Flavour and CP Phenomenology

Miguel Nebot

CFTP - IST Lisbon

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miguel.r.nebot.gomez@tecnico.ulisboa.pt

Disclaimer

- *Before I came here I was confused about this subject.*
  Having listened to your lecture I am still confused.
  But on a higher level.

E. Fermi

CP – Generalities

CP Violation in Neutral Meson Mixing

- Decays $\Delta F = \Delta Q$

$$\begin{array}{ll} M^0 \to X\ell^+ & \bar{M}^0 \nrightarrow X\ell^+ \\ M^0 \nrightarrow X\ell^- & \bar{M}^0 \to X\ell^- \end{array}$$

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f} \quad \text{give} \quad C_{X\ell^\pm} = \pm 1,\ S_{X\ell^\pm} = R_{X\ell^\pm} = 0$$

- Decays $\Leftrightarrow$ Transition Probabilities

$$\left|\langle X\ell^-|\mathcal{T}|M^0(t)\rangle\right|^2 \sim \left|\langle \bar{M}^0|\mathcal{T}|M^0(t)\rangle\right|^2$$
$$\left|\langle X\ell^+|\mathcal{T}|\bar{M}^0(t)\rangle\right|^2 \sim \left|\langle M^0|\mathcal{T}|\bar{M}^0(t)\rangle\right|^2$$

$$\left|\langle f|\mathcal{T}|M^0(t)\rangle\right|^2 = e^{-\Gamma t}\left\{\begin{array}{l} \mathscr{C}_h[M^0,f]\cosh\left(\frac{\Delta\Gamma}{2}t\right) + \mathscr{C}_c[M^0,f]\cos\left(\Delta M t\right) \\ +\mathscr{S}_h[M^0,f]\sinh\left(\frac{\Delta\Gamma}{2}t\right) + \mathscr{S}_c[M^0,f]\sin\left(\Delta M t\right) \end{array}\right\}$$

$$\left|\langle f|\mathcal{T}|\bar{M}^0(t)\rangle\right|^2 = e^{-\Gamma t}\left\{\begin{array}{l} \mathscr{C}_h[\bar{M}^0,f]\cosh\left(\frac{\Delta\Gamma}{2}t\right) + \mathscr{C}_c[\bar{M}^0,f]\cos\left(\Delta M t\right) \\ +\mathscr{S}_h[\bar{M}^0,f]\sinh\left(\frac{\Delta\Gamma}{2}t\right) + \mathscr{S}_c[\bar{M}^0,f]\sin\left(\Delta M t\right) \end{array}\right\}$$

CP Violation in Neutral Meson Mixing

- We have

$$\left|\langle \bar{M}^0|\mathcal{T}|M^0(t)\rangle\right|^2 \sim e^{-\Gamma t}\left\{\begin{array}{l} \mathscr{C}_h[M^0,f]\cosh\left(\frac{\Delta\Gamma}{2}t\right)+\mathscr{C}_c[M^0,f]\cos\left(\Delta M t\right) \\ +\mathscr{S}_h[M^0,f]\sinh\left(\frac{\Delta\Gamma}{2}t\right)+\mathscr{S}_c[M^0,f]\sin\left(\Delta M t\right)\end{array}\right\}$$

$$\left|\langle M^0|\mathcal{T}|\bar{M}^0(t)\rangle\right|^2 \sim e^{-\Gamma t}\left\{\begin{array}{l} \mathscr{C}_h[\bar{M}^0,f]\cosh\left(\frac{\Delta\Gamma}{2}t\right)+\mathscr{C}_c[\bar{M}^0,f]\cos\left(\Delta M t\right) \\ +\mathscr{S}_h[\bar{M}^0,f]\sinh\left(\frac{\Delta\Gamma}{2}t\right)+\mathscr{S}_c[\bar{M}^0,f]\sin\left(\Delta M t\right)\end{array}\right\}$$

with $(\theta = 0)$

$$\mathscr{C}_h[M^0,f]=\frac{\Gamma_f(1-\delta)}{2(1-C_f\delta)}\{2\} \qquad \mathscr{C}_c[M^0,f]=\frac{\Gamma_f(1-\delta)}{2(1-C_f\delta)}\{2C_f\}$$

$$\mathscr{S}_h[M^0,f]=\frac{\Gamma_f(1-\delta)}{2(1-C_f\delta)}\{2R_f\} \qquad \mathscr{S}_c[M^0,f]=\frac{\Gamma_f(1-\delta)}{2(1-C_f\delta)}\{-2S_f\}$$

$$\mathscr{C}_h[\bar{M}^0,f]=\frac{\Gamma_f(1+\delta)}{2(1-C_f\delta)}\{2\} \qquad \mathscr{C}_c[\bar{M}^0,f]=\frac{\Gamma_f(1+\delta)}{2(1-C_f\delta)}\{-2C_f\}$$

$$\mathscr{S}_h[\bar{M}^0,f]=\frac{\Gamma_f(1+\delta)}{2(1-C_f\delta)}\{2R_f\} \qquad \mathscr{S}_c[\bar{M}^0,f]=\frac{\Gamma_f(1+\delta)}{2(1-C_f\delta)}\{2S_f\}$$

