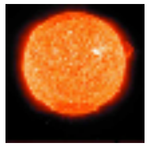
What is the main purpose of Cosmology?

To study the evolution and structure of the large scales in our universe



SUN

$2\ 10^{33}$ g

$7\ 10^{10}$ cm

Galaxies

$2\ 10^{44}$ g

10 kpc=$3\ 10^{22}$ cm

Galaxy Clusters

$2\ 10^{47}$ g

~ Mpc=10^{25} cm

The universe: our "Hubble volume"

$8\ 10^{55}$ g

3000 Mpc=10^{28} cm

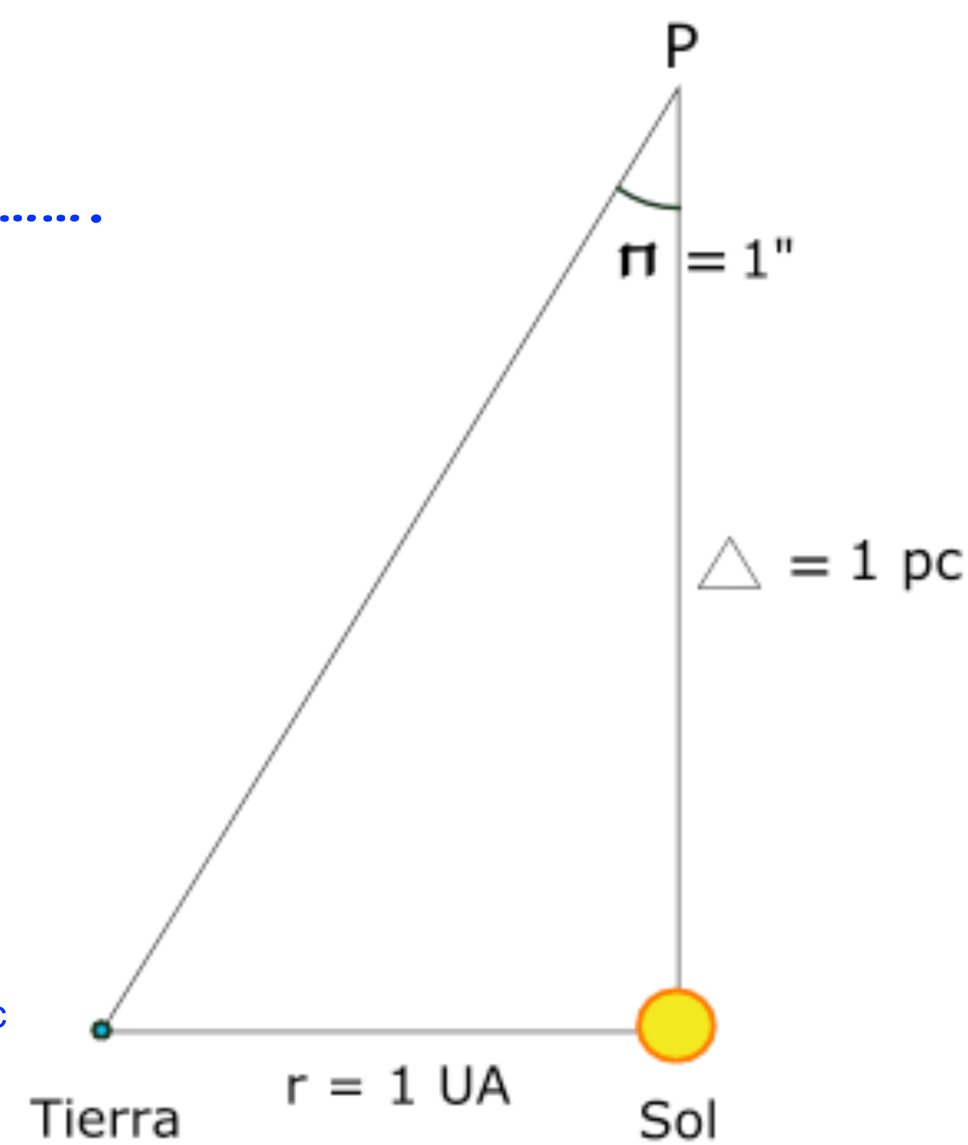
What is a parsec (parallax of one arcsecond)?

A parsec (parallax of one arcsecond) is a length measure commonly used in astrophysics and cosmology. A parsec was defined as the distance at which one astronomical unit subtends an angle of one arc-second.

1 AU= 150 10^9 m

1 pc= 3.08 10^{16} m (3.26 light years)

A parsec amounts to go and come back from the Sun……..



100 000 times!

reminder of some scales

- keep in mind some rough scales when considering galaxies:
  - Sun's distance from centre of Galaxy: ~ 8 kpc
  - diameter of Galaxy: ~ 30 kpc
  - nearest (non-satellite) galaxies: ~750 kpc
  - sizes of groups and clusters: 1-3 Mpc
  - nearest rich clusters: 20-100 Mpc
  - sizes of 'walls' and large-scale structure: 100's Mpc

TEMA 1:

INTRODUCTION:

FUNDAMENTAL INGREDIENTS OF STANDARD COSMOLOGY

Standard Cosmology refers to FLRW Cosmology (FRIEDMANN LEMAITRE ROBERTSON WALKER) and it is based on two basic elements:

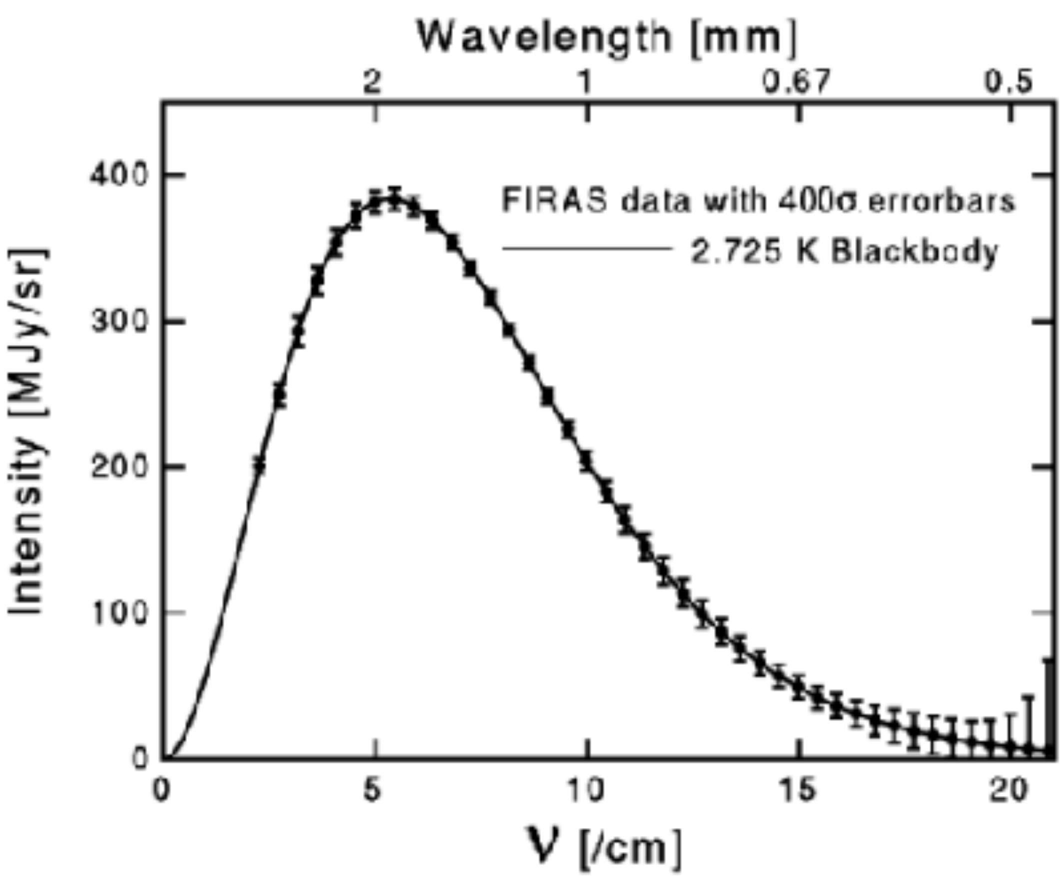
- FLRW Geometry (i.e. the metric, which determines the geodesics)

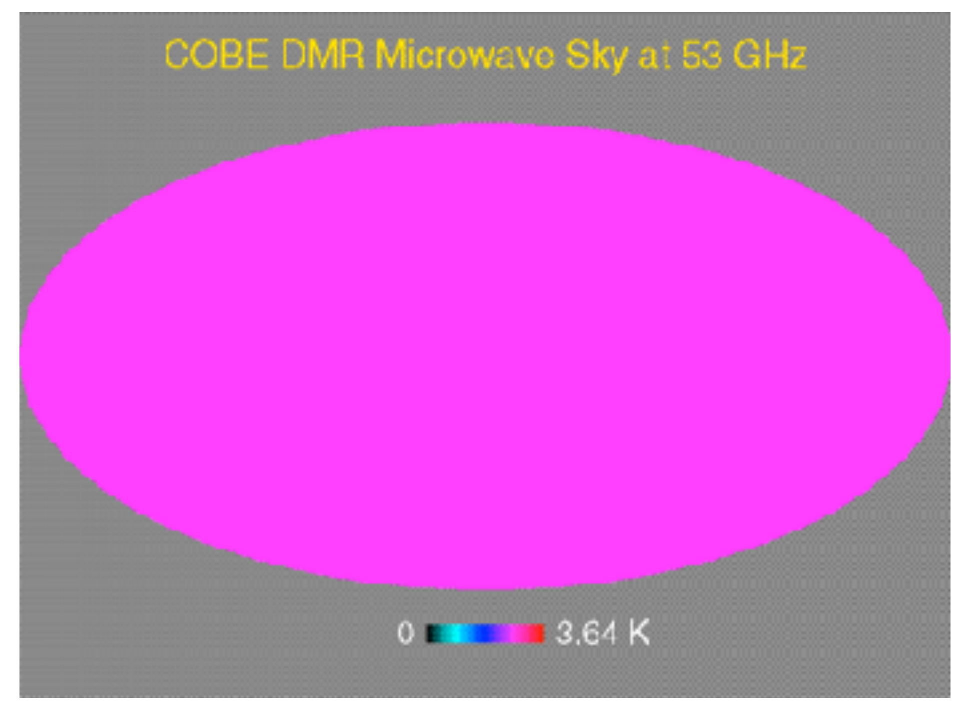
- FLRW Dynamics (Friedmann Equations, which determine the curvature of the space-time)

1.1 FLRW GEOMETRY

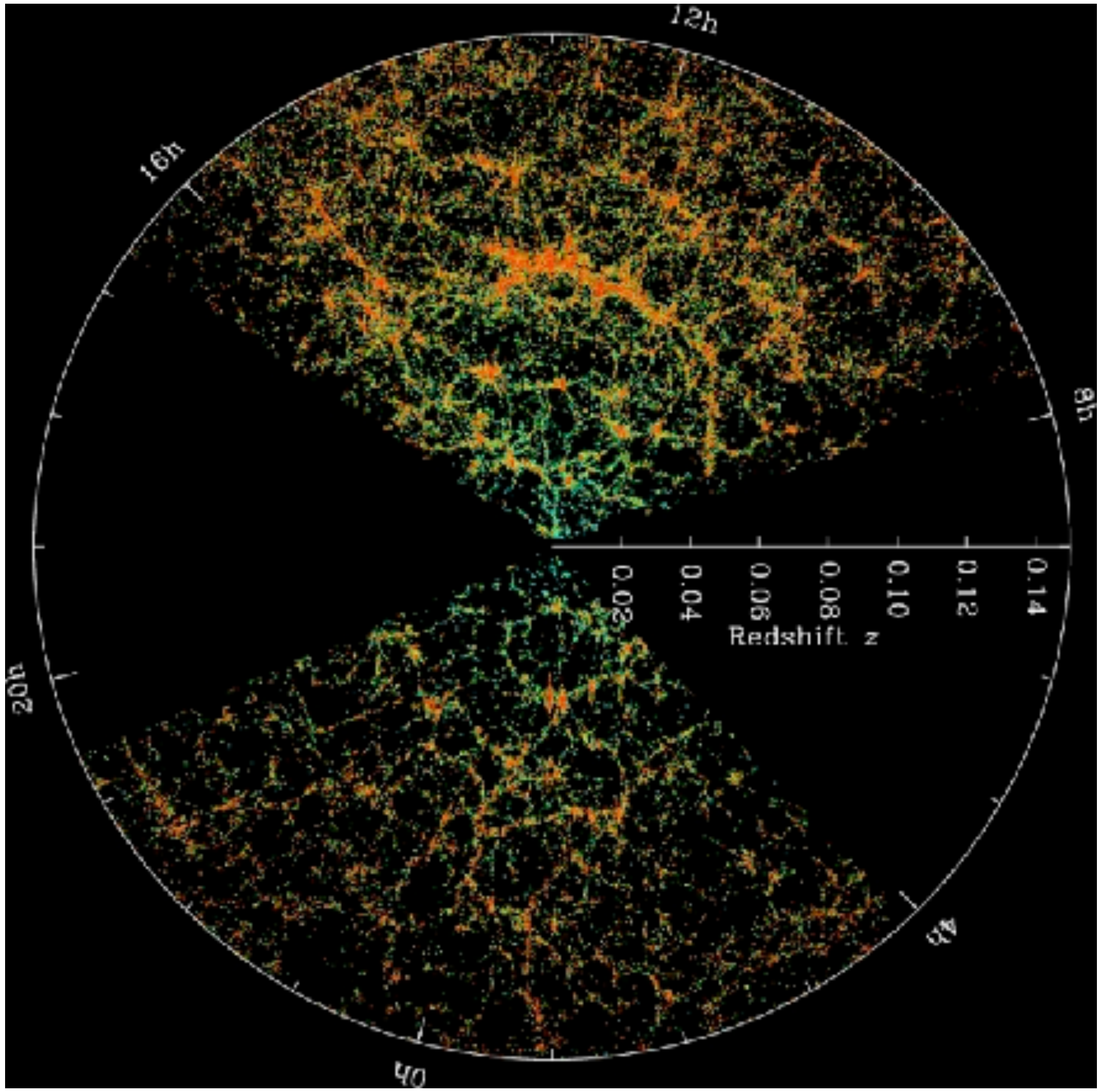
The FLRW geometry asumes that at large scales the universe is homogeneous and isotropic.

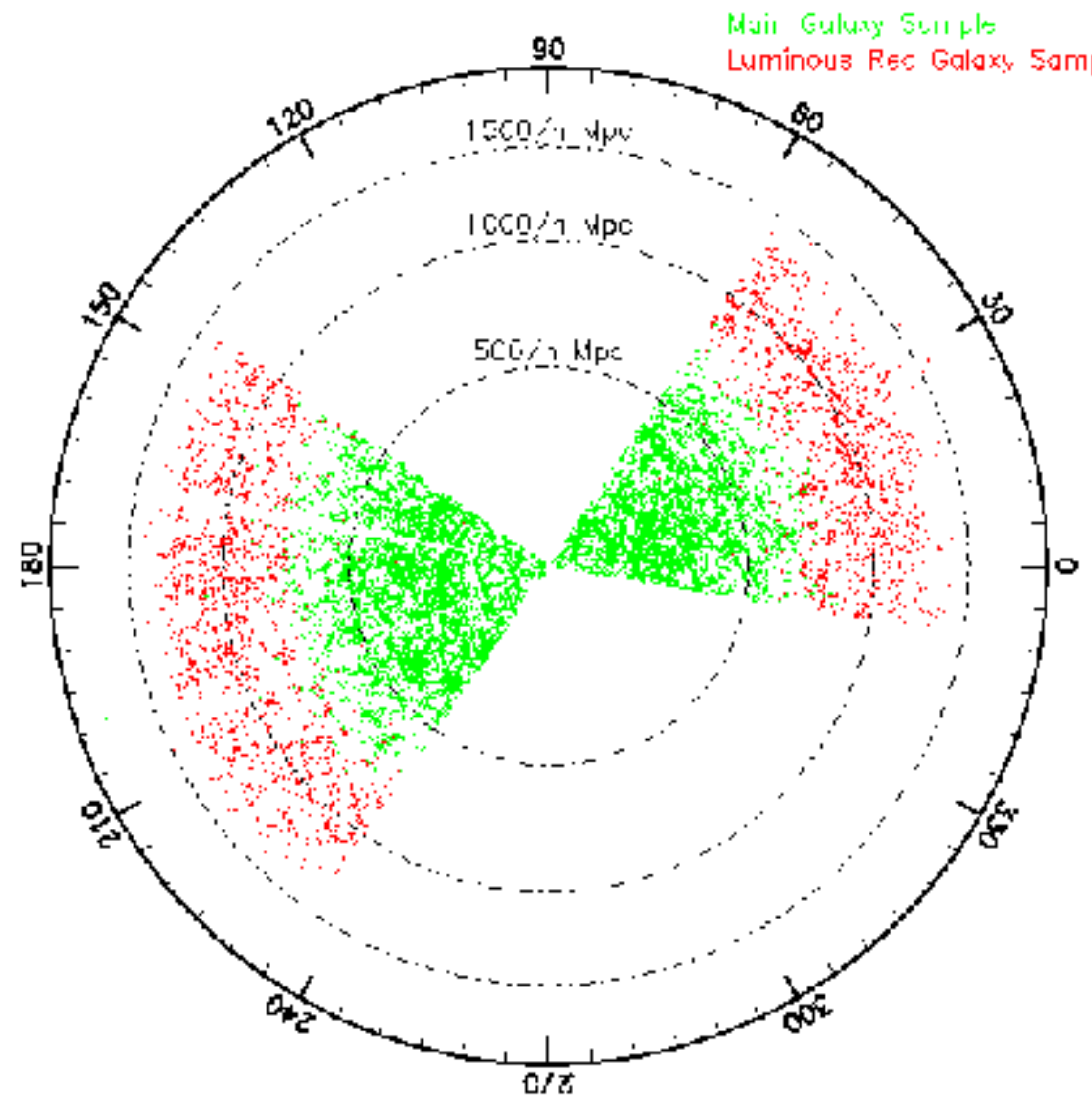
The most robust confirmation of the isotropy of the universe at large scales is provided by the CMB, the Cosmic Microwave Background radiation (Penzias & Wilson'64). When one measures the sky temperature in any direction, one notices that the photons have a thermal black body spectrum with a temperature of 2.725 K. This has been measured with high accuracy by the spectrophotometer FIRAS on the NASA COBE satellite. There are small fluctuations in the temperature across the sky at the level of about 1 part in 100,000 ~(10^{-5})





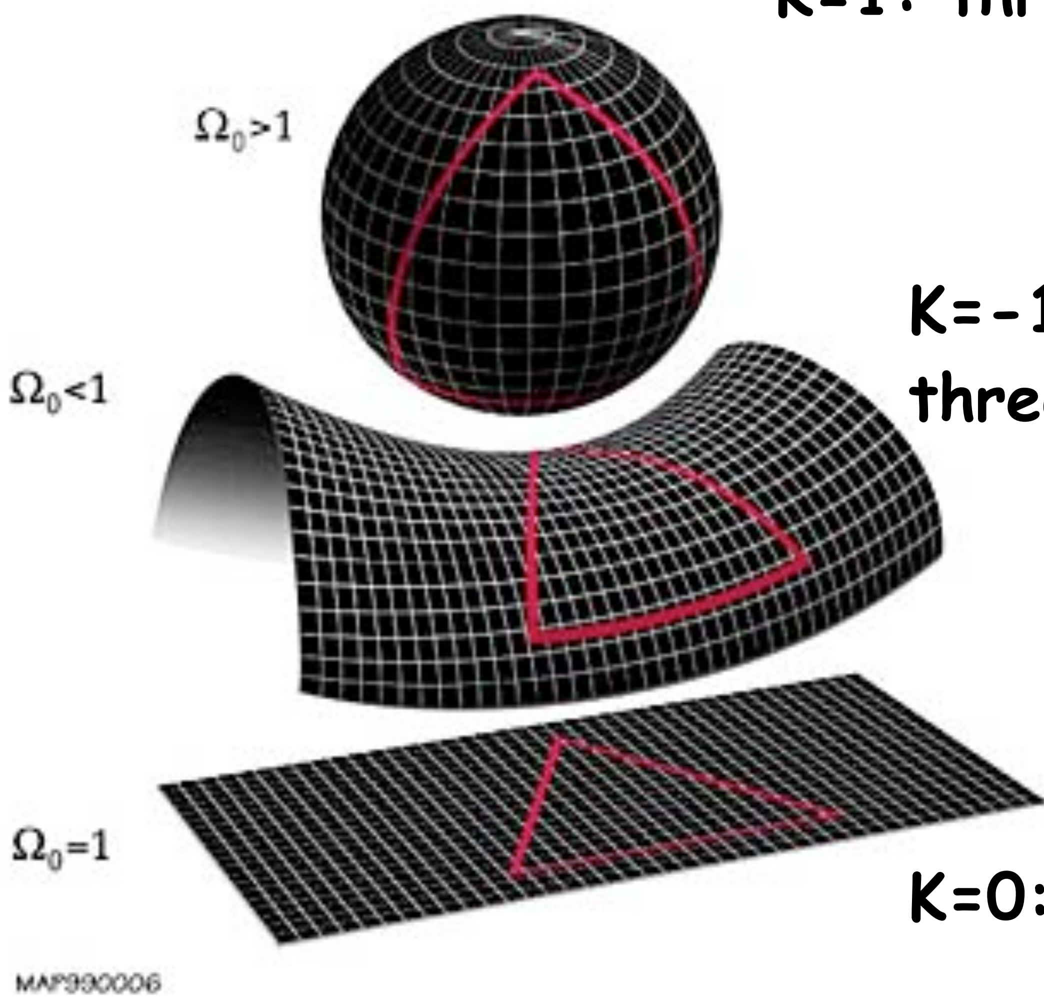
The existence of a CMB, that is, a relic photon bath, was predicted by Alpher & Herman in 1948 while working on BBN. Penzias & Wilson, in 1965, discovered accidentally the CMB while working with a very sensitive radio telescope at Bell Labs in New Jersey. In 1978, Penzias and Wilson were awarded the Nobel Prize for Physics for their joint discovery of the CMB.





At distances larger than 100 Mpc, <u>galaxy survey observations</u> indicate that the universe is homogeneous, that is, galaxies and clusters of galaxies are equally distributed in the sky in all possible directions.

