production of K^0 states, transitions lead to a state K_L containing more K^0 than $\bar{K}^0$. Starting with $\bar{K}^0$ leads to the same K_L with more K^0 than $\bar{K}^0$, $\Delta_\ell = 2\,\mathrm{Re}\,\varepsilon = 3.3\cdot 10^{-3}$.

(2) CP violation in one decay mode, „direct CPV“: Comparing the probability for the decay $K^0 \rightarrow f$ to that of $\bar{K}^0 \rightarrow \bar{f}$ at production time. Requires two different weak amplitudes with different final-state-interaction (FSI) phases.

$$\boxed{\frac{\Gamma\left(K^0 \rightarrow \pi^+\pi^-\right) - \Gamma\left(\bar{K}^0 \rightarrow \pi^+\pi^-\right)}{\Gamma\left(K^0 \rightarrow \pi^+\pi^-\right) + \Gamma\left(\bar{K}^0 \rightarrow \pi^+\pi^-\right)} = 2\,\mathrm{Re}\,\varepsilon' = 6\cdot 10^{-6}.}$$

(3) CP violation in the interplay of transitions and decay: Even with Re ε = Re ε' = 0 there can be large CPV. Best known example $B^0 \rightarrow J/\psi K_S$, Im $\lambda = \sin 2\beta = 0.68$. Also in the K^0 system in the time dependence of $K^0 \rightarrow \pi^+\pi^-$, Im $\eta_{+-} = 1.6\cdot 10^{-3}$.

(4) CP violation in the comparison of two decay modes:

Direct CPV without requiring two different FSI phases, e.g.

$$\boxed{\mathrm{Im}\frac{A\left(K^0 \rightarrow \pi^+\pi^-\right)\cdot A\left(\bar{K}^0 \rightarrow \pi^0\pi^0\right)}{A\left(\bar{K}^0 \rightarrow \pi^+\pi^-\right)\cdot A\left(K^0 \rightarrow \pi^0\pi^0\right)} = 6\,\mathrm{Im}\,\varepsilon' = 1.5\cdot 10^{-5}.}$$

Quantum Mechanics: The Weisskopf-Wigner Formalism

Stable particle: $i\cdot\partial\psi/\partial t = E\psi = m\psi,\quad \psi(t)=\psi(0)\cdot e^{-imt},\quad |\psi|^2 = |\psi(0)|^2 = 1.$

Unstable particle:

$$i\cdot\partial\psi/\partial t = \mu\psi,\quad m=(\mu+\mu^*)/2,\quad \Gamma/2=(\mu-\mu^*)/2i$$

$$i\cdot\dot\psi = (m-i\Gamma/2)\psi,\ \psi = e^{-imt-\Gamma t/2},\ |\psi|^2 = e^{-\Gamma t},\ \Gamma = \sum_f |\langle f|T|\psi\rangle|^2.$$

Unstable two-state system like $\Psi = \psi_1 K^0 + \psi_2 \overline{K}^0$:

7 observables,

$\Phi(m_{12})$ arbitrary,

$\Phi(\Gamma_{12}/m_{12})$ observable.

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \begin{pmatrix}\mu_{11} & \mu_{12}\\ \mu_{21} & \mu_{22}\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix},\qquad \begin{aligned} m_{ij} &= (\mu_{ij}+\mu_{ij}^+)/2\\ \Gamma_{ij}/2 &= (\mu_{ij}-\mu_{ij}^+)/2i\end{aligned}$$

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \left[\begin{pmatrix}m_{11} & m_{12}\\ m_{12}^* & m_{22}\end{pmatrix} - \frac{i}{2}\begin{pmatrix}\Gamma_{11} & \Gamma_{12}\\ \Gamma_{12}^* & \Gamma_{22}\end{pmatrix}\right]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}.$$

m_{12}, m_{21}, and all Γ_{ij} are produced by weak interactions, m_{11} and m_{22} by strong & wk.

$$\Gamma_{11} = \sum_f |\langle f|T_W|K^0\rangle|^2,\ \Gamma_{22} = \sum_f |\langle f|T_W|\overline{K}^0\rangle|^2,\ \Gamma_{12} = \Gamma_{21}^* = \sum_f \langle K^0|T_W|f\rangle\langle f|T_W|\overline{K}^0\rangle.$$

$$m_{ij} = m_0\delta_{ij} + \langle i|T_W|j\rangle + \sum_v \langle i|T_W|v\rangle\langle v|T_W|j\rangle$$

Weisskopf-Wigner: Symmetries of m_{ij} and Γ_{ij}

Using $$\Gamma_{11} = \sum_f \left|\langle f|T_W|K^0\rangle\right|^2, \Gamma_{22} = \sum_f \left|\langle f|T_W|\bar{K}^0\rangle\right|^2, \Gamma_{12} = \Gamma_{21}^* = \sum_f \langle K^0|T_W|f\rangle\langle f|T_W|\bar{K}^0\rangle.$$

$$m_{ij} = m_0\delta_{ij} + \langle i|T_W|j\rangle + \sum_v \langle i|T_W|v\rangle\langle v|T_W|j\rangle$$

and $T_{CP} = (CP)T_W(CP)^+, T_T = T\,T_W T^{-1}, T_{CPT} = (CPT)T_W(CPT)^{-1}$, where *CP* is unitary and *T* is the anti-unitary time-reversal operator in quantum mechanics, defined by exchange of initial and final state and exchange t $\leftrightarrow$ -t in the dynamics, here in T_w,

CP symmetry of T_w leads to $m_{22} = m_{11}, \Gamma_{22} = \Gamma_{11}, \mathrm{Im}(\Gamma_{12}/m_{12}) = 0,$ 4 parameters.

T symmetry to $\mathrm{Im}(\Gamma_{12}/m_{12}) = 0,$ and no effect on the 11 and 22 elements, 6 param.

CPT symmetry to $m_{22} = m_{11}, \Gamma_{22} = \Gamma_{11}$ no effect on the 12 and 21 elements. 5 par.

Braco Lavoura Silva „CP Violation“, Section 6.3

T

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \left[\begin{pmatrix} m_{11} & m_{12}e^{-i\alpha} \\ m_{12}e^{i\alpha} & m_{22}\end{pmatrix} - \frac{i}{2}\begin{pmatrix}\Gamma_{11} & \Gamma_{12}e^{-i\alpha} \\ \Gamma_{12}e^{i\alpha} & \Gamma_{22}\end{pmatrix}\right]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}.$$

m_{12} and Γ_{12} real

CPT

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \left[\begin{pmatrix} m & m_{12} \\ m_{12}^* & m\end{pmatrix} - \frac{i}{2}\begin{pmatrix}\Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma\end{pmatrix}\right]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}.$$

m_{12} and Γ_{12} complex

CP

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \left[\begin{pmatrix} m & m_{12}e^{-i\alpha} \\ m_{12}e^{i\alpha} & m\end{pmatrix} - \frac{i}{2}\begin{pmatrix}\Gamma & \Gamma_{12}e^{-i\alpha} \\ \Gamma_{12}e^{i\alpha} & \Gamma\end{pmatrix}\right]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}.$$

Weisskopf-Wigner: Solutions

$$i\frac{\partial}{\partial t}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}=\left[\begin{pmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix}-\frac{i}{2}\begin{pmatrix}\Gamma_{11} & \Gamma_{12}\\ \Gamma_{21} & \Gamma_{22}\end{pmatrix}\right]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

This Schrödinger eq. for the evolution of $\Psi=\psi_1 K^0+\psi_2\overline{K}^0$ has two solutions with exponential decay laws:

$$K_S^0(t)=\left[p\cdot\sqrt{1+2\delta}\cdot K^0+q\cdot\sqrt{1-2\delta}\cdot\overline{K}^0\right]\cdot e^{-\Gamma_S t/2-\mathrm{i}m_S t}=\left[(1+\varepsilon+\delta)\cdot K^0+(1-\varepsilon-\delta)\cdot\overline{K}^0\right]\cdot e^{-\Gamma_S t/2-\mathrm{i}m_S t}/\sqrt{2}$$

$$K_L^0(t)=\left[p\cdot\sqrt{1-2\delta}\cdot K^0-q\cdot\sqrt{1+2\delta}\cdot\overline{K}^0\right]\cdot e^{-\Gamma_L t/2-\mathrm{i}m_L t}=\left[(1+\varepsilon-\delta)\cdot K^0-(1-\varepsilon+\delta)\cdot\overline{K}^0\right]\cdot e^{-\Gamma_L t/2-\mathrm{i}m_L t}/\sqrt{2}$$

using $p/q=(1+\varepsilon)/(1-\varepsilon)$ and $|\varepsilon| \ll 1$, $|\delta| \ll 1$. Im ε is not observable, only Re ε.

The 7 parameters $m_S = m - \Delta m/2$, $m_L = m + \Delta m/2$, $\Gamma_S = \Gamma + \Delta\Gamma/2$, $\Gamma_L = \Gamma - \Delta\Gamma/2$, Re ε, Re δ and Im δ follow unambiuously from the 7 parameters in the Schrödinger eq.:

$$m=\frac{m_{11}+m_{22}}{2},\ \Delta m\approx 2|m_{12}|,\ \Gamma=\frac{\Gamma_{11}+\Gamma_{22}}{2},\ \ \Delta\Gamma\approx 2|\Gamma_{12}|,$$

$$\mathrm{Re}\,\varepsilon=\frac{\mathrm{Im}(m_{12}^*\Gamma_{12})}{(\Delta m)^2+|\Gamma_{12}|^2},\ \delta=-\frac{1}{2}\cdot\frac{(m_{11}-m_{22})-\mathrm{i}(\Gamma_{11}-\Gamma_{22})/2}{\Delta m-\mathrm{i}\Delta\Gamma/2}.$$

T symmetry of T_W gives Re ε = 0,
CPT symmetry δ = 0,
CP symmetry Re ε = δ = 0.

The $K^0\overline{K}^0$ system is a very small world. Any evolution, any observation is given by only quantum mechanics and by the values of only 7 parameters.

Weisskopf-Wigner: Transition Rates

$$K_S^0(0) = \left[(1+\varepsilon+\delta)\cdot K^0 + (1-\varepsilon-\delta)\cdot \bar{K}^0\right]/\sqrt{2}$$
$$K_L^0(0) = \left[(1+\varepsilon-\delta)\cdot K^0 - (1-\varepsilon+\delta)\cdot \bar{K}^0\right]/\sqrt{2}$$

The scalar product is

$$\left\langle K_L^0 \middle| K_S^0 \right\rangle = 2\,\mathrm{Re}\,\varepsilon + 2\,\mathrm{i}\,\mathrm{Im}\,\delta$$

Solving for K and $\bar{\mathrm{K}}$:

$$K^0 = \left[(1-\varepsilon+\delta)\cdot K_S^0 + (1-\varepsilon-\delta)\cdot K_L^0\right]/\sqrt{2}$$
$$\bar{K}^0 = \left[(1+\varepsilon-\delta)\cdot K_S^0 - (1+\varepsilon+\delta)\cdot K_L^0\right]/\sqrt{2}$$

Control: these expressions fulfill $\langle K|K\rangle = \langle\bar{K}|\bar{K}\rangle = 1$, $\langle K|\bar{K}\rangle = 0$.

