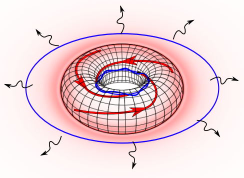
Light ring stability in ultra-compact objects

Pedro Cunha

Physics Department of the University of Aveiro and CIDMA
IST - University of Lisbon, Portugal



Phys. Rev. Lett. **119** 251102,

P. Cunha, E. Berti and C. Herdeiro

What is the nature of Black Hole candidates?

Multiple *uniqueness theorems* establish that equilibrium vacuum BHs of General Relativity (GR) are described by the *Kerr solution* (Israel 1967, Carter 1973, Robinson 1975).

Are astrophysical BHs really described by Kerr?

What is the nature of Black Hole candidates?

Multiple *uniqueness theorems* establish that equilibrium vacuum BHs of General Relativity (GR) are described by the *Kerr solution* (Israel 1967, Carter 1973, Robinson 1975).

Are astrophysical BHs really described by Kerr?

Strong gravity has entered the precision era:

- breakthroughs in gravitational wave astrophysics. LIGO/Virgo, PRL 116, 061102 (2016)
- unveiling of the first black hole (BH) shadow image. EHT, AJ 875 L1 (2019)

To what extent alternative models *could mimic* the Kerr phenomenology?

Event Horizon Telescope

First image (ever) of a BH candidate released on April 10th 2019.

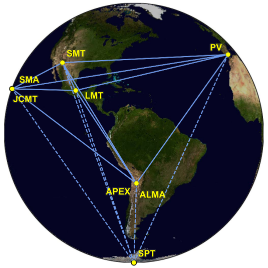
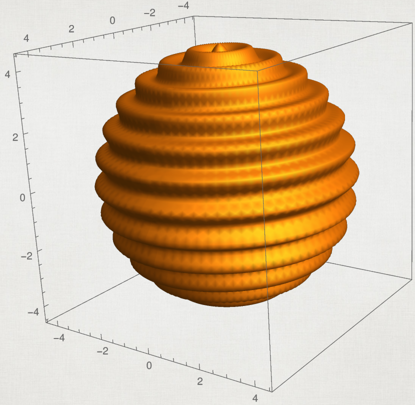


Figure 1. Eight stations of the EHT 2017 campaign over six geographic locations as viewed from the equatorial plane. Solid baselines represent mutual visibility on M87* (+12° declination). The dashed baselines were used for the calibration source 3C279 (see Papers III and IV).

observed image M87* (April 2017) (adapted EHT, AJ 875 L1 (2019))

- Use of global Very Long Baseline Interferometry array at wavelength 1.3 mm.
- Array created an effective telescope with the size of the Earth.
- Consistent with expectations for BH image as predicted by General Relativity.

Image compact star



What is the *expectation* for a BH image? Consider generic academic setup:

- Spherical compact star of mass M and radius R, with some surface texture.
- Take exterior of the star as vacuum $\rightarrow$ described by Schwarzschild solution.

Image compact star

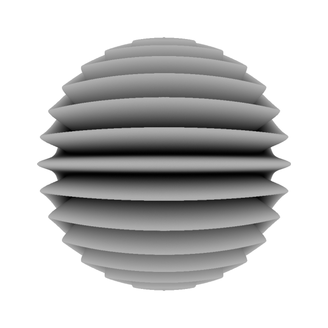


Image of the star ($R > 3M$)

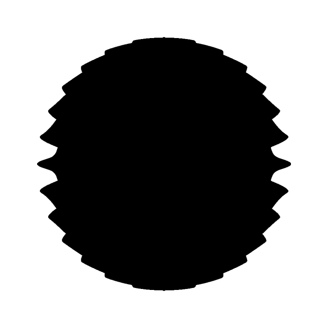


Image of the star (no emission)

We can generate a synthetic star image (left) via numerical ray-tracing.

- For $R > 3M$, the star image is a good measure of surface information.
- Even for *radiation absorbent* surface (right), star outline reveals texture.

