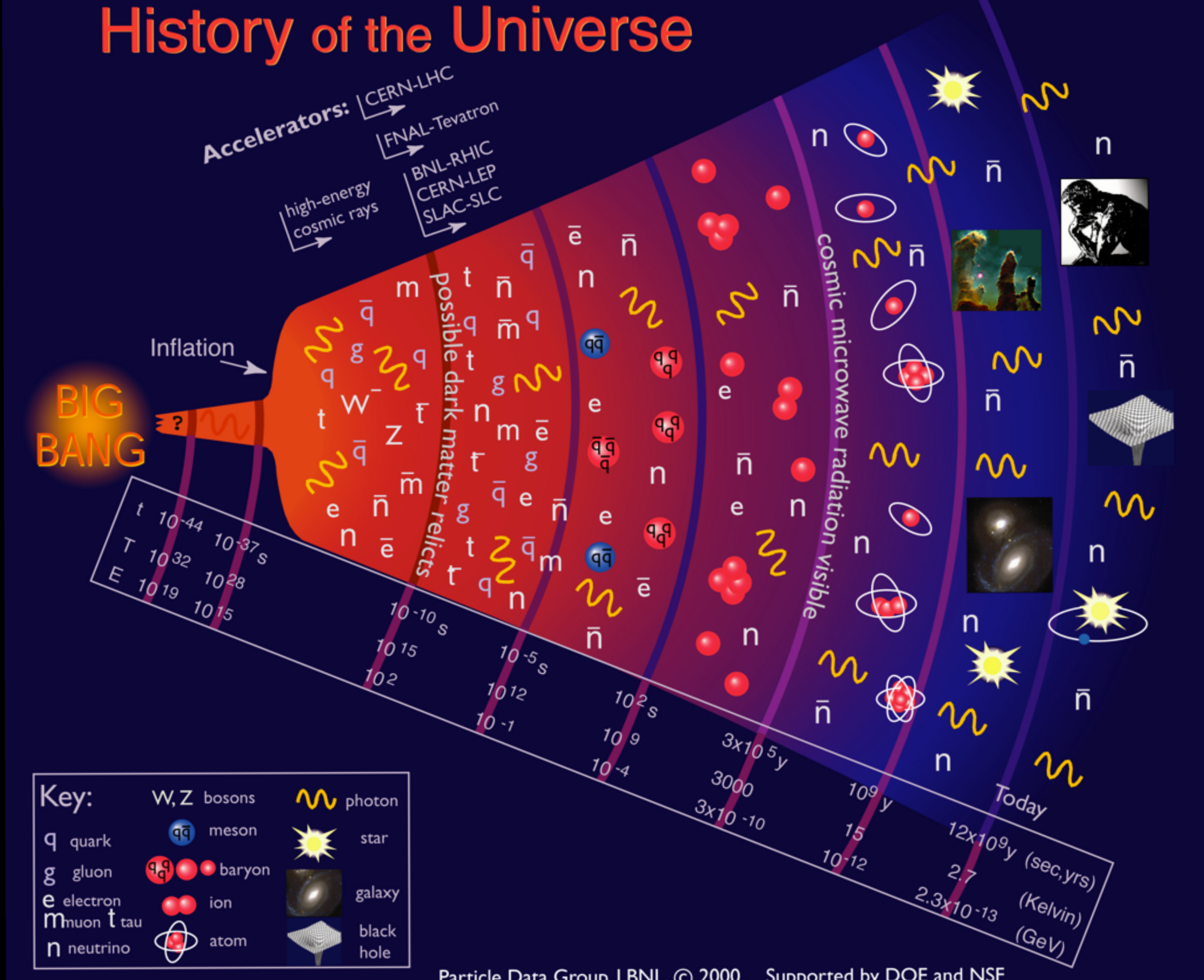
Lesson 2:

Thermal History of our universe


History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10-44 10-37s
T 1032 1028
E 1019 1015
10-10 s
1015
102
10-5s
1012
10-1
102s
109
10-4
3x105y
3000
3x10-10
109y
15
10-12
Today
12x109y (sec,yrs)
2.7 (Kelvin)
2.3x10-13 (GeV)
Key:
q quark
g gluon
e electron
m muon
t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group LBNL © 2000 Supported by DOE and NSF

Event	Time	Redshift	Temperature
Baryogenesis	?	?	?
EW phase transition	$2 \times 10^{-11} s$	10^{15}	$100 GeV$
QCD phase transition	$2 \times 10^{-5} s$	10^{12}	$150 MeV$
Neutrino decoupling	$1 s$	6×10^{9}	$1 MeV$
Electron-positron annihilation	$6 s$	2×10^{9}	$500 keV$
Big bang nucleosynthesis	$3 min$	4×10^{8}	$100 keV$
Matter-radiation equality	$6 \times 10^{4} yrs$	3400	$.75 eV$
Recombination	$2.6 - 3.8 \times 10^{5} yrs$	1100-1400	$.26 - .33 eV$
CMB	$3.8 \times 10^{5} yrs$	1100	$.26 eV$

2.1 Particle distribution functions

- The usual way of describing particles in thermal equilibrium is via their distribution function, indicating the number of particles in the phase space with a given position x and a momentum p. At 0th order, we have the Bose Einstein or the Fermi-Dirac distributions:

- $$f_{BE} = \frac{1}{e^{(E-\mu)/T} - 1} \qquad f_{FD} = \frac{1}{e^{(E-\mu)/T} + 1}$$

