GW Detection Techniques

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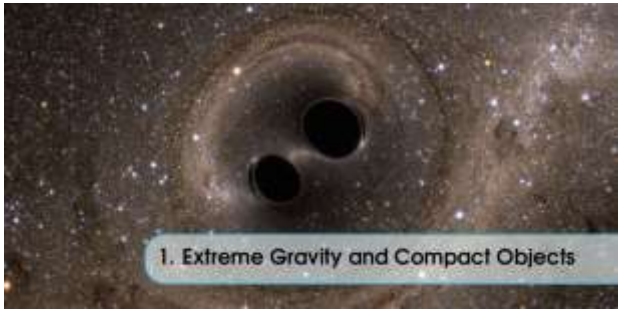
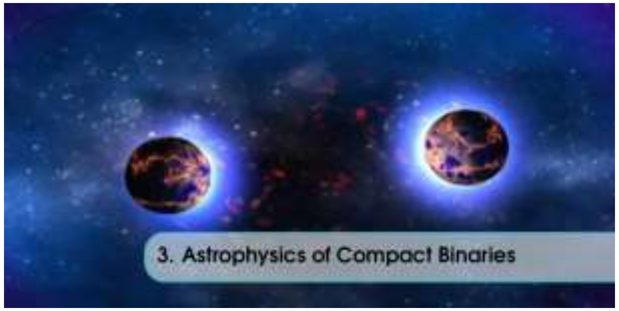
Credits to Jonah Kanner and Peter Shawhan for some slides

A New Window to the Universe





Science with the current and future generation ground-based GW detectors

GW Analysis Tutorials

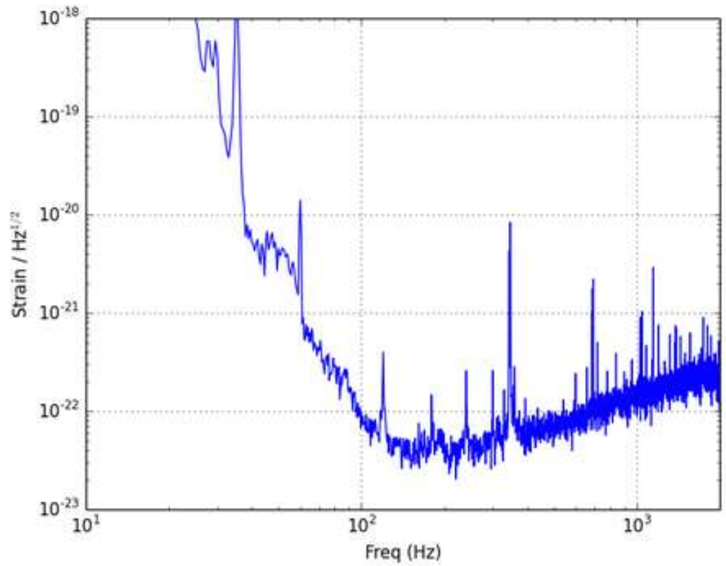
www.gw-openscience.org

Learn how to:

- Read LIGO/Virgo data
- Plots with Python
- PSDs, FFTs, and more

Amplitude Spectral Density (ASD)

The ASD can be obtained by taking the square root of the PSD. This is done to give units that can be more easily compared with the time domain data or FFT. More information about the LIGO ASD can be seen on the Instrumental Lines page.

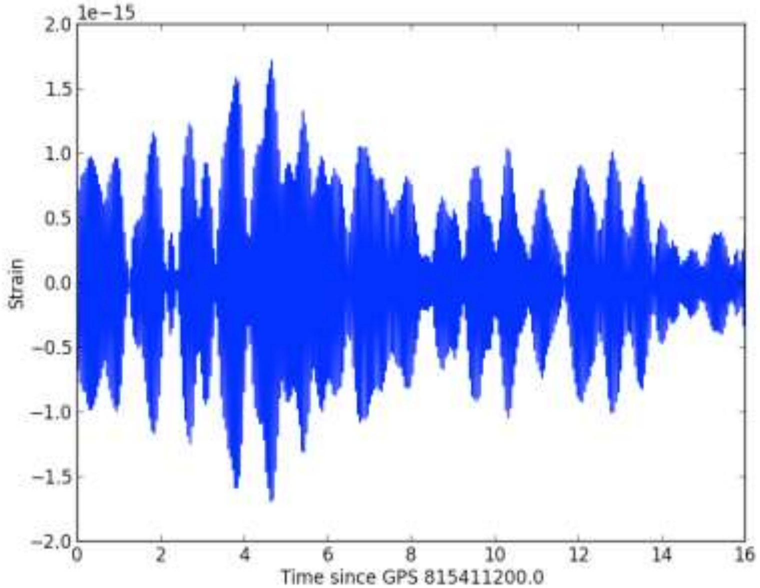


Show the code

```
#------------------------
# Plot the ASD
#------------------------------
Pxx, freqs = mlab.psd(strain_seg, Fs=fs, NFFT=2*fs)
plt.loglog(freqs, np.sqrt(Pxx))
plt.axis([10, 2000, 1e-23, 1e-18])
plt.grid('on')
plt.xlabel('Freq (Hz)')
plt.ylabel('Strain / Hz$^{1/2}$')
```

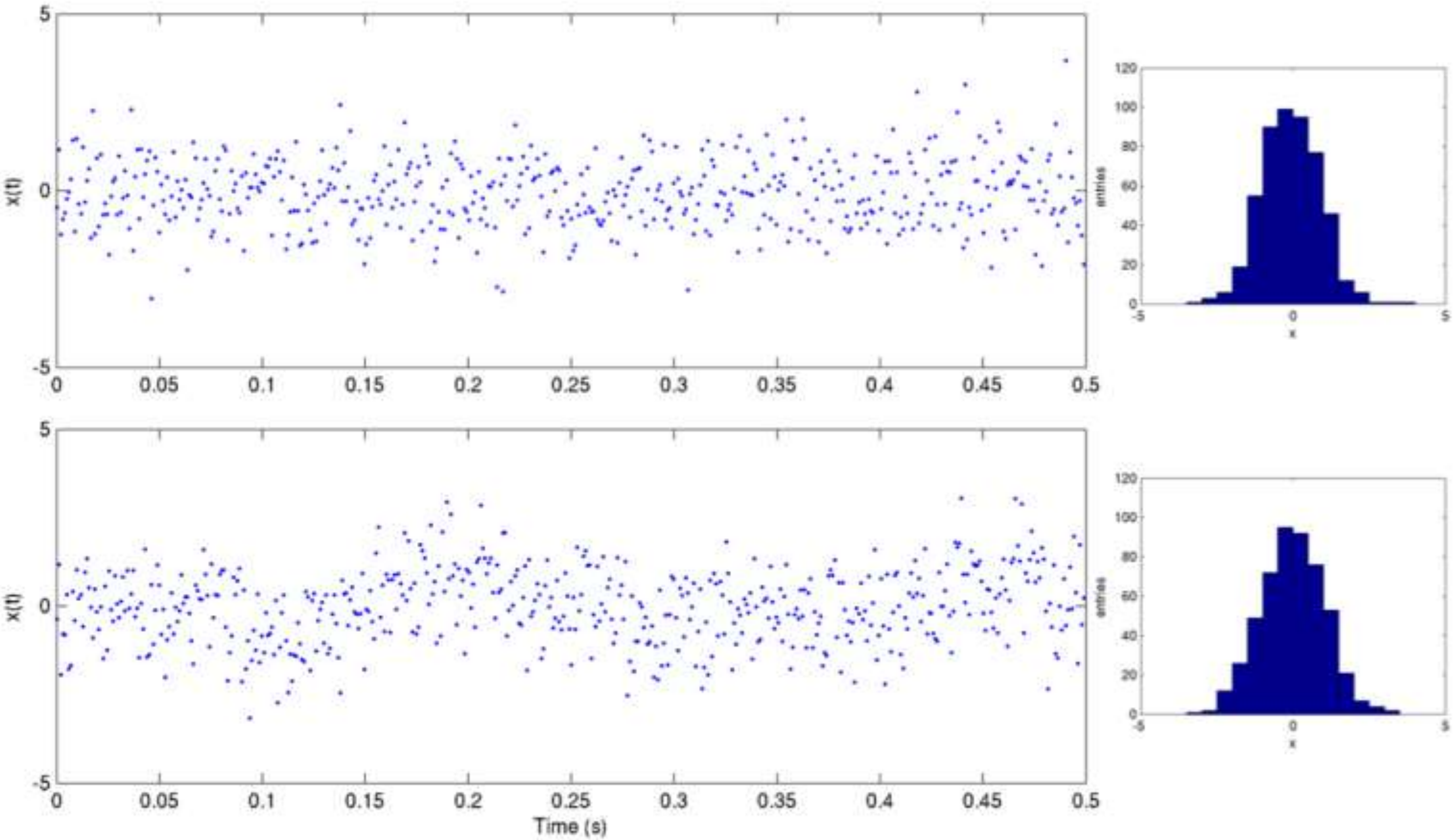
Virgo Data

- Discretely sampled time-series data
- h(t) – calibrated strain
  - ALSO: hundreds of “auxiliary” channels
- h(t) recorded at 20 kHz sample rate
- ~100 MB per hour
- Stored in .gwf “frame” files



Characterization of Random Noise

Noise is random, but its *properties* can be characterized



Possible Properties of Noise

Stationary : statistical properties are independent of time

Ergodic process: time averages are equivalent to ensemble averages

Gaussian : A random variable follows Gaussian distribution

For a single random variable, $$p(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left[-\frac{1}{2}\frac{(x-\mu_x)^2}{\sigma_x^2}\right]$$

More generally, a *set* of random variables (e.g. a time series) is Gaussian if the joint probability distribution is governed by a covariance matrix

$$C_{xij} := \langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle$$

such that

$$p(x_1, x_2, \ldots, x_N) = \frac{1}{(2\pi)^{N/2}\sqrt{\det C_x}} \exp\left[-\frac{1}{2}\sum_{i,j=0}^{N-1} C_{xij}^{-1}(x_i - \mu_{xi})(x_j - \mu_{xj})\right]$$

White : Signal power is uniformly distributed over frequency

$\Rightarrow$ Data samples are uncorrelated

Fourier Transform represents data in the frequency domain

Fourier transform

$$\widetilde{x}(f) = \int_{-\infty}^{\infty} dt\, x(t) e^{-i2\pi f t}$$

$$\Rightarrow \qquad x(t) = \int_{-\infty}^{\infty} df\, \widetilde{x}(f) e^{i2\pi f t}$$

$|\widetilde{x}(f)|^2$ can be interpreted as energy spectral density

Efficient way to calculate complete discrete Fourier Transform:
Fast Fourier Transform (FFT)

Power Spectral Density

Parseval's theorem:

$$\int_{-\infty}^{\infty} dt\, |x(t)|^2 = \int_{-\infty}^{\infty} df\, |\tilde{x}(f)|^2$$

$\Rightarrow$ Total energy in the data can be calculated in either time domain or frequency domain

$|\tilde{x}(f)|^2$ can be interpreted as energy spectral density

When noise (or signal) has infinite extent in time domain, can still define the power spectral density (PSD)

$$\lim_{T\to\infty} \frac{1}{T} \left|\tilde{x}_T(f)\right|^2$$

Watch out for one-sided vs. two-sided PSDs

Estimating the PSD

Simplest approach: FFT the data, calculate square of magnitude of each frequency component – this is a periodogram

For stationary noise, one can show that the frequency components are statistically independent

This estimate is unbiased (has the correct mean), but has a large variance – so average several periodograms

