Physics at LHC:
SUperSYmmetry

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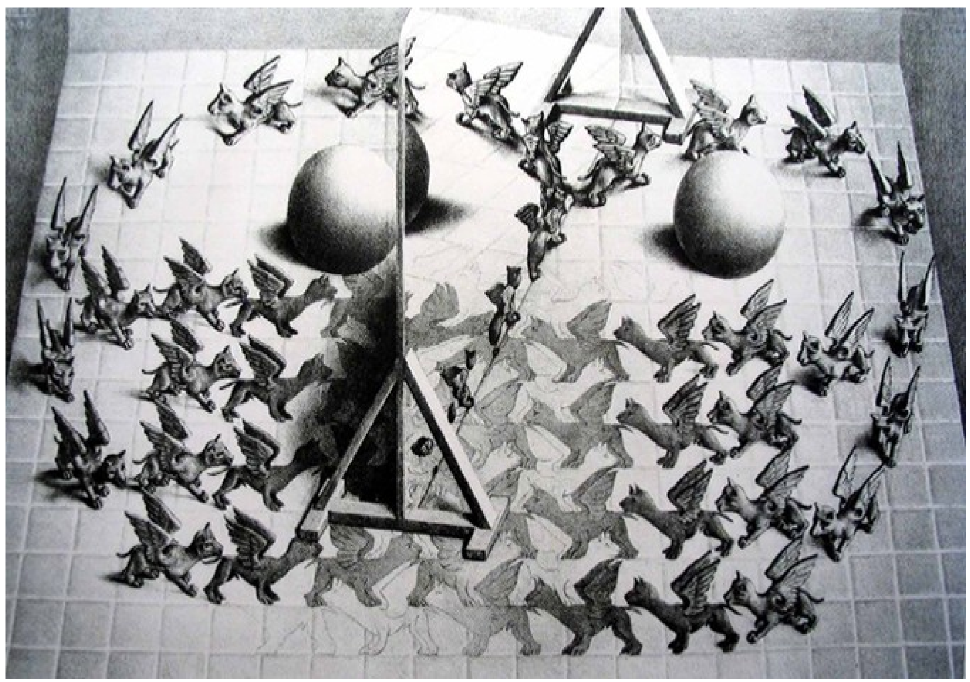
LIP



LIP 17/06/2013

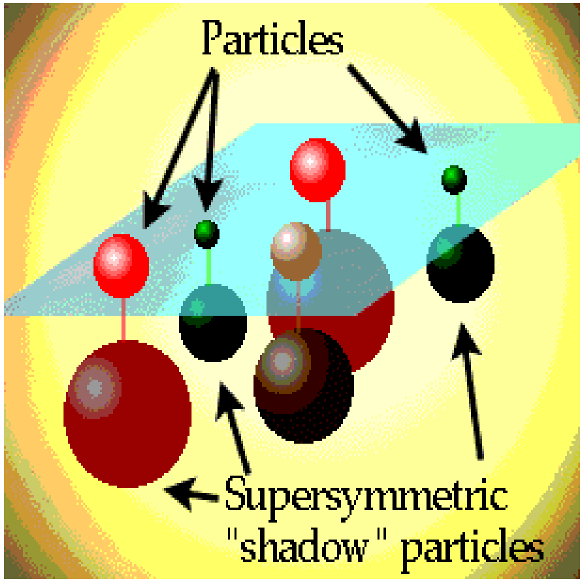
Outline

- *SUperSYmmetry: Brief introduction & Motivations*
- *Reminder of Standard Model (SM) Lagrangian*
- *SUSY phenomenology: Deeper look*
  - *“Constructing” the SUSY Lagrangian*
  - *Different sectors of MSSM:*
    - *Squark & Slepton*
    - *Chargino*
    - *Neutralino*
    - *Higgs*

Advised readings:

- “SUSY & Such” S. Dawson, arxiv:hep-ph/9612229v2
- “A supersymmetry primer” S. P. Martin, arxiv:hep-ph/9709356

Brief introduction & Motivations



Supersymmetry: Introduction words

"Generalize" the spin of known fields

SUperSYmmetry : spin particle ½ ↔ spin partner 0
spin particle 1 ↔ spin partner ½