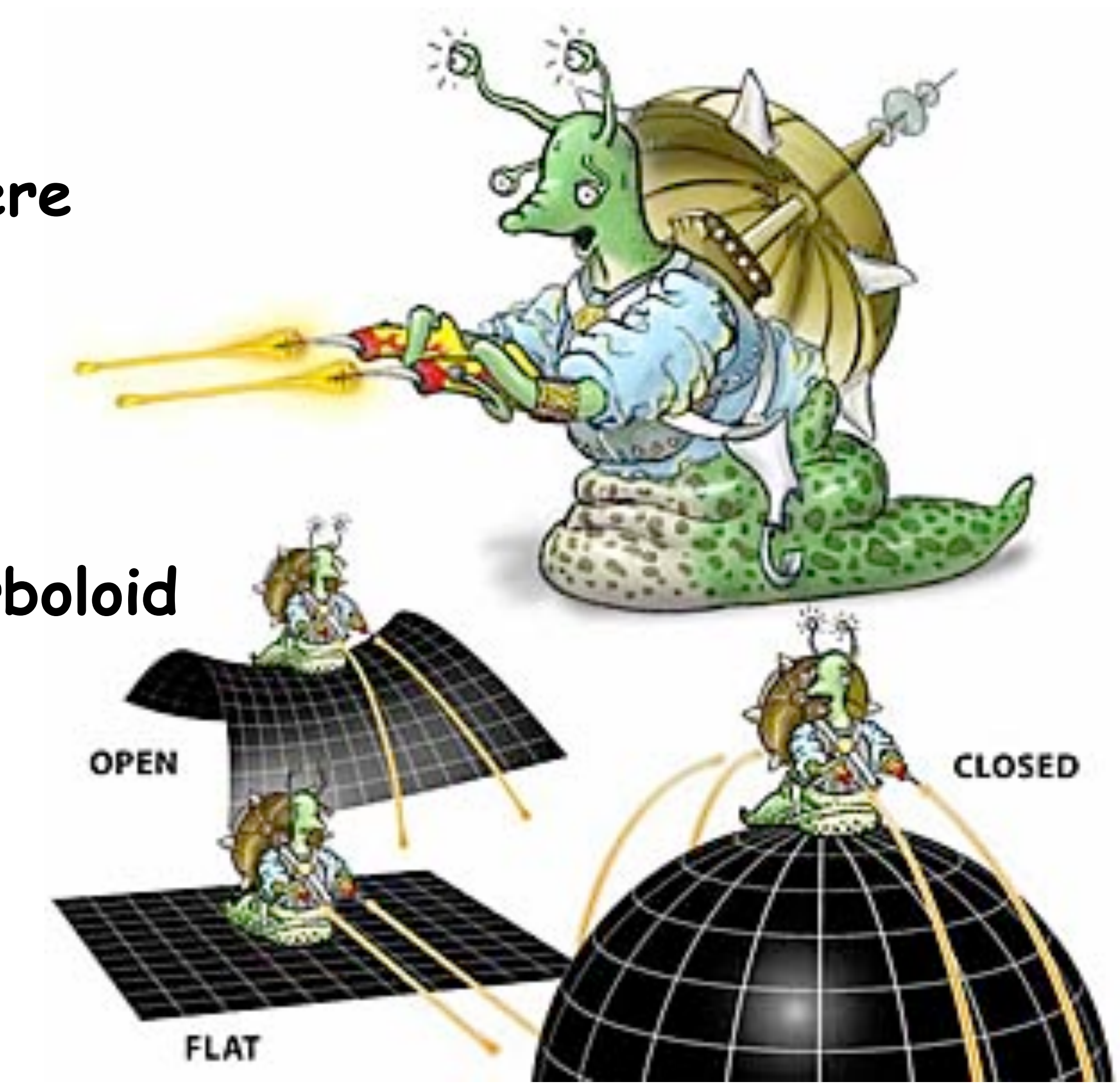
The spatial geometry depends on the curvature, K:



K=1: three-sphere

K=-1:
three-hyperboloid

K=0: flat



The FLRW metric tells us how to measure distances in each of these possible geometries.

The metric $g_{\mu\nu}$ connects the values of the coordinates to the more physical measure of the interval (proper time):

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu} dx^\mu dx^\nu$$

- dx^0 refers to the time-like component, the last three are spatial coordinates.
- $g_{\mu\nu}$ is the metric, necessarily symmetric.
- In special relativity, $g_{\mu\nu}=\eta_{\mu\nu}$ (Minkowski metric)
- In an expanding, homogeneous and isotropic universe the metric is the FLRW one:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

- If the universe is flat (K=0), the FLRW metric, with a(t) the scale factor:

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix}$$

Geodesics

- A geodesic refers to the path followed by a particle in the absence of any forces, (in the Minkowski metric it will be a straight line):

$$\frac{d^2\vec{x}}{dt^2} = 0$$

which should be generalised in the context of an expanding universe to:

$$\frac{d^2 x^\mu}{d\lambda} = -\Gamma^\mu{}_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda}$$

- The Christoffer symbols will be extensively used in the following

$$\Gamma^\mu_{\alpha\beta} = \frac{g^{\mu\nu}}{2} \left(\frac{\partial g_{\alpha\nu}}{\partial x^\beta} + \frac{\partial g_{\beta\nu}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\nu} \right)$$

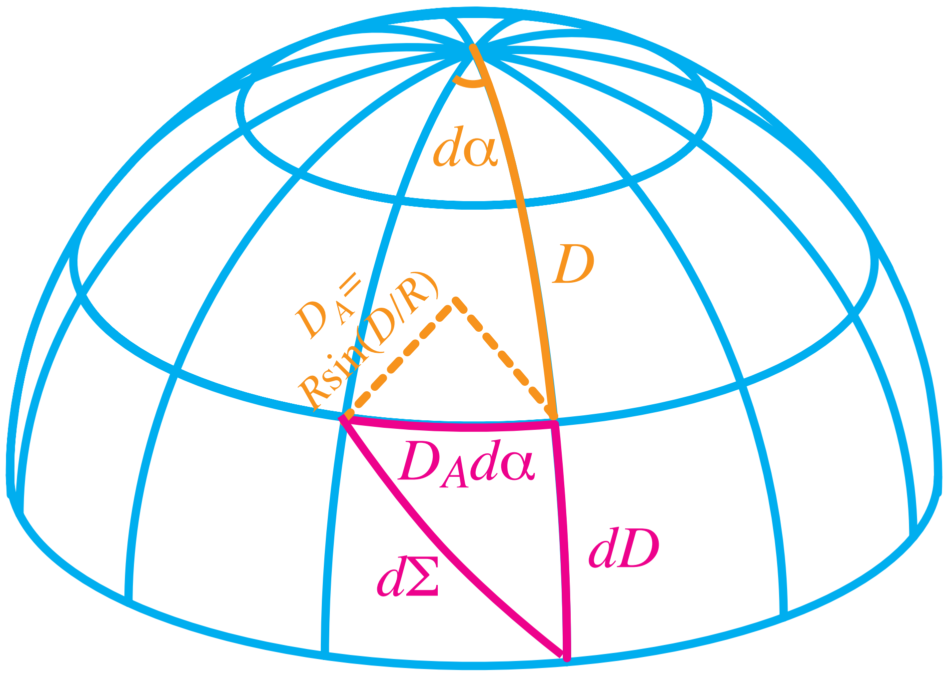
- We can apply the geodesic equation to compute the particle's energy changes as the universe expands:

$$\frac{d}{d\lambda} = \frac{dx^0}{d\lambda} \frac{d}{dx^0} = E \frac{d}{dt} \qquad P^\alpha = (E, \vec{P}) \quad P^\alpha = \frac{dx^\alpha}{d\lambda}$$

- The 0-th component of the geodesic equation reads as:

$$\frac{dE}{dt} + \frac{\dot{a}}{a} E = 0 \qquad E \propto \frac{1}{a}$$

We'll cover two type of distances:



- Radial distance D (photon path length)
- Angular distance D_A (associated to the angle subtended by an object of known physical size)

The volume element is defined as:

$$dV = D_A^2 dD d\Omega$$

We will see some examples of each possible distance/volume element:

- Distance to a Supernova
- Angular size of the universe at photon decoupling
- Galaxy number density

Horizon

- The distance that light has traveled without interactions from t=0 until the present is known as the comoving horizon.

- The horizon ALWAYS increases with time.

- The comoving horizon corresponds to the conformal time (c=1):

$$\eta(t) \equiv \int_0^t \frac{dt'}{a(t')}$$

- The comoving horizon equals to the causal distance. Regions that lie apart from each other by a distance larger than the comoving horizon were never in causal contact.
- This problem is known as "The horizon problem":

- Why at large scales the universe is so homogeneous and isotropic?
- If these regions were never in causal contact, how is it possible that the CMB temperature is so uniform in the sky?

Hubble parameter

- It provides the expansion rate of the universe as a function of time:

$$H \equiv \frac{1}{a}\frac{da}{dt} = \frac{d\ln a}{dt}$$

- The cosmic time reads as:

$$t = \int dt = \int \frac{1}{a}\frac{da}{H(a)}$$

- The conformal time is given by:

$$\eta = \int \frac{dt}{a} = \int \frac{1}{a^2}\frac{da}{H(a)}$$

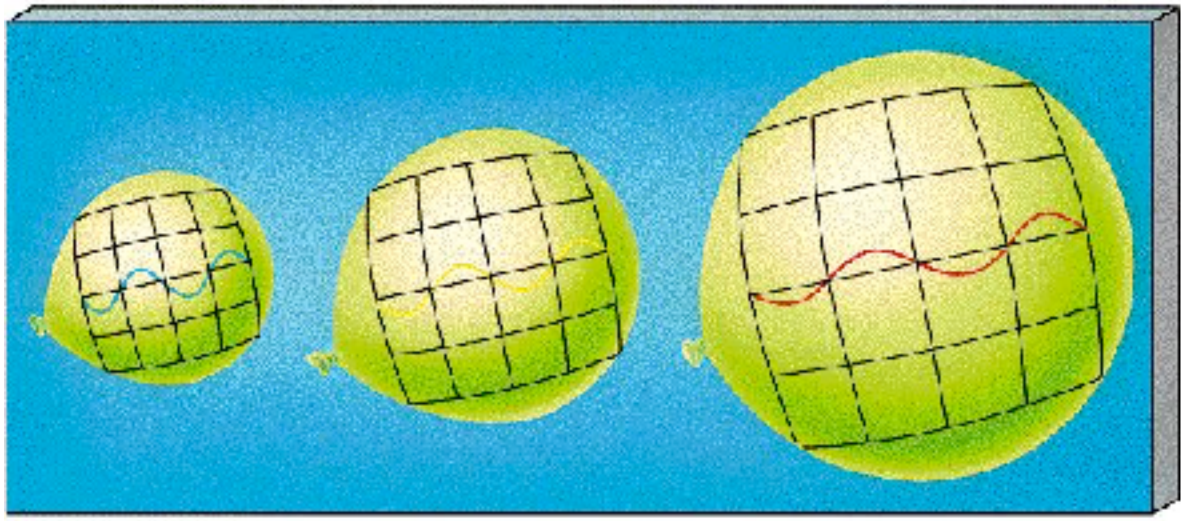
Cosmological redshift

- **Doppler Effect**:

- **Cosmic time: The photon wavelength is stretched with the scale factor as the universe expands.**

$$\lambda = \frac{\lambda_0}{a} = (1 + z)\lambda_0$$

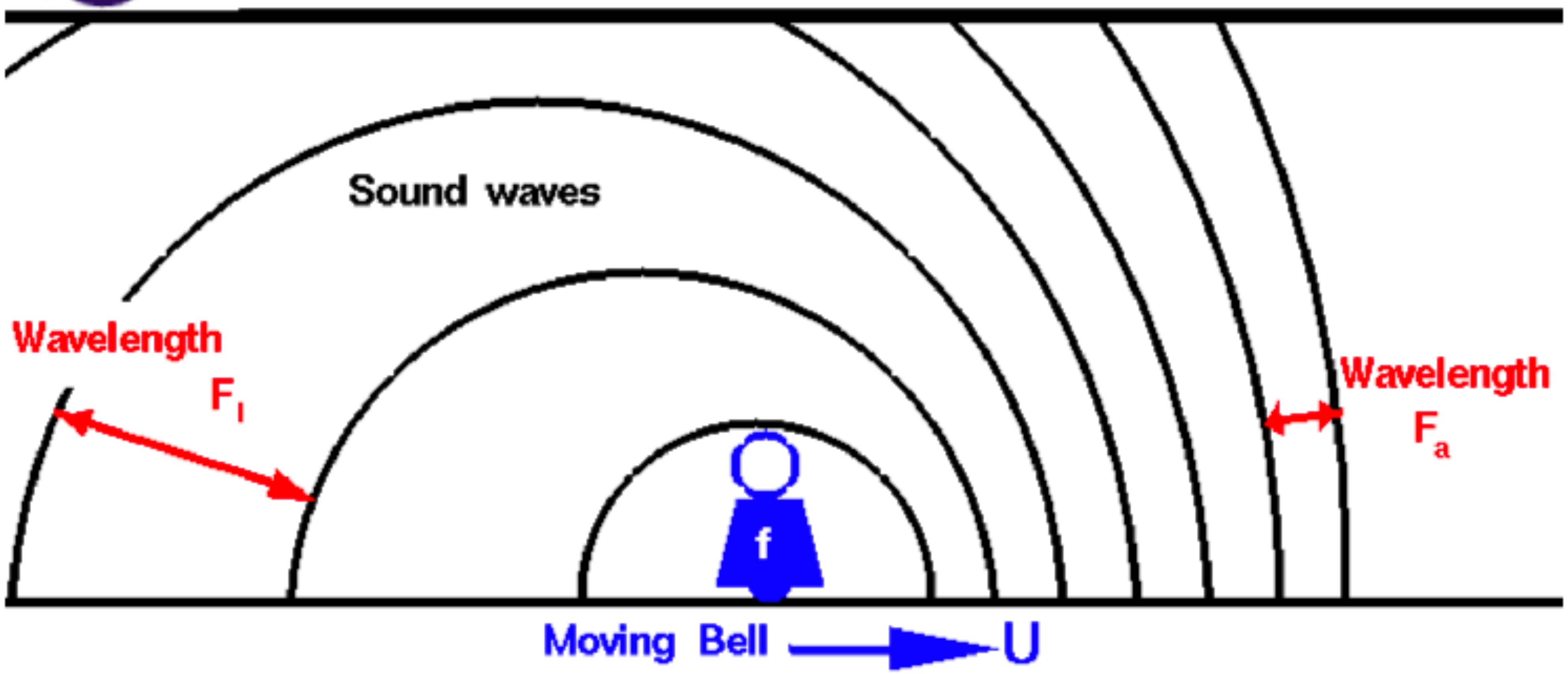
$$a = \frac{1}{1+z}$$



- **If we interpret the redshift z as the Doppler effect, galaxies recede (i.e. they move further away) in an expanding universe.**

Doppler Effect

Glenn Research Center



Wavelength (l) X Frequency (f) = Speed of Sound (a)

Long Wavelength ~ Low Frequency

Leaving: $F_l = f \dfrac{a}{a + U}$

Lower Pitch $F_l < f$

Short Wavelength ~ High Frequency

Approaching: $F_a = f \dfrac{a}{a - U}$

Higher Pitch $F_a > f$

Hubble law

- The comoving distance to an object located at redshift z reads as:

$$D(a) = \int_a^1 \frac{da'}{a'^2 H(a')} \qquad\qquad D(z) = \int_0^z \frac{dz'}{H(z')}$$

- At small redshifts, z≃ v/c.

The Hubble law can be written as:

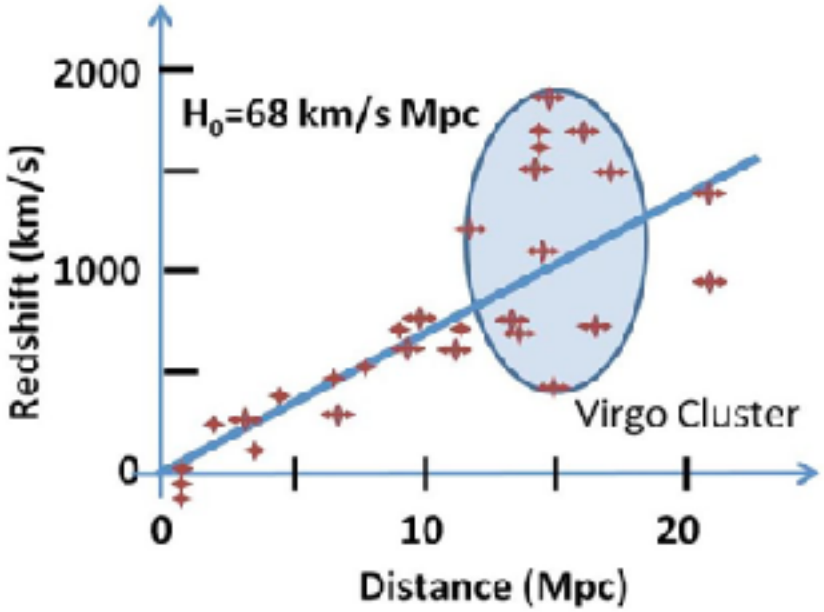
$$lim_{z\to 0} D(z) = \frac{z}{H(z=0)} = \frac{z}{H_0}$$

with the Hubble constant, H_0:

$$H_0 = 100h \text{ km/s/Mpc}$$

- Cosmological observations have determined that h≃0.7

1929: Edwin Hubble measures the spectra of hundred of galaxies and notices that they are redshifted, meaning that they are moving away from our galaxy. Furthermore, the further the galaxy is located, the faster it moves away from our galaxy.



Distances in cosmology

• Warning! you will have to work with these expressions in the "Hands in Cosmology" session

• "Standard ruler": If we have an object of known size the

$$\lambda = a(t)\Lambda \qquad \alpha = \frac{\Lambda}{D_A(z)} = \frac{\lambda}{d_A(z)}$$

$D_A(z)$=comoving angular distance, $d_A(z)$ is the physical angular distance $d_A = a\, D_A$

• "Standard candle": If we have an object of well-known luminosity, the flux (energy/unit of time) observed is:

$$F = \frac{L}{4\pi D_A^2(z)} \frac{1}{(1+z)^2} \equiv \frac{L}{4\pi d_L^2}$$

where a factor (1+z) comes from the redshift in the photon energy: $E_\gamma \propto \dfrac{1}{\lambda}$

and the additional (1+z) factor comes from the photon propagation

"Luminosity distance": $$d_L = (1+z)D_A = (1+z)^2 d_A$$

• At very small redshifts, the three distances are the same:

$$z \to 0, d_L = d_A = D_A$$

Distances in cosmology

- "Standard Candles": Are celestial objects with well-known luminosities. Their apparent magnitude is given by

$$m(z) = M + 5\log_{10}(d_L(z)/\mathrm{Mpc}) + 25$$

- Therefore, if we can determine the object's redshift (spectrography) we can learn about the luminosity distance of that object as a function of the redshift.

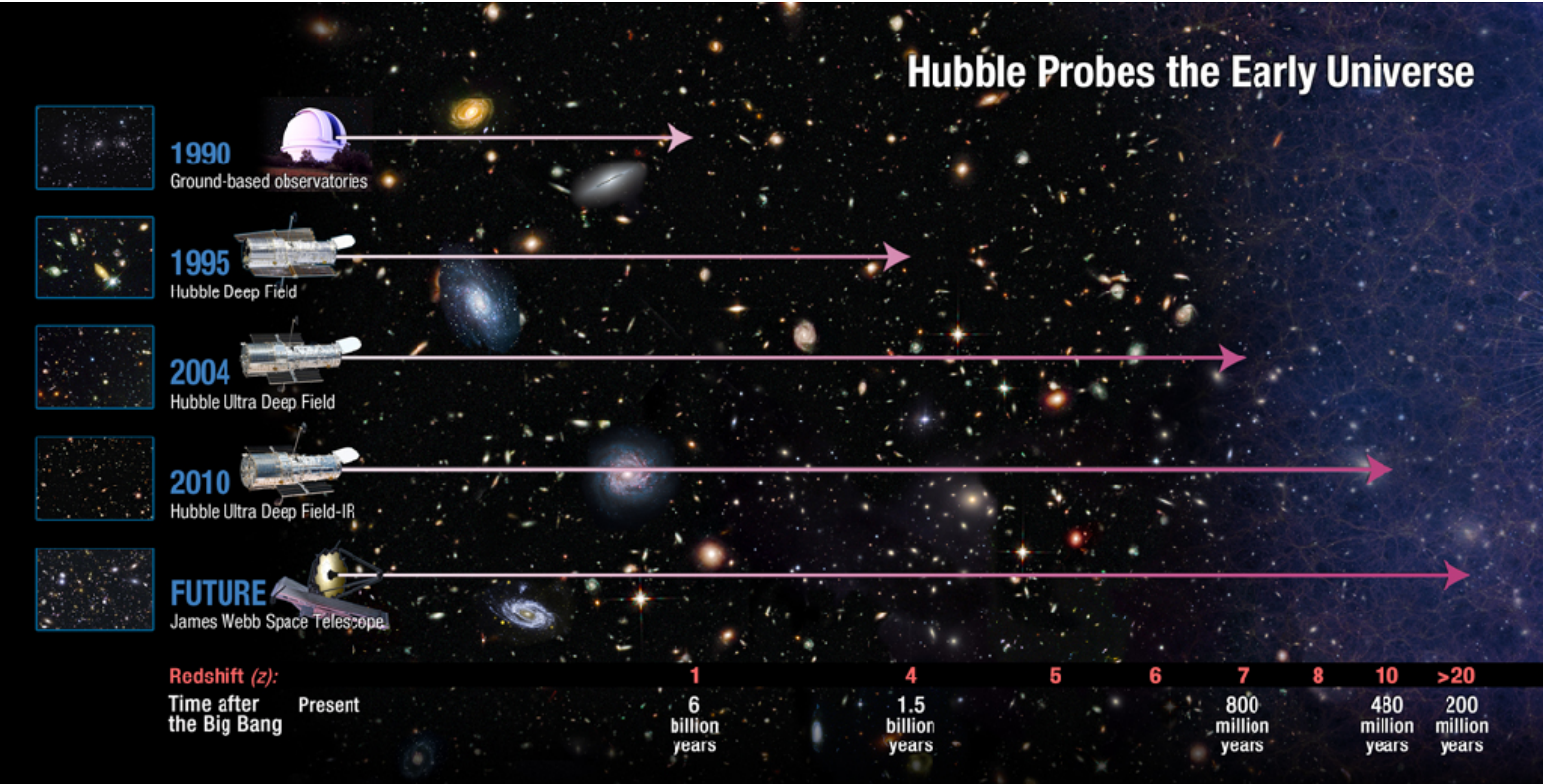
- At small redshifts:

$$d_L \simeq \frac{z}{H_0}$$

- Therefore, the Hubble constant extraction is possible via low-redshift measurements of the apparent magnitude of objects whose absolute magnitude M is known.