- The number and energy densities and the pressure read as:

$$n = \frac{g}{(2\pi)^3} \int f(\vec{x}, \vec{p}) d^3x d^3p$$

- Warning! you will have to work with this expression in the "Hands in Cosmology" session

$$\rho = \frac{g}{(2\pi)^3} \int E(\vec{p}) f(\vec{x}, \vec{p}) d^3x d^3p$$

$$p = \frac{g}{(2\pi)^3} \int \frac{p^2}{3E(p)} f(\vec{x}, \vec{p}) d^3x d^3p$$

- While the entropy density is

$$s \equiv \frac{\rho + p}{T}$$

2.2 BOLTZMANN EQUATIONS

- Throughout the universe's history, particles remain in thermal equilibrium until their interaction rate is equal or larger than the expansion rate of the universe. Then, the particle will decouple from the thermal bath. Of course this is an approximation:

$$\Gamma \lesssim H$$

- The accurate calculation requires to solve the Boltzmann equation:

$$L f = C f$$

- where f is the distribution function, L is the Liouville operator, and C contains all the collision terms.

- In classical mechanics: $\hat{L} = \frac{d}{dt} + \vec{v} \cdot \vec{\nabla}_x + \vec{F}/m \cdot \vec{\nabla}_v$

- The relativistic version is: $\hat{L} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}$ $\quad P^\alpha = (E, \vec{P}) \quad P^\alpha = \frac{dx^\alpha}{d\lambda}$

- FRW geometry: $\hat{L} f = E \frac{\partial f}{\partial t} - H p^2 \frac{\partial f}{\partial E}$

$$\frac{dn}{dt} + 3Hn = \frac{g}{(2\pi)^3} \int C f \frac{d^3p}{E}$$

2.2 BOLTZMANN EQUATIONS

- **The non-relativistic Boltzmann equation: the Liouville operator is just the total time derivative**

$$\frac{f(\mathbf{x}+\frac{\mathbf{p}}{m}dt,\mathbf{p}+\mathbf{F}dt,t+dt)-f(\mathbf{x},\mathbf{p},t)}{dt} = \frac{\partial}{\partial t}f(\mathbf{x},\mathbf{p},t)+\frac{p^i}{m}\frac{\partial}{\partial x^i}f(\mathbf{x},\mathbf{p},t)+F^i\frac{\partial}{\partial p^i}f(\mathbf{x},\mathbf{p},t)$$

$$L_{NR}=\frac{\partial}{\partial t}+\frac{dx^i}{dt}\frac{\partial}{\partial x^i}+\frac{dp^i}{dt}\frac{\partial}{\partial p^i}=\frac{d}{dt}$$

$$\hat{L}=\frac{d}{dt}+\vec{v}\cdot\vec{\nabla}_x+\vec{F}/m\cdot\vec{\nabla}_v$$

2.2 BOLTZMANN EQUATIONS

- **The relativistic version is:**

$$\hat{L} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha} \qquad P^\alpha = (E, \vec{P}) \quad P^\alpha = \frac{dx^\alpha}{d\lambda}$$

- **FLRW geometry, the only non-vanishing component is α = 0:**

$$\hat{L}f = E\frac{\partial f}{\partial t} - Hp^2 \frac{\partial f}{\partial E}$$

- **We can also write the Boltzmann equation in terms of the number density:**

$$n_A = 4\pi \int dpp^2 f_A(E,t)$$

- **Dividing by the energy and integrating over the momentum:**

$$4\pi \int dpp^2 \frac{\hat{L}[f_A]}{E} = \frac{dn_A}{dt} - H4\pi \int dp \frac{p^4}{E}\frac{\partial f_A}{\partial E} = \frac{dn_A}{dt} + H4\pi \int dp \frac{\partial (p^3)}{\partial p} f_A$$

$$\boxed{\frac{dn}{dt} + 3Hn = \frac{g}{(2\pi)^3} \int Cf \frac{d^3p}{E}}$$

2.2 BOLTZMANN EQUATIONS

- Simplifying the possible processes (1+2 ↔3+4):

In an expanding universe, the number of particles gets diluted!

In the absence of interactions, $n \propto a^{-3}$

$$\frac{dn}{dt} + 3Hn = \int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4}$$

Particle Physics

$$\times (2\pi)^4 \delta^3(p^1 + p^2 - p^3 - p^4)\delta(E^1 + E^2 - E^3 - E^4)|\mathcal{M}|^2$$

Energy-momentum tensor conservation

$$\times (f_3 f_4 - f_1 f_2)$$

Loss rate of 1 is proportional to the occupation numbers of 1 and 2

Production rate of 1 is proportional to the occupation numbers of 3 and 4

2.2 BOLTZMANN EQUATIONS

- At temperatures smaller than $E - \mu$: $f(E) \rightarrow e^{\mu/T} e^{-E/T}$

- Therefore :

$$f_3 f_4 - f_1 f_2 \rightarrow e^{-(E_1+E_2)/T} \left(e^{(\mu_3+\mu_4)/T} - e^{(\mu_1+\mu_2)/T} \right)$$

- Using the following definitions for the number density and the equilibrium number density of species as:

$$n_i = g_i e^{\mu_i/T} \int \frac{d^3p}{(2\pi)^3} e^{-E_i/T} \qquad n_i^{(0)} \equiv g_i \int \frac{d^3p}{(2\pi)^3} e^{-E_i/T}$$

- Using these two expressions:

$$f_3 f_4 - f_1 f_2 \rightarrow e^{-(E_1+E_2)/T} \left(\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right)$$

