Flavour and CP Phenomenology

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Disclaimer

- *Never underestimate the joy people derive from hearing something they already know*

E. Fermi

The Standard Model (Reminder)

The Standard Model (SM)

Ingredients

- Gauge symmetry principle
  - Invariance under local $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ transformations
  - $\Rightarrow$ interactions mediated by vector gauge bosons $\{G_\mu^A, W_\mu^a, B_\mu\}$

Gauge bosons

- Matter content
  - Fermions
  - "Higgs"
- Vacuum of the theory

SM - Matter content: fermions

- Quarks
  - Left-handed: $Q^0_{Lj} \sim \left(\mathbf{3},\mathbf{2},\frac{1}{6}\right)$
  - Right-handed: $u^0_{Rj} \sim \left(\mathbf{3},\mathbf{1},\frac{2}{3}\right)$, $d^0_{Rj} \sim \left(\mathbf{3},\mathbf{1},-\frac{1}{3}\right)$
- Leptons
  - Left-handed: $L^0_{Lj} \sim \left(\mathbf{1},\mathbf{2},-\frac{1}{2}\right)$
  - Right-handed: $\ell^0_{Rj} \sim (\mathbf{1},\mathbf{1},-1)$
- $\mathcal{L}^{\uparrow}_{+}$: Left-handed $(\frac{1}{2},0)$, Right-handed $(0,\frac{1}{2})$
- Three copies (generations)

SM - Matter content: fermions

- Gauge interactions from $\mathscr{L}^0_{\text{kin}} \to \mathscr{L}_{\text{kin}} + \mathscr{L}_{Vf\bar{f}}$

$$\mathscr{L}^0_{\text{kin}} = \sum_{j=1}^{3} i\left[\bar{Q}^0_{Lj}\partial\!\!\!/ Q^0_{Lj} + \bar{u}^0_{Rj}\partial\!\!\!/ u^0_{Rj} + \bar{d}^0_{Rj}\partial\!\!\!/ d^0_{Rj} + \bar{L}^0_{Lj}\partial\!\!\!/ L^0_{Lj} + \bar{\ell}^0_{Rj}\partial\!\!\!/ \ell^0_{Rj}\right]$$

$$(\partial\!\!\!/ = \gamma^\mu \partial_\mu) \quad \partial_\mu \to D_\mu = \partial_\mu - ig' B_\mu \mathrm{Y} - igW^a_\mu \mathrm{T}_a - ig_s G^A_\mu \lambda_A$$

SM - The Higgs

At this stage all gauge bosons and fermions would be massless

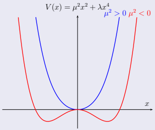
The Higgs mechanism

- Scalar $\Phi \sim \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}\right)$

$$\mathscr{L}_\Phi = (D_\mu \Phi)(D^\mu \Phi)^\dagger - V(\Phi)$$

- Potential

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$



$$\longrightarrow \quad \Phi = \begin{pmatrix} G^+ \\ (v + h + iG^0)/\sqrt{2} \end{pmatrix}$$

- Spontaneous symmetry breaking (SSB)

SM – Fermion masses

- Yukawa interactions ($\tilde{\Phi} = i\sigma_2\Phi^*$)

$$\mathscr{L}_{\text{Y}} = -\bar{Q}^0_{Lj}(Y_d)_{jk}d^0_{Rk}\Phi - \bar{Q}^0_{Lj}(Y_u)_{jk}u^0_{Rk}\tilde{\Phi} + \text{H.c.}$$

$$Q^0_{Lj} = \begin{pmatrix} u^0_{Lj} \\ d^0_{Lj} \end{pmatrix} \qquad \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ h/\sqrt{2} \end{pmatrix} + \begin{pmatrix} G^+ \\ iG^0/\sqrt{2} \end{pmatrix}$$

$$\mathscr{L}_{\text{Y}} = \mathscr{L}_{\text{M}} + \mathscr{L}_{h\bar{f}f} + \mathscr{L}_{G\bar{f}f}$$

- Mass terms

$$\mathscr{L}_{\text{M}} = -\bar{d}^0_{Lj}(\mathscr{M}^0_d)_{jk}d^0_{Rk} - \bar{u}^0_{Lj}(\mathscr{M}^0_u)_{jk}u^0_{Rk} + \text{H.c.},$$

$$\mathscr{M}^0_d \equiv \frac{v}{\sqrt{2}}Y_d, \quad \mathscr{M}^0_u \equiv \frac{v}{\sqrt{2}}Y_u$$

SM – Mass eigenstates

- Diagonalisation

$$\mathcal{U}_{q_L}^\dagger \mathcal{M}_q^0 \mathcal{M}_q^{0\,\dagger} \mathcal{U}_{q_L} = \mathrm{diag}(m_{q_j}^2), \qquad \mathcal{U}_{q_R}^\dagger \mathcal{M}_q^{0\,\dagger} \mathcal{M}_q^0 \mathcal{U}_{q_R} = \mathrm{diag}(m_{q_j}^2)$$

$$\mathcal{U}_{q_L}^\dagger \mathcal{M}_q^0 \mathcal{U}_{q_R} = \mathcal{M}_q = \mathrm{diag}(m_{q_j})$$

- Mass eigenstates

$$d_L^0 = \mathcal{U}_{d_L} d_L, \quad d_R^0 = \mathcal{U}_{d_R} d_R, \qquad u_L^0 = \mathcal{U}_{u_L} u_L, \quad u_R^0 = \mathcal{U}_{u_R} u_R$$

- Rephasing

$$R_q = \mathrm{diag}(e^{i\varphi_{q_j}}), \quad R_q R_q^\dagger = \mathbf{1}, \quad R_q \mathcal{M}_q R_q^\dagger = \mathcal{M}_q$$

$$\mathcal{U}_{q_L} \mapsto \mathcal{U}_{q_L} R_q, \quad \mathcal{U}_{q_R} \mapsto \mathcal{U}_{q_R} R_q$$

Gauge interactions

- Covariant derivative, EW sector

$$D_\mu = \partial_\mu - ig' B_\mu \mathrm{Y} - ig W_\mu^a \mathrm{T_a} - i g_s G_\mu^A \lambda_A$$

- For $(\mathbf{2}, y)$, $\mathrm{T_a} = \frac{\sigma_a}{2}$

$$[D_\mu]_{(\mathbf{2},y)} = \mathbf{1}\partial_\mu - i \begin{pmatrix} g' y B_\mu + \frac{g}{2} W_\mu^3 & \frac{g}{\sqrt{2}} W_\mu^+ \\ \frac{g}{\sqrt{2}} W_\mu^- & g' y B_\mu - \frac{g}{2} W_\mu^3 \end{pmatrix}$$

with charged $W_\mu^\pm$

$$W_\mu^a \mathrm{T_a} = \frac{1}{\sqrt{2}} \left[W_\mu^+ \mathrm{T}_+ + W_\mu^- \mathrm{T}_- \right] + W_\mu^3 \mathrm{T}_3$$

$$\mathrm{T}_\pm = \mathrm{T}_1 \pm i \mathrm{T}_2,\ W_\mu^\pm = \frac{1}{\sqrt{2}} \left[W_\mu^1 \mp i W_\mu^2 \right],\ (W_\mu^\pm)^\dagger = W_\mu^\mp$$

- Mass terms

$$(D_\mu\langle\Phi\rangle)^\dagger(D^\mu\langle\Phi\rangle) = \frac{v^2g^2}{4}W_\mu^- W^{+\mu} + \frac{v^2}{2}\left(g' y B_\mu - \frac{g}{2}W_\mu^3\right)\left(g' y B^\mu - \frac{g}{2}W^{3\mu}\right)$$

- Fix $y=1/2$, identify massive Z in last term

$$Z = c_\mathrm{W} W^3 - s_\mathrm{W} B, \qquad c_\mathrm{W} = \frac{g}{\sqrt{g^2+g'^2}}, \quad s_\mathrm{W} = \frac{g'}{g}c_\mathrm{W}$$

$$\frac{v^2}{2}\left(g' y B_\mu - \frac{g}{2}W_\mu^3\right)\left(g' y B^\mu - \frac{g}{2}W^{3\mu}\right) = \frac{1}{2}\frac{v^2g^2}{4c_\mathrm{W}^2}Z_\mu Z^\mu$$

$$(D_\mu\langle\Phi\rangle)^\dagger(D^\mu\langle\Phi\rangle) = M_W^2 W_\mu^- W^{+\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu$$

$$M_W = \frac{vg}{2}, \quad M_Z = M_W/c_\mathrm{W}$$

- Massless photon A

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} c_{\mathrm{W}} & -s_{\mathrm{W}} \\ s_{\mathrm{W}} & c_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}$$

- Back to D_μ

$$-ig' B_\mu \mathrm{Y} - ig W^3_\mu \mathrm{T}_3 = \frac{-i}{\sqrt{g^2+g'^2}} \left\{ Z_\mu \left[g^2 \mathrm{T}_3 - g'^2 \mathrm{Y} \right] + A_\mu g g' \left[\mathrm{T}_3 + \mathrm{Y} \right] \right\}$$

$$\mathrm{T}_3 + \mathrm{Y} = \mathrm{Q}, \qquad \frac{gg'}{\sqrt{g^2+g'^2}} = e$$

$$D_\mu = \partial_\mu - ieA_\mu \mathrm{Q} - i\frac{g}{c_{\mathrm{W}}} \left[Z_\mu \mathrm{T}_3 - s^2_{\mathrm{W}} \mathrm{Q} \right] - i\frac{g}{\sqrt{2}} \left[W^+_\mu \mathrm{T}_+ + W^-_\mu \mathrm{T}_- \right]$$

- Interactions

$$\mathscr{L}_{\text{kin.}} \to \mathscr{L}_{\text{kin.}} + \mathscr{L}_{\text{gauge}} = \bar{Q}^0_{Lj} i \not{D} Q^0_{Lj} + \bar{d}^0_{Rj} i \not{D} d^0_{Rj} + \bar{u}^0_{Rj} i \not{D} u^0_{Rj}$$

$$\mathscr{L}_{\text{gauge}} = \mathscr{L}_{\text{EM}} + \mathscr{L}_{\text{NC}} + \mathscr{L}_{\text{CC}}$$

QED

- $\mathscr{L}_{\text{EM}} = e J^\mu_{\text{EM}} A_\mu$

$$J^\mu_{\text{EM}} = Q_u[\bar{u}^0_{Lj}\gamma^\mu u^0_{Lj} + \bar{u}^0_{Rj}\gamma^\mu u^0_{Rj}] + Q_d[\bar{d}^0_{Lj}\gamma^\mu d^0_{Lj} + \bar{d}^0_{Rj}\gamma^\mu d^0_{Rj}]$$

$$\text{Q}[u^0_L] = \text{Q}[u^0_R] = Q_u = +\frac{2}{3}, \qquad \text{Q}[d^0_L] = \text{Q}[d^0_R] = Q_d = -\frac{1}{3}$$

- With mass eigenstates

$$J^\mu_{\text{EM}} = Q_u[\bar{u}_{Lj}\gamma^\mu u_{Lj} + \bar{u}_{Rj}\gamma^\mu u_{Rj}] + Q_d[\bar{d}_{Lj}\gamma^\mu d_{Lj} + \bar{d}_{Rj}\gamma^\mu d_{Rj}]$$

$$\mathscr{L}_{\text{EM}} = e\left[Q_u\,\bar{u}_j\gamma^\mu u_j + Q_d\,\bar{d}_j\gamma^\mu d_j\right]A_\mu$$

Neutral Currents (NC)

- $\mathscr{L}_{\text{NC}} = \frac{g}{c_{\text{W}}} J^\mu_{\text{Z}} Z_\mu$

$$J^\mu_{\text{Z}} = t_3^{(u)} \bar{u}^0_{Lj}\gamma^\mu u^0_{Lj} + t_3^{(d)} \bar{d}^0_{Lj}\gamma^\mu d^0_{Lj} - s^2_{\text{W}}\, J^\mu_{\text{EM}}$$

$$\text{T}_3[u^0_L] = t_3^{(u)} = +\frac{1}{2}, \quad \text{T}_3[d^0_L] = t_3^{(d)} = -\frac{1}{2}$$

