Higgs and physics Beyond the SM

2.- Supersymmetry

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SM Theoretical "Problems"

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- Assignment of matter Quantum Numbers
- Unification of Gauge Couplings
- Origin of Spontaneous Symmetry Breaking
- Accomodate Quantum Gravity
- Large number of Flavour Parameters

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We DO need Physics Beyond the SM !!

Supersymmetry

• Only possible extension of symmetry beyond Lie Symmetries (Coleman–Mandula Theorem).

• Correct Unification of Gauge couplings at M_{GUT}, GUT assignment of Quantum numbers (anomaly cancellation).

• Solution of the Hierarchy Problem, strong motivation for low-energy SUSY.

• "Natural" Mechanism of Electroweak Symmetry Breaking , Radiative Symmetry Breaking.

• SUSY is a necessary ingredient in String Theory. Local Supersymmetry $\Leftrightarrow$ Supergravity.

Coleman–Mandula Theorem

In the 60's attempts to combine internal and Lorentz symmetries ...

> The only conserved quantities that transform as tensors under Lorentz transformations in a theory with non-zero scattering amplitudes in 4D are the generators of the Poincare group and Lorentz invariant quantum numbers (scalar charges).

2×2 spinless particle scattering, bosonic conserved charge, $\Sigma_{\mu\nu}$,

$$\langle 1|\Sigma_{\mu\nu}|1\rangle = \alpha p^1_\mu p^1_\nu + \beta g_{\mu\nu}$$

So, in the scattering process,

$$p^1_\mu p^1_\nu + p^2_\mu p^2_\nu = p^3_\mu p^3_\nu + p^4_\mu p^4_\nu \quad \& \quad p^1_\mu + p^2_\mu = p^3_\mu + p^4_\mu$$

Not possible in a theory with non-zero scattering.

However: Coleman–Mandula theorem does not forbid conserved spinor charges, Q_α (transforming like fermions under Lorentz)

$$Q_\alpha|\text{Boson}\rangle = |\text{Fermion}\rangle \qquad Q_\alpha|\text{Fermion}\rangle = |\text{Boson}\rangle$$
$$[Q_\alpha, H] = 0 \qquad [\{Q_\alpha, \bar{Q}_{\dot\beta}\}, H] = 0$$

Supersymmetry Algebra

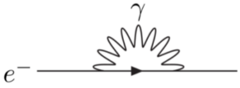
$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \qquad \{Q_\alpha, Q_\beta\} = 0 \qquad \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0$$

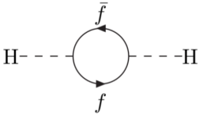
Supersymmetry relates particles of different spin with equal quantum numbers and identical masses.

$$\left(\begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}\right) \sim \left(\begin{pmatrix} q \text{ (quark)} \\ \tilde{q} \text{ (squark)} \end{pmatrix}\right) \qquad \left(\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}\right) \sim \left(\begin{pmatrix} g \text{ (gluon)} \\ \tilde{g} \text{ (gluino)} \end{pmatrix}\right)$$

Chiral supermultiplet — Gauge supermultiplet

Hierarchy Problem





$$\delta m_e = 2\frac{\alpha_{em}}{\pi} m_e \log \frac{\Lambda}{m_e}$$

with $m_e \to 0$, chiral symmetry

$$\delta m_H^2(f) = -2N_f \frac{|\lambda_f|^2}{16\pi^2}[\Lambda^2 - 2m_f^2 \ln \frac{\Lambda}{m_f}]$$

No symmetry protects m_H^2 ...

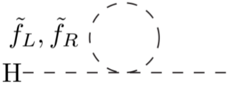
Quadratic div.!!

$$\Lambda = 10^{19} \text{ GeV}$$

$$\delta m_e = 0.24 \; m_e$$

$$\delta m_H^2 \simeq 10^{30} \text{ GeV}$$

Supersymmetry $\Rightarrow$ $+$



$$\delta m_H^2(\tilde{f}) = -2N_{\tilde{f}}\frac{\lambda_{\tilde{f}}}{16\pi^2}[\Lambda^2 - \ 2m_{\tilde{f}}^2 \ln\frac{\Lambda}{m_{\tilde{f}}} \ + \dots]$$

$\delta m_H^2(f) + \delta m_H^2(\tilde{f}) = 0$ if $N_f = N_{\tilde{f}}, \ |\lambda_f|^2 = -\lambda_{\tilde{f}}$ and $m_f = m_{\tilde{f}}$

$\Rightarrow$ Supersymmetry + Chiral symmetry solve hierarchy problem.

