About the sign of the effective gravitational coupling constant

Ismael Ayuso et al,
Galaxies **7** (2019) 38
[arXiv:1903.07604 [gr-qc]]

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Outline

1. Introduction
   - Coupling between gravity and matter
   - Modified gravity
2. Is possible to obtain a negative coupling?
   - What about a negative G?
   - Generalized Brans-Dicke theory
   - Study of the dynamical system
     - ✓ Without potential
     - ✓ With a quadratic potential
3. Mechanism for a positive gravitational coupling
4. Summary and conclusions

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A brief history about G

- 1687 Newton's work Philosophiæ Naturalis Principia Mathematica is published:

$$F \propto \frac{m_1 m_2}{r^2}$$

- 1798 H. Cavendish measured this proportionality with a torsion balance.
- 1799 P. S. Laplace introduced the constant for the first time as:

$$F = -k^2 \frac{m_1 m_2}{r^2}$$

- 1890 C. V. Boys introduced the modern notation in which appears G
- 1905 Albert Einstein published the General Relativity

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \mathcal{L}_m \Longleftrightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

G has the role of coupling the geometry to the matter and is taken to be positive and constant.

Cosmological Model

GR as gravitational theory + The Universe is considered homogeneous and isotropic → FLRW- metric + Observations

↓ ΛCDM model

- Big Bang
- Cold Dark Matter (CDM) to explain the velocity curves of galaxies and the structure formation
- Dark Energy (DE) related with the Cosmological Constant (CC) to explain the current acceleration of the Universe
- The Universe is (nearly) flat
- There are three epochs dominated by radiation, matter and CC respectively
- Early acceleration to solve the horizon, flatness and monopole problem

Results

Achievements

- GR has passed all precision tests
  - Anomalous perihelion advance of Mercury
  - Gravitational lensing
  - Gravitational time dilation
  - …
- GR predicts gravitational waves
- ΛCDM is able to explain almost all observations until now

Problems

- Singularities in GR
- GR can not be quantized
- CDM has not been detected (directly)
- About the Cosmological Constant:
  - The CC Problem, related with theoretical predictions of its value
  - The Coincidence Problem
  - The Fine-Tuning Problem

P. Bull et al. Phys. Dark Univ. **12** (2016) [arXiv:1512.05356 [astro- ph.CO]]

Attempts to explain the current acceleration of the Universe without a cosmological constant

Introducing a new theory of gravity

Modified gravity

Emanuele Berti et al, Class.Quant.Grav. **32** (2015) 243001 [arXiv:0706.2151 [astro-ph]]

Maintaining GR as gravitational theory:

Changing the structure of the space-time

Introducing inhomogeneities and anisotropies

Backreaction models

Modifying the matter content

Attempts to explain the current acceleration of the Universe without a cosmological constant

Introducing a new theory of gravity

Maintaining GR as gravitational theory:

Modified gravity

A possibility: the gravitational coupling G is not a constant and it evolves.

Changing the structure of the space-time

Modifying the matter content

Introducing inhomogeneities and anisotropies

Backreaction models

G as a dynamical variable

- 1938 Dirac put forward that G may evolve with the Hubble rate (motivated by the disparity between gravitational and electromagnetic forces) → First time of the variation of some fundamental constant.

- 1961 C. H. Brans and R. H. Dicke: 1/G is replaced by a scalar field which can vary from place to place and with time:

$$S = \frac{c^4}{16\pi}\int d^4x\sqrt{-g}\left(\phi R - \frac{\omega}{\phi}\partial_\mu\phi\partial^\mu\phi\right) + \int d^4x\sqrt{-g}\mathcal{L}_m$$

$$G \Longleftrightarrow 1/\phi$$

- The scalar field may be seen as the gravitational permittivity of the space-time.
- This coupling could be negative.

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Generalized Brans-Dicke theory

$$S = \int d^4x\sqrt{-g}\left(\phi R - \frac{\omega\left(\phi\right)}{\phi}\partial_\mu\phi\partial^\mu\phi - 2\phi\lambda(\phi)\right) + S_m$$

$$G \Longleftrightarrow 1/\phi$$

Field equations

$$R_{\alpha\beta} - \frac{1}{2}\,g_{\alpha\beta}\,R + \lambda(\phi)\,g_{\alpha\beta} = \frac{\omega(\phi)}{\phi^2}\left[\phi_{;\alpha}\phi_{;\beta} - \frac{1}{2}\,g_{\alpha\beta}\phi_{;\gamma}\phi^{;\gamma}\right] + \frac{1}{\phi}\,\left[\phi_{;\alpha\beta} - g_{\alpha\beta}\phi_{;\gamma}{}^{;\gamma}\right] + 8\pi\,\frac{T_{\alpha\beta}}{\phi}$$

$$\Box\phi - \frac{2\phi^2\lambda'(\phi) - 2\phi\lambda(\phi)}{2\omega(\phi)+3} = \frac{1}{2\omega(\phi)+3}\left[8\pi\,T - \omega'(\phi)\phi_{;\gamma}\phi^{;\gamma}\right]$$