With $\gamma_j = \Gamma_j/2 + \mathrm{i}\,m_j$, we obtain for initial K^0 and initial $\bar{K}^0$:

$$\Psi_K(t) = \left[(1+2\delta)\mathrm{e}^{-\gamma_S t} + (1-2\delta)\mathrm{e}^{-\gamma_L t}\right]\frac{K^0}{2} + (1-2\varepsilon)\left(\mathrm{e}^{-\gamma_S t} - \mathrm{e}^{-\gamma_L t}\right)\frac{\bar{K}^0}{2}$$
$$\Psi_{\bar{K}}(t) = (1+2\varepsilon)\left(\mathrm{e}^{-\gamma_S t} - \mathrm{e}^{-\gamma_L t}\right)\frac{K^0}{2} + \left[(1-2\delta)\mathrm{e}^{-\gamma_S t} + (1+2\delta)\mathrm{e}^{-\gamma_L t}\right]\frac{\bar{K}^0}{2}$$

Projecting and squaring gives the four time-dependent rates:

$$R(K \to K) = \left[(1+4\,\mathrm{Re}\,\delta)\mathrm{e}^{-\Gamma_S t} + (1-4\,\mathrm{Re}\,\delta)\mathrm{e}^{-\Gamma_L t} + 2\,\mathrm{e}^{-\Gamma t}\left(\cos\Delta m t - 4\,\mathrm{Im}\,\delta\cdot\sin\Delta m t\right)\right]/4$$
$$R(\bar{K} \to \bar{K}) = \left[(1-4\,\mathrm{Re}\,\delta)\mathrm{e}^{-\Gamma_S t} + (1+4\,\mathrm{Re}\,\delta)\mathrm{e}^{-\Gamma_L t} + 2\,\mathrm{e}^{-\Gamma t}\left(\cos\Delta m t + 4\,\mathrm{Im}\,\delta\cdot\sin\Delta m t\right)\right]/4$$
$$R(K \to \bar{K}) = (1-4\,\mathrm{Re}\,\varepsilon)\left(\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} - 2\,\mathrm{e}^{-\Gamma t}\cos\Delta m t\right)/4$$
$$R(\bar{K} \to K) = (1+4\,\mathrm{Re}\,\varepsilon)\left(\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} - 2\,\mathrm{e}^{-\Gamma t}\cos\Delta m t\right)/4$$

Weisskopf-Wigner: Transition Asymmetries

$$R_1 = R\left(K \to K\right) = \left[\left(1+4\,\mathrm{Re}\,\delta\right)e^{-\Gamma_S t} + \left(1-4\,\mathrm{Re}\,\delta\right)e^{-\Gamma_L t} + 2\,e^{-\Gamma t}\left(\cos\Delta m t - 4\,\mathrm{Im}\,\delta\cdot\sin\Delta m t\right)\right]/\,4$$

$$R_2 = R\left(\bar{K} \to \bar{K}\right) = \left[\left(1-4\,\mathrm{Re}\,\delta\right)e^{-\Gamma_S t} + \left(1+4\,\mathrm{Re}\,\delta\right)e^{-\Gamma_L t} + 2\,e^{-\Gamma t}\left(\cos\Delta m t + 4\,\mathrm{Im}\,\delta\cdot\sin\Delta m t\right)\right]/\,4$$

$$R_3 = R\left(K \to \bar{K}\right) = \left(1-4\,\mathrm{Re}\,\varepsilon\right)\left(e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\,e^{-\Gamma t}\cos\Delta m t\right)/\,4$$

$$R_4 = R\left(\bar{K} \to K\right) = \left(1+4\,\mathrm{Re}\,\varepsilon\right)\left(e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\,e^{-\Gamma t}\cos\Delta m t\right)/\,4$$

Note that R_1 and R_2 depend only on δ, R_3 and R_4 only on ε.

$$\boxed{A_T = \frac{R\left(\bar{K}\to K\right) - R\left(K\to\bar{K}\right)}{R\left(\bar{K}\to K\right) + R\left(K\to\bar{K}\right)} = 4\,\mathrm{Re}\,\varepsilon.}$$

measured by CPLEAR 1998 at CERN in a classical motion reversal experiment:

Start with a $\bar{K}^0$ at t = 0, let it evolve, look for appearance of a K^0 at time t, and test if the probability is the same as for the appearance of a $\bar{K}^0$ when starting with a K^0.

The motion-reversal experiment tests if the transition dynamics is T-symmetric.

$$\boxed{A_{CPT} = \frac{R\left(\bar{K}\to\bar{K}\right) - R\left(K\to K\right)}{R\left(\bar{K}\to\bar{K}\right) + R\left(K\to K\right)} = \frac{2\,\mathrm{Re}\,\delta\cdot\left(e^{-\Gamma_L t} - e^{-\Gamma_S t}\right) + 4\,\mathrm{Im}\,\delta\cdot e^{-\Gamma t}\sin\Delta m t}{\left(e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2\,e^{-\Gamma t}\cos\Delta m t\right)/\,2} \to +4\,\mathrm{Re}\,\delta}$$

CPLEAR also 1998

But for minimizing systematic uncertainties, they determined Re δ and Im δ in a slightly more refined way. Both measurements together determine all 3 asymmetry parameters.

Motion Reversal, Time Reversal, T Symmetry

Our experience with the „arrow of time" is valid in both the macro- and the micro-cosmos. We explain it by the 2nd law of thermodynamics in the multi-particle world which is not relevant in the few-particle world. But causality is relevant in both: An event can never influence an earlier one. We influence the future, never the past.

Time is not reversable.

What means „time-reversal"? I would like to avoid the term in this lecture and only use „motion reversal" and „T symmetry" (you may call that time-reversal symmetry).

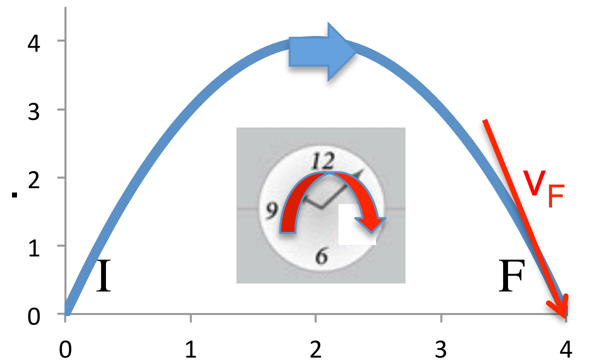
T symmetry = invariance of the dynamics (Hamilton, Lagrange, S-matrix ...) under the exchange $t \leftrightarrow -t$.

Motion reversal in classical mechanics: Comparing two motions with exchanges of start point $\leftrightarrow$ end point and $v \leftrightarrow -v$.

Motion reversal in quantum mechanics: Comparing two transition probabilities with exchanges of initial $\leftrightarrow$ final state, $v \leftrightarrow -v$, $s \leftrightarrow -s$. (some text books call that time reversal). In $K^0 \bar{K}^0$ transitions (W.W.): T invariance $\Leftrightarrow$ Motion-reversal symmetry.

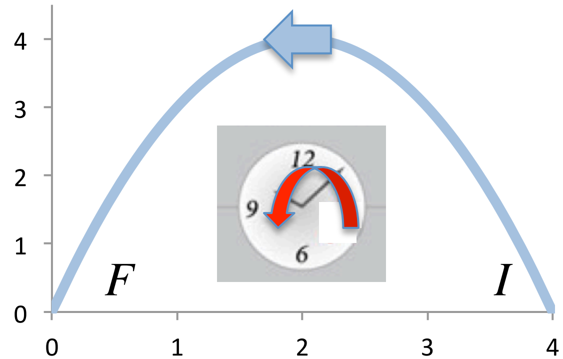
Motion Reversal in Classical Mechanics

Motion of a ball on the earth:



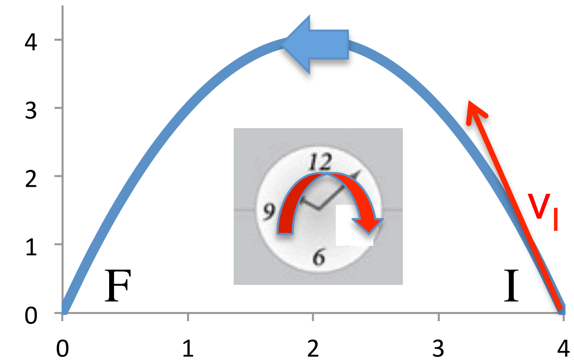
We record motion and clock

and replay it backwards:



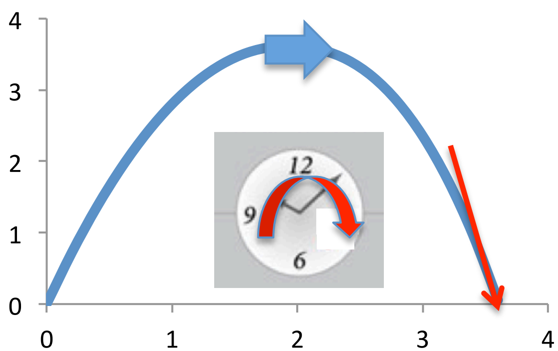
This is unobservable

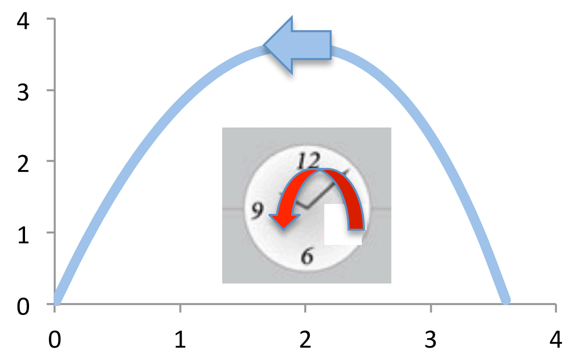
But with $I \leftrightarrow F$ and $v_I = - v_F$:

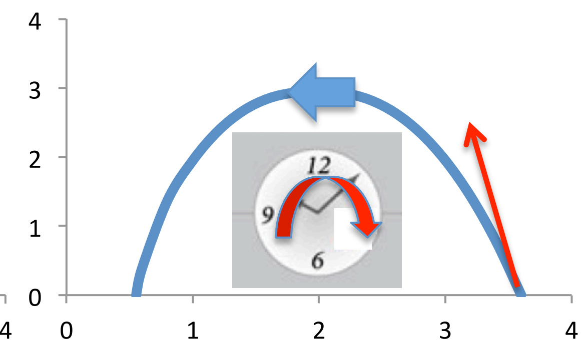


Observed reversed motion

When the orbit of the reversed motion coincides with that of the original motion, we have motion reversal symmetry. It follows from T symmetry of the law for the motion; here: $m\ddot{\vec{x}} = m\vec{g}$. This is invariant under the exchange t → - t. If we now modify the law of motion to $m\ddot{\vec{x}} = m\vec{g} - m\dot{\vec{x}}$ which is not invariant under t → - t:

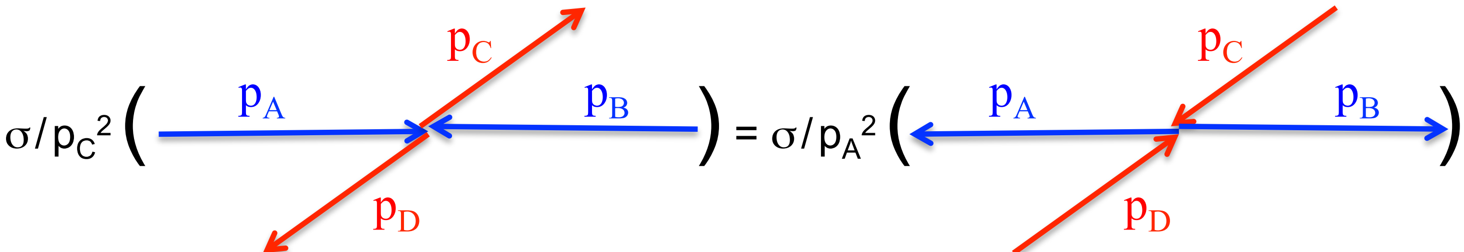






Examples for Motion Reversal in Particle Physics

(1) Reactions between stable particles: Comparing the nuclear reactions $A+B \rightarrow C+D$ and $C+D \rightarrow A+B$, Richter, v.Brentano, v.Witsch 1968: $^{27}\mathrm{Al}(p,\alpha)^{24}\mathrm{Mg} \leftrightarrow {}^{24}\mathrm{Mg}(\alpha,p)^{27}\mathrm{Al}$



for spinless particles and integration over all angles

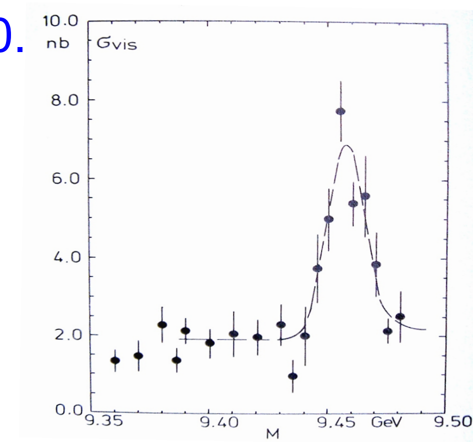
➔ T symmetry better than 1 %.