PRD 97 no.8, 084020

Image compact star

Image of the star ($R < 3M$)

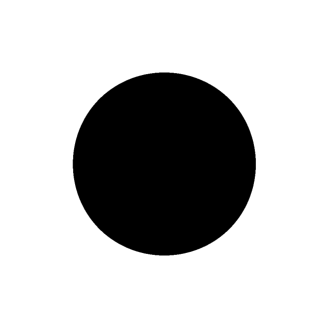


Image of the star (no emission)

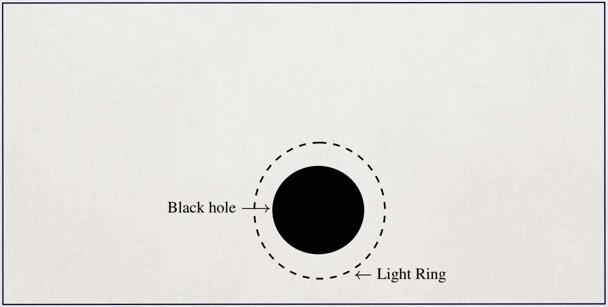
- For $R < 3M$, the star's edge becomes circular (no surface texture display).
- A radiation absorbent star is indistinguishable from BH $\rightarrow$ BH *shadow*.
- The edge is actually an image of the *Light Ring* orbit ($r = 3M$).

PRD 97 no.8, 084020

Light Ring

A scattering light ray around a BH can:

- *Escape* to infinity, *fall* into the BH, or *approach* a bound state orbit.

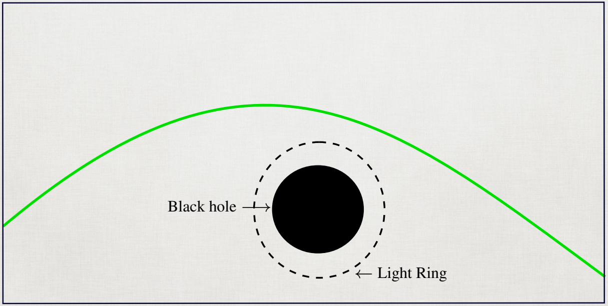


In spherical symmetry, the bound orbit is the *Light Ring* (LR), *a.k.a.* “photon sphere”: a *planar* photon orbit that encircles the BH forever.

Light Ring

A scattering light ray around a BH can:

- *Escape* to infinity, *fall* into the BH, or *approach* a bound state orbit.

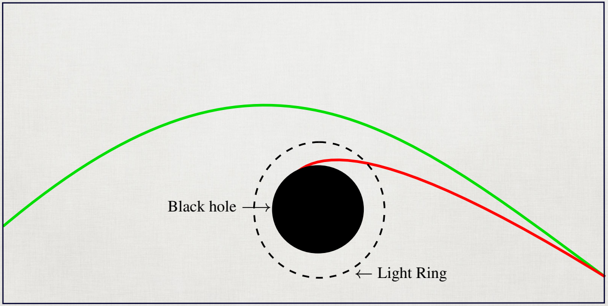


In spherical symmetry, the bound orbit is the *Light Ring* (LR), *a.k.a.* "photon sphere": a *planar* photon orbit that encircles the BH forever.

Light Ring

A scattering light ray around a BH can:

- *Escape* to infinity, *fall* into the BH, or *approach* a bound state orbit.

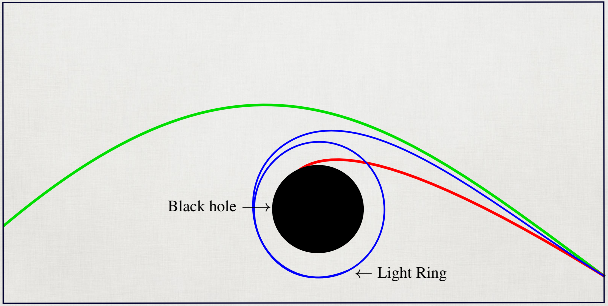


In spherical symmetry, the bound orbit is the *Light Ring* (LR), *a.k.a.* “photon sphere”: a *planar* photon orbit that encircles the BH forever.

Light Ring

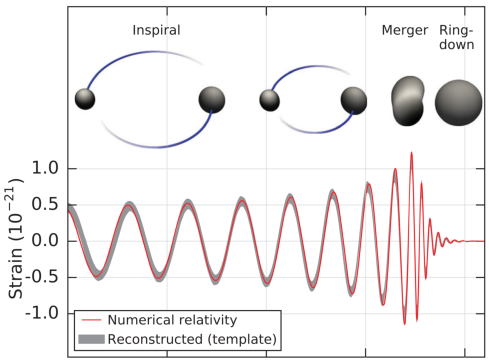
A scattering light ray around a BH can:

- *Escape* to infinity, *fall* into the BH, or *approach* a bound state orbit.



