

Top Quark Physics @ LHC

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Topics covered in this lecture:

- ▶ Introduction
- ▶ The Wtb vertex structure (within and beyond the SM)
- ▶ Single Top quark (SM and beyond)
- ▶ Rare decays of top quarks and Flavor Changing Neutral Currents (FCNC)

Introduction

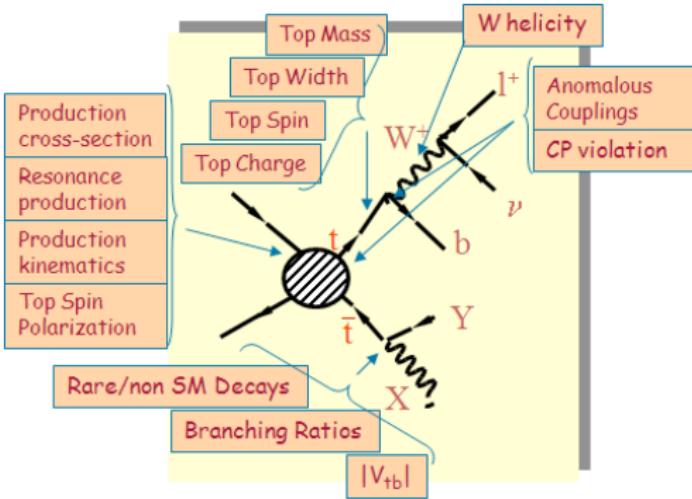
- Top quark completes the 3 family structure of the SM
 - top is the weak-isospin partner of the b -quark
 - spin = $1/2$
 - charge = $+2/3 |e|$
- Top quark is the heaviest known quark ($m_t = 173.34 \pm 0.76$ GeV, ATLAS+CMS+CDF+D0)
- Top decays (almost exclusively) through $t \rightarrow bW$
 $BR(t \rightarrow sW) \leq 0.18\%$, $BR(t \rightarrow dW) \leq 0.02\%$
- $\Gamma_t^{SM} = 1.42$ GeV
(including m_b , m_W , α_s , EW corrections)
 - $\Lambda_{QCD}^{-1} = (100 \text{ MeV})^{-1} = 10^{-23} \text{ s}$ (hadronization time)
 - $\tau_t \ll 10^{-23} \text{ s}$
⇒ top decays before hadronization

Quarks	I	II	III	
Leptons	u	c	t	γ
	d	s	b	g
	ν_e	ν_μ	ν_τ	Z
	e	μ	τ	W

Three Generations of Matter

Top quark @ LHC

- $t\bar{t}$ production
 - $\sigma_{t\bar{t}}$
 - Mass
 - the Wtb vertex struct.
(W polarization,
 $t \rightarrow bW$ decay and
anomalous couplings)
 - FCNC
 - Charge Asymmetry
- Single top production
 - cross section



The Wtb vertex structure

Why is it necessary a precise
model-independent measurement of the Wtb
vertex structure?

- It may reveal physics beyond the Standard Model
 - V_{tb} could be different from the Standard Model value
 - Anomalous couplings may appear at the vertex
- It may help understand possible other new physics beyond the Standard Model
 - top quarks decay almost exclusively to $t \rightarrow W^+ b$
 - understanding the structure of the Wtb vertex helps revealing possible non-standard $t\bar{t}$ production at LHC, $Zt\bar{t}/\gamma t\bar{t}$ couplings at ILC, etc.
 - important for B and K physics (indirect limits on anomalous couplings, see later)

The Wtb vertex must be determined by a global fit to several observables:

- Several, theoretically equivalent, observables studied for $t\bar{t}$ production at LHC (not all explored yet @ LHC)
- Single top cross section useful (sensitive to V_{tb} and anomalous couplings)
- Indirect limits from $b \rightarrow s\gamma$ available (not used)
- The most general CP-conserving vertex for top quarks on-shell is used
- All couplings are allowed to vary freely in TopFit to find the allowed regions for a given CL

The Wtb vertex structure

Effective Wtb vertex from dim-6 operators

$$\begin{aligned}\mathcal{L} = & -\frac{g}{\sqrt{2}} \bar{b} \gamma^\mu (V_L P_L + \textcolor{red}{V_R} P_R) t W_\mu^- \\ & -\frac{g}{\sqrt{2}} \bar{b} \frac{i \sigma^{\mu\nu} q_\nu}{M_W} (\textcolor{red}{g_L} P_L + \textcolor{red}{g_R} P_R) t W_\mu^- + \text{h.c.}\end{aligned}$$

$V_L \equiv V_{tb} \sim 1$ (within SM)

$V_R, g_R, g_L \Rightarrow$ anomalous couplings

[EPJC50 (2007) 519, NPB804 (2008) 160, NPB812 (2009) 181]

How to probe anomalous couplings in the Wtb vertex?

- indirect limits from B -physics
- measurements of single top quark production: cross-section and angular distributions
- measurements of $t\bar{t}$ production: angular distributions of top quark decays

The Wtb vertex structure

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B -physics constraints to Wtb vertex

$$BR(\bar{B} \rightarrow X_s \gamma) = (3.55 \pm 0.24 {}^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}$$

[hep-ex/0603003]

$$\begin{aligned} BR(B \rightarrow X_s \gamma) \times 10^4 &= (3.15 \pm 0.23) - 4.14 (V_L - V_{tb}) + 411 V_R \\ &- 53.9 g_L - 2.12 g_R - 8.03 C_7^{(p)}(\mu_0) \\ &+ \mathcal{O} \left[(V_L - V_{tb}, V_R, g_L, g_R, C_7^{(p)})^2 \right] \end{aligned}$$

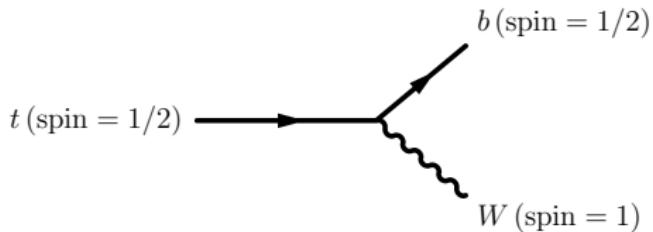
$$\mathcal{O} \left[(V_L - V_{tb}, V_R, \dots)^2 \right] \simeq 1.32(V_L - V_{tb})^2 - 262(V_L - V_{tb})V_R + 12970V_R^2 + \dots$$

	$V_L - V_{tb}$	V_R	g_L	g_R	$C_7^{(p)}(\mu_0)$
upper bound	0.04	0.0024	0.003	0.08	0.02
lower bound	-0.24	-0.0004	-0.018	-0.46	-0.12

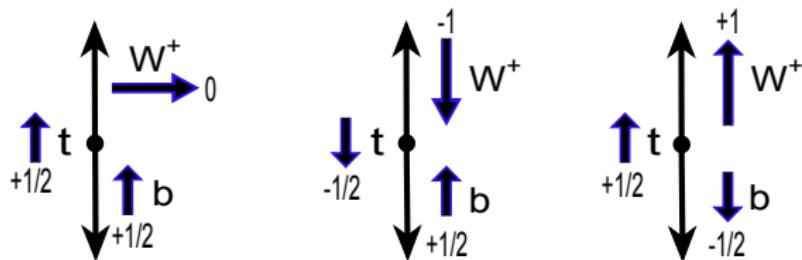
[EPJC57 (2008) 183]

The Wtb vertex structure

[PRD 45 (1992) 124]



W helicity fractions ($F_0 = \Gamma_0/\Gamma$, $F_L = \Gamma_L/\Gamma$, $F_R = \Gamma_R/\Gamma$):



SM (L0): $F_0 = 0.6966$

$F_L = 0.3030$

$F_R = 0.0004$

The Wtb vertex structure

● [arXiv:hep-ph0605190v2 18 Mar 2007]

Probing anomalous Wtb couplings in top pair decays

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Abstract

We investigate several quantities, defined in the decays of top quark pairs, which can be used to explore non-standard Wtb interactions. Two new angular asymmetries are introduced in the leptonic decay of top (anti)quarks. Both are very sensitive to anomalous Wtb couplings, and their measurement allows for a precise determination of the W helicity fractions. We also examine other angular and energy asymmetries, the W helicity fractions and their ratios, as well as spin correlation asymmetries, analysing their dependence on anomalous Wtb couplings and identifying the quantities which are most sensitive to them. It is explicitly shown that spin correlation asymmetries are less sensitive to new interactions in the decay of the top quark; therefore, when combined with the measurement of other observables, they can be used to determine the $t\bar{t}$ spin correlation even in the presence of anomalous Wtb couplings. We finally discuss some asymmetries which can be used to test CP violation in $t\bar{t}$ production and complex phases in the effective Wtb vertex.