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Soft Supersymmetry breaking

• Preserve cancellation of Quadratic divergencies requires dimensionless couplings still supersymmetric: $|\lambda_f|^2 = -\lambda_{\tilde{f}}$

• SUSY only broken in couplings with possitive mass dimension: Soft Breaking $m_{\tilde{f}}^2 = m_f^2 + \delta^2$

$$\delta m_H^2(f) + \delta m_H^2(\tilde{f}) \simeq 2N_f \frac{|\lambda_f|^2}{16\pi^2} \delta^2 \ln \frac{\Lambda}{m_{\tilde{f}}} + \dots$$

to solve hierarchy problem $\delta \lesssim 1$ TeV

GUT and coupling unification

Grand Unification

- Simple gauge group unifying $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$
- All matter multiplets in 1 generation unified in a single (two) representation of the gauge group:

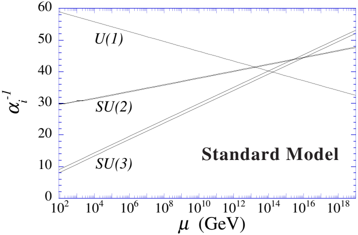
$$SU(5) \qquad \bar{\mathbf{5}} = \left(\begin{pmatrix} d^c \\ d^c \\ d^c \\ e^- \\ \nu_e \end{pmatrix} \right)_L \qquad \mathbf{10} = \left(\begin{pmatrix} 0 & u^c & u^c & u & d \\ & 0 & u^c & u & d \\ & & 0 & u & d \\ & & & 0 & e^c \\ & & & & 0 \end{pmatrix} \right)_L$$

$$S0(10) \qquad \mathbf{16} = \mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}$$

$\Rightarrow$Explains the assignment of quantum numbers in the SM

RGE evolution of gauge couplings Running the couplings to high energies they come close at $M_{GUT} \simeq 10^{16}$ GeV

SM



Using $SU(3)$ and $SU(2)_L$ to predict $\sin\theta_W$ with correct unification :

$$\sin\theta_W^{th} = 0.214 \pm 0.004$$
$$\sin\theta_W^{exp} = 0.23149 \pm 0.00017$$

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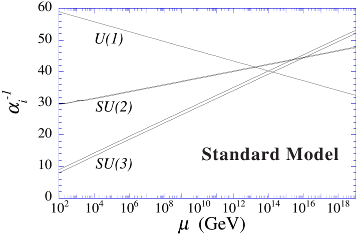

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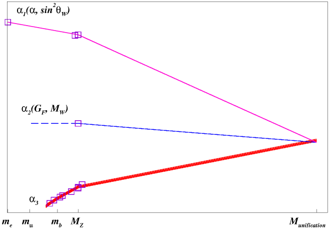
MSSM



Much better agreement:

$$\sin\theta_W^{th} \simeq 0.232$$
$$\sin\theta_W^{exp} = 0.23149 \pm 0.00017$$

iif ($M_{SUSY} \simeq 1$ TeV).

$\Rightarrow$ Strongly suggests Supersymmetric Grand Unification !!

Radiative Symmetry Breaking

- In MSSM many scalars but (typically) only Higgs gets a vev
- All soft masses possitive $\mathcal{O}(M_W)$ at M_{GUT}
- $\mu H_1 H_2 \in W$, $\mu \sim \mathcal{O}(M_W)$ (SUSY μ problem)
- Approx. RGE evolution from M_{GUT} to M_W:

$$
\begin{aligned}
16\pi^2 \frac{dm^2_{H_2}}{dt} &= 6Y_t^2(m^2_{H_2} + m^2_{Q_3} + m^2_{U_3}) - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2, \\
16\pi^2 \frac{dm^2_{H_1}}{dt} &= -6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2, \\
16\pi^2 \frac{dm^2_{Q_3}}{dt} &= 2Y_t^2(m^2_{H_2} + m^2_{Q_3} + m^2_{U_3}) - \frac{32}{3} g_3^2 M_3^2 - \\
&\quad 6g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2,
\end{aligned}
$$

$\Rightarrow$ $m^2_{H_2}$ pushed down by Y_t and no $SU(3)$ coupling

EW symmetry breaking occurs naturally as a radiative effect

Gravity and strings ...

Global bosonic sym. $\xrightarrow{Local}$ Gauge Theory

Global SUSY $\xrightarrow{Local}$ Supergravity

- SUSY transformation parameters ξ^{α} deppend on space-time position. Anticonmutator of 2 SUSY transformations is a translation

$$\{Q_{\alpha}, \bar{Q}_{\dot{\beta}}\} = 2\sigma^{\mu}_{\alpha\dot{\beta}} P_{\mu}$$

$$\Downarrow$$

Local SUSY implies local coordinate tranformations: Gravity

- Superstring can unify gravity with gauge interactions.