- With mass eigenstates

$$J^\mu_{\text{Z}} = \frac{1}{2}\bar{u}_{Lj}\gamma^\mu u_{Lj} - \frac{1}{2}\bar{d}_{Lj}\gamma^\mu d_{Lj} - s^2_{\text{W}}\, J^\mu_{\text{EM}}$$

$$\mathscr{L}_{\text{NC}} = \frac{g}{c_{\text{W}}}\left[\frac{1}{2}\bar{u}_{Lj}\gamma^\mu u_{Lj} - \frac{1}{2}\bar{d}_{Lj}\gamma^\mu d_{Lj} - s^2_{\text{W}}\, J^\mu_{\text{EM}}\right] Z_\mu$$

Charged Currents (CC)

- $\mathscr{L}_{\text{CC}} = \frac{g}{\sqrt{2}} J^\mu_{\text{W}} W^+_\mu + \text{H.c.}$

$$J^\mu_{\text{W}} = \bar{u}^0_{Lj} \gamma^\mu d^0_{Lj}$$

- With mass eigenstates

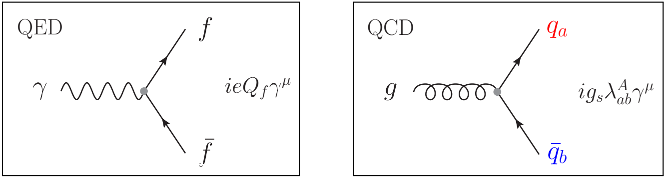
$$J^\mu_{\text{W}} = \bar{u}_{La} [\mathcal{U}^\dagger_{u_L}]_{aj} \gamma^\mu [\mathcal{U}_{d_L}]_{jb} d_{Lb} = \bar{u}_{La} \gamma^\mu V_{ab} d_{Lb}$$

$$V \equiv \mathcal{U}^\dagger_{u_L} \mathcal{U}_{d_L}, \qquad V V^\dagger = V^\dagger V = \mathbf{1}$$

$$\mathscr{L}_{\text{CC}} = \frac{g}{\sqrt{2}} \left\{ \bar{u}_{La} \gamma^\mu V_{ab} d_{Lb} W^+_\mu + \bar{d}_{Lb} \gamma^\mu V^*_{ab} u_{La} W^-_\mu \right\}$$

Summary

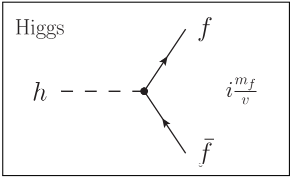
- QED & QCD



  - Universal
  - Non-chiral
  - Flavour conserving

Summary

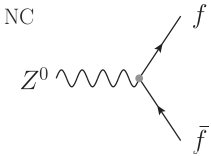
- Higgs



  - Non-Universal
  - Scalar
  - Flavour conserving

Summary

- Neutral Currents

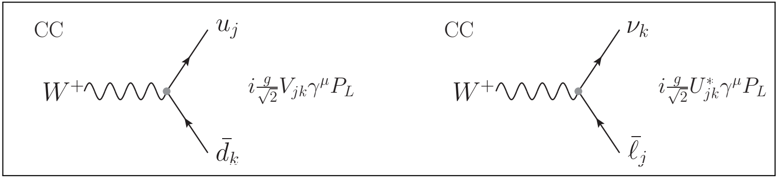


$$i\frac{g}{2c_W}\gamma^\mu(\mathrm{T}_3[f] - 2Q_f s_W^2 - \mathrm{T}_3[f]\gamma_5)$$

  - Universal
  - Chiral
  - Flavour conserving

Summary

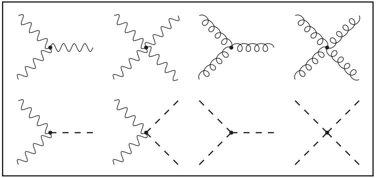
- Charged Currents



  - Universal
  - Chiral
  - Flavour changing

Summary

- Gauge+Higgs



Discrete Transformations

Discrete transformations

Discrete transformations

- Space reflection or Parity P: $x = (t, \vec{x}) \mapsto \tilde{x} = (t, -\vec{x})$
  [N.B. in odd # of space dim.]
- Time reversal T: $x = (t, \vec{x}) \mapsto Tx = (-t, \vec{x})$
- Charge Conjugation C: $\mathbf{n} \mapsto \mathbf{n}^*$, Q $\mapsto$ −Q

Discrete transformations

Operators

Field operator

$$O(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left\{ \mathbf{a}(\vec{p},s)[\text{w.f.}]e^{-ip\cdot x} + \mathbf{b}^\dagger(\vec{p},s)[\text{w.f.}]e^{+ip\cdot x} \right\}$$

- For P and C, unitary **P** and **C**:

$$\mathbf{P}\,\mathbf{a}(\vec{p},s)\,\mathbf{P} = \eta_a \mathbf{a}(-\vec{p},s) \qquad \mathbf{C}\,\mathbf{a}(\vec{p},s)\,\mathbf{C} = \varphi_a \mathbf{b}(\vec{p},s)$$
$$\mathbf{P}\,\mathbf{b}(\vec{p},s)\,\mathbf{P} = \eta_b \mathbf{b}(-\vec{p},s) \qquad \mathbf{C}\,\mathbf{b}(\vec{p},s)\,\mathbf{C} = \varphi_b \mathbf{a}(\vec{p},s)$$
$$\mathbf{P}^2 = \mathbf{1}, \quad \eta_a^2 = \eta_b^2 = 1 \qquad \mathbf{C}^2 = \mathbf{1}, \quad \varphi_a \varphi_b = 1$$

- (For T, antiunitary **T**, not here)
- Wave functions $C_\pm(\vec{p},s)$ from free solutions

Discrete transformations

Operators

Field operator

$$O(x) = \sum\!\!\!\!\!\!\int \left\{ \mathbf{a}(\vec{p},s) C_-(\vec{p},s) e^{-ip\cdot x} + \mathbf{b}^\dagger(\vec{p},s) C_+(\vec{p},s) e^{+ip\cdot x} \right\}$$

- For P and C, unitary **P** and **C**:

$$\begin{aligned}
&\mathbf{P}\,\mathbf{a}(\vec{p},s)\,\mathbf{P} = \eta_a \mathbf{a}(-\vec{p},s) \qquad && \mathbf{C}\,\mathbf{a}(\vec{p},s)\,\mathbf{C} = \varphi_a \mathbf{b}(\vec{p},s) \\
&\mathbf{P}\,\mathbf{b}(\vec{p},s)\,\mathbf{P} = \eta_b \mathbf{b}(-\vec{p},s) \qquad && \mathbf{C}\,\mathbf{b}(\vec{p},s)\,\mathbf{C} = \varphi_b \mathbf{a}(\vec{p},s) \\
&\mathbf{P}^2 = \mathbf{1}, \quad \eta_a^2 = \eta_b^2 = 1 \qquad && \mathbf{C}^2 = \mathbf{1}, \quad \varphi_a \varphi_b = 1
\end{aligned}$$

- (For T, antiunitary **T**, not here)
- Wave functions $C_\pm(\vec{p},s)$ from free solutions

Discrete transformations

The CPT Theorem

Any quantum field theory which is local, Lorentz invariant, and which respects the spin-statistics connection,

is invariant under the product CPT

Some consequences

- Particle/Antiparticle have equal mass and width
- CP Violation $\Leftrightarrow$ T Violation

[N.B. It is a property of QFT]

Sakharov conditions

In order to obtain a net baryon asymmetry in the Universe

- baryon number violation
- departure from thermal equilibrium
- *C and CP violation*

Discrete transformations – Scalar Φ

$$\Phi(x) = \sum\!\!\!\!\!\!\int \left\{\mathbf{a}(\vec{p},s)e^{-ip\cdot x} + \mathbf{b}^\dagger(\vec{p},s)e^{ip\cdot x}\right\}$$

[No wavefunction!]

- Parity

$$\mathbf{P}\Phi(x)\mathbf{P} = \sum\!\!\!\!\!\!\int \left\{\eta_a \mathbf{a}(-\vec{p},s)e^{-ip\cdot x} + \eta_b^* \mathbf{b}^\dagger(-\vec{p},s)e^{ip\cdot x}\right\} =$$

$$\sum\!\!\!\!\!\!\int \left\{\eta_a \mathbf{a}(\vec{p},s)e^{-i\tilde{p}\cdot x} + \eta_b^* \mathbf{b}^\dagger(\vec{p},s)e^{i\tilde{p}\cdot x}\right\} =$$

$$\eta_S \sum\!\!\!\!\!\!\int \left\{\mathbf{a}(\vec{p},s)e^{-ip\cdot \tilde{x}} + \mathbf{b}^\dagger(\vec{p},s)e^{ip\cdot \tilde{x}}\right\}$$

with $\eta_b^* = \eta_a = \eta_S$ and $\tilde{p}\cdot x = p\cdot \tilde{x}$

$$\mathbf{P}\Phi(x)\mathbf{P} = \eta_S \Phi(\tilde{x})$$

Discrete transformations – Scalar Φ

$$\Phi(x) = \sum\!\!\!\!\!\!\int \left\{ \mathbf{a}(\vec{p},s) e^{-ip\cdot x} + \mathbf{b}^\dagger(\vec{p},s) e^{ip\cdot x} \right\}$$

- Charge Conjugation

$$\mathbf{C}\Phi(x)\mathbf{C} = \sum\!\!\!\!\!\!\int \left\{ \varphi_a \mathbf{b}(\vec{p},s) e^{-ip\cdot x} + \varphi_b^* \mathbf{a}^\dagger(\vec{p},s) e^{ip\cdot x} \right\} =$$

$$\varphi_S \sum\!\!\!\!\!\!\int \left\{ \mathbf{a}(\vec{p},s) e^{-ip\cdot x} + \mathbf{b}^\dagger(\vec{p},s) e^{ip\cdot x} \right\}^\dagger$$

with $\varphi_b^* = \varphi_a = \varphi_S$

$$\mathbf{C}\Phi(x)\mathbf{C} = \varphi_S [\Phi(x)]^\dagger$$

Discrete transformations – Vector V^μ

$$V_\mu(x) = \sum\!\!\!\!\!\!\int \left\{\mathbf{a}(\vec{p},s)\epsilon_\mu(\vec{p},s)e^{-ip\cdot x} + \mathbf{b}^\dagger(\vec{p},s)\epsilon^*_\mu(\vec{p},s)e^{ip\cdot x}\right\}$$

- Polarisation vector:

$$\epsilon^\mu(\vec{p},s) = L^\mu_\nu(\vec{p})\epsilon^\nu(\vec{0},s), \qquad L^\mu_\nu(\vec{p}) \text{ boosts } \vec{0} \mapsto \vec{p}$$

$$\epsilon^\mu(\vec{0},0) = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \quad \epsilon^\mu(\vec{0},+1) = \frac{-1}{\sqrt{2}}\begin{pmatrix}0\\1\\i\\0\end{pmatrix} \quad \epsilon^\mu(\vec{0},-1) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-i\\0\end{pmatrix}$$

$$\epsilon^\mu(-\vec{p},s) = -\epsilon_\mu(\vec{p},s)$$

Discrete transformations – Vector V^μ

- Parity

$$\mathbf{P}V^\mu(x)\mathbf{P} =$$

$$\sum\!\!\!\!\!\!\!\int\left\{\eta_a \mathbf{a}(-\vec{p},s)\epsilon^\mu(\vec{p},s)e^{-ip\cdot x} + \eta_b \mathbf{b}^\dagger(-\vec{p},s)\epsilon^{\mu *}(\vec{p},s)e^{ip\cdot x}\right\} =$$

$$\sum\!\!\!\!\!\!\!\int\left\{\eta_a \mathbf{a}(\vec{p},s)\epsilon^\mu(-\vec{p},s)e^{-i\tilde{p}\cdot x} + \eta_b \mathbf{b}^\dagger(\vec{p},s)\epsilon^{\mu *}(-\vec{p},s)e^{i\tilde{p}\cdot x}\right\} =$$

$$-\eta_V \sum\!\!\!\!\!\!\!\int\left\{\mathbf{a}(\vec{p},s)\epsilon_\mu(\vec{p},s)e^{-ip\cdot\tilde{x}} + \mathbf{b}^\dagger(\vec{p},s)\epsilon^*_\mu(\vec{p},s)e^{ip\cdot\tilde{x}}\right\}$$

with $\eta_b = \eta_a = \eta_V$ and $\tilde{p}\cdot x = p\cdot\tilde{x}$

$$\mathbf{P}V^\mu(x)\mathbf{P} = -\eta_V V_\mu(\tilde{x})$$

Discrete transformations – Vector V^μ

- Charge Conjugation

$$\mathbf{C}V^\mu(x)\mathbf{C} =$$

$$\sum\!\!\!\!\!\!\int \left\{\varphi_a \mathbf{b}(\vec{p},s)\epsilon^\mu(\vec{p},s)e^{-ip\cdot x} + \varphi_b^* \mathbf{a}^\dagger(\vec{p},s)\epsilon^{\mu*}(\vec{p},s)e^{ip\cdot x}\right\} =$$

$$\varphi_V \sum\!\!\!\!\!\!\int \left\{\mathbf{a}(\vec{p},s)\epsilon^\mu(\vec{p},s)e^{-ip\cdot x} + \mathbf{b}^\dagger(\vec{p},s)\epsilon^{\mu*}(\vec{p},s)e^{ip\cdot x}\right\}^\dagger$$

with $\varphi_b^* = \varphi_a = \varphi_V$

$$\mathbf{C}V^\mu(x)\mathbf{C} = \varphi_V [V^\mu(x)]^\dagger$$

Fermions – Reminder

- Dirac: $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$
- Weyl representation

