Critical phenomena: A semi-linear wave model

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Motivation

Weak Cosmic Censorship Conjecture

Generically, there is global existance outside a black hole.

There is no hope, right now, to prove it in general for General Relativity. → **Counterexamples**

Critical phenomena in General Relativity

Discovered by M. Choptuik in 1993 [1] in simulations of spherically symmetric collapse of a scalar field.

We consider one-parameter families of initial data that takes a critical value ***a**** and divides the solutions space between th

Solutions near the critical point show critical behaviour: Universality, power-law mass scaling of the BH and self-similar be

Self-Similarity of the critical solution

Type of scale-invariant symmetry.

The critical solution in GR shows Discrete Self-Similarity.

There is no known simple model that has DSS threshold solutions [2].

[1] Phys. Rev. Lett. 70, 9
[2] arXiv:1602.08059

Self-similarity and scale-invariance

Continuos self-similarity (CSS)

Continuos set of λ

Scale-invariance

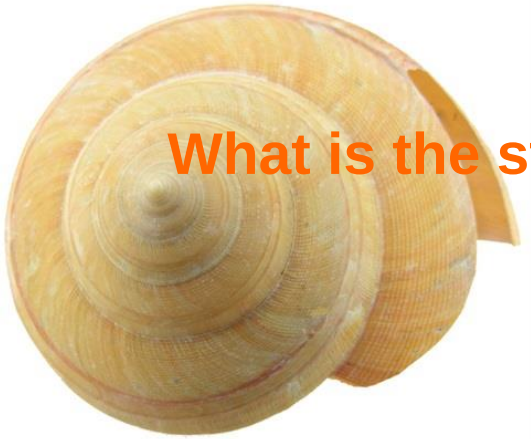
$$f(\lambda x) = \lambda^{\Delta} f(x)$$

$$\Delta = cte$$

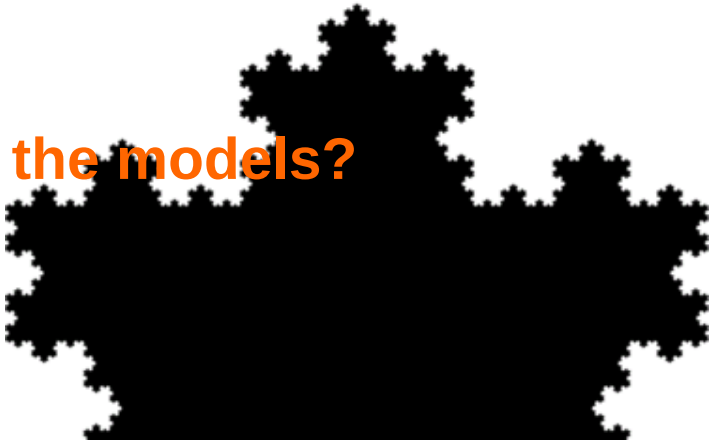
Discrete self-similarity (DSS)

Discrete set of λ

What is the structural difference behind the models?



Seashell as example of CSS
(allowing rotations)



Koch curve as example of infinite DSS

Semi-linear wave equations

Wave equation

$$\Box\varphi = 0$$

Solution in one dimension

$$\varphi = \frac{f(t+r) - f(t-r)}{r}$$

Deformation

$$\phi = D[\varphi]$$

Working...

$$\Box\varphi(\phi) = 0$$

What do we need?

- A solution to the wave equation
- A deformation function
- Some mathematical work
- We work in one dimension

Semilinear wave equation

$$\Box\phi - \frac{D'[\varphi]}{D''[\varphi]}\partial_\alpha\phi\partial^\alpha\phi = 0$$

What do we get?

- Semilinear wave equation
- Analytical solution
- Self-similar solutions
- Same structure as GR