The Wtb vertex structure

- [arXiv:hep-ph/0605190v2 18 Mar 2007]

2 W helicity fractions and ratios

The polarisation of the W bosons emitted in the top decay is sensitive to non-standard couplings [17]. The W bosons can be produced with positive (right-handed), negative (left-handed) or zero helicity, with corresponding partial widths Γ_R , Γ_L , Γ_0 , being $\Gamma \equiv \Gamma(t \rightarrow W^+ b) = \Gamma_R + \Gamma_L + \Gamma_0$. The Γ_R component vanishes in the $m_b = 0$ limit because the b quarks produced in top decays have left-handed chirality, and for vanishing m_b the helicity and the chirality states coincide. The three partial widths can be calculated for a general Wtb vertex as parameterised in Eq. (I), yielding

$$\begin{aligned}\Gamma_0 = & \frac{g^2 |\vec{q}|}{32\pi} \left\{ \frac{m_t^2}{M_W^2} \left[|V_L|^2 + |V_R|^2 \right] (1 - x_W^2 - 2x_b^2 - x_W^2 x_b^2 + x_b^4) - 4x_b \operatorname{Re} V_L V_R^* \right. \\ & + \left[|g_L|^2 + |g_R|^2 \right] (1 - x_W^2 + x_b^2) - 4x_b \operatorname{Re} g_L g_R^* \\ & - 2 \frac{m_t}{M_W} \operatorname{Re} [V_L g_R^* + V_R g_L^*] (1 - x_W^2 - x_b^2) \\ & \left. + 2 \frac{m_t}{M_W} x_b \operatorname{Re} [V_L g_L^* + V_R g_R^*] (1 + x_W^2 - x_b^2) \right\},\end{aligned}$$

being $x_W = M_W/m_t$, $x_b = m_b/m_t$ and

$$|\vec{q}| = \frac{1}{2m_t} (m_t^4 + M_W^4 + m_b^4 - 2m_t^2 M_W^2 - 2m_t^2 m_b^2 - 2M_W^2 m_b^2)^{1/2}$$

The Wtb vertex structure

● [arXiv:hep-ph0605190v2 18 Mar 2007]

$$\begin{aligned}\Gamma_{R,L} = & \frac{g^2 |\vec{q}|}{32\pi} \left\{ \left[|V_L|^2 + |V_R|^2 \right] (1 - x_W^2 + x_b^2) - 4x_b \operatorname{Re} V_L V_R^* \right. \\ & + \frac{m_t^2}{M_W^2} \left[|g_L|^2 + |g_R|^2 \right] (1 - x_W^2 - 2x_b^2 - x_W^2 x_b^2 + x_b^4) - 4x_b \operatorname{Re} g_L g_R^* \\ & - 2 \frac{m_t}{M_W} \operatorname{Re} [V_L g_R^* + V_R g_L^*] (1 - x_W^2 - x_b^2) \\ & \left. + 2 \frac{m_t}{M_W} x_b \operatorname{Re} [V_L g_L^* + V_R g_R^*] (1 + x_W^2 - x_b^2) \right\} \\ & \pm \frac{g^2}{64\pi} \frac{m_t^3}{M_W^2} \left\{ -x_W^2 \left[|V_L|^2 - |V_R|^2 \right] + \left[|g_L|^2 - |g_R|^2 \right] (1 - x_b^2) \right. \\ & + 2x_W \operatorname{Re} [V_L g_R^* - V_R g_L^*] + 2x_W x_b \operatorname{Re} [V_L g_L^* - V_R g_R^*] \} \\ & \times (1 - 2x_W^2 - 2x_b^2 + x_W^4 - 2x_W^2 x_b^2 + x_b^4) , \end{aligned} \tag{2}$$

The Wtb vertex structure

● [arXiv:hep-ph0605190v2 18 Mar 2007]

the modulus of the W boson three-momentum in the top quark rest frame. The total top width is

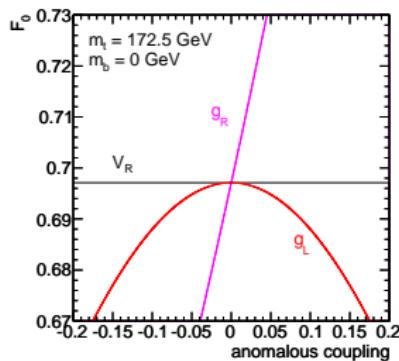
$$\begin{aligned} \Gamma = & \frac{g^2 |\vec{q}|}{32\pi} \frac{m_t^2}{M_W^2} \left\{ \left[|V_L|^2 + |V_R|^2 \right] \left(1 + x_W^2 - 2x_b^2 - 2x_W^4 + x_W^2 x_b^2 + x_b^4 \right) \right. \\ & - 12x_W^2 x_b \operatorname{Re} V_L V_R^* + 2 \left[|g_L|^2 + |g_R|^2 \right] \left(1 - \frac{x_W^2}{2} - 2x_b^2 - \frac{x_W^4}{2} - \frac{x_W^2 x_b^2}{2} + x_b^4 \right) \\ & - 12x_W^2 x_b \operatorname{Re} g_L g_R^* - 6x_W \operatorname{Re} [V_L g_R^* + V_R g_L^*] \left(1 - x_W^2 - x_b^2 \right) \\ & \left. + 6x_W x_b \operatorname{Re} [V_L g_L^* + V_R g_R^*] \left(1 + x_W^2 - x_b^2 \right) \right\}. \end{aligned} \quad (4)$$

The Wtb vertex structure

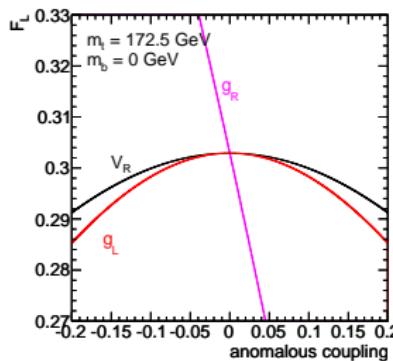
[EPJC50 (2007) 519]

anomalous couplings \Rightarrow deviations in W helicity fractions

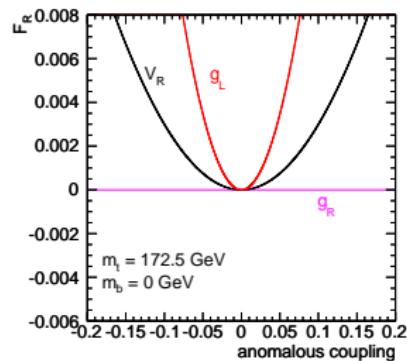
F_0



F_L



F_R

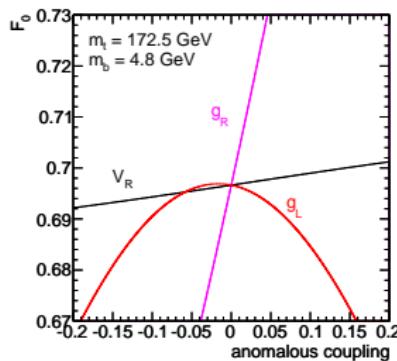


The Wtb vertex structure

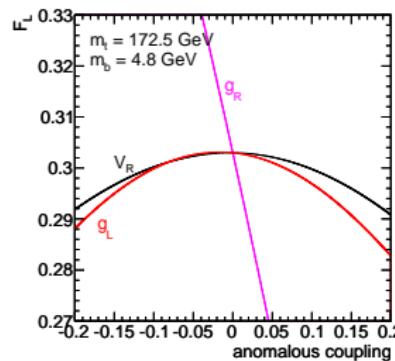
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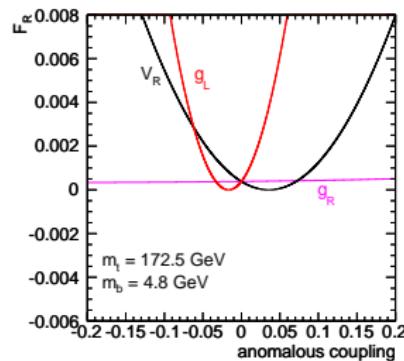
F_0



F_L

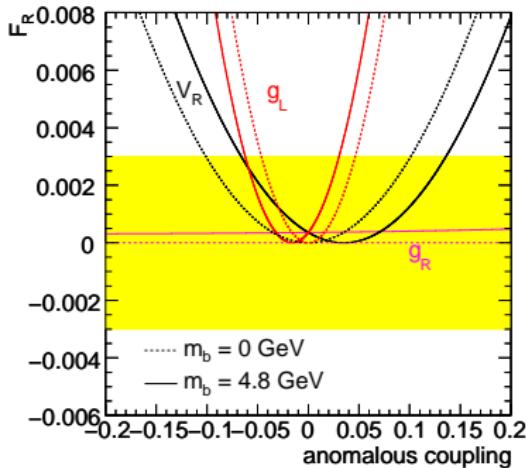


F_R



☞ correct m_b has to be considered!

