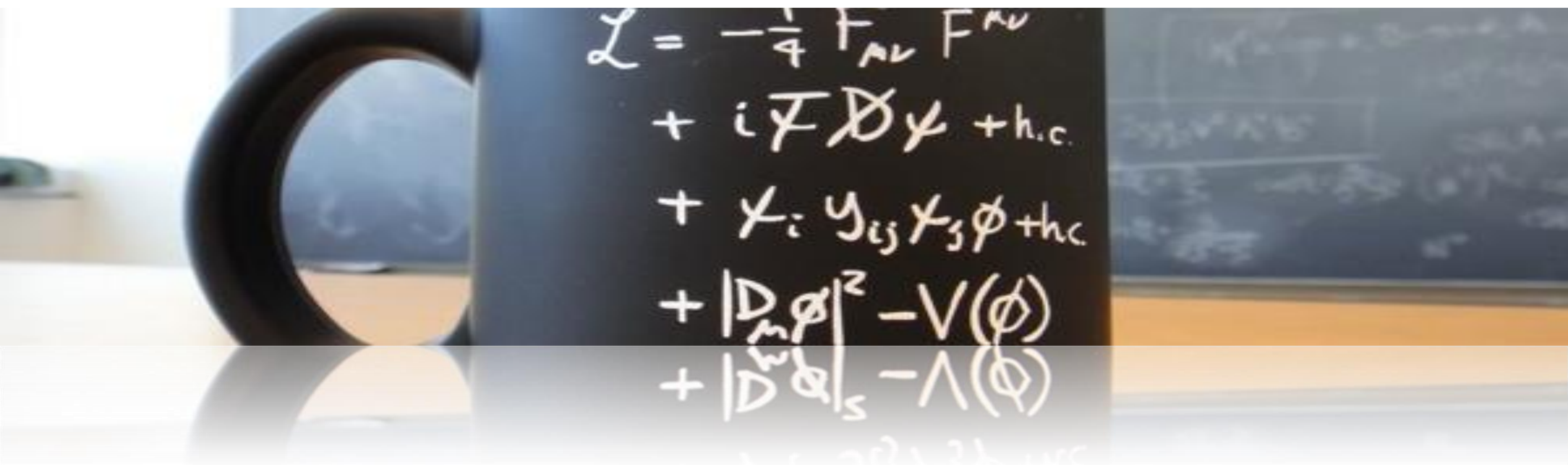
Electroweak theory
(an ultra short introduction)



Electroweak Theory

Unified theory of electromagnetic and weak interactions

Non-abelian gauge group: $SU(2)_T \times U(1)_Y$
[T: weak isospin ➛ coupling g, Y: hypercharge ➛ coupling g']

Pure Yang-Mills theory:
Massless gauge bosons $W^{1,2,3}$, B^0

Electroweak symmetry breaking:
Masses for gauge bosons and fermions [Higgs mechanism]

Three generations of quarks and leptons

T = ½

Left-handed doublets: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$

Right-handed singlets: $e_R, \quad \mu_R, \quad \tau_R, \quad u_R, d_R, \; c_R, s_R, \; t_R, b_R$

T = 0

Rich flavor phenomenology ...

The SM Lagrangian

Free Fields

Interaction

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

Gauge Bosons

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

Fermions

$$\mathcal{L}' = e\bar{\psi}\gamma^\mu A_\mu\psi$$

Fermion-Boson
Coupling

$$eA_\mu = \frac{g_s}{2}\lambda_\nu G^\nu_\mu + \frac{g}{2}\vec{\tau}\,\vec{W}_\mu + \frac{g'}{2}YB_\mu$$

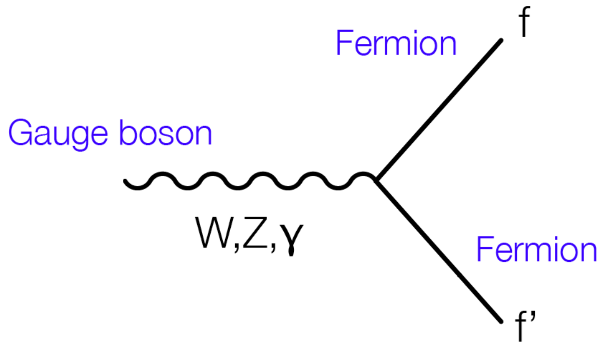
$$F_{\mu\nu}F^{\mu\nu} = G_{\mu\nu}G^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$$