In spherical symmetry, the bound orbit is the *Light Ring* (LR), *a.k.a.* "photon sphere": a *planar* photon orbit that encircles the BH forever.

LIGO events

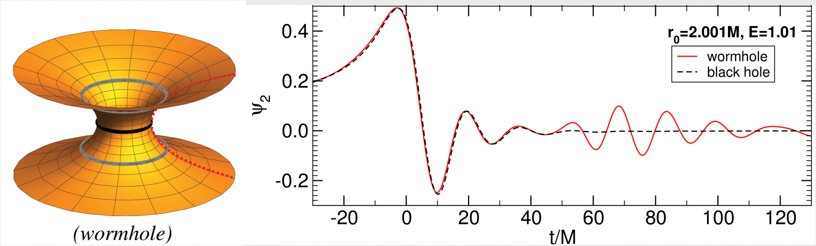


PRL 116, 061102 (2016)

- The LIGO collaboration has detected multiple GW events.
- Several events are consistent with merger of a Kerr BH binary.
- The GW waveform is divided in: *inspiral*, *merger*, and *ringdown*.

Ringdown as an horizon probe?

From Cardoso+2016:

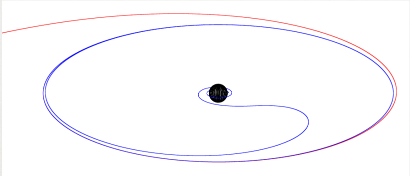


(wormhole)

source: PRL**117**, 089902

- It is claimed GW ringdown has *signature* of the Light Ring (LR).
- Case study: wormhole (not a BH) has an unstable LR.
- When perturbed, wormhole vibrates like a BH (initially).
- In principle, it could mimic a BH ringdown...

Light Rings in astrophysics



- Light Rings have important **astrophysical signatures**.
- *Electromagnetic channel* $\rightarrow$ shadow, *GW channel* $\rightarrow$ BH ringdown.
- An alternative Ultra-Compact Object (UCO), *i.e.* with a LR, could mimic Kerr!

Alternatives to Kerr

What could be an alternative to Kerr BHs?

- BHs in GR with matter fields, *e.g.* BHs with synchronized hair PRL 112, 221101 (2014)
- BHs in alternative gravity theories, *e.g.* Einstein-dilaton-Gauss-Bonnet PRD 93 (2016) 044047
- Compact objects with no horizon, *e.g.* gravastars, Boson/Proca stars PRD 96 (2017) no.10, 104040

Alternatives to Kerr

What could be an alternative to Kerr BHs?

- BHs in GR with matter fields, *e.g.* BHs with synchronized hair PRL 112, 221101 (2014)
- BHs in alternative gravity theories, *e.g.* Einstein-dilaton-Gauss-Bonnet PRD 93 (2016) 044047
- Compact objects with no horizon, *e.g.* gravastars, Boson/Proca stars PRD 96 (2017) no.10, 104040

What are the criteria for a *viable* alternative object?

- Arise in a consistent and well motivated (effective field) theory of gravity.
- Have a dynamical formation mechanism.
- Be (sufficiently) stable.

Focus: spherical horizonless objects

Consider a spherical *horizonless* object. The radial motion of light rays is 1D:

$$g_{rr}\dot{r}^2 + V(r) = 0, \qquad V(r) = \frac{E^2}{g_{tt}} + \frac{L^2}{r^2}$$

E is photon's energy and L its angular momemtum.

Conditions for a Light Ring: $V = 0$ and $V'(r) = 0$.

Focus: spherical horizonless objects

Consider a spherical *horizonless* object. The radial motion of light rays is 1D:

$$g_{rr}\dot{r}^2 + V(r) = 0, \qquad V(r) = \frac{E^2}{g_{tt}} + \frac{L^2}{r^2}$$

E is photon's energy and L its angular momemtum.

Conditions for a Light Ring: $V = 0$ and $V'(r) = 0$.

Smooth deformation of metric fixing:

- asymptotic behavior (asymptotic flatness), *i.e.* $g_{tt} \to -1$.
- near origin behavior (smoothness), *i.e.* $g_{tt} \neq 0$.

$\Longrightarrow$ Light Rings are created in pairs.