(2) Transitions beween stable particles: Neutrino "Oscillations" with three ν species

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{ij} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad P(\nu_e \rightarrow \nu_\mu) = \left| \sum_{j=1}^{3} U_{ej} U^*_{\mu j} \, e^{-i m_j^2 L/2E} \right|^2, \; P(\nu_\mu \rightarrow \nu_e) = \left| \sum_{j=1}^{3} U_{\mu j} U^*_{ej} \, e^{-i m_j^2 L/2E} \right|^2$$

If rates differ, T is violated because $J(U_{ij}) \neq 0$.

(3) Decays of unstable particles: Very difficult, not impossible, my example $\Upsilon(1S) \rightarrow \ell^+ \ell^-$, discovered by Lederman et al 1977, reversed by DASP-2 1978 in $e^+ e^- \rightarrow \Upsilon(1S)$. T symmetry not tested, but principally possible by comparing $\Gamma \cdot BF$ and $\sigma_{\mathrm{formation}}$.

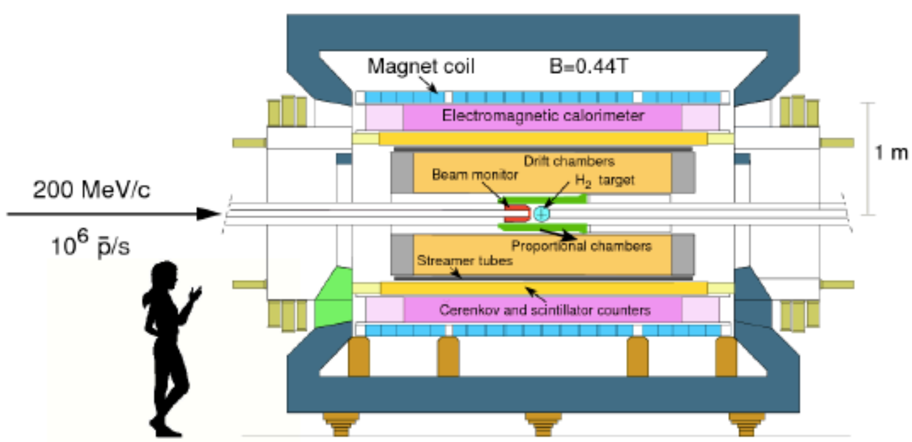


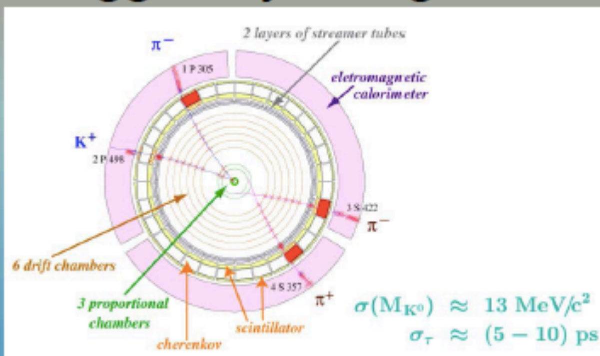
Next Example: A_T Measurement of CPLEAR

The CPLEAR Experiment PS195

- Data taking from ~1990 until 8th July 1996, 08:21:33
- Using antiprotons from CERN de-accelarator LEAR
- 1MHz $\bar{p}$ rate, stopped in hydrogen target, $\bar{p}p \to K^+\pi^-\bar{K}^0,\ K^-\pi^+K^0$
- Flavour of neutral kaon at production tagged by charge of charged kaon

The CPLEAR Detector





from T. Ruf 04/13

CPLEAR's A_T 1998

Physics Letters B 444 (1998) 43–51

$$A_T = \frac{R(\bar{K} \to K) - R(K \to \bar{K})}{R(\bar{K} \to K) + R(K \to \bar{K})} = 4\,\mathrm{Re}\,\varepsilon.$$

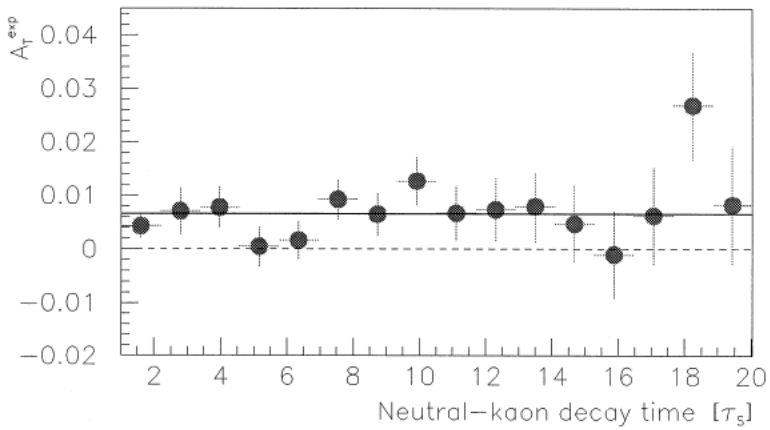
$$R_3 = R(K \to \bar{K}) = (1 - 4\,\mathrm{Re}\,\varepsilon)\left(e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\Gamma t}\cos\Delta m t\right)/4$$

$$R_4 = R(\bar{K} \to K) = (1 + 4\,\mathrm{Re}\,\varepsilon)\left(e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\Gamma t}\cos\Delta m t\right)/4$$

Count $N_3 = N[\bar{p}p \to (K^-\pi^+)(\pi^+ e^- \nu)]$ and $N_4 = N[\bar{p}p \to (K^+\pi^-)(\pi^- e^+ \nu)]$, correct for all detector asymmetries, use the „$\Delta S = \Delta Q$" results $\Gamma(K^0 \to \pi^+ e^- \nu) = \Gamma(\bar{K}^0 \to \pi^- e^+ \nu) = 0$ and CPT symmetry in decays $\Gamma(K^0 \to \pi^- e^+ \nu) = \Gamma(\bar{K}^0 \to \pi^+ e^- \nu)$:

$$A_T = \frac{N_4 - N_3}{N_4 + N_3}.$$

As function of t CPLEAR finds:



$A_T = (6.6 \pm 1.3 \pm 1.0)\ 10^{-3}$

5.1 σ from zero (stat), 4.0 σ (stat + sys),

Re $\varepsilon = (1.65 \pm 0.33 \pm 0.25)\ 10^{-3}$

A_T is both T- and CP-violating, since MR $(K \to \bar{K}) = \bar{K} \to K$, CP $(K \to \bar{K}) = \bar{K} \to K$. But from A_T we have only obtained half of the answer where CP violation in $K^0\bar{K}^0$ transition comes from; one portion is CP & T violating, the other portion is CP & CPT violating. CPLAR also determined the 2nd.

CPLEAR's A_{CPT} 1998

Physics Letters B 444 (1998) 52–60 after the A_T article: Physics Letters B 444 (1998) 43–51

$$A_{CPT} = \frac{R(\bar{K} \to \bar{K}) - R(K \to K)}{R(\bar{K} \to \bar{K}) + R(K \to K)} = \frac{2\,\mathrm{Re}\,\delta \cdot \left(e^{-\Gamma_L t} - e^{-\Gamma_S t}\right) + 4\,\mathrm{Im}\,\delta \cdot e^{-\Gamma t} \sin \Delta m t}{\left(e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2 e^{-\Gamma t} \cos \Delta m t\right)/2} \to +4\,\mathrm{Re}\,\delta$$

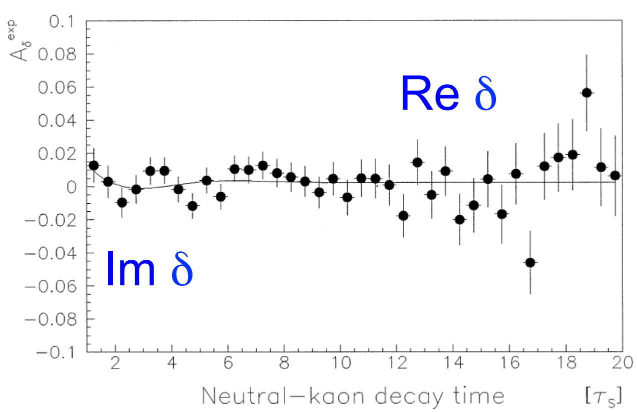
sensitive to Re δ and Im δ

CPLEAR could count $N_2 = N[\bar{p}p \to (K^+\pi^-)(\pi^+e^-\nu)]$ and $N_1 = N[\bar{p}p \to (K^-\pi^+)(\pi^-e^+\nu)]$, correct for all detector asymmetries, use the „$\Delta S=\Delta Q$“ results $\Gamma(K^0 \to \pi^+e^-\nu) = \Gamma(\bar{K}^0 \to \pi^-e^+\nu) = 0$ and CPT symmetry in decays $\Gamma(K^0 \to \pi^-e^+\nu) = \Gamma(\bar{K}^0 \to \pi^+e^-\nu)$ ➔

$$A_{CPT} = \frac{N_2 - N_1}{N_2 + N_1}.$$

But they have chosen an assumption-independent way:

$$A_\delta = \frac{N_2 - (1+\alpha)N_1}{N_2 + (1+\alpha)N_1} + \frac{N_4 - (1+\alpha)N_3}{N_4 + (1+\alpha)N_3} = \cdots \xrightarrow[t \cdot \Gamma_S >> 1]{} +8\,\mathrm{Re}\,\delta.$$



where $\alpha = 4\,\mathrm{Re}(\varepsilon-\delta) \cdot f$(detector asymmetries) is a measured quantity. Result:

Re δ = $(0.30 \pm 0.33 \pm 0.06)\ 10^{-3} \approx 1\,\sigma(\mathrm{Re}\ \varepsilon)$,

Im δ = $(-15 \pm 23 \pm 3)\ 10^{-3}$.

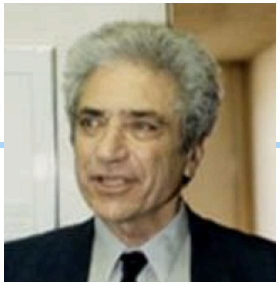
CPTV only in $m_{22} - m_{11}$ ➔ Im $\delta \approx$ - Re δ, $|\delta| < 0.6 \cdot 10^{-3}$

CPV($K^0 \leftrightarrow \bar{K}^0$) is T- violating and less CPT-violating.