Hubble constant measurements

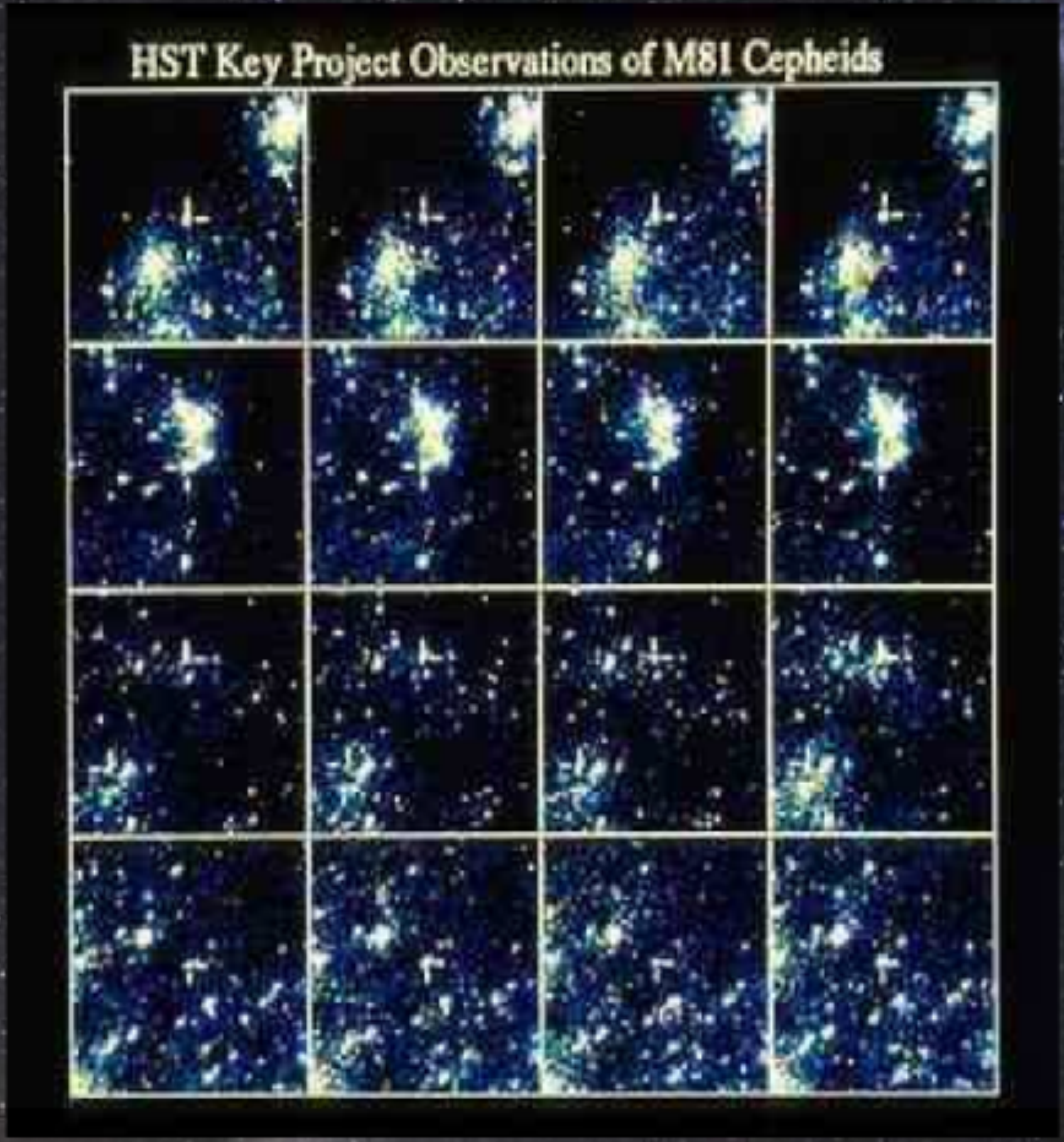
• Traditionally, the ideal objects are the Cepheids Variables, whose luminosity follows a very precise and regular period. Knowing such a relation, one can extract the luminosity distance to the galaxy where the Cepheid is located.



Hubble constant measurements

Advantages of Cepheids

- Among brightest stellar indicators
- Abundant in spiral galaxies
- Long lifetimes
- Small scatter in PL relation
- Studied and modeled extensively



Hubble constant measurements

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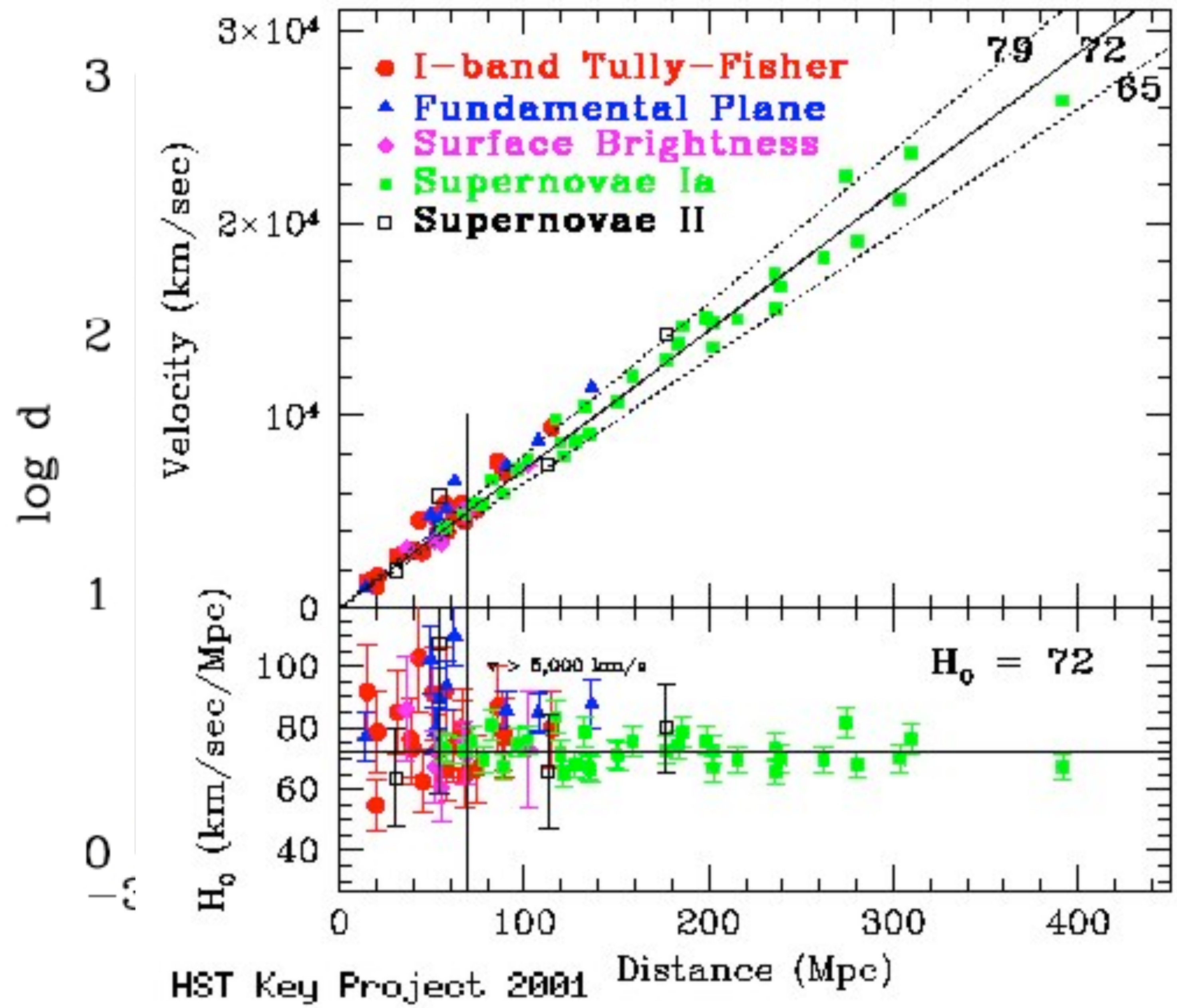

Freedman et al (2001)

Hubble constant measurements

HST Cepheids measurements have lead to a 2.4% determination of the Hubble constant:

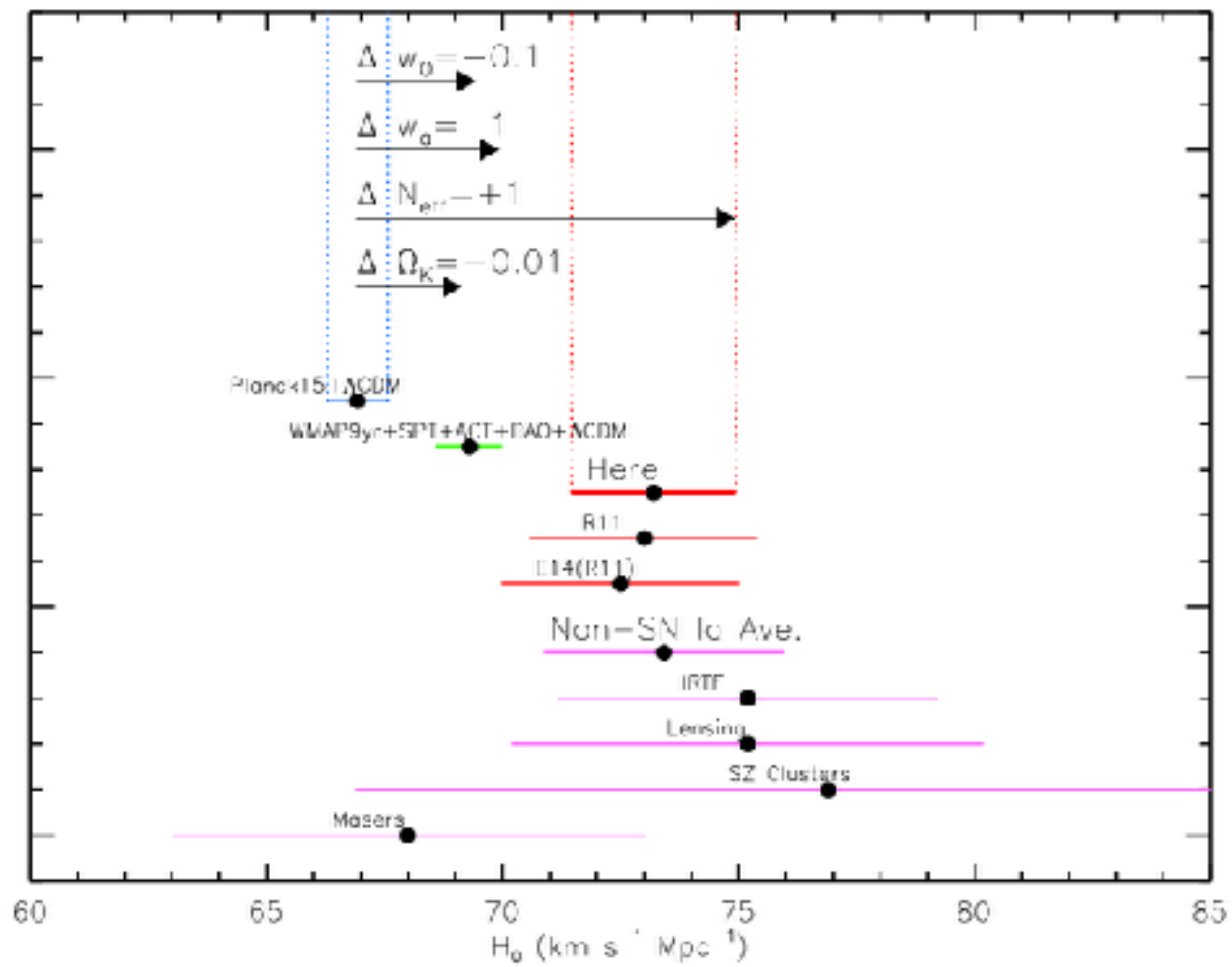

Fig. 13.— Local measurements of H_0 compared to values predicted by CMB data in conjunction with ΛCDM. We show 4 SN Ia-independent values selected for comparison by Planck Collaboration et al. (2014) and their average, the primary fit from R11, its reanalysis by Efstathiou (2014) and the results presented here. The 3.4σ difference between *Planck*+ΛCDM (Planck Collaboration et al. 2016) and our result motivates the exploration of extensions to ΛCDM.

A 2.4% Determination of the Local Value of the Hubble Constant[1]

Adam G. Riess[2,3], Lucas M. Macri[4], Samantha L. Hoffmann[4], Dan Scolnic[3,5], Stefano Casertano[3], Alexei V. Filippenko[6], Brad E. Tucker[6,7], Mark J. Reid[8], David O. Jones[2], Jeffrey M. Silverman[9], Ryan Chornock[10], Peter Challis[8], Wenlong Yuan[4], Peter J. Brown[4], and Ryan J. Foley[11,12]

ABSTRACT

We use the Wide Field Camera 3 (WFC3) on the *Hubble Space Telescope (HST)* to reduce the uncertainty in the local value of the Hubble constant from 3.3% to 2.4%. The bulk of this improvement comes from new, near-infrared observations of Cepheid variables in 11 host galaxies of recent type Ia supernovae (SNe Ia), more than doubling the sample of reliable SNe Ia having a Cepheid-calibrated distance to a total of 19; these in turn leverage the magnitude-redshift relation based on ~ 300 SNe Ia at $z < 0.15$. All 19 hosts as well as the megamaser system NGC 4258 have been observed with WFC3 in the optical and near-infrared, thus nullifying cross-instrument zeropoint errors in the relative distance estimates from Cepheids. Other noteworthy improvements include a 33% reduction in the systematic uncertainty in the maser distance to NGC 4258, a larger sample of Cepheids in the Large Magellanic Cloud (LMC), a more robust distance to the LMC based on late-type detached eclipsing binaries (DEBs), *HST* observations of Cepheids in M31, and new *HST*-based trigonometric parallaxes for Milky Way (MW) Cepheids.

$$H_0 = 73.24 \pm 1.74 \text{ km/s/Mpc}$$

Hubble constant measurements

HST Cepheids measurements have lead to a 2.4% determination of the Hubble constant:

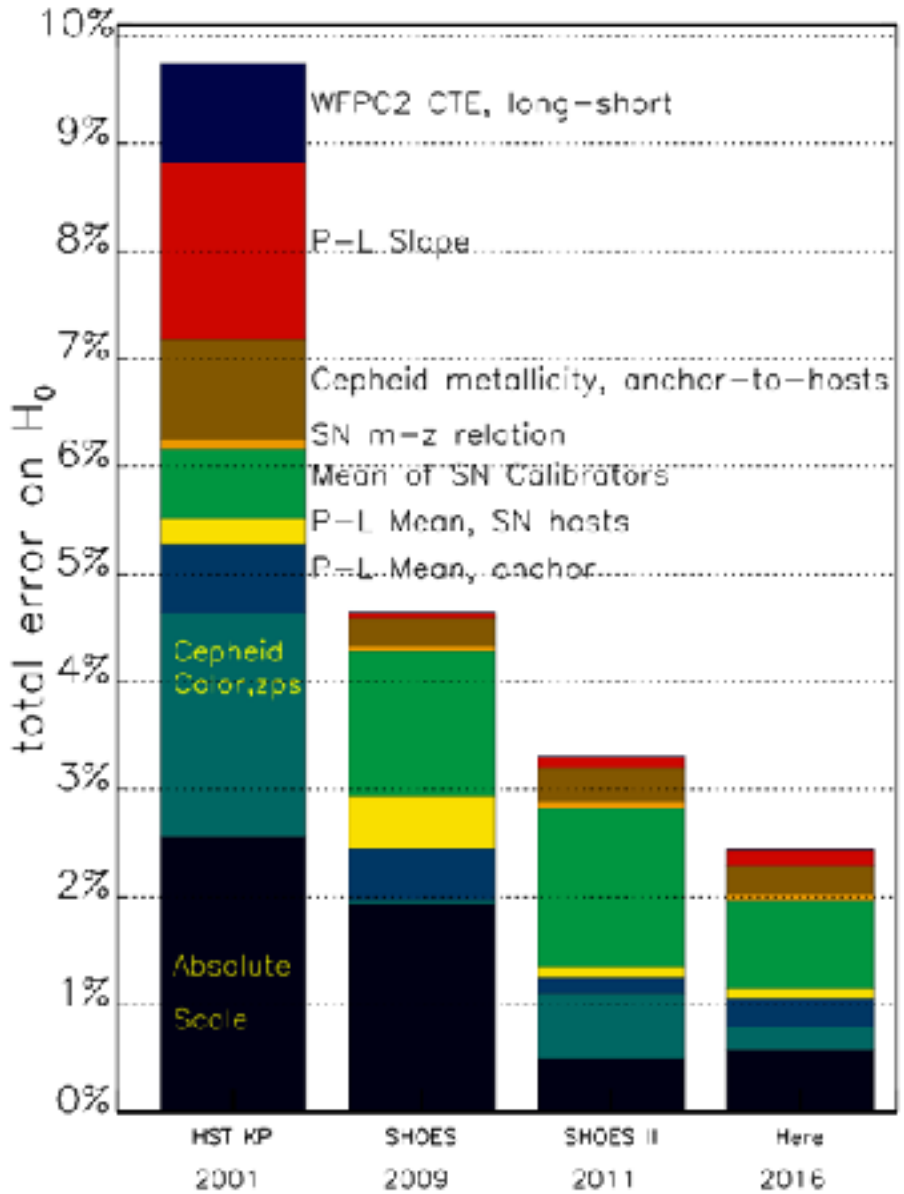

Fig. 1.— Uncertainties in the determination of H_0. Uncertainties are squared to show their individual contribution to the quadrature sum. These terms are given in Table 7.

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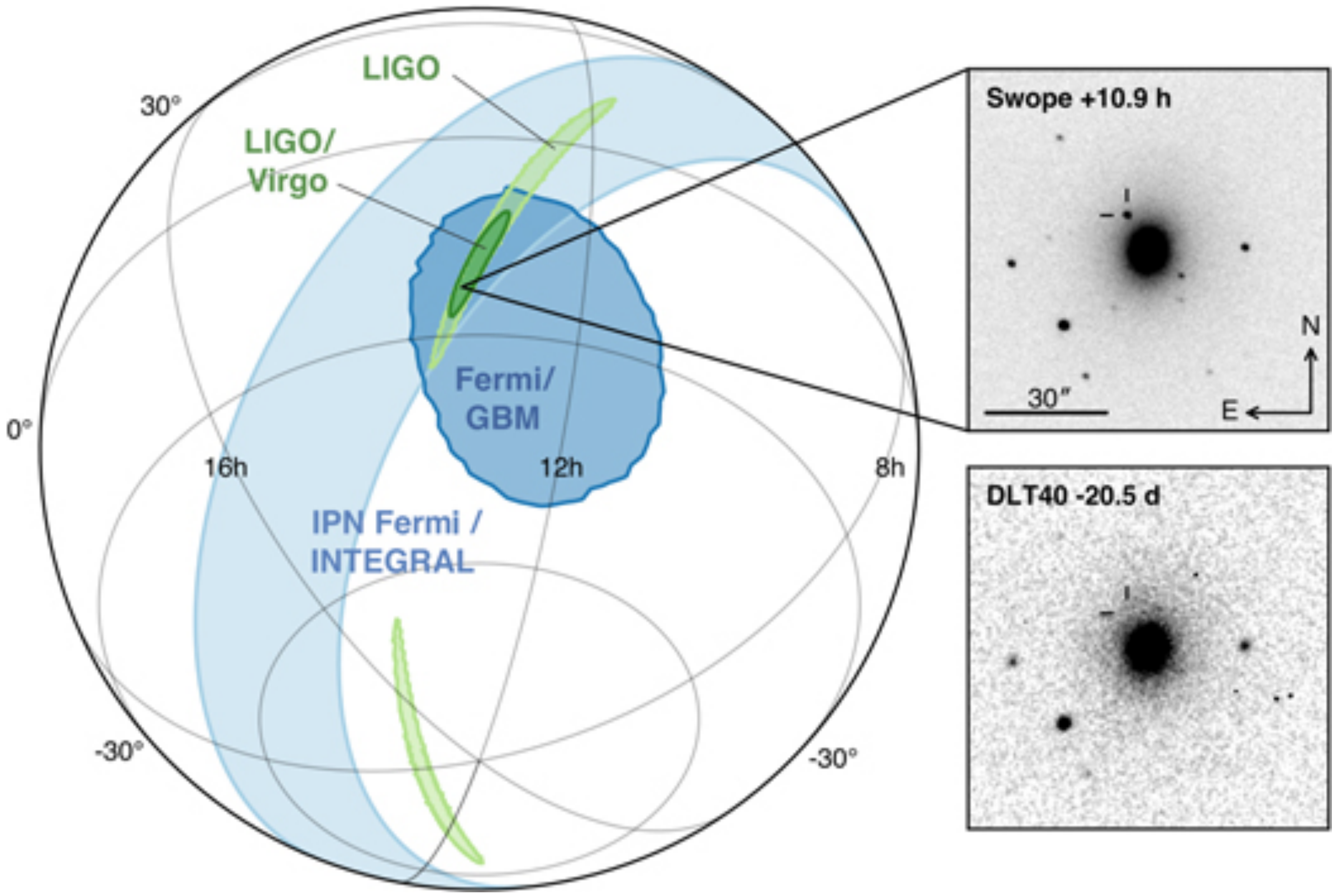
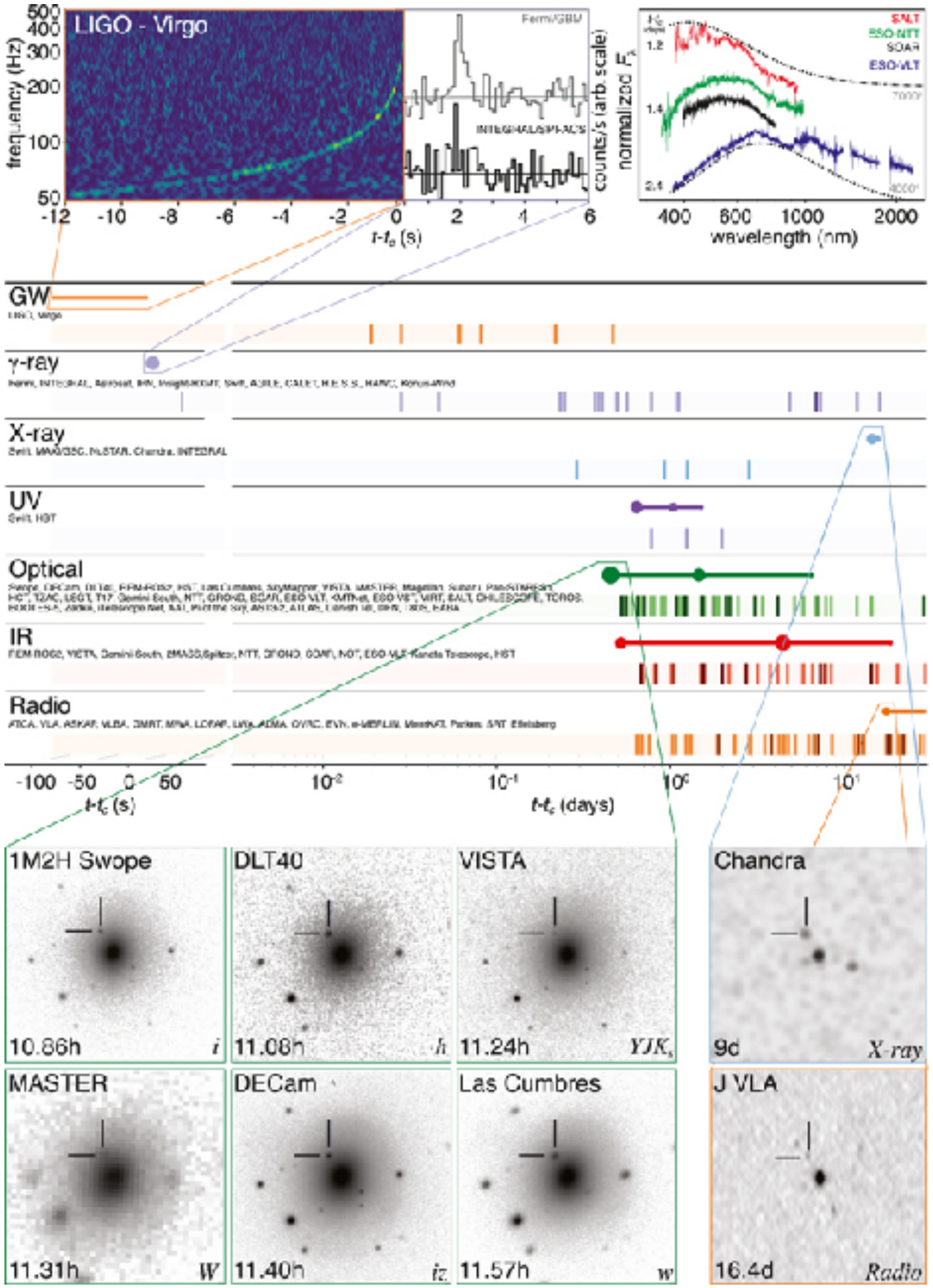
GW170817

The detection of GW170817 in both gravitational waves and electromagnetic waves represents the first 'multi-messenger' astronomical observation



Localization of the gravitational-wave, gamma-ray, and optical signals.
The left panel shows an orthographic projection of the 90% credible regions from LIGO (light green),
the initial LIGO-Virgo localization (dark green),
IPN triangulation from the time delay between Fermi an
INTEGRAL (light blue), and Fermi-GBM (dark blue).
The inset shows the location of the apparent host galaxy NGC 4993 in the Swope optical discovery image at 10.9 hr after the merger (top right)
and the DLT40 pre-discovery image from 20.5 days prior to merger (bottom right).
The reticle marks the position of the transient in both images.