2.2 BOLTZMANN EQUATIONS

- **Defining the thermally averaged cross-section as:**

$$\langle \sigma v \rangle \equiv e^{-(E_1+E_2)/T} \left(\frac{1}{n_1^{(0)} n_2^{(0)}} \right) \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \int \frac{d^3 p_3}{(2\pi)^3 2E_3} \int \frac{d^3 p_4}{(2\pi)^3 2E_4}$$

$$\times (2\pi)^4 \delta^3 (p^1 + p^2 - p^3 - p^4) \delta(E^1 + E^2 - E^3 - E^4) |\mathcal{M}|^2$$

2.2 BOLTZMANN EQUATIONS

- **After defining the thermally-averaged cross section, as we have seen:**

$$\frac{dn}{dt} + 3Hn = n_1^0 n_2^0 \langle \sigma v \rangle \left(\frac{n_3 n_4}{n_3^0 n_4^0} - \frac{n_1 n_2}{n_1^0 n_2^0} \right)$$

$$g_i (m_i T/2\pi)^{3/2} e^{-m_i/T} \quad m_i \gg T$$

- **where the equilibrium density:** $n_i^0 \equiv g_i \int \frac{d^3p}{(2\pi)^3} e^{-E_i/T}$

$$g_i \frac{T^3}{\pi^2} \quad m_i \ll T$$

2.2 BOLTZMANN EQUATIONS

$$\frac{dn}{dt} + 3Hn = n_1^0 n_2^0 \langle \sigma v \rangle \left(\frac{n_3 n_4}{n_3^0 n_4^0} - \frac{n_1 n_2}{n_1^0 n_2^0} \right)$$

	1	2	3	4
• **Neutron-Proton ratio**	n	ν_e / e^+	p	$e^- / \bar{\nu}_e$
• **Recombination**	e	p	H	γ
• **Dark matter production**	X	X	l	l

- **When looking into the DM annihilating case, XX ↔ ll, 3 and 4 will not couple anymore and therefore:**

$$n_3 n_4 = n_3^0 n_4^0$$

$$\frac{dn_X}{dt} + 3Hn_X = \langle \sigma v \rangle \left((n_X^0)^2 - n_X^2 \right)$$

$$Y \equiv \frac{n_X}{T^3} \qquad \frac{dY}{dt} = T^3 \langle \sigma v \rangle \left(Y_{EQ}^2 - Y^2 \right)$$

2.3 Particle decoupling in the early universe: Neutrinos

See Sergio Pastor's lectures

- We have seen that a very easy and straightforward hand-waving rule to compute a particle decoupling time in the early universe is:

$$\Gamma \lesssim H$$

- Neutrinos only interact via weak interactions, with a rate:

$$\Gamma_\nu = n\sigma v \simeq T^3 G_F^2 T^2 \sim G_F^2 T^5$$

- While the expansion rate of the universe is given by the Hubble factor:

$$H^2 = \frac{8\pi G}{3}\rho \sim T^4/m_{pl}^2$$

$$\Gamma_\nu/H \sim \left(\frac{T}{1 \text{ MeV}}\right)^3$$

- Therefore neutrinos decouple from the thermal bath around 1 MeV.

2.3 Particle decoupling in the early universe: Neutrinos

See Sergio Pastor's lectures

- The entropy density is: $s \equiv \dfrac{\rho + p}{T}$

¿How are related the photon and the neutrino temperatures?

- Electron positron annihilation takes place AFTER neutrino decoupling.

- In an expanding universe the entropy density pero comoving volume is conserved:

- Boson's entropy contribution: $2\pi^2 T^3/45$
- Fermion's entropy contribution: $7/8 \times 2\pi^2 T^3/45$

- Before electron/positron annihilation= electrons (g=2), positrons (g=2), neutrinos (3), antineutrinos (3) and photons (g=2) therefore:

$$s(a_1) = 2\pi^2 T_1^3/45(2 + 7/8(2 + 2 + 3 + 3))$$

- After, only neutrinos, antineutrinos and photons but at different temperature!

$$s(a_2) = 2\pi^2/45(2T_\gamma^3 + 7/8(3 + 3)T_\nu^3)$$

$$s(a_1)a_1^3 = s(a_2)a_2^3 \qquad a_1 T_1 = a_2 T_\nu \longrightarrow \left(\frac{T_\nu}{T_\gamma}\right) = \left(\frac{4}{11}\right)^{1/3}$$

2.4 Big Bang Nucleosynthesis (BBN)

Nuclear binding energies are of a few MeV, that's why BBN occurs when the temperature of the universe is around 1 MeV, being the universe made of:

- Relativistic particles in equilibrium: photons, electrons and positrons ($e^+e^- \leftrightarrow \gamma\gamma$)
- Decoupled relativistic particles: neutrinos
- Non-relativistic particles (baryons):

$$\eta_b \equiv \frac{n_b}{n_\gamma} = 5.5 \times 10^{-10} \left(\frac{\Omega_b h^2}{0.020} \right)$$

- There are way less baryons than photons!