CP Violation in Neutral Meson Mixing

- We have

$$\left|\langle \bar{M}^0|\mathcal{T}|M^0(t)\rangle\right|^2 \sim e^{-\Gamma t}\left\{\begin{array}{l}\mathscr{C}_h[M^0,f]\cosh\left(\frac{\Delta\Gamma}{2}t\right)+\mathscr{C}_c[M^0,f]\cos\left(\Delta M t\right)\\ +\mathscr{S}_h[M^0,f]\sinh\left(\frac{\Delta\Gamma}{2}t\right)+\mathscr{S}_c[M^0,f]\sin\left(\Delta M t\right)\end{array}\right\}$$

$$\left|\langle M^0|\mathcal{T}|\bar{M}^0(t)\rangle\right|^2 \sim e^{-\Gamma t}\left\{\begin{array}{l}\mathscr{C}_h[\bar{M}^0,f]\cosh\left(\frac{\Delta\Gamma}{2}t\right)+\mathscr{C}_c[\bar{M}^0,f]\cos\left(\Delta M t\right)\\ +\mathscr{S}_h[\bar{M}^0,f]\sinh\left(\frac{\Delta\Gamma}{2}t\right)+\mathscr{S}_c[\bar{M}^0,f]\sin\left(\Delta M t\right)\end{array}\right\}$$

with $(\theta = 0)$

$$\mathscr{C}_h[M^0,\bar{M}^0] = \frac{\Gamma(1-\delta)}{(1+\delta)} \qquad \mathscr{C}_c[M^0,\bar{M}^0] = -\frac{\Gamma(1-\delta)}{(1+\delta)}$$

$$\mathscr{S}_h[M^0,\bar{M}^0] = 0 \qquad \mathscr{S}_c[M^0,\bar{M}^0] = 0$$

$$\mathscr{C}_h[\bar{M}^0,M^0] = \frac{\Gamma(1+\delta)}{(1-\delta)} \qquad \mathscr{C}_c[\bar{M}^0,M^0] = -\frac{\Gamma(1+\delta)}{(1-\delta)}$$

$$\mathscr{S}_h[\bar{M}^0,M^0] = 0 \qquad \mathscr{S}_c[\bar{M}^0,M^0] = 0$$

CP Violation in Neutral Meson Mixing

- We have with $(\theta = 0)$

$$\left|\langle\bar{M}^0|\mathcal{T}|M^0(t)\rangle\right|^2 \sim e^{-\Gamma t}\Gamma\left\{\cosh\left(\frac{\Delta\Gamma}{2}t\right) - \cos\left(\Delta M t\right)\right\}\frac{(1-\delta)}{(1+\delta)}$$

$$\left|\langle M^0|\mathcal{T}|\bar{M}^0(t)\rangle\right|^2 \sim e^{-\Gamma t}\Gamma\left\{\cosh\left(\frac{\Delta\Gamma}{2}t\right) - \cos\left(\Delta M t\right)\right\}\frac{(1+\delta)}{(1-\delta)}$$

CP Violation in Neutral Meson Mixing

- We have with $(\theta = 0)$
- CP Violating asymmetry

$$\left|\langle \bar{M}^0|\mathcal{T}|M^0(t)\rangle\right|^2 - \left|\langle M^0|\mathcal{T}|\bar{M}^0(t)\rangle\right|^2 \sim \frac{-4\delta}{1-\delta^2} e^{-\Gamma t}\Gamma\left[\cosh\left(\frac{\Delta\Gamma}{2}t\right) - \cos\left(\Delta M t\right)\right]$$

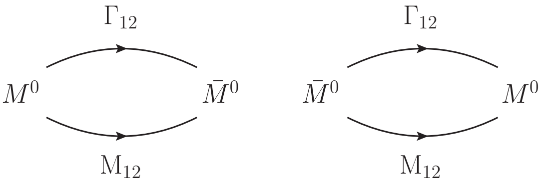
CP Violation in Neutral Meson Mixing

- H with $\theta = 0$

$$\mathrm{H} = \begin{pmatrix} \mu & \frac{p}{q}\frac{\Delta\mu}{2} \\ \frac{q}{p}\frac{\Delta\mu}{2} & \mu \end{pmatrix}$$