Fermion-Boson Interaction

$$i\,\bar{\psi}\gamma^{\mu}\mathbf{D}_{\mu}\psi$$

Fermion-Boson Interaction

$$= i\,\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + \mathcal{L}_{\text{int}}$$



using

$$\mathbf{D}_{\mu} = \partial_{\mu} + ig\mathbf{W}^{a}_{\mu}\mathbf{T}^{a} + ig'\mathbf{B}_{\mu}\mathbf{Y}$$

$$\mathcal{L}_{\text{int}} = -\bar{\psi}\gamma^{\mu}(g\mathbf{W}^{a}_{\mu}\mathbf{T}^{a} + g'\mathbf{B}_{\mu}\mathbf{Y})\psi$$

Weak Isospin

Hypercharge

Fermion-Boson Interaction

$$\mathcal{L}_{\text{int}} = -\bar{\psi}\gamma^{\mu}(g\mathbf{W}_{\mu}^{a}\mathbf{T}^{a} + g'\mathbf{B}_{\mu}\mathbf{Y})\psi$$

with a = 1,2,3

$$\mathbf{W}_{\mu}^{\pm} = \frac{1}{\sqrt{2}}(\mathbf{W}_{\mu}^{1} \mp i\mathbf{W}_{\mu}^{2})$$

$$\mathbf{A}_{\mu} = \frac{1}{\sqrt{g^2 + g'^2}}(g\mathbf{W}_{\mu}^{3} + g'\mathbf{B}_{\mu}) = \mathbf{W}_{\mu}^{3}\cos\theta_W + \mathbf{B}_{\mu}\sin\theta_W$$

$$\mathbf{Z}_{\mu} = \frac{1}{\sqrt{g^2 + g'^2}}(g\mathbf{W}_{\mu}^{3} - g'\mathbf{B}_{\mu}) = \mathbf{W}_{\mu}^{3}\cos\theta_W - \mathbf{B}_{\mu}\sin\theta_W$$

Weinberg angle θ_W

$$s_W = \sin\theta_W$$
$$c_W = \cos\theta_W$$
$$e = g\sin\theta_W = g'\cos\theta_W$$

$$\mathcal{L}_{\text{int}} = -e\left[\,\mathbf{A}_{\mu}\mathcal{J}_{\text{em}}^{\mu} + (s_W c_W)^{-1}\mathbf{Z}_{\mu}\mathcal{J}_{\text{NC}}^{\mu} + (\sqrt{2}s_W)^{-1}(\mathbf{W}_{\mu}^{+}\mathcal{J}_{\text{CC}}^{\mu} + \mathbf{W}_{\mu}^{-}\mathcal{J}_{\text{CC}}^{\mu\dagger})\,\right]$$

e.m. current

neutral current

charged current

Fermion-Boson Interaction

$$\mathcal{L}_{\text{int}} = -e\,[\,\mathbf{A}_\mu \mathcal{J}^\mu_{\text{em}} + (s_W c_W)^{-1} \mathbf{Z}_\mu \mathcal{J}^\mu_{\text{NC}}$$
$$+(\sqrt{2}s_W)^{-1}(\mathbf{W}^+_\mu \mathcal{J}^\mu_{\text{CC}} + \mathbf{W}^-_\mu \mathcal{J}^{\mu\dagger}_{\text{CC}})\,]$$

$$\mathcal{J}^\mu_{\text{em}} = \bar{\psi}\gamma^\mu(\mathbf{T}_3 + \mathbf{Y})\psi = \bar{\psi}\gamma^\mu \mathbf{Q}\psi$$

charge

3rd isospin component

$$\mathcal{J}^\mu_{\text{NC}} = \bar{\psi}\gamma^\mu(\mathbf{T}_3 - \sin^2\theta_W(\mathbf{T}_3 + \mathbf{Y}))\psi = \bar{\psi}\gamma^\mu(\mathbf{T}_3 - \sin^2\theta_W \mathbf{Q})\psi$$

$$\mathcal{J}^\mu_{\text{CC}} = \bar{\psi}\gamma^\mu(\mathbf{T}_1 + i\mathbf{T}_2)\psi$$

isospin raising operator

Coupling strengths:

$$\text{"}ff\gamma\text{"} : e\mathbf{Q} \qquad \text{"}ffZ\text{"} : e(s_W c_W)^{-1}(\mathbf{T}_3 - \sin^2\theta_W \mathbf{Q})$$

$$\text{"}\ell\nu W\text{"}, \text{"}udW\text{"} : e(\sqrt{2}s_W)^{-1}$$

[left-handed only]