Supersymmetry necessary ingredient of consistent String Theory

Non-interacting Wess–Zumino

Weyl fermion + complex boson   Ref: Martin, hep-ph/9709356

$$S = \int d^4xL \; = \; \int d^4x \left(i\bar{\psi}\bar{\sigma}^\mu \partial_\mu \psi \; + \; \partial^\mu \phi^* \partial_\mu \phi \right)$$

SUSY transformation: scalar $\leftrightarrow$ fermion, with ξ Weyl spinor ($[\xi] = -1/2$) transf. parameter

$$\delta\phi = \sqrt{2}\xi^\alpha \psi_\alpha \equiv \sqrt{2}\xi\psi \qquad \delta\phi^* = \sqrt{2}\bar{\xi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} \equiv \sqrt{2}\bar{\xi}\bar{\psi}$$

$$\delta L_{scal} = +\sqrt{2}\xi \; \partial^\mu\psi \; \partial_\mu\phi^* + \sqrt{2}\bar{\xi} \; \partial^\mu\bar{\psi} \; \partial_\mu\phi$$

for a fermion, comparing δL_{scal} and L_{ferm} must be:

$$\delta\psi_\alpha = i\sqrt{2}(\sigma^\mu\bar{\xi})_\alpha \partial_\mu\phi \qquad \delta\bar{\psi}_{\dot{\alpha}} = -i\sqrt{2}(\xi\sigma^\mu)_{\dot{\alpha}}\partial_\mu\phi^*$$

$$\delta L_{fer} = \sqrt{2}(\xi \; \sigma^\mu\bar{\sigma}^\nu\partial_\nu\psi)\partial_\mu\phi^* - \sqrt{2}(\bar{\psi} \; \bar{\sigma}^\nu\sigma^\mu\bar{\xi})\partial_\nu\partial_\mu\phi$$

So, we arrive at

$$\delta L_{fermion} = -\sqrt{2}\xi\ \partial^\mu\psi\ \partial_\mu\phi^* - \sqrt{2}\bar{\xi}\ \partial^\mu\bar{\psi}\ \partial_\mu\phi +$$
$$\sqrt{2}\ \partial^\mu\left[\bar{\xi}\ \bar{\psi}\ \partial_\mu\phi + \xi\psi\partial_\mu\phi^* + (\xi\ \sigma^\nu\bar{\sigma}^\mu\psi)\ \partial_\mu\phi^*\right]$$
$$\Rightarrow \delta S = \int d^4x(\delta L_{scalar} + \delta L_{fermion}) = 0$$

Theory is invariant only if the algebra closes, *i.e.* if the conmutator of two supersymmetric transformations is a symmetry of the theory scalar

$$(\delta_{\xi_2}\delta_{\xi_1} - \delta_{\xi_1}\delta_{\xi_2})\phi = 2i\left(\xi_1\sigma^\mu\bar{\xi}_2 - \xi_2\sigma^\mu\bar{\xi}_1\right)\partial_\mu\phi$$

But, for fermion, only on-shell, $\bar{\sigma}^\mu\partial_\mu\psi = 0$, it closes. Off-shell we must introduce auxiliary field, F with $L_{aux} = F^*F$ and dimensions $mass^2$. It does not propagate, the eqs. motion are $F = F^* = 0$

we modify the transformation properties

$$\delta\phi = \sqrt{2}\xi\psi \qquad \delta\phi^* = \sqrt{2}\bar{\xi}\bar{\psi}$$

$$\delta\psi_\alpha = i\sqrt{2}\ (\sigma^\mu\bar{\xi})_\alpha\partial_\mu\phi\ +\ \sqrt{2}\ \xi_\alpha F$$

$$\delta\bar{\psi}_{\dot{\alpha}} = -i\sqrt{2}\ (\xi\sigma^\mu)_{\dot{\alpha}}\partial_\mu\phi^*\ +\ \sqrt{2}\bar{\xi}_{\dot{\alpha}}F^*$$

$$\delta F = i\sqrt{2}\ \bar{\xi}\bar{\sigma}^\mu\partial_\mu\psi \qquad \delta F^* = -i\sqrt{2}\ \partial_\mu\bar{\psi}\bar{\sigma}^\mu\xi$$

Now theory is Supersymmetry invariant even off-shell:

$$L_{WZ} = i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi\ +\ \partial^\mu\phi^*\partial_\mu\phi + F^*F$$

$$(\delta_{\xi_2}\delta_{\xi_1} - \delta_{\xi_1}\delta_{\xi_2})X = 2i\left(\xi_1\sigma^\mu\bar{\xi}_2 - \xi_2\sigma^\mu\bar{\xi}_1\right)\partial_\mu X$$