Generalized Brans-Dicke theory

$$S = \int d^4x\sqrt{-g}\left(\phi R - \frac{\omega\left(\phi\right)}{\phi}\partial_\mu\phi\partial^\mu\phi - 2\phi\lambda(\phi)\right) + S_m$$

$$G \Longleftrightarrow 1/\phi$$

Field equations for FLRW

$$3\left(\frac{\dot{a}}{a}\right)^2 + 3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + 3\frac{k}{a^2} = \lambda(\phi) + \frac{\omega(\phi)}{2}\frac{\dot{\phi}^2}{\phi^2} + 8\pi\frac{\rho}{\phi}$$

$$2\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{a}}{a}\right) + 3\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{k}{a^2} = \lambda(\phi) - \frac{\omega(\phi)}{2}\frac{\dot{\phi}^2}{\phi^2} - 8\pi\frac{p}{\phi} - \frac{\ddot{\phi}}{\phi}$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{2\phi^2\lambda'(\phi) - 2\phi\lambda(\phi)}{2\omega(\phi)+3} = -\frac{1}{2\omega(\phi)+3}\left[8\pi(3p-\rho) + \omega'(\phi)\dot{\phi}^2\right]$$

Study of the dynamical system

$$S = \int d^4x\sqrt{-g}\left(\phi R - \frac{\omega\left(\phi\right)}{\phi}\partial_\mu\phi\partial^\mu\phi - 2\phi\lambda(\phi)\right) + S_m$$

We are going to introduce the redefined variables:

$$X = \frac{\phi}{\phi_0}\,a^2 \qquad\qquad Y' = \sqrt{\frac{2\omega(\phi)+3}{3}}\,\frac{\phi'}{\phi}$$

where

- we have used conformal time: $d\eta = dt/a$
- X preserves the sign of ϕ i.e.

$$X < 0 \quad \textit{when} \quad \phi < 0 \quad \textit{or when} \quad G < 0$$

this allows us to extend the study of the dynamics into the region with a negative coupling

Study of the dynamical system

$$S=\int d^4x\sqrt{-g}\left(\phi R-\frac{\omega\left(\phi\right)}{\phi}\partial_\mu\phi\partial^\mu\phi-2\phi\lambda(\phi)\right)+S_m$$

We are going to introduce the redefined variables:

$$X=\frac{\phi}{\phi_0}\,a^2 \qquad\qquad Y'=\sqrt{\frac{2\omega(\phi)+3}{3}}\,\frac{\phi'}{\phi}$$

The FLRW equations are then:

$$(X')^2+4\,k\,X^2-(Y'\,X)^2=4M\;X\left(\frac{X}{\phi}\right)^{\frac{4-3\gamma}{2}}+\frac{4}{3}\left(\frac{\lambda(\phi)}{\phi}\right)X^3$$

$$[Y'\,X]'=M(4-3\gamma)\sqrt{\frac{3}{2\omega+3}}\left(\frac{X}{\phi}\right)^{\frac{4-3\gamma}{2}}-\frac{2\,X^2}{\sqrt{2\omega(\phi)+3}}\left(\frac{\mathrm{d}\lambda}{\mathrm{d}\phi}-\frac{\lambda(\phi)}{\phi}\right)$$

$$X''+4\,k\;X=3\,M(2-\gamma)\left(\frac{X}{\phi}\right)^{\frac{4-3\gamma}{2}}+2X^2\left(\frac{\lambda(\phi)}{\phi}\right)$$

Study of the dynamical system

$$S = \int d^4x\sqrt{-g}\left(\phi R - \frac{\omega\left(\phi\right)}{\phi}\partial_\mu\phi\partial^\mu\phi - 2\phi\lambda(\phi)\right) + S_m$$

We are going to introduce the redefined variables:

$$X = \frac{\phi}{\phi_0}\,a^2 \qquad\qquad Y' = \sqrt{\frac{2\omega(\phi)+3}{3}}\,\frac{\phi'}{\phi}$$

The FLRW equations for a quadratic potential $\lambda(\phi) = \lambda_0\phi$

$$(X')^2 + 4\,k\,X^2 - (Y'\,X)^2 = 4M\;X\;\left(\frac{X}{\phi}\right)^{\frac{4-3\gamma}{2}} + \frac{4}{3}\,\lambda_0\,X^3$$

$$[Y'\,X]' = M(4-3\gamma)\,\sqrt{\frac{3}{2\omega+3}}\;\left(\frac{X}{\phi}\right)^{\frac{4-3\gamma}{2}}$$

$$X'' + 4\,k\;X = 3\,M(2-\gamma)\;\left(\frac{X}{\phi}\right)^{\frac{4-3\gamma}{2}} + 2\lambda_0\,X^2$$