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \qquad \sigma^\mu = (1, \vec{\sigma}),\ \bar{\sigma}^\mu = (1, -\vec{\sigma}) \qquad \text{reducible}$$

$$\sigma^1 = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right) \quad \sigma^2 = \left(\begin{smallmatrix} 0 & -i \\ i & 0 \end{smallmatrix}\right) \quad \sigma^3 = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$$

$$(\sigma^j)^\dagger = \sigma^j, \qquad \sigma^2\,\sigma^j\,\sigma^2 = -(\sigma^j)^*$$

- Infinitesimal rotation $\vec{\theta}$, boost $\vec{\beta}$,

$$\psi_L \mapsto (\mathbf{1} - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2} - \vec{\beta}\cdot\frac{\vec{\sigma}}{2})\psi_L$$

$$\psi_R \mapsto (\mathbf{1} - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2} + \vec{\beta}\cdot\frac{\vec{\sigma}}{2})\psi_R$$

Fermions – Reminder

- Transformation of $\sigma^2\psi_L^*$

$$\sigma^2\psi_L^* \mapsto \sigma^2(\mathbf{1} + i\vec{\theta}\cdot\frac{\vec{\sigma}^*}{2} - \vec{\beta}\cdot\frac{\vec{\sigma}^*}{2})\psi_L^* = (\mathbf{1} - i\vec{\theta}\cdot\frac{\vec{\sigma}}{2} + \vec{\beta}\cdot\frac{\vec{\sigma}}{2})\sigma^2\psi_L^*$$

$\sigma^2\psi_L^*$ transforms like ψ_R, $\sigma^2\psi_R^*$ transforms like ψ_L

- Dirac equation

$$(i\gamma^\mu\partial_\mu - m\mathbf{1})\begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix} = 0 \qquad \begin{pmatrix}-m\mathbf{1} & i\sigma^\mu\partial_\mu\\ i\bar{\sigma}^\mu\partial_\mu & -m\mathbf{1}\end{pmatrix}\begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix} = 0$$

$$i\sigma^\mu\partial_\mu\psi_R = m\psi_L \qquad i\bar{\sigma}^\mu\partial_\mu\psi_L = m\psi_R$$

for $m = 0$ decoupled

- Massless fermions: helicity $\Leftrightarrow$ chirality

Fermions – Reminder

- Plane wave solutions $u(p)e^{-ip\cdot x}$, $v(p)e^{+ip\cdot x}$

$$s=1,2 \qquad u^s(p)=\begin{pmatrix}\sqrt{p\cdot\sigma}\xi^s\\ \sqrt{p\cdot\bar{\sigma}}\xi^s\end{pmatrix} \qquad v^s(p)=\begin{pmatrix}\sqrt{p\cdot\sigma}\eta^s\\ -\sqrt{p\cdot\bar{\sigma}}\eta^s\end{pmatrix}$$

- γ_5

$$\gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3, \qquad \gamma_5=(\gamma_5)^\dagger, \qquad (\gamma_5)^2=\mathbf{1}, \qquad \{\gamma_5,\gamma^\mu\}=0$$

$$\gamma_5=\begin{pmatrix}-\mathbf{1} & 0\\ 0 & \mathbf{1}\end{pmatrix} \qquad P_L=\frac{1}{2}(1-\gamma_5), \qquad P_R=\frac{1}{2}(1+\gamma_5)$$

- Quantized Dirac field

$$\Psi(x)=\sum\!\!\!\!\!\!\int\left\{\mathbf{a}(\vec{p},s)u^s(p)e^{-ip\cdot x}+\mathbf{b}^\dagger(\vec{p},s)v^s(p)e^{+ip\cdot x}\right\}$$

Discrete transformations – Fermion

$$\Psi(x) = \sum\!\!\!\!\!\!\int \left\{ \mathbf{a}(\vec{p},s) u^s(p) e^{-ip\cdot x} + \mathbf{b}^\dagger(\vec{p},s) v^s(p) e^{+ip\cdot x} \right\}$$

- Parity

$$\mathbf{P}\Psi(x)\mathbf{P} = \sum\!\!\!\!\!\!\int \left\{ \eta_a \mathbf{a}(-\vec{p},s) u^s(p) e^{-ip\cdot x} + \eta_b^* \mathbf{b}^\dagger(-\vec{p},s) v^s(p) e^{+ip\cdot x} \right\} =$$

$$\sum\!\!\!\!\!\!\int \left\{ \eta_a \mathbf{a}(\vec{p},s) u^s(\tilde{p}) e^{-i\tilde{p}\cdot x} + \eta_b^* \mathbf{b}^\dagger(\vec{p},s) v^s(\tilde{p}) e^{+i\tilde{p}\cdot x} \right\} =$$

$$\sum\!\!\!\!\!\!\int \left\{ \eta_a \mathbf{a}(\vec{p},s) u^s(\tilde{p}) e^{-ip\cdot \tilde{x}} + \eta_b^* \mathbf{b}^\dagger(\vec{p},s) v^s(\tilde{p}) e^{+ip\cdot \tilde{x}} \right\}$$

Discrete transformations – Fermion

- Spinors

$$u^s(\tilde{p}) = \begin{pmatrix} \sqrt{\tilde{p}\cdot\sigma}\xi^s \\ \sqrt{\tilde{p}\cdot\bar{\sigma}}\xi^s \end{pmatrix} = \begin{pmatrix} \sqrt{p\cdot\bar{\sigma}}\xi^s \\ \sqrt{p\cdot\sigma}\xi^s \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\bar{\sigma}}\xi^s \end{pmatrix} = \gamma^0 u^s(p)$$

$$v^s(\tilde{p}) = \begin{pmatrix} \sqrt{\tilde{p}\cdot\sigma}(-i\sigma^2\xi^s)^* \\ -\sqrt{\tilde{p}\cdot\bar{\sigma}}(-i\sigma^2\xi^s)^* \end{pmatrix} = \begin{pmatrix} \sqrt{p\cdot\bar{\sigma}}(-i\sigma^2\xi^s)^* \\ -\sqrt{p\cdot\sigma}(-i\sigma^2\xi^s)^* \end{pmatrix} =$$

$$- \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \sqrt{p\cdot\sigma}(-i\sigma^2\xi^s)^* \\ -\sqrt{p\cdot\bar{\sigma}}(-i\sigma^2\xi^s)^* \end{pmatrix} = -\gamma^0 v^s(p)$$

Discrete transformations – Fermion

- Back to $\mathbf{P}\Psi(x)\mathbf{P}$, with $\eta_a = -\eta_b^* = \eta_P$

$$\mathbf{P}\Psi(x)\mathbf{P} = \eta_P \gamma^0 \Psi(\tilde{x})$$

- For Ψ_L

$$\mathbf{P} P_L \Psi(x) \mathbf{P} = \eta_P \gamma^0 P_L \Psi(\tilde{x}) = P_R \eta_P \gamma^0 \Psi(\tilde{x}) \qquad \text{right-handed}$$

and similarly the parity transformed of a right-handed fermion is left-handed

Discrete transformations – Fermion

$$\Psi(x) = \sum\!\!\!\!\!\!\int \left\{\mathbf{a}(\vec{p},s)u^s(p)e^{-ip\cdot x} + \mathbf{b}^\dagger(\vec{p},s)v^s(p)e^{+ip\cdot x}\right\}$$

- Charge Conjugation

$$\mathbf{C}\Psi(x)\mathbf{C} = \sum\!\!\!\!\!\!\int \left\{\varphi_a\mathbf{b}(\vec{p},s)u^s(p)e^{-ip\cdot x} + \varphi_b^*\mathbf{b}^\dagger(\vec{p},s)v^s(p)e^{+ip\cdot x}\right\}$$

Consider now

$$[\Psi(x)]^* = \sum\!\!\!\!\!\!\int \left\{\mathbf{a}^\dagger(\vec{p},s)u^s(p)^*e^{+ip\cdot x} + \mathbf{b}(\vec{p},s)v^s(p)^*e^{-ip\cdot x}\right\}$$

Discrete transformations – Fermion

- Spinors (again)

$$u^s(p)^* = \begin{pmatrix} \sqrt{p\cdot\sigma^*}(\xi^s)^* \\ \sqrt{p\cdot\bar{\sigma}^*}(\xi^s)^* \end{pmatrix} = \begin{pmatrix} \sqrt{p\cdot\sigma^*}\,i\sigma^2(-i\sigma^2\xi^s)^* \\ \sqrt{p\cdot\bar{\sigma}^*}\,i\sigma^2(-i\sigma^2\xi^s)^* \end{pmatrix} =$$

$$\begin{pmatrix} i\sigma^2\sqrt{p\cdot\bar{\sigma}}(-i\sigma^2\xi^s)^* \\ i\sigma^2\sqrt{p\cdot\sigma}(-i\sigma^2\xi^s)^* \end{pmatrix} =$$

$$\begin{pmatrix} \mathbf{0} & -i\sigma^2 \\ i\sigma^2 & \mathbf{0} \end{pmatrix} \begin{pmatrix} \sqrt{p\cdot\sigma}(-i\sigma^2\xi^s)^* \\ -\sqrt{p\cdot\bar{\sigma}}(-i\sigma^2\xi^s)^* \end{pmatrix} = -i\gamma^2 v^s(p)$$

$$v^s(p)^* = \begin{pmatrix} \sqrt{p\cdot\sigma^*}(-i\sigma^2\xi^s) \\ -\sqrt{p\cdot\bar{\sigma}^*}(-i\sigma^2\xi^s) \end{pmatrix} = \begin{pmatrix} -i\sigma^2\sqrt{p\cdot\bar{\sigma}}\xi^s \\ i\sigma^2\sqrt{p\cdot\sigma}\xi^s \end{pmatrix} =$$

$$\begin{pmatrix} \mathbf{0} & -i\sigma^2 \\ i\sigma^2 & \mathbf{0} \end{pmatrix} \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\bar{\sigma}}\xi^s \end{pmatrix} = -i\gamma^2 u^s(p)$$

Discrete transformations – Fermion

- Back to $[\Psi(x)]^*$

$$[\Psi(x)]^* = -i\gamma^2 \sum\!\!\!\!\!\!\!\int \left\{ \mathbf{b}(\vec{p},s)u^s(p)e^{-ip\cdot x} + \mathbf{a}^\dagger(\vec{p},s)v^s(p)e^{+ip\cdot x} \right\}$$

with $\varphi_a = \varphi_b^* = \varphi_C$ and

$$\mathcal{C} = -i\gamma^2, \qquad \mathcal{C} = \mathcal{C}^*,\ \mathcal{C}^T = -\mathcal{C},\ \mathcal{C}^2 = \mathbf{1}$$

$$\Psi_c = \mathcal{C}\Psi^*, \qquad (\Psi_c)_c = \Psi$$

- Back to $\mathbf{C}$

$$\mathbf{C}\Psi(x)\mathbf{C} = \varphi_C \Psi_c(x) = \varphi_C \mathcal{C}\Psi(x)^*$$

