"How to kick a Q-Ball"

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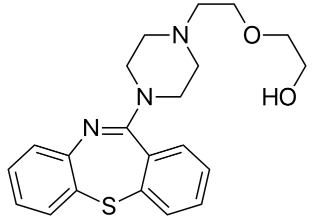
Nickolas WILDSTAR
For Governor of California 2018



What’s a Q-Ball?

"Q-balls are nontopological solitons arising in field theories admitting a conserved charge Q, associated to continuous internal symmetry "

A soliton is a nonlinear and nondissipative solution of a field theory.

Noether charge ensures existence and stability.

Bound states of scalar particles, stable classical solutions carrying a rotating time dependent internal phase.

Why they studied them?

Interpretate Q as the baryon-number.

Q-balls are allowed in supersymmetric extensions of the SM.

(Stable) Q-balls, formed in the early universe, can contribute to its dark matter content.

Why I'm studying them?

Action resembling some limit of massive gravity theory.

Scalar field bound objects toy-model, like Oscillons, Boson stars, etc. but easier!

How to build a Q-Ball?

$$S \equiv \int d^4x\sqrt{-\eta}\left(\frac{1}{2}\eta^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi^* + \mathcal{U}(|\Phi|^2)\right) ,$$

Considering the spherically symmetric ansatz: $\Phi(t, r) = e^{-i\Omega t}\Psi(r)$ and a nonlinear potential such as,

$$\mathcal{U} = \frac{1}{2}\mu^2|\Phi|^2 - \lambda_4^2|\Phi|^4 + \lambda_6^2|\Phi|^6 ,$$

(i) The absolute minimum of $\mathcal{U}$ is at $\Phi = 0$.

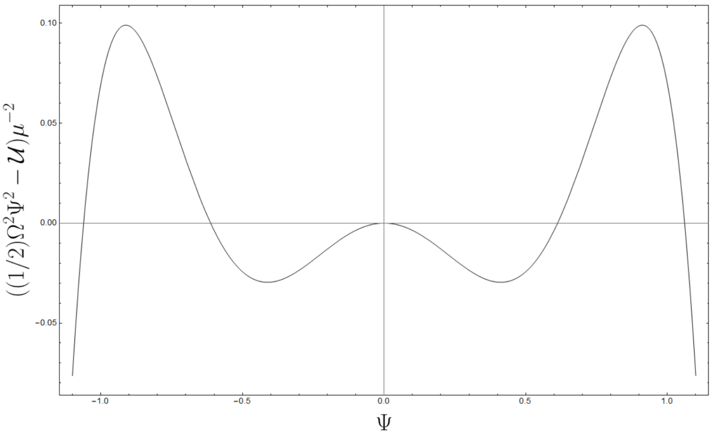
(ii) There exists a $\Phi_c > 0$, such that,

$$0 \leq \min\left(2\mathcal{U}/|\Phi|^2\right) = 2\mathcal{U}_c/\Phi_c^2 < \mu^2 .$$

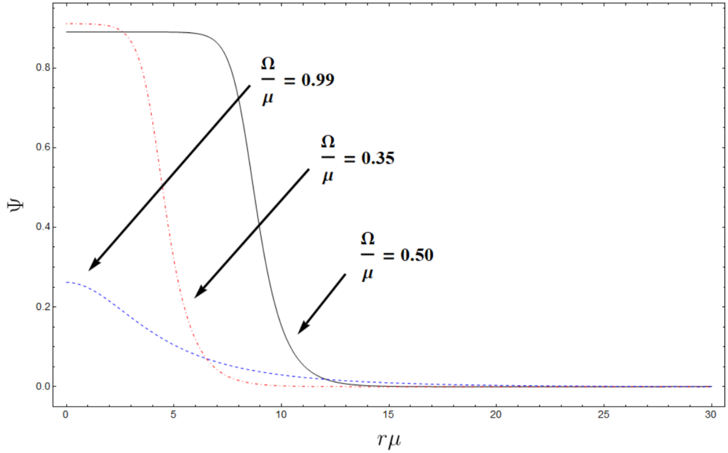
Then, there exist bound-state nondissipative solutions which are absolute minima of the energy for fixed Q.

Equation of motion (EoM)

$$\partial_r^2\Psi + \frac{2}{r}\partial_r\Psi + \left[\Omega^2 - 2\mathcal{U}_2'(\Psi^2)\right]\Psi = 0\,,$$



Example of Q-Ball profile



How to perturb a Q-Ball?

Hunt for modes

Assume a background and add small pertubations:

$$\Phi = \Phi_0 + \delta\Phi$$

Choose ansatzes:

$$\delta\Phi = \sum_{l,m} \int \frac{d\omega}{\sqrt{2\pi}} e^{-i\omega t} \frac{Z_1(r;\omega,l,m)}{r} Y_{lm}(\vartheta,\varphi)\,,$$
$$\delta\Phi^* = \sum_{l,m} \int \frac{d\omega}{\sqrt{2\pi}} e^{-i(\omega-2\Omega)t} \frac{Z_2(r;\omega,l,m)}{r} Y_{lm}(\vartheta,\varphi)\,,$$

Linearize the EoMs and choose boundary conditions

Solve the eigenmodes problem

$$Z'' + (\omega^2 - U(r))Z = 0$$

Coupling with a source

Equations of motion

$$\Box\Phi - 2\mathcal{U}_2'\Phi = T\,,$$
$$\Box\Phi^* - 2\mathcal{U}_2'\Phi^* = T\,,$$

In the thin-wall limit,

$$Z'' + \left(\omega^2 - \frac{l(l+1)}{r^2} - \tilde{\mu}^2\right) Z = \frac{2T_{lm}}{r}\,,$$

where the matter moments are:

$$r^2 T = \sum_{lm} \int \frac{d\omega}{\sqrt{2\pi}} e^{-i\omega t} T_{lm} Y_{lm}(\vartheta, \varphi)\,.$$

Again, solve the eigenproblem (Green's functions etc.), but focusing on the field values!