The Wtb vertex structure

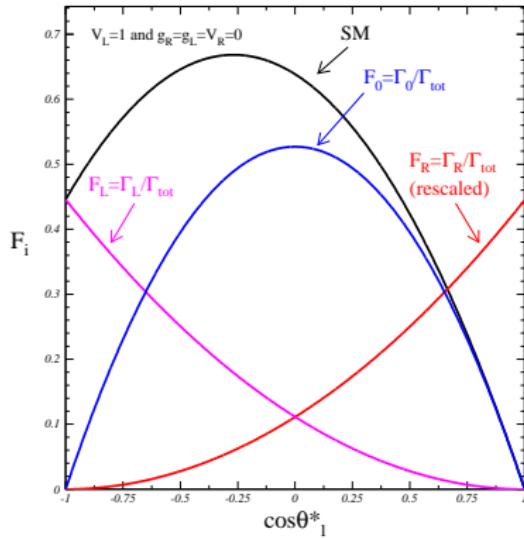
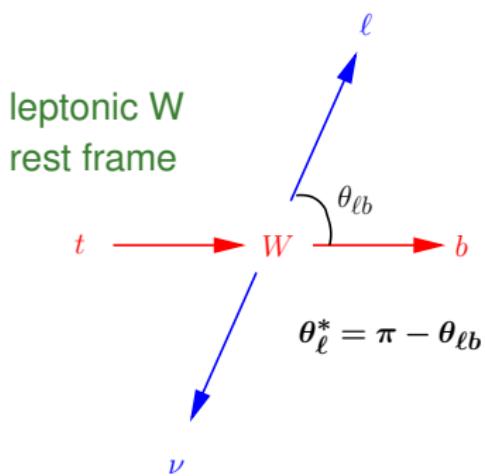


example: $|F_R| < 0.003$ can be converted into a V_R constraint using the intersection method:

- 👉 $-0.101 < V_R < 0.101$ ($m_b = 0.0$ GeV)
- 👉 $-0.067 < V_R < 0.136$ ($m_b = 4.8$ GeV)

Measuring the W helicity states

$$\frac{1}{N} \frac{dN}{d \cos \theta_\ell^*} = \frac{3}{2} \left[F_0 \left(\frac{\sin \theta_\ell^*}{\sqrt{2}} \right)^2 + F_L \left(\frac{1 - \cos \theta_\ell^*}{2} \right)^2 + F_R \left(\frac{1 + \cos \theta_\ell^*}{2} \right)^2 \right]$$



Measuring the W helicity states

W polarisation can be measured by:

- ➊ Fitting $\cos \theta_\ell^*$ to obtain the W helicity fractions (F_0, F_L, F_R)
- ➋ Fitting $\cos \theta_\ell^*$ to obtain the W helicity ratios:
 - ↳ $\rho_L = F_L/F_0 = 0.435$ (SM, LO)
 - ↳ $\rho_R = F_R/F_0 = 5.5 \times 10^{-4}$ (SM, LO)
- ➌ Computing angular asymmetries: $A_t = \frac{N(\cos \theta_\ell^* > t) - N(\cos \theta_\ell^* < t)}{N(\cos \theta_\ell^* > t) + N(\cos \theta_\ell^* < t)}$

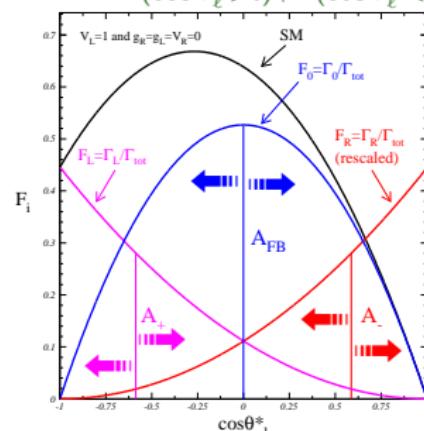
$$A_{FB} = 3/4[F_R - F_L]$$
$$= -0.2227 \text{ (SM, LO)}$$

$$A_+ = 3\beta[F_0 + (1 + \beta)F_R]$$
$$= 0.5436 \text{ (SM, LO)}$$

$$A_- = -3\beta[F_0 + (1 + \beta)F_L]$$
$$= -0.8409 \text{ (SM, LO)}$$

$$(\beta = 2^{1/3} - 1)$$

[EPJC50 (2007) 519]



The Wtb vertex structure

- [arXiv:hep-ph0605190v2 18 Mar 2007]
- How do $\rho_{L,R}$ behave?

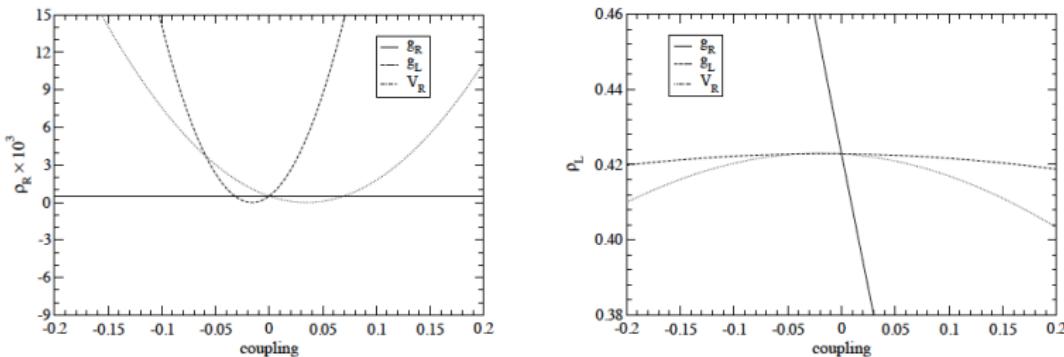


Figure 3: Dependence of the helicity ratios $\rho_{R,L} = \Gamma_{R,L}/\Gamma_0$ on the anomalous couplings in Eq. (1), in the CP-conserving case.

The Wtb vertex structure

- [arXiv:hep-ph0605190v2 18 Mar 2007]
- How do A_{FB} , A_+ and A_- behave?

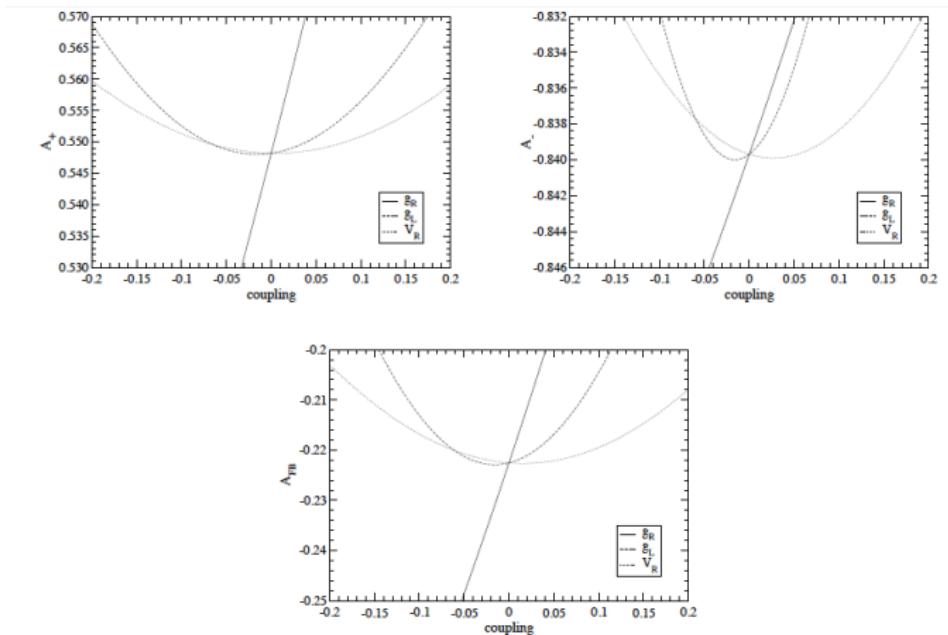


Figure 4: Dependence of the asymmetries A_+ , A_- and A_{FB} on the couplings g_L , g_R and V_R , for the CP-conserving case.

The Wtb vertex structure

- [arXiv:hep-ph0605190v2 18 Mar 2007]
- A rough comparison between results

	F_i	ρ_i
V_R	[-0.062, 0.13]	[-0.029, 0.099]
g_L	[-0.060, 0.028]	[-0.046, 0.013]
g_R	[-0.023, 0.021]	[-0.025, 0.026]

Table 1: 1σ bounds of anomalous couplings obtained from the measurement of helicity fractions F_i and ratios ρ_i .

	A_+	A_-	A_{FB}
V_R	[-0.15, 0.15]	[-0.056, 0.11]	[-0.12, 0.15]
g_L	[-0.12, 0.082]	[-0.057, 0.026]	[-0.092, 0.062]
g_R	[-0.019, 0.018]	[-0.024, 0.022]	[-0.027, 0.025]

Table 2: 1σ bounds on anomalous couplings obtained from the measurement of angular asymmetries.

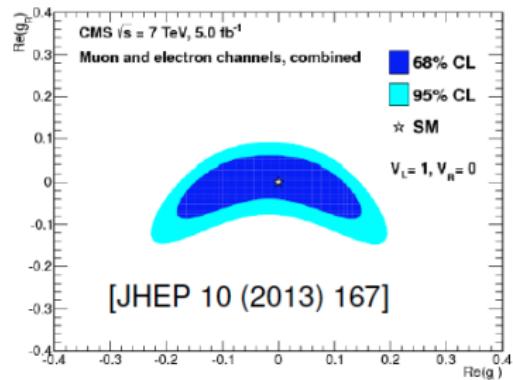
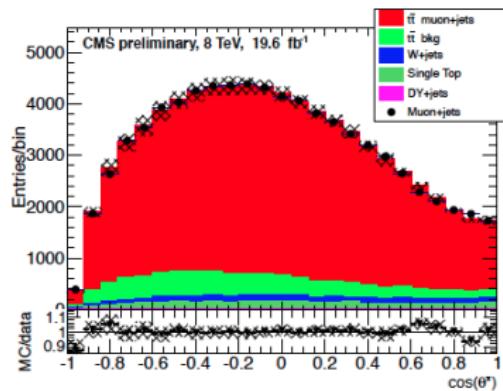
The Wtb vertex structure

W polarization in $t \rightarrow bW$ decays

	CMS [CMS PAS TOP-13-008]	ATLAS [JHEP 1206 (2012) 088]
F_0	$0.659 \pm 0.015(\text{stat}) \pm 0.023(\text{syst})$	$0.67 \pm 0.03(\text{stat}) \pm 0.06(\text{syst})$
F_L	$0.350 \pm 0.010(\text{stat}) \pm 0.024(\text{syst})$	$0.32 \pm 0.02(\text{stat}) \pm 0.03(\text{syst})$
F_R	$-0.009 \pm 0.006(\text{stat}) \pm 0.020(\text{syst})$	$0.01 \pm 0.01(\text{stat}) \pm 0.04(\text{syst})$