  - $\frac{\mathrm{H}_{12}}{\mathrm{H}_{21}} = \frac{q^2}{p^2}$

$$\left|\frac{q}{p}\right|^2 = \frac{\left|\mathrm{M}_{12}^* - \frac{i}{2}\Gamma_{12}^*\right|^2}{\left|\mathrm{M}_{12} - \frac{i}{2}\Gamma_{12}\right|^2} \Leftrightarrow \delta = -\frac{\mathrm{Im}(\mathrm{M}_{12}\Gamma_{12}^*)}{|\mathrm{M}_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2}$$



Direct CP Violation

- CP transformation

$$\mathbf{CP}|f\rangle = e^{i\varphi_f}|\bar{f}\rangle \qquad \mathbf{CP}|\bar{f}\rangle = e^{-i\varphi_f}|f\rangle$$

- Invariance under CP of transition operator $\mathcal{T}$

$$\mathbf{CP}\,\mathcal{T}\,\mathbf{CP} = \mathcal{T}$$

- CP invariance of $f \to g$ transition

$$\langle g|\mathcal{T}|f\rangle = e^{i(\varphi_f - \varphi_g)}\langle \bar{g}|\mathcal{T}|\bar{f}\rangle$$

- Simplest CP asymmetry

$$\mathcal{A}_{CP}(f \to g) = |\langle g|\mathcal{T}|f\rangle|^2 - |\langle \bar{g}|\mathcal{T}|\bar{f}\rangle|^2$$

$\mathcal{A}_{CP}(f \to g) \neq 0$ in decays of CP conjugate channels: *direct* CP Violation

Direct CP Violation

- In general

$$\langle g|\mathcal{T}|f\rangle = A\, e^{i\delta}\, e^{i\phi} \qquad \langle \bar{g}|\mathcal{T}|\bar{f}\rangle = \bar{A}\, e^{i\delta}\, e^{-i\phi}$$

δ: *strong* phase, ϕ: *weak* phase

$$\mathcal{A}_{CP}(f \to g) = A^2 - \bar{A}^2 \qquad A \neq \bar{A} \;\Rightarrow\; \text{CP Violation}$$

but in SM, from $\mathscr{L}_{CC}$, expect $A = \bar{A}$ at tree level

- Consider two amplitudes

$$\langle g|\mathcal{T}|f\rangle = A_1\, e^{i(\delta_1+\phi_1)} + A_2\, e^{i(\delta_2+\phi_2)}$$
$$\langle \bar{g}|\mathcal{T}|\bar{f}\rangle = A_1\, e^{i(\delta_1-\phi_1)} + A_2\, e^{i(\delta_2-\phi_2)}$$

$$\mathcal{A}_{CP}(f \to g) = -4A_1A_2\, \sin(\delta_2 - \delta_1)\, \sin(\phi_2 - \phi_1)$$

Indirect CP Violation

- Indirect CP Violation
  - Interference CP Violation
  - Mixing $\times$ Decay
- CP Violating asymmetry

$$\left|\langle f|\mathcal{T}|M^0(t)\rangle\right|^2 - \left|\langle \bar{f}|\mathcal{T}|\bar{M}^0(t)\rangle\right|^2$$

- If No CP Violation

$$\langle f|\mathcal{T}|M^0\rangle = e^{i(\varphi_B - \varphi_f)}\langle \bar{f}|\mathcal{T}|\bar{M}^0\rangle \qquad \langle f|\mathcal{T}|\bar{M}^0\rangle = e^{-i(\varphi_B + \varphi_f)}\langle \bar{f}|\mathcal{T}|M^0\rangle$$

$$\langle M^0|\mathrm{H}|\bar{M}^0\rangle = e^{-i2\varphi_B}\langle \bar{M}^0|\mathrm{H}|M^0\rangle$$

Indirect CP Violation

- That is

$$A_f = e^{i(\varphi_B - \varphi_f)} \bar{A}_{\bar{f}} \qquad \bar{A}_f = e^{-i(\varphi_B + \varphi_f)} A_{\bar{f}}$$

$$\frac{p}{q} = e^{-i2\varphi_B} \frac{q}{p}$$

- Combine

$$\frac{q}{p}\frac{\bar{A}_f}{A_f} = \frac{p}{q}\frac{A_{\bar{f}}}{\bar{A}_{\bar{f}}} \Leftrightarrow \lambda_f \lambda_{\bar{f}} = 1$$

$$C_f = -C_{\bar{f}} \qquad S_f = -S_{\bar{f}} \qquad R_f = R_{\bar{f}}$$

- For a CP eigenstate f_{CP}

$$\lambda_{f_{CP}} = \pm 1$$

$$C_{f_{CP}} = 0 \qquad S_{f_{CP}} = 0 \qquad R_{f_{CP}} = \pm 1$$

Rephasing

- Arbitrary rephasings
  - States: $|M^0\rangle \mapsto e^{i\omega}|M^0\rangle$, $|\bar{M}^0\rangle \mapsto e^{i\bar{\omega}}|\bar{M}^0\rangle$
  - States: $|f\rangle \mapsto e^{i\omega_f}|f\rangle$
  - CP definition: $\mathbf{CP}|f\rangle = e^{i\varphi_f}|\bar{f}\rangle$, $\mathbf{CP}|\bar{f}\rangle = e^{-i\varphi_f}|f\rangle$
- Decay amplitudes