- For Ψ_L

$$\mathbf{C}P_L\Psi(x)\mathbf{C} = \varphi_C P_R \Psi_c(x) \qquad \text{right-handed}$$

Discrete transformations – SM Interactions

- QED is C and P invariant

$$\mathscr{L}_{\text{EM}} = e\left[Q_u\,\bar{u}_j\gamma^\mu u_j + Q_d\,\bar{d}_j\gamma^\mu d_j\right]A_\mu$$

- Neutral currents (Z) violate C and P, but CP invariant

$$\mathscr{L}_{\text{NC}} = \frac{g}{c_{\text{W}}}\left[\frac{1}{2}\bar{u}_{Lj}\gamma^\mu u_{Lj} - \frac{1}{2}\bar{d}_{Lj}\gamma^\mu d_{Lj} - s_{\text{W}}^2\,J_{\text{EM}}^\mu\right]Z_\mu$$

- Charged currents $(W^\pm)$ violate C, P and CP

$$\mathscr{L}_{\text{CC}} = \frac{g}{\sqrt{2}}\left\{\bar{u}_{La}\gamma^\mu V_{ab} d_{Lb}W_\mu^+ + \bar{d}_{Lb}\gamma^\mu V_{ab}^* u_{La}W_\mu^-\right\}$$

Cabibbo-Kobayashi-Maskawa

CKM

The Cabibbo-Kobayashi-Maskawa matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad VV^\dagger = V^\dagger V = \mathbf{1}$$

Rephasings $R_u = \mathrm{diag}(e^{i\varphi_{u_j}})$, $R_d = \mathrm{diag}(e^{i\varphi_{d_j}})$

$$V = \mathcal{U}^\dagger_{u_L}\mathcal{U}_{d_L} \Rightarrow V \mapsto R_u^\dagger V R_d, \quad \text{i.e.} \quad V_{jk} \mapsto e^{i(\varphi_{d_k}-\varphi_{u_j})}V_{jk}$$

Observables: rephasing invariant

$$|V_{jk}|, \qquad Q_{j_1 j_2 k_1 k_2} = V_{j_1 k_1} V_{j_2 k_2} V^*_{j_1 k_2} V^*_{j_2 k_1}$$

$$\mathrm{Im}\big(V_{j_1 k_1} V_{j_2 k_2} V^*_{j_1 k_2} V^*_{j_2 k_1}\big) \quad j_1 \neq j_2, k_1 \neq k_2 \qquad \text{unique } (\pm)$$

Parameter count

- Unitary matrix $n \times n$, $A = e^{i\theta_a H_a}$, with H_a hermitian
  - n diagonal real
  - $\frac{n(n-1)}{2}$ off-diagonal complex

$$n + \frac{n(n-1)}{2} = \frac{n(n+1)}{2} \text{ phases}, \quad \frac{n(n-1)}{2} \text{ angles}$$

- In V:
  - $\frac{n(n-1)}{2}$ rotation angles
  - $2n-1$ rephasings

$$\frac{n(n+1)}{2} - (2n-1) = \frac{(n-1)(n-2)}{2} \text{ phases}$$

- 2 generations (Cabibbo): 1 rotation angle, 0 phases
- 3 generations (Kobayashi-Maskawa): 3 rotation angles, 1 phase

Unitary parametrisations

- Kobayashi-Maskawa

$$V=\begin{pmatrix}1&0&0\\0&c_2&s_2\\0&-s_2&c_2\end{pmatrix}\begin{pmatrix}c_1&s_1&0\\-s_1&c_1&0\\0&0&e^{i\delta}\end{pmatrix}\begin{pmatrix}1&0&0\\0&c_3&s_3\\0&-s_3&c_3\end{pmatrix}$$

$$V=\begin{pmatrix}c_1&-s_1c_3&-s_1s_3\\s_1c_2&c_1c_2c_3-s_2s_3e^{i\delta}&c_1c_2s_3+s_2c_3e^{i\delta}\\s_1s_2&c_1s_2c_3+c_2s_3e^{i\delta}&c_1s_2c_3-c_2c_3e^{i\delta}\end{pmatrix}$$

- Chau-Keung (PDG)

$$V=\begin{pmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&0&s_{13}e^{-i\delta_{13}}\\0&1&0\\-s_{13}e^{i\delta_{13}}&0&c_{13}\end{pmatrix}\begin{pmatrix}c_{12}&s_{12}&0\\-s_{12}&c_{12}&0\\0&0&1\end{pmatrix}$$

$$V=\begin{pmatrix}c_{12}c_{13}&s_{12}c_{13}&s_{13}e^{-i\delta_{13}}\\-s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}}&c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}}&s_{23}c_{13}\\s_{12}c_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}}&-c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}}&c_{23}c_{13}\end{pmatrix}$$

- Wolfenstein

$$V \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

with $\lambda \simeq 0.22$ and $\rho, \eta, A \sim \mathcal{O}(1)$

$$|V| \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

At the end of the day, $\mathcal{O}(1)$ means

$$\rho \sim 0.12 \qquad \eta \sim 0.35 \qquad A \sim 0.8$$

- Rephasing invariant phases

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \qquad \beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$
$$\beta_s = \arg\left(-\frac{V_{cb}V_{cs}^*}{V_{td}V_{ts}^*}\right) \qquad \chi' = \arg\left(-\frac{V_{us}V_{ud}^*}{V_{cs}V_{cd}^*}\right)$$

- Convenient phase convention

$$\arg(V) = \begin{pmatrix} 0 & \chi' & -\gamma \\ \pi & 0 & 0 \\ -\beta & \pi+\beta_s & 0 \end{pmatrix} \qquad \alpha \equiv \pi - \gamma - \beta$$

with

$$\gamma \sim \mathcal{O}(1) \qquad \beta \sim \mathcal{O}(1) \qquad \beta_s \sim \mathcal{O}(\lambda^2) \qquad \chi' \sim \mathcal{O}(\lambda^4)$$

- N.B. "Belle" convention: $\phi_1 = \beta$, $\phi_2 = \alpha$, $\phi_3 = \gamma$

Unitarity relations, $VV^\dagger = \mathbf{1}$, $V^\dagger V = \mathbf{1}$

- Diagonal

$$(u):\ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
$$(c):\ |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$
$$(t):\ |V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$$

$$(d):\ |V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1$$
$$(s):\ |V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1$$
$$(b):\ |V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$$

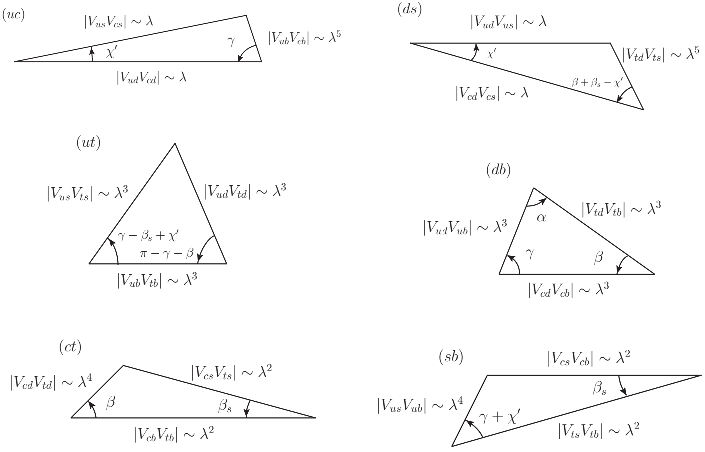
Unitarity relations $VV^\dagger = \mathbf{1}$, $V^\dagger V = \mathbf{1}$

- Non-diagonal

$$(uc): V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0, \qquad \mathcal{O}(\lambda) + \mathcal{O}(\lambda) + \mathcal{O}(\lambda^5) = 0$$
$$(ut): V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0, \qquad \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0$$
$$(ct): V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0, \qquad \mathcal{O}(\lambda^4) + \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^2) = 0$$

$$(ds): V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \qquad \mathcal{O}(\lambda) + \mathcal{O}(\lambda) + \mathcal{O}(\lambda^5) = 0$$
$$(db): V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \qquad \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0$$
$$(sb): V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \qquad \mathcal{O}(\lambda^4) + \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^4) = 0$$

Unitarity triangles



Tree level dominated

- PDG

12. CKM quark-mixing matrix **1**

12. CKM Quark-Mixing Matrix

Revised January 2016 by A. Ceccucci (CERN), Z. Ligeti (LBNL), and Y. Sakai (KEK).

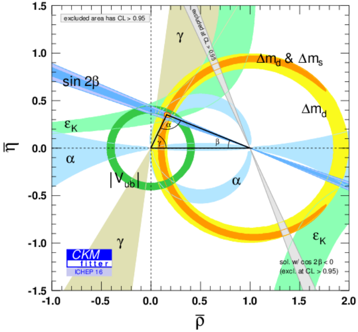
- Moduli $|V_{ij}|$

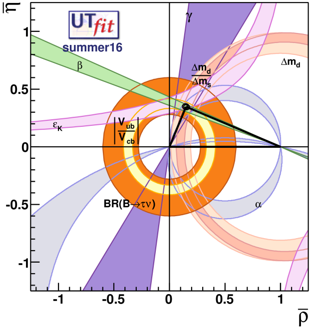
$$|V| = \begin{pmatrix} 0.97417 \pm 0.00021 & 0.2248 \pm 0.0006 & (4.09 \pm 0.39)10^{-3} \\ 0.220 \pm 0.005 & 0.995 \pm 0.016 & (4.05 \pm 0.15)10^{-2} \\ * & * & 1.009 \pm 0.031 \end{pmatrix}$$

- γ ($B \to DK$ decays, GLW,ADS,Dalitz)

$$\gamma = (73.2 \pm 7.0)^\circ$$

CKM Fits


excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δmd & Δms
sin 2β
Δmd
εK
α
β
|Vub|
α
εK
γ
CKM fitter
ICHEP 16
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
η̄
ρ̄


UTfit
summer16
γ
β
Δmd/Δms
Δmd
εK
|Vub/Vcb|
BR(B→τν)
α
η̄
ρ̄

Neutral Meson Systems

Neutral meson systems

- 3 generations of quarks:

$$\begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix}$$

- Confinement
- Bound states quark-antiquark: $M^0 = q_1\bar{q}_2$, $\bar{M}^0 = \bar{q}_1 q_2$
  N.B. $Q(q_1) = Q(q_2)$, $q_1 \neq q_2$ i.e. $M^0 \neq \bar{M}^0$
- Best known cases, lightest pseudoscalar mesons
  $K^0 = \bar{s}d$, $D^0 = \bar{c}u$, $B_d^0 = \bar{b}d$, $B_s^0 = \bar{b}s$
- M^0 and $\bar{M}^0$ have the same strong and electromagnetic properties
- Strong and electromagnetic interactions conserve flavour (i.e. no $M^0 \leftrightarrows \bar{M}^0$ transitions)
- As mentioned, *this is not the case for weak interactions*
  - they violate C, P, CP, T
  - they induce flavour transitions

The Weisskopf-Wigner approximation

- Quantum mechanical description of the two-state system $\{|M^0\rangle, |\bar{M}^0\rangle\}$
- Times much larger than strong interaction times
- System described by an effective Hamiltonian

$$\mathrm{H} = \mathrm{M} - \frac{i}{2}\Gamma, \quad \mathrm{M} = \mathrm{M}^\dagger,\ \Gamma = \Gamma^\dagger$$

- Dispersive M and absorptive Γ parts
- Schrödinger equation

$$i\hbar\frac{d}{dt}|\Psi(t)\rangle = \mathrm{H}|\Psi(t)\rangle$$

- Dispersive M and absorptive Γ parts
- H_{ij} from 2nd order perturbation theory (in the weak interaction)

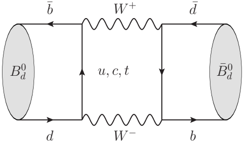
$$\mathrm{M}_{ij} = M_0\delta_{ij} + \langle i|\mathcal{H}_{\mathrm{w}}|j\rangle + \sum_n \mathrm{P}\frac{\langle i|\mathcal{H}_{\mathrm{w}}|n\rangle\langle n|\mathcal{H}_{\mathrm{w}}|j\rangle}{M_0 - E_n}$$

$$\Gamma_{ij} = 2\pi\sum_n \delta(M_0 - E_n)\,\langle i|\mathcal{H}_{\mathrm{w}}|n\rangle\langle n|\mathcal{H}_{\mathrm{w}}|j\rangle$$