GW170817


LIGO - Virgo
frequency (Hz)
counts/s (arb. scale)
Fermi/GBM
INTEGRAL/SPI-ACS
normalized F_λ
wavelength (nm)
SALT
ESO-NTT
SOAR
ESO-VLT
GW
γ-ray
X-ray
UV
Optical
IR
Radio
$t-t_c$ (s)
$t-t_c$ (days)
1M2H Swope
10.86h
DLT40
11.08h
VISTA
11.24h
Chandra
9d
X-ray
MASTER
11.31h
DECam
11.40h
Las Cumbres
11.57h
J VLA
16.4d
Radio

日本語要約

A gravitational-wave standard siren measurement of the Hubble constant

The LIGO Scientific Collaboration and The Virgo Collaboration, The 1M2H Collaboration, The Dark Energy Camera GW-EM Collaboration and the DES Collaboration, The DLT40 Collaboration, The Las Cumbres Observatory Collaboration, The VINROUGE Collaboration & The MASTER Collaboration

Affiliations | Contributions | Corresponding authors

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Citation | Rights & permissions | Article metrics

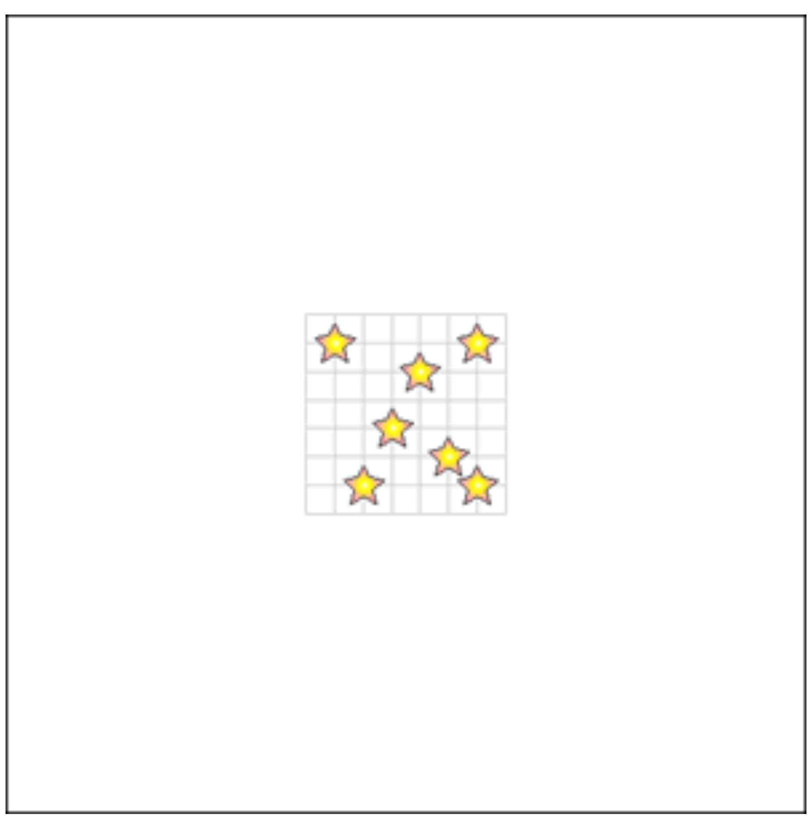
On 17 August 2017, the Advanced LIGO[1] and Virgo[2] detectors observed the gravitational-wave event GW170817—a strong signal from the merger of a binary neutron-star system[3]. Less than two seconds after the merger, a γ-ray burst (GRB 170817A) was detected within a region of the sky consistent with the LIGO–Virgo-derived location of the gravitational-wave source[4, 5, 6]. This sky region was subsequently observed by optical astronomy facilities[7], resulting in the identification[8, 9, 10, 11, 12, 13] of an optical transient signal within about ten arcseconds of the galaxy NGC 4993. This detection of GW170817 in both gravitational waves and electromagnetic waves represents the first 'multi-messenger' astronomical observation. Such observations enable GW170817 to be used as a 'standard siren'[14, 15, 16, 17, 18] (meaning that the absolute distance to the source can be determined directly from the gravitational-wave measurements) to measure the Hubble constant. This quantity represents the local expansion rate of the Universe, sets the overall scale of the Universe and is of fundamental importance to cosmology. Here we report a measurement of the Hubble constant that combines the distance to the source inferred purely from the gravitational-wave signal with the recession velocity inferred from measurements of the redshift using the electromagnetic data. In contrast to previous measurements, ours does not require the use of a cosmic 'distance ladder'[19]: the gravitational-wave analysis can be used to estimate the luminosity distance out to cosmological scales directly, without the use of intermediate astronomical distance measurements. We determine the Hubble constant to be about 70 kilometres per second per megaparsec. This value is consistent with existing measurements[20, 21], while being completely independent of them. Additional standard siren measurements from future gravitational-wave sources will enable the Hubble constant to be constrained to high precision.

Subject terms: High-energy astrophysics · Cosmology

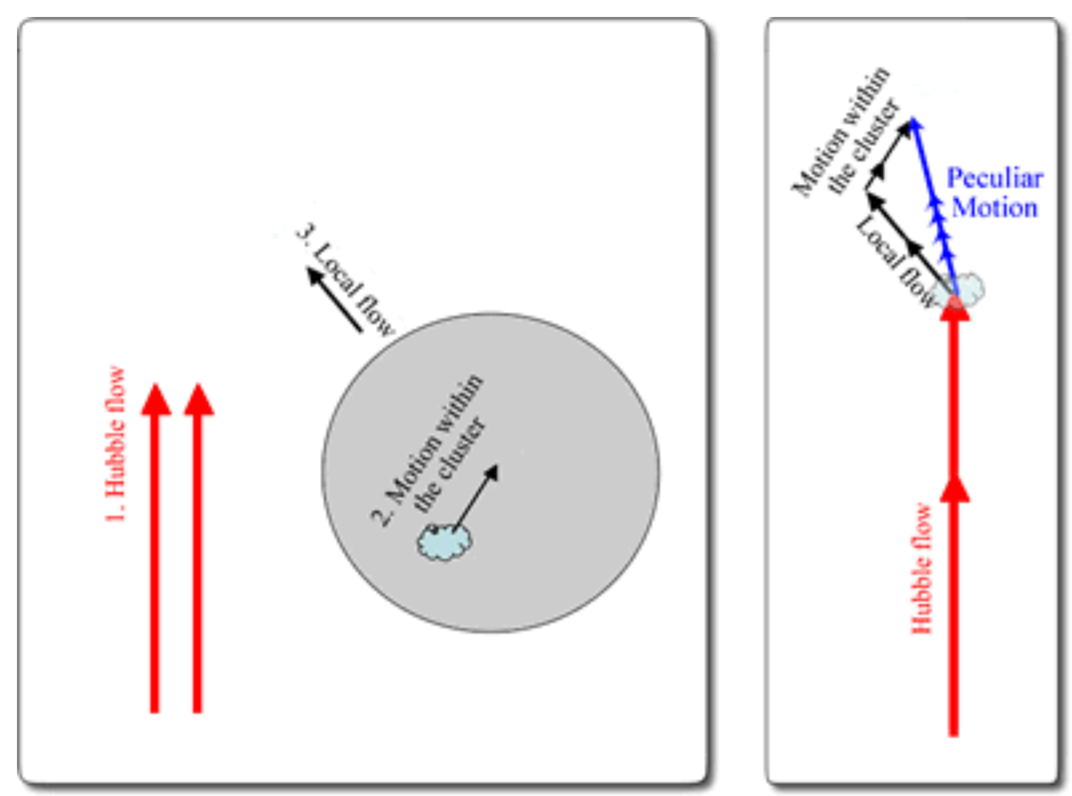
The method combines the distance to the source inferred purely from the gravitational-wave signal with the recession velocity inferred from measurements of the redshift using electromagnetic data.

$$v_H = H_0 d \qquad d = 43.8^{+2.9}_{-6.9}\,\mathrm{Mpc}$$

Using the optical identification of the host galaxy NGC 4993, they derive the Hubble flow velocity. PROBLEM: the random relative motion of galaxies (peculiar velocity) needs to be taken into account! In practice, the motions of galaxies are influenced by more than just the Hubble flow: the local flow, and the motion of the galaxy within its **cluster** and/or **group** environment. These deviations from the pure Hubble flow are referred to as peculiar motions. The peculiar velocity is about 10% of the measured recessional velocity.



The **Hubble flow** causes all **galaxies** to receed from each other.



The **local flow** and the motion of the galaxy within its **cluster environment** also contribute.

NGC 4993 is part of a collection of galaxies, ESO 508, which has a center-of-mass recession velocity relative to the frame of the cosmic CMB of 3 327 ± 72 km/s. The authors correct the group velocity by 310 km/s, due to the local gravitational fields.

The standard error on their estimate of the peculiar velocity is 69 km/s, but recognizing that this value may be sensitive to details of the bulk flow motion, in their analysis adopt a more conservative estimate of 150 km/s for the uncertainty on the peculiar velocity at the location of NGC 4993 and fold this in their estimate of the uncertainty on v_H.
From this, they obtain a Hubble velocity v_H= 3 017 ± 166 km/s.

Using this recessional velocity, one can find H_0= 68.9 km/s, which is very close to the value obtained through the more refined statistical method you will work on tomorrow afternoon in the "Hands in cosmology" session.

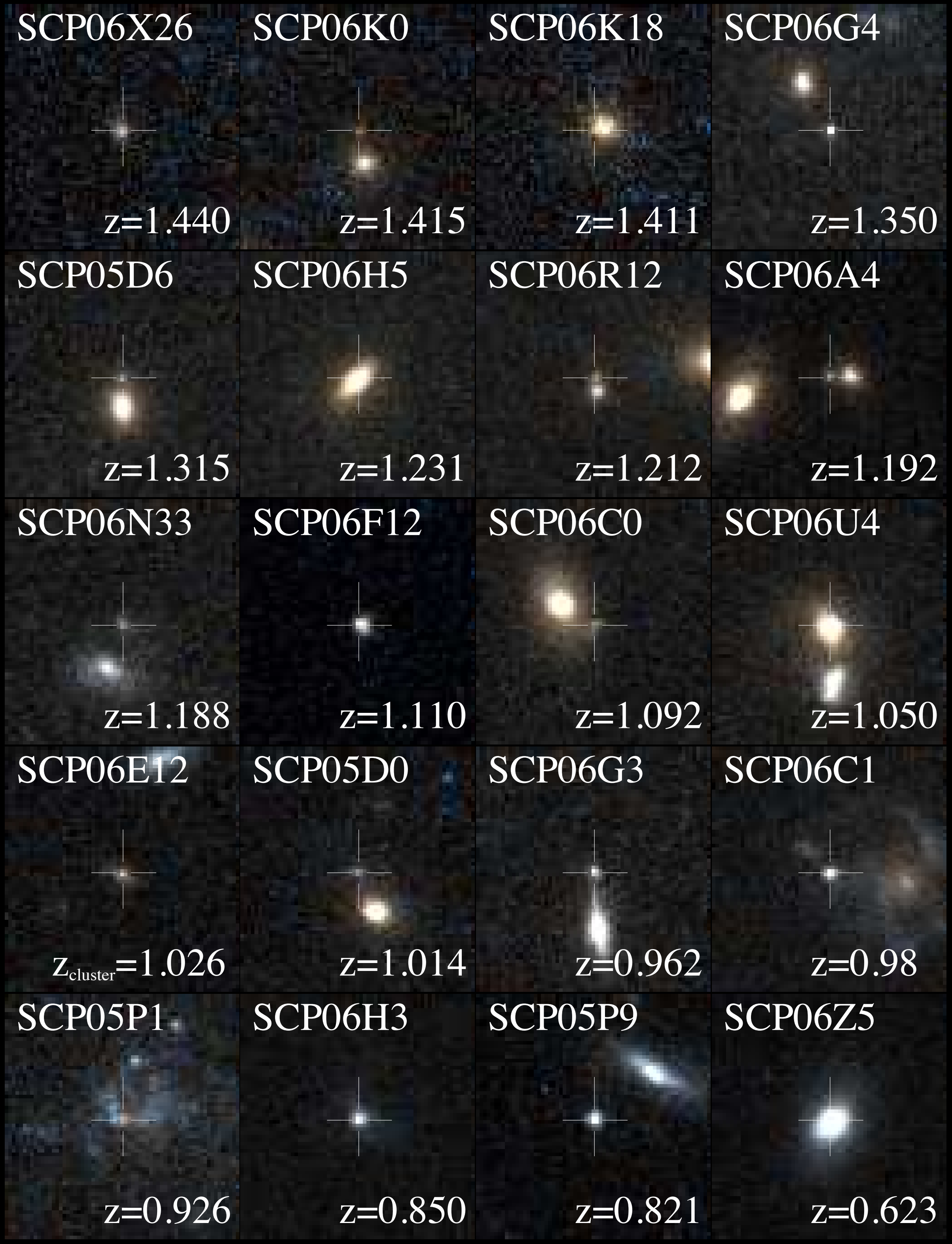
SNIa: Standard Candles

Supernova: star that dies causing a very violent nuclear explosion. It emits the energy equivalent to a full galaxy!

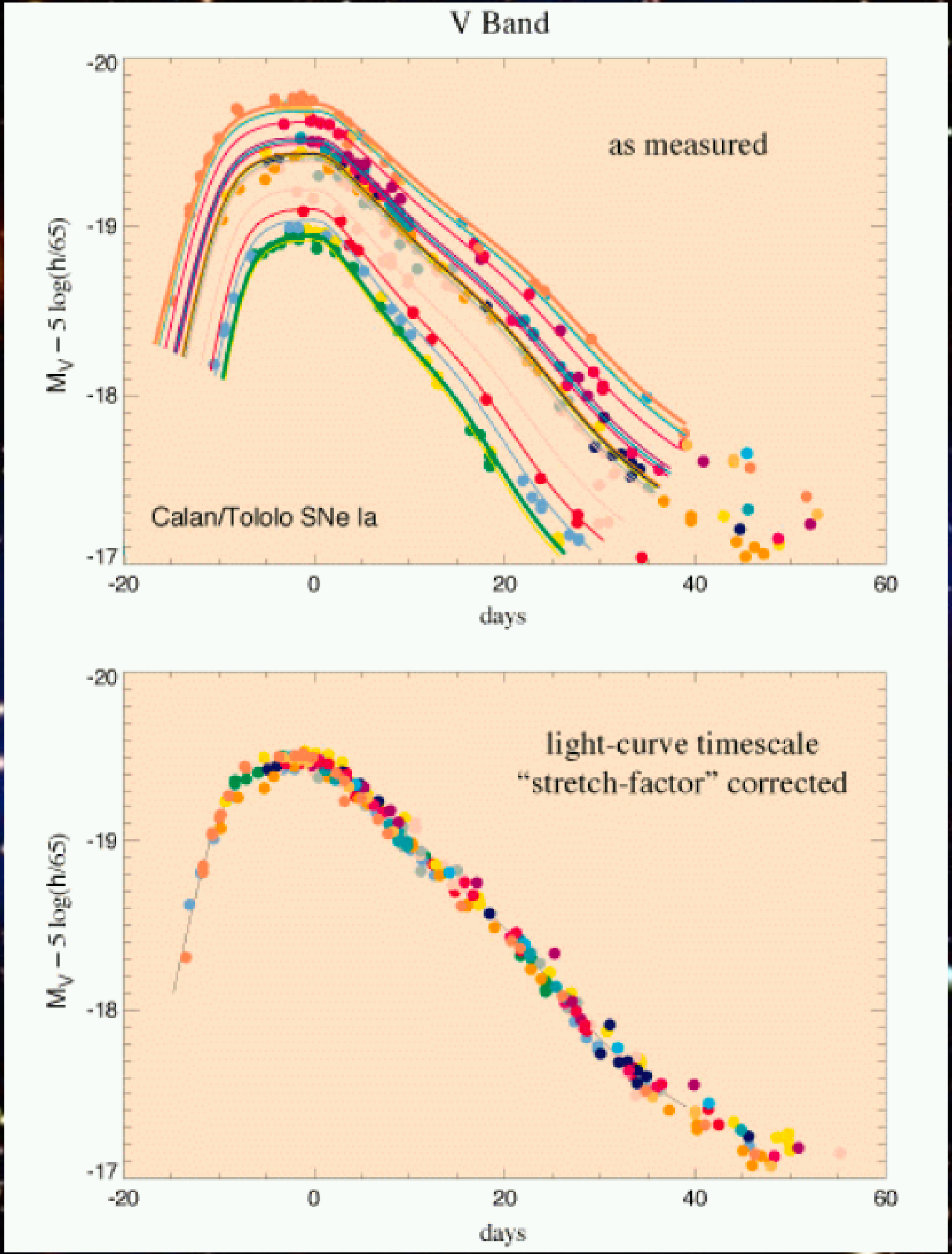
SN Ia: objects of very well-known luminosity. If we know their apparent luminosity, we can determine their distance:

Standard candles

2011 HST: 14 Supernovae Ia extra and with z>1!



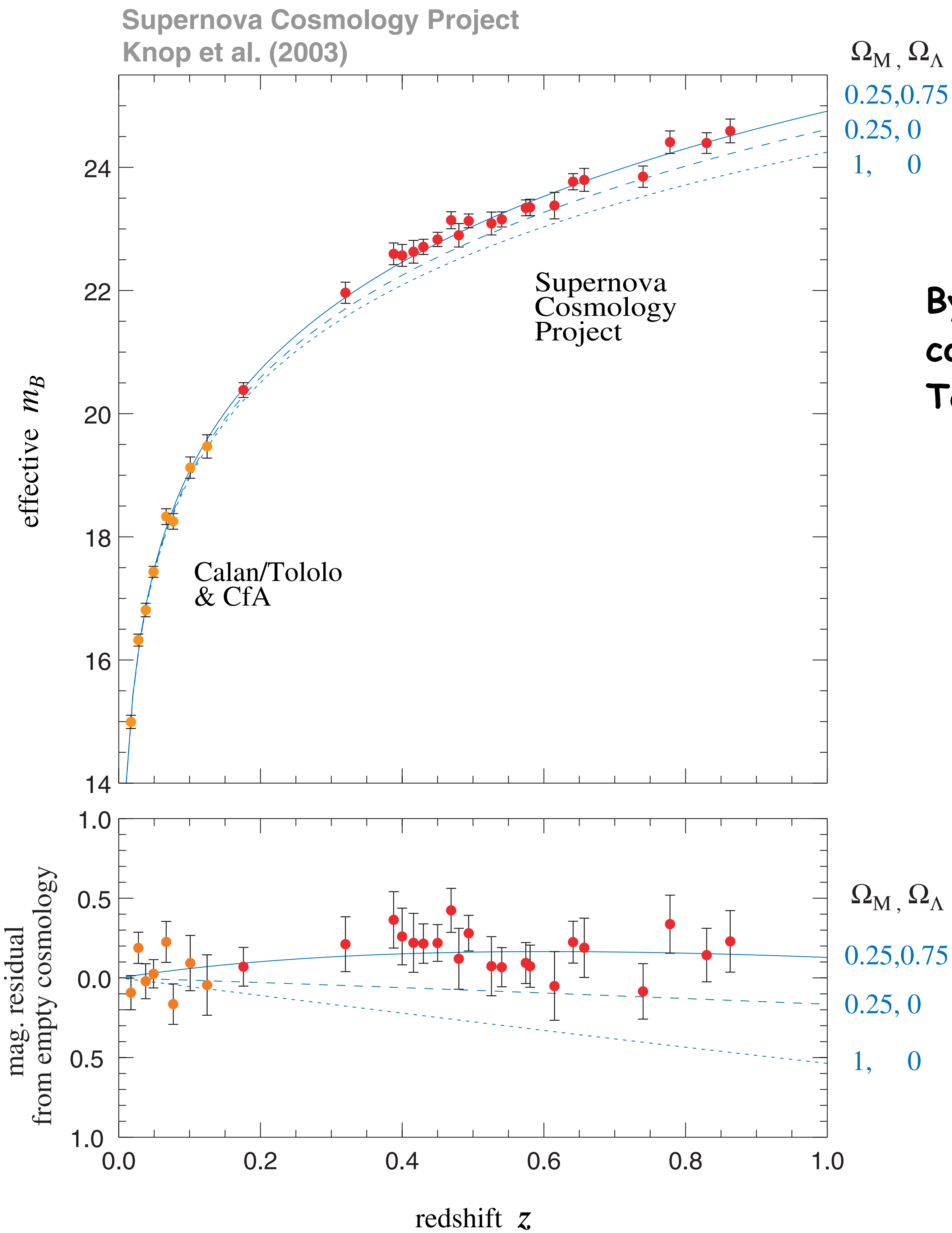
580 Supernovae Type Ia!