$$\left.\begin{array}{l}\langle f|\mathcal{T}|M^0\rangle = A_f \mapsto e^{i\omega}e^{-i\omega_f}A_f \\ \langle f|\mathcal{T}|\bar{M}^0\rangle = \bar{A}_f \mapsto e^{i\bar{\omega}}e^{-i\omega_f}\bar{A}_f\end{array}\right\} \ |A_f|, |\bar{A}_f| \text{ are invariant}$$

- Meson Mixing $\mathrm{H}_{21} \mapsto e^{i(\omega-\bar{\omega})}\mathrm{H}_{21}$

$$\frac{q}{p} \mapsto e^{i(\omega-\bar{\omega})}\frac{q}{p} \ \Rightarrow \ |q/p| \text{ is invariant } (\Leftrightarrow \delta)$$

- Meson Mixing $\times$ Decay amplitudes

$$\frac{q}{p}\frac{\langle f|\mathcal{T}|\bar{M}^0\rangle}{\langle f|\mathcal{T}|M^0\rangle} = \frac{q}{p}\frac{\bar{A}_f}{A_f} = \lambda_f \text{ is invariant}$$

Neutral Meson Mixing (again)

- H with $\theta = 0$

$$\mathrm{H} = \begin{pmatrix} \mu & \frac{p}{q}\frac{\Delta\mu}{2} \\ \frac{q}{p}\frac{\Delta\mu}{2} & \mu \end{pmatrix}$$

  - $\mathrm{H}_{12}\mathrm{H}_{21} = \frac{1}{4}(\Delta\mu)^2$

$$\mathrm{H}_{12}\mathrm{H}_{21} = |\mathrm{M}_{12}|^2 - \frac{1}{4}|\Gamma_{12}|^2 - i\mathrm{Re}(\mathrm{M}_{12}\Gamma^*_{12})$$
$$\frac{(\Delta\mu)^2}{4} = \frac{1}{4}\left[(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 - i(\Delta M)(\Delta\Gamma)\right]$$

  - $\frac{\mathrm{H}_{12}}{\mathrm{H}_{21}} = \frac{q^2}{p^2}$

$$\left|\frac{q}{p}\right|^2 = \frac{\left|\mathrm{M}^*_{12} - \frac{i}{2}\Gamma^*_{12}\right|^2}{\left|\mathrm{M}_{12} - \frac{i}{2}\Gamma_{12}\right|^2} \Leftrightarrow \delta = -\frac{\mathrm{Im}(\mathrm{M}_{12}\Gamma^*_{12})}{|\mathrm{M}_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2}$$

B_d, B_s systems

$$|\mathrm{M}_{12}|^2 - \frac{1}{4}|\Gamma_{12}|^2 - i\mathrm{Re}(\mathrm{M}_{12}\Gamma_{12}^*) = \frac{1}{4}\left[(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 - i(\Delta M)(\Delta\Gamma)\right]$$

$$\delta = -\frac{\mathrm{Im}(\mathrm{M}_{12}\Gamma_{12}^*)}{|\mathrm{M}_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2}$$

- If $|\Gamma_{12}| \ll |\mathrm{M}_{12}|$

$$4|\mathrm{M}_{12}|^2 = (\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2, \qquad 0 = (\Delta M)(\Delta\Gamma), \qquad \delta = 0$$

  - No CP Violation in mixing: $\delta = 0 \Leftrightarrow \left|\frac{q}{p}\right| = 1$
  - $\Delta\Gamma = 0$ and $2|\mathrm{M}_{12}| = \Delta M$

$$\frac{q}{p} = e^{-i\,\arg(\mathrm{M}_{12})} = \frac{\mathrm{M}_{12}^*}{|\mathrm{M}_{12}|} = \frac{p^*}{q^*}$$

Kaons

$$|\mathrm{M}_{12}|^2 - \frac{1}{4}|\Gamma_{12}|^2 - i\mathrm{Re}(\mathrm{M}_{12}\Gamma_{12}^*) = \frac{1}{4}\left[(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 - i(\Delta M)(\Delta\Gamma)\right]$$

$$\delta = -\frac{\mathrm{Im}(\mathrm{M}_{12}\Gamma_{12}^*)}{|\mathrm{M}_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2}$$

- Relevant fact 1:

$$|\Delta\Gamma| \simeq 2|\Delta M| \;\Rightarrow\; 2|\mathrm{M}_{12}| \simeq |\Gamma_{12}|$$