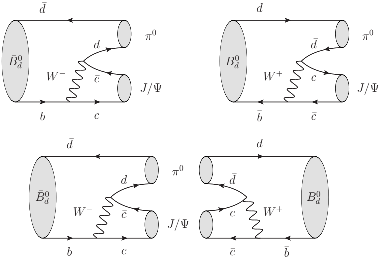
$$(\text{Rel. Norm.})\{|M^0\rangle, |\bar{M}^0\rangle\} = \{|1\rangle, |2\rangle\}$$

- Dispersive M and absorptive Γ parts
- H_{ij} from 2nd order perturbation theory (in the weak interaction)

$M_{21} \rightarrow$



$\Gamma_{12} \rightarrow$



Effective Hamiltonian for Neutral Meson Mixing

Eigenvectors[1]:

$$\mathrm{H}|M_{\mathrm{H}}\rangle = \mu_H|M_{\mathrm{H}}\rangle, \qquad |M_{\mathrm{H}}\rangle = p_H|M^0\rangle + q_H|\bar{M}^0\rangle,$$
$$\mathrm{H}|M_{\mathrm{L}}\rangle = \mu_L|M_{\mathrm{L}}\rangle, \qquad |M_{\mathrm{L}}\rangle = p_L|M^0\rangle - q_L|\bar{M}^0\rangle.$$

$$\langle M^0|M^0\rangle = \langle \bar{M}^0|\bar{M}^0\rangle = 1, \quad \langle M^0|\bar{M}^0\rangle = 0$$

Eigenvalues:

$$\mu = \frac{\mu_H + \mu_L}{2} \equiv M - \frac{i}{2}\Gamma, \quad \Delta\mu = \mu_H - \mu_L \equiv \Delta M - \frac{i}{2}\Delta\Gamma,$$

Evolution

$$i\hbar\frac{d}{dt}|\Psi(t)\rangle = \mathrm{H}|\Psi(t)\rangle \;\Rightarrow\; |\Psi(t)\rangle = e^{-i\mathrm{H}t/\hbar}|\Psi(0)\rangle$$

[1] N.B. “H” and “L” correspond to the “heavy” and “light” states respectively, $\Delta M > 0$ and the sign of $\Delta\Gamma$ is not a matter of convention

Effective Hamiltonian for Neutral Meson Mixing

Hamiltonian:

$$\mathrm{H} = \begin{pmatrix} \mu - \frac{\Delta\mu}{2}\theta & \frac{p}{q}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} \\ \frac{q}{p}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} & \mu + \frac{\Delta\mu}{2}\theta \end{pmatrix}.$$

Mixing parameters $\theta, q/p \in \mathbb{C}$:

$$\frac{q_H}{p_H} = \frac{q}{p}\sqrt{\frac{1+\theta}{1-\theta}}, \qquad \frac{q_L}{p_L} = \frac{q}{p}\sqrt{\frac{1-\theta}{1+\theta}}, \qquad \delta = \frac{1-|q/p|^2}{1+|q/p|^2}.$$

$$\theta = \frac{\mathrm{H}_{22}-\mathrm{H}_{11}}{\Delta\mu}, \qquad \left(\frac{q}{p}\right)^2 = \frac{\mathrm{H}_{21}}{\mathrm{H}_{12}}.$$

- θ is CP and CPT violating
- δ is CP and T violating

Effective Hamiltonian for Neutral Meson Mixing

Hamiltonian:

$$\mathrm{H} = \begin{pmatrix} \mu - \frac{\Delta\mu}{2}\theta & \frac{p}{q}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} \\ \frac{q}{p}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} & \mu + \frac{\Delta\mu}{2}\theta \end{pmatrix}.$$

- θ is CP and CPT violating
- δ is CP and T violating

$$\mathrm{H_O} = (O)^\dagger\,\mathrm{H}\,(O)$$

$$\mathrm{CP}|M^0\rangle = e^{+i\alpha}|\bar{M}^0\rangle,\ \mathrm{TCP}|M^0\rangle = e^{i\beta}|\bar{M}^0\rangle,$$
$$\mathrm{CP}|M^0\rangle = e^{-i\alpha}|\bar{M}^0\rangle,\ \mathrm{TCP}|\bar{M}^0\rangle = e^{i\beta}|M^0\rangle.$$

Effective Hamiltonian for Neutral Meson Mixing

Hamiltonian:

$$\mathrm{H} = \begin{pmatrix} \mu - \frac{\Delta\mu}{2}\theta & \frac{p}{q}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} \\ \frac{q}{p}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} & \mu + \frac{\Delta\mu}{2}\theta \end{pmatrix}.$$

- θ is CP and CPT violating
- δ is CP and T violating

For example

$$\frac{\left|\langle M^0|\mathrm{H}|\bar{M}^0\rangle\right|}{\left|\langle M^0|\mathrm{H}_{\mathrm{CP}}|\bar{M}^0\rangle\right|} = \frac{\left|\langle M^0|\mathrm{H}|\bar{M}^0\rangle\right|}{\left|\langle \bar{M}^0|\mathrm{H}|M^0\rangle\right|} = \left|\frac{p}{q}\right|^2 = \frac{1+\delta}{1-\delta}$$

$$\langle M^0|(\mathrm{H}-\mathrm{H}_{\mathrm{CP}})|M^0\rangle = \langle M^0|\mathrm{H}|M^0\rangle - \langle \bar{M}^0|\mathrm{H}|\bar{M}^0\rangle = -\Delta\mu\,\theta$$

$$\langle M^0|(\mathrm{H}-\mathrm{H}_{\mathrm{TCP}})|M^0\rangle = \langle M^0|\mathrm{H}|M^0\rangle - \langle \bar{M}^0|(T)^\dagger\,\mathrm{H}\,T|\bar{M}^0\rangle = \langle M^0|\mathrm{H}|M^0\rangle - \langle \bar{M}^0|\,\mathrm{H}\,|\bar{M}^0\rangle^* = -\mathrm{Re}(\Delta\mu\,\theta)$$

Some facts [N.B. $\hbar = 6.58 \times 10^{-22}$ MeV s $\Rightarrow$ 1 ps$^{-1} \simeq 0.658$ meV]

	Quarks	M	Γ	ΔM	$\Delta\Gamma$
K^0–$\bar{K}^0$	sd	0.498 GeV	5.56 ns^{-1}	5.30 ns^{-1}	5.54 ns^{-1}
D^0–$\bar{D}^0$	cu	1.864 GeV	2.439 ps^{-1}	9.5 ns^{-1}	3.15 ns^{-1}
B_d^0–$\bar{B}_d^0$	bd	5.280 GeV	0.658 ps^{-1}	0.506 ps^{-1}	$\simeq 0$
B_s^0–$\bar{B}_s^0$	bs	5.367 GeV	0.664 ps^{-1}	17.76 ps^{-1}	0.08 ps^{-1}

	$\Delta M/\Gamma$	$\Delta\Gamma/\Gamma$
K^0–$\bar{K}^0$	0.953	0.996
D^0–$\bar{D}^0$	$3.9 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$
B_d^0–$\bar{B}_d^0$	0.769	0
B_s^0–$\bar{B}_s^0$	26.75	0.12

The discovery of CP Violation

If CP is a good symmetry in the neutral kaon system

- Eigenstates of the Hamiltonian are also CP eigenstates:

$$CP|K^0\rangle = |\bar{K}^0\rangle, \qquad (CP)^2 = \mathbf{1},$$

$$|K^0_\pm\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle \pm |\bar{K}^0\rangle\right], \qquad CP|K^0_\pm\rangle = \pm|K^0_\pm\rangle$$

- Decays

$$K^0_+ \to \pi\pi, \quad K^0_- \nrightarrow \pi\pi, \qquad K^0_- \to \pi\pi\pi$$

but $m_K \sim 500$ GeV and $m_\pi \sim 140$ GeV

$\Rightarrow$ much more phase space for $\pi\pi$ than $\pi\pi\pi$

- K^0_+ short-lived, K^0_- long-lived

The discovery of CP Violation

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EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

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(Received 10 July 1964)

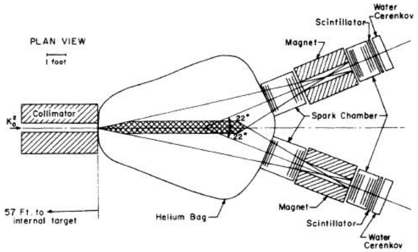


FIG. 1. Plan view of the detector arrangement.

The discovery of CP Violation

The Christenson, Cronin, Fitch & Turlay experiment

- Prepare a beam of kaons
- Propagate: the short-lived component disappears
- If CP is a good symmetry, no decays $\to \pi\pi$ should be observed
- ... *they were observed!*

Time evolution

- General case

$$|\Psi(0)\rangle = \alpha_0 |M^0\rangle + \beta_0 |\bar{M}^0\rangle \;\mapsto\; |\Psi(t)\rangle = {\color{red}?}$$

$$|M^0(t)\rangle = e^{-i\mathrm{H}t}|M^0\rangle = ? \quad |\bar{M}^0(t)\rangle = e^{-i\mathrm{H}t}|\bar{M}^0\rangle = ?$$

- Time evolution of $|M^0\rangle$, $|\bar{M}^0\rangle$:

$$\{M^0, \bar{M}^0\} \xrightarrow{\mathbf{1}} \{M_{\mathrm{H}}, M_{\mathrm{L}}\} \xrightarrow{\mathbf{2}} e^{-i\mathrm{H}t}\{M_{\mathrm{H}}, M_{\mathrm{L}}\} \xrightarrow{\mathbf{3}} \{M^0, \bar{M}^0\}$$

  - Step **1**

$$\begin{pmatrix}|M^0\rangle\\ |\bar{M}^0\rangle\end{pmatrix} = \frac{1}{p_H q_L + p_L q_H}\begin{pmatrix} q_L & q_H\\ p_L & -p_H\end{pmatrix}\begin{pmatrix}|M_{\mathrm{H}}\rangle\\ |M_{\mathrm{L}}\rangle\end{pmatrix}$$

  - Step **2**

$$e^{-i\mathrm{H}t}|M_{\mathrm{H}}\rangle = e^{-i\mu_H t}|M_{\mathrm{H}}\rangle, \quad e^{-i\mathrm{H}t}|M_{\mathrm{L}}\rangle = e^{-i\mu_L t}|M_{\mathrm{L}}\rangle$$

  - Step **3**

$$\begin{pmatrix}|M_{\mathrm{H}}\rangle\\ |M_{\mathrm{L}}\rangle\end{pmatrix} = \begin{pmatrix} p_H & q_H\\ p_L & -q_L\end{pmatrix}\begin{pmatrix}|M^0\rangle\\ |\bar{M}^0\rangle\end{pmatrix}$$

Time evolution of $|M^0\rangle$, $|\bar{M}^0\rangle$

- $M^0(t)$, $\bar{M}^0(t)$

$$|M^0(t)\rangle = \frac{e^{-i\mu t}}{2}\left\{[g_+(t) - \theta g_-(t)]\,|M^0\rangle + \frac{q}{p}\sqrt{1-\theta^2}g_-(t)|\bar{M}^0\rangle\right\}$$

$$|\bar{M}^0(t)\rangle = \frac{e^{-i\mu t}}{2}\left\{\frac{p}{q}\sqrt{1-\theta^2}g_-(t)|M^0\rangle + [g_+(t) + \theta g_-(t)]\,|\bar{M}^0\rangle\right\}$$

$$e^{-i\mu t} = e^{-\Gamma t/2}e^{-iMt}, \quad g_\pm(t) = e^{-\Delta\Gamma t/2}e^{-i\Delta M t} \pm e^{+\Delta\Gamma t/2}e^{+i\Delta M t}$$