- Neutrons and protons interact via weak interactions:

$$p + \bar{\nu} \leftrightarrow n + e^+ \qquad p + e^- \leftrightarrow n + \nu \qquad n \leftrightarrow p + e^- + \bar{\nu}$$

- Light elements are formed via nuclear interactions:

$$p + n \rightarrow D + \gamma \qquad D + D \rightarrow n + {}^3He \qquad {}^3He + D \rightarrow p + {}^4He$$

- Calculations will be way simpler because we will assume that:
- We neglect elements heavier than Helium/Lithium.
- At temperatures above 0.1 MeV only free protons and neutrons exist (at higher temperatures, any time a nucleus is formed, it is destroyed by a high-energy photon)

2.4 Big Bang Nucleosynthesis (BBN)

$$p + n \rightarrow D + \gamma$$

- **In equilibrium:**

$$\frac{dn}{dt} + 3Hn = n_1^0 n_2^0 \langle\sigma v\rangle \left(\frac{n_3 n_4}{n_3^0 n_4^0} - \frac{n_1 n_2}{n_1^0 n_2^0}\right) \xrightarrow{n_\gamma = n_\gamma^0} \frac{n_D}{n_n n_p} = \frac{n_D^0}{n_n^0 n_p^0}$$

- **We have seen that:**

$$n_i^0 \equiv g_i \int \frac{d^3p}{(2\pi)^3} e^{-E_i/T} \qquad g_i (m_i T/2\pi)^{3/2} e^{-m_i/T} \qquad m_i \gg T$$

- **Therefore**

$$\longrightarrow \frac{n_D}{n_n n_p} = \frac{3}{4} \left(\frac{2\pi m_D}{m_n m_p T}\right)^{3/2} e^{(m_n + m_p - m_D)/T}$$

- The factor 3/4 refers to the spin states (g). Simplifying a bit more:

$$n_n, n_p \propto n_b \longrightarrow \frac{n_D}{n_b} \sim \eta_b \left(\frac{T}{m_p}\right)^{3/2} e^{(B_D)/T}$$

where B_D=2.2 MeV. If the condition $T \ll B_D$ does not apply, all baryons are protons and neutrons: no light elements! This is what happens at T> 0.1 MeV. For T < 0.1 MeV, deuterium and helium are produced, but not heavier elements, as there are not heavy stable isotopes with mass number A=5 to be produced as: $^4He + p \rightarrow X$

In stars, the process:

$$^4He + ^4He + ^4He \rightarrow ^{12}C$$

takes place but in the early universe densities are far too low to allow for such a process!

2.4 Big Bang Nucleosynthesis (BBN)

$$\frac{n_D}{n_b} \sim \eta_b \left(\frac{T}{m_p}\right)^{3/2} e^{(B_D)/T}$$

The deuterium equilibrium abundance is of order of the baryon abundance if:

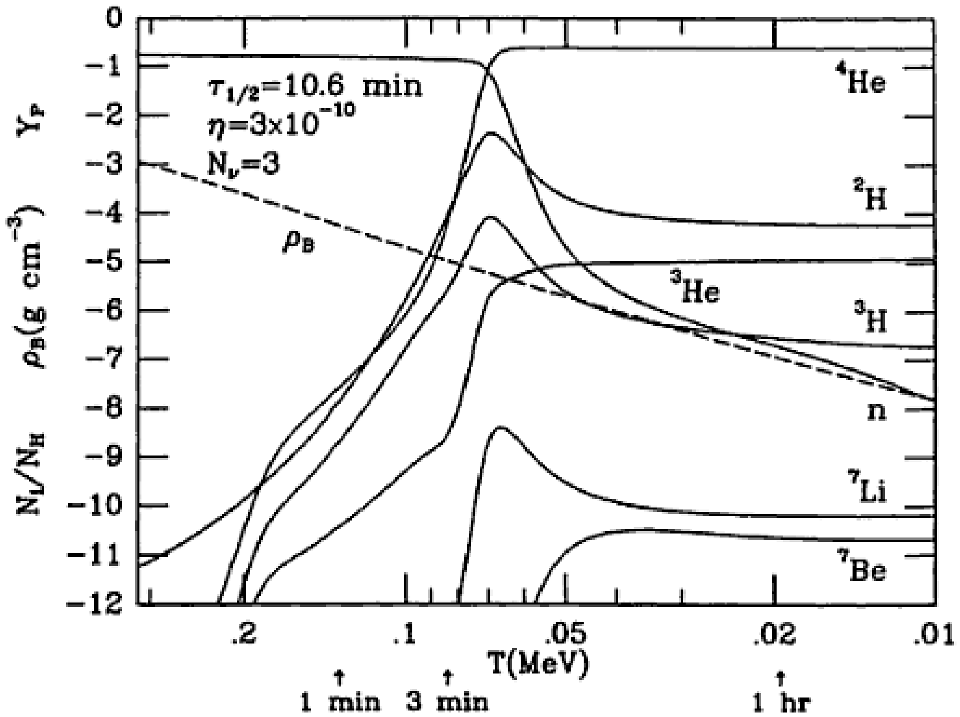
$$\ln(\eta_b) + 3/2\ln(T/m_p) \simeq -B_D/T$$