Study of the dynamical system

$$S = \int d^4x\sqrt{-g}\left(\phi R - \frac{\omega\left(\phi\right)}{\phi}\partial_\mu\phi\partial^\mu\phi - 2\phi\lambda(\phi)\right) + S_m$$

$$X = \frac{\phi}{\phi_0}\,a^2$$

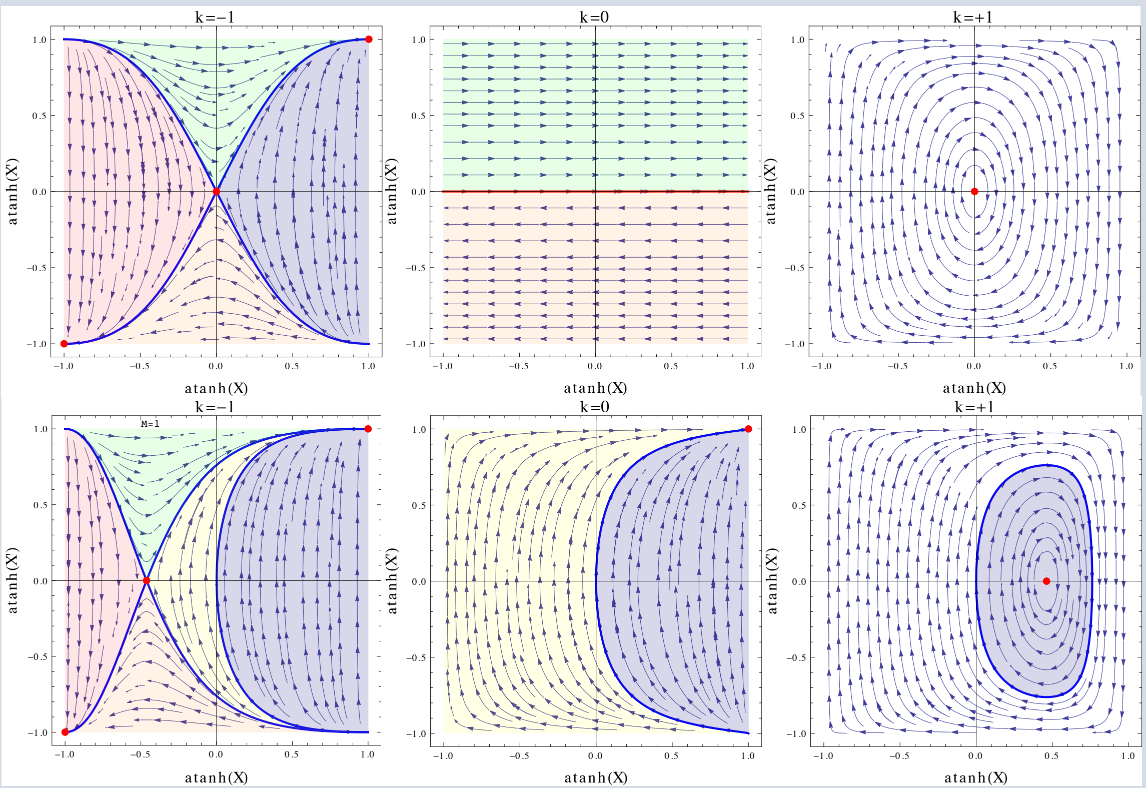
$$X'' + 4\,k\;X = 3\,M(2-\gamma)\;\left(\frac{X}{\phi}\right)^{\frac{4-3\gamma}{2}} + 2\lambda_0\,X^2$$

Without potential $\lambda_0 = 0$		With potential $\lambda_0 \neq 0$	
Vacuum and still fluid $M=0$ $\gamma=2$	Radiation $\gamma = 4/3$	Vacuum and still fluid $M=0$ $\gamma=2$	Radiation $\gamma = 4/3$
$X' = W$ $W' = -4kX$	$X' = W$ $W' = 2M - 4kX$	$X' = W$ $W' = 2\lambda_0 X^2 - 4kX$	$X' = W$ $W' = 2M + 2\lambda_0 X^2 - 4kX$

Study of the dynamical system without potential

$$S = \int d^4x\sqrt{-g}\left(\phi R - \frac{\omega(\phi)}{\phi}\partial_\mu\phi\partial^\mu\phi - 2\phi\lambda(\phi)\right) + S_m$$

$$X = \frac{\phi}{\phi_0}\,a^2 \qquad X'' + 4\,k\,X = 3\,M(2-\gamma)\left(\frac{X}{\phi}\right)^{\frac{4-3\gamma}{2}} + 2\lambda_0 X^2$$