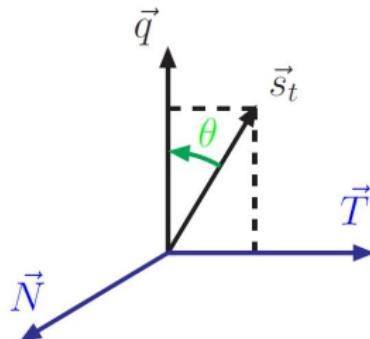
dominant uncertainties: $t\bar{t}$ modelling (ATLAS+CMS), JES and template stat. (ATLAS)

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \bar{b} \gamma^\mu (V_L P_L + V_R P_R) t W_\mu^- - \frac{g}{\sqrt{2}} \bar{b} \frac{i \sigma^{\mu\nu} q_\nu}{M_W} (g_L P_L + g_R P_R) t W_\mu^-$$



W polarisation beyond helicity fractions

- New idea to study top decays: [NPB840 (2010) 349]
 - ➡ consider transverse and normal directions



\vec{q} → W mom in t rest frame
 \vec{s}_t → top spin

$$\vec{N} = \vec{s}_t \times \vec{q}$$

$$\vec{T} = \vec{q} \times \vec{N}$$

meaningful for polarised t decays
(e.g. in single top production)

- | | |
|---|---|
| θ_ℓ^*
θ_ℓ^T
θ_ℓ^N | <ul style="list-style-type: none"> → angle between ℓ, \vec{q}
determine F_+, F_0, F_- → angle between ℓ, \vec{T}
determine F_+^T, F_0^T, F_-^T → angle between ℓ, \vec{N}
determine F_+^N, F_0^N, F_-^N |
|---|---|

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_\ell^X} = \frac{3}{8} (1 + \cos \theta_\ell^X)^2 \textcolor{red}{F_+^X} + \frac{3}{8} (1 - \cos \theta_\ell^X)^2 \textcolor{red}{F_-^X} + \frac{3}{4} \sin^2 \theta_\ell^X \textcolor{red}{F_0^X}$$

$$A_{\text{FB}}^N = \frac{3}{4} [F_+^N - F_-^N]$$

$$A_{\text{FB}}^N \simeq 0.64 P \text{Im } g_R$$

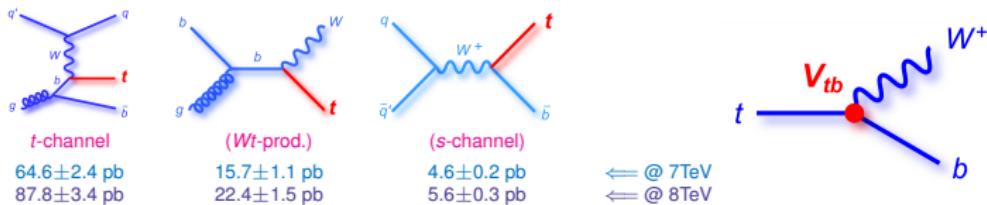
Single top quark production

Single top quark production

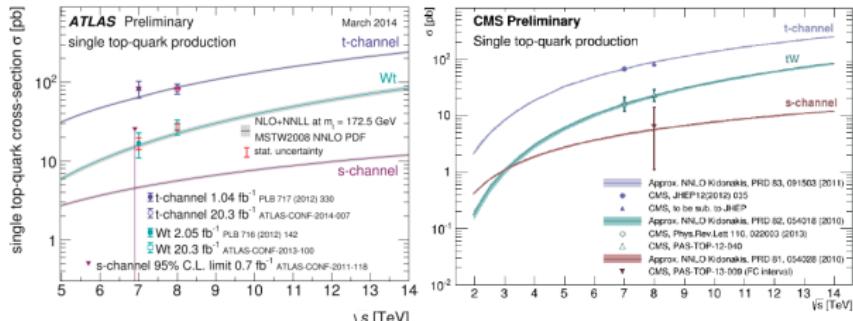
- Production Cross section (several channels)

V_{tb} @ LHC

- Single top quark cross section @ LHC:

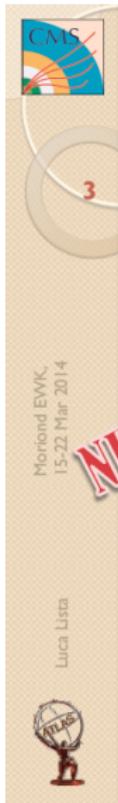


- Powerfull probe of V_{tb} ($\delta V_{tb} / V_{tb}$ few % @ LHC) and Test of physics BSM (FCNC in t-channel; W' in s-channel)
- CMS and ATLAS results within SM expectations:



Single top quark production

- Dominant production @ LHC

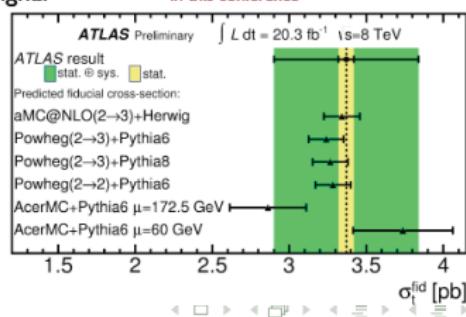
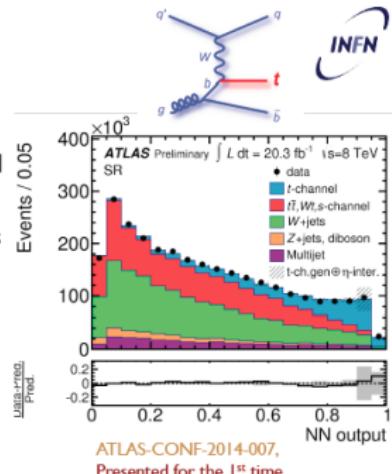


t channel: ATLAS

- ATLAS: new update with 20 fb^{-1} at 8 TeV, NN analysis using 14 discriminating variables
- Signal selection: one e or μ , 2jets-1tag events
- Multijet background rate determined from data (ME_T fit)
- Fiducial cross-section determined within detector acceptance:
 - $\sigma_{t\text{-ch. fid.}} = 3.37 \pm 0.05 \text{ (stat)} \pm 0.47 \text{ (syst)} \pm 0.09 \text{ (lumi) pb}$
- Largest systematics: jet energy scale, signal generator (ACERMC vs aMC@NLO)

Fiducial volume:

Object	Cut
Electrons	$p_T > 25 \text{ GeV}$ and $ \eta < 2.5$
Muons	$p_T > 25 \text{ GeV}$ and $ \eta < 2.5$
Jets	$p_T > 30 \text{ GeV}$ and $ \eta < 4.5$
Lepton (ℓ), Jets (j_1)	$p_T > 35 \text{ GeV}$, if $2.75 < \eta < 3.5$ $\Delta R(\ell, j_1) > 0.4$
E_T^{miss}	$E_T^{\text{miss}} > 30 \text{ GeV}$
Transverse W-boson mass	$m_T(W) > 50 \text{ GeV}$
Lepton (ℓ), jet with the highest p_T (j_1)	$p_T(\ell) > 40 \text{ GeV} \left(1 - \frac{\sigma_{j_1}(j_1, \ell)}{\pi}\right)$

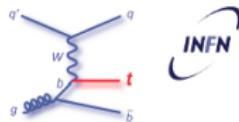


Single top quark production

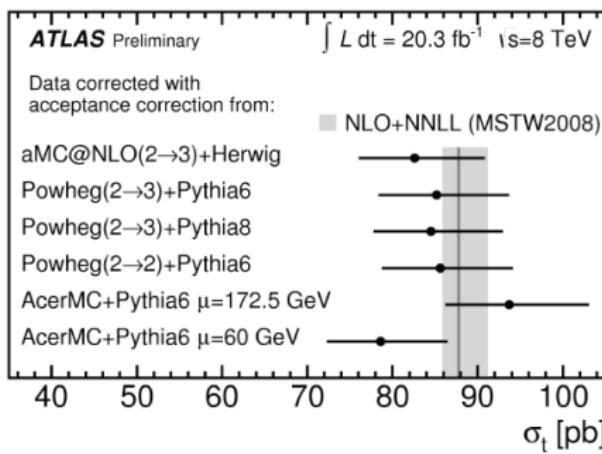
- Dominant production @ LHC



t channel: ATLAS



- Extrapolated to the entire phase space using various generator assumptions
- Assuming aMC@NLO + Herwig:
 - $\sigma_{t\text{-ch.}} = 82.6 \pm 1.2(\text{stat}) \pm 11.4(\text{syst}) \pm 3.1(\text{PDF}) \pm 2.3(\text{lumi}) \text{ pb}$