The Bell-Steinberger Unitarity Relation

John Bell

$$\boxed{(\Gamma - \mathrm{i}\,\Delta m)\langle K_L | K_S\rangle = \sum_f \langle f | T | K_L\rangle^* \langle f | T | K_S\rangle}$$



Jack Steinberger

The derivation is straightforward, „pure algebra“, see Branco Lavoura Silva, Sec.6.5

$$\Psi = \chi_S \mathrm{e}^{-\gamma_S t}|K_S\rangle + \chi_L \mathrm{e}^{-\gamma_L t}|K_L\rangle, \quad |\Psi|^2 = |\chi_S|^2 \mathrm{e}^{-\Gamma_S t} + |\chi_L|^2 \mathrm{e}^{-\Gamma_L t} + 2\,\mathrm{Re}\left(\chi_L^* \chi_S \mathrm{e}^{-\Gamma t - i\Delta m t}\langle K_L | K_S\rangle\right),$$

$$-\mathrm{d}|\Psi|^2/\mathrm{d}t(t=0) = \Gamma_S|\chi_S|^2 + \Gamma_L|\chi_L|^2 + 2\,\mathrm{Re}\left(\chi_L^*\chi_S(\Gamma - \mathrm{i}\,\Delta m)\langle K_L|K_S\rangle\right) = \sum_f \left|\langle f|T|\Psi(t=0)\rangle\right|^2$$

$$= \sum_f \left|\chi_S\langle f|T|K_S\rangle + \chi_L\langle f|T|K_L\rangle\right|^2 \quad \Rightarrow \quad \Gamma_S = \sum_f \left|\langle f|T|K_S\rangle\right|^2, \quad \Gamma_L = \sum_f \left|\langle f|T|K_L\rangle\right|^2,$$

$$2\,\mathrm{Re}\left(\chi_L^*\chi_S(\Gamma - \mathrm{i}\,\Delta m)\langle K_L|K_S\rangle\right) = 2\,\mathrm{Re}\left(\chi_L^*\chi_S \sum_f \langle f|T|K_L\rangle^* \langle f|T|K_S\rangle\right),$$

$$(\Gamma - \mathrm{i}\,\Delta m)\langle K_L|K_S\rangle = \sum_f \langle f|T|K_L\rangle^* \langle f|T|K_S\rangle.$$

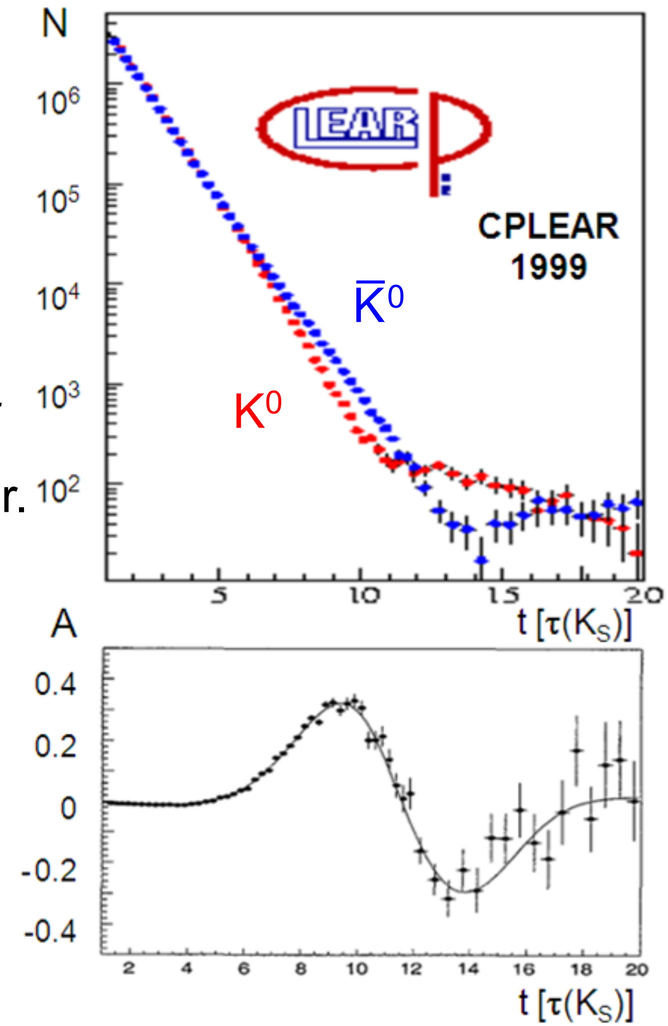
Published in the 1965 Oxford Conference Proceedings. Since $\langle K_L^0 | K_S^0\rangle = 2\,\mathrm{Re}\,\varepsilon + 2\,\mathrm{i}\,\mathrm{Im}\,\delta$, we can determine $\mathrm{Re}\,\varepsilon$ and $\mathrm{Im}\,\delta$ from the measurement of all decay amplitudes for the final states that can be reached from both K_S and K_L

The two Dominant Modes for Bell-Steinberger

$$\mathrm{Re}\,\varepsilon - \mathrm{Im}\,\delta = \Sigma \frac{\langle f|T|K_L\rangle \langle f|T|K_S\rangle^*}{(\Gamma + i\Delta m)} = \sum_f \frac{\langle f|T|K_L\rangle}{\langle f|T|K_S\rangle} \cdot \frac{BF(K_S \to f)\cdot \Gamma_S}{(\Gamma_S + 2i\Delta m)}$$

Common decay modes of K_S and K_L: (1) $\pi^+\pi^-$, the discovery 1964. Here a nicer graph 1999:

This interference is described by the CP-violating parameter $\eta_{+-} = \langle \pi^+\pi^-|T|K_L\rangle / \langle \pi^+\pi^-|T|K_S\rangle$. In 1968 we had $|\eta_{+-}| = (1.90 \pm 0.06)\,10^{-3}$ and $\phi_{+-} = (65 \pm 11)^o$, the phase was measured in the interference pattern of $\pi^+\pi^-$ decays from the state $K_L + \rho\, K_S$ after a regenerator.



If $\pi^+\pi^-$ would be the only CP-violating mode and $\phi_{+-} \approx 45^o$, then we would have Im $\delta \approx 0$. But:

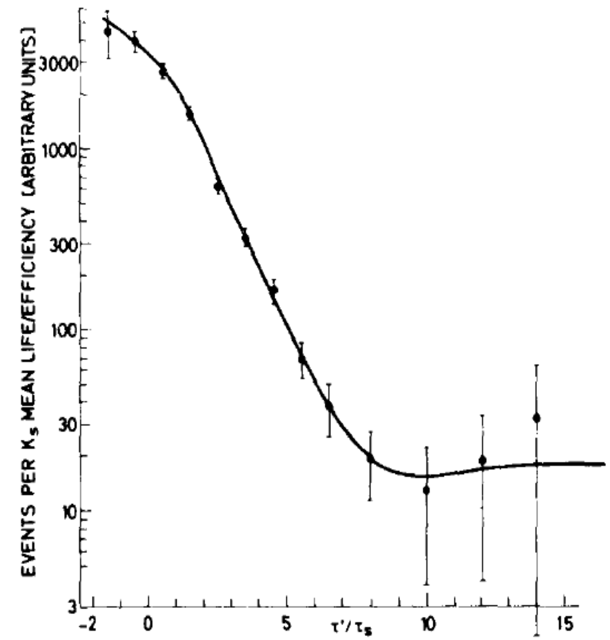
Mode (2) $\pi^0\pi^0$ with $\eta_{00} = \langle \pi^0\pi^0|T|K_L\rangle / \langle \pi^0\pi^0|T|K_S\rangle$ and $BF \approx BF(\pi^+\pi^-)/2$ gave 1968: $|\eta_{00}| = (4 \pm 1)\,10^{-3}$.

Without ϕ_{00} no chance to obtain Re ε and Im δ.

1970

OBSERVATION OF THE INTERFERENCE BETWEEN K_L^0 AND K_S^0 IN THE $\pi^0\pi^0$ DECAY MODE

J. C. CHOLLET *, J.-M. GAILLARD, M. R. JANE **, T. J. RATCLIFFE ***, J.-P. REPELLIN *, K. R. SCHUBERT and B. WOLFF *
CERN, Geneva, Switzerland



$|\eta_{00}| = (3.3 \pm 0.7)\ 10^{-3}$, $\phi_{00} = (51 \pm 30)^{\circ}$

This result completed the measurement of CPV in the $\pi\pi$ modes and the way was free for a first Bell-Steinberger analysis:

THE PHASE OF η_{00} AND THE INVARIANCES *CPT* AND *T*

K. R. SCHUBERT, B. WOLFF *, J. C. CHOLLET *, J.-M. GAILLARD, M. R. JANE **, T. J. RATCLIFFE *** and J.-P. REPELLIN *
CERN, Geneva, Switzerland

Received 7 April 1970

$$\left(1 + \frac{2\,\mathrm{i}\,\Delta m}{\Gamma_S}\right)\cdot\left(\mathrm{Re}\,\varepsilon - \mathrm{i}\,\mathrm{Im}\,\delta\right) = \frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00} + \sum_{f\neq\pi\pi}\alpha_f$$

In addition to η_{00}, the analysis used $2\Delta m/\Gamma_S = 0.983 \pm 0.030$, $\eta_{+-} = (1.92 \pm 0.05)\,10^{-3}\,e^{i(44\pm5)^{\circ}}$, $\alpha_{\pi\pi\pi,I=1} = [0.3 \pm 1.7 + i\,(-0.8 \pm 2.5)]\,10^{-4}$, $\alpha_{\pi\mu\nu+e\mu\nu} = [-2.4 \pm 2.1 + i\,(0.9 \pm 3.4)]\,10^{-4}$, $\alpha_{\pi\pi\pi,I=3} = \alpha_{\pi\pi\gamma} = \alpha_{\gamma\gamma} = 0$, resulting in

Re ε = $(1.68 \pm 0.30)\,10^{-3}$, T violation with 5 σ, Im δ = $(-0.30 \pm 0.45)\,10^{-3}$. Also Re ε:

η_{+-} and η_{00}

$$\eta_{+-}=\frac{\langle\pi^+\pi^-|T|K_L\rangle}{\langle\pi^+\pi^-|T|K_S\rangle},\ \eta_{00}=\frac{\langle\pi^0\pi^0|T|K_L\rangle}{\langle\pi^0\pi^0|T|K_S\rangle},\quad \eta_{I=0}=\frac{\langle\pi\pi,I=0|T|K_L\rangle}{\langle\pi\pi,I=0|T|K_S\rangle}=\frac{2}{3}\eta_{+-}+\frac{1}{3}\eta_{00}.$$

$\eta_{+-}=\eta_{I=0}+\varepsilon'$ and $\eta_{00}=\eta_{I=0}-2\varepsilon'$ are both measured in modulus and phase, the results 1970 show that ε' is at least as small as $\eta_{I=0}$. With $A_0=\langle\pi\pi,I=0|T|K^0\rangle$, $\bar{A}_0=\langle\pi\pi,I=0|T|\bar{K}^0\rangle$:

$$\eta_{+-}=\frac{p(1-\delta)A_0-q(1+\delta)\bar{A}_0}{p(1+\delta)A_0+q(1-\delta)\bar{A}_0}+\varepsilon'=\frac{1-\delta-(1+\delta)\lambda}{1+\delta+(1-\delta)\lambda}+\varepsilon' \text{ with } \lambda=\frac{q\bar{A}_0}{pA_0}.$$

q/p and $\bar{\mathrm{A}}_0/\mathrm{A}_0$ are phase-convention dependent, but λ and η_{+-} are observable. Without CPT in decays, $\alpha=\left(|\bar{A}_0|-|A_0|\right)/\left(|\bar{A}_0|+|A_0|\right)$, and with small values of $\mathrm{Re}\,\varepsilon$, δ, $-2\Phi=\mathrm{phase}(\lambda)$ and α:

$$\lambda=\left|\frac{q}{p}\right|\cdot\left|\frac{\bar{A}_0}{A_0}\right|\cdot e^{-2i\Phi}=(1-2\mathrm{Re}\,\varepsilon)(1+2\alpha)(1-2i\Phi)=1-2\mathrm{Re}\,\varepsilon+2\alpha-2i\Phi,$$

$$\eta_{+-}=\frac{1-\delta-(1+\delta-2\mathrm{Re}\,\varepsilon+2\alpha-2i\Phi)}{1+\delta+(1-\delta-2\mathrm{Re}\,\varepsilon+2\alpha-2i\Phi)}+\varepsilon'=(\mathrm{Re}\,\varepsilon-\delta)-\alpha+i\Phi+\varepsilon'.$$

Both λ and η have 3 parts: mixing, decay and interplay.