- Relevant fact 2:

$$\mathrm{Re}(\mathrm{M}_{12}\Gamma_{12}^*) \simeq |\mathrm{M}_{12}\Gamma_{12}| = (\Delta M)(\Delta\Gamma)$$

$$\Rightarrow\; \mathrm{Re}(\mathrm{M}_{12}\Gamma_{12}^*) \simeq \frac{1}{2}(\Delta M)^2 \qquad 2|\mathrm{M}_{12}| \simeq \Delta M$$

Kaons

$$2|\mathrm{M}_{12}| \simeq |\Gamma_{12}| \simeq \Delta M$$

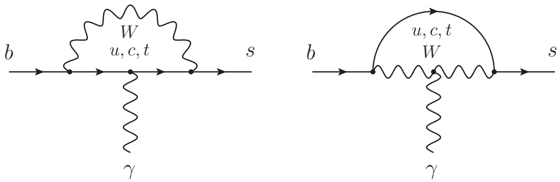
- CP Violation in mixing

$$\delta = -\frac{\mathrm{Im}(\mathrm{M}_{12}\Gamma^*_{12})}{|\mathrm{M}_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2} = -\frac{\mathrm{Im}\left(\mathrm{M}_{12}e^{-i\arg(\Gamma_{12})}\right)}{\Delta M}$$

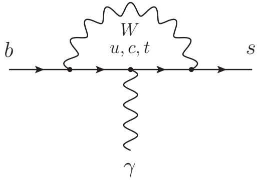
- In kaons, since $K_S \sim K_+$ and $K_L \sim K_-$, the "traditional" notation is not q and p:

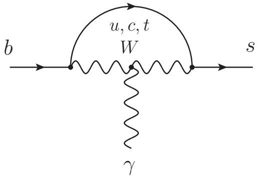
$$\frac{q}{p} = \frac{1-\epsilon_K}{1+\epsilon_K} \Rightarrow \delta = \frac{2\mathrm{Re}(\epsilon_K)}{1+|\epsilon_K|^2}$$

FCNC $b \to s\gamma$

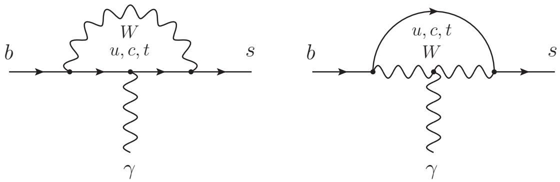


e

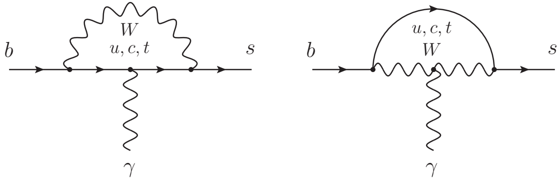




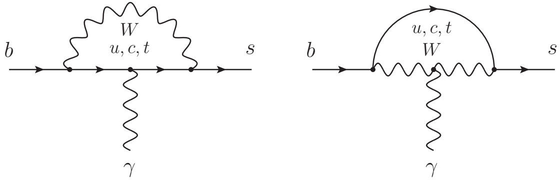
$e\, g^2$



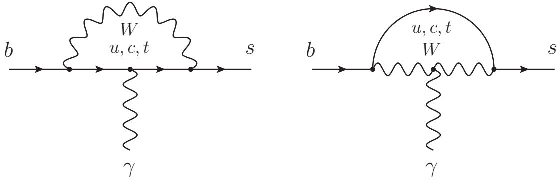
$$e\, g^2\, V_{jb} V_{js}^*$$



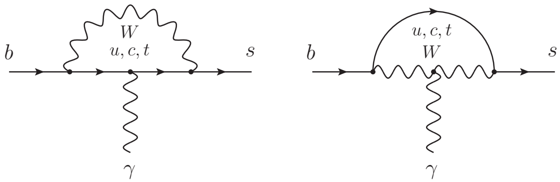
$$e\, g^2\, V_{jb} V_{js}^* \,\frac{1}{16\pi^2}$$



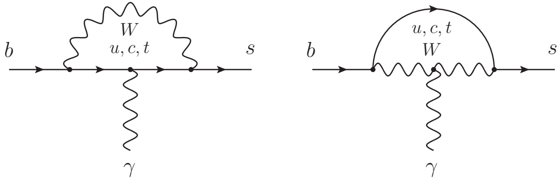
$$e\,g^2\,V_{jb}V_{js}^*\,\frac{1}{16\pi^2}\,F_{\mu\nu}$$