V Band
as measured
Calan/Tololo SNe Ia
$M_V - 5\log(h/65)$
days
light-curve timescale
"stretch-factor" corrected
$M_V - 5\log(h/65)$
days

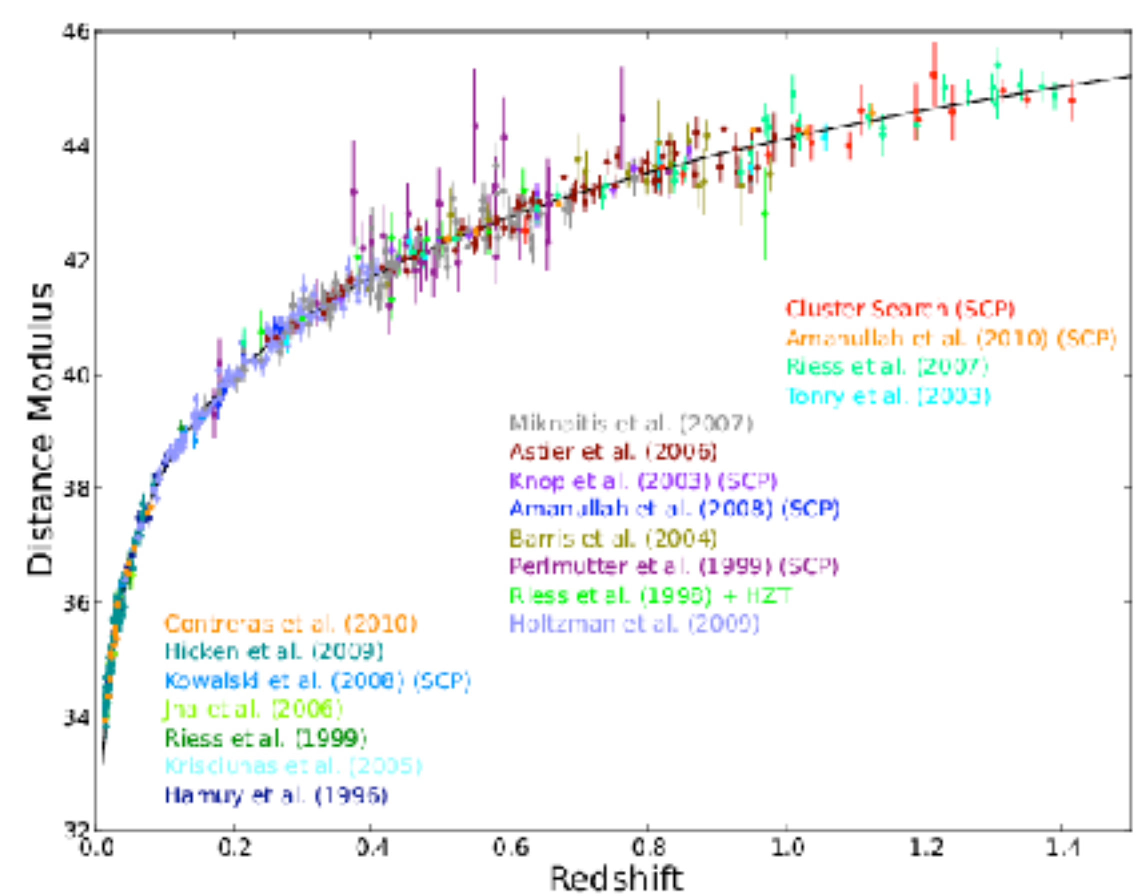
Distance modulus

$$\mu = m - M = 5\log_{10}(d_L(z)/\mathrm{Mpc}) + 25$$



$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

By knowing μ, we can obtain the luminosity distance and compare to the theoretical predictions.
To which curve match data better?



Imagine an object with well-known luminosity L: standard candle

If we measure the flux S and we know L, we can determine d_L and compare to what we expect from theory:

$$S \equiv \frac{L}{4\pi d_L^2}$$

$$d_L^2 \propto \int \frac{1}{H(z)}$$

What we observe from the SNIa data is that the measured d_L is larger than what one estimates in a universe with only matter. It seems that light has been travelled since longer ago, or for a larger distance, or, maybe, that the universe is not only expanding, but it is doing so in an accelerated way!

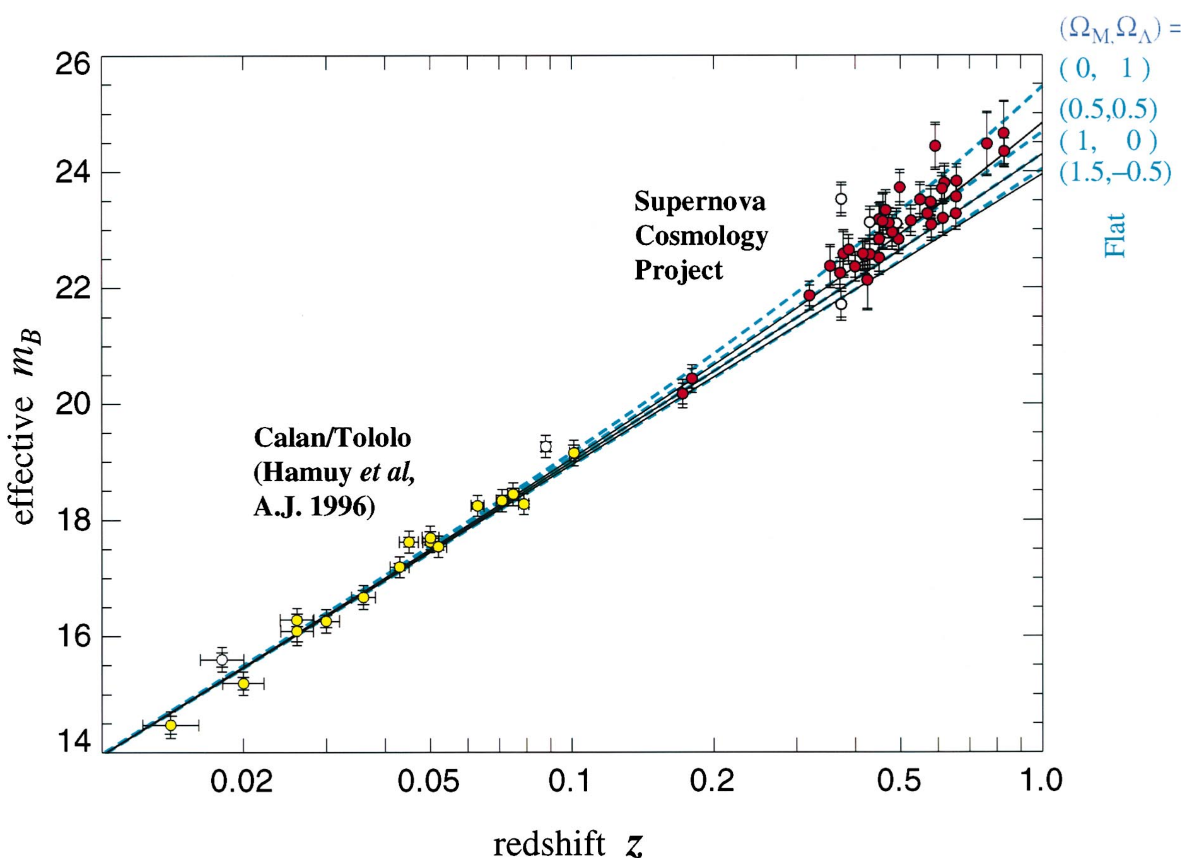
2011 Phsyics Nobel Prize

Saul Perlmutter

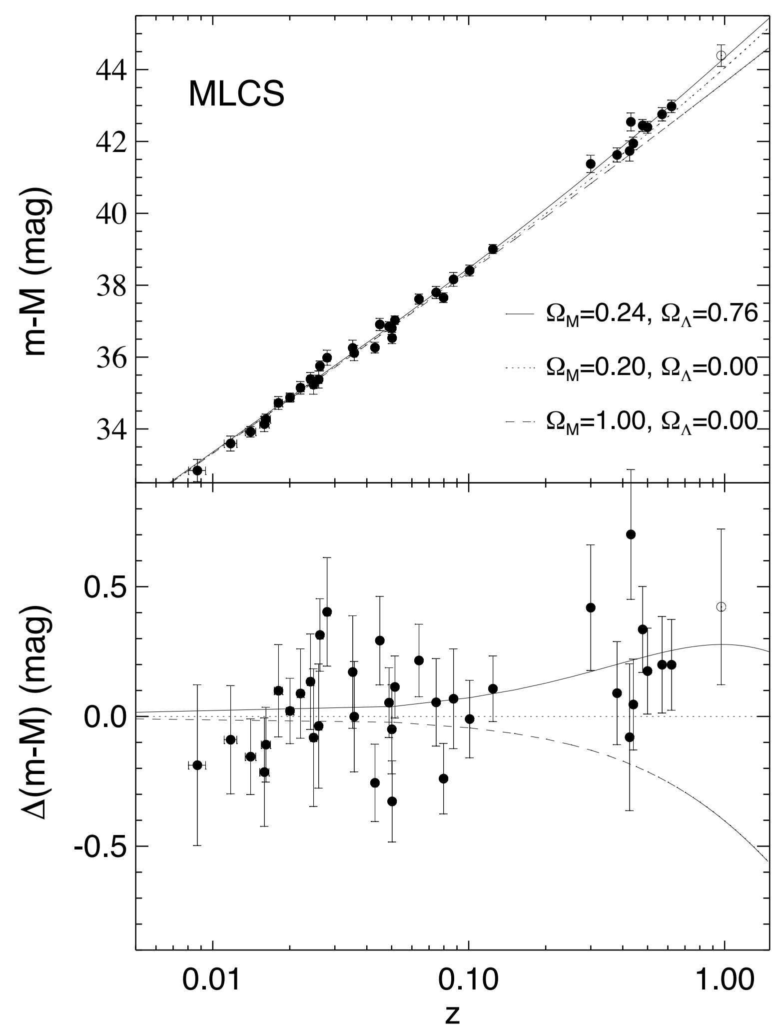
Brian P. Schmidt

Adam Riess

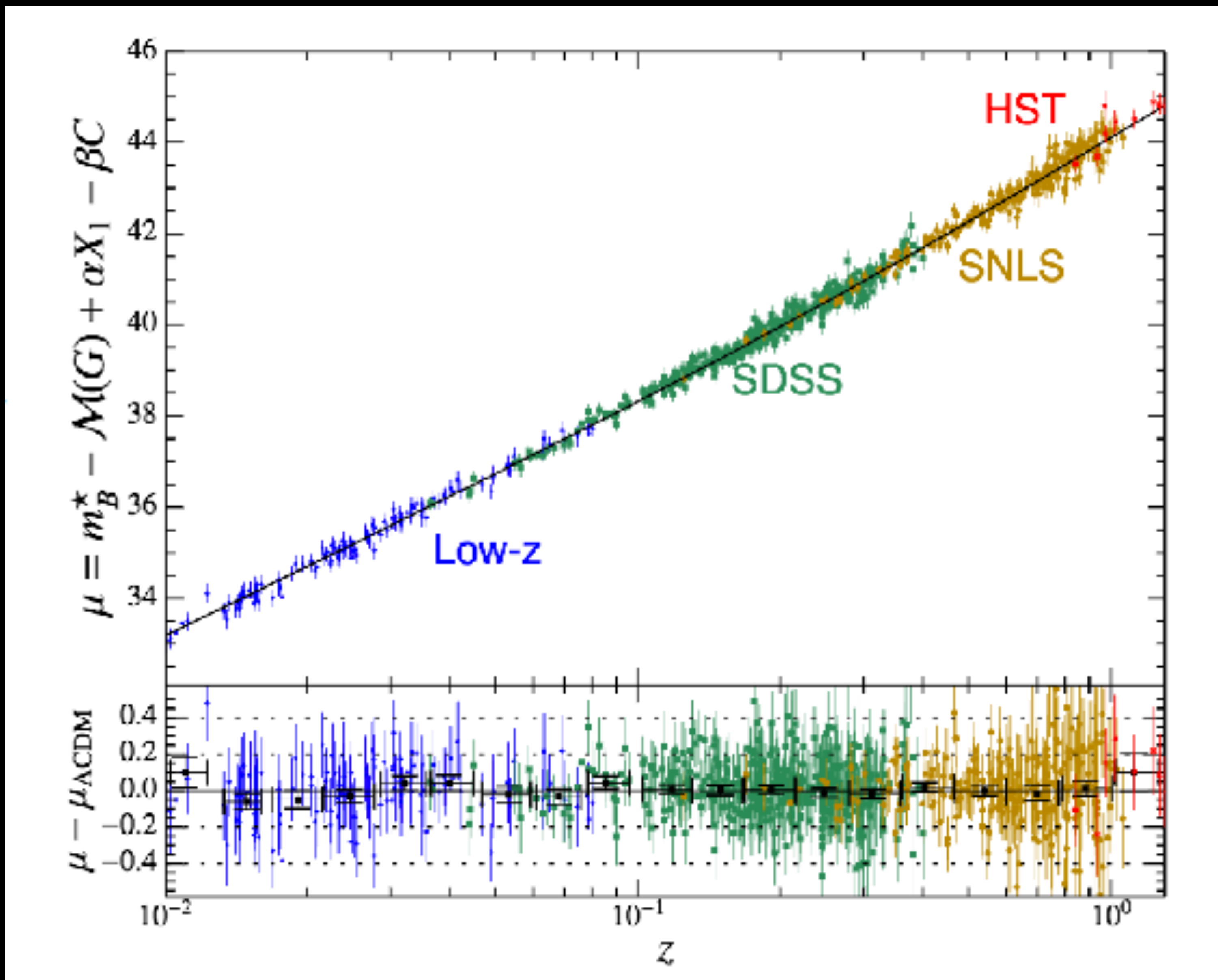
- **Supernova Cosmology Project**



- **High-Z Supernova Search**



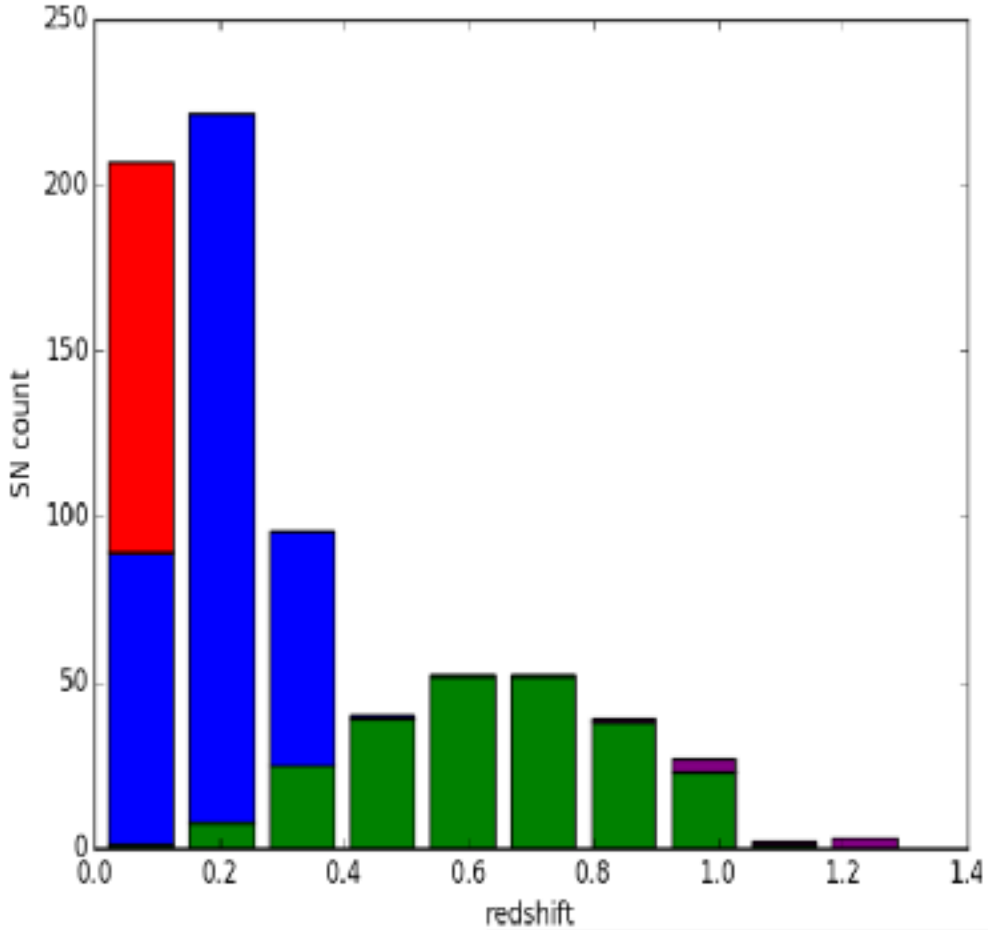
The JLA SDSS-SNLS joint SNIa sample

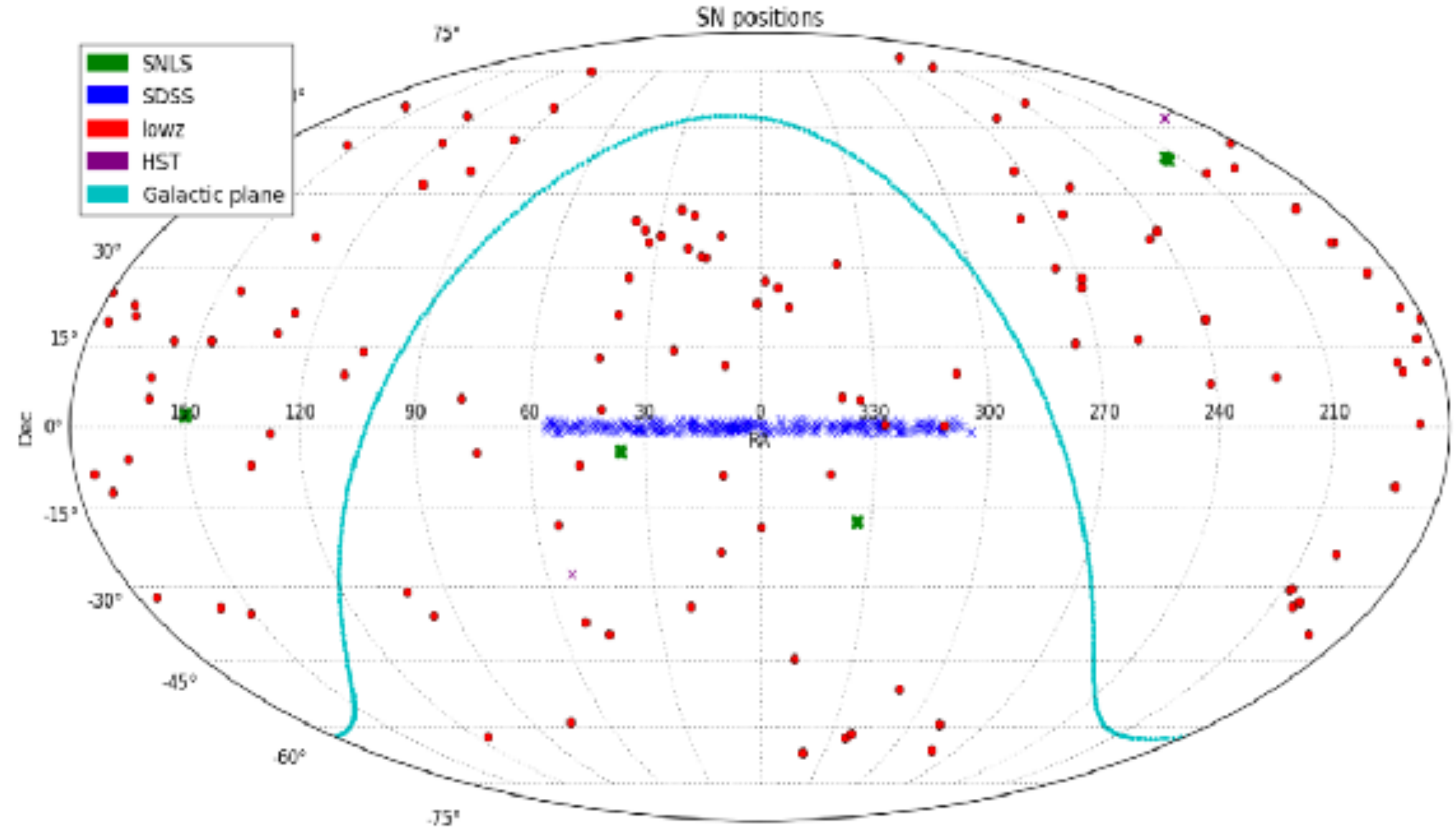


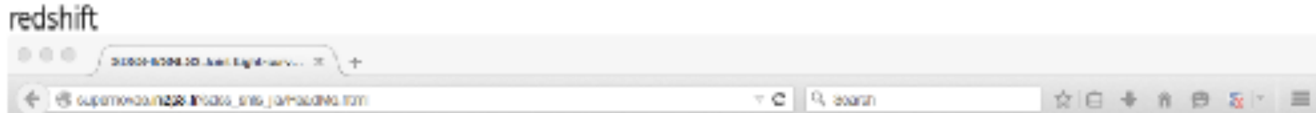
Source	Number
Cálan/Tololo	17
CfAI	7
CfAII	15
CfAIII[a]	55
CSP[a]	13
Other low-z	11
SDSS[a]	374
SNLS	239
HST	9
Total	740

740 Supernovae Ia!

Joint Lightcurve Analysis data (740 SNe)







This page contains links to data associated with the SDSS-II/SNLS3 Joint Light-Curve Analysis (Betoule et al. 2014, submitted to A&A).