Assuming $\alpha=0$, i.e. CPT symmetry in $\pi\pi$,I=0 decays, we can determine $\mathrm{Re}\,\delta$ from η_{+-}

More Bell-Steinberger Results

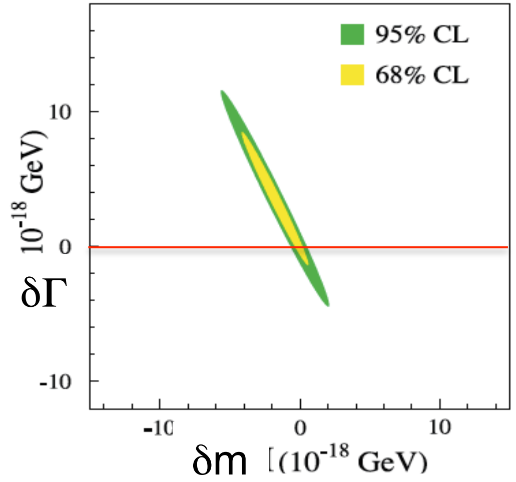
1970: Re ε = $(1.68 \pm 0.30)\,10^{-3}$, Im δ = $(-0.30 \pm 0.45)\,10^{-3}$, Re δ = $(0.07 \pm 0.43)\,10^{-3}$,
CPT conserved, T violated with 5 σ.

1984: Re ε = $(1.64 \pm 0.04)\,10^{-3}$, Im δ = $(0.10 \pm 0.10)\,10^{-3}$, Re δ = $(0.22 \pm 0.10)\,10^{-3}$,
CPT conserved, T violated with „40 σ“, Barmin, Okun et al

1999: Re ε = $(1.649 \pm 0.025)\,10^{-3}$, Im δ = $(0.02 \pm 0.05)\,10^{-3}$, Re $\delta = (0.24 \pm 0.28)\,10^{-3}$
CPT conserved, T violated with „65 σ“, CPLEAR, Re δ from $\pi\ell\nu$ decays

2006: Re ε = $(1.596 \pm 0.013)\,10^{-3}$, Im δ = $(0.00 \pm 0.02)\,10^{-3}$, Re $\delta = (0.23 \pm 0.27)\,10^{-3}$
CPT conserved, T violated with „120 σ“, KLOE

2012: Re ε = $(1.611 \pm 0.005)\,10^{-3}$, Im δ = $(0.7 \pm 1.4)\,10^{-5}$, Re $\delta = (0.24 \pm 0.23)\,10^{-3}$
CPT conserved, T violated with „320 σ“,
Antonelli and d‘Ambrosio, PDG, combining all results from CPLEAR, KLOE, KTeV, NA48.
With $\delta\Gamma = 0$, $|\delta m| = |m(K^0) - m(\bar{K}^0)| < 4{\cdot}10^{-19}$ GeV.
$\delta m/m < 8{\cdot}10^{-18}$. „Best test of CPT symmetry“.



Unitarity condition: No decays into invisible final states!

„Direct“ CP Violation

CPT: $$\eta_{+-} = \mathrm{Re}\,\varepsilon\left(1+\mathrm{i}\,2\Delta m/\Delta\Gamma\right)+\varepsilon' = \varepsilon_W + \varepsilon'$$

$$\eta_{00} = \varepsilon_W - 2\,\varepsilon'$$

$$\varepsilon' = \frac{\langle \pi\pi, I=2|T|K_L\rangle}{\langle \pi\pi, I=0|T|K_S\rangle} = \frac{\mathrm{i}}{\sqrt{2}}\mathrm{e}^{i(\delta_2-\delta_0)}\,\mathrm{Im}\frac{A_2}{A_0}$$

$$A_I = \langle \pi\pi, I|T|K^0\rangle$$

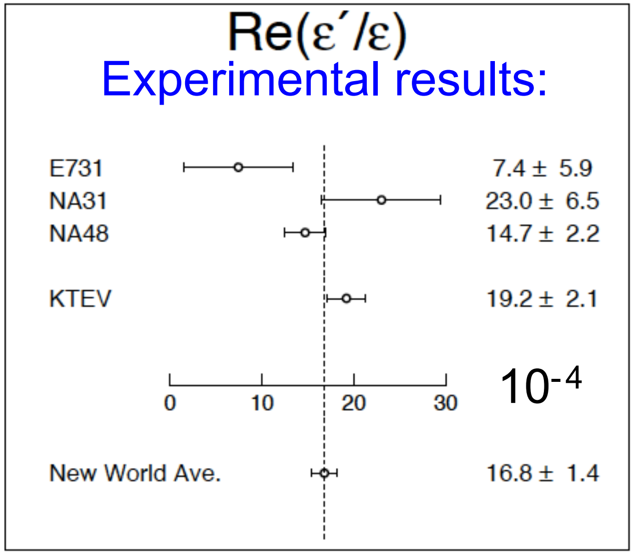
$$\boxed{\frac{\Gamma\left(K_L\to\pi^+\pi^-\right)}{\Gamma\left(K_S\to\pi^+\pi^-\right)}\Big/\frac{\Gamma\left(K_L\to\pi^0\pi^0\right)}{\Gamma\left(K_S\to\pi^0\pi^0\right)} = \left|\frac{\eta_{+-}}{\eta_{00}}\right|^2 = 1+6\,\mathrm{Re}\frac{\varepsilon'}{\varepsilon_W}}$$

$\Phi(\varepsilon_W) = (43.5 \pm 0.1)^\circ$

$\Phi(\varepsilon') = (37 \pm 5)^\circ$

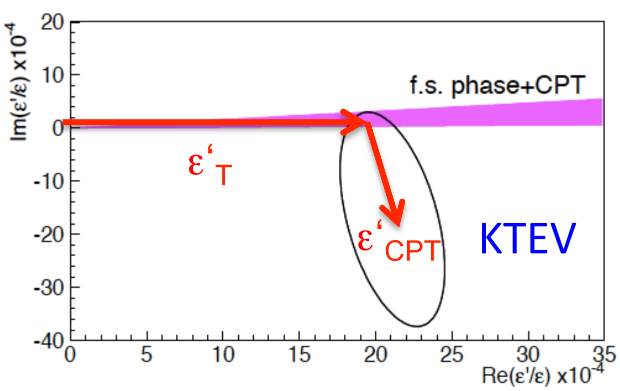
$\varepsilon'/\varepsilon_W$ = real.

1993
2001
2011



This means direct CP violation and direct T violation ($\varepsilon'_T/\varepsilon_W$) with 12$\sigma$. Is CPT symmetry tested?

KTEV 2011 measures η_{+-} and η_{00} in modulus and phase. Result: $\Phi_{+-}-\Phi_{00} = 3\,\mathrm{Im}(\varepsilon'/\varepsilon_W) = (-0.30\pm0.35)^\circ$



In spite of the great precision on $\Phi_{+-}-\Phi_{00}$, the CPT-violating part $\varepsilon'_{CPT}/\varepsilon_W$ could be as big as $\varepsilon'_T/\varepsilon_W$.

CP Violation in the Interplay of Transitions and Decays

Completely different from CP violation in the $B^0 \bar{B}^0$ system, all CP violation in the $K^0 \bar{K}^0$ system has within a precision of 10^{-3} only one source: $|q/p| = 1 - 2\,\mathrm{Re}\,\varepsilon \neq 1$ because of the phase between m_{12} and Γ_{12}. Since $\Gamma_{12} = A_0^* \bar{A}_0$ within even better precision, $A_0 = \langle \pi\pi, I=0 | T | K^0 \rangle$, the interplay CP violation $q\bar{A}_0/pA_0$ is strongly bound to Re ε: the T violations in Im η_{+-} and Im η_{00} follow from unitarity and T violation in transitions.

KTEV 2011 presents the result $\Phi(2\,\eta_{+-} + \eta_{00}) - \mathrm{atan}(2\Delta m/\Delta\Gamma) = (-0.40 \pm 0.56)^\circ$ as a test of CPT symmetry.
It tests CPT, but not independent of the Bell-Steinberger unitarity result.

Summary

Motivated by a 1963 experiment on „anomaluous regeneration“ of K_1 meson-states in a K_2-state beam passing through liquid hydrogen,

Christenson Cronin Fitch Turlay discovered 1964 that CP symmetry is broken in $K \to \pi^+ \pi^-$ decays, 7 years after the discovery of P and C violation. In 1967, the full CP violation is seen in $K^0\overline{K}^0$ transitions.

Since CPT = CP·T and Quantum Mechanics does not require CPT symmetry, there was an interest to find out from data what are the fractions of T and of CPT violation.

Using the Bell-Steinberger relation, there were enough data 1970 to find with 5 σ that T violation dominates and that CPT violation is smaller than 20% of the CP violation.

With more data, we know this today with „320 σ“, and no CPT violation is found, also not in direct CP violation and not in the interplay of transitions and decays.

Appendix 1: Δm from Regeneration

Regeneration = Matter-induced appearance of K_S states in a K_L beam.

Stronger than CP violation, therefore $K_S \approx K_1$, $K_L \approx K_2$.

Origin: K^0 and $\bar{K}^0$ in K_2 have different strong interactions, the amplitude f(0) for elastic scattering of a K^0 on a nucleus under 0° is different from $\bar{f}(0)$ for a $\bar{K}^0$. After a thin regenerator (maximally one appearance), the state K_2 changes into $K_2 + \rho\, K_1$:

$$\rho = i\pi \frac{f(0) - \bar{f}(0)}{k} \cdot \frac{dN}{dA} \quad \text{with } k = p/\hbar \text{ and } \frac{dN}{dA} = \frac{\text{number of nuclei}}{\text{area}}$$

Good 1961 worked with thick regenerators where more than one K_1 appeares leading to ρ depending on Δm. This can be easily understood for two thin regenerators separated by a distance D. At the end of the first regenerator we have $\Psi = K_2 + \rho_1 K_1$. At the end of the second: $\Psi = \rho_1 K_1 \mathrm{e}^{-im_s T - \Gamma_S T/2} + \mathrm{e}^{-im_L T}\left(K_2 + \rho_2 K_1\right)$ where we have used $\Gamma_L \approx 0$. Since only the two K_1 states lead to $\pi\pi$ decays, we obtain for $\rho_1 = \rho_2 = \rho$ a $\pi\pi$ rate proportional to $\left|\rho\, \mathrm{e}^{-im_s T - \Gamma_S T/2} + \rho\right|^2 = |\rho|^2 \left(1 + \mathrm{e}^{-\Gamma_S T} + 2\mathrm{e}^{-\Gamma_S T/2} \cos \Delta m\, t\right)$ where T = D/$\beta\gamma$ = D m/p. For determining Δm, this method has been used by Christenson 1965 with various D and by Geweniger 1974 with fixed D and various p.