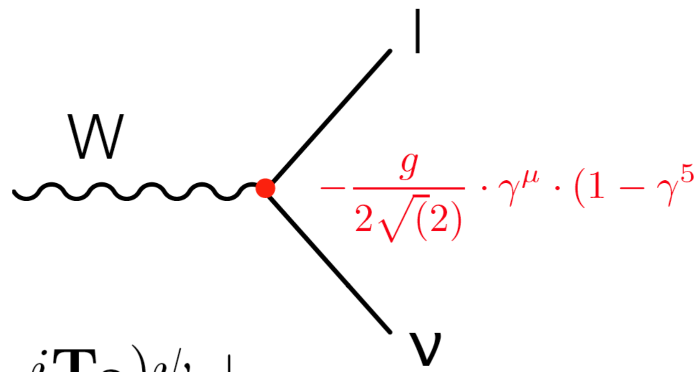
Flavor Quantum Numbers

			T	T_3	Y	Q
$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$	1/2 1/2	1/2 – 1/2	– 1/2 – 1/2	0 – 1
e_R	μ_R	τ_R	0	0	– 1	– 1
$\begin{pmatrix} u_L \\ d'_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s'_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b'_L \end{pmatrix}$	1/2 1/2	1/2 – 1/2	1/6 1/6	2/3 – 1/3
u_R d_R	c_R s_R	t_R b_R	0 0	0 0	2/3 – 1/3	2/3 - 1/3

T : Weak Isospin
T_3 : 3rd Isospin Component
Y : Hypercharge
Q : Charge [=T_3-Y]

W-boson interaction

$$"\ell\nu W", "udW" : e(\sqrt{2}s_W)^{-1}$$



CC interaction:
[e,ν only]

$$\mathcal{L}^W_{\text{int}} = -e(\sqrt{2}s_W)^{-1}\left[\mathbf{W}^+_\mu\ \bar{\psi}\gamma^\mu(\mathbf{T_1}+i\mathbf{T_2})\psi + \right.$$

$$\left. + \mathbf{W}^-_\mu\ \bar{\psi}\gamma^\mu(\mathbf{T_1}-i\mathbf{T_2})\psi\right]$$

$$= -{}^e/\sqrt{2}s_W\left[\mathbf{W}^+_\mu\ (\bar{\nu}_e)_L\gamma^\mu e_L + \mathbf{W}^-_\mu\ \bar{e}_L\gamma^\mu(\nu_e)_L\right]$$

propagator

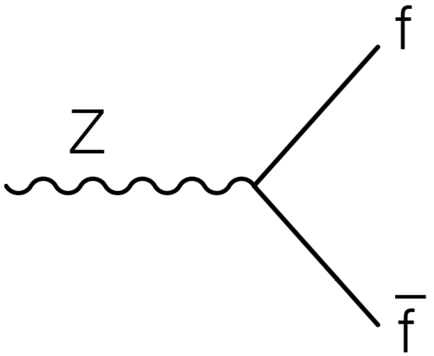
left-handed

$$\mathcal{L}^W_{\text{int}} = -{}^g/\sqrt{2}\left[\mathbf{W}^+_\mu\ (\bar{\nu}_e)_L\gamma^\mu e_L + \mathbf{W}^-_\mu\ \bar{e}_L\gamma^\mu(\nu_e)_L\right]$$

Fermions with $T \neq 0$ only

Z-boson interaction

$$\text{"}ffZ\text{"} : e(s_W c_W)^{-1}(\mathbf{T}_3 - \sin^2\theta_W \mathbf{Q})$$



NC interaction:

$$\mathcal{L}_{\text{int}}^{Z} = -e(s_W c_W)^{-1}\mathbf{Z}_\mu \;\; \bar{\psi}\,\gamma^\mu(\mathbf{T}_3 - s_W^2\mathbf{Q})\psi$$

$$= -e(s_W c_W)^{-1}\mathbf{Z}_\mu \;\; \bar{\psi}\,\gamma^\mu[\,{}^1\!/_2(1-\gamma^5)\mathbf{T}_3 - s_W^2\mathbf{Q}\,]\,\psi$$

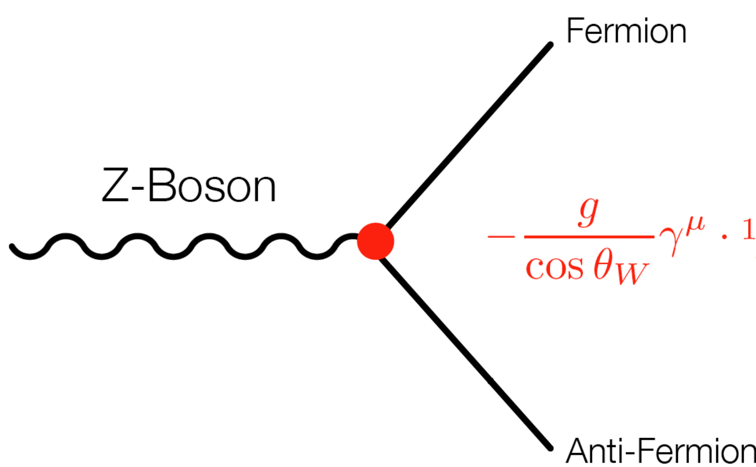
$$= -{}^g\!/_{c_W}\cdot\mathbf{Z}_\mu\cdot{}^1\!/_2\cdot(\;\bar{\psi}\,\gamma^\mu[\,\mathbf{T}_3 - 2s_W^2\mathbf{Q}\,]\psi - \;\bar{\psi}\,\gamma^\mu\gamma^5\;\mathbf{T}_3\;\psi)$$