Alternately, smooth periodogram; give up frequency resolution either way

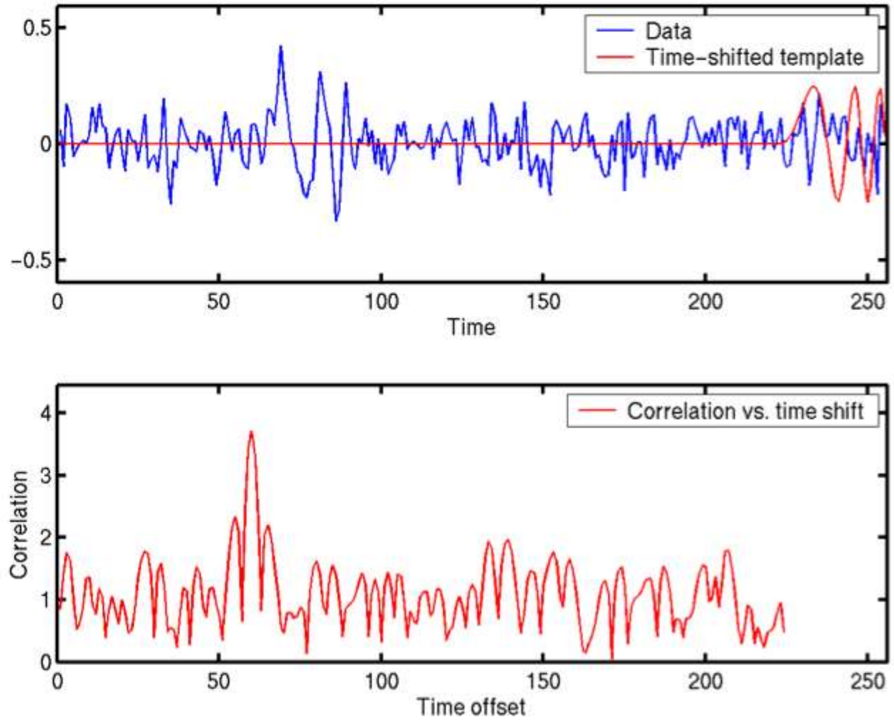
Generally apply a "window" to the data to avoid spectral leakage

Leakage arises from the assumption that the data is periodic!

Tapered window forces data to go to zero at ends of time interval

Welch's method of estimating a PSD averages periodograms calculated from windowed data

Cross correlation
Slide template against data



$$C(t) = \int_{-\infty}^{\infty} dt' \, s(t') \, h(t' - t)$$

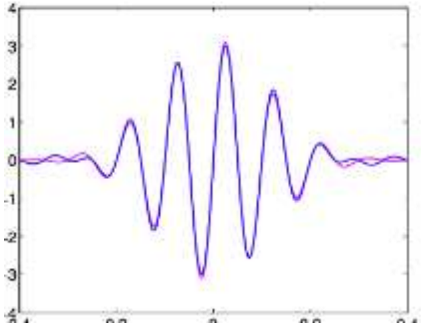
Time offset

Data

Template with time offset

Burst searches look for excess power

$$X(\tau,\phi,Q) = \int_{-\infty}^{+\infty} x(t)\, w(t-\tau,\phi,Q)\, e^{-i2\pi\phi t}\, dt,$$

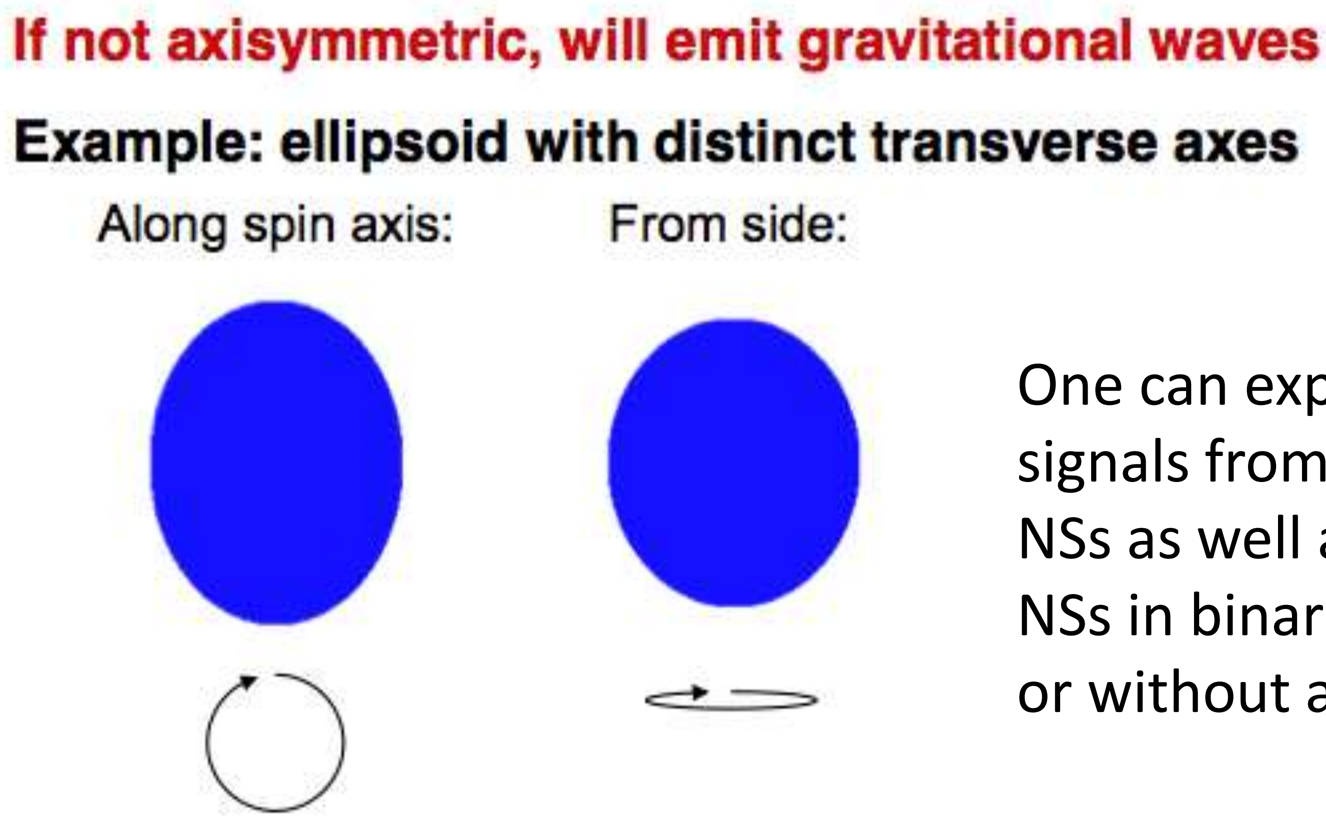


"Transform" to a time-frequency basis:

Cross-correlate data with wavelets to get energy in each time-frequency pixel

Similar to how Fourier Transform cross correlates against sine waves to get energy in each frequency bin

Continuous Wave sources from spinning neutron stars



One can expect GW signals from isolated NSs as well as from NSs in binaries with or without accretion.

Continuous Wave signals

Start with a sinusoidal signal with spin-down term(s)

Polarization content depends on orientation/inclination of spin axis

Amplitude modulation

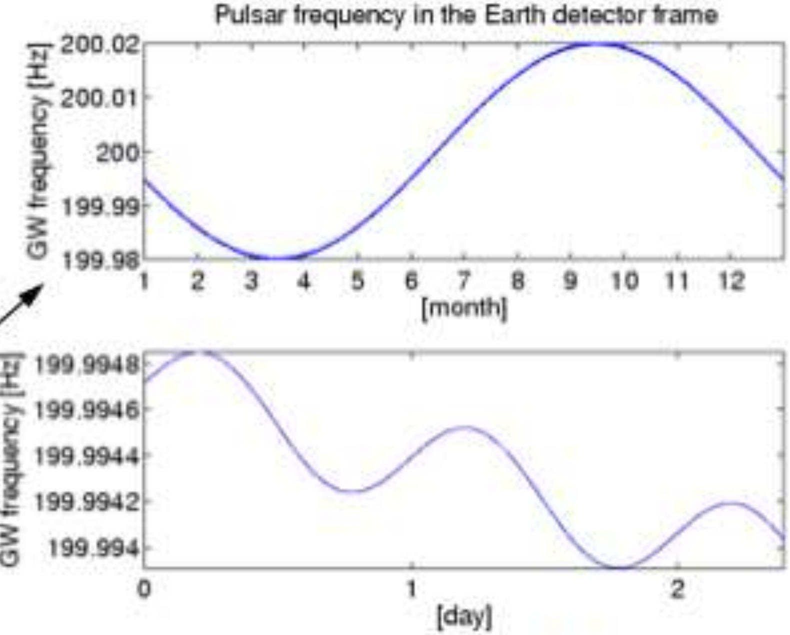
Polarization projection changes over a sidereal day

Doppler shift

$$\frac{\Delta f}{f} = \frac{\mathbf{v} \cdot \mathbf{n}}{c}$$

Annual variation: up to $\sim 10^{-4}$

Daily variation: up to $\sim 10^{-6}$



GW signals from binary systems are more complicated !

Additional Doppler shift due to orbital motion of neutron star

Varying gravitational redshift if orbit is elliptical

Shapiro time delay if GW passes near companion

CW – Wide parameter space

Method: matched filtering with a bank of templates

Parameters:

Sky position
Spin axis inclination and azimuthal angle
Frequency, spindown, initial phase
Binary orbit parameters (if in a binary system)

Can use a detection statistic, $\mathcal{F}$, which analytically maximizes over spin axis inclination & azimuthal angle and initial phase

Even so, computing cost scales as $\sim T^6$

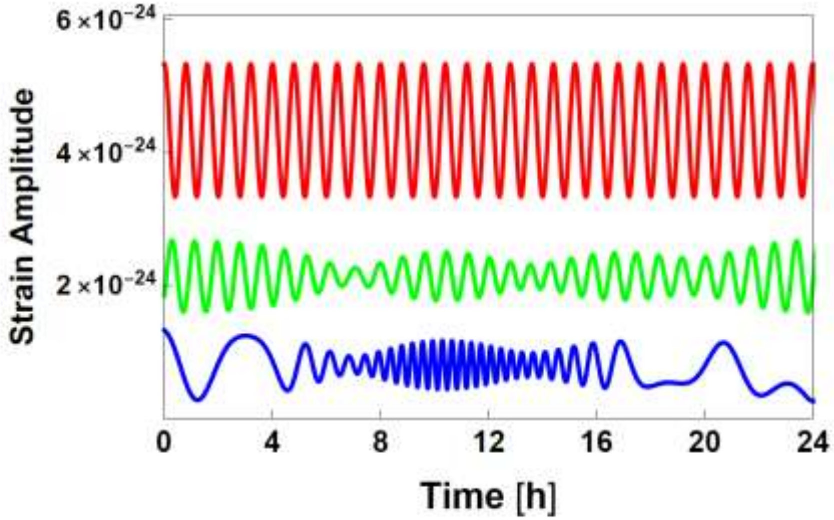
Detection threshold also must increase with number of templates