The sign of Δm from Regeneration

Regeneration has be used for determining that $\Delta m = m_L - m_S > 0$ by placing a Regenerator at short distance D behind a target in which pure K^0 states are produced by the charge-exchange reaction $K^+n \to K^0p$ in a nuclear target.

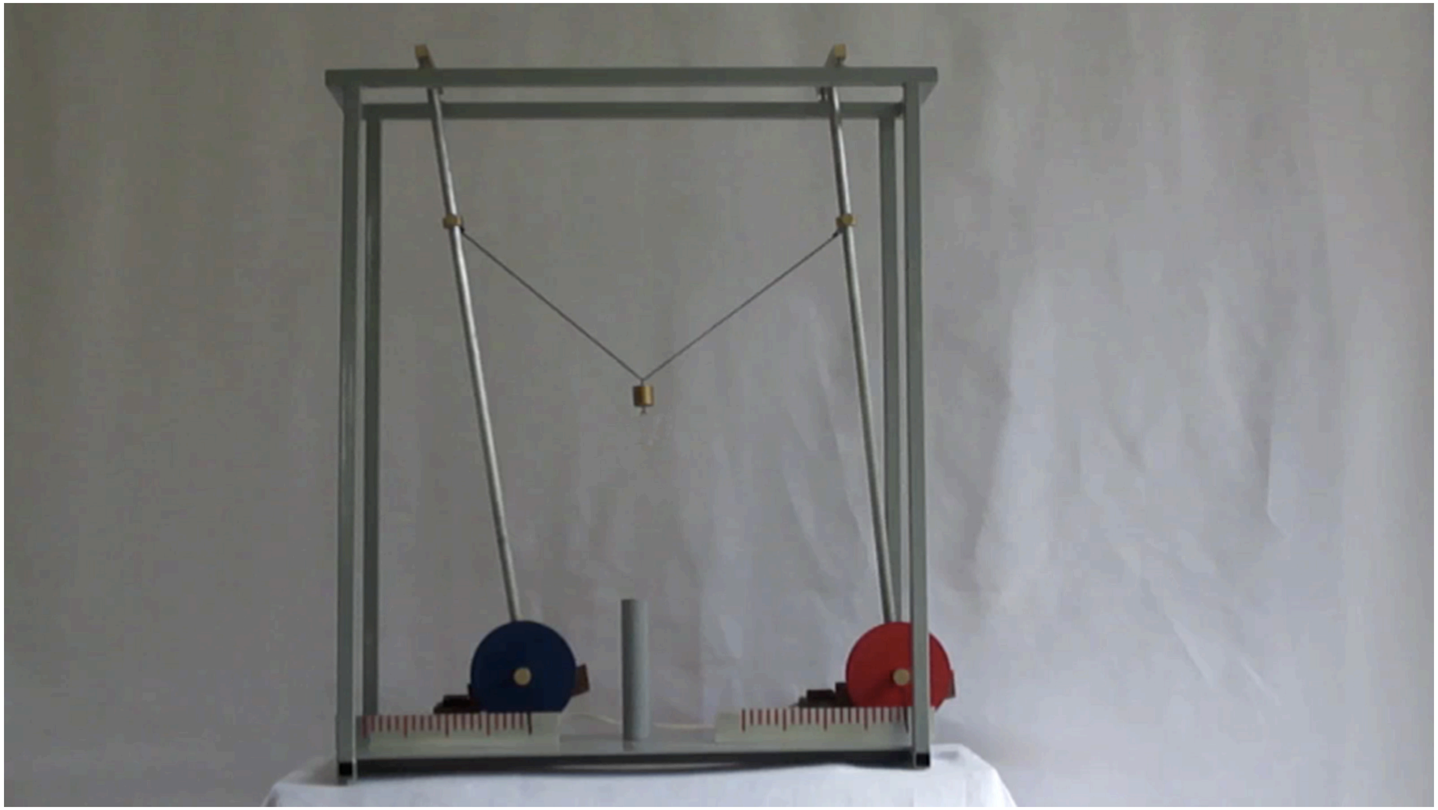
At production we have $\Psi = K^0 = K_1 + K_2$. At the entrance of the regenerator we have $\Psi = K_1 e^{-im_S T - \Gamma_S T/2} + K_2 e^{-im_L T}$.

After a thin regenerator we have $\Psi = K_1 e^{-im_S T - \Gamma_S T/2} + e^{-im_L T}\left(K_2 + \rho K_1\right)$ and the ππ rate behind the regenerator is proportional to

$$\left| e^{-im_s T - \Gamma_S T/2} + \rho\, e^{-im_L T} \right|^2 = e^{-\Gamma_S T} + |\rho|^2 + 2e^{-\Gamma_S T/2}\left(\mathrm{Re}\,\rho \cdot \cos\Delta m\, t + \mathrm{Im}\,\rho \cdot \sin\Delta m\, t\right).$$

In the experiment of Mehlhop 1968, the phase of $i[f(0) - \bar{f}(0)]$ is known to be -60° from K^+ and K^- scattering on their charge-exchange target. This knowledge allows to determine both cosΔmt and sinΔmt, even if their use of a thick regenerator requires a more detailed determination of ψ after the regenerator. The reason for a sinΔmt term remains valid, the interference of pure K^0 states with regenerated K_1 states, and also in a thick regenarator the phase of ρ is given by the phase of $i[f(0) - \bar{f}(0)]$.

Appendix 2: A Model for $K^0\,\overline{K}^0$ Transitions



blue = $\overline{K}^0$ red = K^0

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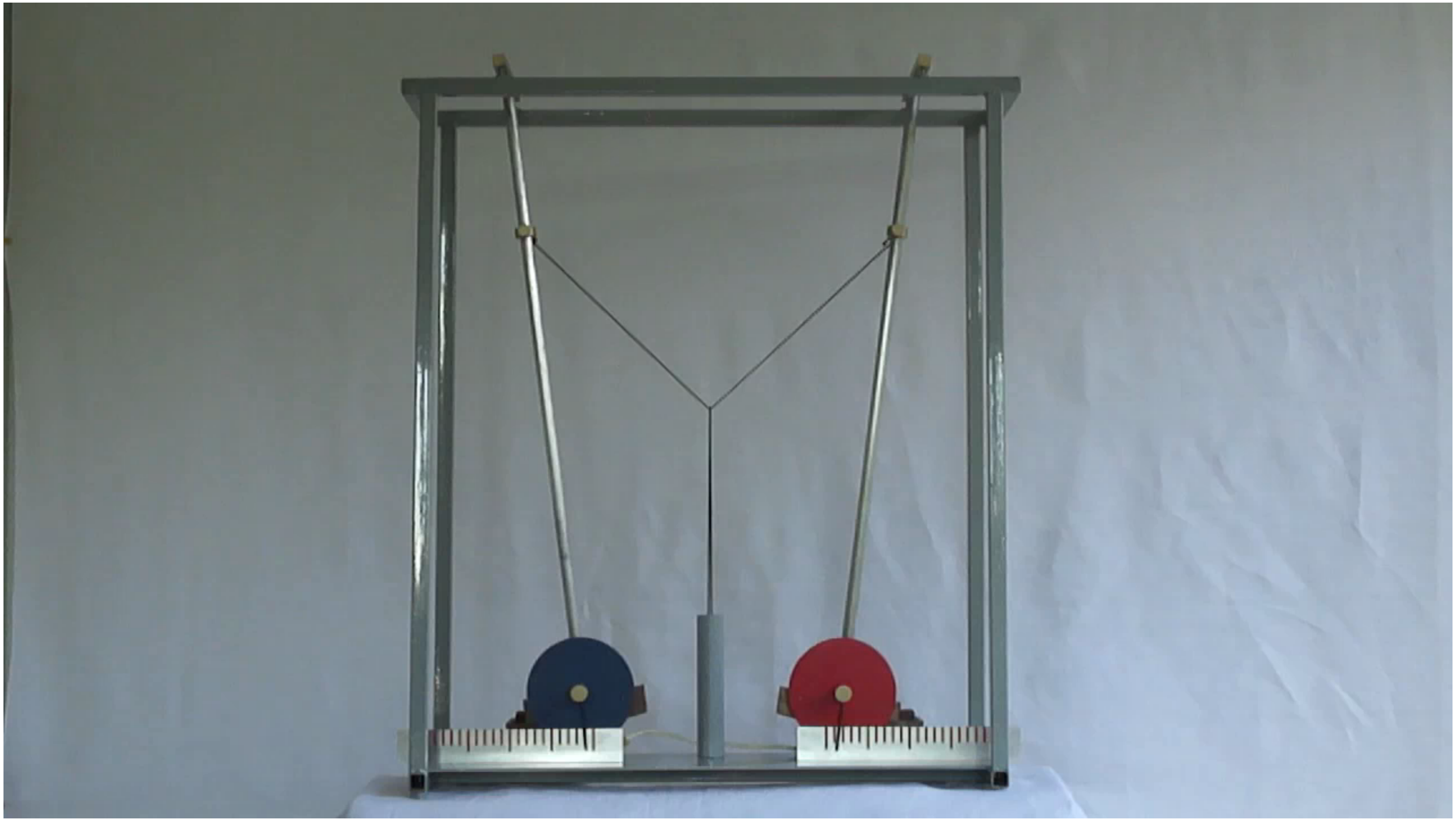
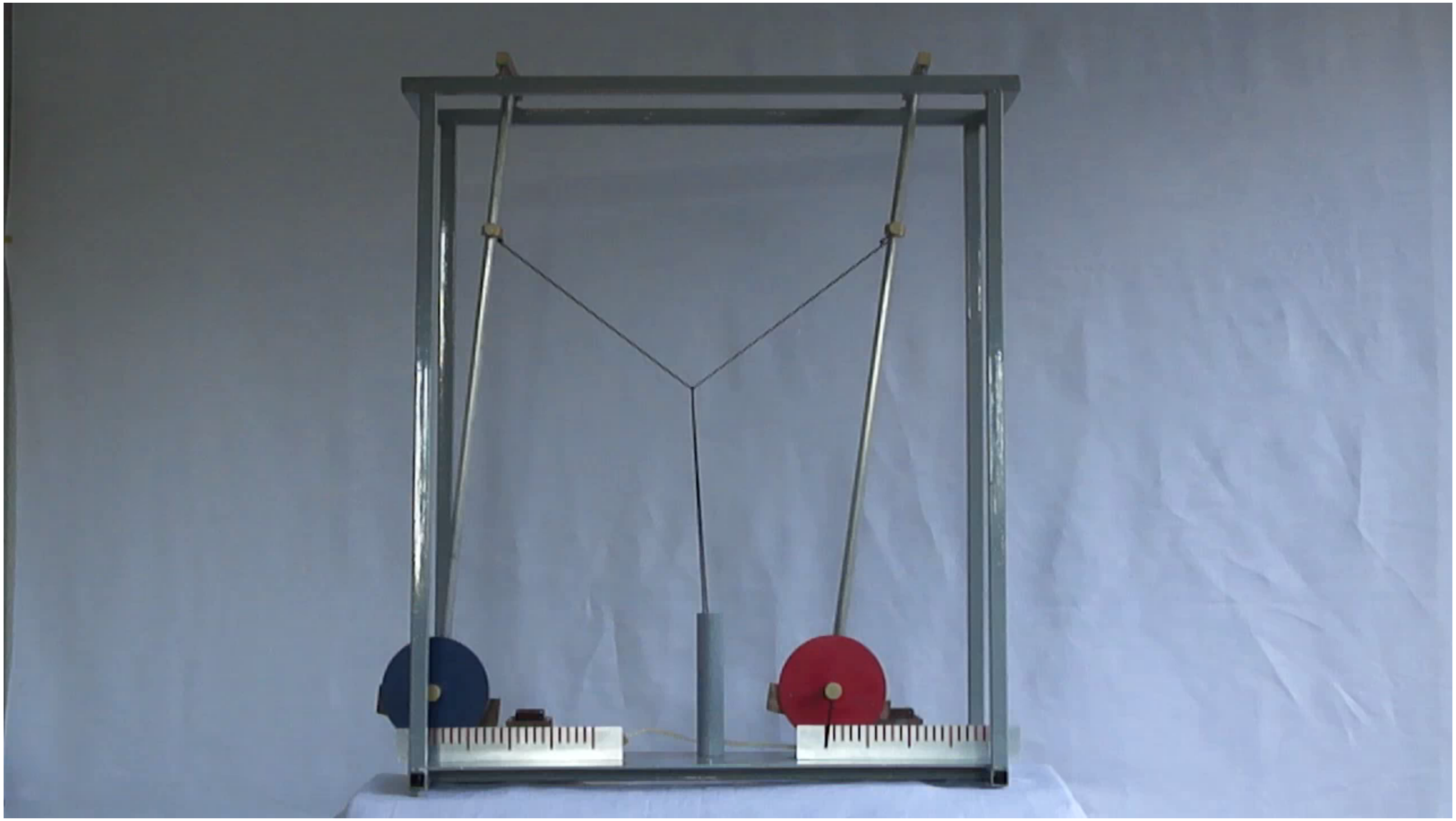
built 1987