propagator — vector coupling — axial coupling

$$\mathcal{L}_{\text{int}}^{Z} = -{}^g\!/_{c_W}\cdot\mathbf{Z}_\mu\cdot{}^1\!/_2\cdot(\;\bar{\psi}\,\gamma^\mu g_V\,\psi - \;\bar{\psi}\,\gamma^\mu\gamma^5 g_A\,\psi)$$

Z-boson interaction

Couplings
to the Z-Boson:



$$-\frac{g}{\cos\theta_W}\gamma^\mu \cdot 1/2 \cdot (g_V^f - g_A^f\gamma^5)$$

$g_V = T_3 - 2Q\sin^2\theta_W$
$g_A = T_3$

Standard Model	g_V	g_A
ν	$1/2$	$1/2$
ℓ^-	$-1/2 + 2\sin^2\theta_W$	$-1/2$
$u-quark$	$+1/2 - 4/3\sin^2\theta_W$	$1/2$
$d-quark$	$-1/2 + 2/3\sin^2\theta_W$	$-1/2$

Couplings to
left/right handed fermions:

$g_L = ½(g_V+g_A)$
$g_R = ½(g_V-g_A)$

Electroweak Theory

W and Z masses: connected via weak mixing angle

$$m_W^2 = \frac{g^2 v^2}{4}, \quad m_Z^2 = \frac{v^2}{4}(g^2 + g'^2) \quad \rightarrow \quad \frac{m_W^2}{m_Z^2 \cos\theta_W} = 1$$

g : $SU(2)_T$ coupling
g' : $U(1)_Y$ coupling
θ_W : Weinberg angle
v : vacuum expectation value

Couplings to W and Z
[here: leptons only]

$$\mathcal{L}^{CC} = -\frac{g}{\sqrt{2}}\left[J_\mu^{+CC} W^{\mu,-} + J_\mu^{-CC} W^{\mu,+}\right]$$

$$= -\frac{g}{\sqrt{2}}\left[\left(\bar{\nu}_e \gamma_\mu \frac{1}{2}(1-\gamma_5)e\right) W^{\mu,-} + \left(\bar{e}\gamma_\mu \frac{1}{2}(1-\gamma_5)\nu_e\right) W^{\mu,+}\right]$$

Charged current: always flavor-changing
[quarks: mass eigenstates ≠ EW eigenstates → CKM matrix]

$$\mathcal{L}^{NC} = -\frac{g}{2\cos\theta_W} J_\mu^{NC} Z^\mu$$

$$= -\frac{g}{2\cos\theta_W}\left[\bar{\nu}_e\gamma_\mu \frac{1}{2}(1-\gamma_5)\nu_e - \bar{e}\gamma_\mu \frac{1}{2}(1-\gamma_5)e + 2\sin^2\theta_W(\bar{e}\gamma_\mu e)\right] Z^\mu$$

Neutral current: always flavor-conserving

Gauge Boson Self-Couplings

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi$$

$$F_{\mu\nu}F^{\mu\nu} = W_{\mu\nu}W^{\mu\nu} + B_{\mu\nu}B^{\mu\nu}$$

[electroweak only]

Transition to
covariant derivative ...

$$\partial_\mu \rightarrow \mathbf{D}_\mu$$

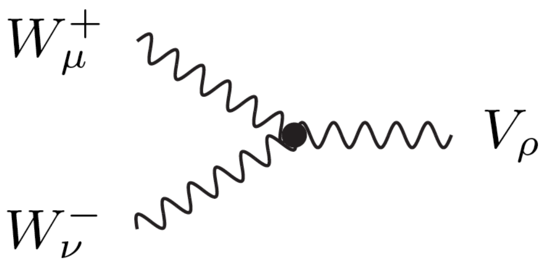
with $\mathbf{D}_\mu = \partial_\mu + ig\mathbf{W}_\mu^a\mathbf{T}^a + ig'\mathbf{B}_\mu\mathbf{Y}$

yields ...