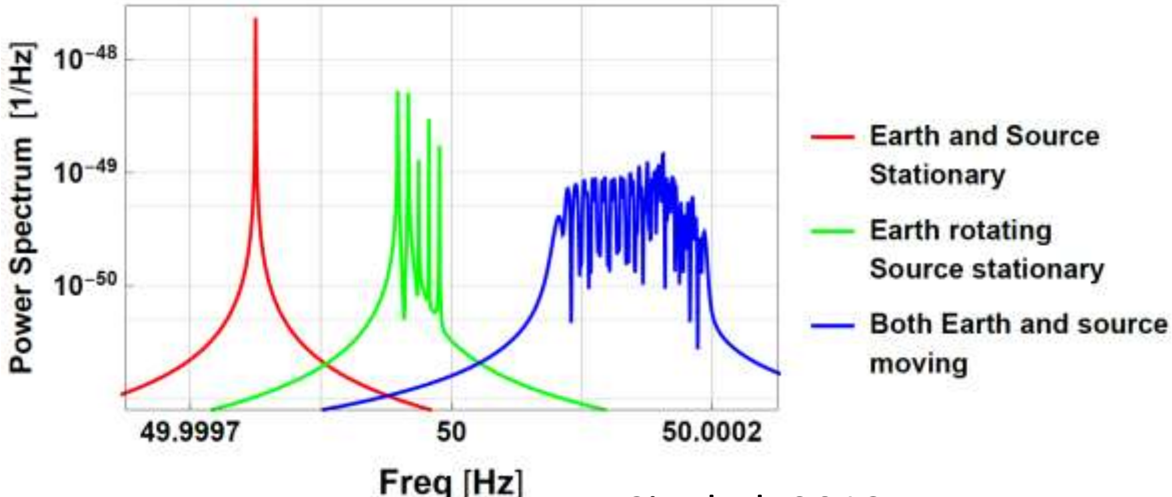
Check for signal consistency in multiple detectors

Problem: huge number of templates needed

Even using clever semi-coherent analysis methods

Qualitative Comparison





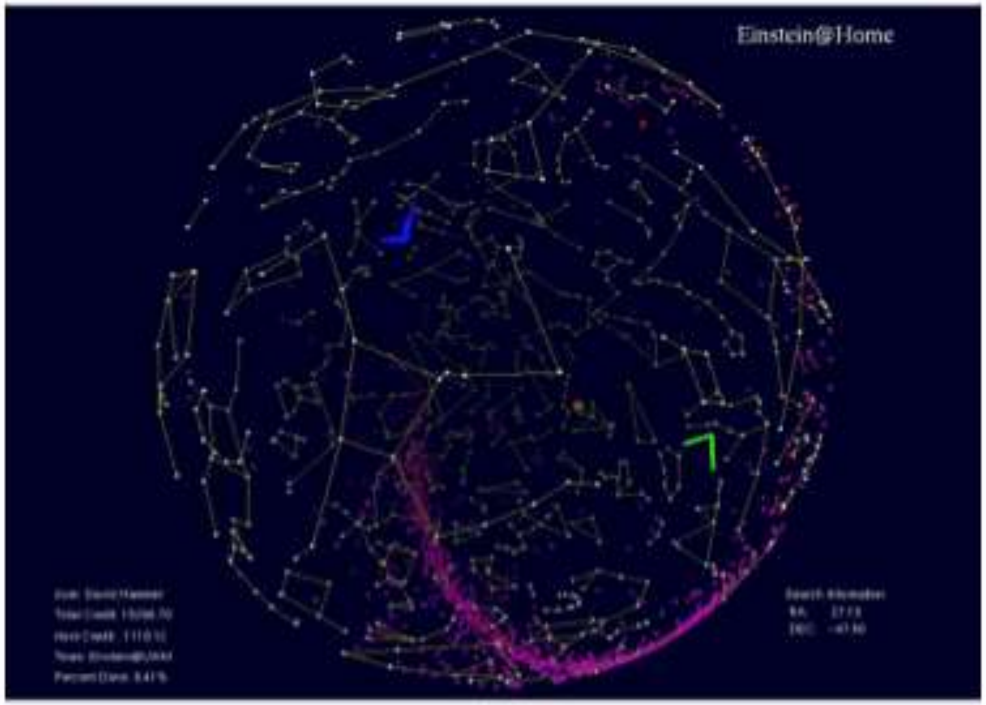
Singhal, 2019

Citizen Science

Public distributed computing project: Einstein@Home

Small bits of data distributed for processing;
results collected, verified, and post-processed

einsteinathome.org

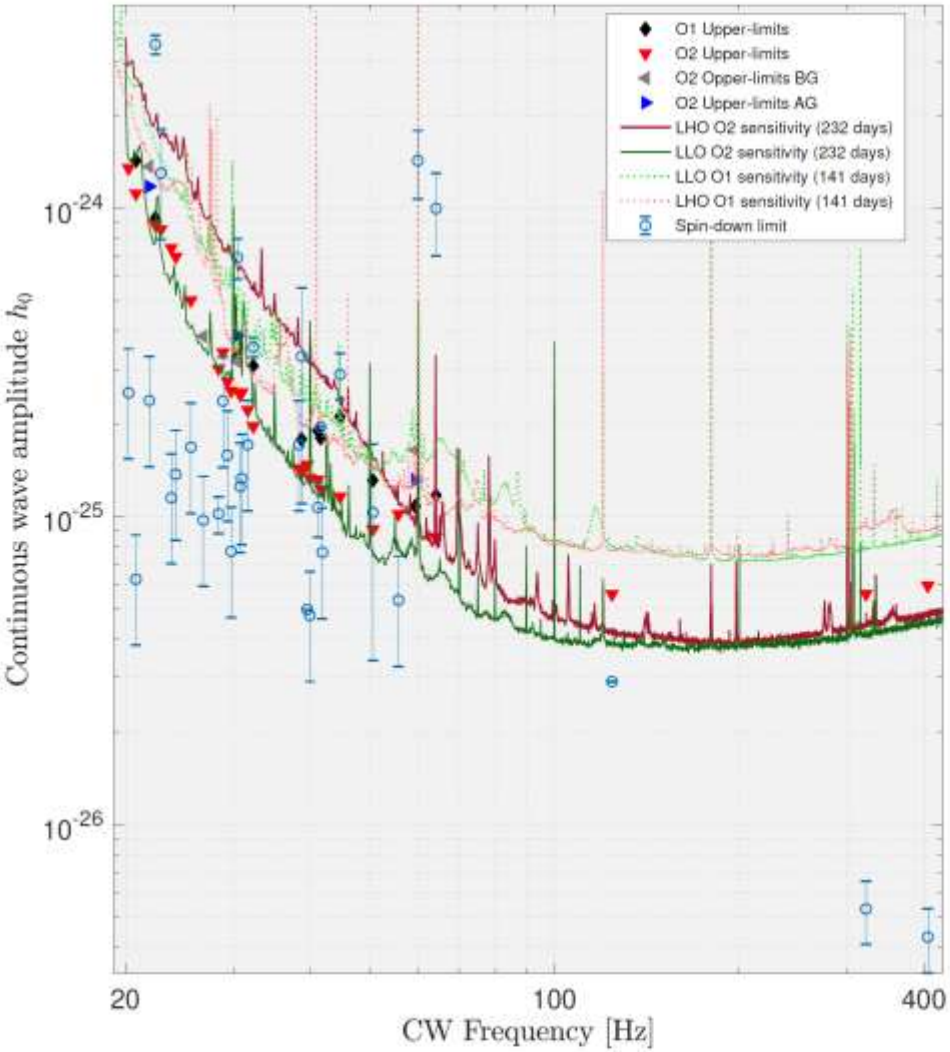


Searching for CW signals in LIGO+Virgo data

Also searching for millisecond pulsars in data from Arecibo, Parkes, and the Fermi satellite

Upper Limits

https://arxiv.org/abs/1902.08442



Spin-down limits assume that all of the observed df/dt is due to emission of GWs.

Stochastic Signals

Random signal from sum of unresolved sources

From the early universe, or from astrophysical sources since then

Usual assumptions about the signal:

Stationary

Gaussian

Unpolarized

Power-law frequency dependence, probably (e.g. f^{-3})

May be isotropic, or not

Looks basically like extra noise in each detector !

Use cross-correlation to find stochastic signals

Use cross-correlation between GW data streams

- No time delay for all-sky isotropic search – will affect correlation
- For anisotropic ("radiometer") search, fix time delay between streams

Include a filter function in the cross-correlation

$$Y := \int_{-T/2}^{T/2} dt_1 \int_{-T/2}^{T/2} dt_2\, x_1(t_1) x_2(t_2) K(t_1 - t_2)$$

$$\Downarrow$$

$$Y = \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} df'\, \delta_T(f - f') \tilde{x}_1^*(f) \tilde{x}_2(f') \widetilde{K}(f')$$

Filter function optimizes the detection statistic, accounting for two effects:

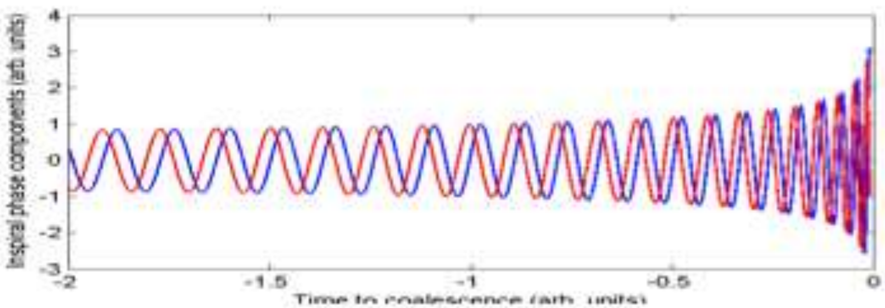
- Power spectrum of the signal being searched for
- Expected correlation between detectors, which depends on frequency due to their separation

CBC: Compact Binary Coalescence

Inspiral source parameters

- Masses (m1, m2)
- Spins → Negligible for neutron stars, at least
- Orbital phase at coalescence → Maximize analytically when filtering
- Inclination of orbital plane
- Sky location
  → Simply multiplicative for a given detector (long-wavelength limit)
- Distance → Simply multiplicative
- Coalescence time

Filter with orthogonal templates, take quadrature sum



➔ Only have to explicitly search over masses and coalescence time ("intrinsic parameters")

Matched Filtering in the frequency domain

$$C(t) = \int_{-\infty}^{\infty} dt'\, s(t')\, h(t'-t)$$

Time offset

Data

Template with time offset

Rewrite correlation integral using Fourier transforms…

$$\Rightarrow \quad C(t) = 4\int_{0}^{\infty} \tilde{s}(f)\, \tilde{h}^{*}(f)\, e^{2\pi i f t}\, df$$

Correlate in the frequency domain

"Phase factor" represents the time offsets

Optimal Matched Filtering

FFT of data

Template; can be generated in frequency domain using stationary phase approximation

$$C(t) = 4\int_0^\infty \frac{\tilde{s}(f)\ \tilde{h}^*(f)}{S_n(f)}\, e^{2\pi i f t}\, df$$

Noise power spectral density

Look for maximum of $|C(t)|$ above some threshold → trigger

Putting the noise PSD in the denominator “down weights” places where the noise is high