- and decay into $|f\rangle$

$$A_f = \langle f|\mathcal{T}|M^0\rangle, \quad \bar{A}_f = \langle f|\mathcal{T}|\bar{M}^0\rangle$$

$$\langle f|\mathcal{T}|M^0(t)\rangle = \frac{e^{-i\mu t}}{2}\left\{[g_+(t) - \theta g_-(t)]\,A_f + \frac{q}{p}\sqrt{1-\theta^2}g_-(t)\bar{A}_f\right\}$$

$$\langle f|\mathcal{T}|\bar{M}^0(t)\rangle = \frac{e^{-i\mu t}}{2}\left\{\frac{p}{q}\sqrt{1-\theta^2}g_-(t)A_f + [g_+(t) + \theta g_-(t)]\,\bar{A}_f\right\}$$

Time dependent rates $\left|\langle f|\mathcal{T}|M^0(t)\rangle\right|^2$, $\left|\langle f|\mathcal{T}|\bar{M}^0(t)\rangle\right|^2$

- Time dependence

$$|g_\pm(t)|^2 = 2\left[\cos(\Delta M t) \pm \cosh\left(\frac{\Delta\Gamma}{2}t\right)\right]$$

$$g_+(t)^* g_-(t) = 2\sinh\left(\frac{\Delta\Gamma}{2}t\right) + i\,2\sin(\Delta M t)$$

- Introduce "Mixing × Decay" λ_f

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f},\quad C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2},\quad R_f = \frac{2\mathrm{Re}(\lambda_f)}{1+|\lambda_f|^2},\quad S_f = \frac{2\mathrm{Im}(\lambda_f)}{1+|\lambda_f|^2}$$

- Collect time dependences

$$\left|\langle f|\mathcal{T}|M^0(t)\rangle\right|^2 = e^{-\Gamma t}\left\{\begin{array}{l}\mathscr{C}_h[M^0,f]\cosh\left(\frac{\Delta\Gamma}{2}t\right) + \mathscr{C}_c[M^0,f]\cos(\Delta M t)\\ +\mathscr{S}_h[M^0,f]\sinh\left(\frac{\Delta\Gamma}{2}t\right) + \mathscr{S}_c[M^0,f]\sin(\Delta M t)\end{array}\right\}$$

$$\left|\langle f|\mathcal{T}|\bar{M}^0(t)\rangle\right|^2 = e^{-\Gamma t}\left\{\begin{array}{l}\mathscr{C}_h[\bar{M}^0,f]\cosh\left(\frac{\Delta\Gamma}{2}t\right) + \mathscr{C}_c[\bar{M}^0,f]\cos(\Delta M t)\\ +\mathscr{S}_h[\bar{M}^0,f]\sinh\left(\frac{\Delta\Gamma}{2}t\right) + \mathscr{S}_c[\bar{M}^0,f]\sin(\Delta M t)\end{array}\right\}$$

N.B. $\Gamma_f \equiv (|A_f|^2 + |\bar{A}_f|^2)/2$

$$\mathscr{C}_h[M^0, f] = \frac{\Gamma_f(1-\delta)}{2(1-C_f\delta)} \left\{ \begin{array}{c} (1+C_f)(1+|\theta|^2) + (1-C_f)|1-\theta^2| \\ -2R_f\mathrm{Re}\left(\theta^*\sqrt{1-\theta^2}\right) + 2S_f\mathrm{Im}\left(\theta^*\sqrt{1-\theta^2}\right) \end{array} \right\}$$

$$\mathscr{C}_c[M^0, f] = \frac{\Gamma_f(1-\delta)}{2(1-C_f\delta)} \left\{ \begin{array}{c} (1+C_f)(1-|\theta|^2) - (1-C_f)|1-\theta^2| \\ +2R_f\mathrm{Re}\left(\theta^*\sqrt{1-\theta^2}\right) - 2S_f\mathrm{Im}\left(\theta^*\sqrt{1-\theta^2}\right) \end{array} \right\}$$

$$\mathscr{S}_h[M^0, f] = \frac{\Gamma_f(1-\delta)}{2(1-C_f\delta)} \left\{ \begin{array}{c} -2(1+C_f)\mathrm{Re}(\theta) \\ +2R_f\mathrm{Re}\left(\sqrt{1-\theta^2}\right) - 2S_f\mathrm{Im}\left(\sqrt{1-\theta^2}\right) \end{array} \right\}$$

$$\mathscr{S}_c[M^0, f] = \frac{\Gamma_f(1-\delta)}{2(1-C_f\delta)} \left\{ \begin{array}{c} 2(1+C_f)\mathrm{Im}(\theta) \\ -2S_f\mathrm{Re}\left(\sqrt{1-\theta^2}\right) - 2R_f\mathrm{Im}\left(\sqrt{1-\theta^2}\right) \end{array} \right\}$$

Similarly

$$\mathscr{C}_h[\bar{M}^0,f]=\frac{\Gamma_f(1+\delta)}{2(1-C_f\delta)}\left\{\begin{matrix}(1-C_f)(1+|\theta|^2)+(1+C_f)|1-\theta^2|\\ +2R_f\mathrm{Re}\left(\theta^*\sqrt{1-\theta^2}\right)+2S_f\mathrm{Im}\left(\theta^*\sqrt{1-\theta^2}\right)\end{matrix}\right\}$$

$$\mathscr{C}_c[\bar{M}^0,f]=\frac{\Gamma_f(1+\delta)}{2(1-C_f\delta)}\left\{\begin{matrix}(1-C_f)(1-|\theta|^2)-(1+C_f)|1-\theta^2|\\ -2R_f\mathrm{Re}\left(\theta^*\sqrt{1-\theta^2}\right)-2S_f\mathrm{Im}\left(\theta^*\sqrt{1-\theta^2}\right)\end{matrix}\right\}$$

$$\mathscr{S}_h[\bar{M}^0,f]=\frac{\Gamma_f(1+\delta)}{2(1-C_f\delta)}\left\{\begin{matrix}2(1-C_f)\mathrm{Re}(\theta)\\ +2R_f\mathrm{Re}\left(\sqrt{1-\theta^2}\right)+2S_f\mathrm{Im}\left(\sqrt{1-\theta^2}\right)\end{matrix}\right\}$$

$$\mathscr{S}_c[\bar{M}^0,f]=\frac{\Gamma_f(1+\delta)}{2(1-C_f\delta)}\left\{\begin{matrix}-2(1-C_f)\mathrm{Im}(\theta)\\ +2S_f\mathrm{Re}\left(\sqrt{1-\theta^2}\right)-2R_f\mathrm{Im}\left(\sqrt{1-\theta^2}\right)\end{matrix}\right\}$$

With $\theta = 0$

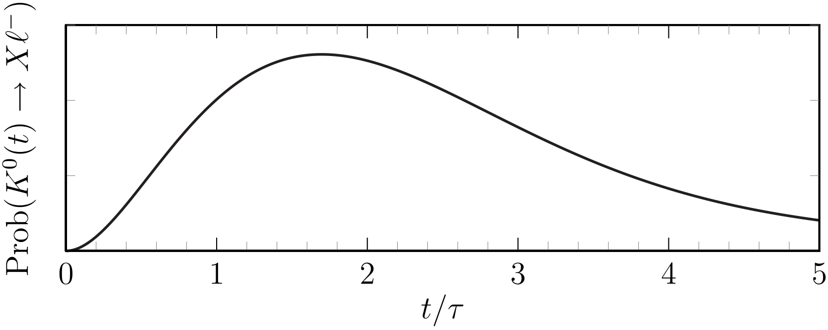
$$\mathscr{C}_h[M^0, f] = \frac{\Gamma_f(1-\delta)}{2(1-C_f\delta)}\{2\} \qquad \mathscr{C}_c[M^0, f] = \frac{\Gamma_f(1-\delta)}{2(1-C_f\delta)}\{2C_f\}$$

$$\mathscr{S}_h[M^0, f] = \frac{\Gamma_f(1-\delta)}{2(1-C_f\delta)}\{2R_f\} \qquad \mathscr{S}_c[M^0, f] = \frac{\Gamma_f(1-\delta)}{2(1-C_f\delta)}\{-2S_f\}$$

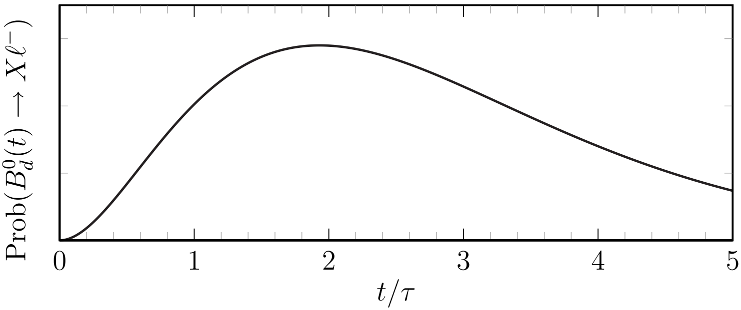
$$\mathscr{C}_h[\bar{M}^0, f] = \frac{\Gamma_f(1+\delta)}{2(1-C_f\delta)}\{2\} \qquad \mathscr{C}_c[\bar{M}^0, f] = \frac{\Gamma_f(1+\delta)}{2(1-C_f\delta)}\{-2C_f\}$$

$$\mathscr{S}_h[\bar{M}^0, f] = \frac{\Gamma_f(1+\delta)}{2(1-C_f\delta)}\{2R_f\} \qquad \mathscr{S}_c[\bar{M}^0, f] = \frac{\Gamma_f(1+\delta)}{2(1-C_f\delta)}\{2S_f\}$$

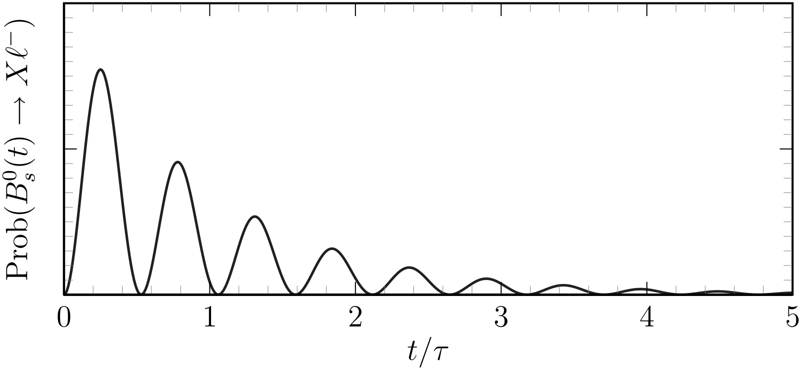
$\Pr(K^0(t) \to \bar{K}^0)$ (from $\left|\langle f|\mathcal{T}|K^0(t)\rangle\right|^2$)



$\Pr(B_d^0(t) \to \bar{B}_d^0)$ (from $\left|\langle f|\mathcal{T}|B_d^0(t)\rangle\right|^2$)



$\Pr(B_s^0(t) \to \bar{B}_s^0)$ (from $\left|\langle f|\mathcal{T}|B_s^0(t)\rangle\right|^2$)