Single top quark production

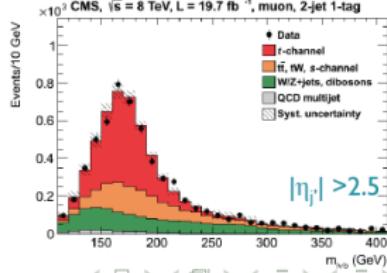
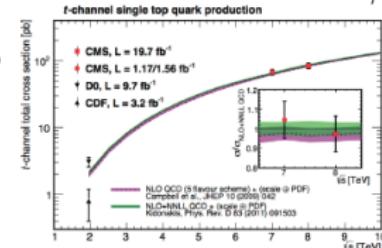
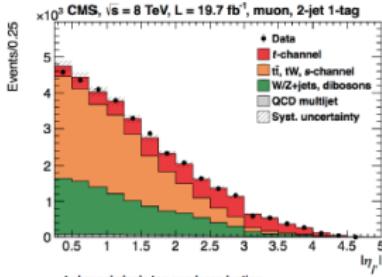
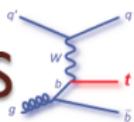
● Dominant production @ LHC



t channel: CMS

- CMS: updated 8 TeV analysis with the entire dataset
- Signal region: one e or μ , 2jets-1 tag events, reconstr. top mass window: $130 < m_{\text{lb}} < 220$ GeV
- Shapes for W+jets and $t\bar{t}$ are determined from control regions in data (m_{lb} SB, 3jet-2tag)
- Signal, W+jets, $t\bar{t}$ yields determined from a fit to the $|\eta_j|$ distribution
 - $\sigma_{t\text{-ch}} = 83.6 \pm 2.3(\text{stat}) \pm 7.4(\text{syst})$ pb
 - $R_{8/7} = 1.24 \pm 0.08(\text{stat}) \pm 0.12(\text{syst})$
- Largest uncertainty: signal modeling (POWHEG vs COMPHEP), jet energy scale
- LHC Combination: (TOPLHCWG, using BLUE), using preliminary result (CMS PAS TOP-12-011, ATLAS-CONF-2012-132):
 - $\sigma_{t\text{-ch}} = 85 \pm 6(\text{stat}) \pm 11(\text{syst}) \pm 3(\text{lumi})$ pb
 $= 85 \pm 12$ pb
- To be updated with latest results!

CMS PAS TOP-12-011 → to be sub. to JHEP
LHC combination:
CMS PAS TOP-12-002/ATLAS-CONF-2013-061



Single top quark production

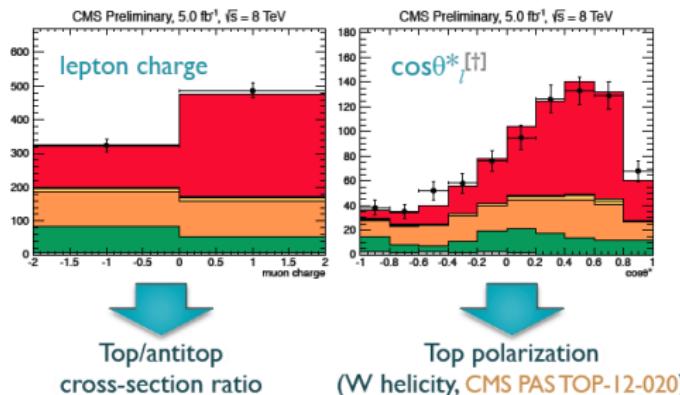
- Dominant production @ LHC



t channel: distributions



- The t-channel data sample is large enough to study distributions
 - → differential cross sections
- Signal can be enhanced by requiring e.g.: large forward jet pseudorapidity: $|\eta_j| > 2.0$



$[\dagger] \theta_W^* = \text{angle between lepton in } W \text{ rest frame and the } W \text{ in top rest frame.}$

**What can single top
production say about the Wtb
vertex structure beyond V_{tb} ?**

Single top quark production



Moniord EVVK,
15-22 Mar 2014

Luca Lista



$|V_{tb}|$ from single top

- The $|V_{tb}|$ measurement in single-top events provides a unique opportunity to directly probe the top production Wtb vertex: $|V_{tb}| = (\sigma/\sigma^{\text{th}}(|V_{tb}|=1))^{1/2}$, assuming $|V_{tb}| \gg |V_{ts}|, |V_{td}|$ or equivalently $B(t \rightarrow Wb) = 1$
 - Deviations from the SM are potentially sensitive to new physics
- Eight measurements in the t channel and in tW, the latter with less precision

• ATLAS:

- 7 TeV: $|V_{tb}| = 1.13^{+0.14}_{-0.13}$ (t-ch., 11.9%)
 $|V_{tb}| = 1.03^{+0.16}_{-0.19}$ (tW, 17.0%)
- 8 TeV: $|V_{tb}| = 0.97 \pm 0.01 \text{ (stat)}^{+0.06}_{-0.07} \text{ (syst)} \pm 0.6 \text{ (gen+PDF)}^{+0.02}_{-0.01} \text{ (th)} \pm 0.01 \text{ (lumi)}$
 $= 0.97^{+0.09}_{-0.10}$ (t-ch., 9.8%)
 $|V_{tb}| = 1.10 \pm 0.12 \text{ (exp)} \pm 0.03 \text{ (th)}$ (tW, 11.2%)

NEW

• CMS:

- 7 TeV: $|V_{tb}| = 1.020 \pm 0.046 \text{ (exp)} \pm 0.017 \text{ (th)}$ (t-ch. 4.8%)
 $|V_{tb}| = 1.01^{+0.16}_{-0.13} \text{ (exp)}^{+0.03}_{-0.04} \text{ (th)}$ (tW, 14.8%)
- 8 TeV: $|V_{tb}| = 0.979 \pm 0.045 \text{ (exp)} \pm 0.016 \text{ (th)}$ (t-ch. 4.9%)
 $|V_{tb}| = 1.03 \pm 0.12 \text{ (exp)} \pm 0.04 \text{ (th)}$ (tW 12.3%)

NEW



$$|V_{tb}| = 0.998 \pm 0.038 \text{ (exp)} \pm 0.016 \text{ (th)}$$

(7+8 TeV t-ch., comb.: 4.1%)

- Considering ATLAS+CMS combination with future updates

Single top quark production

● [arXiv:hep-ph/0605190v2 18 Mar 2007]

Single top quark production at LHC with anomalous Wtb couplings

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Abstract

We investigate single top production in the presence of anomalous Wtb couplings. We explicitly show that, if these couplings arise from gauge invariant effective operators, the only relevant couplings for single top production and decay are the usual γ^μ and $\sigma^{\mu\nu}q_\nu$ terms, where q is the W boson momentum. This happens even in the single top production processes where the Wtb interaction involves off-shell top and/or bottom quarks. With this parameterisation for the Wtb vertex, we obtain expressions for the dependence on anomalous couplings of the single top cross sections, for (i) the t -channel process, performing a matching between $t\bar{q}$ and $t\bar{b}j$ production, where j is a light jet; (ii) s -channel $t\bar{b}$ production; (iii) associated tW^- production, including the correction from tW^-b . We use these expressions to estimate, with a fast detector simulation, the simultaneous limits which the measurement of single top cross sections at LHC will set on V_{tb} and possible anomalous couplings. Finally, a combination with top decay asymmetries and angular distributions is performed, showing how the limits can be improved when the latter are included in a global fit to Wtb couplings.

Single top quark production

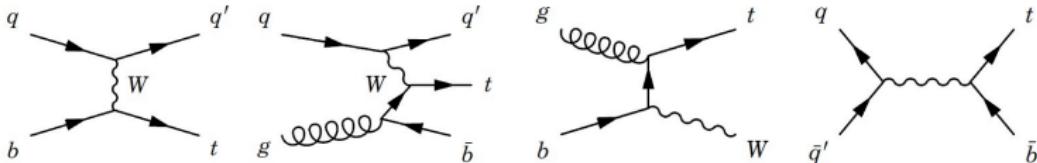
• [arXiv:hep-ph/0605190v2 18 Mar 2007]

New physics beyond the Standard Model (SM) is expected to affect especially the top quark, and, in particular, it may modify its charged current interaction with its $SU(2)_L$ partner the bottom quark. For on-shell t , b and W , the most general Wtb vertex involving terms up to dimension five can be written as [5]

$$\begin{aligned}\mathcal{L}_{Wtb}^{\text{OS}} = & -\frac{g}{\sqrt{2}} \bar{b} \gamma^\mu (V_L P_L + V_R P_R) t W_\mu^- \\ & -\frac{g}{\sqrt{2}} \bar{b} \frac{i \sigma^{\mu\nu} q_\nu}{M_W} (g_L P_L + g_R P_R) t W_\mu^- + \text{H.c.},\end{aligned}\quad (1)$$

with $q \equiv p_t - p_b$ (being p_t and p_b the momenta of the top and b quark, respectively, following the fermion flow), which equals the W boson momentum. Additional $\sigma^{\mu\nu} k_\nu$ and k^μ terms, where $k \equiv p_t + p_b$, can be absorbed into this Lagrangian using Gordon identities. If the W boson is on its mass shell or it couples to massless external fermions we have $q^\mu \epsilon_\mu = 0$, where ϵ_μ is the polarisation vector of the W boson, so that terms proportional to q^μ can be dropped from the effective vertex. Within the SM, the only Wtb interaction term at the tree level is given by the left-handed γ^μ term, with $V_L \equiv V_{tb} \simeq 1$. The rest of couplings are called “anomalous” and vanish at the tree level, although they can be generated by radiative corrections. They are not necessarily constants but rather “form factors”, usually approximated by the constant term (as we will do in this work). If we assume that CP is conserved in the Wtb interaction then $V_{L,R}$ and $g_{L,R}$ are real, and V_L can be taken to be positive without loss of generality.