The release consists in:

1. The end products of the analysis and a C++ code to compute the likelihood of this data associated to a cosmological model. The code enables both evaluations of the *complete* likelihood, and *fast* evaluations of an *approximate* likelihood (see Betoule et al. 2014, Appendix E).
2. The version 2.4 of the SALT2 light-curve model used for the analysis plus 200 random realizations usable for the propogation of model uncertainties.
3. The exact set of Supernovae light-curves used in the analysis.

We also deliver presentation material.

Since March 2014, the JLA likelihood plugin is included in the official release of cosmomc. For older versions, the plugin is still availalble (see below: *Installation of the cosmomc plugin*).

To analyze the JLA sample with SNANA, see $SNDATA_ROOT/sample_input_files/JLA2014/AAA_README.

1. Release history
V1 (January 2014, paper submitted):
V2 (March 2014):
V3 (April 2014, paper accepted):
V4 (June 2014):
V5 (March 2015):
V6 (March 2015):
2. Installation of the C++ likelihood code
Installation of the cosmomc plugin
3. SALT2 model
4. Error propagation
Error decomposition
SALT2 light-curve model uncertainties

1 Release history

V1 (January 2014, paper submitted):

First arxiv version.

V2 (March 2014):

Same as v1 with additionnal information (R.A., Dec. and bias correction) in the file of light-curve parameters.

V3 (April 2014, paper accepted):

Same as v2 with the addition of a C++ likelihood code in an independant archive (jla_likelihood_v3.tgz).

Data publicly available

Betoule *et al*, 1401.4064

1.1 DYNAMICS FLRW

General Relativity relates the metric with the matter and energy content in the universe. The sale factor a(t) will evolve in time accordingly to the matter-energy content of the universe.

In other words, matter and energy will tell us how the geometry of the space-time is curved via the Einstein equations.

Einstein Equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = 8\pi T_{\mu\nu}$$

- $R_{\mu\nu}$ is the Ricci tensor, depending on the metric $g_{\mu\nu}$ and its derivatives:

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\alpha}_{\beta\alpha}\Gamma^{\beta}_{\mu\nu} - \Gamma^{\alpha}_{\beta\nu}\Gamma^{\beta}_{\mu\alpha}$$

(It seems tedious but there are only two components different from 0, the 00 and the ii ones)

- R is the Ricci scalar, $R=g^{\mu\nu}R_{\mu\nu}$.

- $T_{\mu\nu}$ is the energy-momentum tensor.

- The Christoffel symbols:

$$\Gamma^{\mu}_{\alpha\beta} = \frac{g^{\mu\nu}}{2}\left(\frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}}\right)$$

Einstein Equations

$$\Gamma^{\mu}_{\alpha\beta} = \frac{g^{\mu\nu}}{2}\left(\frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}}\right)$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix}$$

$$\Gamma^{0}_{\alpha\beta} = -\frac{1}{2}\left(\frac{\partial g_{\alpha 0}}{\partial x^{\beta}} + \frac{\partial g_{\beta 0}}{\partial x^{\beta}} - \frac{\partial g_{\alpha\beta}}{\partial x^{0}}\right) = \frac{1}{2}\left(\frac{\partial g_{\alpha\beta}}{\partial x^{0}}\right)$$

- **EXERCISE, Check that:**

$$\Gamma^{0}_{00} = 0 \qquad \Gamma^{0}_{0i} = \Gamma^{0}_{i0} = 0$$

$$\boxed{\begin{aligned} \Gamma^{0}_{ij} &= \delta_{ij}\dot{a}a \\ \Gamma^{i}_{0j} &= \Gamma^{i}_{j0} = \delta^{i}_{j}\frac{\dot{a}}{a} \end{aligned}}$$

Einstein Equations

$$\boxed{\begin{gathered}\Gamma^0_{ij} = \delta_{ij}\dot{a}a \\ \Gamma^i_{0j} = \Gamma^i_{j0} = \delta^i_j \frac{\dot{a}}{a}\end{gathered}}$$

- **Lets compute the 00 component for the Einstein equations:**

$$R_{00} - \frac{1}{2} g_{00} \mathcal{R} = 8\pi G T_{00}$$

$$R_{00} = \Gamma^\alpha_{00,\alpha} - \Gamma^\alpha_{0\alpha,0} + \Gamma^\alpha_{\beta\alpha}\Gamma^\beta_{00} - \Gamma^\alpha_{\beta 0}\Gamma^\beta_{0\alpha}$$

- **But we know that $\Gamma^\alpha_{00}=0$, therefore:**

$$R_{00} = -\Gamma^i_{0i,0} - \Gamma^i_{j0}\Gamma^j_{0i} = -\frac{\partial}{\partial t}\left(\frac{\dot{a}}{a}\right)\delta_{ii} - \left(\frac{\dot{a}}{a}\right)^2 \delta_{ii} = -3\left(\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2\right) - 3\left(\frac{\dot{a}}{a}\right)^2 = -3\frac{\ddot{a}}{a}$$

- **EXERCISE, Check that:**

$$R_{ij} = \delta_{ij}\left(2\dot{a}^2 + a\ddot{a}\right)$$

- **And consequently:**

$$\mathcal{R} \equiv g^{\mu\nu}R_{\mu\nu} = -R_{00} + \frac{1}{a^2}R_{ii} = 6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right)$$

- **Finally we find that:**

$$\boxed{R_{00} - \frac{1}{2} g_{00} \mathcal{R}} = 8\pi G T_{00}$$

$$3\left(\frac{\dot{a}}{a}\right)^2$$

Einstein Equations

$$3\left(\frac{\dot{a}}{a}\right)^2$$

$$\boxed{R_{00} - \frac{1}{2}g_{00}\mathcal{R}} = 8\pi G T_{00}$$

- $T_{\mu\nu}$ is the energy-momentum tensor, that in the case of a isotropic perfect fluid:

$$T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho \qquad H^2(a) = \frac{8\pi G}{3}\rho$$

Friedmann Equation (1)

- Exercise! From:

$$R_{ij} - \frac{1}{2}g_{ij}\mathcal{R} = 8\pi G T_{ij}$$

- Derive the Friedmann Equation (2):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Friedmann Equations

- First Friedmann Equation reads as:

$$H^2(a) = H_0^2 \frac{\rho(a)}{\rho_{crit}} \qquad \rho_{crit} \equiv \frac{3H_0^2}{8\pi G}$$

- The second Friedmann Equation reads as:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

and it determines the accelerated processes in our universe's expansion. In order to have such an accelerated expansion it is required that:

$$\rho + 3p < 0$$

i.e. a negative pressure fluid!

Energy-momentum tensor conservation

- Time evolution of the $T_{\mu\nu}$ components

- In the absence of external forces, the energy momentum tensor is conserved.
- In an expanding universe, the energy momentum tensor conservation implies that its covariant derivative equals zero.

$$T^{\mu}{}_{\nu;\mu} \equiv \frac{\partial T^{\mu}{}_{\nu}}{\partial x^{\mu}} + \Gamma^{\mu}{}_{\alpha\mu} T^{\alpha}{}_{\nu} - \Gamma^{\alpha}{}_{\nu\mu} T^{\mu}{}_{\alpha} \qquad T^{\mu}{}_{\nu;\mu} = 0$$

$$T^{\mu}{}_{0;\mu} = 0 \qquad \frac{\partial T^{\mu}{}_{0}}{\partial x^{\mu}} + \Gamma^{\mu}{}_{\alpha\mu} T^{\alpha}{}_{0} - \Gamma^{\alpha}{}_{0\mu} T^{\mu}{}_{\alpha}$$

$$\boxed{\Gamma^{i}_{0j} = \Gamma^{i}_{j0} = \delta^{i}_{j}\frac{\dot{a}}{a}}$$

$$-\frac{\partial \rho}{\partial t} - \Gamma^{\mu}{}_{0\mu}\rho - \Gamma^{\alpha}{}_{0\mu} T^{\mu}{}_{\alpha}$$

$$\frac{\partial \rho}{\partial t} + \frac{\dot{a}}{a}(3\rho + 3p) = 0$$

$$\frac{\partial \rho}{\partial t} + 3H\rho(1 + w) = 0$$

Equation of state

- Matter (either cold dark matter or baryonic one) has zero pressure: $\rho_m \propto a^{-3}$
- Radiation is characterised by p=ρ/3: $\rho_r \propto a^{-4}$
- While dark energy should behave as:

$$\rho + 3p < 0 \qquad w < -1/3 \qquad \rho_{de} \propto a^{-3(1+w)}$$

Friedmann Equations

- **The first Friedmann equation can be written as:**

$$H^2(a) = H_0^2 \frac{\rho(a)}{\rho_{crit}} \qquad \rho_{crit} \equiv \frac{3H_0^2}{8\pi G}$$

$$H_0 = 100h \text{ km/s/Mpc}$$

$$\rho_{crit} = 1.879h^2 \times 10^{-29} \text{g cm}^{-3}$$

Friedmann Equations

- **The first Friedmann equation can be written as:**

$$H^2(a) = H_0^2 \frac{\rho(a)}{\rho_{crit}} \qquad \rho_{crit} \equiv \frac{3H_0^2}{8\pi G}$$

$$H_0 = 100h \text{ km/s/Mpc}$$
$$\rho_{crit} = 1.879h^2 \times 10^{-29}\text{g cm}^{-3}$$

$$H^2(a) = H_0^2 \left(\Omega_m(a) + \Omega_r(a) + \Omega_{de}(a)\right)$$

$$\Omega_m(a) = \rho_m(a)/\rho_{crit} = \rho_{m,0}a^{-3}/\rho_{crit} = \Omega_{m,0}a^{-3} = (\Omega_{dm,0} + \Omega_{b,0})a^{-3}$$

$$\Omega_r(a) = \rho_r(a)/\rho_{crit} = \rho_{r,0}a^{-4}/\rho_{crit} = \Omega_{r,0}a^{-4} = (\Omega_{\gamma,0} + \Omega_{\nu,0})a^{-4}$$

$$\Omega_{de}(a) = \rho_{de}(a)/\rho_{crit} = \rho_{de,0}a^{-3(1+w)}/\rho_{crit} = \Omega_{de,0}a^{-3(1+w)}$$

- **These expressions are valid for a FLAT universe. In case the universe is not flat:**

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2}$$

$$H^2(a) = H_0^2 \left(\Omega_m(a) + \Omega_r(a) + \Omega_{de}(a) + \Omega_K(a)\right)$$

$$\Omega_K(a) = -Ka^{-2}/H_0^2 = \Omega_{K,0}a^{-2}$$

“Cosmic sum rule”

$$\Omega_{m,0} + \Omega_{r,0} + \Omega_{de,0} + \Omega_{K,0} = 1$$

- In a flat universe, K=0, therefore:

$$\Omega_{m,0} + \Omega_{r,0} + \Omega_{de,0} = 1$$

- In an open universe, K=-1, therefore the curvature contribution is positive:

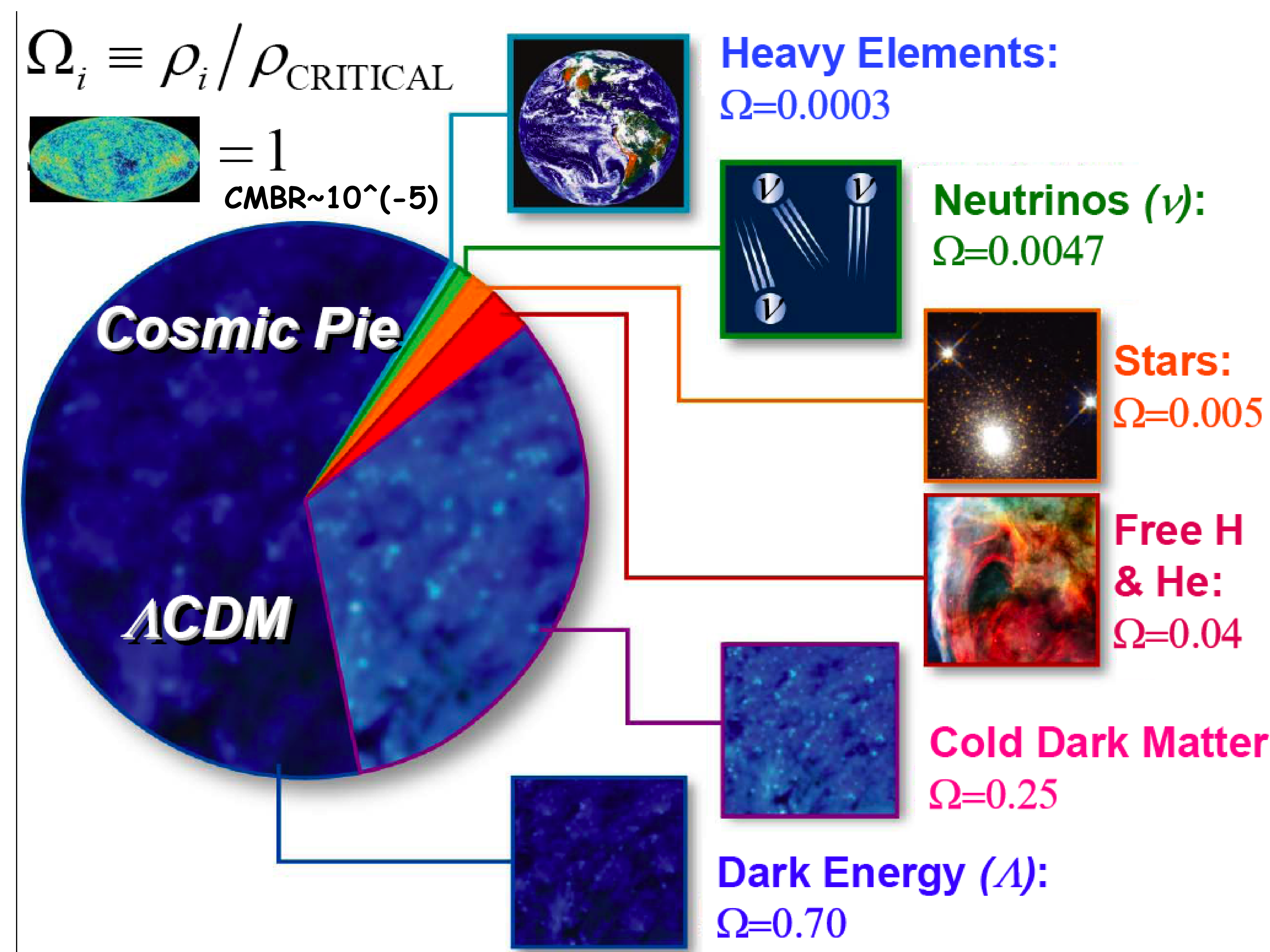
$$\Omega_{m,0} + \Omega_{r,0} + \Omega_{de,0} < 1$$

- In a close universe, K=+1, therefore the curvature contribution is negative:

$$\Omega_{m,0} + \Omega_{r,0} + \Omega_{de,0} > 1$$

Current cosmological observations indicate that the universe as a geometry very, very <u>close to the FLAT one:</u>

$$\Omega_K = -0.037^{+0.043}_{-0.049}$$


$\Omega_i \equiv \rho_i / \rho_{CRITICAL}$
$= 1$
CMBR~10^(-5)
Cosmic Pie
ΛCDM
Heavy Elements:
$\Omega=0.0003$
Neutrinos (ν):
$\Omega=0.0047$
Stars:
$\Omega=0.005$
Free H & He:
$\Omega=0.04$
Cold Dark Matter
$\Omega=0.25$
Dark Energy (Λ):
$\Omega=0.70$

Radiation: photons and neutrinos

• **Photons:** The cosmic microwave background radiation temperature is 2.725 K, measured with a precision of 50 parts in a million. The energy of such a photon bath is given by the integral of the Bose-Einstein distribution times E=p (massless):

$$\rho_\gamma = 2\int \frac{d^3p}{(2\pi)^3}\frac{1}{e^{p/T}-1}p \qquad x \equiv p/T \qquad \rho_\gamma = \frac{8\pi T^4}{(2\pi)^3}\int_0^\infty \frac{x^3 dx}{e^x-1}p = \frac{\pi^2}{15}T^4$$

$$\Omega_\gamma(a) = \frac{\rho_\gamma}{\rho_{crit}} = \frac{\pi^2}{15}\left(\frac{2.725\ \mathrm{K}}{a}\right)^4 \frac{1}{\rho_{crit}} = \frac{2.47\times 10^{-5}}{a^4 h^2} = \frac{4.75\times 10^{-5}}{a^4}$$

• **Neutrinos:** Neutrinos are fermions and therefore follow the Fermi-Dirac statistics. As we shall soon see, neutrinos decouple from the thermal bath before electron positron annihilation and therefore they did not share in the entropy release, being their temperature lower than that of photons:

$$\Omega_\nu(a) = \frac{\rho_\nu}{\rho_{crit}} = \frac{1.68\times 10^{-5}}{a^4 h^2} \quad (m_\nu = 0) \qquad \left(\frac{T_\nu}{T_\gamma}\right) = \left(\frac{4}{11}\right)^{1/3}$$

But neutrinos are massive particles!

$$n_\nu(T) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{e^{p/T}-1} = \frac{3}{22}n_\gamma(T)$$

$$\Omega_\nu(a) = \frac{m_\nu n_\nu}{\rho_{crit}} = \frac{\sum m_\nu}{94\ \mathrm{eV}h^2}\frac{1}{a^3}$$

Data tell us....

$$0.0006 \lesssim \Omega_{\nu,0}h^2 \lesssim 0.0025$$

Matter: baryons and dark matter

- **Baryons**: The baryon density can not be inferred from temperature measurements. Currently we know that:

$$\Omega_b h^2 = 0.02205^{+0.00056}_{-0.00055}$$

from the CMB anisotropies. Other methods to extract the present baryonic mass-energy density are light element abundances, quasar spectra or the gas population in galaxies.

- **Dark matter**

A number of observations (galaxy rotation curves, galaxy clusters, gravitational lensing, large scale structure and the CMB anisotropies) indicate that the majority of the matter in the universe is unknown: dark matter

$$\Omega_{dm} h^2 = 0.1199^{+0.0053}_{-0.0052}$$

Furthermore, observations of the large scale structure of our universe tell us that a COLD dark matter component provides an excellent fit to data.

There is way more matter in the universe than that we can see (stars, gas, planets...)

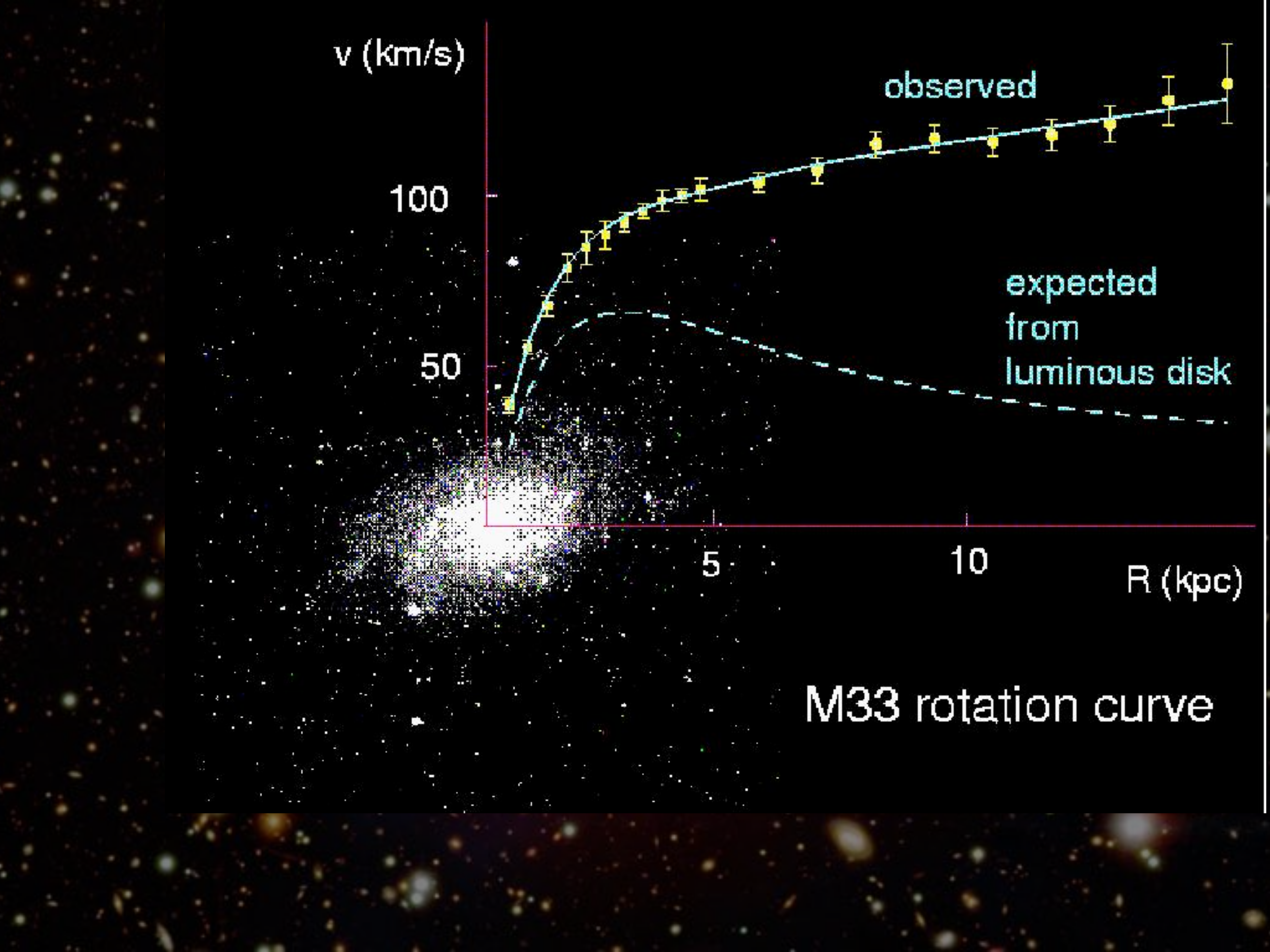
Dark matter

We know that should be there due to its gravitational effects:

Nothing escapes from gravity!