CBC Template Banks

Want to be able to detect any signal in a *space* of possible signals

All with different phase evolution

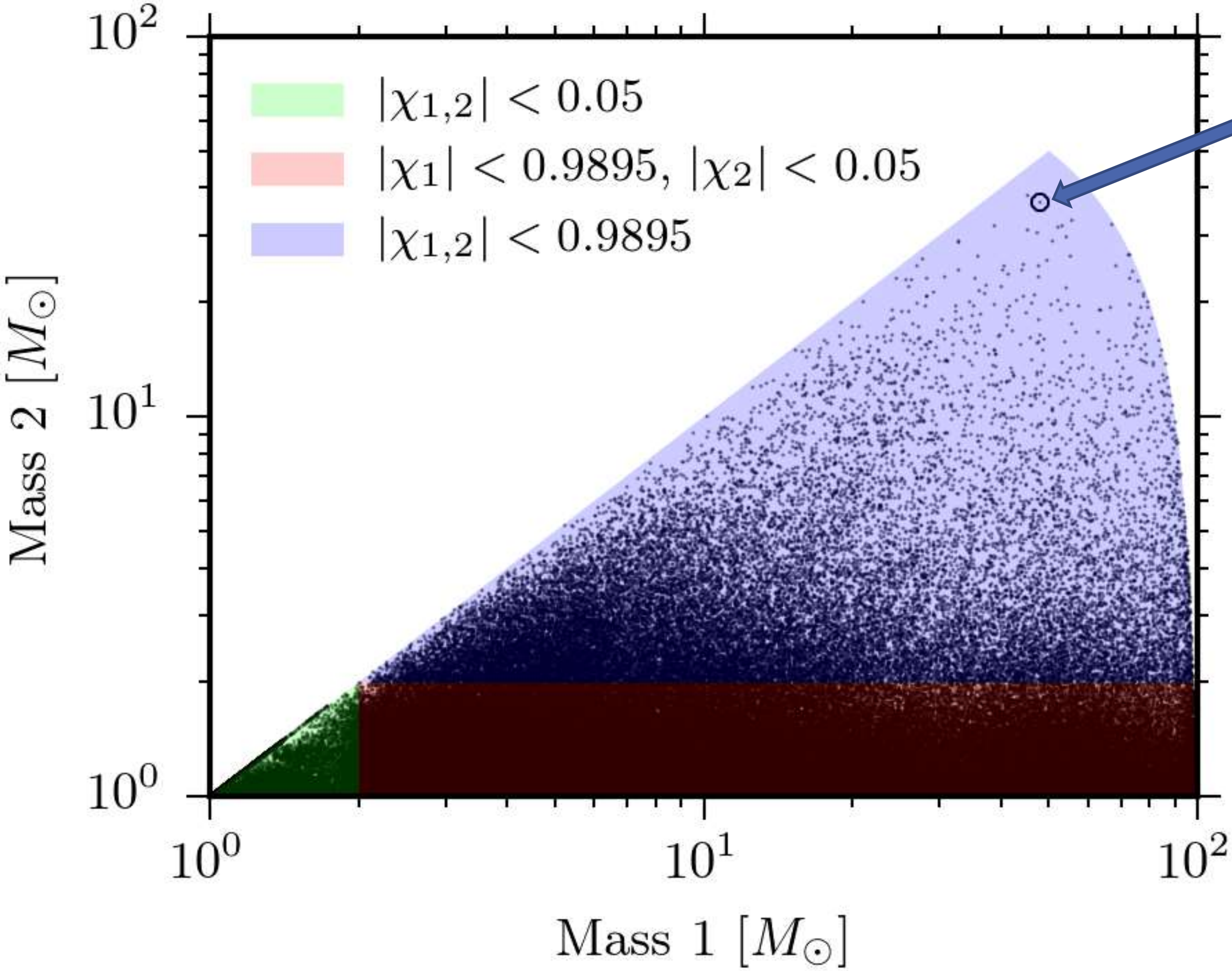
… but do it with a finite set of templates!

So make sure there is a "close enough" template for every part of the signal space

Require a minimum overlap between signal and template, e.g. 0.97

Often can calculate a "metric" which parametrizes the mismatch for small mismatches

CBC Template Banks in "Mass Space"



Best match with GW150914

GW150914

Selected for a Viewpoint in *Physics*

PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

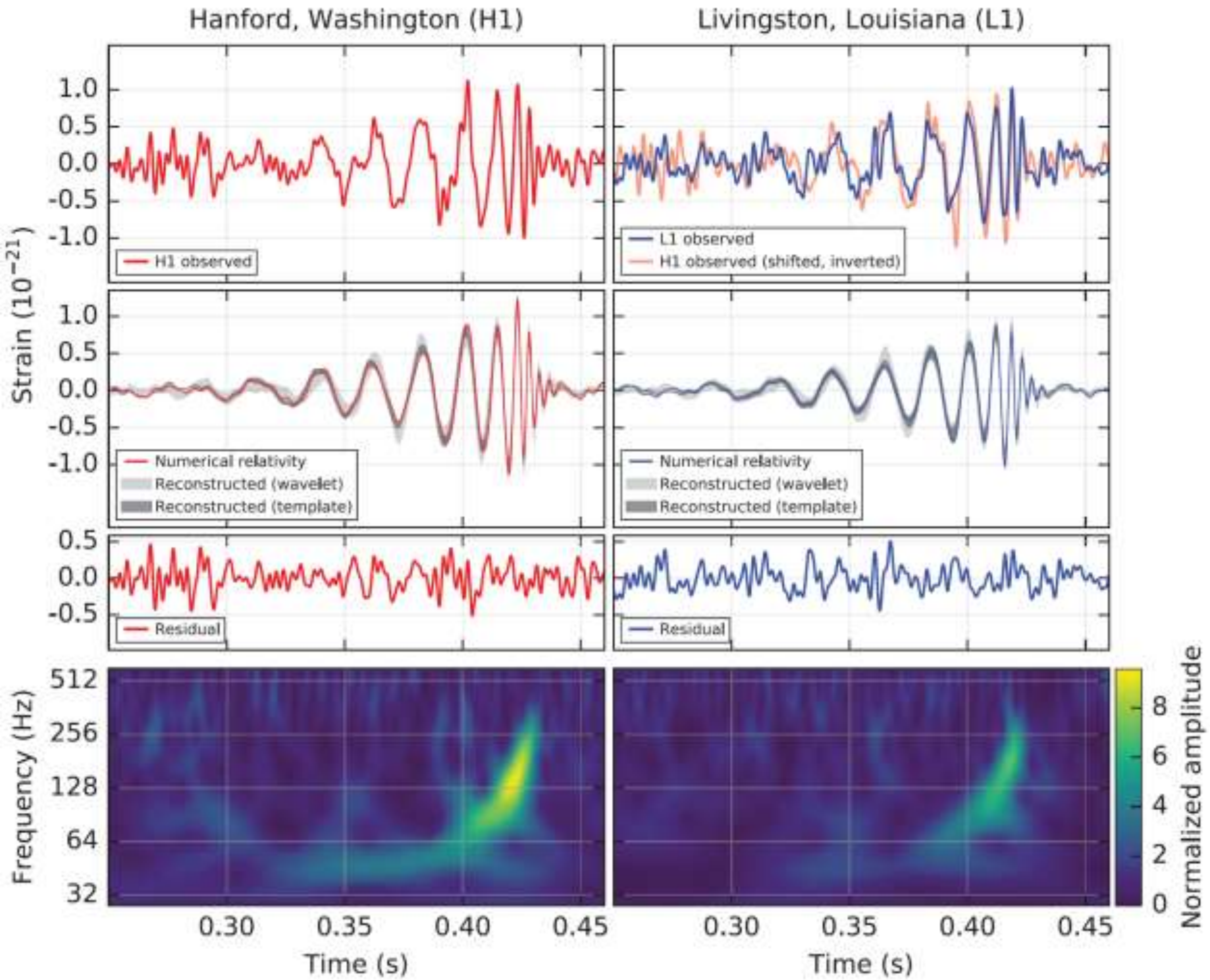
B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

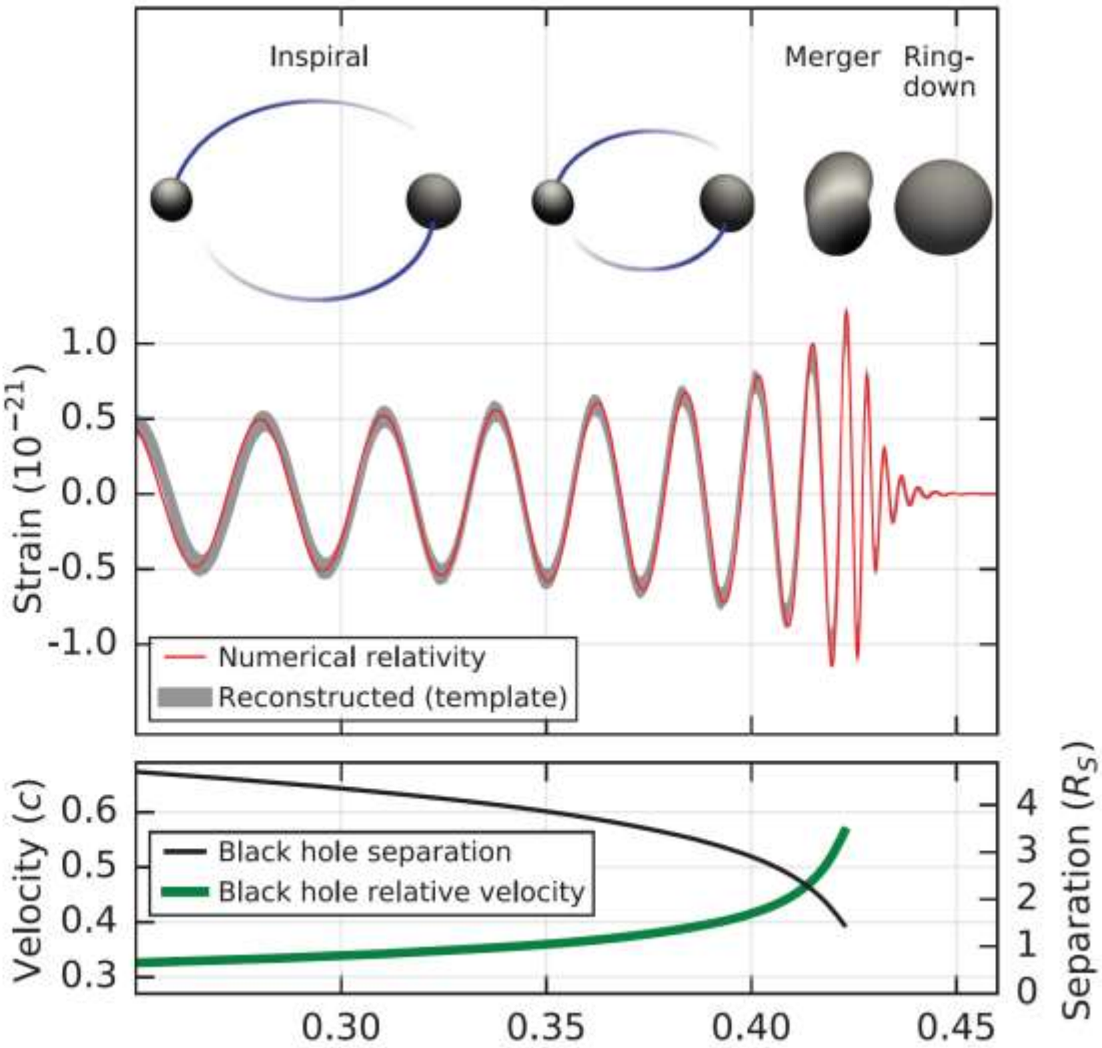
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$, and the final black hole mass is $62^{+4}_{-4} M_\odot$, with $3.0^{+0.5}_{-0.5} M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