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$$\delta\phi = \sqrt{2}\xi\psi \qquad \delta\phi^* = \sqrt{2}\bar{\xi}\bar{\psi}$$
$$\delta\psi_\alpha = i\sqrt{2}\,(\sigma^\mu\bar{\xi})_\alpha\partial_\mu\phi \;+\; \sqrt{2}\,\xi_\alpha F$$
$$\delta\bar{\psi}_{\dot\alpha} = -i\sqrt{2}\,(\xi\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^* \;+\; \sqrt{2}\bar{\xi}_{\dot\alpha}F^*$$
$$\delta F = i\sqrt{2}\,\bar{\xi}\bar{\sigma}^\mu\partial_\mu\psi \qquad \delta F^* = -i\sqrt{2}\,\partial_\mu\bar{\psi}\bar{\sigma}^\mu\xi$$

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$$L_{WZ} = i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi \;+\; \partial^\mu\phi^*\partial_\mu\phi + F^*F$$
$$(\delta_{\xi_2}\delta_{\xi_1} - \delta_{\xi_1}\delta_{\xi_2})X = 2i\left(\xi_1\sigma^\mu\bar{\xi}_2 - \xi_2\sigma^\mu\bar{\xi}_1\right)\partial_\mu X \qquad \text{for } X = \phi, \psi, F$$

Supersymmetry Algebra

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta}P_\mu \qquad [P_\mu, Q_\alpha] = 0$$
$$\{Q_\alpha, Q_\beta\} = 0 \qquad \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0$$

Chiral Supermultiplet$= \psi, \phi, F$ with four fermionic d.o.f. (2 complex components of ψ) and four bosonic d.o.f. (2 complex scalars, ϕ and F)

Matter Interactions

General set of renormalizable matter SUSY interactions is given by,

$$L_{int} = -\frac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i + c.c.$$

with W^{ij}, W^i functions of bosonic fields $[W^{ij}] = M$ and $[W^i] = M^2$ ($[L] = 4$, W^{ij} and W^i cannot be functions of ψ and F)

- L_{int} must be SUSY invariant by itself

$\Rightarrow$ **Superpotential W**

Analitic function of the (complex) scalar fields (not of ϕ^*), at most cubic in ϕ and dimensions of mass^3

$$W = \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} \lambda^{ijk} \phi_i \phi_j \phi_k, \qquad W^i = \frac{\delta W}{\delta \phi_i} \qquad W^{ij} = \frac{\delta W}{\delta \phi_i \delta \phi_j}$$

F_i, F^{i*} eliminated using equations of motion: $F_i = -W_i^*$ $F^{i*} = -W^i$, i.e. functions of scalar fields, no derivatives. Then Lagrangian,

$$L_{chiral} = L_{WZ} + L_{int} = i\bar{\psi}\bar{\sigma}^\mu \partial_\mu \psi + \partial^\mu \phi^* \partial_\mu \phi + - \frac{1}{2} W^{ij} \psi_i \psi_j - \frac{1}{2} W^{ij*} \bar{\psi}_i \bar{\psi}_j - W^i W_i^*$$

The scalar potential $V(\phi, \phi^*)$ is given by

$$V(\phi,\phi^*) = W^i W_i^* = F^i F_i^* = M_{ji}^* M^{ik} \phi^{j*} \phi_k + \frac{1}{2} M^{ik} \lambda_{jnk}^* \phi_i \phi^{j*} \phi^{n*} + \frac{1}{2} M_{ik}^* \lambda^{jnk} \phi^{i*} \phi_j \phi_n + \frac{1}{4} \lambda^{ijn} \lambda_{kln}^* \phi_i \phi_j \phi^{k*} \phi^{l*}$$

Superpotential: general SUSY invariant matter interactions

Example: top Yukawa

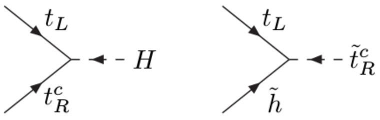
$W \; = \; h_t \; Q_L H t_R^c$ Gives rise to the Lagrangian,

$$L_{int} \; = \; -\frac{1}{2} W^{ij} \; \psi_i \psi_j \; - \; \frac{1}{2} W^{ij*} \; \bar{\psi}_i \bar{\psi}_j \; - \; W^i W_i^*$$

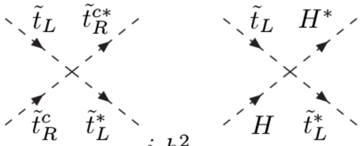
$$= -\frac{1}{2} \; h_t \left[H \; Q_L t_R^c + \tilde{Q}_L \; \tilde{h} t_R^c + \tilde{t}_R^c \; Q_l \tilde{h} \right] + c.c.$$

$$- \; h_t^2 \left(|H \tilde{t}_R^c|^2 + |H \tilde{Q}_L|^2 + |\tilde{Q}_L \tilde{t}_R^c|^2 \right)$$

and the Feynman rules



$- \; i \; h_t \; L$



$- \; i \; h_t^2$

Vector Superfields

Gauge multilplet: massless vector boson A_μ^a + Weyl gaugino λ^a + auxiliary field D^a

$$L_{gauge} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + i\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c \quad D_\mu \lambda^a = \partial_\mu \lambda^a - g f^{abc} A_\mu^b \lambda^c$$