CP Violation

- Consider now a decay channel which is a CP *eigenstate*

$$CP|f_{CP}\rangle = \eta_{f_{CP}}|f_{CP}\rangle$$

e.g. $\pi^+\pi^-$, K^+K^-, $J/\Psi\Phi$, etc

- If there is matter-antimatter symmetry, i.e. CP invariance,

$$\Pr(M^0(t) \to f_{CP}) = \Pr(\bar{M}^0(t) \to f_{CP})$$

- that is, if

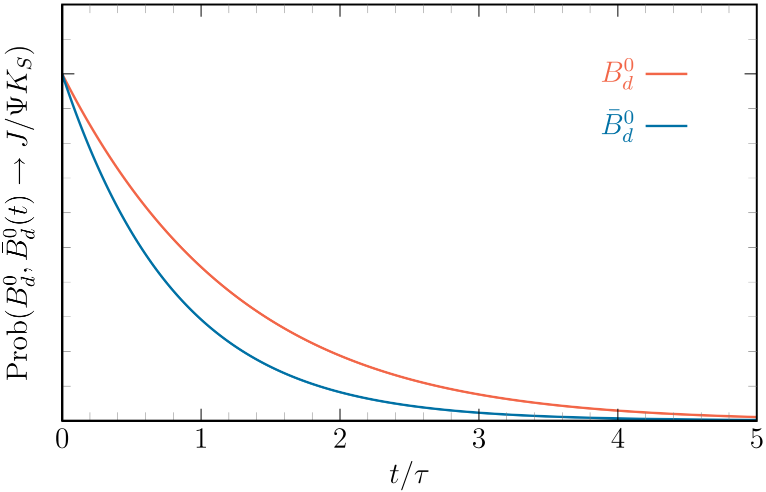
$$\Pr(M^0(t) \to f_{CP}) - \Pr(\bar{M}^0(t) \to f_{CP}) \neq 0$$

we have CP Violation, *matter-antimatter asymmetry!*

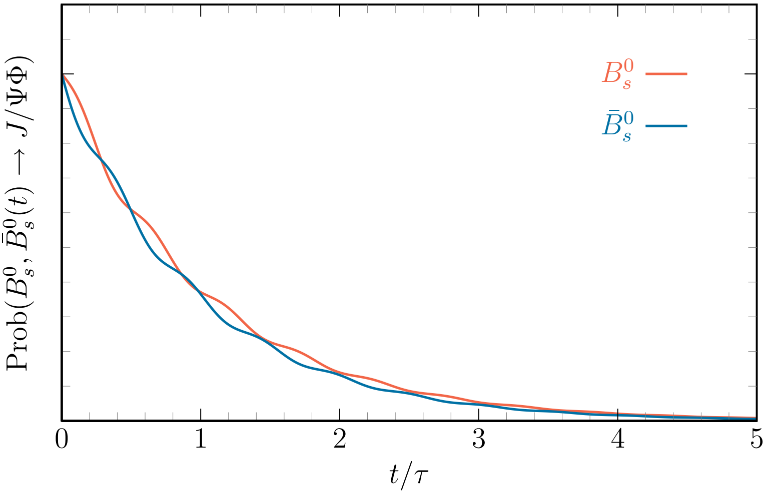
$$\left|\langle f|\mathcal{T}|M^0(t)\rangle\right|^2-\left|\langle f|\mathcal{T}|\bar{M}^0(t)\rangle\right|^2=$$
$$e^{-\Gamma t}\left\{\begin{array}{c}\Delta\mathscr{C}_h\cosh\left(\frac{\Delta\Gamma}{2}t\right)+\Delta\mathscr{C}_c\cos\left(\Delta M t\right)\\ +\Delta\mathscr{S}_h\sinh\left(\frac{\Delta\Gamma}{2}t\right)+\Delta\mathscr{S}_c\sin\left(\Delta M t\right)\end{array}\right\}$$

$$\Delta\mathscr{C}_h=\frac{\Gamma_f}{1-C_f\delta}\left\{-2\delta\right\}\qquad \Delta\mathscr{C}_c=\frac{\Gamma_f}{1-C_f\delta}\left\{2C_f\right\}$$
$$\Delta\mathscr{S}_h=\frac{\Gamma_f}{1-C_f\delta}\left\{-2\delta R_f\right\}\qquad \Delta\mathscr{S}_c=\frac{\Gamma_f}{1-C_f\delta}\left\{-2S_f\right\}$$

$\Pr(B_d^0, \bar{B}_d^0(t) \to f_{CP})$



$\mathrm{Pr}(B_s^0, \bar{B}_s^0(t) \to f_{CP})$



Time-dependent CP asymmetry in B_d^0–$\bar{B}_d^0$ (BaBar)

MEASUREMENT OF TIME-DEPENDENT *CP* ASYMMETRY ... PHYSICAL REVIEW D **79**, 072009 (2009)

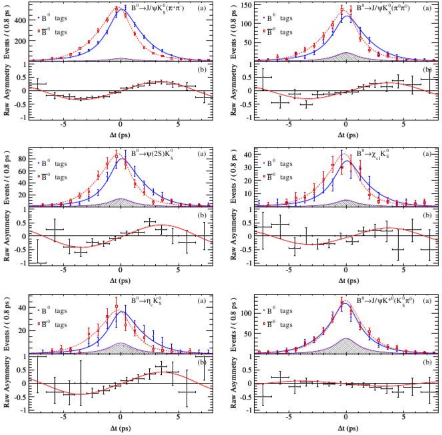


FIG. 4 (color online). (a) Number of B^0 candidates in the signal region with a B^0 tag (N_{B^0}) and with a $\bar{B}^0$ tag ($N_{\bar{B}^0}$), and (b) the corresponding raw asymmetry, $(N_{B^0} - N_{\bar{B}^0})/(N_{B^0} + N_{\bar{B}^0})$, as functions of Δt for each B^0 decay mode. The solid (dashed) curves represent the fit projections in Δt for $B^0(\bar{B}^0)$ tags. The right-hatched (left-hatched) shaded regions represent the estimated background contributions in $\bar{B}^0(B^0)$ tagged events.

Central question

In the previous discussion

- the t evolution starts with $|M^0\rangle$ or $|\bar{M}^0\rangle$, but how do you *prepare* these initial states?

A different (apparently unrelated) question

For a two-meson state

$$|\Psi_2(0)\rangle = \begin{pmatrix} a_{00}(0)|M^0\rangle|M^0\rangle + a_{0\bar{0}}(0)|M^0\rangle|\bar{M}^0\rangle \\ +a_{\bar{0}0}(0)|\bar{M}^0\rangle|M^0\rangle + a_{\bar{0}\bar{0}}(0)|\bar{M}^0\rangle|\bar{M}^0\rangle \end{pmatrix}$$

the question is:

$$e^{-i\mathrm{H}\,t}|\Psi_2(0)\rangle = |\Psi_2(t)\rangle = \begin{pmatrix} a_{00}(t)|M^0\rangle|M^0\rangle + a_{0\bar{0}}(t)|M^0\rangle|\bar{M}^0\rangle \\ +a_{\bar{0}0}(t)|\bar{M}^0\rangle|M^0\rangle + a_{\bar{0}\bar{0}}(t)|\bar{M}^0\rangle|\bar{M}^0\rangle \end{pmatrix} = \ ?$$

The "technology" is already in place:
Two meson states

$$\begin{pmatrix} |M^0\rangle|M^0\rangle \\ |M^0\rangle|\bar{M}^0\rangle \\ |\bar{M}^0\rangle|M^0\rangle \\ |\bar{M}^0\rangle|\bar{M}^0\rangle \end{pmatrix} = \frac{1}{\mathrm{D}^2} \begin{pmatrix} q_L^2 & q_L q_H & q_L q_H & q_H^2 \\ p_L q_L & -p_H q_L & p_L q_H & -p_H q_H \\ p_L q_L & p_L q_H & -p_H q_L & -p_H q_H \\ p_L^2 & -p_L p_H & -p_L p_H & p_H^2 \end{pmatrix} \begin{pmatrix} |M_\mathrm{H}\rangle|M_\mathrm{H}\rangle \\ |M_\mathrm{H}\rangle|M_\mathrm{L}\rangle \\ |M_\mathrm{L}\rangle|M_\mathrm{H}\rangle \\ |M_\mathrm{L}\rangle|M_\mathrm{L}\rangle \end{pmatrix}$$

with $\mathrm{D} = p_L q_H + p_H q_L$

The "technology" is already in place:
Two meson states

$$\begin{pmatrix} |M^0\rangle|M^0\rangle \\ |M^0\rangle|\bar{M}^0\rangle \\ |\bar{M}^0\rangle|M^0\rangle \\ |\bar{M}^0\rangle|\bar{M}^0\rangle \end{pmatrix} = \frac{1}{\mathrm{D}^2} \begin{pmatrix} q_L^2 & q_L q_H & q_L q_H & q_H^2 \\ p_L q_L & -p_H q_L & p_L q_H & -p_H q_H \\ p_L q_L & p_L q_H & -p_H q_L & -p_H q_H \\ p_L^2 & -p_L p_H & -p_L p_H & p_H^2 \end{pmatrix} \begin{pmatrix} |M_{\mathrm{H}}\rangle|M_{\mathrm{H}}\rangle \\ |M_{\mathrm{H}}\rangle|M_{\mathrm{L}}\rangle \\ |M_{\mathrm{L}}\rangle|M_{\mathrm{H}}\rangle \\ |M_{\mathrm{L}}\rangle|M_{\mathrm{L}}\rangle \end{pmatrix}$$

with $\mathrm{D} = p_L q_H + p_H q_L$

$$|S\rangle \equiv \frac{1}{\sqrt{2}}\left[|M^0\rangle|\bar{M}^0\rangle + |\bar{M}^0\rangle|M^0\rangle\right], \quad |A\rangle \equiv \frac{1}{\sqrt{2}}\left[|M^0\rangle|\bar{M}^0\rangle - |\bar{M}^0\rangle|M^0\rangle\right]$$

The "technology" is already in place:
Two meson states

$$\begin{pmatrix} |M^0\rangle|M^0\rangle \\ |S\rangle \\ |\bar{M}^0\rangle|\bar{M}^0\rangle \\ |A\rangle \end{pmatrix} =$$

$$\frac{1}{\mathrm{D}^2}\begin{pmatrix} q_L^2 & q_L q_H & q_L q_H & q_H^2 \\ \sqrt{2}p_L q_L & \frac{p_L q_H - p_H q_L}{\sqrt{2}} & \frac{p_L q_H - p_H q_L}{\sqrt{2}} & -\sqrt{2}p_H q_H \\ p_L^2 & -p_L p_H & -p_L p_H & p_H^2 \\ {\color{blue}0} & -\frac{p_L q_H + p_H q_L}{\sqrt{2}} & \frac{p_L q_H + p_H q_L}{\sqrt{2}} & {\color{blue}0} \end{pmatrix} \begin{pmatrix} |M_{\mathrm{H}}\rangle|M_{\mathrm{H}}\rangle \\ |M_{\mathrm{H}}\rangle|M_{\mathrm{L}}\rangle \\ |M_{\mathrm{L}}\rangle|M_{\mathrm{H}}\rangle \\ |M_{\mathrm{L}}\rangle|M_{\mathrm{L}}\rangle \end{pmatrix}$$

$$e^{-i\mathrm{H}\,t}\begin{pmatrix}|M_{\mathrm{H}}\rangle|M_{\mathrm{H}}\rangle\\|M_{\mathrm{H}}\rangle|M_{\mathrm{L}}\rangle\\|M_{\mathrm{L}}\rangle|M_{\mathrm{H}}\rangle\\|M_{\mathrm{L}}\rangle|M_{\mathrm{L}}\rangle\end{pmatrix}=e^{-i2\mu t}\begin{pmatrix}e^{-i\Delta\mu\,t}&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&e^{+i\Delta\mu\,t}\end{pmatrix}\begin{pmatrix}|M_{\mathrm{H}}\rangle|M_{\mathrm{H}}\rangle\\|M_{\mathrm{H}}\rangle|M_{\mathrm{L}}\rangle\\|M_{\mathrm{L}}\rangle|M_{\mathrm{H}}\rangle\\|M_{\mathrm{L}}\rangle|M_{\mathrm{L}}\rangle\end{pmatrix}$$

$$\begin{aligned}e^{-i\mathrm{H}\,t}\left\{a_{00}(0)|M^0\rangle|M^0\rangle+a_S(0)|S\rangle+a_{\bar{0}\bar{0}}(0)|\bar{M}^0\rangle|\bar{M}^0\rangle+a_A(0)|A\rangle\right\}\\=a_{00}(t)|M^0\rangle|M^0\rangle+a_S(t)|S\rangle+a_{\bar{0}\bar{0}}(t)|\bar{M}^0\rangle|\bar{M}^0\rangle+a_A(t)|A\rangle\end{aligned}$$

$$\begin{pmatrix}a_{00}(t)\\a_S(t)\\a_{\bar{0}\bar{0}}(t)\\a_A(t)\end{pmatrix}=\begin{pmatrix}&&\\&G^{\mathrm{Fl.}}(t)&\\&&\end{pmatrix}\begin{pmatrix}a_{00}(0)\\a_S(0)\\a_{\bar{0}\bar{0}}(0)\\a_A(0)\end{pmatrix}$$

$$G^{\mathrm{Fl.}}(t)=e^{-i2\mu\,t}\left\{e^{-i\Delta\mu\,t}G^{\mathrm{Fl.}}_{-}+e^{+i\Delta\mu\,t}G^{\mathrm{Fl.}}_{+}+G^{\mathrm{Fl.}}_{0}\right\}$$