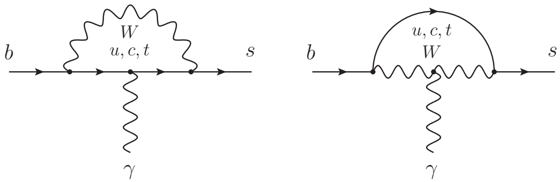
$$e\, g^2\, V_{jb} V_{js}^* \,\frac{1}{16\pi^2} F_{\mu\nu} \bar{s}_L \qquad b_L$$



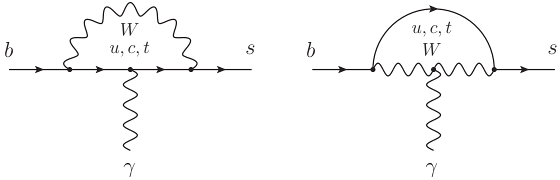
$$e\, g^2\, V_{jb} V_{js}^* \,\frac{1}{16\pi^2}\, F_{\mu\nu}\, \bar{s}_L \sigma^{\mu\nu} b_L$$



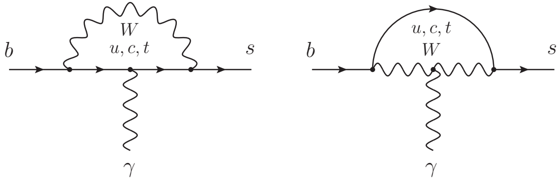
$$e\, g^2\, V_{jb} V_{js}^* \,\frac{1}{16\pi^2}\, F_{\mu\nu}\, \bar{s}_L \sigma^{\mu\nu} b_L$$



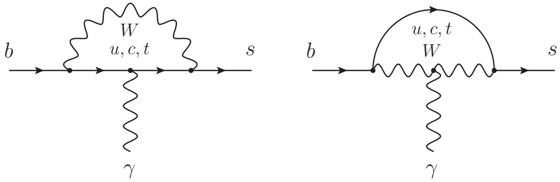
$$e\, g^2\, V_{jb} V_{js}^* \,\frac{1}{16\pi^2}\, F_{\mu\nu}\, \bar{s}_L \sigma^{\mu\nu} b_R\, m_b$$



$$e\, g^2\, V_{jb} V_{js}^* \, \frac{1}{16\pi^2} F_{\mu\nu} \bar{s}_L \sigma^{\mu\nu} b_R \, m_b \, \hat{I}(m_{u_j}^2, M_W^2)$$



$$e\, g^2\, V_{jb} V_{js}^* \, \frac{1}{16\pi^2} F_{\mu\nu} \bar{s}_L \sigma^{\mu\nu} b_R\, m_b\, \hat{I}(x_j)\, M_W^{-2}$$



$$\sum_{j=u,c,t} \frac{eg^2}{16\pi^2 M_W^2} V_{jb} V_{js}^* \hat{I}(x_j)\, m_b\, F_{\mu\nu}\, \bar{s}_L \sigma^{\mu\nu} b_R\,, \qquad x_j = \frac{m_{u_j}^2}{M_W^2}$$

Glashow-Iliopoulos-Maiani (in $\Delta F = 1$)

- Loop function $\hat{I}(x_j)$ in

$$\sum_{j=1}^{3} V_{jb} V_{js}^{*} \hat{I}(x_j)$$

- CKM unitarity

$$\sum_{j=1}^{3} V_{jb} V_{js}^{*} = 0 \qquad \sum_{j=1}^{3} V_{jb} V_{js}^{*} \hat{I}(0) = 0$$

- Then

$$\sum_{j=1}^{3} V_{jb} V_{js}^{*} \hat{I}(x_j) = \sum_{j=1}^{3} V_{jb} V_{js}^{*} I(x_j)$$

with

$$I(x_j) \equiv \hat{I}(x_j) - \hat{I}(0), \qquad \lim_{x \to 0} I(x) = 0$$

- In $b \to s\gamma$, t contribution dominant

FCNC and CP in $\Delta F = 2$

GIM (in $\Delta F = 2$)

- Loop function $\hat{S}(x_j, x_k)$

$$\sum_{j,k=1}^{3} V_{j\alpha} V_{j\beta}^* V_{k\alpha} V_{k\beta}^* \hat{S}(x_j, x_k) \qquad \text{sym. } j \leftrightarrows k$$

- CKM unitarity *twice*

$$\sum_{k=1}^{3} V_{k\alpha} V_{k\beta}^* \left[\sum_{j=1}^{3} V_{j\alpha} V_{j\beta}^* \hat{S}(0, x_k) \right] = 0 \qquad \left[\sum_{j=1}^{3} V_{j\alpha} V_{j\beta}^* \right] \left[\sum_{k=1}^{3} V_{k\alpha} V_{k\beta}^* \right] \hat{S}(0,0) = 0$$