Galaxy M33

located at 3 million light years


v (km/s)
100
50
observed
expected
from
luminous disk
5
10
R (kpc)
M33 rotation curve

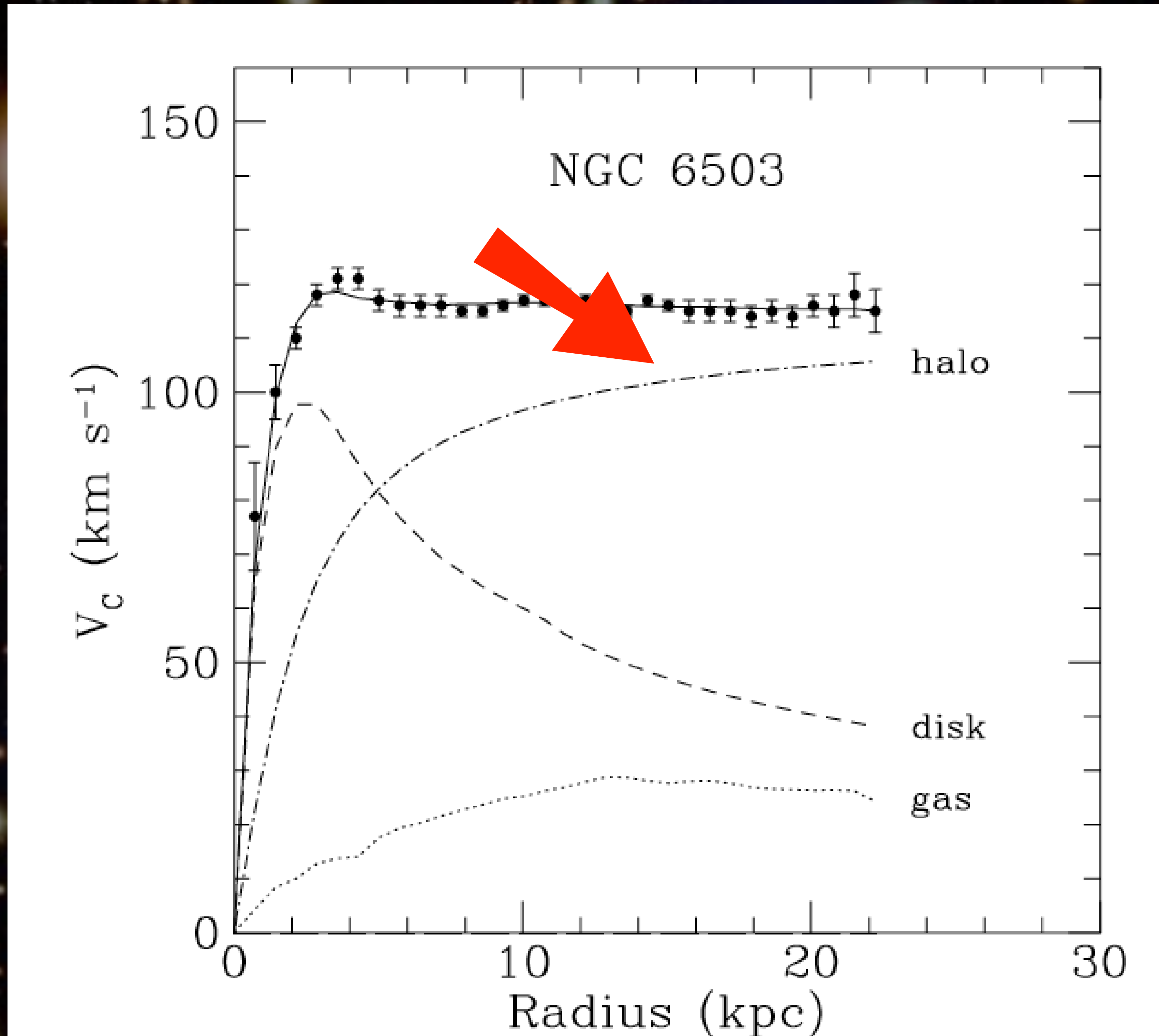
Newton tells us that velocity must decrease with the radius!

$$v(r) = \sqrt{\frac{GM(r)}{r}}$$

If it would exist, furthermore, an amount of additional matter whose distribution:

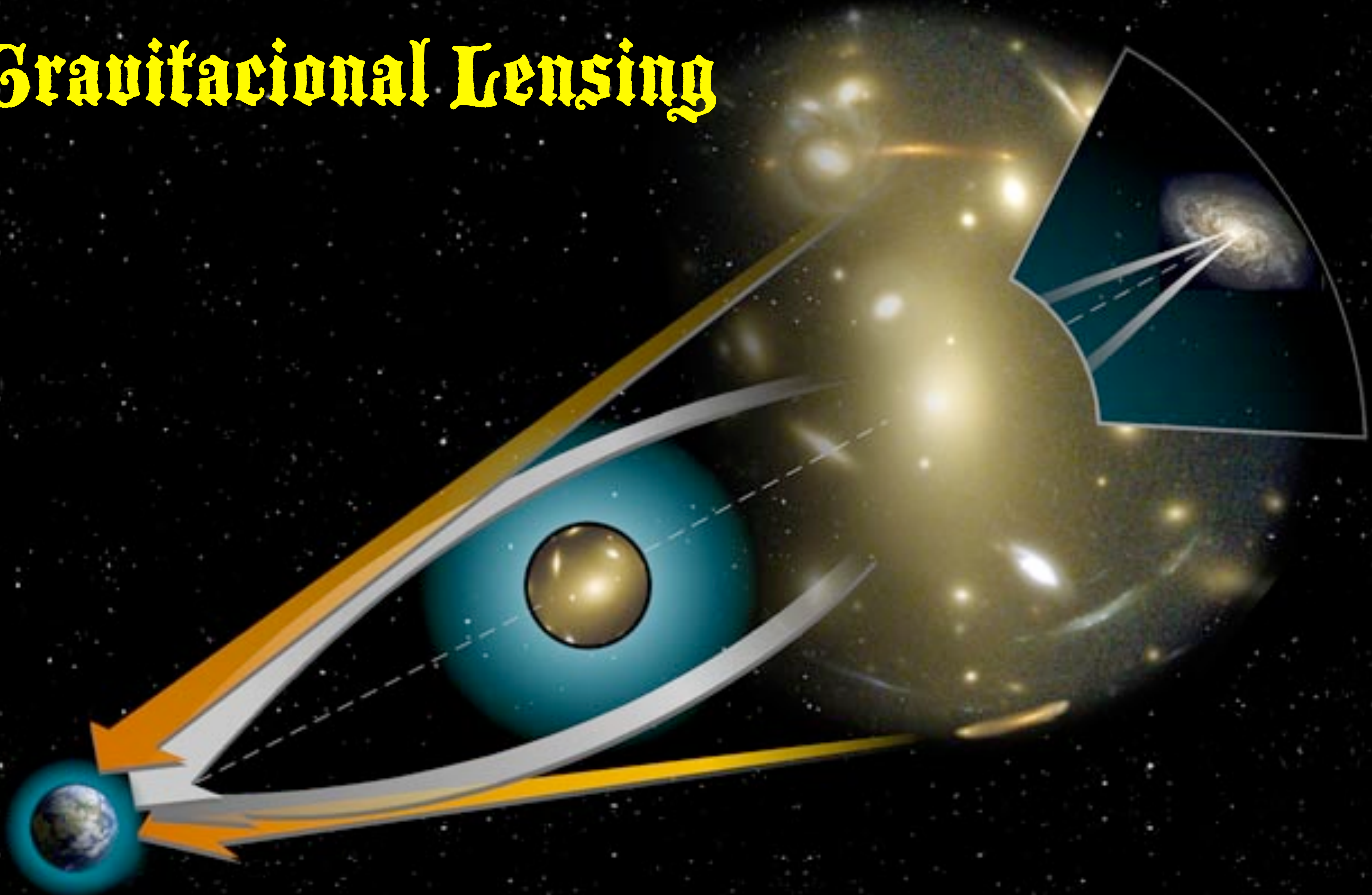
$$M(r) \propto r \qquad \rho \propto 1/r^2$$

then, we could explain the galaxy rotation curves



$M(r) \propto r$ $\Rightarrow$ $\rho \propto 1/r^2$

Gravitacional Lensing



Einstein's relativity predicts that the presence of a massive body will curve space time, distorting the light trajectory. The shape of the background objects will change/ multiplied by the presence of intervening galaxies.

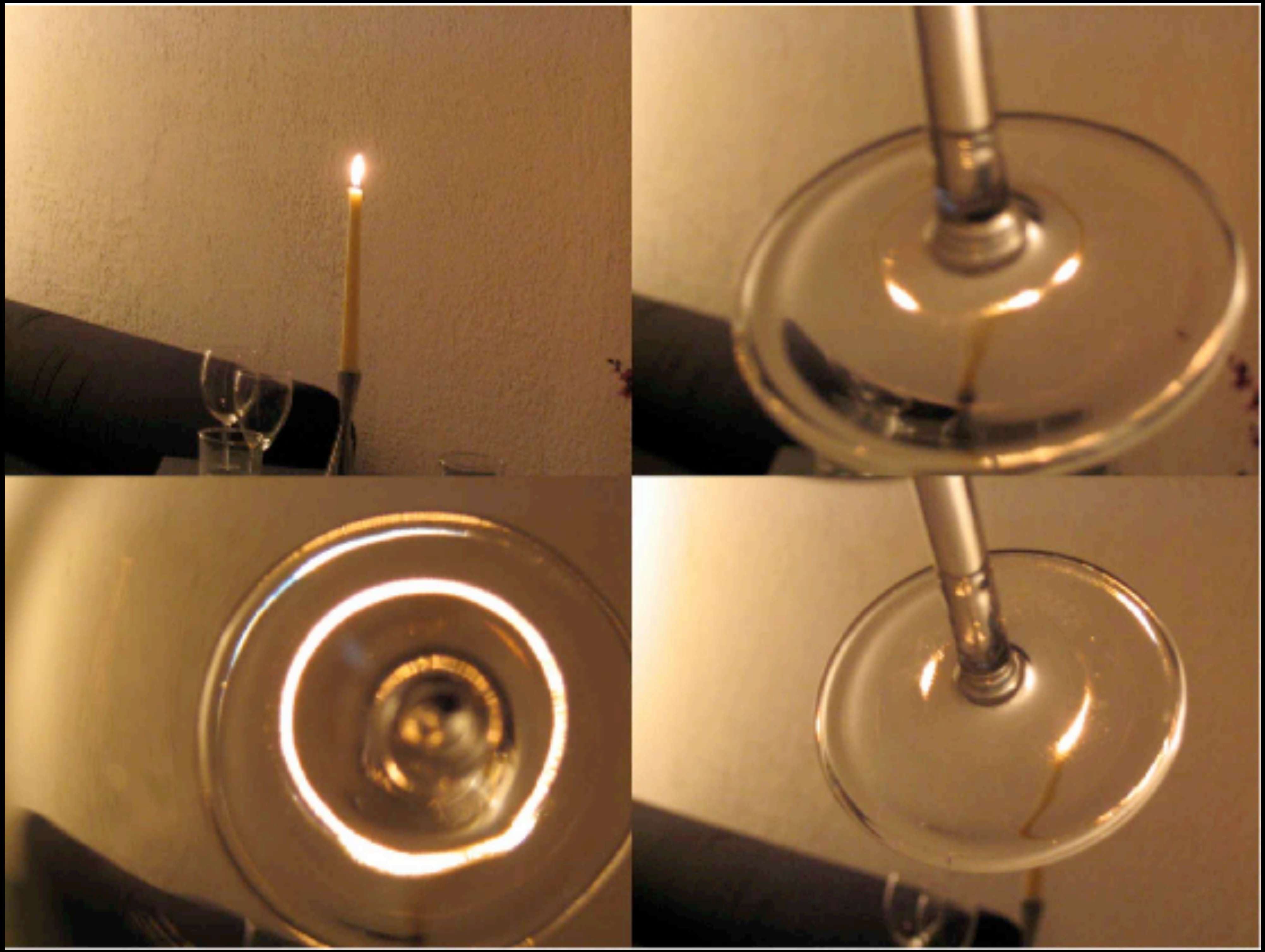
Einstein rings: Perfect alignment

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{d_{LS}}{d_L d_S}},$$

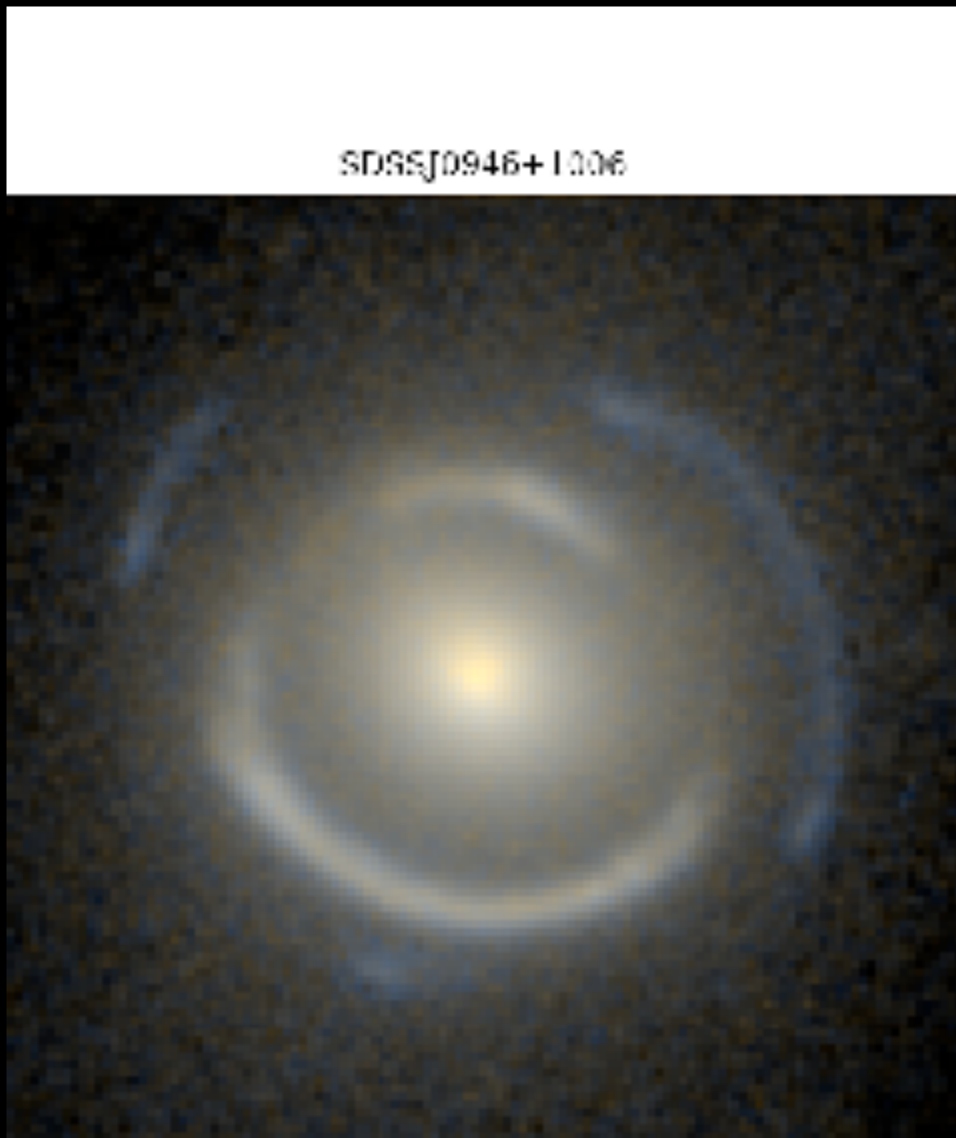
Lensing Galaxy

This movie shows a spiral galaxy acting as a lense of a background quasar (Quasi-stellar radio source) moving behind the galaxy. When the alignment source-lens-observer is perfect, we see the formation of the Einstein ring!