Names		spin 0	spin 1/2
squarks, quarks	Q	$(\widetilde{u}_L \quad \widetilde{d}_L)$	$(u_L \quad d_L)$
(×3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$
sleptons, leptons	L	$(\widetilde{\nu} \quad \widetilde{e}_L)$	$(\nu \quad e_L)$
(×3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$
Higgs, higgsinos	H_u	$(H_u^+ \quad H_u^0)$	$(\widetilde{H}_u^+ \quad \widetilde{H}_u^0)$
	H_d	$(H_d^0 \quad H_d^-)$	$(\widetilde{H}_d^0 \quad \widetilde{H}_d^-)$

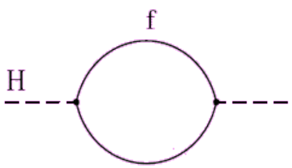
Names	spin 1/2	spin 1
gluino, gluon	$\widetilde{g}$	g
winos, W bosons	$\widetilde{W}^\pm \quad \widetilde{W}^0$	$W^\pm \quad W^0$
bino, B boson	$\widetilde{B}^0$	B^0

Observed SUSY particles with same mass than Standard-Model partners ? No !

SUSY : A broken symmetry !
Physical sParticles:
Mixture of super-partners

- Charginos ($\chi^\pm$) / Neutralinos (χ^0) : Bino/Wino ↔ Higgs (charged/neutral)
- Squarks, Sleptons : Mixture of $f_L \leftrightarrow f_R$

- Admitting existence of a Higgs Boson
  - Considering Gauge boson scatterings at High-Energy
  - **Requiring Unitarity of scattering amplitudes**
    - **$m_H \sim O(100\ \mathrm{GeV}/c^2)$**
- **Consider Higgs mass correction from fermionic loop:**



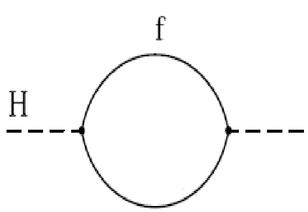
$$\longrightarrow \quad \Delta m_H^2 = \frac{\lambda_f^2}{16\pi^2} \cdot \left[-2\Lambda_{UV}^2 + \ldots\right]$$

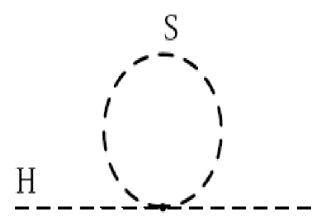
Λ_{UV}: Energy-scale at which new physics alters the Standard-Model (momentum cut-off regulating the loop-integral)

If $\Lambda_{UV} \sim M_P$ → $\Delta m^2{}_H \sim O(10^{30})$ larger than m_H !!!

And all Standard-Model masses indirectly sensitive to Λ_{UV} !!!

$$\Delta m_H^2 = \frac{\lambda_f^2}{16\pi^2} \cdot \left[-2\Lambda_{UV}^2 + \ldots\right]$$





$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2} \cdot \left[\Lambda_{UV}^2 - \ldots\right]$$

$\Delta m^2{}_H$ quadratic divergence cancelled :

Hierarchy problem naturally solved !

Supersymmetry & Coupling constants

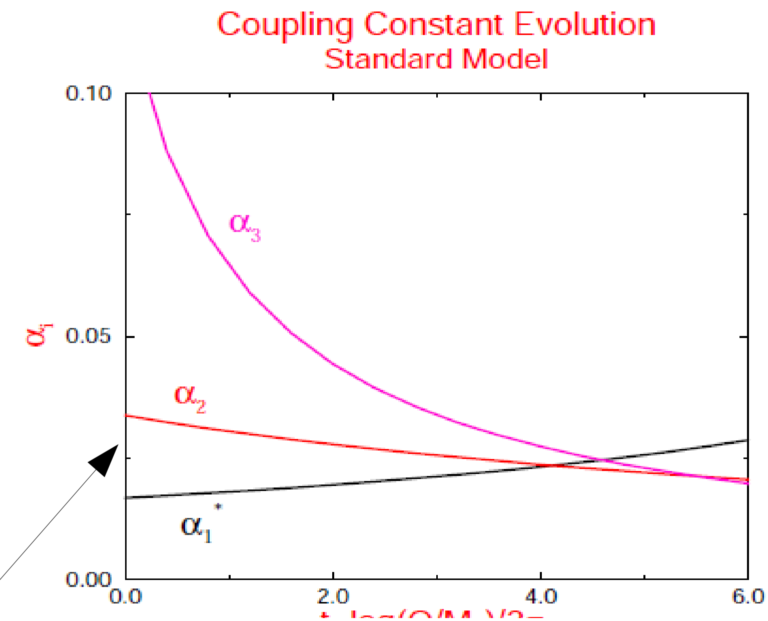
In Gauge theories :
Predict coupling constants at a scale Q once we measured them at another:

$$\mathbf{1/\alpha_i(Q) = 1/\alpha_i(M_Z) + (b_i/2)\log[M_Z/Q]}$$

b_i: Function of N_g(=3) and N_H(Number of Higgs doublets)

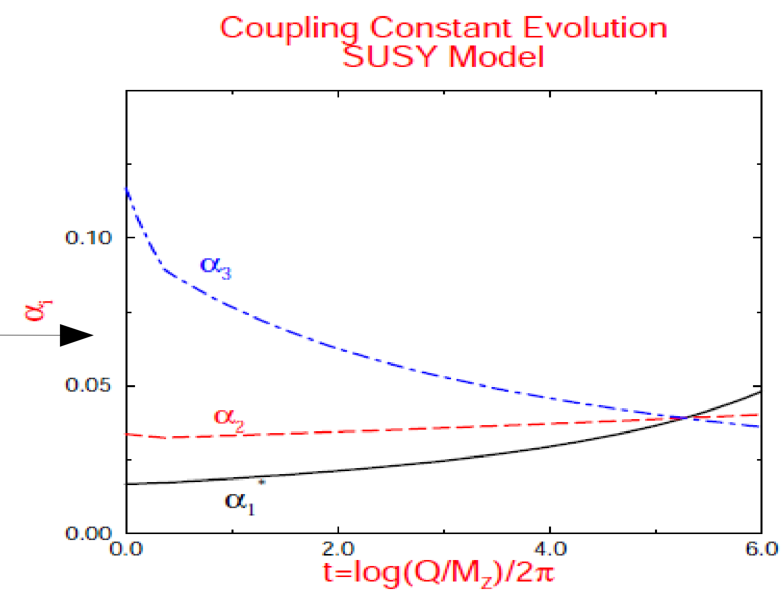
In Standard-Model : N_H=1

-> b_i's *such* that ...



In SUSY: N_H=2 + New particles contributing to a different evolution of coupling constants

-> b_i's *such* that !



SUSY can naturally be incorporated into Grand Unified Theories

Supersymmetry & Dark Matter

Most general SUSY lagrangian allows interactions leading to Baryon- & Lepton-number violation !

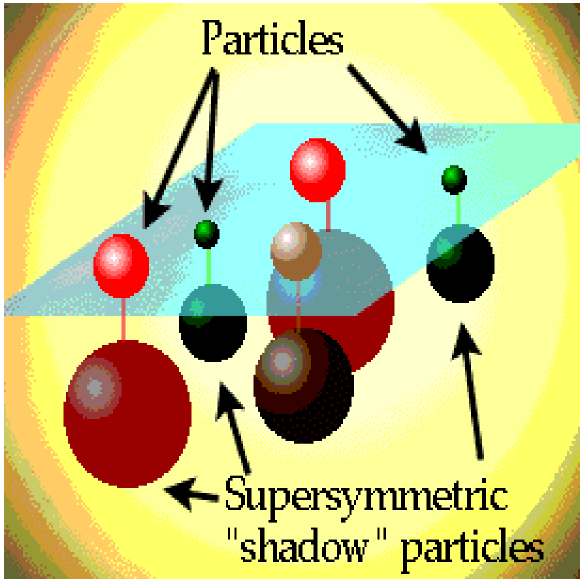
Now if sParticles were to exist at TeV scale:
Such interactions seriously restricted by experimental observation !