- Then

$$\sum_{j,k=1}^{3} V_{j\alpha} V_{j\beta}^* V_{k\alpha} V_{k\beta}^* \hat{S}(x_j, x_k) = \sum_{j,k=1}^{3} V_{j\alpha} V_{j\beta}^* V_{k\alpha} V_{k\beta}^* S(x_j, x_k)$$

$$S(x_j, x_k) \equiv \hat{S}(x_j, x_k)$$

- Largest function: $S(x_t, x_t) \gg S(x_j, x_{!t})$

GIM (in $\Delta F = 2$)

- Loop function $\hat{S}(x_j, x_k)$

$$\sum_{j,k=1}^{3} V_{j\alpha} V_{j\beta}^* V_{k\alpha} V_{k\beta}^* \hat{S}(x_j, x_k) \qquad \text{sym. } j \leftrightarrows k$$

- CKM unitarity *twice*

$$\sum_{k=1}^{3} V_{k\alpha} V_{k\beta}^* \left[\sum_{j=1}^{3} V_{j\alpha} V_{j\beta}^* \hat{S}(0, x_k)\right] = 0 \qquad \left[\sum_{j=1}^{3} V_{j\alpha} V_{j\beta}^*\right] \left[\sum_{k=1}^{3} V_{k\alpha} V_{k\beta}^*\right] \hat{S}(0,0) = 0$$

- Then

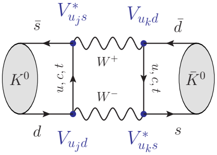
$$\sum_{j,k=1}^{3} V_{j\alpha} V_{j\beta}^* V_{k\alpha} V_{k\beta}^* \hat{S}(x_j, x_k) = \sum_{j,k=1}^{3} V_{j\alpha} V_{j\beta}^* V_{k\alpha} V_{k\beta}^* S(x_j, x_k)$$

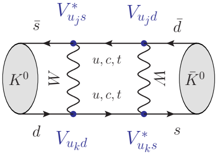
$$S(x_j, x_k) \equiv \hat{S}(x_j, x_k) - \hat{S}(x_j, 0) - \hat{S}(0, x_k) + \hat{S}(0,0), \qquad \lim_{x,y\to 0} S(x,y) = 0$$

- Largest function: $S(x_t, x_t) \gg S(x_j, x_{!t})$

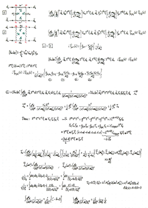
ϵ_K, δ in kaons

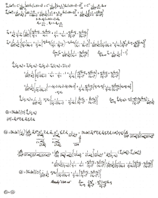
- For M_{12}

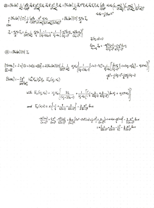




$$\mathrm{M}_{12} \propto \sum_{j,k=1}^{3} V_{js} V_{jd}^{*} V_{ks} V_{kd}^{*} \, L(x_j, x_k)$$





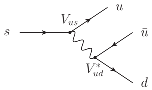


ϵ_K, δ in kaons

- For Γ_{12}

$$\Gamma_{12} = 2\pi \sum_n \delta(M_0 - E_n) \langle i|\mathcal{H}_\mathrm{w}|n\rangle\langle n|\mathcal{H}_\mathrm{w}|j\rangle$$

  - $\pi\pi$ decays



$$\to \quad \Gamma_{12} \propto (V_{us}V_{ud}^*)^2$$

- Back to $\delta = -\frac{\mathrm{Im}\left(\mathrm{M}_{12} e^{-i\arg(\Gamma_{12})}\right)}{\Delta M}$

$$\delta \propto \mathrm{Im}\left(\sum_{j,k=1}^{3} V_{js}V_{jd}^* V_{ks}V_{kd}^* \, L(x_j, x_k) \frac{(V_{us}^* V_{ud})^2}{|V_{us}^* V_{ud}|^2}\right)$$

ϵ_K, δ in kaons

- Loop function

$$L(x_u) \simeq 1.4 \times 10^{-9} \qquad L(x_c) \simeq 7.4 \times 10^{-5} \qquad L(x_t) \simeq 2.4$$

$$L(x_u, x_c) \simeq 4.2{\times}10^{-8} \qquad L(x_u, x_t) \simeq 3.0{\times}10^{-8} \qquad L(x_c, x_t) \simeq 8.0{\times}10^{-4}$$

- CKM's with Wolfenstein parameters

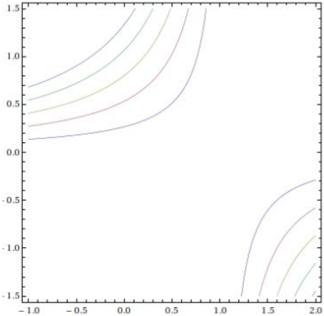
$$V_{cs}V_{cd}^* = -\lambda(1-\lambda^2/2) \qquad V_{ts}V_{td}^* = -A^2\lambda^5(1-\rho+i\eta)$$