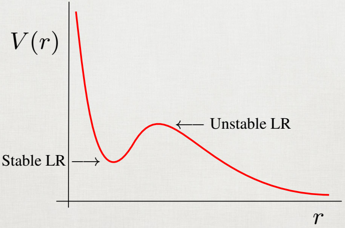
Focus: spherical horizonless objects

Consider a spherical *horizonless* object. The radial motion of light rays is 1D:

$$g_{rr}\dot{r}^2 + V(r) = 0, \qquad V(r) = \frac{E^2}{g_{tt}} + \frac{L^2}{r^2}$$

E is photon's energy and L its angular momemtum.

Conditions for a Light Ring: $V = 0$ and $V'(r) = 0$.



Smooth deformation of metric fixing:

- asymptotic behavior (asymptotic flatness), *i.e.* $g_{tt} \to -1$.
- near origin behavior (smoothness), *i.e.* $g_{tt} \neq 0$.

$\Longrightarrow$ Light Rings are created in pairs.

Non-linear instability

Paper by Keir: (J. Keir, CQG 33 (2016) no.13, 135009); Benomio arXiv:1809.07795

- Scalar linear waves ϕ can be treated as a model for nonlinear perturbations:

$$\Box_g \phi + F(r)\,\phi = 0, \qquad (arbitrary\ \ F(r) > 0)$$

- In proving *non-linear stability*, one usually requires uniform fast decay:

$$\mathcal{E}[\phi](t) \lesssim \frac{1}{t^2}\,\mathcal{E}[\phi](0), \qquad (\textit{polynomial decay})$$

where $\mathcal{E}[\phi](t)$ is integrated "energy" of wave ϕ across hypersurface Σ_t.

Non-linear instability

Paper by Keir: (J. Keir, CQG 33 (2016) no.13, 135009); Benomio arXiv:1809.07795

- Scalar linear waves ϕ can be treated as a model for nonlinear perturbations:

$$\Box_g \phi + F(r)\,\phi = 0, \qquad (\textit{arbitrary}\ \ F(r) > 0)$$

- In proving *non-linear stability*, one usually requires uniform fast decay:

$$\mathcal{E}[\phi](t) \lesssim \frac{1}{t^2}\,\mathcal{E}[\phi](0), \qquad (\textit{polynomial decay})$$

where $\mathcal{E}[\phi](t)$ is integrated "energy" of wave ϕ across hypersurface Σ_t.

- If *stable* Light Ring exists, lower bound of uniform decay rate is slow:

$$\mathcal{E}[\phi](t) \lesssim \frac{1}{[\log(2+t)]^2}\,\mathcal{E}[\phi](0), \qquad (\textit{logarithmic decay})$$

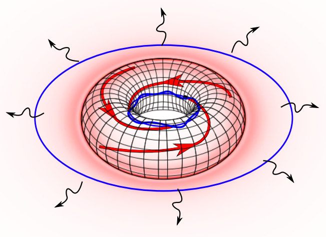
- This slow decay is highly suggestive of a *non-linear instability*.

Implications

If one has an horizonless (spherical) star that would mimic the BH observations:

- It has an unstable Light Ring $\Longrightarrow$ *stable* LR also exists.
- The latter could trigger a non-linear *spacetime instability*.
- Is this a feature restricted to spherical symmetry? No, it can be generalized!

Outline of the argument



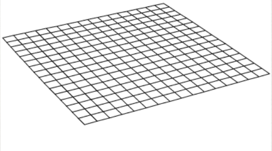
$\Longrightarrow$ horizonless UCOs might be *unstable*, within generic conditions.

- Reasonable assumptions, *e.g.* smoothness, causality and axial symmetry.
- *Topological argument*: LRs come in pairs $\rightarrow$ one is *stable* (NEC satisfied).
- Stable LR traps radiation $\rightarrow$ destabilizes object.

PRL 119 (2017) no.25, 251102

Assumptions for the spacetime

$\simeq$ flat spacetime



$\longrightarrow$

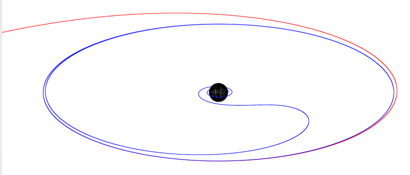
We assume:

- dynamical formation from gravitational collapse.
- initial flat spacetime and causality $\Longrightarrow$ topological triviality (Geroch).

- UCO is stationary, axially-symmetric and asymptotically flat.
- There is **no** event horizon; $\mathbb{Z}_2$ reflection symmetry **not** required.
- The metric is *smooth*.