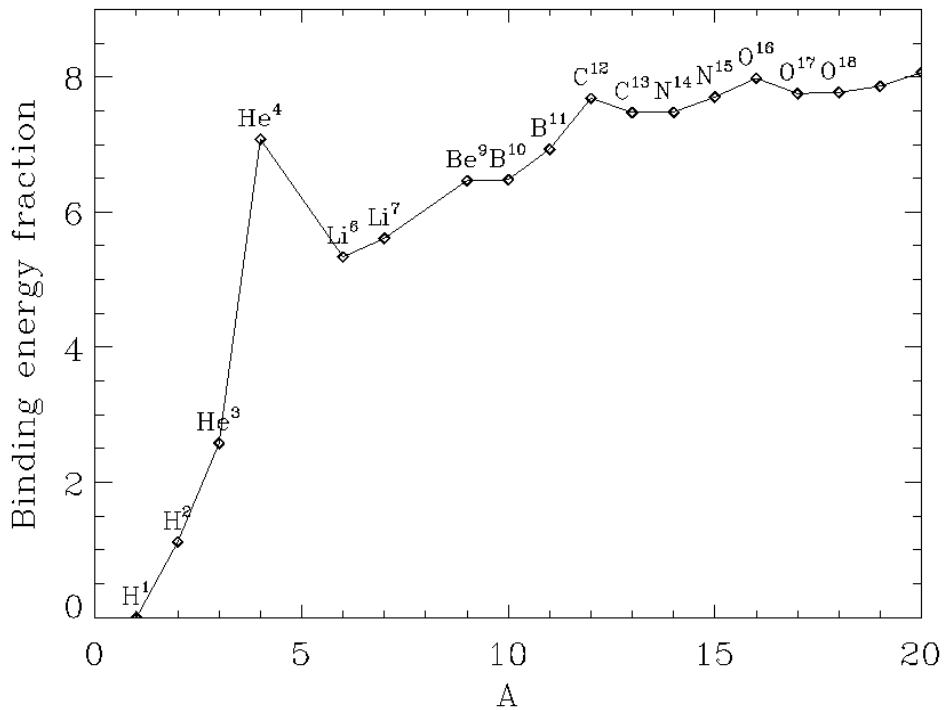
suggesting that deuterium production starts at T= 0.07 MeV or so.

Helium is favoured over Deuterium, as its binding energy is larger!
Almost all remaining neutrons at 0.07 MeV ended up into Helium. Since two neutrons go into Helium, the final Helium abundance is half the neutron abundance at that temperature, with a final abundance of 0.24 or so.

Not all Deuterium is processed into Helium, though!

2.4 BBN





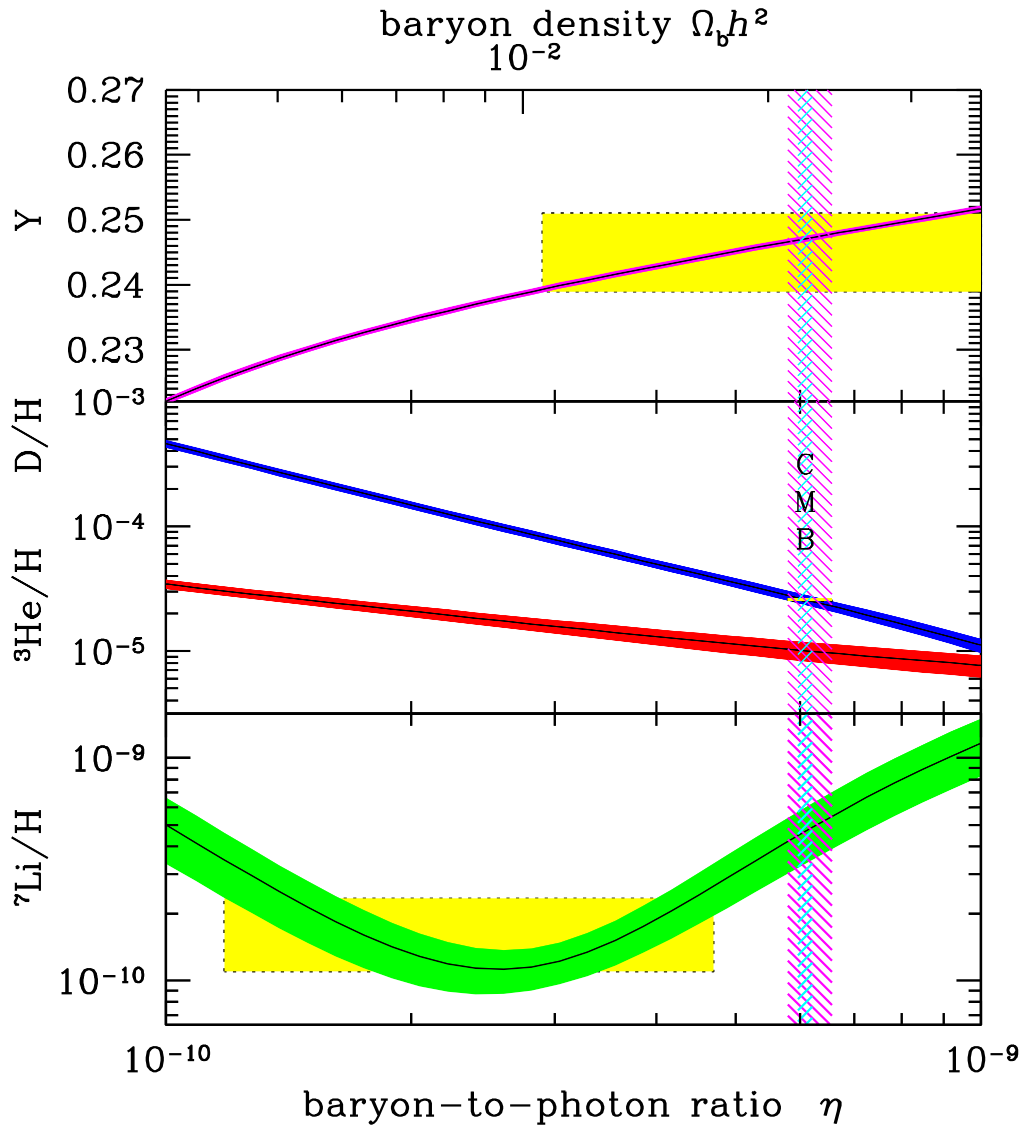


Figure 24.1: The primordial abundances of ^{4}He, D, ^{3}He, and ^{7}Li as predicted by the standard model of Big-Bang nucleosynthesis — the bands show the 95% CL range [5]. Boxes indicate the observed light element abundances. The narrow vertical band indicates the CMB measure of the cosmic baryon density, while the wider band indicates the BBN D+^{4}He concordance range (both at 95% CL).