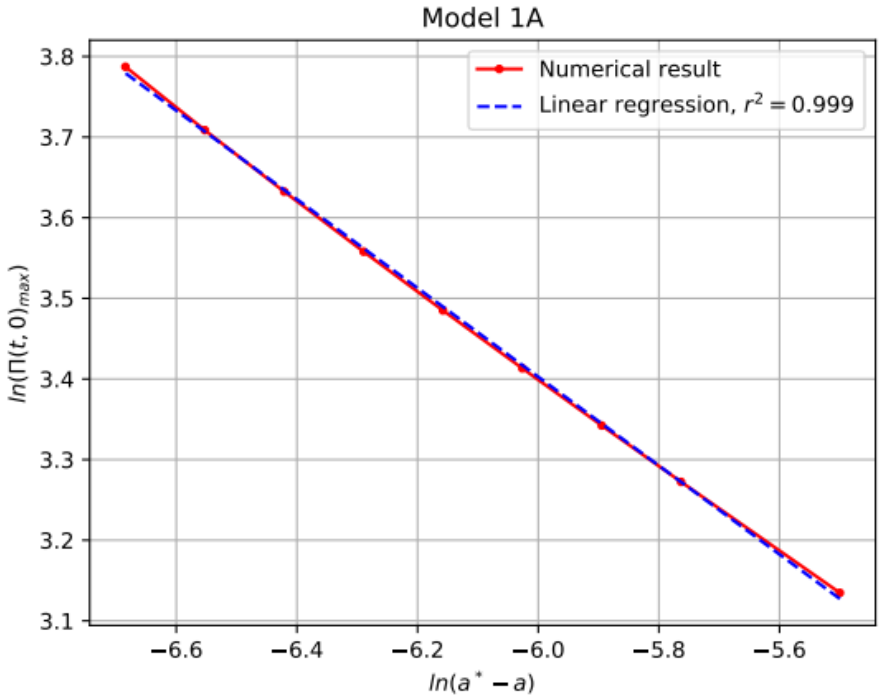
Model 1 – Results

The model 1

$$\Box\phi - \partial_\alpha\phi\partial^\alpha\phi = 0$$

Analytical solution

$$\phi = -\ln(\varphi + 1)$$



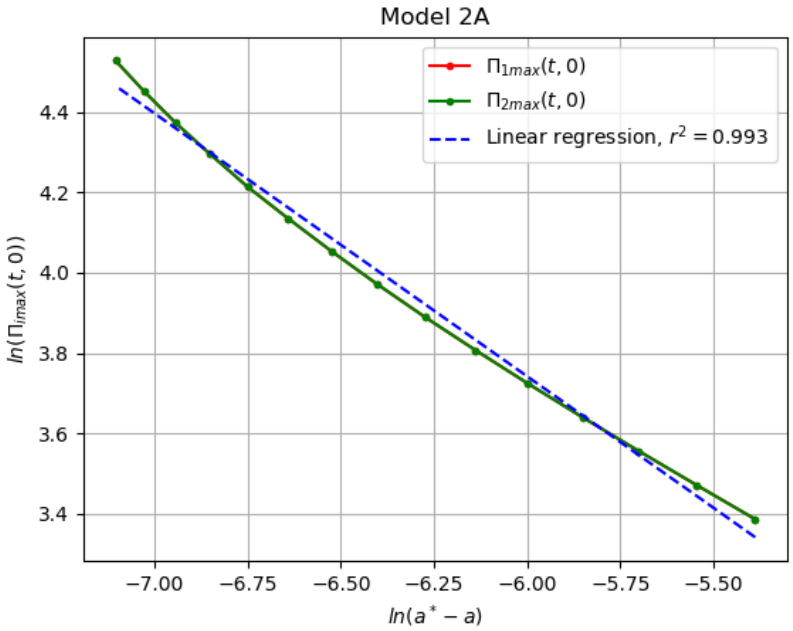
What do we get?

- Analytical solution to a non-linear equation
- Global solutions for small data
- CSS threshold solutions

Model 2 - Universality

The model 2

$$\Box\phi_1 - \partial_\alpha\phi_2\partial^\alpha\phi_2 = 0$$
$$\Box\phi_2 - \partial_\alpha\phi_1\partial^\alpha\phi_1 = 0$$



What do we get?

- This model does not come from deformation of the wave e
- Global solutions for small data
- CSS threshold solutions
- Universal behaviour near the
- critical point
- Solutions of model 1 are embedded in its solution space.
- The threshold solution **always** seems to correspond to the

Towards DSS

Deformation

$$\phi = D[\varphi]$$

Working...

$$\Box\varphi(\phi) = 0$$

Defining

$$D[\varphi] \equiv P \circ C[\varphi]$$

What do we need?

In a DSS solution it is the **derivative** of the field what blows up.

Our massless scalar field itself has to be **bounded**.

Careful definition of the deformation function

Where

P: Bounded periodic function of period Δ.

C: Compactifying function.

Will we have DSS now?