Video 1: The K_S



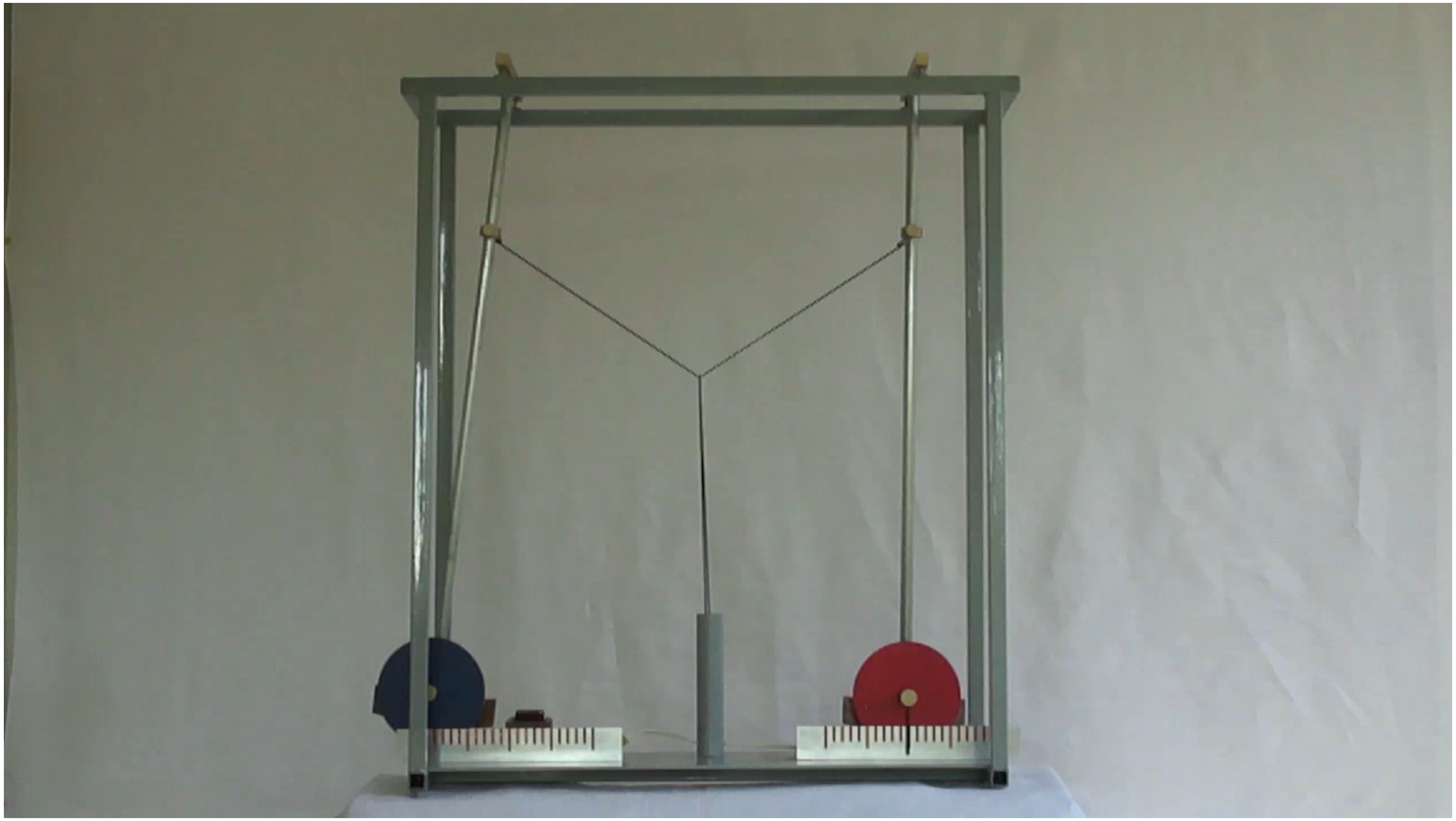
blue = $\overline{K}^0$ red = K^0

Video 2: The K_L



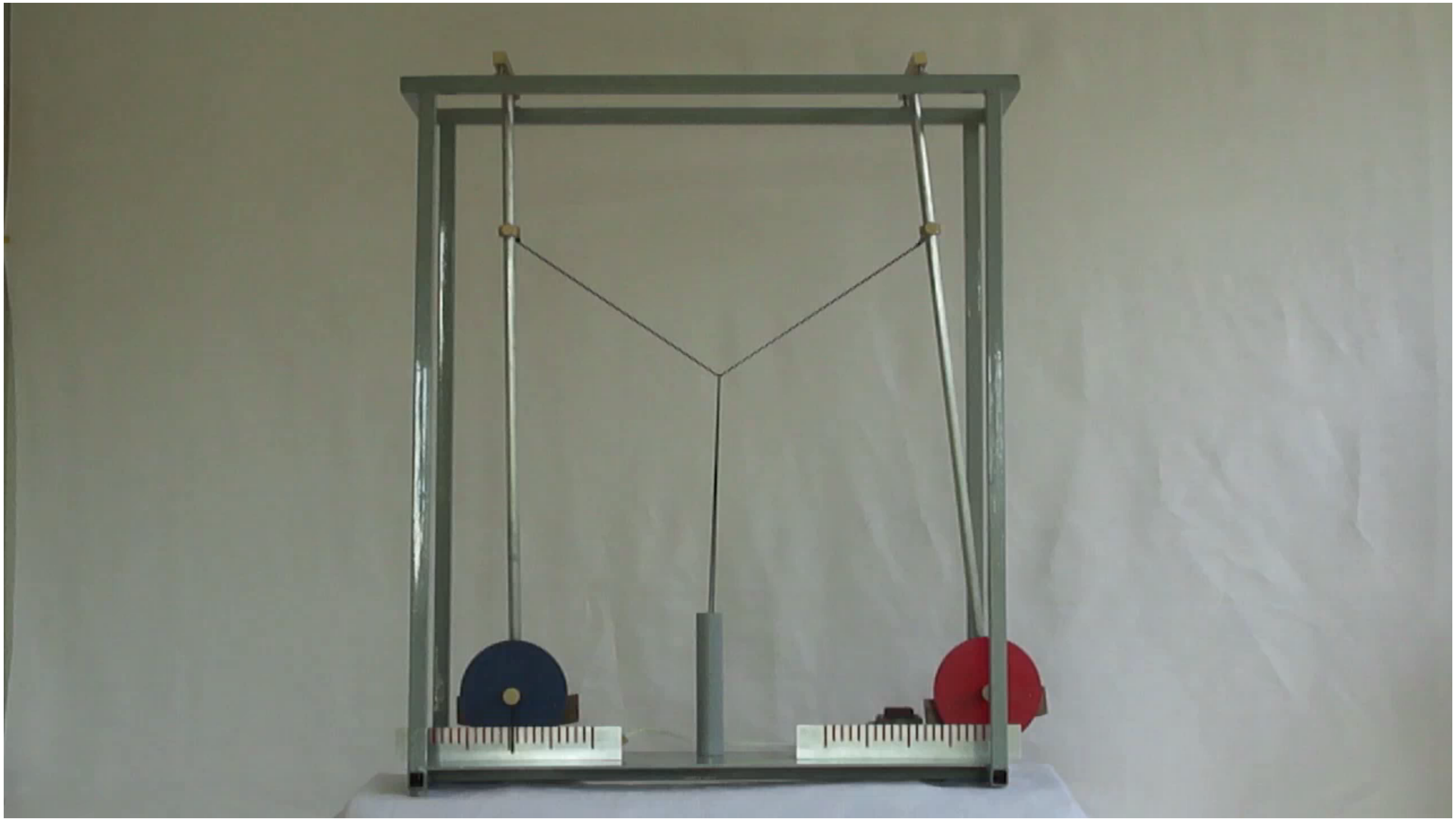
blue = $\overline{K}^0$ red = K^0

Video 3: The initial $\overline{K}^0$



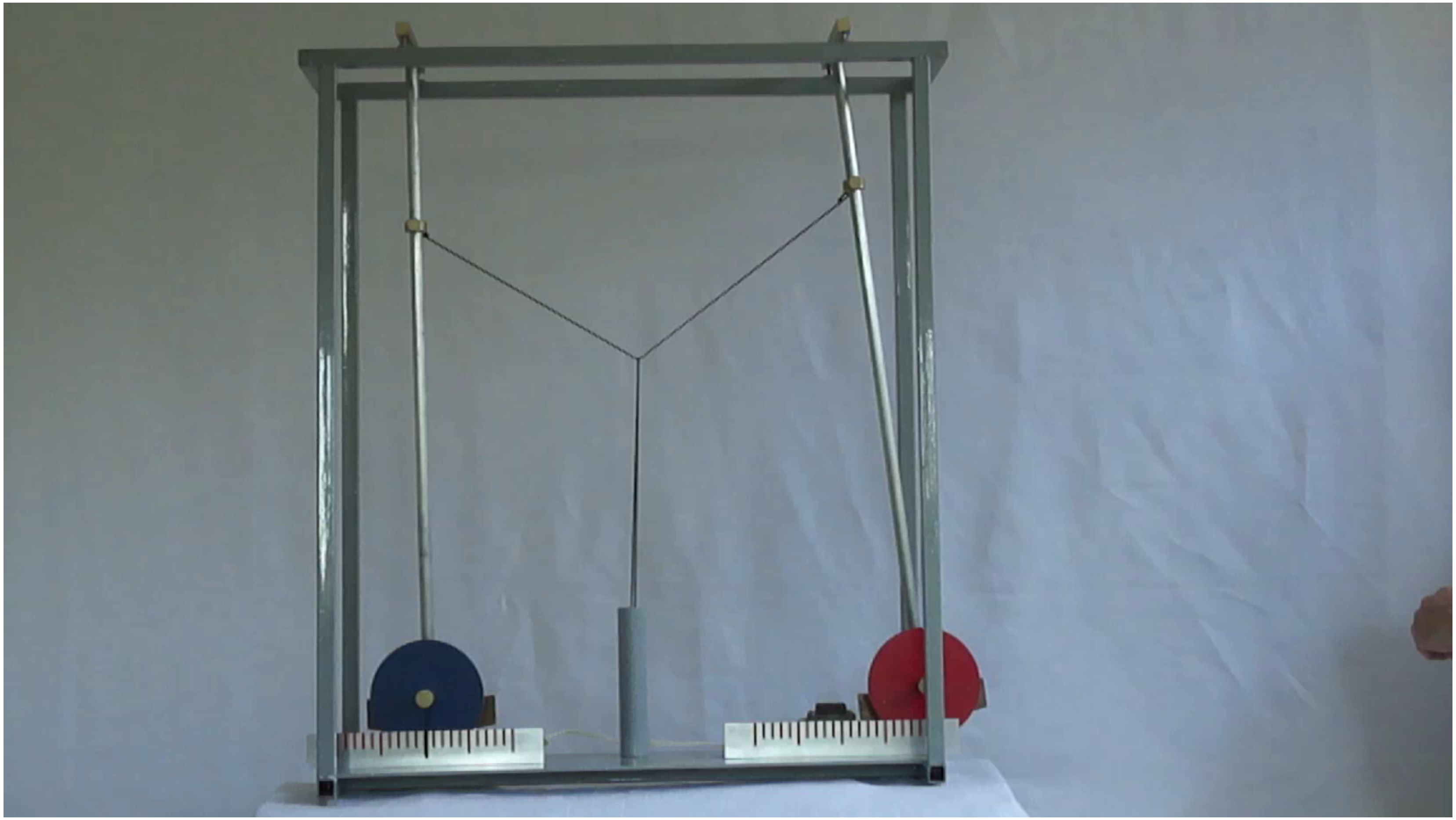
blue = $\overline{K}^0$ red = K^0

Video 4: The initial K^0



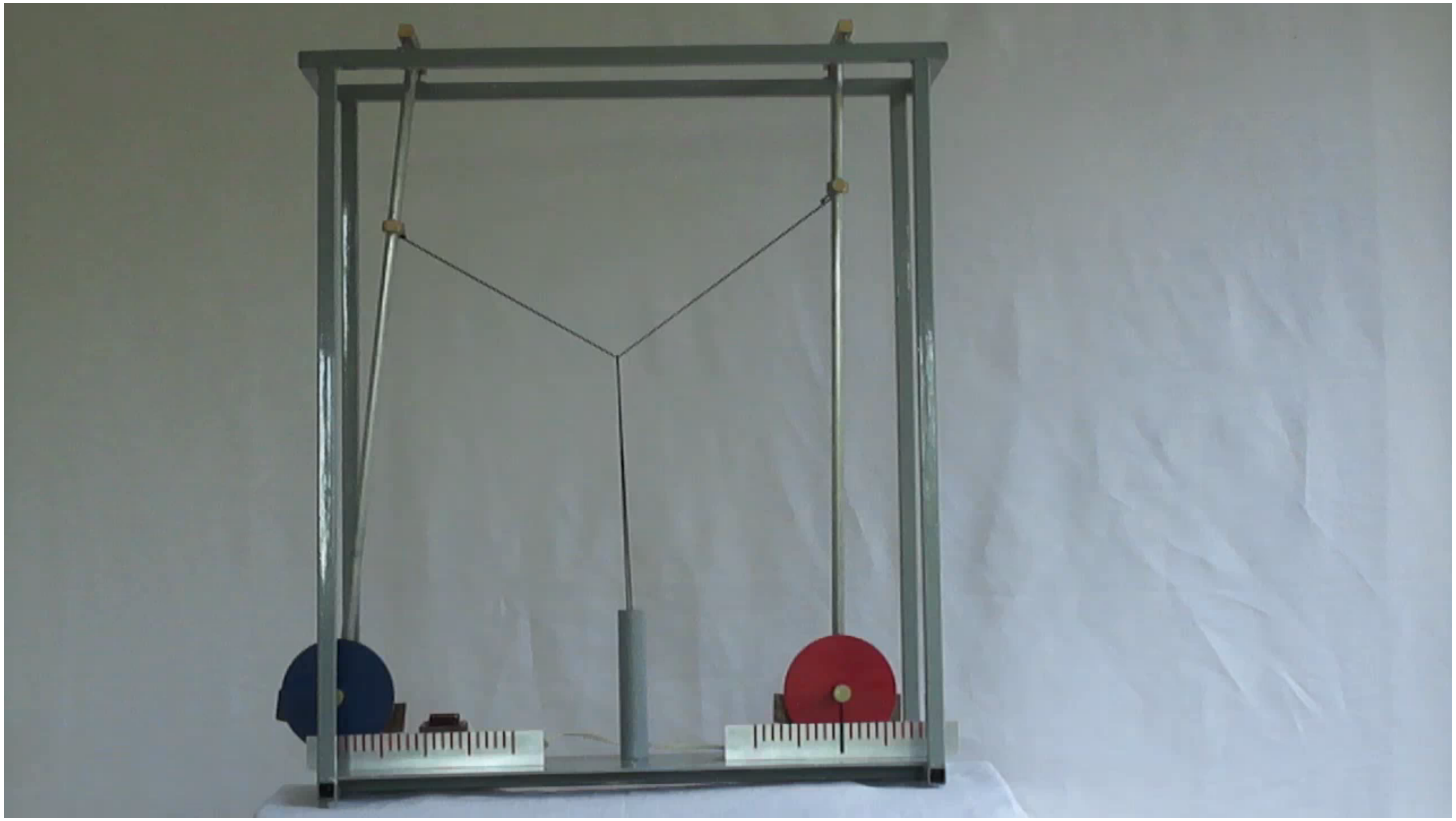
blue = $\overline{K}^0$ red = K^0

Video 5: CP Violation, K^0 starts



blue = $\overline{K}^0$ red = K^0

Video 6: CP Violation, $\overline{K}^0$ starts



blue = $\overline{K}^0$ red = K^0

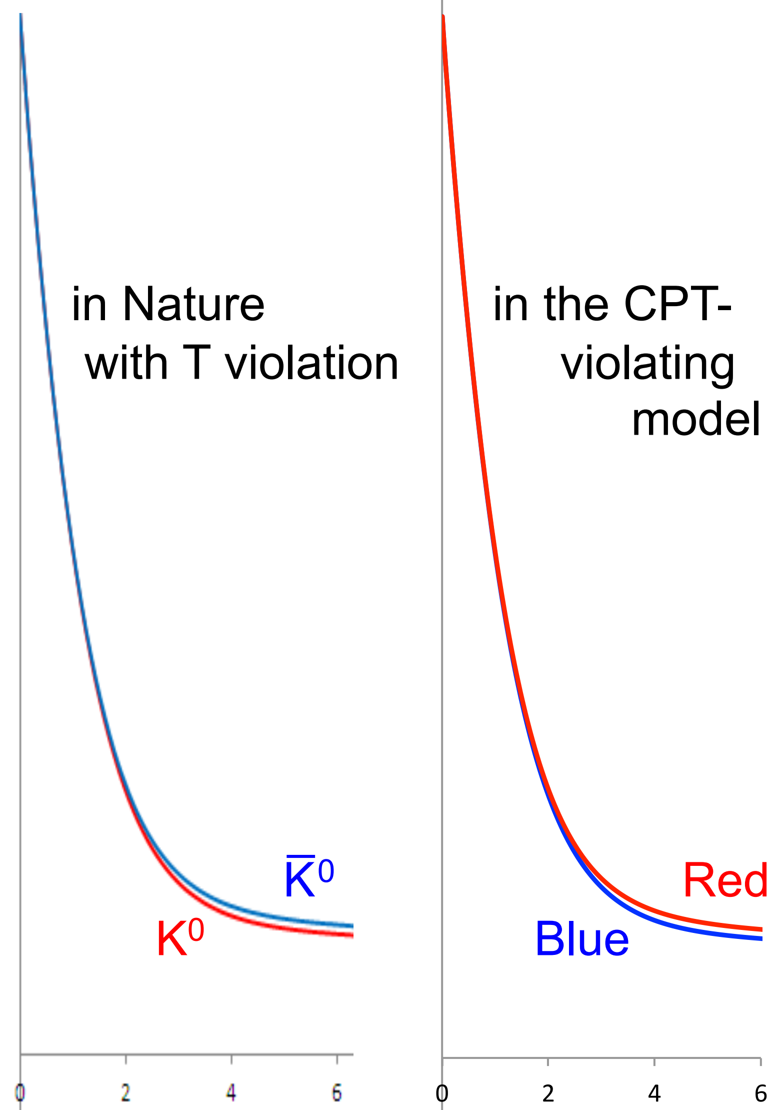
Does this Model represent T or CPT violation?

You may look at the differential equations, since the equations for the coupled pendulums are the same as those for the real parts of ψ_1 and ψ_2.