Geroch J.Math.Phys. 8, 782 (1967)

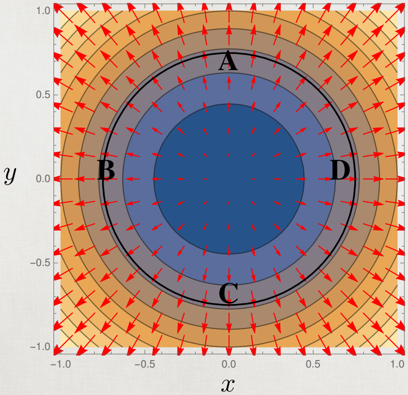
Light rings

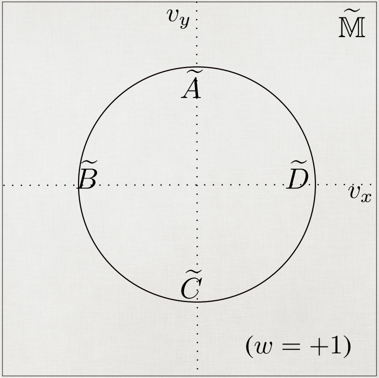


- *Light ring* (LR) is a planar FPO $\to$ it is tangent to Killing vectors $\partial_t, \partial_\varphi$.
- It is possible to assign a **topological charge** to a LR.
- We can introduce 2D effective potential $U(r, \theta)$.
- Along trajectory $p_r = p_\theta = U = 0$ and $\dot{p}_\mu = 0$.
- $2\,\dot{p}_\mu = -\partial_\mu U + \mathcal{O}(p_r,\, p_\theta)$.

At a LR: $\Longrightarrow$ $\boxed{U = \nabla U = 0}$

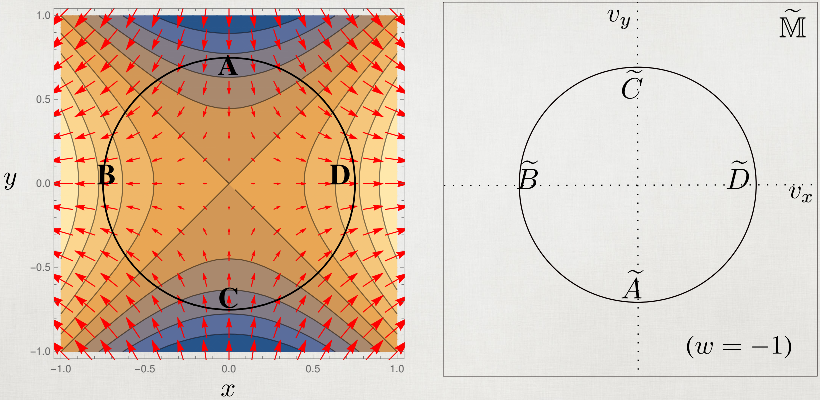
Winding number





- Consider a closed 2D contour with a 2D field (v_x, v_y), *e.g.* a gradient.
- Each point of contour is mapped to an auxiliary space $\widetilde{\mathbb{M}} : (v_x, v_y)$.
- The winding number of new curve around origin is a topological quantity w.

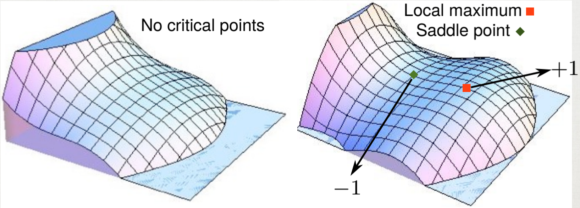
Winding number



- Consider a closed 2D contour with a 2D field (v_x, v_y), *e.g.* a gradient.
- Each point of contour is mapped to an auxiliary space $\widetilde{\mathbb{M}} : (v_x, v_y)$.
- The winding number of new curve around origin is a topological quantity w.