The Signal Waveform

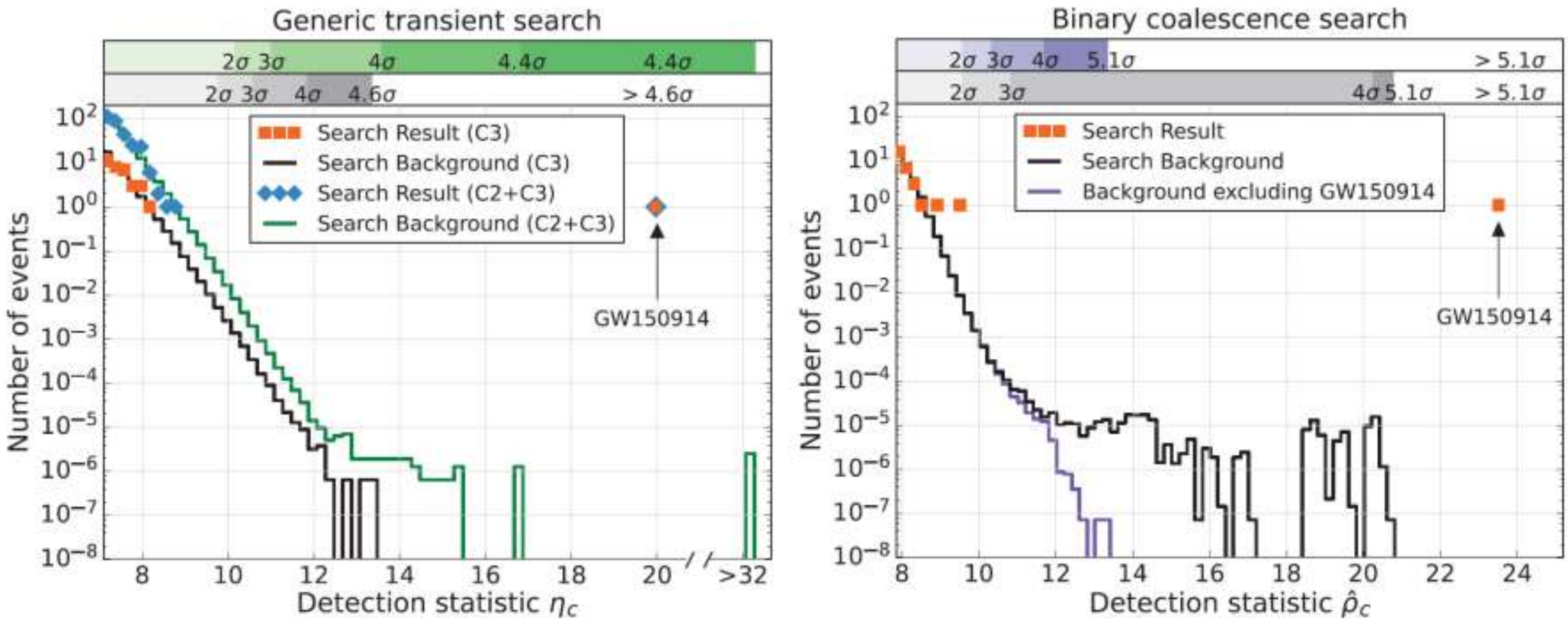


A Chirp



Inspiral
Merger
Ring-
down
1.0
0.5
0.0
-0.5
-1.0
Strain (10^{-21})
Numerical relativity
Reconstructed (template)
0.6
0.5
0.4
0.3
Velocity (c)
Black hole separation
Black hole relative velocity
4
3
2
1
0
Separation (R_S)
0.30
0.35
0.40
0.45

Noise Background



Posterior Distributions

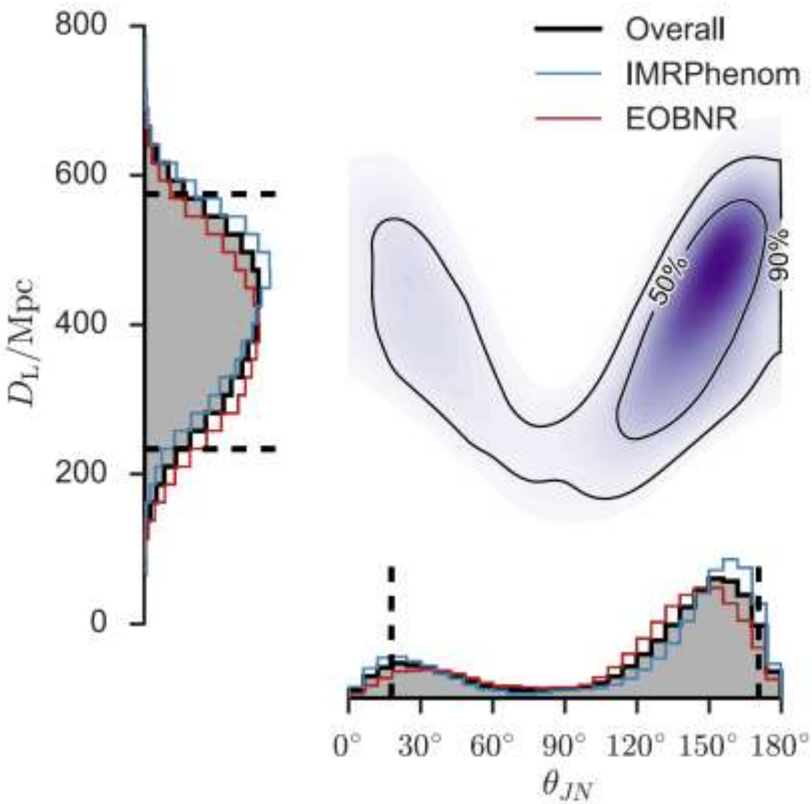
C Berry

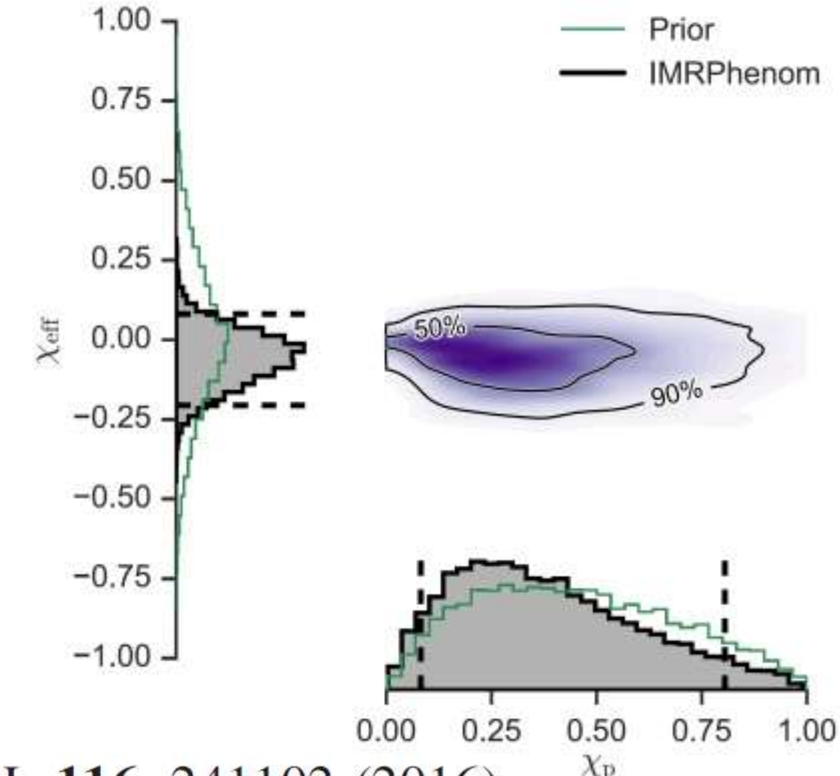
Likelihood Prior

$$\boxed{p(\theta|d)} = \frac{\boxed{p(d|\theta)}\,\boxed{p(\theta)}}{\boxed{p(d)}}$$

Posterior Evidence

$$p(d|\theta) \propto \exp\left[-\frac{1}{2}\sum_k \langle h_k(\theta) - d_k | h_k(\theta) - d_k \rangle\right]$$





PRL **116,** 241102 (2016)

GW170817

Selected for a Viewpoint in *Physics*

PRL **119,** 161101 (2017) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

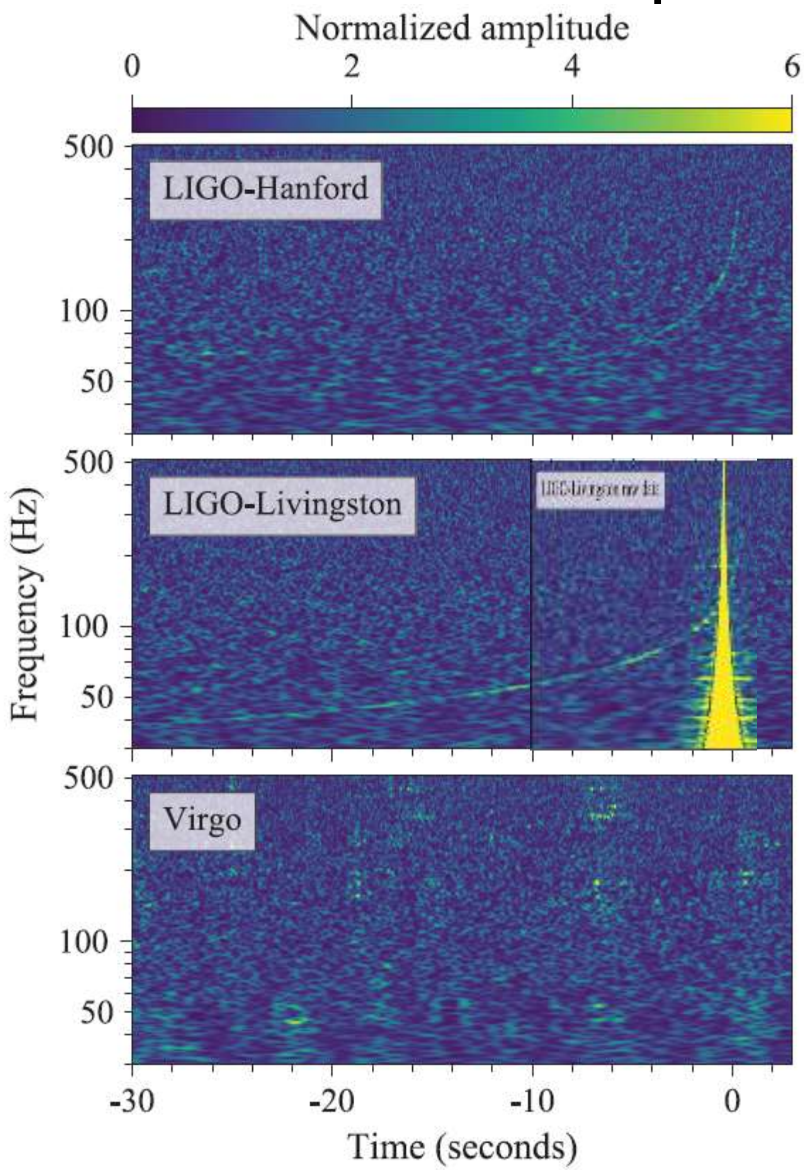
B. P. Abbott *et al.*[*]

(LIGO Scientific Collaboration and Virgo Collaboration)