1. Invariance under local gauge transformation
2. Gauge-boson self-couplings ...

Gauge Boson Self-Couplings

Triple gauge-boson couplings:

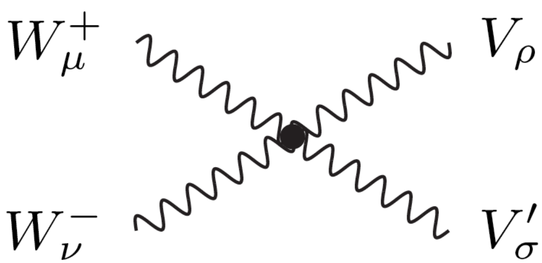


V_ρ = Z,γ:

$$\mathrm{i}eC_{WWV}\Big[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu\Big]$$

with $C_{WW\gamma} = 1, \quad C_{WWZ} = -\frac{c_\mathrm{W}}{s_\mathrm{W}}$

Quartic gauge-boson couplings:



V_ρ,$V_{\rho'}$ = (W,W),(Z,Z),(Z,γ),(γ,γ):

$$\mathrm{i}e^2C_{WWVV'}\Big[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho}\Big]$$

with $C_{WW\gamma\gamma} = -1, \qquad C_{WW\gamma Z} = \frac{c_\mathrm{W}}{s_\mathrm{W}},$

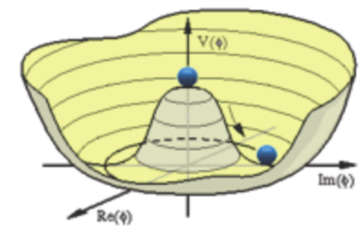
$$C_{WWZZ} = -\frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}, \quad C_{WWWW} = \frac{1}{s_\mathrm{W}^2}$$

The Higgs mechanism

Yukawa Couplings

Higgs Field

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}' + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi'}$$



$$\mathcal{L}_\phi = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - V(\phi)$$

Higgs Potential

$$\mathcal{L}_{\text{Yuk}} = c_f(\bar{\psi}_L \psi_R \phi + \bar{\psi}_R \psi_L \phi)$$

Higgs Fermion Interaction

Gauge Boson masses: $i\partial_\mu \rightarrow i(\partial_\mu - ieA_\mu)$

Fermion masses: $c_f \bar{\psi}\psi\phi$

and $\phi' = \phi - \rho_0$

Vacuum expectation value

SM parameters

3 Couplings	$g_s,\ e,\ \sin\theta_W$
4 CKM parameters	$\vartheta_1,\ \vartheta_2,\ \vartheta_3,\ \delta$
2 Boson masses	$m_{\mathrm{Z}},\ m_{\mathrm{H}}$
3 Lepton masses	$m_{\mathrm{e}},\ m_{\mu},\ m_{\tau}$
6 Quark masses	$m_{\mathrm{u}},\ m_{\mathrm{d}},\ m_{\mathrm{s}},\ m_{\mathrm{c}},\ m_{\mathrm{t}},\ m_{\mathrm{b}}\,.$

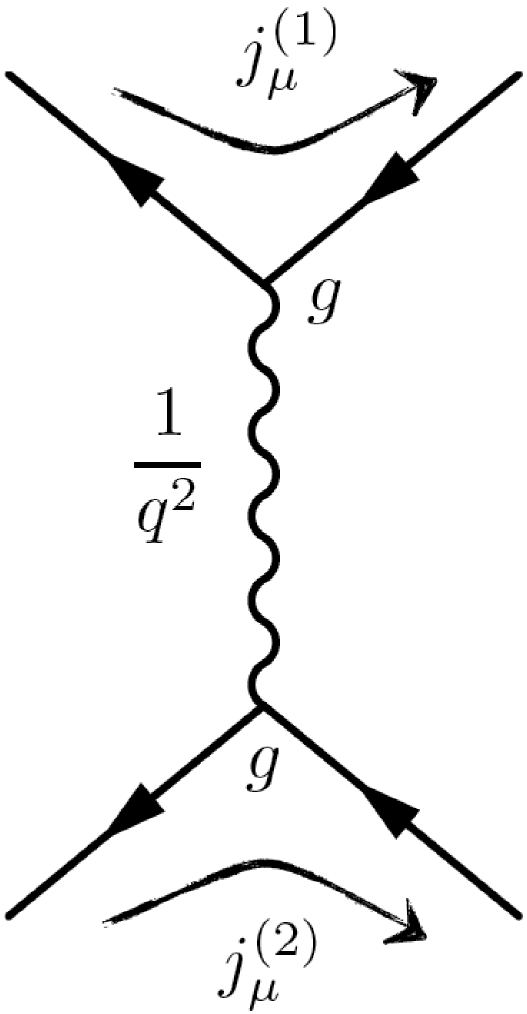
18 free SM parameters
no neutrino masses

$$\begin{aligned} m_{\mathrm{W}}^2 &= \tfrac{1}{2}\, g^2\, \rho_0^2 \\ m_{\mathrm{Z}}^2 &= \tfrac{1}{2}\, (g^2 + g'^2)\, \rho_0^2 \\ m_{\mathrm{H}}^2 &= 4\,\lambda \rho_0^2 \end{aligned}$$