2.5 Recombination

• At temperatures above 1 eV, electrons and photons interact via Compton, while electrons and protons via Coulomb scattering

$$e^- + p \leftrightarrow H + \gamma$$

• Some hydrogen atoms are formed, however photons are much more numerous than baryons and they are ionised very fast!

$$\frac{dn}{dt} + 3Hn = n_1^0 n_2^0 \langle \sigma v \rangle \left(\frac{n_3 n_4}{n_3^0 n_4^0} - \frac{n_1 n_2}{n_1^0 n_2^0} \right)$$

$$X_e \equiv \frac{n_p}{n_p + n_H} = \frac{n_e}{n_e + n_H}$$ Universe's neutrality

$$\frac{dn}{dt} + 3Hn = n_b \langle \sigma v \rangle \left((1 - X_e)(m_e T / 2\pi)^{3/2} e^{-(m_e + m_p - m_H)/T} - X_e^2 n_b \right)$$

$$n_e = n_b X_e$$

$$\frac{dX_e}{dt} = \left((1 - X_e)\beta - X_e^2 n_b \alpha^{(2)} \right)$$

$$\beta \equiv \langle \sigma v \rangle (m_e T / 2\pi)^{3/2} e^{-\epsilon_0 / T}$$ **Ionisation**

$$\alpha^{(2)} \equiv \langle \sigma v \rangle$$ **Recombination rate to n=2 (n=1 is irrelevant as it leads to the production of a photon that will immediately ionise a neutral atom!)**

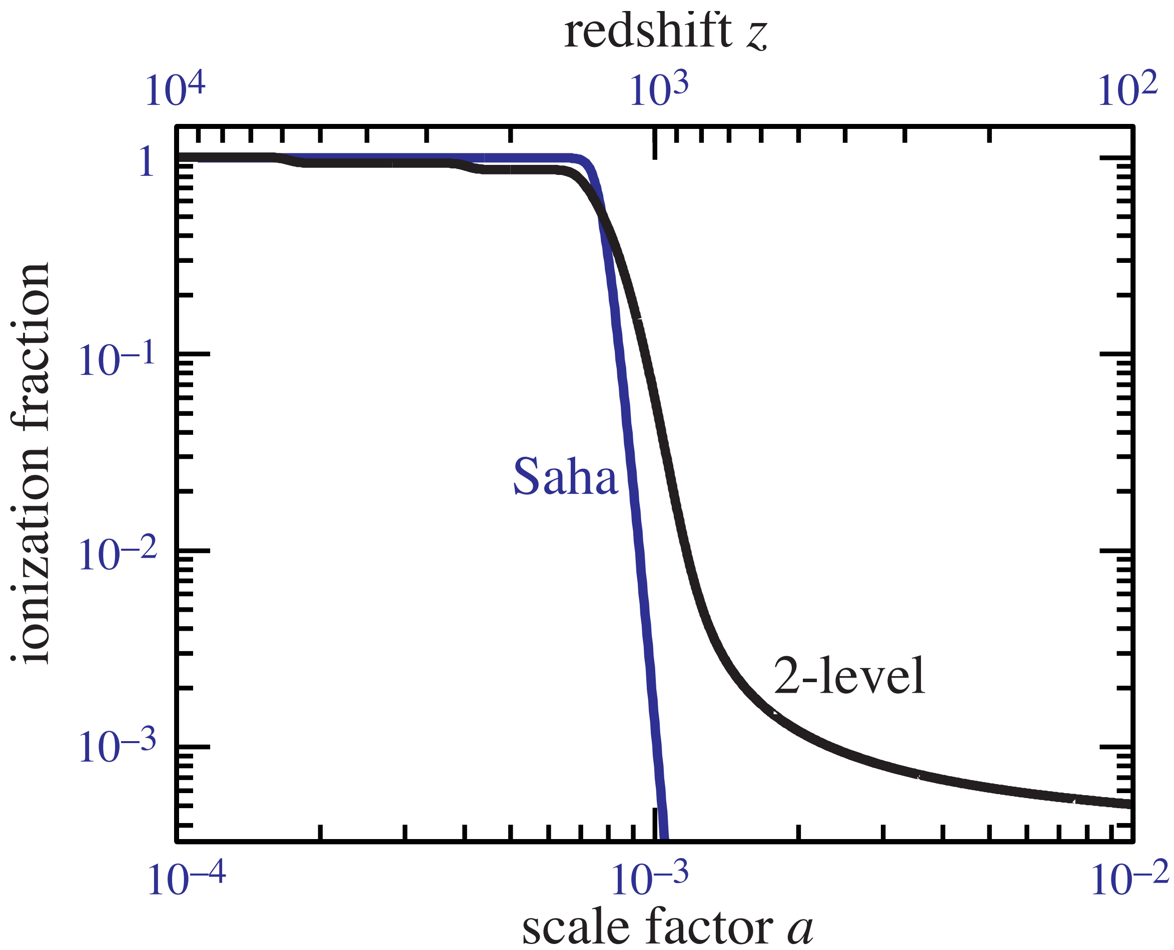
$$\alpha^{(2)} = 9.78 \alpha^2 / m_e^2 (\epsilon_0 / T)^{1/2} \ln(\epsilon_0 / T)$$