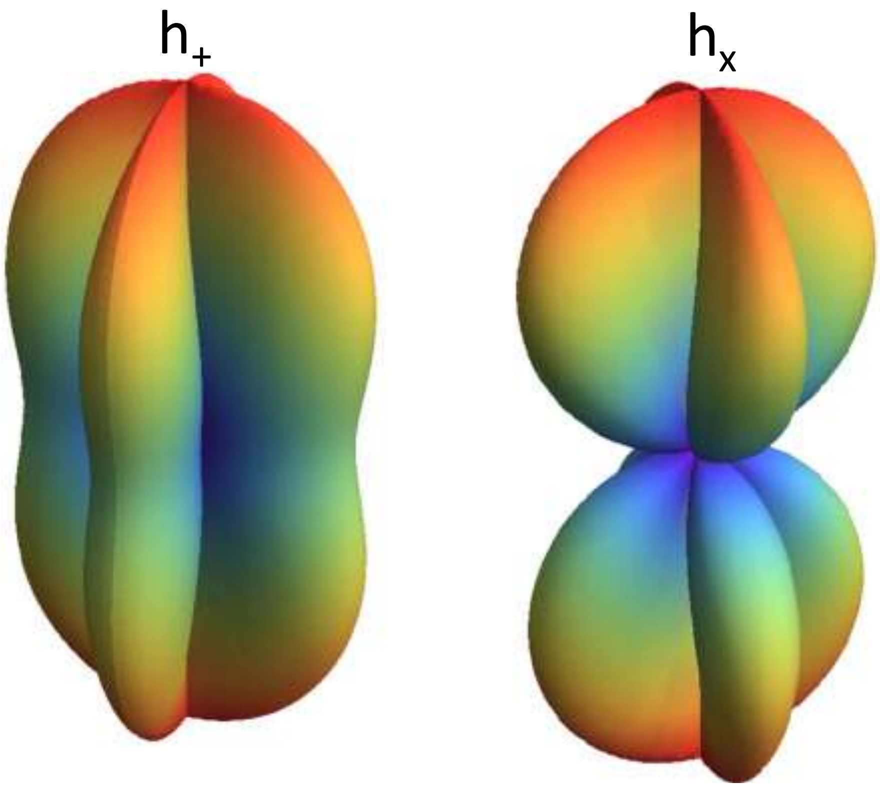
(Received 26 September 2017; revised manuscript received 2 October 2017; published 16 October 2017)

On August 17, 2017 at 12:41:04 UTC the Advanced LIGO and Advanced Virgo gravitational-wave detectors made their first observation of a binary neutron star inspiral. The signal, GW170817, was detected with a combined signal-to-noise ratio of 32.4 and a false-alarm-rate estimate of less than one per 8.0×10^4 years. We infer the component masses of the binary to be between 0.86 and 2.26 $M_\odot$, in agreement with masses of known neutron stars. Restricting the component spins to the range inferred in binary neutron stars, we find the component masses to be in the range 1.17–1.60 $M_\odot$, with the total mass of the system $2.74^{+0.04}_{-0.01} M_\odot$. The source was localized within a sky region of 28 deg^2 (90% probability) and had a luminosity distance of 40^{+8}_{-14} Mpc, the closest and most precisely localized gravitational-wave signal yet. The association with the γ-ray burst GRB 170817A, detected by Fermi-GBM 1.7 s after the coalescence, corroborates the hypothesis of a neutron star merger and provides the first direct evidence of a link between these mergers and short γ-ray bursts. Subsequent identification of transient counterparts across the electromagnetic spectrum in the same location further supports the interpretation of this event as a neutron star merger. This unprecedented joint gravitational and electromagnetic observation provides insight into astrophysics, dense matter, gravitation, and cosmology.

Spectrogram

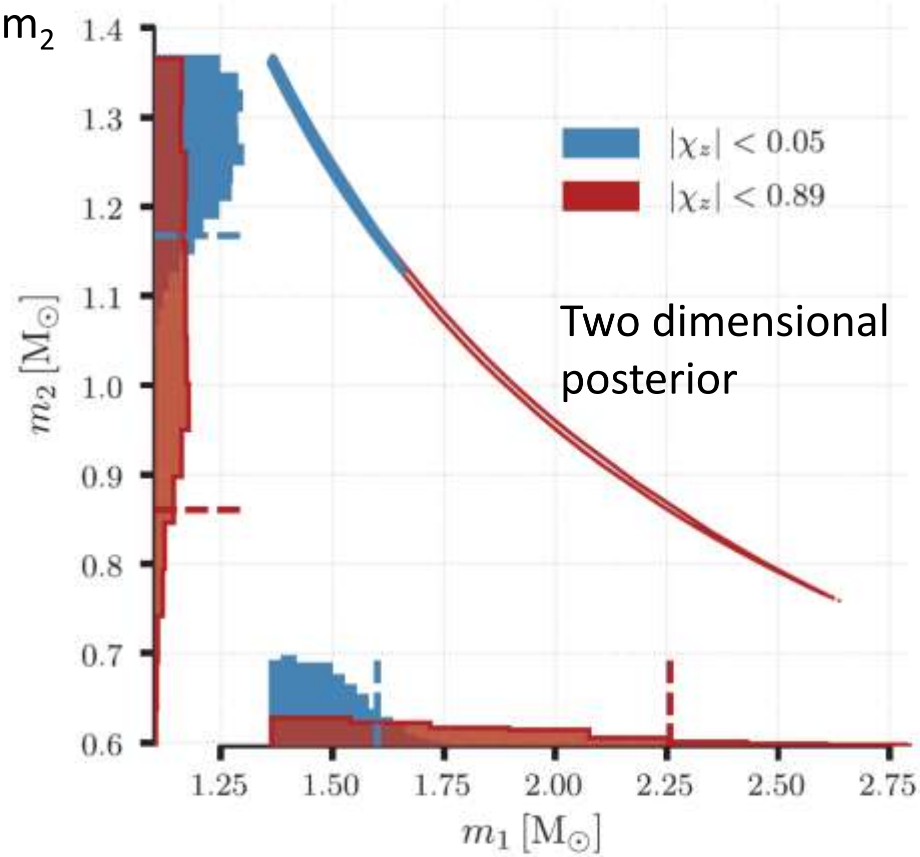


Spectrograms of GW170817 in the three detectors.



Posterior distribution of component masses

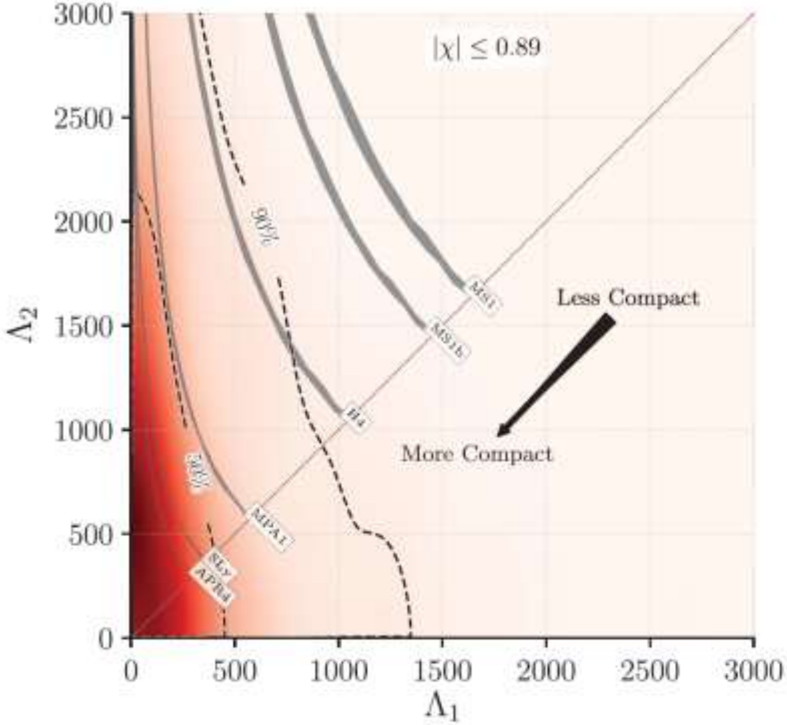
Marginal distribution of m_2

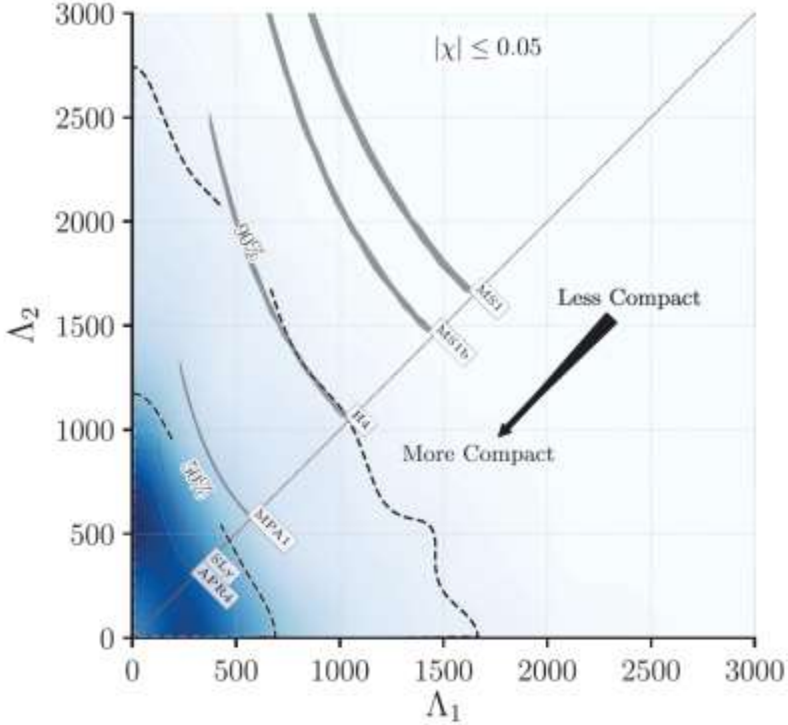


Two dimensional posterior

Marginal distribution of m_1

Tidal Deformability





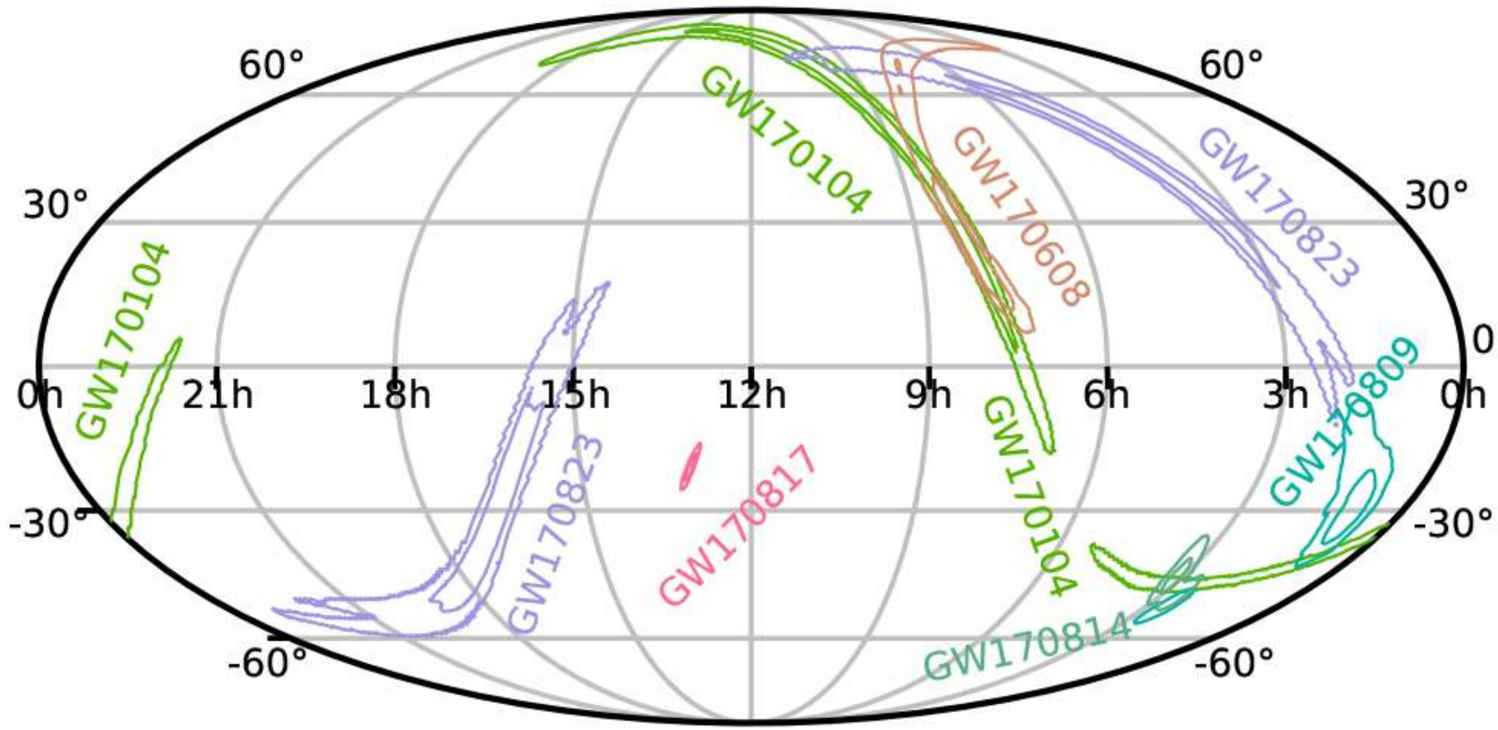
Cross-correlations are used in all types of searches