$$\begin{aligned} g &= e/\sin\theta_W \\ g' &= e/\cos\theta_W \end{aligned}$$

$$m_f = c_f\, \rho_0$$

Cross section: using Feynman diagrams



Fermi's Golden Rule

$$W_{\text{fi}} = 2\pi \, |M_{\text{fi}}|^2 \cdot \frac{dN}{dE_{\text{f}}}$$

Transition probability

Matrix element

Phase space

4-vector current

$$M_{\text{fi}} = -i \int j_\mu^{(1)} \cdot \left(\frac{1}{q^2} \right) \cdot j_\mu^{(2)} d^4x$$

Propagator

$$\sigma \;\sim\; |M_{\text{fi}}|^2$$

$$\sim\; g^4 \cdot \left(\frac{1}{q^4} \right)$$

From the Lagrangian to cross sections

$$\sigma \sim \langle f \,|\mathbf{S}|\, i \rangle^2$$

Inelastic
Cross Section
[for $|i\rangle \neq |f\rangle$]

$$[\mathrm{Def.}: \; |t = +\infty\rangle \equiv \mathbf{S}|t = -\infty\rangle\,]$$

Time Evolution

From Schrödinger-Equation
[Dirac picture]

$$|t\rangle = |t_0\rangle - i\int_{t_0}^{t} \mathrm{d}t'\, \mathbf{H}'(t')|t'\rangle$$

$$\mathbf{H}'(t) = -\int \mathcal{L}'(x,t)\,\mathrm{d}^3x$$

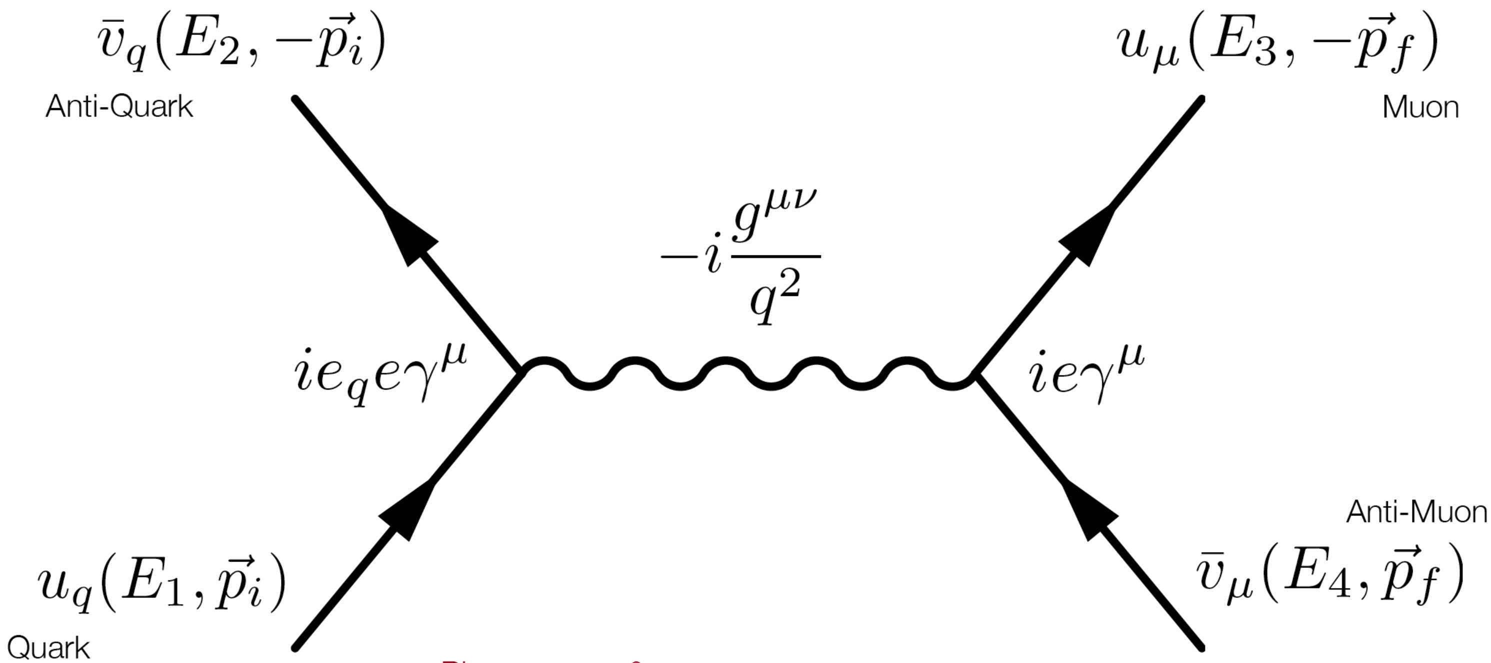
Lagrangian
of Interaction

Matrix element

$$\langle f\,|\mathbf{S}|\,i\rangle \cong \delta_{fi} - i\int_{-\infty}^{\infty} \mathrm{d}t' \langle f\,|\mathbf{H}'(t')|\,i\rangle$$