Single top quark production



$$\sigma = \sigma_{\text{SM}} (V_L^2 + \kappa^{V_R} V_R^2 + \kappa^{V_L V_R} V_L V_R + \kappa^{g_L} g_L^2 + \kappa^{g_R} g_R^2 + \kappa^{g_L g_R} g_L g_R + \dots)$$

- the κ factors determine the dependence on anomalous couplings
- the κ factors are, in general, different for t and \bar{t} production
- the measurement of the single top production cross-section allows to obtain a measurement of V_L ($\equiv V_{tb}$) and bounds on anomalous couplings

Single top quark production

• t-channel

• [arXiv:hep-ph/0605190v2 18 Mar 2007]

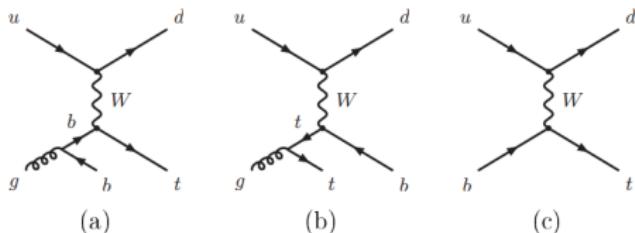


Figure 2: Sample Feynman diagrams for single top production in the t -channel process. Additional diagrams are obtained by crossing the light quark fermion line, and/or replacing (u, d) by (c, s) . The diagrams for antitop production are the charge conjugate ones.

Single top quark production

• t-channel

[arXiv:hep-ph/0605190v2 18 Mar 2007]

	tj				$\bar{t}j$			
	κ	ΔQ	Δm_t	Δm_b	κ	ΔQ	Δm_t	Δm_b
V_R^2	$0.916 - 0.923$	$+0.$ $-0.$	$+0.$ $-0.$	$+0.$ $-0.$	$1.082 - 1.084$	$+0.$ $-0.$	$+0.$ $-0.$	$+0.$ $-0.$
g_L^2	$1.75 - 1.79$	$+0.044$ -0.038	$+0.007$ -0.035	$+0$ -0.027	$2.16 - 2.17$	$+0.035$ -0.022	$+0.014$ -0.032	$+0.$ $-0.$
g_R^2	2.18	$+0.042$ -0.033	$+0.014$ -0.034	$+0.$ -0.022	$1.75 - 1.77$	$+0.042$ -0.033	$+0.007$ -0.033	$+0.$ -0.025
$V_L g_R$	$-(0.348 - 0.365)$	$+0.007$ -0.011	$+0.$ $-0.$	$+0.$ $-0.$	$-(0.038 - 0.040)$	$+0.010$ -0.009	$+0.$ $-0.$	$+0.$ $-0.$
$V_R g_L$	$-(0.006 - 0.008)$	$+0.006$ -0.005	$+0.$ $-0.$	$+0.$ $-0.$	$-(0.399 - 0.408)$	$+0.$ -0.008	$+0.$ $-0.$	$+0.$ $-0.$

Table 1: Representative κ factors for the tj and $\bar{t}j$ processes and their uncertainties, explained in the text. Errors smaller than 0.005 are omitted.

	$t\bar{b}j$				$\bar{t}bj$			
	κ	ΔQ	Δm_t	Δm_b	κ	ΔQ	Δm_t	Δm_b
V_R^2	$0.927 - 0.932$	$+0.005$ $-0.$	$+0.$ $-0.$	$+0.$ $-0.$	$1.068 - 1.069$	$+0.$ -0.005	$+0.$ $-0.$	$+0.$ $-0.$
$V_L V_R$	-0.117	$+0.$ $-0.$	$+0.$ $-0.$	$+0.005$ -0.005	-0.126	$+0.$ $-0.$	$+0.$ $-0.$	$+0.006$ -0.006
g_L^2	$1.96 - 2.01$	$+0.070$ -0.056	$+0.005$ -0.005	$+0.$ $-0.$	$2.98 - 3.00$	$+0.040$ -0.040	$+0.014$ -0.014	$+0.$ $-0.$
g_R^2	$2.97 - 2.98$	$+0.056$ -0.043	$+0.013$ -0.013	$+0.$ $-0.$	$2.08 - 2.11$	$+0.056$ -0.045	$+0.006$ -0.007	$+0.$ $-0.$
$V_L g_R$	$-(0.539 - 0.550)$	$+0.012$ -0.010	$+0.$ $-0.$	$+0.$ $-0.$	$-(0.169 - 0.172)$	$+0.010$ -0.010	$+0.014$ -0.013	$+0.$ $-0.$
$V_R g_L$	$-(0.121 - 0.134)$	$+0.009$ -0.011	$+0.$ $-0.$	$+0.$ $-0.$	$-(0.567 - 0.571)$	$+0.014$ -0.013	$+0.$ $-0.$	$+0.$ $-0.$

Table 2: Representative κ factors for the $t\bar{b}j$ and $\bar{t}bj$ processes and their uncertainties, explained in the text. Errors smaller than 0.005 are omitted.

- (ii) The coefficient of the V_R^2 term is different for single top and single antitop production, but the differences cancel to a large extent in the total cross section. This property makes the ratio $R(\bar{t}/t) = \sigma(\bar{t})/\sigma(t)$ more sensitive to a V_R component than the total cross section itself. A purely left-handed interaction yields

Single top quark production

- **tW associated production**
- [arXiv:hep-ph/0605190v2 18 Mar 2007]

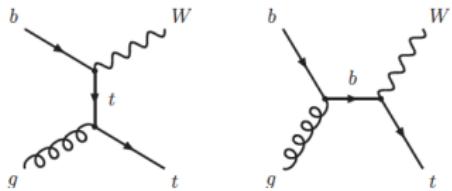


Figure 5: Feynman diagrams for single top production in the $gb \rightarrow tW^-$ process.

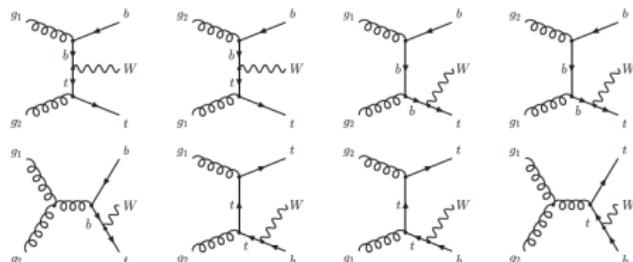
Single top quark production

• tW associated prod.

[arXiv:hep-ph/0605190v2 18 Mar 2007]

	κ	ΔQ	Δm_t	Δm_b
V_R^2	1	—	—	—
g_L^2, g_R^2	$3.46 - 3.57$	$+0.23$ -0.11	$+0.015$ -0.015	$+0.009$ -0.008
$V_L g_R, V_R g_L$	1	—	—	—

Table 7: Representative κ factors for the tW^- and $\bar{t}W^+$ processes and their uncertainties, explained in the text. Errors smaller than 0.005 are omitted.

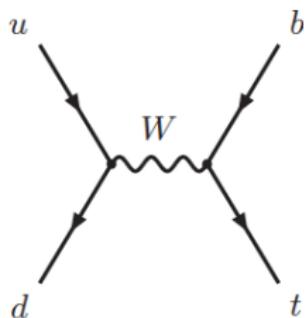


	κ	ΔQ	Δm_t	Δm_b
V_R^2	1	—	—	—
g_L^2, g_R^2	$4.51 - 4.73$	$+0.19$ -0.04	$+0.009$ -0.027	$+0.030$ $-0.$
$V_L g_R, V_R g_L$	$1.21 - 1.23$	$+0.014$ -0.003	$+0.005$ -0.007	$+0.$ $-0.$

Table 8: Representative κ factors for the tW^-b and $\bar{t}W^+b$ processes and their uncertainties, explained in the text. Errors smaller than 0.005 are omitted.

Single top quark production

- **s-channel**



Single top quark production

• s-channel

[arXiv:hep-ph/0605190v2 18 Mar 2007]

$t\bar{b}$				$\bar{t}b$					
	κ	ΔQ	Δm_t	Δm_b		κ	ΔQ	Δm_t	Δm_b
V_R^2	1	—	—	—		1	—	—	—
$V_L V_R$	0.121	+0. −0.	+0. −0.	+0.005 −0.005		0.127	+0. −0.	+0. −0.	+0.006 −0.006
g_L^2, g_R^2	13.06 – 13.10	+0.25 −0.21	+0.26 −0.26	+0. −0.	12.22 – 12.28	+0.21 −0.18	+0.25 −0.24	+0. −0.	
$g_L g_R$	1.23	+0.007 −0.008	+0.012 −0.012	+0.055 −0.055		1.25	+0.008 −0.009	+0.013 −0.013	+0.056 −0.056
$V_L g_L, V_R g_R$	−0.415	+0. −0.	+0. −0.	+0.018 −0.018		−0.426	+0. −0.	+0. −0.	+0.019 −0.019
$V_L g_R, V_R g_L$	−5.51	+0.009 −0.010	+0.057 −0.057	+0. −0.		−5.48	+0.008 −0.010	+0.057 −0.056	+0. −0.

Table 5: κ factors for the $t\bar{b}$ and $\bar{t}b$ processes and their uncertainties, explained in the text. Errors smaller than 0.005 are omitted.

- The κ factors of g_L^2 and g_R^2 are a factor of four larger than for the t -channel process, because in $t\bar{b}$ production the s -channel W boson carries a larger momentum, and so the q_ν factor in the $\sigma^{\mu\nu}$ vertex gives a larger enhancement.
- For $t\bar{b}$ and $\bar{t}b$ production the factors are very similar, although not equal (the difference is not due to Monte Carlo statistics, which is very high). Then, the measurement of the ratio $\sigma(\bar{t}b)/\sigma(t\bar{b})$ is not as useful as in the t -channel process.
- Interferences among couplings are again important, in particular between V_L and g_R , and between V_R and g_L .