- CBC searches cross-correlate data against a template bank of merging NS-NS or BH-BH
- Burst searches cross-correlate against wavelets to transform to time-frequency space
- Continuous Wave searches cross-correlate against sine-waves with doppler shifts
- Stochastic searches cross-correlate data from 2 detectors

GWTC-1

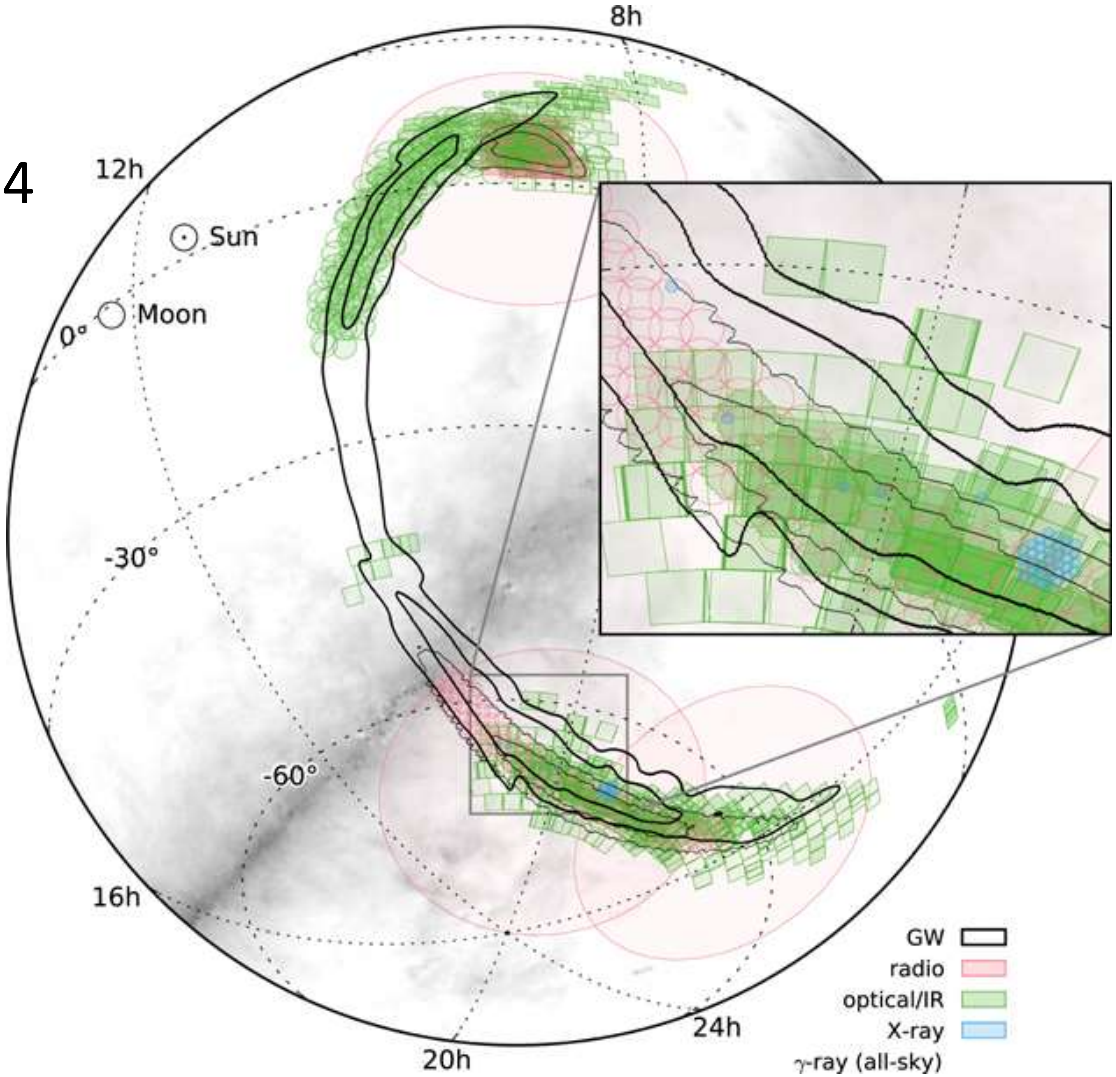
Event	$m_1/\mathrm{M}_\odot$	$m_2/\mathrm{M}_\odot$	$\mathcal{M}/\mathrm{M}_\odot$	χ_{eff}	$M_{\mathrm{f}}/\mathrm{M}_\odot$	a_{f}	$E_{\mathrm{rad}}/(\mathrm{M}_\odot c^2)$	$\ell_{\mathrm{peak}}/(\mathrm{erg\,s}^{-1})$	d_L/Mpc	z	$\Delta\Omega/\mathrm{deg}^2$
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.6}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.3}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$3.1^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4} \times 10^{56}$	430^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	180
GW151012	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.0}_{-1.1}$	$0.04^{+0.28}_{-0.19}$	$35.7^{+9.9}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	$1.5^{+0.5}_{-0.5}$	$3.2^{+0.8}_{-1.7} \times 10^{56}$	1060^{+540}_{-480}	$0.21^{+0.09}_{-0.09}$	1555
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	$8.9^{+0.3}_{-0.3}$	$0.18^{+0.20}_{-0.12}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	$1.0^{+0.1}_{-0.2}$	$3.4^{+0.7}_{-1.7} \times 10^{56}$	440^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$	1033
GW170104	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	$21.5^{+2.1}_{-1.7}$	$-0.04^{+0.17}_{-0.20}$	$49.1^{+5.2}_{-3.9}$	$0.66^{+0.08}_{-0.10}$	$2.2^{+0.5}_{-0.5}$	$3.3^{+0.6}_{-0.9} \times 10^{56}$	960^{+430}_{-410}	$0.19^{+0.07}_{-0.08}$	924
GW170608	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	$7.9^{+0.2}_{-0.2}$	$0.03^{+0.19}_{-0.07}$	$17.8^{+3.2}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	$0.9^{+0.05}_{-0.1}$	$3.5^{+0.4}_{-1.3} \times 10^{56}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$	396
GW170729	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	$35.7^{+6.5}_{-4.7}$	$0.36^{+0.21}_{-0.25}$	$80.3^{+14.6}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	$4.8^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-1.5} \times 10^{56}$	2750^{+1350}_{-1320}	$0.48^{+0.19}_{-0.20}$	1033
GW170809	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	$25.0^{+2.1}_{-1.6}$	$0.07^{+0.16}_{-0.16}$	$56.4^{+5.2}_{-3.7}$	$0.70^{+0.08}_{-0.09}$	$2.7^{+0.6}_{-0.6}$	$3.5^{+0.6}_{-0.9} \times 10^{56}$	990^{+320}_{-380}	$0.20^{+0.05}_{-0.07}$	340
GW170814	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	$24.2^{+1.4}_{-1.1}$	$0.07^{+0.12}_{-0.11}$	$53.4^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	$2.7^{+0.4}_{-0.3}$	$3.7^{+0.4}_{-0.5} \times 10^{56}$	580^{+160}_{-210}	$0.12^{+0.03}_{-0.04}$	87
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$	$0.00^{+0.02}_{-0.01}$	≤ 2.8	≤ 0.89	≥ 0.04	$\geq 0.1 \times 10^{56}$	40^{+10}_{-10}	$0.01^{+0.00}_{-0.00}$	16
GW170818	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	$26.7^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$	$59.8^{+4.8}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	$2.7^{+0.5}_{-0.5}$	$3.4^{+0.5}_{-0.7} \times 10^{56}$	1020^{+430}_{-360}	$0.20^{+0.07}_{-0.07}$	39
GW170823	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	$29.3^{+4.2}_{-3.2}$	$0.08^{+0.20}_{-0.22}$	$65.6^{+9.4}_{-6.6}$	$0.71^{+0.08}_{-0.10}$	$3.3^{+0.9}_{-0.8}$	$3.6^{+0.6}_{-0.9} \times 10^{56}$	1850^{+840}_{-840}	$0.34^{+0.13}_{-0.14}$	1651