In SUSY: $N_{B,L}$ conservation *can* be "protected" by new symmtery R_P:

- **Eigenvalue: $(-1)^{3(B-L)+s}$**
  - +1 / -1 for SM / SUSY particles
- **If R_P conserved: Lightest Supersymmetric Particle (LSP) is stable**
  In most SUSY scenarios, LSP is either:
  - The lightest neutralino $\tilde{\chi}^0$ (mixture of neutral Higgsinos / Bino / Wino)
  - Scalar neutrinos
- ...In all cases a weakly interacting neutral particle

SUSY *can* have a natural candidate for the observed Cold Dark Matter

Revisiting SM Lagrangian



SM Lagrangian

Let's put the QCD part aside & have a look at the EW part only

$$\mathbf{L}_{\mathbf{EW}} = \mathbf{L}_{\mathbf{free+interaction}} + \mathbf{L}_{\mathbf{gauge}} + \mathbf{L}_{\mathbf{higgs}} + \mathbf{L}_{\mathbf{yukawa}}$$

SM Lagrangian: Free & Interaction parts

$$\mathbf{L_{free+interaction} = \Sigma_f\, i[\bar{\psi}_f^{\,L}\, \gamma^\mu\, D^L_\mu\, \psi_f^{\,L} + \bar{\psi}_f^{\,R}\, \gamma^\mu\, D^R_\mu\, \psi_f^{\,R}]}$$

→ $\psi_f^{L,R}$: Left and Right fermion, CC, Dirac spinors

→ Gauge-invariant derivatives:

$$D^L_\mu = \delta_\mu - i\, g\, (\tau_a/2)\, W^a_\mu - i\, g'\, (Y_L/2)\, B_\mu$$

$$D^R_\mu = \delta_\mu \qquad\qquad\qquad\qquad - i\, g'\, (Y_R/2)\, B_\mu$$

→ g, g': Weak-isospin & -hypercharge couplings

→ W^a_μ, B_μ: Weak-isospin & -hypercharge fields

→ τ_a,$Y_{L,R}$: Weak-isospin & -hypercharge quantum numbers, matrices

SM Lagrangian: The gauge part

$$\mathbf{L_{gauge} = -(1/4)\, W^a_{\mu\nu} W^{a\mu\nu} - (1/4)\, B_{\mu\nu} B^{\mu\nu}}$$

→ Gauge-invariant Weak-isospin & -hypercharge fields:

$$W^a_{\mu\nu} = \delta_\mu W^a_\nu - \delta_\nu W^a_\nu + g\, \varepsilon_{abc}\, W^b_\mu W^c_\nu$$

$$B_{\mu\nu} = \delta_\mu B_\nu - \delta_\nu B_\nu$$

2nd term of $W^a_{\mu\nu}$: Self-interacting character of Weak-isospin interaction → *This is the term allowing tri-boson couplings in SM*

A similar term exists in QCD sector of SM: QCD is also non-abelian → Allows self-coupling

SM Lagrangian: The Higgs part

$\mathbf{L}_{\mathbf{higgs}} \mathbf{=} (D_\mu \phi)^+(D^\mu \phi) - V(\phi)$

D_μ : Same gauge-invariant derivatives as before

→ $V(\phi)$: Pure Higgs interaction:

Mass: $m_H = \sqrt{-2\mu^2} = \sqrt{2\lambda v^2}$

Coupling: Calculate :-D

→ 1st term: Higgs↔Boson interaction:

Gives Boson masses

Gives Higgs↔Boson couplings

The lagrangian has to be SU(2)xU(1) invariant

→ 4 scalar <u>real</u> fields: $\phi = (\phi^+,\phi^0)$

$\phi^+=(1/\sqrt{2})(\phi_1 + i\phi_2)$

$\phi^0=(1/\sqrt{2})(\phi_3 + i\phi_4)$

SM Lagrangian: Yukawa

$$\mathbf{L_{yukawa} = -G_d\,(\bar{u},\bar{d})_L\,(\phi^+,\phi^0)\,d_R - G_u\,(\bar{u},\bar{d})_L\,(-\bar{\phi}^0,\phi^-)\,u_R}$$

$$\mathbf{+\ hermitian\text{-}conjugate}$$

(u,d): Up & Down doublets of <u>quarks or leptons</u>

Once Higgs sector is EW-broken:

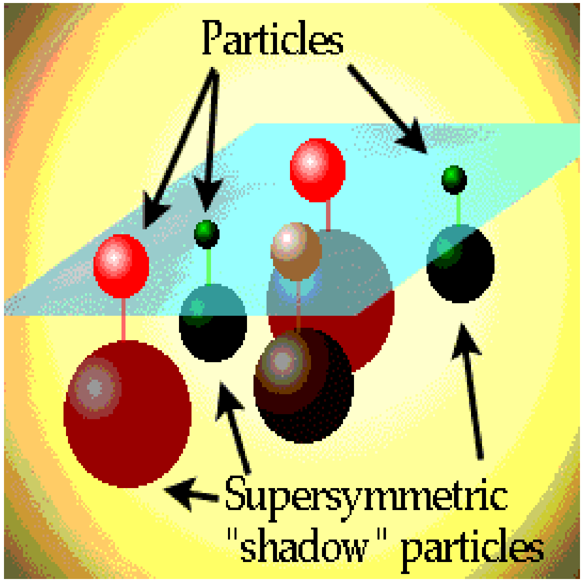
$\phi = (1/\sqrt{2})(0,v+H) \rightarrow$ "Confers" mass to fermions:

$L_{yukawa} = -m_d\,\bar{d}_L d_R\,(1+H/v) - m_u\,\bar{u}_L u_R\,(1+H/v)$

because: $m_f = G_f v/\sqrt{2}$

For neutrinos: $m = G_\nu v/\sqrt{2} = 0$:-D

“Constructing” the SUSY Lagrangian



Recipe to build the particle content and Lagrangian:

- Each SM fermion f has 2 chiral superpartners: f_L & f_R
- SM fermions and SUSY sfermions are regrouped in **superfields**

$$Q = \begin{pmatrix} u \\ d \end{pmatrix}_L \longrightarrow \tilde{Q} = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix} \quad \begin{matrix} \overline{u}_R \\ \overline{d}_R \end{matrix} \quad \begin{matrix} \tilde{u}^*_R \\ \tilde{d}^*_R \end{matrix}$$

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \longrightarrow \tilde{L} = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix} \quad \overline{e}_R \quad \tilde{e}^*_R$$

SM **MSSM**

- **Gauge superfields:** “Simply” containing the SM gauge fields and their SUSY partners
- Gauge superfields: Respecting the SU(3) x $SU_L(2)$ x U(1)

Superfields of Gauge & Matter, by definition, respect the gauge symmetries extended from the SM

Superfield	SU(3)	$SU(2)_L$	$U(1)_Y$	Particle Content
$\hat{Q}$	3	2	$\frac{1}{6}$	$(u_L, d_L),\ (\tilde{u}_L, \tilde{d}_L)$
$\hat{U}^c$	$\overline{3}$	1	$-\frac{2}{3}$	$\overline{u}_R,\ \tilde{u}^*_R$
$\hat{D}^c$	$\overline{3}$	1	$\frac{1}{3}$	$\overline{d}_R,\ \tilde{d}^*_R$
$\hat{L}$	1	2	$-\frac{1}{2}$	$(\nu_L, e_L),\ (\tilde{\nu}_L, \tilde{e}_L)$
$\hat{E}^c$	1	1	1	$\overline{e}_R,\ \tilde{e}^*_R$
$\hat{H}_1$	1	2	$-\frac{1}{2}$	$(H_1, \tilde{h}_1)$
$\hat{H}_2$	1	2	$\frac{1}{2}$	$(H_2, \tilde{h}_2)$

Superfield	SU(3)	$SU(2)_L$	$U(1)_Y$	Particle Content
$\hat{G}^a$	8	1	0	$g,\ \tilde{g}$
$\hat{W}^i$	1	3	0	$W_i,\ \tilde{\omega}_i$
$\hat{B}$	1	1	0	$B,\ \tilde{b}$

MSSM: Writing the Lagrangian

The interaction part:

$$\mathcal{L}_{int} = -\sqrt{2}\sum_{i,A} g_A \left[\boxed{S_i^*} T^A \overline{\psi}_{iL} \boxed{\lambda_A} + \text{h.c.}\right] - \frac{1}{2}\sum_A \left(\sum_i g_A S_i^* T^A S_i\right)^2$$

- Interaction-specific quantum number
- S_i: Scalar fields: Squarks & Sleptons
- ψ_i: Higgsinos
- λ_A: Gauge fermions

The gauge invariant derivative part: As same as introduced in SM, but generalized to superfields