Constraints on anomalous couplings

● Limits from single top

[arXiv:hep-ph/0605190v2 18 Mar 2007]

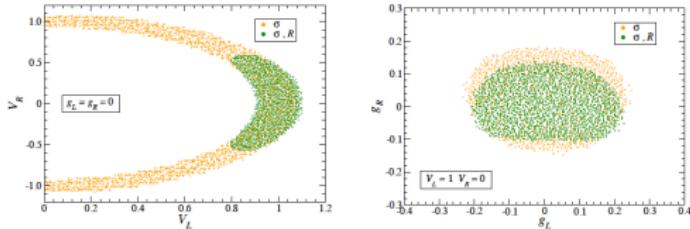


Figure 9: Estimated two-dimensional limits (with 68.3% CL) on (V_L, V_R) and (g_L, g_R) , obtained from measurement of single top cross sections, with and without the ratio $R(\bar{t}/t)$ for the tj final state.

● Using $t\bar{t}$ observables

[arXiv:hep-ph/0605190v2 18 Mar 2007]

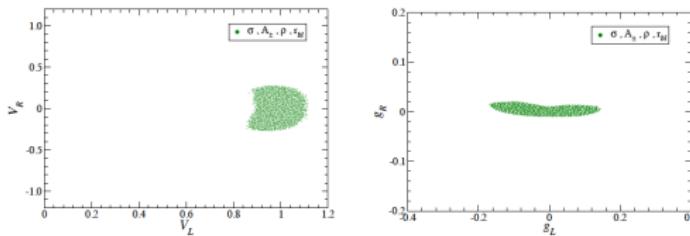


Figure 11: Combined limits on Wtb couplings from single top cross section measurements (excluding $R(\bar{t}/t)$) and top decay observables $A_{\pm}, \rho_{R,L}, r_{bl}$. The two graphs correspond to different projections of the 4-dimensional allowed region (with 68.3% CL).

**A first publication to show
what happens...**

Phys. Rev. D 83, 117301 (2011)

Constraining Wtb anomalous couplings: TopFit

- Constraints on Wtb vertex:

- combine the information of the most sensitive observables (taking into account the correlations)
- evaluate 95% CL allowed regions considering the dependence of these observables with V_R , g_L and g_R

☞ this is the purpose of



<http://www-ftae.ugr.es/topfit>

Observables from LHC

- Top decay (in $t\bar{t}$ events): angular asymmetries

☞ ATLAS Collaboration [ATLAS-CONF-2011-037]:

$$A_+ = 0.50 \pm 0.10 \text{ (stat)} \pm 0.06 \text{ (syst)} \quad (e)$$

$$A_- = -0.85 \pm 0.07 \text{ (stat)} \pm 0.05 \text{ (syst)} \quad (e)$$

$$A_+ = 0.50 \pm 0.08 \text{ (stat)} \pm 0.04 \text{ (syst)} \quad (\mu)$$

$$A_- = -0.87 \pm 0.04 \text{ (stat)} \pm 0.03 \text{ (syst)} \quad (\mu)$$

with $\rho(A_+, A_-) = 0.16$, assuming $m_t = 172.5 \text{ GeV}$

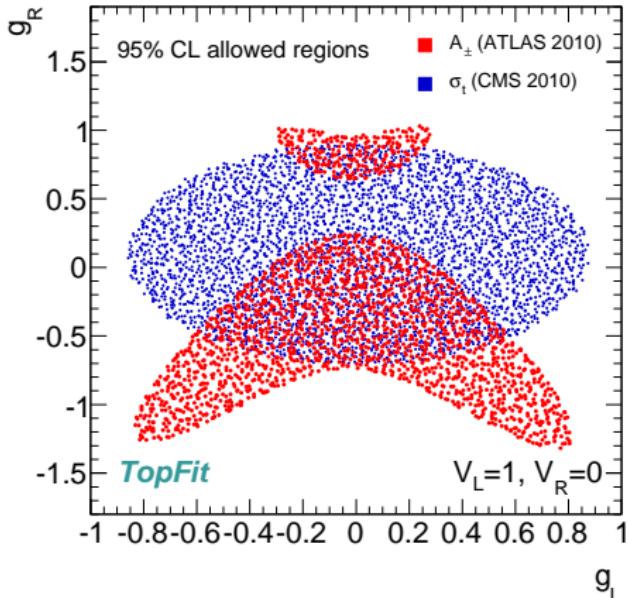
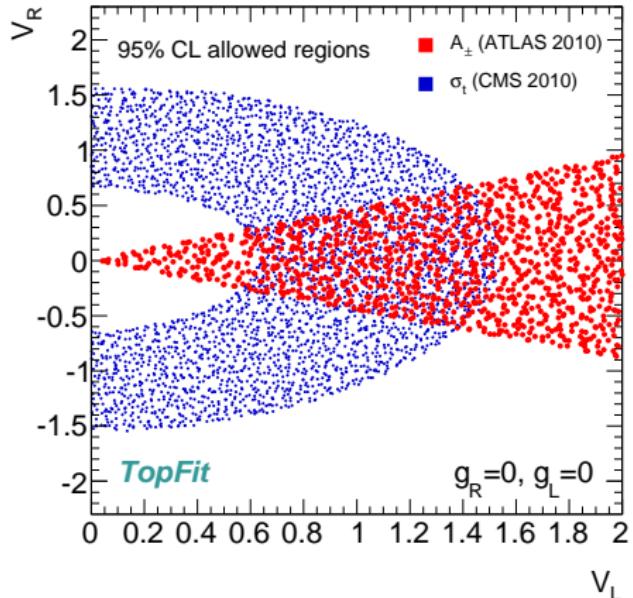
- Single top production [CMS-PAS-TOP-10-008]:

☞ CMS Collaboration:

$$\sigma_t = 83.6 \pm 30.0 \text{ pb}$$

assuming $m_t = 172.5 \text{ GeV}$

Constraints on the Wtb vertex from early LHC data



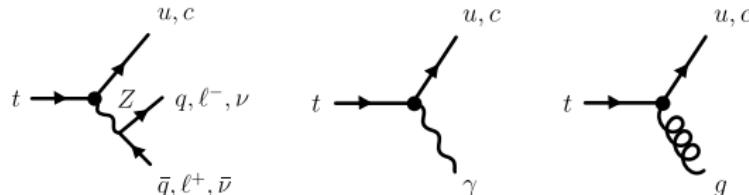
(anomalous couplings assumed to be real)

Top quarks and Flavor Changing Neutral Currents (FCNC)

FCNC @ decay

FCNC decays ($t \rightarrow qX$, $X = \gamma, Z$)

- Several $t\bar{t}$ FCNC Decay Channels Studied @ LHC:



Theoretical predictions for the BR of FCNC top quark decays

Process	SM	QS	2HDM	FC 2HDM	MSSM	R SUSY	TC2	RS
$t \rightarrow u\gamma$	3.7×10^{-16}	7.5×10^{-9}	—	—	2×10^{-6}	1×10^{-6}	—	$\sim 10^{-11}$
$t \rightarrow uZ$	8×10^{-17}	1.1×10^{-4}	—	—	2×10^{-6}	3×10^{-5}	—	$\sim 10^{-9}$
$t \rightarrow ug$	3.7×10^{-14}	1.5×10^{-7}	—	—	8×10^{-5}	2×10^{-4}	—	$\sim 10^{-11}$
$t \rightarrow c\gamma$	4.6×10^{-14}	7.5×10^{-9}	$\sim 10^{-6}$	$\sim 10^{-9}$	2×10^{-6}	1×10^{-6}	$\sim 10^{-6}$	$\sim 10^{-9}$
$t \rightarrow cZ$	1×10^{-14}	1.1×10^{-4}	$\sim 10^{-7}$	$\sim 10^{-10}$	2×10^{-6}	3×10^{-5}	$\sim 10^{-4}$	$\sim 10^{-5}$
$t \rightarrow cg$	4.6×10^{-12}	1.5×10^{-7}	$\sim 10^{-4}$	$\sim 10^{-8}$	8×10^{-5}	2×10^{-4}	$\sim 10^{-4}$	$\sim 10^{-9}$

Acta Phys. Polon. B35 (2004) 2695

- In the SM flavour changing neutral currents (FCNC) are forbidden at tree level and **much smaller** than the dominant decay mode ($t \rightarrow bW$) at one loop level
- BSM models predict **higher BR** for top FCNC decays
☞ powerful probe for new physics



New physics in single top

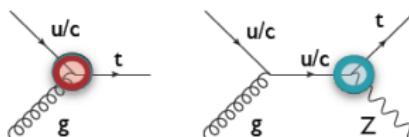


- FCNC in single-top production may arise from several new physics scenarios affecting both production ($u/c \rightarrow t$) and decay (e.g: $u/c \rightarrow tZ$, $t\gamma$, tg)

$$\mathcal{L} = \sum_{q=u,c} \left[\sqrt{2} g_s \frac{\kappa_{gqt}}{\Lambda} \bar{t} \sigma^{\mu\nu} T_a (f_q^L P_L + f_q^R P_R) q G_{\mu\nu}^a \right] \\ + \frac{g}{\sqrt{2} c_W} \frac{\kappa_{Zqt}}{\Lambda} \bar{t} \sigma^{\mu\nu} (\bar{f}_q^L P_L + \bar{f}_q^R P_R) q Z_{\mu\nu} \right] + h.c.$$

gut, gct

Zut, Zct



- ATLAS searches for FCNC in single top production with SM $t \rightarrow Wb$ decay ([ATLAS-CONF-2013-063, 8 TeV](#))
- CMS looked for FCNC in associated tZ production ([CMS PAS TOP-12-021, 7 TeV](#))
- ATLAS also looked for CP violation in the Wtb vertex using lepton angular distribution in single-top ([ATLAS-CONF-2013-032, 7 TeV](#))
- No deviation from SM prediction spotted so far