Source Localization

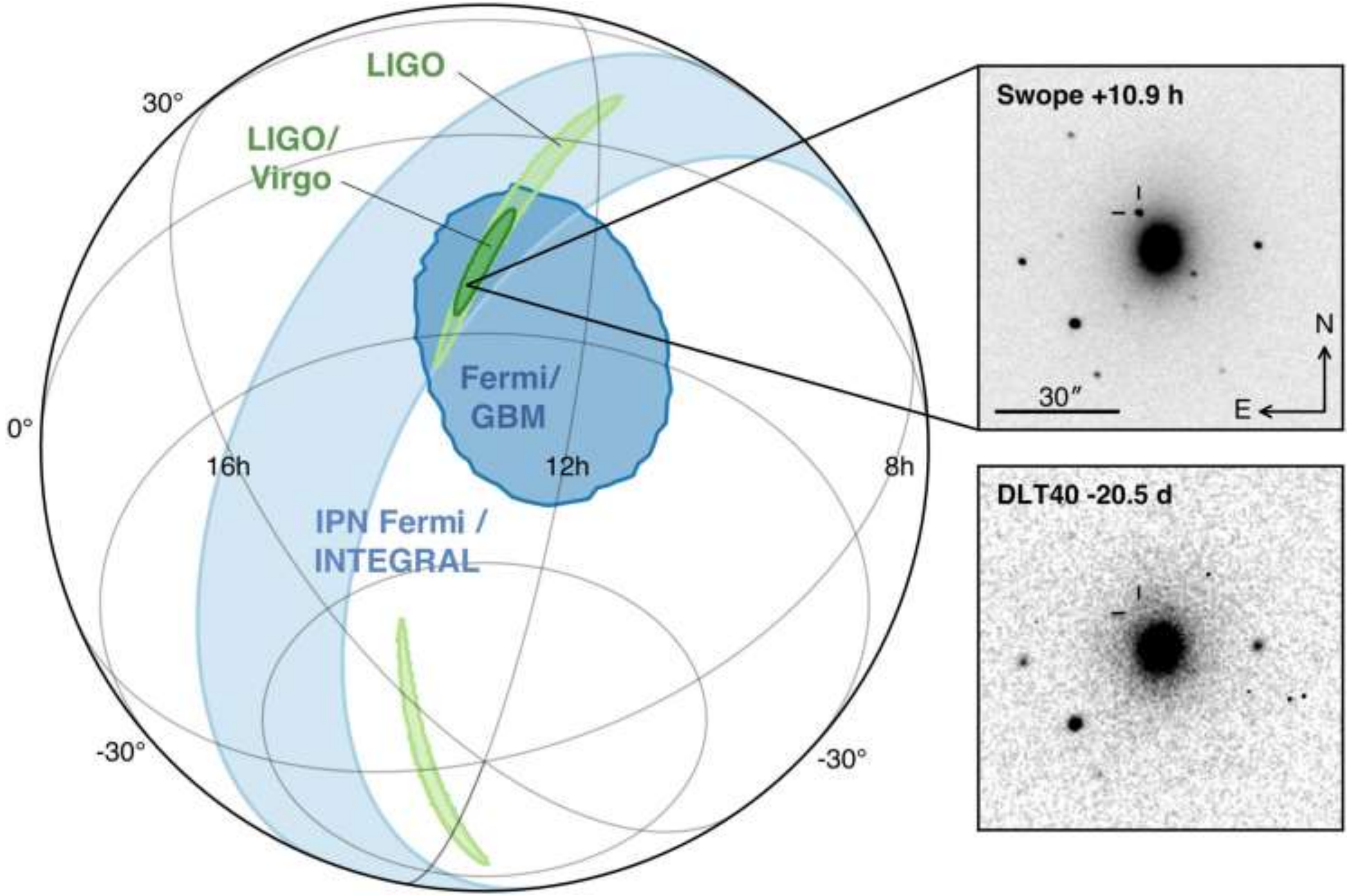


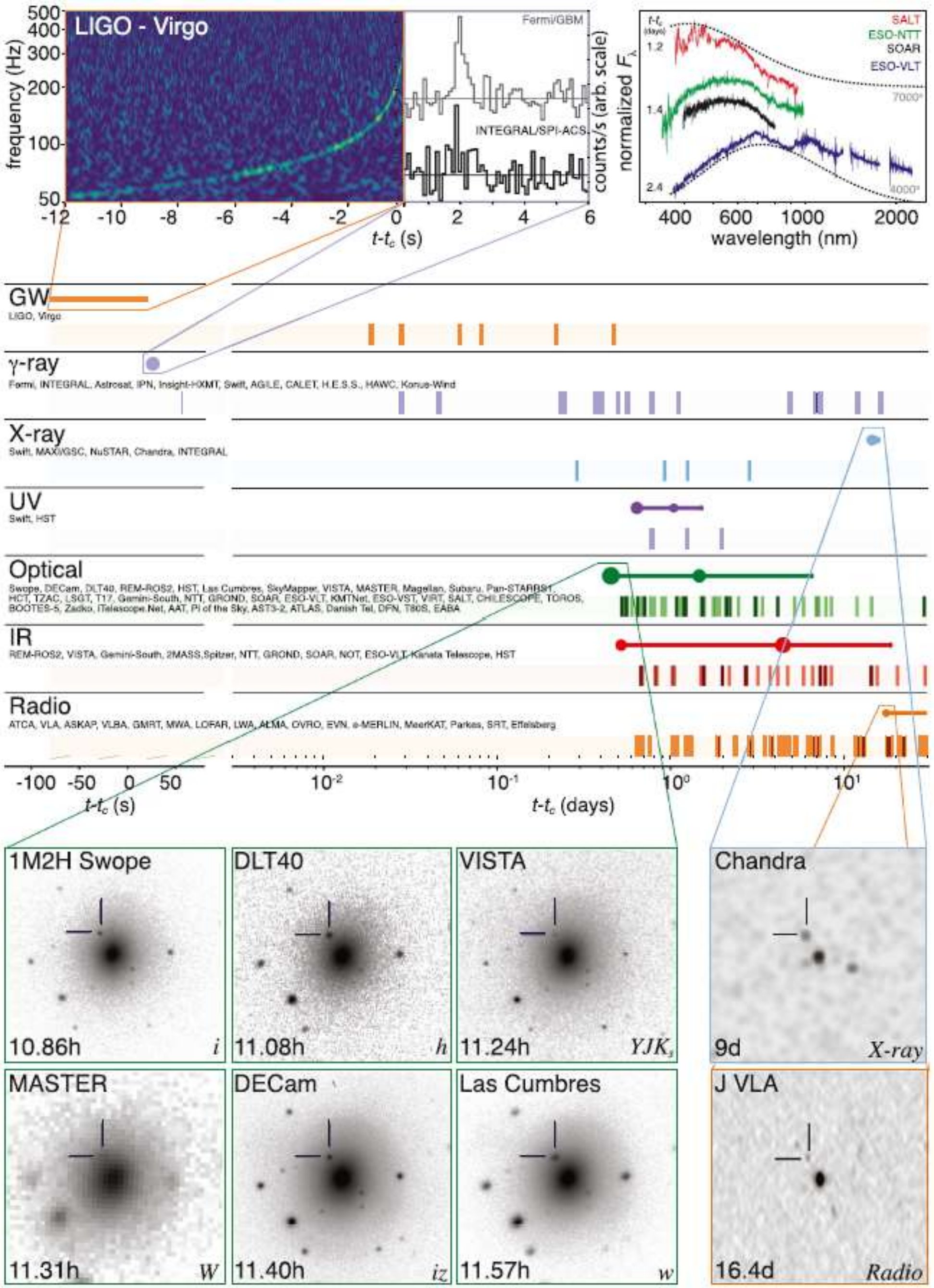
Multi-Messenger Campaigns

GW150914



The Birth of a New Field in Science

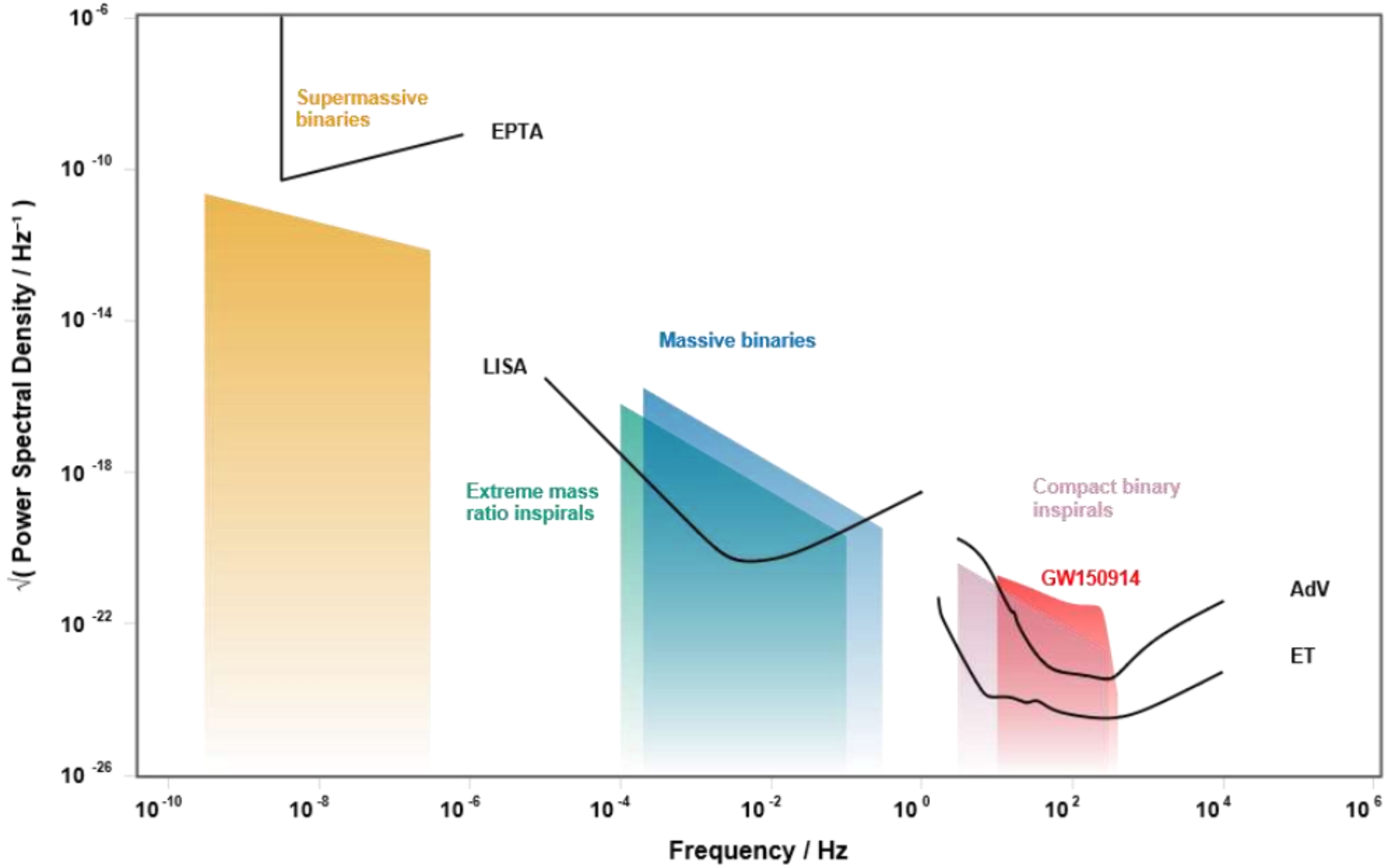




GW170817

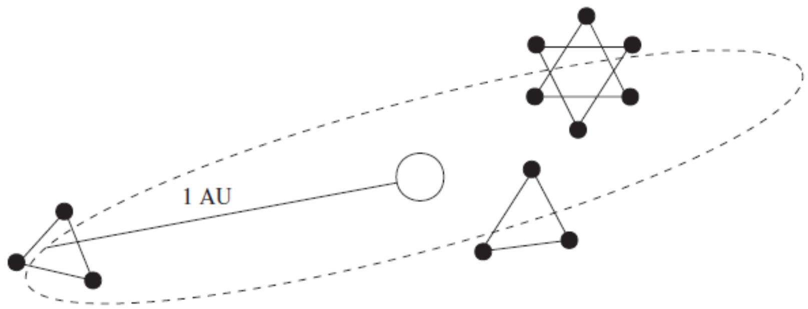
The largest scientific observation campaign in human history.

Observation Bands

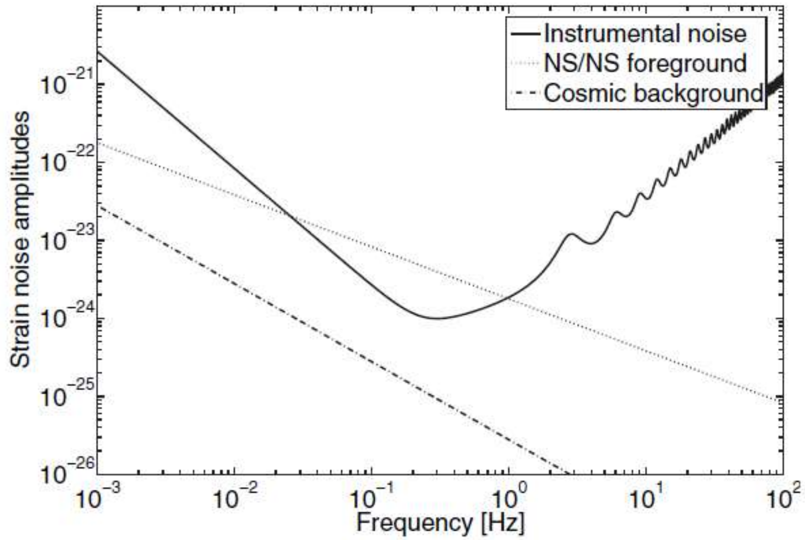


gwplotter.com

The Big Bang Observer



BBO is formed by 4 LISA-type configurations for GW observations at 10mHz – 10Hz.



Distribution of detectors around Sun makes sure that all compact binaries can be detected with high SNR.

The waveforms of all detected binaries need to be subtracted from the data streams.

A pair of collocated detectors is used for the final stochastic GW search.