The kinetic part:

$$\mathcal{L}_{KE} = \sum_i \left\{ (D_\mu \boxed{S_i^*})(D^\mu \boxed{S_i}) + i\overline{\psi}_i D\psi_i \right\} + \sum_A \left\{ -\frac{1}{4} F^A_{\mu\nu} F^{\mu\nu A} + \frac{i}{2} \boxed{\overline{\lambda}_A} D \boxed{\lambda_A} \right\}$$

MSSM: SM ↔ MSSM correspondance

Fermion	**Scalar**	**Gauge field**

SM

$i\,\bar{f}\,\gamma^{\mu} D_{\mu} f \quad + \quad (D_{\mu}\,\phi)^{+}(D^{\mu}\,\phi) \quad - \quad (1/4)\, F_{\mu\nu}F^{\mu\nu}$

SM: Higgs

MSSM (includes what is above)

$i\,\bar{\psi}\,\gamma^{\mu} D_{\mu}\,\psi \quad + \quad (D_{\mu}\,S_{i})^{+}(D^{\mu}\,S_{i}) \quad - \quad (1/4)\, F_{\mu\nu}F^{\mu\nu}$

MSSM: Higgsinos

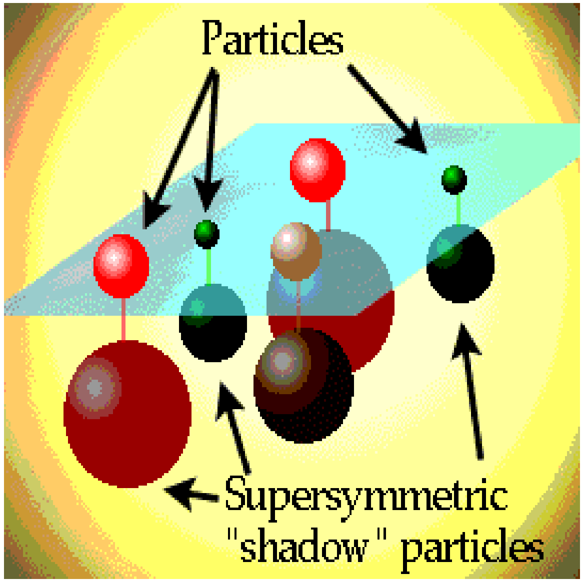
Squarks & Sleptons

This is the same as above

$+(i/2)\,\bar{\lambda}_{A}\,\gamma^{\mu} D_{\mu}\,\lambda_{A}$

Gauge fermions

SUSY: Let's minimally break it: Broken & effective MSSM



SUSY breaking

<u>How is it broken</u> ? We don't know... did not discover it (yet)...

How we *think* it's broken: Models/Implications by/for the theorists/experimentalists

mSUGRA Spontaneous Super-Gravity breaking: More constrained → 5 parameters @ breaking scale -> RGEs → Our mass spectrum

- m_0: Scalar mass
- $m_{½}$: Fermion mass
- μ: Higgs parameter ($\mu H_1 H_2$)
- A: Tri-linear squark/slepton mixing term
- $\tan\beta = \langle H^0_2 \rangle / \langle H^0_1 \rangle$

MSSM Parametrizing our ignorance of SUSY breaking, i.e. no hypothesis: Un-constrained → 124 parameters

- $\tan\beta$ / μ / M_A (pseudoscalar Higgs boson mass)
- $M_{L1,2,3}$: Controls slepton masses
- $M_{Q1,2,3}$: Controls squark masses
- $M_{1,2}$: Controls neutralino/chargino sectors
- ...

This is the most general Lagrangian we can write, hence the large number of unknowns: Only the spin hypothesis has been made