ATLAS:

$$\kappa_{\text{gut}}/\Lambda < 5.1 \times 10^{-3} \text{ TeV}^{-1}$$

$$\kappa_{\text{gct}}/\Lambda < 1.1 \times 10^{-2} \text{ TeV}^{-1}$$

CMS:

$$\kappa_{\text{Zut}}/\Lambda < 0.45 \text{ TeV}^{-1}$$

$$\kappa_{\text{Zct}}/\Lambda < 2.27 \text{ TeV}^{-1}$$



$$B(t \rightarrow gu) < 3.1 \times 10^{-5}$$

$$B(t \rightarrow gc) < 1.6 \times 10^{-5}$$

$$B(t \rightarrow Zu) < 5.1 \times 10^{-3}$$

$$B(t \rightarrow Zc) < 0.1140 \\ (95\% \text{ CL})$$

$$A_{FB}^N = 0.031 \pm 0.065(\text{stat.})^{+0.029}_{-0.031}(\text{syst.})$$

anomalous tensor coupling:
 $-0.2 < \Im(g_R) < 0.3, 95\% \text{ CL}$

What about couplings to other bosons?

Couplings to other Bosons



$t\bar{t}$ + W, Z, γ

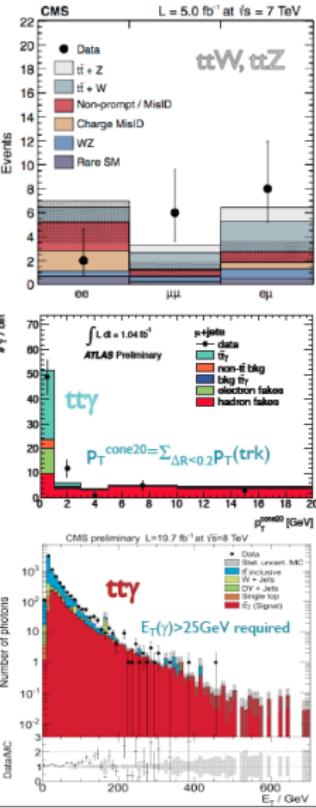
ttZ/W: CMS: Inclusive search for same-sign dilepton from ttV,
V=W, Z, exclusive trilepton search from ttZ:

- $\sigma(t\bar{t}V) = 0.43^{+0.17}_{-0.15}(\text{stat})^{+0.09}_{-0.07}(\text{syst}) \text{ pb (3.3}\sigma)$
SM: $0.306^{+0.031}_{-0.053} \text{ pb}$ (Garzelli et al.; JHEP11(2012)056)
- $\sigma(t\bar{t}Z) = 0.28^{+0.14}_{-0.11}(\text{stat})^{+0.06}_{-0.03}(\text{syst}) \text{ pb (3.3}\sigma)$
SM: $0.137^{+0.012}_{-0.016} \text{ pb}$ (Campbell, Ellis; JHEP07(2012)052)

ttγ: 1+jets used to detect tt pair; photon fake rate estimated from template fit of photon/ch. hadron isolation

- ATLAS, 7 TeV:
 $\sigma(t\bar{t}\gamma) = 2.0 \pm 0.5(\text{stat}) \pm 0.7(\text{syst}) \pm 0.1(\text{lumi}) \text{ pb}$
SM: $2.1 \pm 0.4 \text{ pb}, E_T(\gamma) > 8 \text{ GeV}$
(W. Kilian et al.: EPJC71(2011)1742)
- CMS, 8 TeV:
 $\sigma(t\bar{t}\gamma)/\sigma(t\bar{t}) = (1.07 \pm 0.07(\text{stat}) \pm 0.27(\text{syst})) \times 10^{-2}$
 $\rightarrow \sigma(t\bar{t}\gamma) = 2.4 \pm 0.2(\text{stat}) \pm 0.6(\text{syst}) \text{ pb}$
SM: $1.8 \pm 0.5 \text{ pb}, E_T(\gamma) > 20 \text{ GeV}, \Delta R(\gamma, b) > 0.1$
(K. Melnikov, et al., PRD83(2011)074013)
- Results compatible within uncertainties with NLO calculations

CMS PRL110.172002: ttV, 7 TeV
ATLAS-CONF-2011-153: ttγ, 7 TeV
CMS PASTOP-13-011: ttγ, 8 TeV



- Combination of production and decay observables is crucial to constrain the Wtb couplings
 - Should be done not only within a single experiment, but including all the available data from different experiments
- First publication done show limits improve significantly
 - Increase of collected luminosity at the LHC should allow to have stringent bounds on the Wtb vertex  rapidly become a precision physics field
- Global fit to the general complex Wtb vertex requires not only more data but also a complete set of observables (TopFit available to experiments \Rightarrow use it!)
- Couplings to bosons (γ, Z, W, H) is the next thing to do
- Studies @ LHC of FCNC processes both at decay and production are promising (already best results in the world)

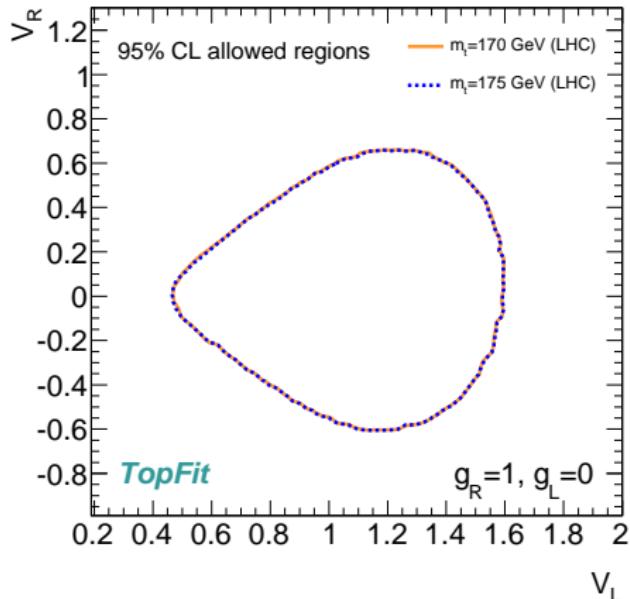


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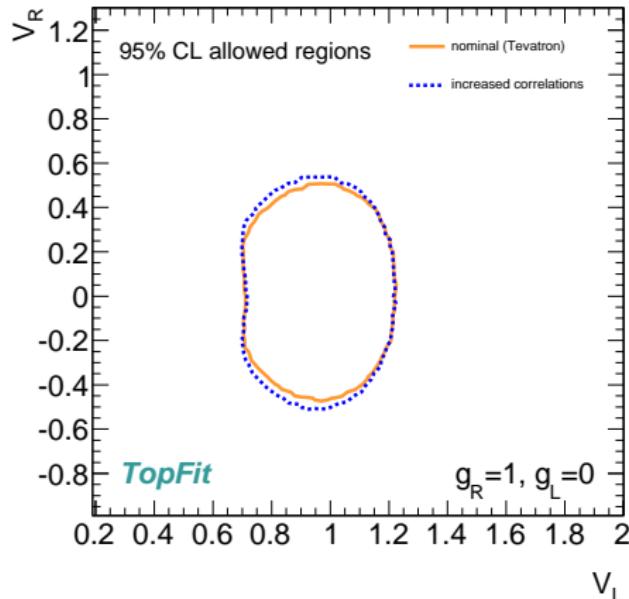
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Backup Slides

Constraints on the Wtb vertex: effect of m_t and correlations



different m_t were considered



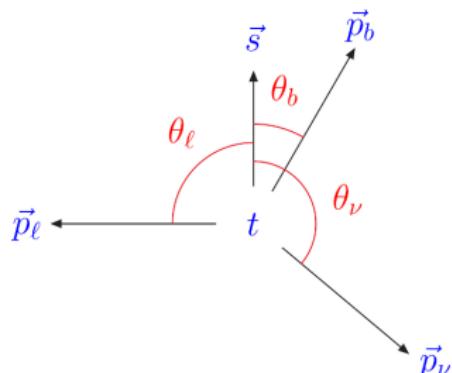
(increased correlations between W
helicity fractions were introduced)

Probing the Wtb vertex: spin asymmetries

- polarised top decays

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_X} = \frac{1 + \alpha_X \cos \theta_X}{2}$$

☞ α_X depends on the anomalous couplings



X = top decay product → \vec{p}_X = momentum in t rest frame
 \vec{p}_j = jet momentum in t rest frame

$$Q = \cos(\vec{p}_X, \vec{p}_j) \quad \rightarrow \quad \begin{aligned} A_X &\equiv \frac{N(Q > 0) - N(Q < 0)}{N(Q > 0) + N(Q < 0)} \\ &= \frac{1}{2} P \alpha_X \quad [P = 0.95 \text{ (}t\text{)} \quad P = -0.93 \text{ (}\bar{t}\text{)}] \end{aligned}$$

[PLB 476 (2000) 323]