2.5 Recombination

Saha equation fails to describe the recombination process once that equilibrium becomes difficult to maintain: $e^- + p \leftrightarrow H + \gamma$

$$\frac{dn}{dt} + 3Hn = n_1^0 n_2^0 \langle \sigma v \rangle \left(\frac{n_3 n_4}{n_3^0 n_4^0} - \frac{n_1 n_2}{n_1^0 n_2^0} \right) \qquad n_\gamma = n_\gamma^0$$

$$\frac{n_e n_p}{n_H} = \frac{n_e^0 n_p^0}{n_H^0} \longrightarrow \frac{X_e^2}{1 - X_e} = \frac{1}{n_e + n_H} \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-(m_e + m_p - m_H)/T}$$



2.5 Photon decoupling

- Photons decoupled from electrons when the Compton interaction rate is below the universe's expanding rate:

$$n_e \sigma_T = X_e n_b \sigma_T = H$$

Thomson cross section= 0.665 10^{-24} cm^2

$$X_e \sigma_T \Omega_b h^2 a^{-3} \rho_{crit} m_p = H$$

$$H^2(a) = H_0^2 \left(\Omega_m(a) + \Omega_r(a) + \Omega_{de}(a)\right)$$

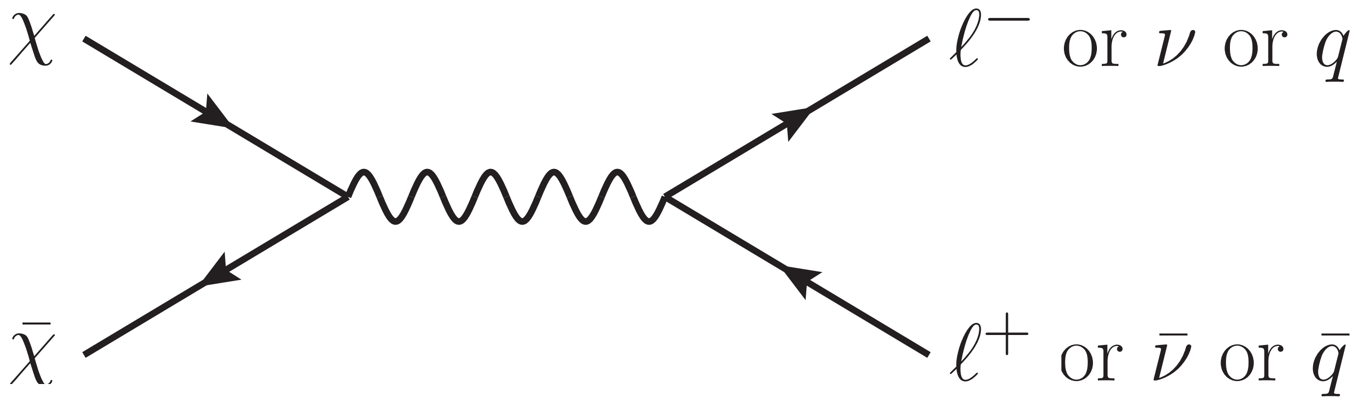
$$\frac{n_e \sigma_T}{H} = 113 X_e (\Omega_b h^2/0.02)(0.15/\Omega_m h^2)^{1/2} \left(\frac{1+z}{1000}\right)^{3/2} \left(1 + \frac{1+z_{eq}}{3600}\frac{0.15}{\Omega_m h^2}\right)^{-1/2}$$

The redshift at which decoupling takes place is z=1000, and therefore it happens during the recombination epoch!

2.6 Particle decoupling in the early universe:

See Sergio Palomares' lectures

Dark Matter decoupling and freeze-out



$$\frac{dn}{dt} + 3Hn = n_1^0 n_2^0 \langle \sigma v \rangle \left(\frac{n_3 n_4}{n_3^0 n_4^0} - \frac{n_1 n_2}{n_1^0 n_2^0} \right) \qquad \begin{array}{l} n_3 = n_3^{(0)} \\ n_4 = n_4^{(0)} \end{array}$$

$$\frac{dn_X}{dt} + 3Hn_X = \langle \sigma v \rangle \left((n_X^0)^2 - n_X^2 \right)$$

$$Y \equiv \frac{n_X}{T^3} \qquad\qquad \frac{dY}{dt} = T^3 \langle \sigma v \rangle \left(Y_{EQ}^2 - Y^2 \right)$$

2.6 Particle decoupling in the early universe: Dark Matter decoupling and freeze-out

See Sergio Palomares' lectures

$$\frac{dY}{dt} = T^3 \langle \sigma v \rangle \left(Y_{EQ}^2 - Y^2 \right) \quad \longleftrightarrow_{x \equiv m/T} \quad \frac{dY}{dx} = \frac{\lambda}{x^2} \left(Y^2 - Y_{EQ}^2 \right)$$

$$\lambda \equiv \frac{m^3 \langle \sigma v \rangle}{H(m)}$$

- This equation is a Riccatti-like one (with no analytical solutions)
- For very high temperatures, x <<1, reactions happen really FAST! $Y = Y_{EQ}$.