- Imaginary part

$$\mathrm{Im}\left(\mathrm{M}_{12}e^{-i\arg(\Gamma_{12})}\right) \propto A^2\lambda^6\,\eta\,\left[2L(x_c,x_t) + A^2\lambda^4\,L(x_t)(1-\rho)\right]$$
$$\sim \eta\,[0.15 + 3.6(1-\rho)]$$

ϵ_K, δ in kaons

Contours $2L(x_c, x_t) + A^2\lambda^4\, L(x_t)(1-\rho) = [\text{Const.}]$



$\sin 2\beta$

- CP Asymmetry in $B_d^0 \to J\Psi K_S$
  - Decays:

$$\langle J/\Psi K_S|\mathcal{T}|B_d^0\rangle = +\frac{1}{2p_K}\langle J/\Psi K^0|\mathcal{T}|B_d^0\rangle$$
$$\langle J/\Psi K_S|\mathcal{T}|\bar{B}_d^0\rangle = -\frac{1}{2q_K}\langle J/\Psi \bar{K}^0|\mathcal{T}|\bar{B}_d^0\rangle$$

  - Tree level: $b \to c\bar{c}s \propto V_{cb}V_{cs}^*$
  - Kaon mixing: $\frac{q_K}{p_K} = \frac{V_{cs}^* V_{cd}}{V_{cs}V_{cd}^*}e^{-i2\chi'}$
  - $\delta = 0$ in B_d^0–$\bar{B}_d^0$: $\frac{q}{p}\frac{\mathrm{M}_{12}^*}{|\mathrm{M}_{12}|} = \frac{(V_{tb}^*V_{td})^2}{|V_{tb}^*V_{td}|^2}$
- Then:

$$\lambda_{J/\Psi K_S} = \frac{q}{p}\frac{\bar{A}_{J/\Psi K_S}}{A_{J/\Psi K_S}} = -\frac{(V_{tb}^*V_{td})^2}{|V_{tb}^*V_{td}|^2}\frac{(V_{cb}V_{cs}^*)^2}{|V_{cb}V_{cs}^*|^2}\frac{(V_{cs}V_{cd}^*)^2}{|V_{cs}V_{cd}^*|^2}e^{i2\chi'} = -e^{-i2(\beta-\chi')} \simeq -e^{-i2\beta}$$

$$C_{J/\Psi K_S} = 0 \qquad S_{J/\Psi K_S} = \sin 2\beta \qquad R_{J/\Psi K_S} = -\cos 2\beta$$

$\sin 2\beta$

- Time dependent rates

$$\left|\langle J/\Psi K_S|\mathcal{T}|B_d^0(t)\rangle\right|^2 = e^{-\Gamma t}\Gamma_{J/\Psi K_S}\left\{1 - \sin 2\beta\, \sin\left(\Delta M t\right)\right\}$$

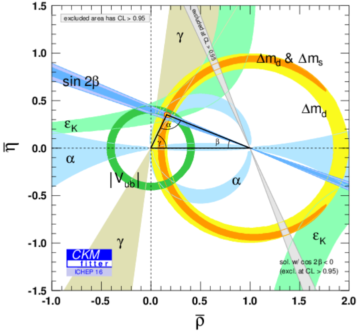
$$\left|\langle J/\Psi K_S|\mathcal{T}|\bar{B}_d^0(t)\rangle\right|^2 = e^{-\Gamma t}\Gamma_{J/\Psi K_S}\left\{1 + \sin 2\beta\, \sin\left(\Delta M t\right)\right\}$$

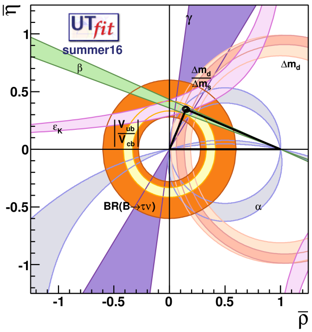
- CP Violating Asymmetry

$$\frac{\left|\langle J/\Psi K_S|\mathcal{T}|B_d^0(t)\rangle\right|^2 - \left|\langle J/\Psi K_S|\mathcal{T}|\bar{B}_d^0(t)\rangle\right|^2}{\left|\langle J/\Psi K_S|\mathcal{T}|B_d^0(t)\rangle\right|^2 + \left|\langle J/\Psi K_S|\mathcal{T}|\bar{B}_d^0(t)\rangle\right|^2} = -2\, \sin 2\beta\, \sin\left(\Delta M t\right)$$

- Experimentally: $\sin 2\beta \simeq 0.68 \pm 0.02$

Remember CKM Fits?





More stuff

- Effective approach
- QCD effects, RGE
- New Physics sensitivity of flavour processes