L_{gauge} is already Supersymmetric with the transformations (Wess-Zumino gauge)

$$\delta A_\mu^a = i\bar{\xi}\bar{\sigma}_\mu \lambda^a - i\bar{\lambda}^a \bar{\sigma}_\mu \xi$$

$$\delta \lambda_\alpha^a = -\frac{1}{2} (\sigma^\mu \bar{\sigma}^\nu \xi)_\alpha F_{\mu\nu}^a - i\xi_\alpha D^a$$

$$\delta D^a = -\bar{\xi}\bar{\sigma}^\mu D_\mu \lambda^a + D_\mu \bar{\lambda}^a \bar{\sigma}^\mu \xi$$

Gauge Interactions

Replacing $\partial_\mu \to D_\mu = \partial_\mu + igT^a A^a_\mu$ obtain Gauge invariant Lagrangian, but not SUSY invariant, we need gaugino and D interactions,

$$\phi^* T^a \psi\, \lambda^a \qquad \bar{\lambda}^a\, \bar{\psi} T^a \phi \qquad \phi^* T^a \phi\, D^a$$

Then the full Lagrangian is Supersymmetric replacing also in the SUSY transformations derivatives by covariant derivatives,

$$L = L_{gauge} + L_{chiral} + i\sqrt{2}\, g\, \left[\phi^* T^a \psi\, \lambda^a + \bar{\lambda}^a\, \bar{\psi} T^a \phi\right] + g\, \phi^* T^a \phi\, D^a$$

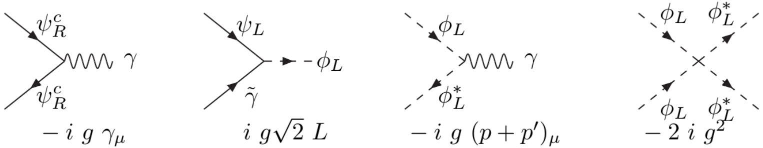
where W must also be Gauge invariant by itself. Moreover D^a can be eliminated using eqs of motion,

$$D^a = -g\, \phi^* T^a \phi$$

The gauge invariant SUSY Lagrangian,

$$L \;=\; i\bar{\psi}_L\bar{\sigma}^\mu D_\mu\psi_L \;+\; D^\mu\phi_L^* D_\mu\phi_L + (L\to R^c) \;+\; \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}$$
$$-\, i\;\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a \;+\; i\,\sqrt{2}\,g\;\left[\phi_i^* T^a\psi_i\;\lambda^a \;+\; \bar{\lambda}^a\;\bar{\psi}_i T^a\phi_i\right]$$
$$-\,\frac{1}{2}W^{ij}\;\psi_i\psi_j \;-\; \frac{1}{2}W^{ij*}\;\bar{\psi}_i\bar{\psi}_j \;-\; W^iW_i^* \;-\frac{1}{2}\;g^2\;(\textstyle\sum_i\phi_i^* T^a\phi_i)^2$$

therefore the gauge interactions are ($U(1)$),



SUSY Lagrangian

- All interactions determined by gauge quantum numbers and Superpotential
- Write all kinetic terms with covariant derivativesplus gaugino interactions and D–terms
- W must be a gauge invariant analitic function of the scalar fields
- Matter interactions determined by $W^i W_i^*$ and $W^{ij}\psi_i\psi_j$

$\Longrightarrow$ Lagrangian is Supersymmetric

Minimal Supersymmetric Standard Model

- Must include all Standard Model particles and interactions
- Supersymmetric partners and SUSY SM interactions
- Supersymmetry must be softly broken

	LH SM fermion	$\leftrightarrow$	Scalar partner
Chiral supermultiplets	$Q, u_R^c, d_R^c, L, e_R^c$		$\tilde{Q}, \tilde{u}_R^c, \tilde{d}_R^c, \tilde{L}, \tilde{e}_R^c$
	2 Higgs	$\leftrightarrow$	fermionic part.
	$H_1, \quad H_2$		$\tilde{H}_1, \quad \tilde{H}_2$
	gauge bosons	$\leftrightarrow$	fermionic part.
Vector supermultiplets	B_μ, W_μ^i, G_μ^a		$\tilde{B}, \tilde{W}^i, \tilde{g}^a$