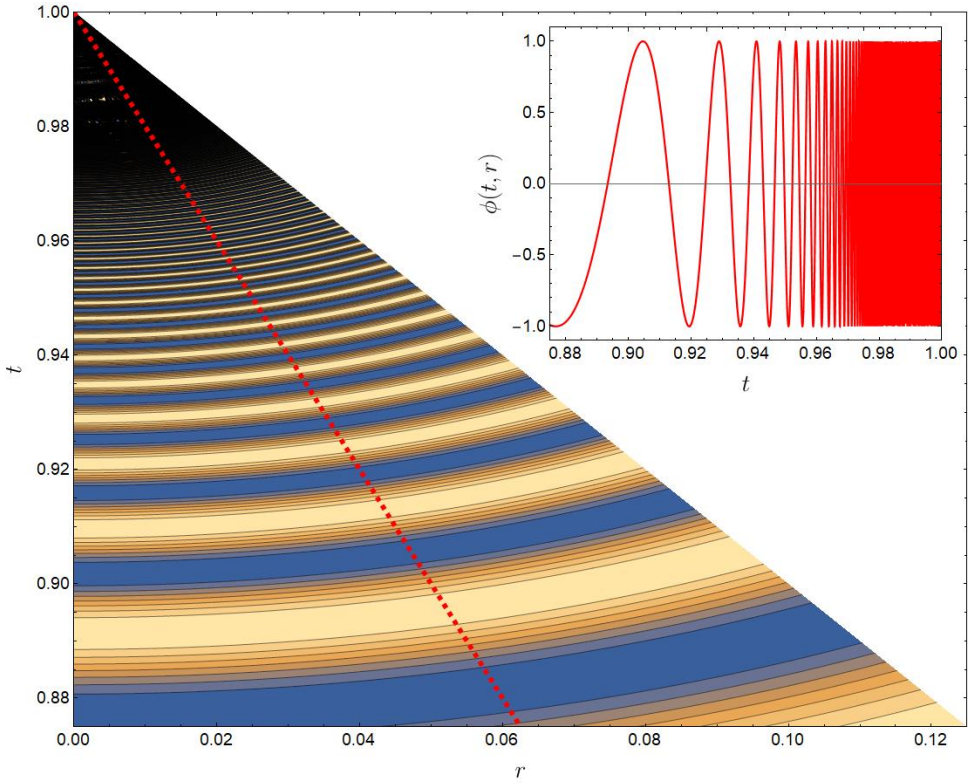
Model 3 – Discrete self-similarity

The model 3

$$\Box\phi - \frac{\phi + \sqrt{1-\phi^2}}{\phi^2 - 1}\left[\partial_\alpha \phi \partial^\alpha \phi\right] = 0$$

Analytical solution

$$\phi = \sin(\ln(\varphi + 1))$$



We do have DSS!

Model 3 – Discrete self-similarity

The model 3

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Analytical solution

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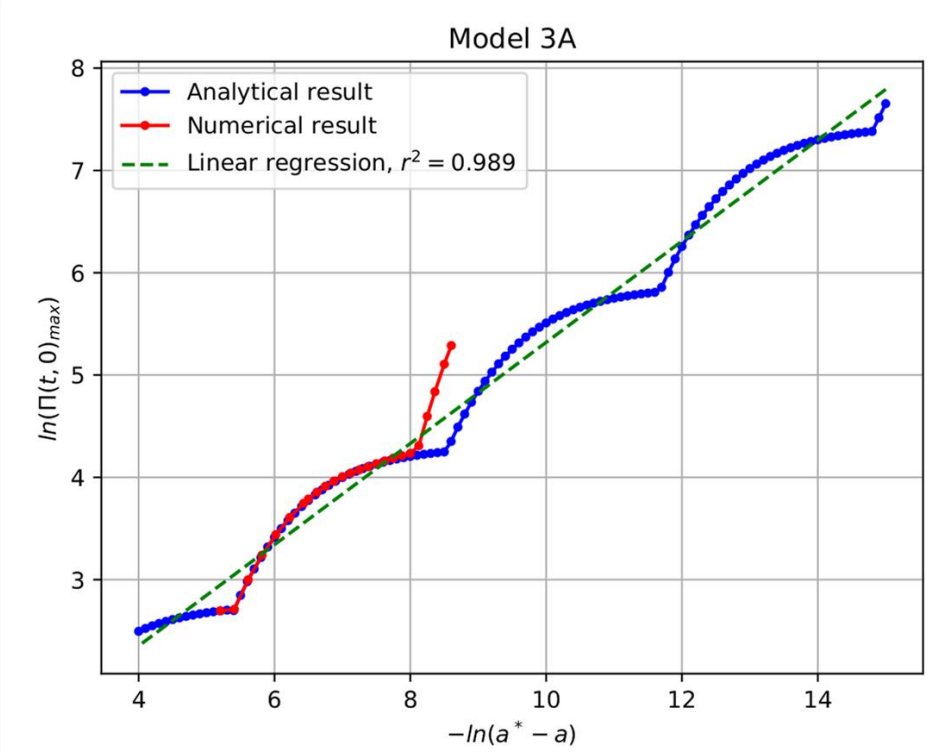

Conclussions & work in progress

Tool for models

We have a tool to obtain non-linear models with known analytical solution and critical phenomena for a massless scalar fi

First simple DSS model

We constructed the first ever simple model with DSS threshold (analytical) solutions.

We have a tool (code) to test models for which we dont know the analytical solution.

Work in progress

We are currently working in modifications of the presented models where the analytical solution is unkown but we can ha

The aim is to fully understand the structure that makes a scale-invariant solution CSS or DSS.

Thanks for your attention

Questions?