➡ Feynman rules

Example: Drell-Yan Process

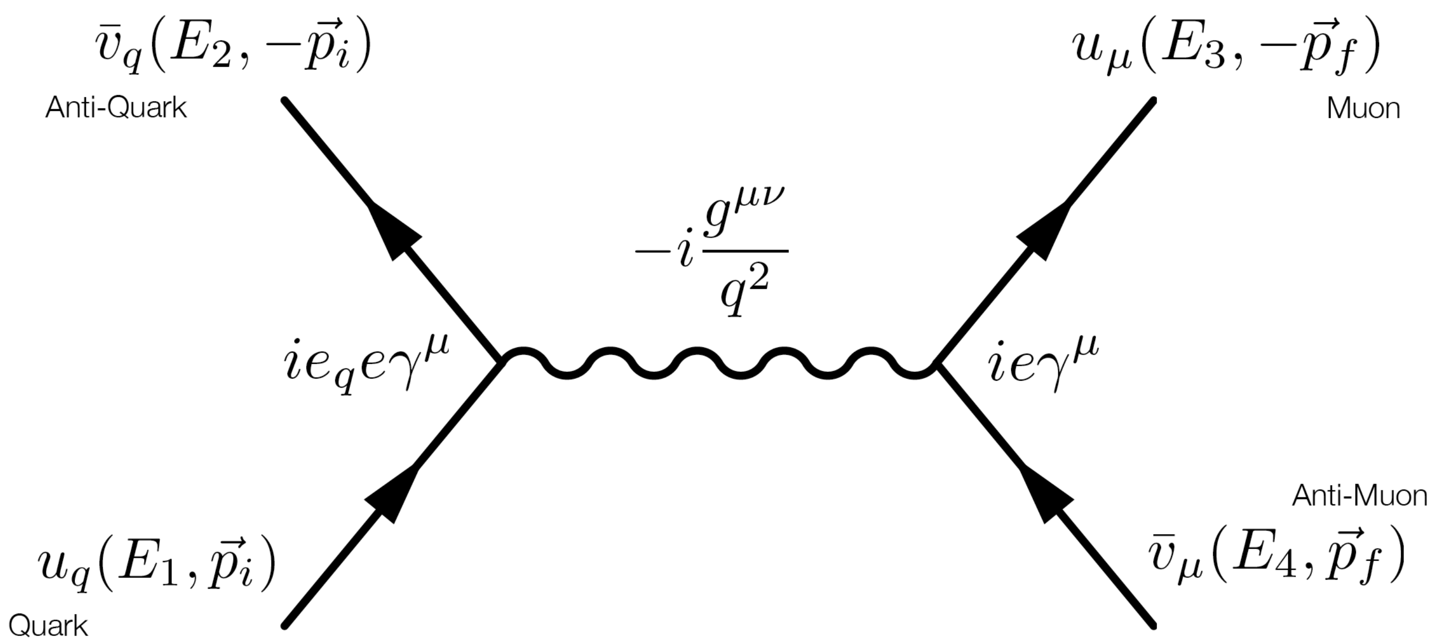


Phase space & delta functions ...

Squared Matrix element

$$\frac{d\sigma}{d\Omega} = \frac{1}{s \cdot 64\pi^2} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|} \cdot \overline{|M_{fi}|^2}$$