MSSM Superpotential

Includes the Yukawa interactions of the SM

$$W = Y_d^{ij} Q_i H_1 d_{Rj}^c + Y_e^{ij} L_i H_1 e_{Rj}^c + Y_u^{ij} Q_i H_2 u_{Rj}^c + \mu H_1 H_2$$

- Need of two Higgs doublets

$$Y_{Q_i} + Y_{d_R^c} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2} \quad \Rightarrow \quad Y_{H_d} = -\frac{1}{2}$$
$$Y_{Q_i} + Y_{u_R^c} = \frac{1}{6} - \frac{2}{3} = -\frac{1}{2} \quad \Rightarrow \quad Y_{H_u} = \frac{1}{2}$$

in the SM $H_u = H$ and $H_d = H^*$. However, a Superpotential containing H^* would be non-supersymmetric.

A second doublet also required by anomaly cancellation.

R – Parity Other Gauge invariant terms can appear in W

$$W_{\Delta L=1} = \lambda^{ijk} L_i L_j e^c_{Rk} + \lambda'^{\,ijk} L_i Q_j d^c_{Rk} + \epsilon^i L_i H_2$$

$$W_{\Delta B=1} = \lambda''^{\,ijk} \; u^c_{Ri} d^c_{Rj} d^c_{Rk}$$

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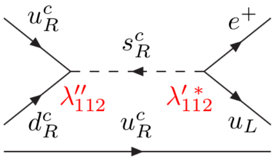
$$W_{\Delta B=1} = \lambda''^{\,ijk}\ u^c_{Ri} d^c_{Rj} d^c_{Rk}$$

violate baryon or lepton number by 1 unit. If both λ' and $\lambda'' \neq 0$
$\Rightarrow$ rapid proton decay!!



$$p^+ \to e^+ \pi^0 \Rightarrow \lambda'^{*}_{112} \cdot \lambda''_{112} \leq 2 \times 10^{-27}$$

New discrete symmetry, R-parity, forbids these terms

$$R_P = (-1)^{3B+L+2S}$$

SM particles and Higgs bosons $R_P = +1$, superpartners $R_P = -1$

R_P conserved in the MSSM

- $W_{\Delta L=1}$ and $W_{\Delta B=1}$ absent in the MSSM.
- Lightest Supersymmetric Particle (LSP) stable (dark matter).
- Any sparticle decays into final state with odd number of LSP.
- In colliders, Supersymmetric particles only produced in pairs.

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It is also possible an R_P violating MSSM with $W_{\Delta L=1}$ or $W_{\Delta B=1}$ (not both) and stable proton.

- L or B is violated
- LSP not stable anymore (not dark matter candidate).
- Single production of SUSY particles possible.

Soft SUSY Breaking

- SUSY must be broken, $m_{\tilde{e}} \neq m_e$, $m_{\tilde{g}} \neq 0$.
- Solve hierarchy problem, broken by terms of possitive mass dimension Soft Supersymmetry Breaking, and $M_{susy} \leq \mathcal{O}(1 \text{ TeV})$.
  - Gaugino masses

$$L^{(1)}_{soft} = \frac{1}{2}\left(M_1\ \tilde{B}\tilde{B} + M_2\ \tilde{W}\tilde{W} + M_3\ \tilde{g}\tilde{g}\right) + h.c.$$

  - Scalar masses

$$L^{(2)}_{soft} = (m^2_{\tilde{Q}})_{ij}\tilde{Q}_i\tilde{Q}^*_j + (m^2_{\tilde{u}})_{ij}\tilde{u}^c_{Ri}\tilde{u}^{c*}_{Rj} + (m^2_{\tilde{d}})_{ij}\tilde{d}^c_{Ri}\tilde{d}^{c*}_{Rj} + (m^2_{\tilde{L}})_{ij}\tilde{L}_i\tilde{L}^*_j + (m^2_{\tilde{e}})_{ij}\tilde{e}^c_{Ri}\tilde{e}^{c*}_{Rj} + (m^2_{H_1})H_1H^*_1 + (m^2_{H_2})H_2H^*_2$$

  - Trilinear couplings and B–term

$$L^{(3)}_{soft} = (Y^A_d)^{ij}\tilde{Q}_iH_1\tilde{d}_{Rj} + (Y^A_e)^{ij}\tilde{L}_iH_1\tilde{e}^c_{Rj} + (Y^A_u)^{ij}\tilde{Q}_iH_2\tilde{u}^c_{Rj} + B\mu H_1H_2$$

CMSSM

- Minimal and simple realization of the MSSM.
- Similar sparticle masses in a general MSSM.
- Any generic MSSM must include at least the CMSSM physics.
- Main difference in FCNC and CP violation observables.
- Representative MSSM example for collider phenomenology.