Vacuum $M = 0$ and still fluid $\gamma = 2$

$$X' = W$$
$$W' = -4kX$$

Radiation $\gamma = 4/3$

$$X' = W$$
$$W' = 2M - 4kX$$

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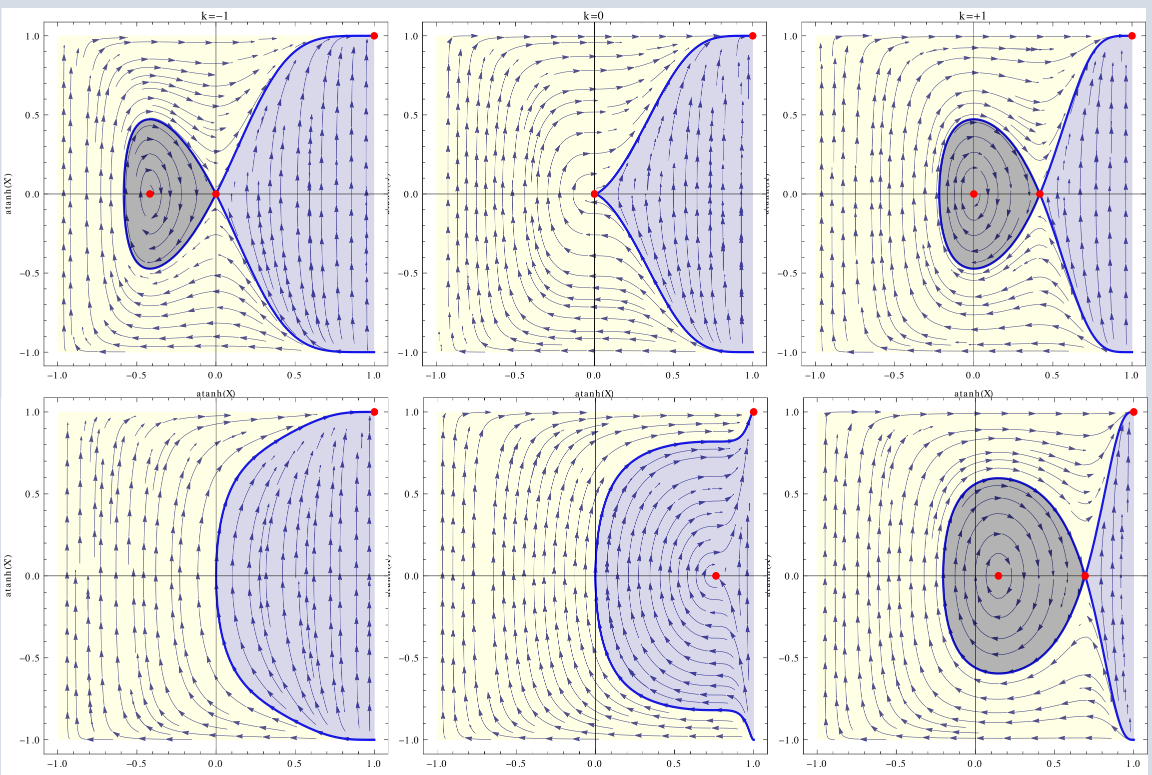
Study of the dynamical system with a quadratic potential

$$S=\int d^4x\sqrt{-g}\left(\phi R-\frac{\omega\left(\phi\right)}{\phi}\partial_\mu\phi\partial^\mu\phi-2\phi\lambda(\phi)\right)+S_m$$

$$X=\frac{\phi}{\phi_0}\,a^2 \qquad\qquad X''+4\,k\,X=3\,M(2-\gamma)\,\left(\frac{X}{\phi}\right)^{\frac{4-3\gamma}{2}}+2\lambda_0\,X^2$$



Vacuum $M=0$ and still fluid $\gamma=2$

$$X'=W$$
$$W'=2\lambda_0X^2-4kX$$

Radiation $\gamma=4/3$

$$X'=W$$
$$W'=2M+2\lambda_0X^2-4kX$$

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Summary and conclusions

- We have investigated a cosmological mechanism that induces the value of the gravitational effective coupling "constant" to be positive in the framework of scalar-tensor theories with and without a cosmological potential.

- In the absence of the cosmological potential, the presence of matter or radiation favours a positive value of the gravitational "constant", when the evolution enters a phase of matter domination.

- However, it is when a quadratic cosmological potential is present that an attracting mechanism towards a positive value of the gravitational running "constant" becomes manifest.

Thank you!

Questions