Example: Drell-Yan Process

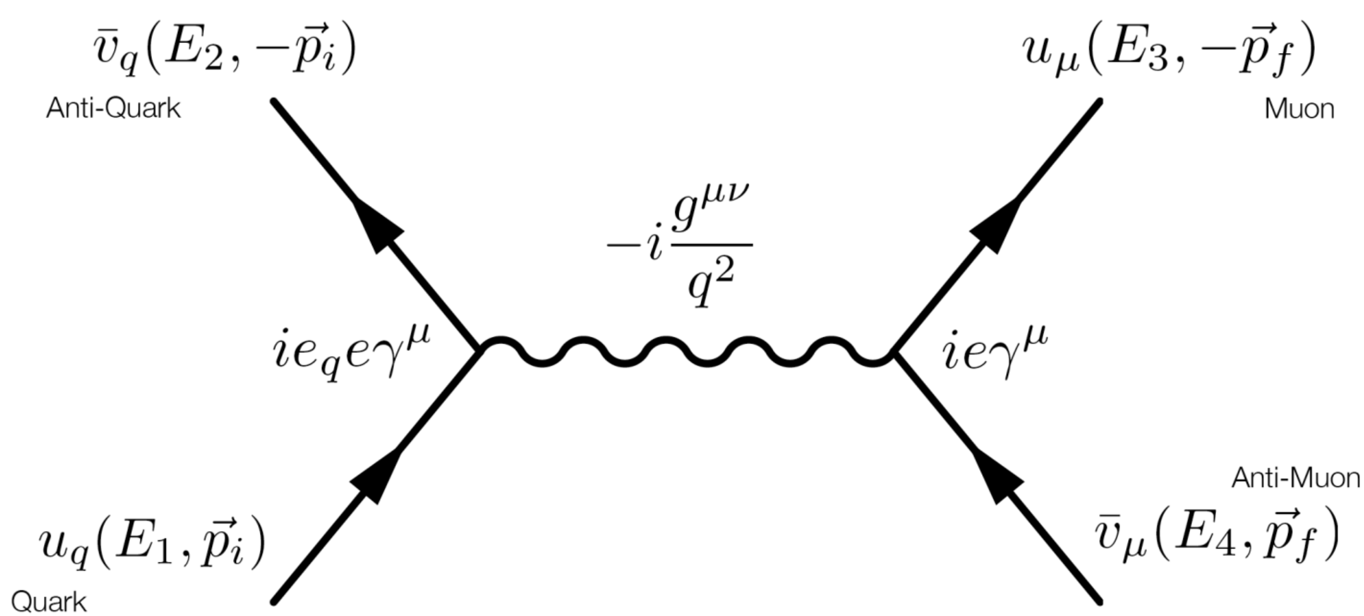


Averaging over initial spins

Summing over initial and final spins

$$\overline{|M_{fi}|^2} = \frac{1}{(2s_q+1)^2} \cdot \sum_{s_q, s'_q} \sum_{s_\mu, s'_\mu} |M_{fi}|^2$$

Example: Drell-Yan Process



Couplings — Anti-Quark — Anti-Muon

$$M_{fi} = -\frac{e_q e^2}{q^2}\bar{v}_q\gamma_\mu u_q \cdot \bar{v}_\mu\gamma^\mu u_\mu$$

Propagator — Quark — Muon

Example: Drell-Yan Process

$$\overline{|M|^2}_{q\bar{q}\to\mu\mu} = 2e_q^2 e^4 \cdot \frac{t^2+u^2}{s^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{32\pi^2} e_q^2 \cdot \frac{1}{s} \cdot \frac{t^2+u^2}{s^2}$$

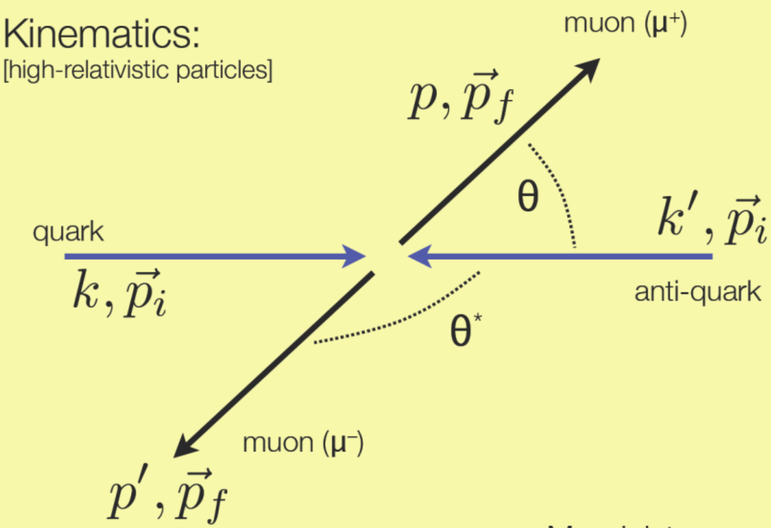
$$= \frac{e^4}{64\pi^2} e_q^2 \cdot \frac{1}{s} \cdot (1 + cos^2\theta)$$

with $e^2 = 4\pi\alpha$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha}{4s} e_q^2 \cdot (1 + cos^2\theta)$$

[θ in CMS frame]

Kinematics:
[high-relativistic particles]



Mandelstam variables

$$s = (k+k')^2 = 4E_i^2$$

$$t = (k-p)^2 \approx -2kp \approx -2E_i^2(1-\cos\theta^*)$$

$$\approx -\frac{s}{2}(1+\cos\theta)$$

$$u = (k-p')^2 \approx -2kp' \approx -2E_i^2(1-\cos\theta)$$

$$\approx -\frac{s}{2}(1-\cos\theta)$$