Minimal number of new SUSY paramenters

m_0^2	$\rightarrow$	Universal scalar mass.	$M_{1/2}$	$\rightarrow$	Common gaugino mass.
A_0	$\rightarrow$	Universal trilinear.	B	$\rightarrow$	Soft Higgs mass.
μ	$\rightarrow$	Susy Higgs mass.	$\tan\beta$	$\rightarrow$	Ratio of Higgs vevs.

Soft breaking parameters defined at $\sim M_{GUT}$. Evolve them to M_W with Renormalization Group Equations (RGE).

- Gauge couplings and gaugino masses

$$\frac{d\alpha_a^{-1}}{dt} = -\frac{b_a}{2\pi} \qquad \frac{dM_a}{dt} = \frac{1}{8\pi^2} b_a g_a^2 M_a \qquad \frac{dM_a/g_a^2}{dt} = 0 \Rightarrow$$

$$\Rightarrow \frac{M_1}{\alpha_1} = \frac{M_2}{\alpha_2} = \frac{M_3}{\alpha_3} \qquad b_a = (\tfrac{33}{5}, 1, -3) \qquad t = \ln\left(\frac{Q}{Q_0}\right)$$

- Yukawa couplings (b–τ unification ...)

$$\begin{aligned}
\frac{dy_t}{dt} &= \frac{y_t}{16\pi^2}\left[6y_t^2 + y_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right] \\
\frac{dy_b}{dt} &= \frac{y_b}{16\pi^2}\left[6y_b^2 + y_t^2 + y_\tau^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right] \\
\frac{dy_\tau}{dt} &= \frac{y_\tau}{16\pi^2}\left[4y_\tau^2 + 3y_b^2 - 3g_2^2 - \frac{9}{5}g_1^2\right]
\end{aligned}$$

- Soft masses

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} m_{H_2}^2 &= 6y_t^2(m_{H_2}^2 + m_{Q_3}^2 + m_{U_3}^2) - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 \\
16\pi^2 \frac{d}{dt} m_{H_1}^2 &= -6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 \\
16\pi^2 \frac{d}{dt} m_{Q_3}^2 &= 2y_t^2(m_{H_2}^2 + m_{Q_3}^2 + m_{U_3}^2) - \frac{32}{3} g_3^2 M_3^2 - 6g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2 \\
16\pi^2 \frac{d}{dt} m_{U_3}^2 &= 4y_t^2(m_{H_2}^2 + m_{Q_3}^2 + m_{U_3}^2) - \frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2
\end{aligned}
$$

Full set of RGEs in literature. Fortran codes, ISASUSY, SOFTSUSY, SPHENO . . .

Mass of H_2 pushed down by top and absence of $SU(3)$ coupling

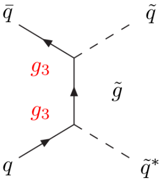
$$\Downarrow$$

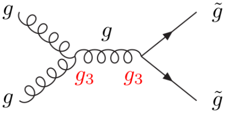
Radiative Symmetry Breaking

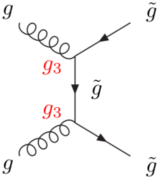
Collider phenomenology

Sparticle production

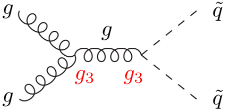
Gluino and Squark production (1st gen.) dominates through

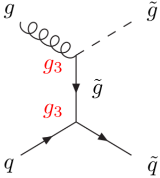






LHC very stringent constraints on gluino and 1st gen. squark masses, produced strong





Squark & gluino decays

- If two body decays with strong coupling are allowed they always dominate

$$\tilde{q} \to q\tilde{g}, \qquad \tilde{g} \to q\tilde{q}$$

Otherwise squarks decay to quark chargino or neutralino through electroweak couplings

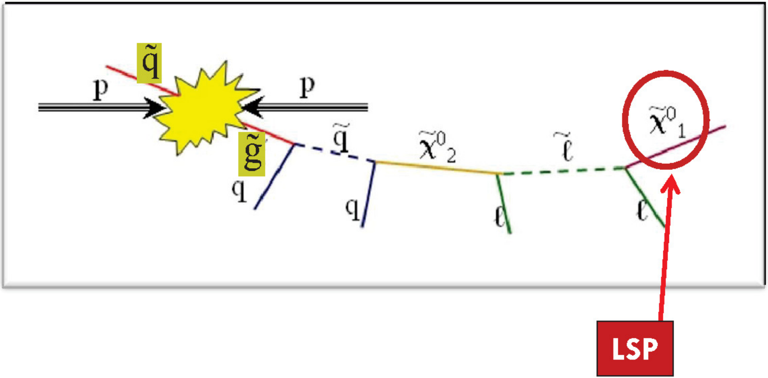
$$\tilde{q} \to q\chi_i^0, \ \ q'\chi_i^-$$