$$G_{-}^{\rm Fl.}=\frac{1}{4}\begin{pmatrix}(1-\theta)^2 & \sqrt{2}\frac{p}{q}(1-\theta)\sqrt{1-\theta^2} & \frac{p^2}{q^2}(1-\theta^2) & 0\\ \sqrt{2}\frac{q}{p}(1-\theta)\sqrt{1-\theta^2} & 2(1-\theta^2) & \sqrt{2}\frac{p}{q}(1+\theta)\sqrt{1-\theta^2} & 0\\ \frac{q^2}{p^2}(1-\theta^2) & \sqrt{2}\frac{q}{p}(1+\theta)\sqrt{1-\theta^2} & (1+\theta)^2 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

$$G_{+}^{\rm Fl.}=\frac{1}{4}\begin{pmatrix}(1+\theta)^2 & -\sqrt{2}\frac{p}{q}(1+\theta)\sqrt{1-\theta^2} & \frac{p^2}{q^2}(1-\theta^2) & 0\\ -\sqrt{2}\frac{q}{p}(1+\theta)\sqrt{1-\theta^2} & 2(1-\theta^2) & -\sqrt{2}\frac{p}{q}(1-\theta)\sqrt{1-\theta^2} & 0\\ \frac{q^2}{p^2}(1-\theta^2) & -\sqrt{2}\frac{q}{p}(1-\theta)\sqrt{1-\theta^2} & (1-\theta)^2 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

$$G_{0}^{\rm Fl.}=\frac{1}{2}\begin{pmatrix}(1-\theta^2) & \sqrt{2}\frac{p}{q}\theta\sqrt{1-\theta^2} & -\frac{p^2}{q^2}(1-\theta^2) & 0\\ \sqrt{2}\frac{q}{p}\theta\sqrt{1-\theta^2} & 2\theta^2 & -\sqrt{2}\frac{p}{q}\theta\sqrt{1-\theta^2} & 0\\ -\frac{q^2}{p^2}(1-\theta^2) & -\sqrt{2}\frac{q}{p}\theta\sqrt{1-\theta^2} & (1-\theta^2) & 0\\ 0 & 0 & 0 & 2\end{pmatrix}$$

One line summary

$$\boxed{e^{-i\mathrm{H}t}|A\rangle = e^{-i2\mu\, t}|A\rangle}$$

For $\theta = 0$

$$G_{-}^{\text{Fl.}} = \frac{1}{4}\begin{pmatrix} 1 & \sqrt{2}\frac{p}{q} & \frac{p^2}{q^2} & 0 \\ \sqrt{2}\frac{q}{p} & 2 & \sqrt{2}\frac{p}{q} & 0 \\ \frac{q^2}{p^2} & \sqrt{2}\frac{q}{p} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$G_{+}^{\text{Fl.}} = \frac{1}{4}\begin{pmatrix} 1 & -\sqrt{2}\frac{p}{q} & \frac{p^2}{q^2} & 0 \\ -\sqrt{2}\frac{q}{p} & 2 & -\sqrt{2}\frac{p}{q} & 0 \\ \frac{q^2}{p^2} & -\sqrt{2}\frac{q}{p} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$G_{0}^{\text{Fl.}} = \frac{1}{2}\begin{pmatrix} 1 & 0 & -\frac{p^2}{q^2} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{q^2}{p^2} & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

One line summary

$$\boxed{e^{-i\text{H}t}|A\rangle = e^{-i2\mu\, t}|A\rangle}$$

Importance

The simple evolution

$$e^{-i\mathrm{H}t}|A\rangle = e^{-i2\mu\, t}|A\rangle$$

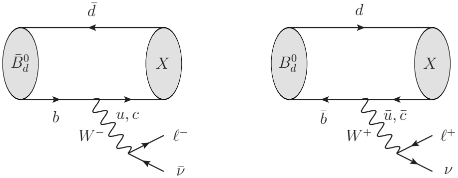
is fundamental for "tagging", i.e. *knowing the initial state*

Antisymmetric entangled state – Flavour Tag

- Time evolution

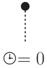
$$e^{-i\mathrm{H}t}|A\rangle = e^{-\Gamma t}e^{-i2Mt}|A\rangle$$

- Flavour Tag: $\bar{B}_d^0$ decays into ℓ^- while B_d^0 decays into ℓ^+



- $\Rightarrow$ controlled state for the evolution of one (anti-)meson

$$\Upsilon(4S) \to B\bar{B}$$

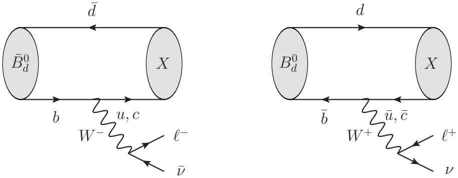


Antisymmetric entangled state – Flavour Tag

- Time evolution

$$e^{-i\mathrm{H}t}|A\rangle = e^{-\Gamma t}e^{-i2Mt}|A\rangle$$

- Flavour Tag: $\bar{B}_d^0$ decays into ℓ^- while B_d^0 decays into ℓ^+



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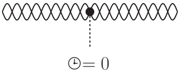

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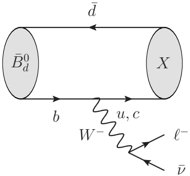

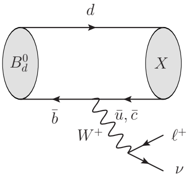

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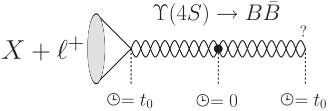

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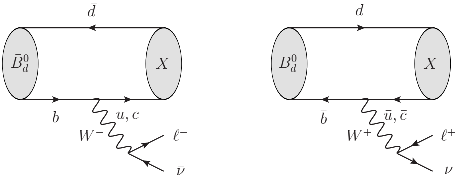

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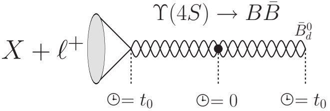

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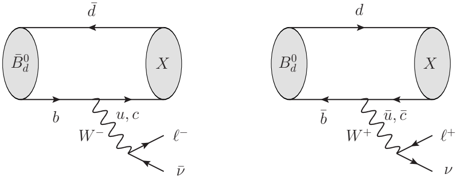

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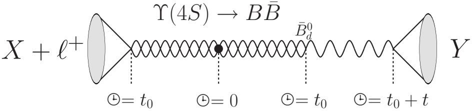
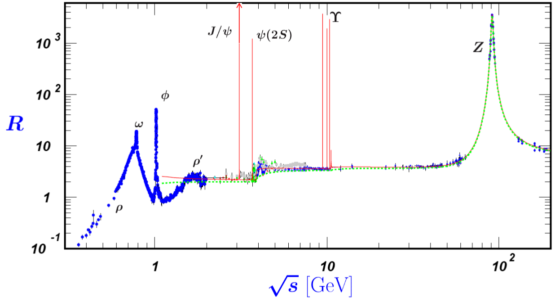
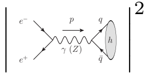
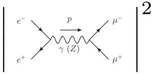


Resonances in e^+e^-

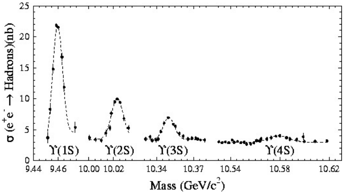


$$R(s=p^2) = \frac{\sigma(e^+e^- \to \text{ hadrons })}{\sigma(e^+e^- \to \mu^+\mu^-)} =$$





Υ Resonances and B mesons



$\Upsilon(3S)$ $\quad I^G(J^{PC}) = 0^-(1^{--})$

$\Upsilon(3S)$ MASS

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
10355.2±0.5	[1] ARTAMONOV 00	MD1	$e^+e^- \to$ hadrons

$\Upsilon(4S)$ or $\Upsilon(10580)$ $\quad I^G(J^{PC}) = 0^-(1^{--})$

$\Upsilon(4S)$ MASS

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
10579.4±1.2 OUR AVERAGE			

BOTTOM MESONS
$(B = \pm 1)$

$B^+ = u\overline{b}$, $B^0 = d\overline{b}$, $\overline{B}^0 = \overline{d}\,b$, $B^- = \overline{u}\,b$, similarly for B^*'s

$B^\pm$ $\quad I(J^P) = \frac{1}{2}(0^-)$

I, J, P need confirmation. Quantum numbers shown are quark-model predictions.

Mass $m_{B^\pm} = 5279.29 \pm 0.15$ MeV (S = 1.1)
Mean life $\tau_{B^\pm} = (1.638 \pm 0.004) \times 10^{-12}$ s
$c\tau = 491.1\ \mu$m

B^0 $\quad I(J^P) = \frac{1}{2}(0^-)$

I, J, P need confirmation. Quantum numbers shown are quark-model predictions.

Mass $m_{B^0} = 5279.61 \pm 0.16$ MeV (S = 1.1)
$m_{B^0} - m_{B^\pm} = 0.32 \pm 0.06$ MeV
Mean life $\tau_{B^0} = (1.520 \pm 0.004) \times 10^{-12}$ s
$c\tau = 455.7\ \mu$m
$\tau_{B^+}/\tau_{B^0} = 1.076 \pm 0.004$ (direct measurements)

B^0-$\overline{B}^0$ mixing parameters

Υ Resonances and B mesons

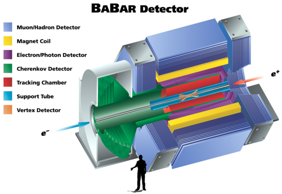
$\Upsilon(3S)$ DECAY MODES

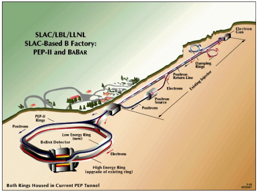
	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	$\Upsilon(2S)$ anything	(10.6 ± 0.8) %	
Γ_2	$\Upsilon(2S)\pi^+\pi^-$	(2.82 ± 0.18) %	S=1.6
Γ_3	$\Upsilon(2S)\pi^0\pi^0$	(1.85 ± 0.14) %	
Γ_4	$\Upsilon(2S)\gamma\gamma$	(5.0 ± 0.7) %	
Γ_5	$\Upsilon(2S)\pi^0$	$< 5.1 \times 10^{-4}$	CL=90%
Γ_6	$\Upsilon(1S)\pi^+\pi^-$	(4.37 ± 0.08) %	
Γ_7	$\Upsilon(1S)\pi^0\pi^0$	(2.20 ± 0.13) %	

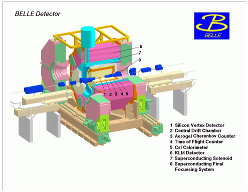
$\Upsilon(4S)$ DECAY MODES

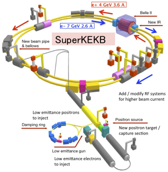
	Mode	Fraction (Γ_i/Γ)	Confidence level
Γ_1	$B\overline{B}$	> 96 %	95%
Γ_2	B^+B^-	(51.4 ± 0.6) %	
Γ_3	D_s^+ anything + c.c.	(17.8 ± 2.6) %	
Γ_4	$B^0\overline{B}^0$	(48.6 ± 0.6) %	
Γ_5	$J/\psi K_S^0 + (J/\psi, \eta_c) K_S^0$	$< 4 \times 10^{-7}$	90%
Γ_6	non-$B\overline{B}$	< 4 %	95%
Γ_7	e^+e^-	$(1.57 \pm 0.08) \times 10^{-5}$	

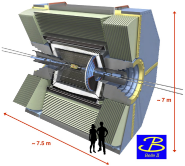
Υ Resonances and B mesons: B-factories











$\Upsilon(4S) \to B\bar{B}\,(B_d^0\bar{B}_d^0, B^+B^-)$

Decay $\Upsilon(4S) \to B\bar{B}$

- $J^{PC}[\Upsilon(4S)] = 1^{--}$, $J^P[B] = J^P[\bar{B}] = 0^-$
- $B\bar{B}$ state with $C = -$

$$|\Psi(0)\rangle \sim \Big(|B_d^0(\vec{p})\rangle|\bar{B}_d^0(-\vec{p})\rangle - |\bar{B}_d^0(\vec{p})\rangle|B_d^0(-\vec{p})\rangle\Big)/\sqrt{2}$$

- Antisymmetric entangled state $|A\rangle$