2.6 Particle decoupling in the early universe:

Dark Matter decoupling and freeze-out

See Sergio Palomares' lectures

$$\frac{dY}{dx} = \frac{\lambda}{x^2}\left(Y^2 - Y_{EQ}^2\right) \qquad x \equiv m/T \qquad \lambda \equiv \frac{m^3\langle\sigma v\rangle}{H(m)}$$

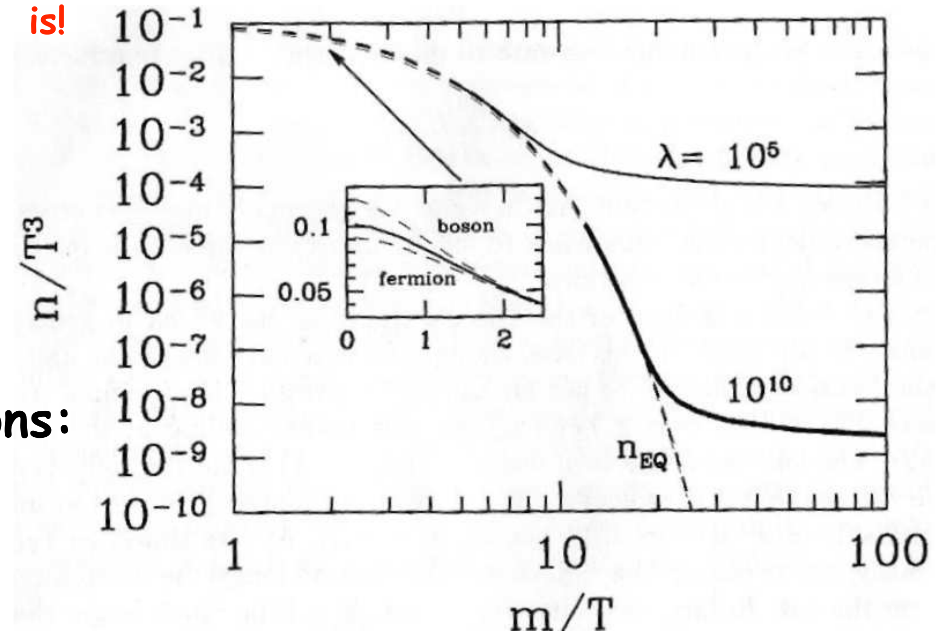
- **For x>>1, Y_{EQ} is exponentially suppressed, and the dark matter number density is so low that the interaction is no longer effective: FREEZE OUT**

$$\frac{dY}{dx} \simeq \frac{\lambda Y^2}{x^2} \qquad \frac{1}{Y_\infty} - \frac{1}{Y_f} = \frac{\lambda}{x_f}$$

$$Y_\infty \simeq \frac{x_f}{\lambda} \longleftrightarrow_{x_f \sim 10} Y_\infty \simeq \frac{10}{\lambda}$$

$$\Omega_X = \frac{H(m) x_f T_0^3}{30 m^2 \langle\sigma v\rangle \rho_{crit}} \qquad \Omega_X h^2 = \frac{3\times 10^{-27}\ \mathrm{cm^3 s^{-1}}}{\langle\sigma v\rangle}$$

The larger the interaction rate, the later it freezes out and the lower the relic abundance is!



- **Assuming weak-like interacting cross sections:**

$$\Omega_X h^2 \sim 0.1$$ "The WIMP miracle"!

Backup slides

$$
\begin{array}{lclcll}
p & + & n & \rightarrow & {}^{2}H & + \gamma \\
{}^{2}H & + & p & \rightarrow & {}^{3}He & + \gamma \\
{}^{2}H & + & {}^{2}H & \rightarrow & {}^{3}H & + p \\
{}^{2}H & + & {}^{2}H & \rightarrow & {}^{3}He & + n \\
n & + & {}^{3}He & \rightarrow & {}^{3}H & + p \\
p & + & {}^{3}H & \rightarrow & {}^{4}He & + \gamma \\
{}^{2}H & + & {}^{3}H & \rightarrow & {}^{4}He & + n \\
{}^{2}H & + & {}^{3}He & \rightarrow & {}^{4}He & + p \\
{}^{4}He & + & {}^{3}He & \rightarrow & {}^{7}Be & + \gamma \\
{}^{4}He & + & {}^{3}H & \rightarrow & {}^{7}Li & + \gamma \\
{}^{7}Be & + & n & \rightarrow & {}^{7}Li & + p \\
{}^{7}Li & + & p & \rightarrow & {}^{4}He & + {}^{4}He
\end{array}
$$

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2.4 Big Bang Nucleosynthesis (BBN)

$$\frac{dn}{dt} + 3Hn = n_1^0 n_2^0 \langle \sigma v \rangle \left(\frac{n_3 n_4}{n_3^0 n_4^0} - \frac{n_1 n_2}{n_1^0 n_2^0} \right)$$

- **Neutron-to-proton ratio** $p + e^- \rightarrow n + \nu_e$

- **In equilibrium, the neutron-to-proton ratio in the non-relativistic limit reads as:**

$$\frac{n_p^{(0)}}{n_n^{(0)}} = e^{Q/T} \qquad Q \equiv m_n - m_p = 1.293 \text{ MeV}$$

- **At high temperatures, there are as many protons as neutrons. As temperature decreases, the neutron fraction goes down.**

$$X_n \equiv \frac{n_n}{n_n + n_p} \qquad \frac{dX_n}{dt} = \lambda_{np} \left((1 - X_n) e^{-Q/T} - X_n \right)$$

$$\lambda_{np} = n_l^{(0)} \langle \sigma v \rangle$$