and gluinos decay through an off-shell squark

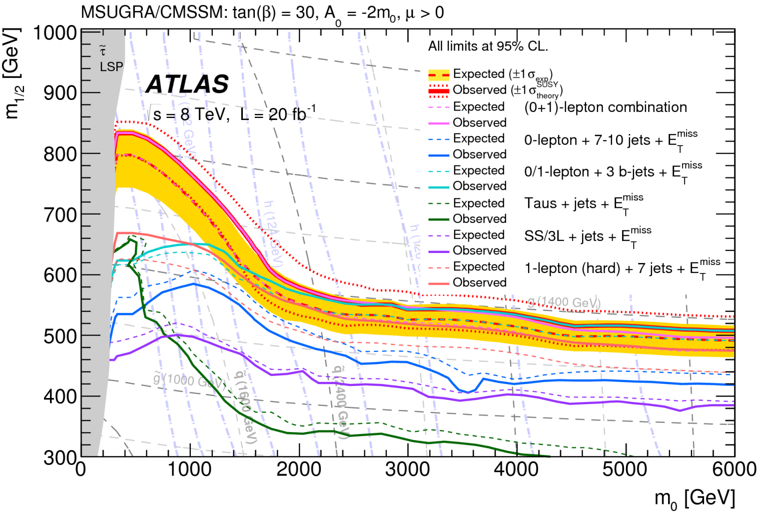
$$\tilde{g} \to qq\chi_i^0, \ \ qq'\chi_i^-$$

LHC constraints

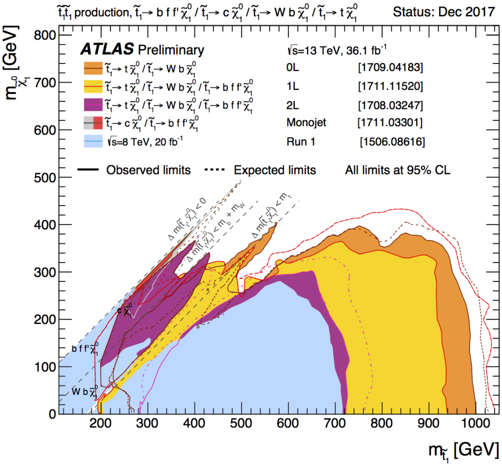
– Production of coloured particles with long decay chains. . .



– Search of jets + missing E_T:



– Stop ($\tilde{t}$) production:



$\Rightarrow$ already ver stringent constraints for minimal models (CMSSM like), but not yet dead!!!

Weyl spinors Susy described in terms of 2-comp. Weyl spinors:

$$\mathcal{L}_{\rm Dirac} = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - M\bar{\Psi}\Psi$$

Weyl repres.

$$\gamma_\mu = \left(\begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}\right) \qquad \gamma_5 = \left(\begin{pmatrix} -I_2 & 0 \\ 0 & I_2 \end{pmatrix}\right)$$

$$(\sigma_\mu)_{\alpha\dot{\alpha}} = (I_2, \sigma_i)_{\alpha\dot{\alpha}} \qquad (\bar{\sigma}_\mu)^{\dot{\alpha}\alpha} = (I_2, -\sigma_i)^{\dot{\alpha}\alpha}$$

$$\psi_D = \left(\begin{pmatrix} \xi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}\right) = \psi_L + \psi_R = \left(\begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix}\right) + \left(\begin{pmatrix} 0 \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}\right),$$

$$\bar{\psi}_D = \psi_D^\dagger\gamma^0 = \left(\left(\chi^\alpha, \ \ \bar{\xi}_{\dot{\alpha}}\right)\right), \quad (\xi_\alpha)^\dagger = \bar{\xi}_{\dot{\alpha}}$$

$$\Rightarrow \qquad \mathcal{L} = i\bar{\xi}\bar{\sigma}^\mu\partial_\mu\xi + i\chi\sigma^\mu\partial_\mu\bar{\chi} - M(\xi\chi + \bar{\xi}\bar{\chi})$$

Dirac → Weyl:

$$\bar{\Psi}_1\gamma^\mu \ L \ \Psi_2 = \bar{\xi}_1\bar{\sigma}_\mu\xi_2 \qquad \bar{\Psi}_1\gamma^\mu \ R \ \Psi_2 = \chi_1\sigma_\mu\bar{\chi}_2$$

$$\bar{\Psi}_1 \ L \ \Psi_2 = \chi_1\xi_2 \qquad \bar{\Psi}_1 \ R \ \Psi_2 = \bar{\xi}_1\bar{\chi}_2$$