How a Q-Ball radiates?

Flux of energy

Far away from the Q-Ball,

$$\frac{dE}{dt} = \lim_{r\to\infty} \left\{ r^2 \int d\vartheta d\varphi \sqrt{-\eta}\, T^r_t \right\},$$

where

$$T_{\mu\nu} = \partial_{(\mu}\Phi^* \partial_{\nu)}\Phi - \frac{1}{2}\eta_{\mu\nu}\left[\partial_\alpha \Phi^* \partial^\alpha \Phi + \mathcal{U}(|\Phi|^2)\right] .$$

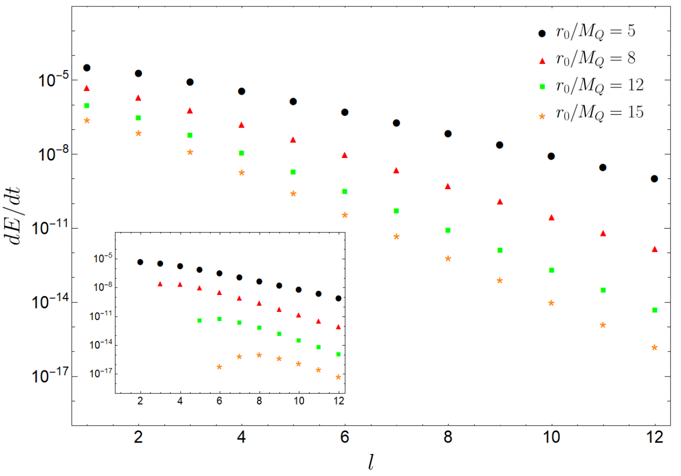
Choose the setup:

- Circular orbits
- Radial infaling

and evaluate either the flux or the spectrum ($\frac{dE}{d\omega}$).

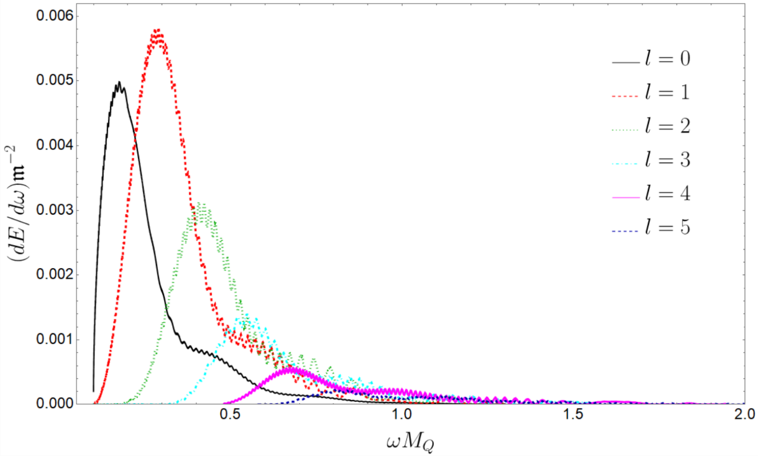
Circular orbits

$$\frac{dE}{dt} = \frac{1}{4\pi} \sum_{l,m} \mathrm{sgn}\,(m\omega_0 + \mu)\, m\omega_0 \sqrt{(m\omega_0)^2 - \mu^2} |Z_\infty|^2 \,.$$



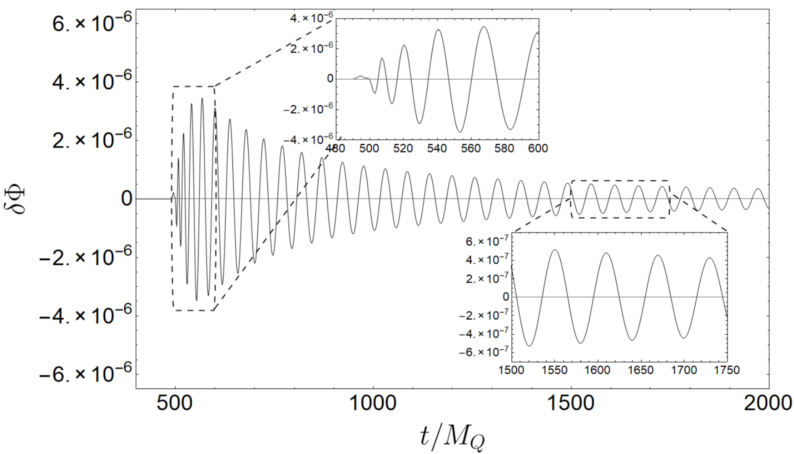
Radial infalling

$$\frac{dE}{d\omega} = \frac{1}{2}\sum_{l,m}\left[\text{sgn}\,(\omega+\mu)\omega\sqrt{\omega^2-\mu^2}\right]|Z_\infty|^2\,.$$



The waveform - time domain

$$\delta\Phi(t,r)_l = \mathrm{Re}\left[2\int_{\mu+\varepsilon}^{+\infty}\frac{d\omega}{\sqrt{2\pi}}e^{-i\omega t}\frac{Z(r;\omega,l,0)}{2r}Y_{l0}(\vartheta,0)\right],$$



Summary

Analyzing Q-Balls:

- Building
- Perturbations
- Radiation

Q-Ball as a gym:

- to get results about scalar objects.
- to understand properties and behaviors of solitons.
- to study massive gravity (there's more to do).