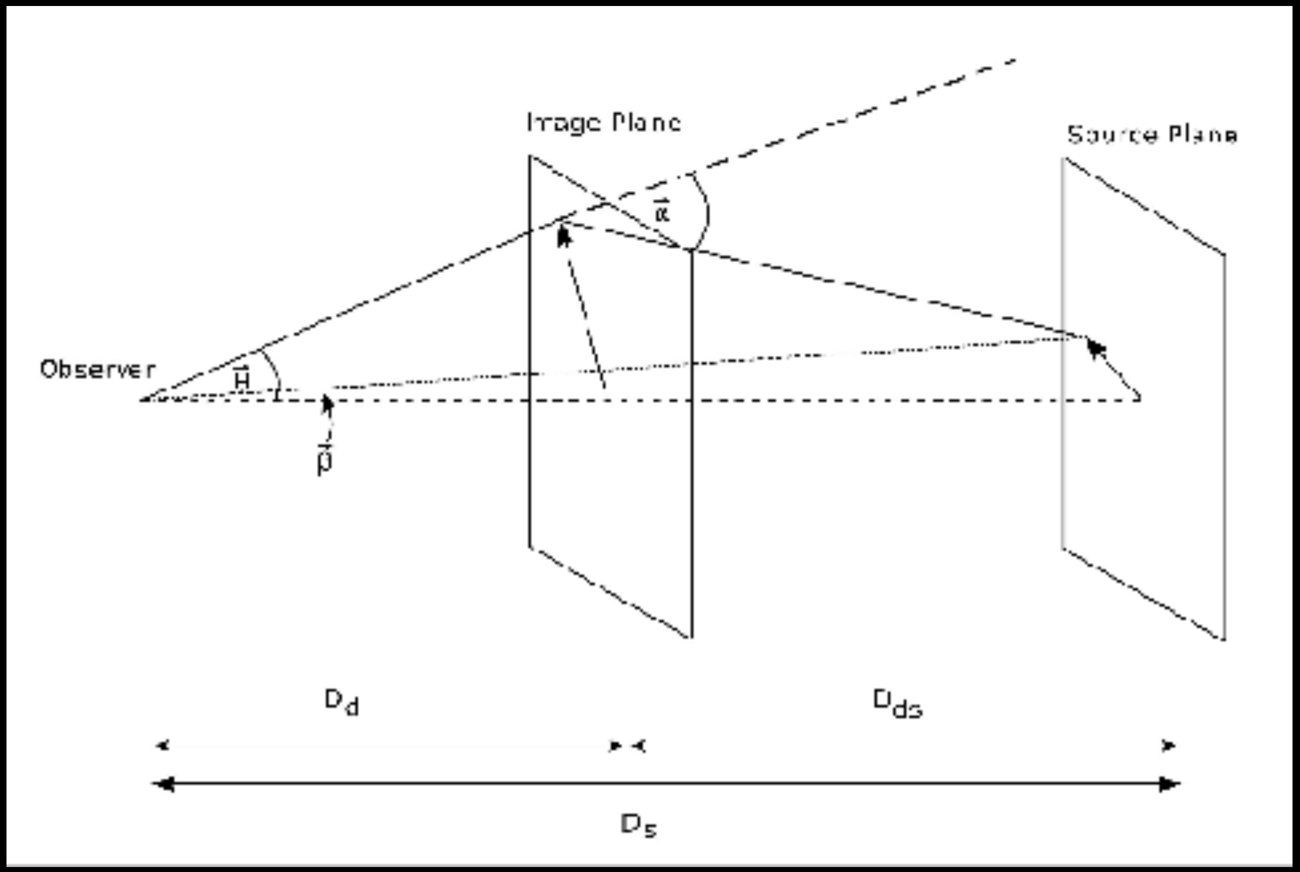
Gravitacional Lensing

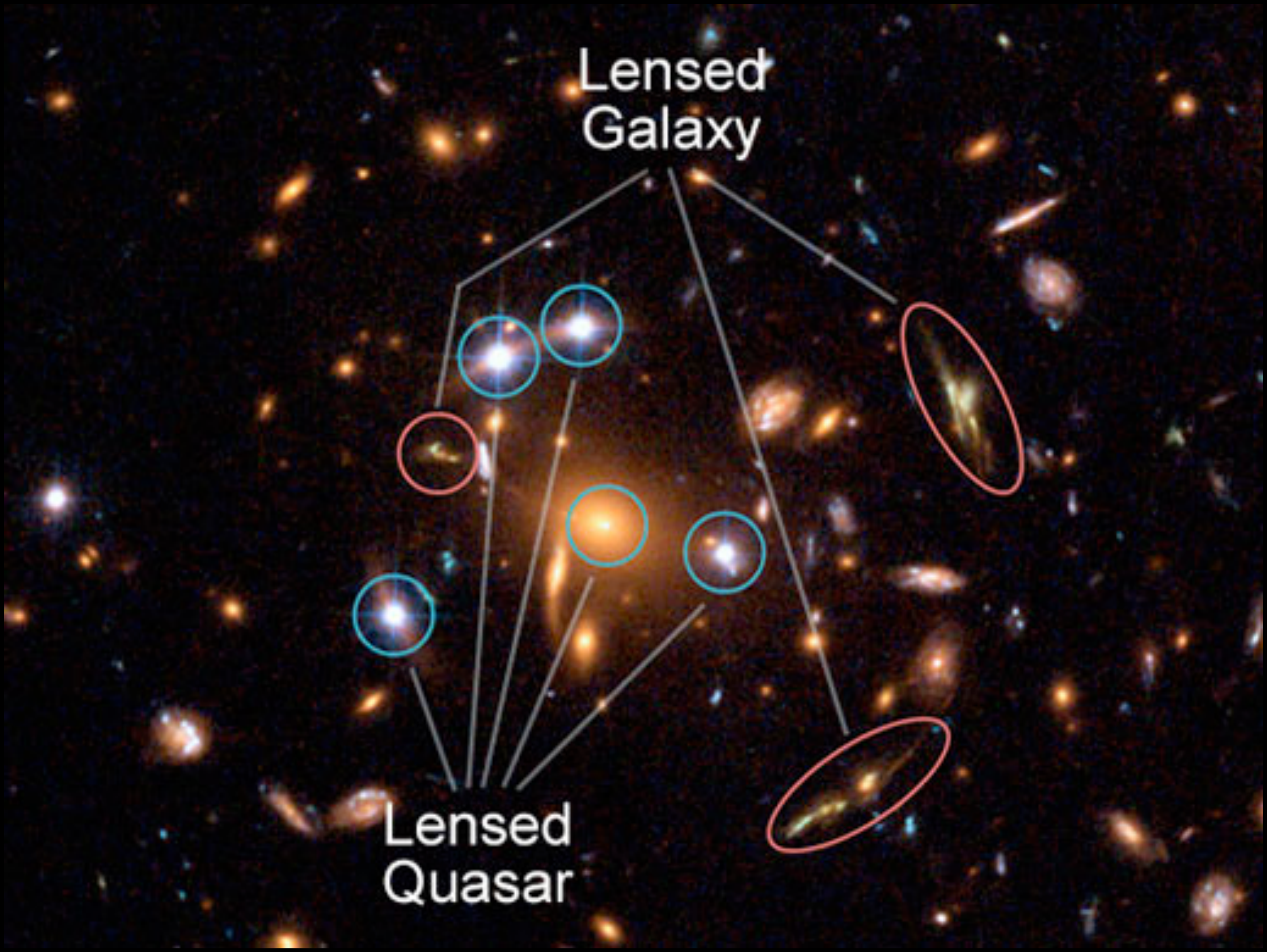


Gravitacional Lensing



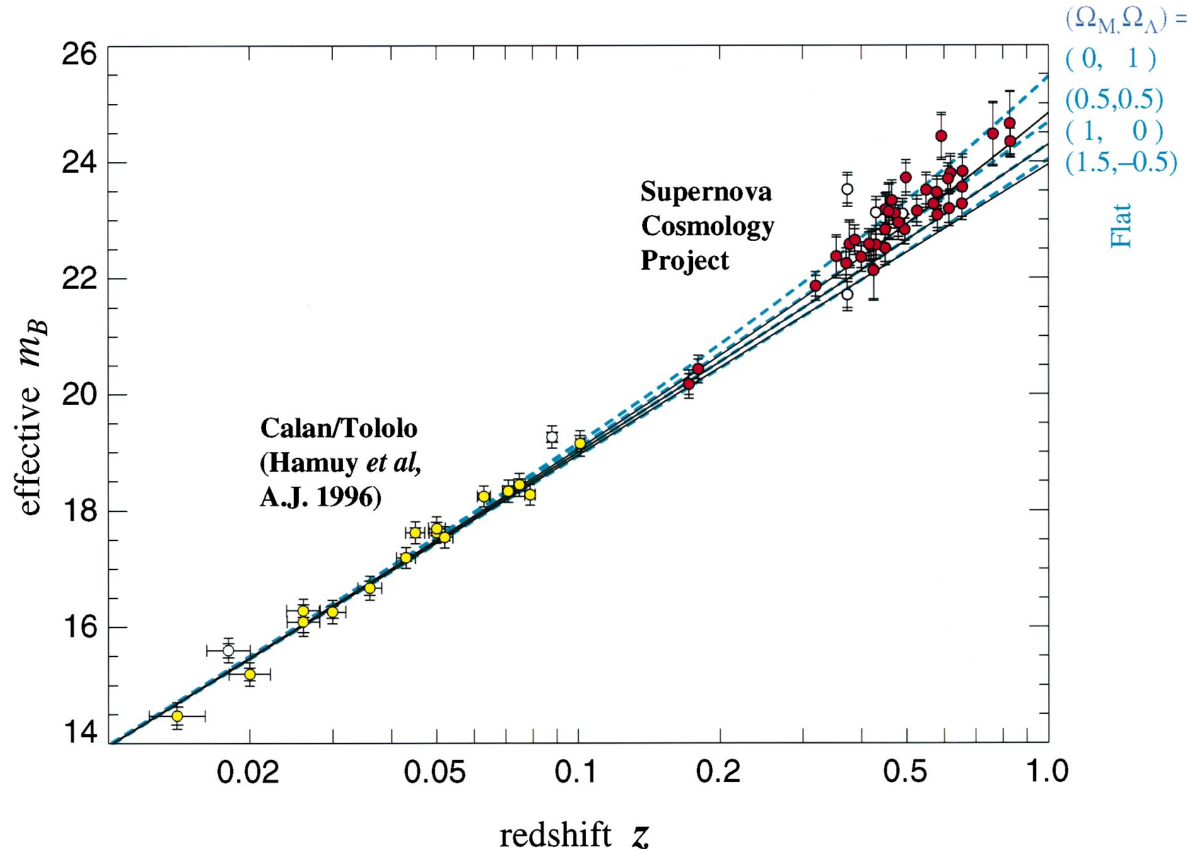
Anillo de Einstein doble! 3 galaxias perfectamente alineadas (posiblemente menos de 100 casos en todo el universo, y hemos visto uno!)





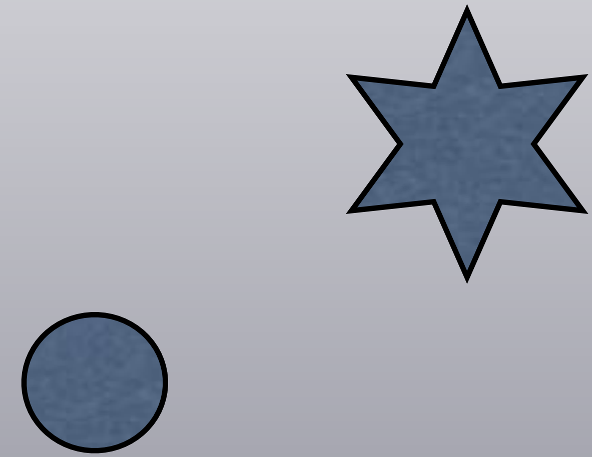
HST Telescope

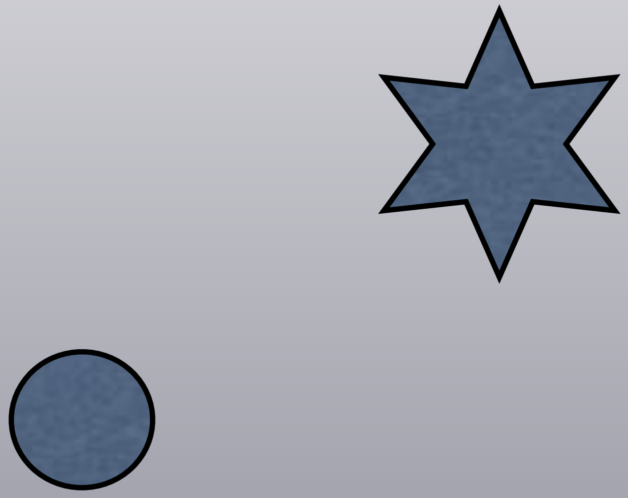
Dark energy

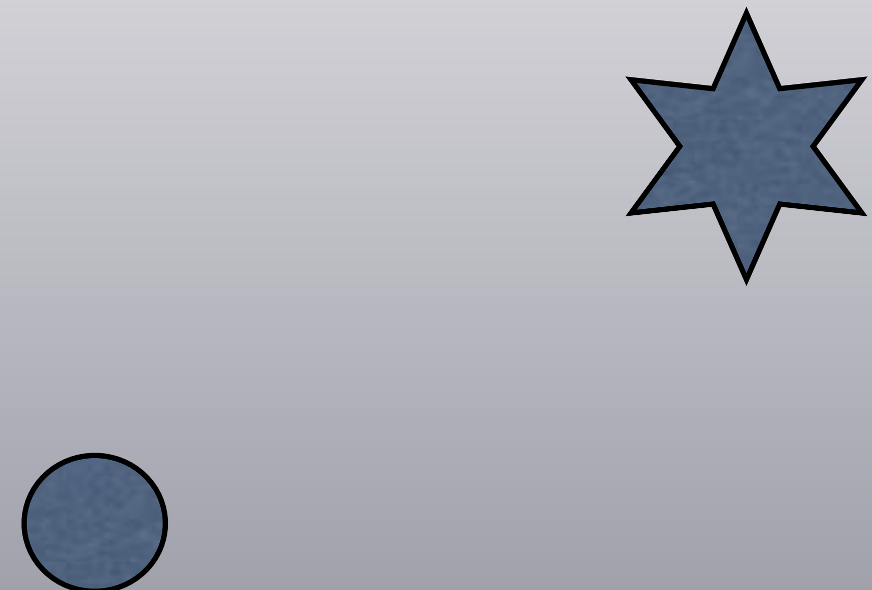


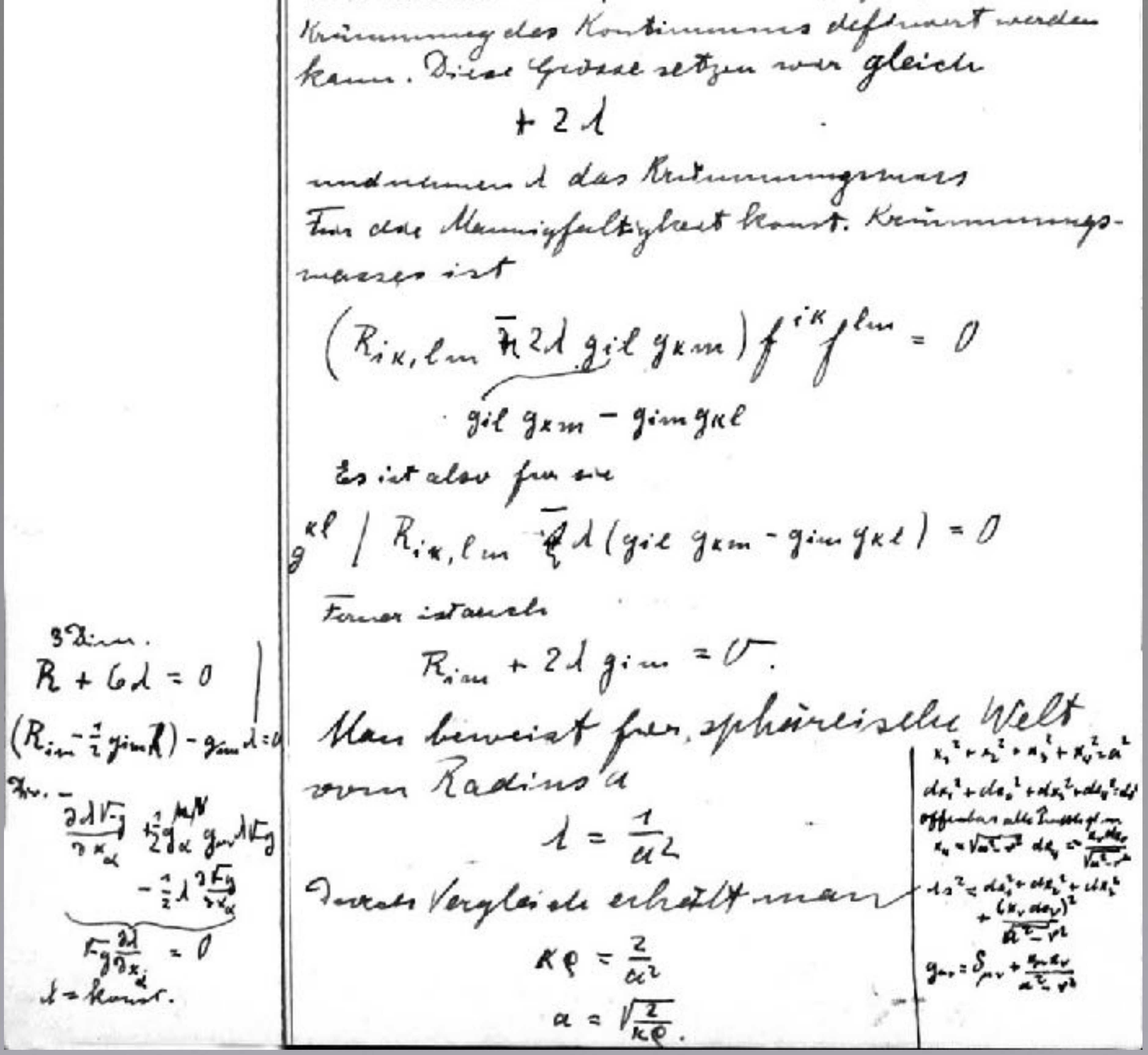
In 1998, two independent groups, observed that type Ia Supernovae were much fainter than what one would expect in a universe with only matter.
An additional ingredient was mandatory to make the universe to expand in an accelerated way!
Today the evidence for an accelerated expansion of the universe is 4.2σ-4.6σ with JLA SNIa data alone, and 11.2σ in a flat universe.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad \rho + 3p < 0$$









Krümmung des Kontinuums definiert werden kann. Diese Grösse setzen wir gleich

$$+2\lambda$$

und nennen λ das Krümmungsmass

Für die Mannigfaltigkeit konst. Krümmungsmasses ist

$$\left(R_{ik,lm} \mp 2\lambda\, g_{il} g_{km}\right) f^{ik} f^{lm} = 0$$

$$g_{il} g_{km} - g_{im} g_{kl}$$

Es ist also für sie

$$g^{kl} \mid R_{ik,lm} \mp \lambda \left(g_{il} g_{km} - g_{im} g_{kl}\right) = 0$$

Ferner ist auch

$$R_{im} + 2\lambda g_{im} = 0.$$

Man beweist für sphärische Welt vom Radius a

$$\lambda = \frac{1}{a^2}$$

Durch Vergleich erhält man

$$\kappa\rho = \frac{2}{a^2}$$

$$a = \sqrt{\frac{2}{\kappa\rho}}.$$

3 Dim.

$$R + 6\lambda = 0$$

$$\left(R_{im} - \tfrac{1}{2} g_{im} R\right) - g_{im}\lambda = 0$$

Div.

$$\underbrace{-\frac{\partial \lambda \sqrt{-g}}{\partial x_\alpha} + \frac{1}{2} g_\alpha^{\mu\nu} g_{\mu\nu} \lambda \sqrt{-g} - \frac{1}{2}\lambda \frac{\partial \sqrt{-g}}{\partial x_\alpha}}_{\sqrt{-g}\,\frac{\partial \lambda}{\partial x_\alpha} = 0}$$

λ = konst.

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = a^2$$

$$dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 = ds^2$$

offenbar alle Punkte gleichw.

$$x_4 = \sqrt{a^2 - r^2} \quad dx_4 = \frac{x_\nu dx_\nu}{\sqrt{a^2 - r^2}}$$

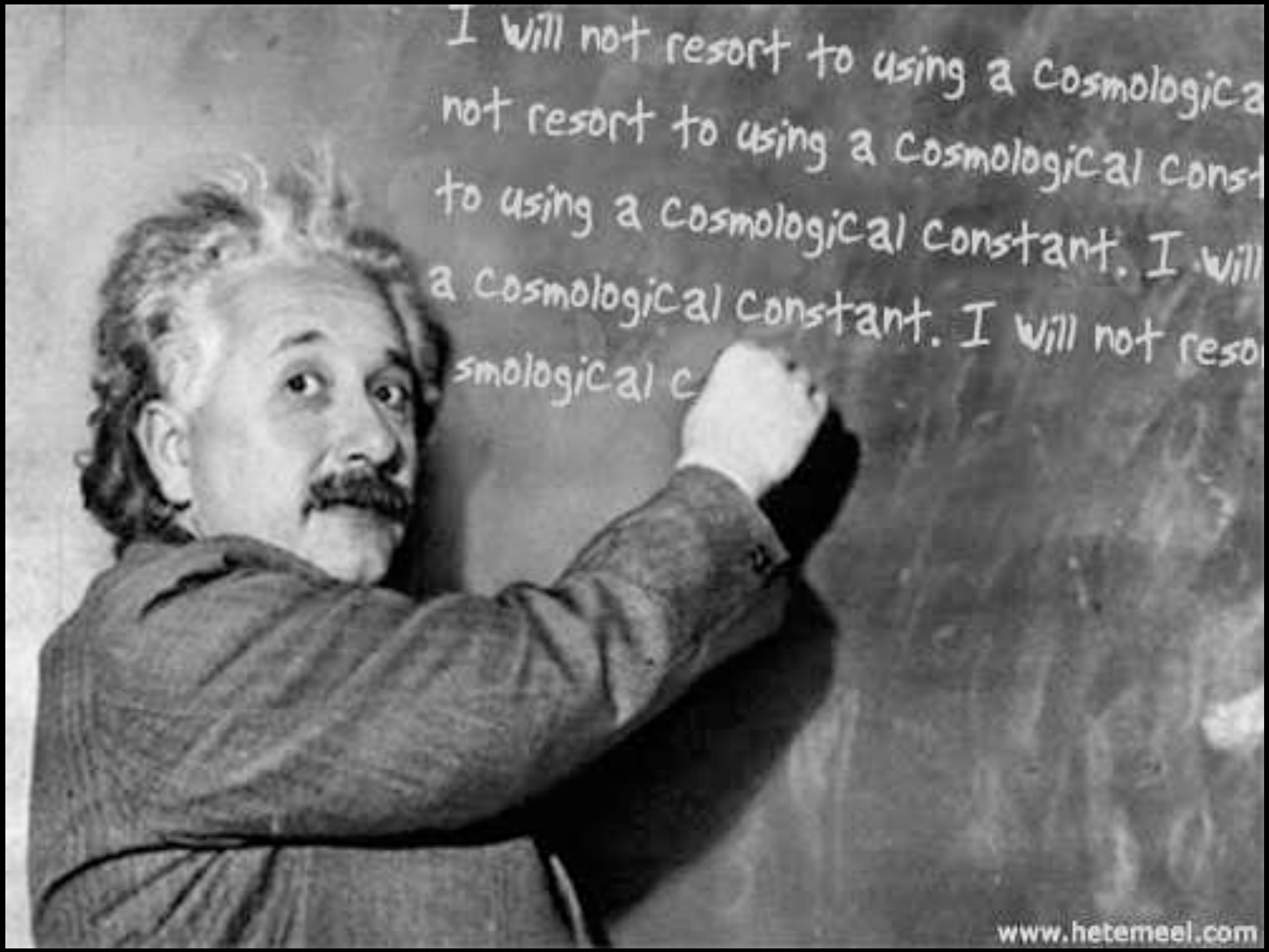
$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + \frac{(x_\nu dx_\nu)^2}{a^2 - r^2}$$

$$g_{\mu\nu} = \delta_{\mu\nu} + \frac{x_\mu x_\nu}{a^2 - r^2}$$

In 1917 Einstein added in his equations a cosmological constant Λ to have a static universe

Hubble, 1929:
The universe is not static

Λ : "My biggest blunder"



Today, the comological constant comes back!

The most economical explanation for the universe's accelerated expansion is to assume that there is an energy associated to the vacuum, with an equation of state w=-1, and that has been constant along the universe's history.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\mathcal{R} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$H^2(a) = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$$

$$H^2(a) = H_0^2(\Omega_m(a) + \Omega_r(a) + \Omega_\Lambda) \qquad \Omega_\Lambda = \frac{\Lambda}{3H_0^2}$$

Or to assume the existence of a dark energy fluid with a w≠ -1 (and also time dependent):

$$H^2(a) = H_0^2(\Omega_m(a) + \Omega_r(a) + \Omega_{de}(a))$$

$$\Omega_{de}(a) = \Omega_{de,0}\, a^{-3(1+w)}$$

Such that: $w < -1/3$

- We know how to compute the vacuum energy:

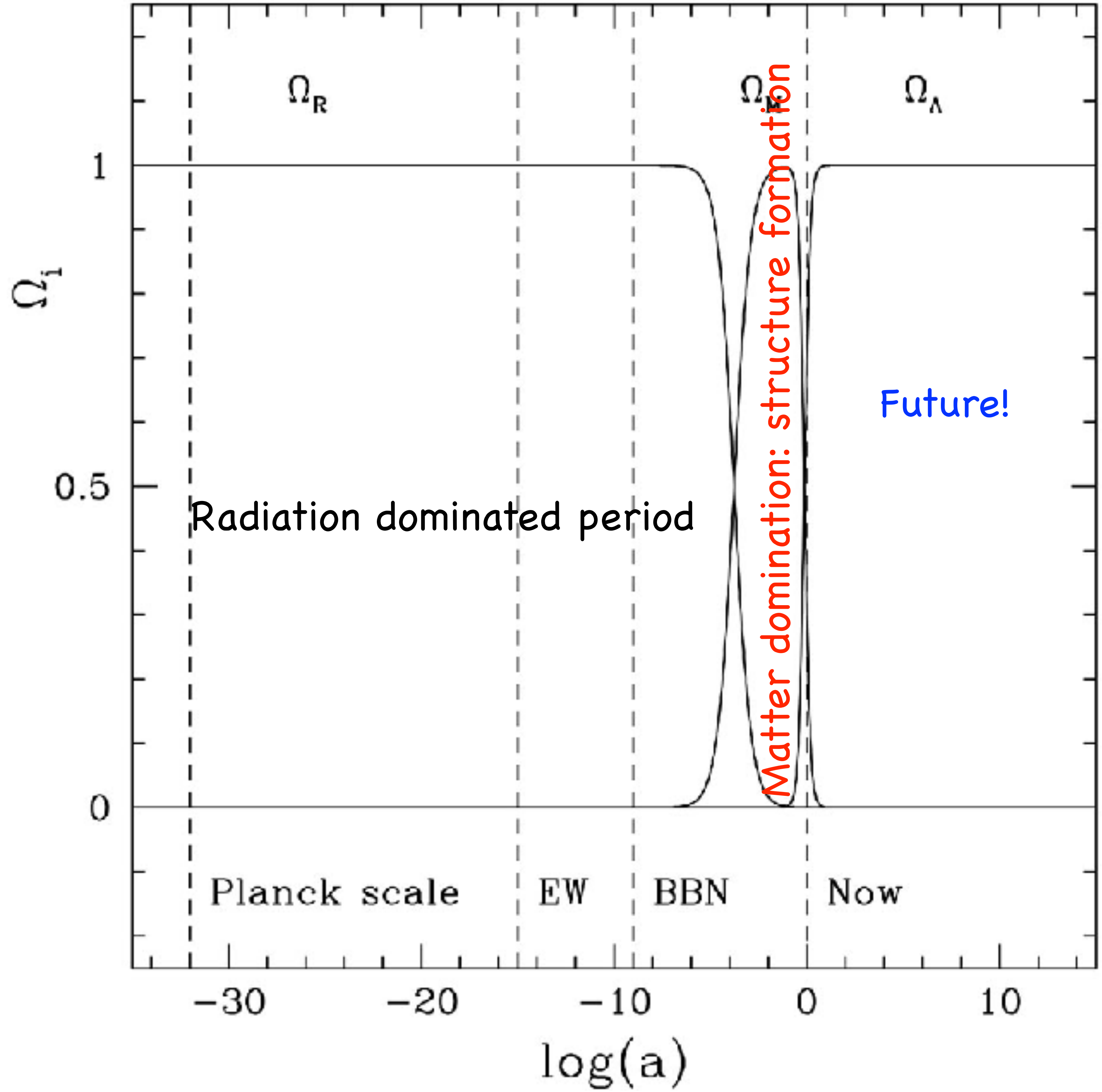
$$\frac{\Lambda}{8\pi G} = \langle T_{00} \rangle_{\mathrm{vac}} \propto \int_0^\infty \sqrt{k^2 + m^2}\, k^2\, dk$$

- But, if we compute it, we get a value which is 1000000000000000000000000000000000000000 0000000000000000000000000000000000000000 0000000000000000000000000000000000000000

- $= 10^{120}$ times larger than the one we measure

- !!!!!!!!!!!! ???????????????

- This is the so-called <u>cosmological constant problem</u>



Exercise: compute the matter-radiation and matter-dark energy transitions