Or, as an experimentalist and good observer, you may see that with the same initial amplitudes A = 10, you find after a certain large time t that
A(Red → Red) = 6, A(Red → Blue) = 5,
A(Blue → Red) = 5, A(Blue → Blue) = 4,
Same asymmetry, but different survival chance, in videos 5 and 6.

This is the equivalent of T symmetry and CPT violation.

The non-exponential decay laws:



Model with T Violation:

A single two-dimensional pendulum on a turntable.
Like for a Foucault pendulum, T violation is provided by the Coriolis force in a rotating frame.

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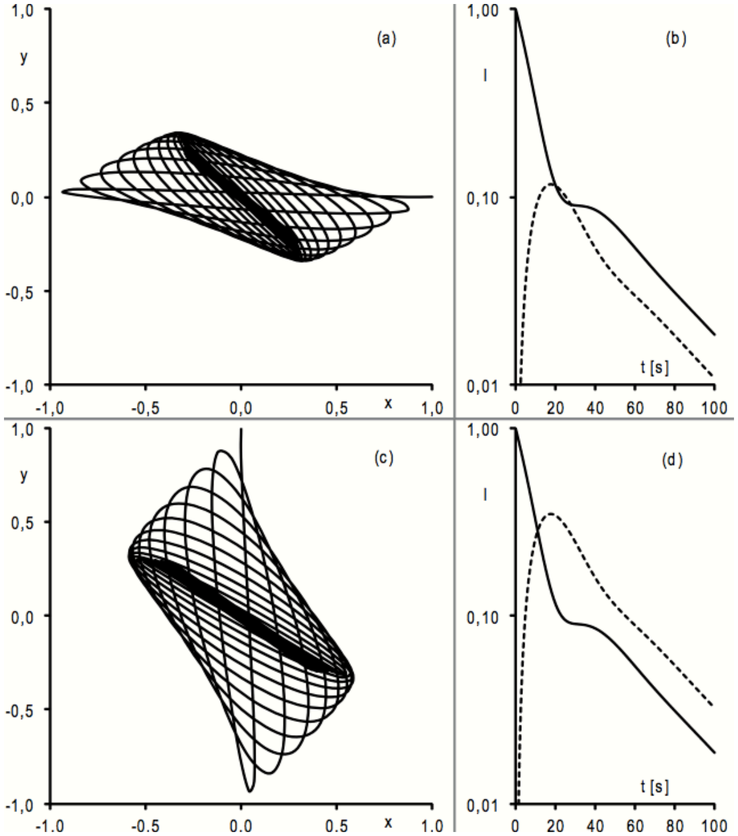
Built 2011 following a proposal of Jon Rosner



$x = K^0$
$y = \overline{K}^0$

$x = K^0$
$y = \overline{K}^0$