Light Rings are created in pairs

Effective Potential H change

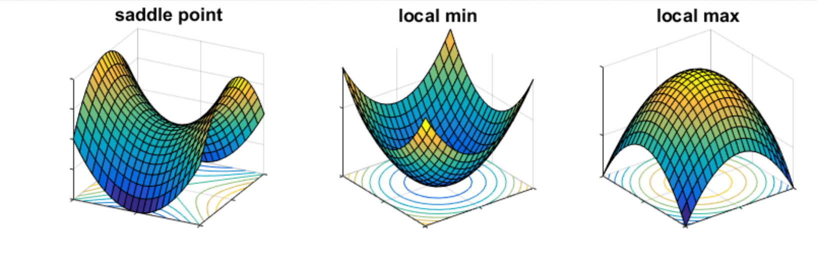


$w = 0$ $w = 0$

Smooth deformation of spacetime (fixed asymptotics):

- The potential H is smoothly deformed.
- *Total* w within curve C is a (topological) constant.
- Light Rings are created in pairs as combinations $w = \{+1, -1\}$.

Light Ring types



$w = -1$ $\quad$ $w = +1$ $\quad$ $w = +1$

Different types of Light Rings:

- Saddle point of U $\rightarrow$ unstable LR $(w = -1)$ $\rightarrow$ GW ringdown (Kerr).
- Local minimum of U $\rightarrow$ stable LR $(w = +1)$ $\rightarrow$ possible spacetime instability.
- Local maximum of U $\rightarrow$ unstable LR $(w = +1)$ $\rightarrow$ violates Null Energy Cond.

Conclusions

Under generic and reasonable physical conditions:

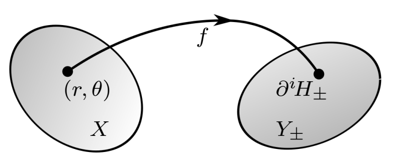
- Light Rings can be associated to a *topological charge*.
- LRs are created in pairs.
- BH mimickers must have a stable LR, if the Null Energy Condition is satisfied.

- BH mimickers are potentially *unstable*.
- The observed BH candidates should really be BHs.

Acknowledgements

- Work is supported by the FCT IDPASC Portugal Ph.D. Grant No. PD/BD/114071/2015 and partially supported by the H2020-MSCA-RISE2015 Grant No. StronGrHEP-690904, the H2020-MSCA-RISE-2017 Grant No. FunFiCO-777740 and by the CIDMA project UID/MAT/04106/2019.

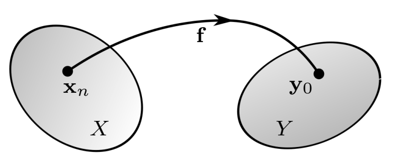
$\nabla H_{\pm}$ as a map



Consider $\nabla H_{\pm}$ as a *map* $f : X \to Y_{\pm}$:

- X is a compact, simply connected 2D region parametrized by (r, θ).
- $Y_{\pm}$ is a 2D space parametrized by the components $\partial^i H_{\pm}, \quad i \in \{r, \theta\}$.
- LR $\to$ origin of $Y_{\pm}$.

Brouwer degree (topology)



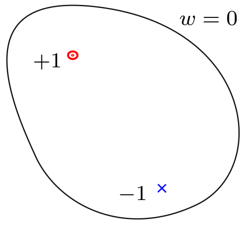
Consider a smooth map $\mathbf{f} : X \to Y$

- take a regular value $\mathbf{y}_0 \in Y$ with finite solutions to $\mathbf{f}(\mathbf{x}_n) = \mathbf{y}_0$.
- the Jacobian $J_n = \det(\partial \mathbf{f} / \partial \mathbf{x}_n) \neq 0$ is computed at each $\mathbf{x}_n$.

The Brouwer degree of $\mathbf{f}$ is: $w = \sum_n \mathrm{sign}(J_n)$.

- It is independent on the choice $\mathbf{y}_0$.
- It is invariant under homotopies (continuous deformations of the map).

Brouwer degree (topology)



Each critical point $\nabla H_{\pm} = 0$:

- is assigned a *topological* charge w.
- sign w depends on the Jacobian $J_n = |\partial^2 H_{\pm}/\partial^2 \mathbf{x}_n|$.

Charge of a critical point:

- maximum/minimum $\implies$ $w = +1$.
- saddle point $\implies$ $w = -1$.

Relation to NEC

- the NEC can be violated at some point other than a LR.

$$\text{Exotic LR} \implies \text{NEC violation}$$

$$\text{NEC violation} \not\Longrightarrow \text{Exotic LR}$$

- Stable and exotic LRs are not possible in vacuum.

$$\boxed{T^{\mu\nu}\, p_\mu\, p_\nu = \frac{1}{16\pi}\partial_i \partial^i U.}$$