MSSM: Effective Lagrangian

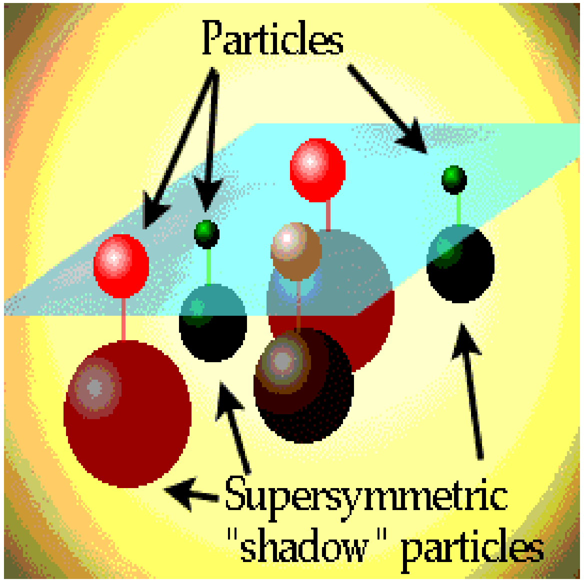
- We don't know how SUSY is broken, but can write the **most general broken effective Lagrangian**
- Soft: The breaking of the symmetry is taken care of by introducing of “soft” mass terms for scalars & gauginos: Soft because no re-introduction of quadratic divergence
- Maximal dimension of soft operators: ≤ 3 → Mass terms, Bilinear & Trilinear terms

$$
\begin{aligned}
-\mathcal{L}_{soft} = {} & m_1^2 \mid H_1 \mid^2 + m_2^2 \mid H_2 \mid^2 - B\mu\epsilon_{ij}(H_1^i H_2^j + \mathrm{h.c.}) + \tilde{M}_Q^2(\tilde{u}_L^* \tilde{u}_L + \tilde{d}_L^* \tilde{d}_L) \\
& + \tilde{M}_u^2 \tilde{u}_R^* \tilde{u}_R + \tilde{M}_d^2 \tilde{d}_R^* \tilde{d}_R + \tilde{M}_L^2(\tilde{e}_L^* \tilde{e}_L + \tilde{\nu}_L^* \tilde{\nu}_L) + \tilde{M}_e^2 \tilde{e}_R^* \tilde{e}_R \\
& + \frac{1}{2}\left[M_3 \bar{\tilde{g}}\tilde{g} + M_2 \bar{\tilde{\omega}}_i \tilde{\omega}_i + M_1 \bar{\tilde{b}}\tilde{b}\right] + \frac{g}{\sqrt{2} M_W}\epsilon_{ij}\left[\frac{M_d}{\cos\beta} A_d H_1^i \tilde{Q}^j \tilde{d}_R^* \right. \\
& \left. + \frac{M_u}{\sin\beta} A_u H_2^j \tilde{Q}^i \tilde{u}_R^* + \frac{M_e}{\cos\beta} A_e H_1^i \tilde{L}^j \tilde{e}_R^* + \mathrm{h.c.}\right] .
\end{aligned}
$$

Trilinear terms: As you might guess, that's where the real fun is :-D

Specificity of SUSY: Writing the most general Lagrangian, generalizing the spins of fields, SUCH that quadratic divergences are always shut down

Squark & Slepton sector



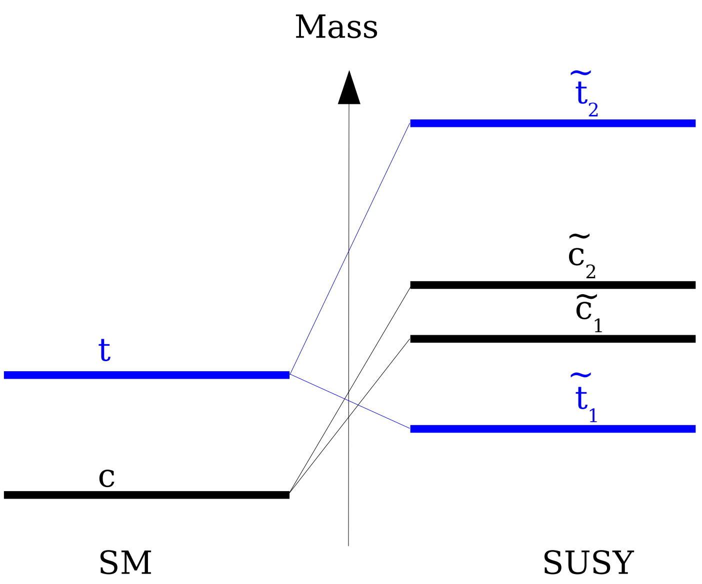
MSSM: Squark & Slepton sector

Physical states are 2 scalar mass-eigenstates: Mixtures of left- & -right chiral superpartners (scalars) of SM quark and leptons

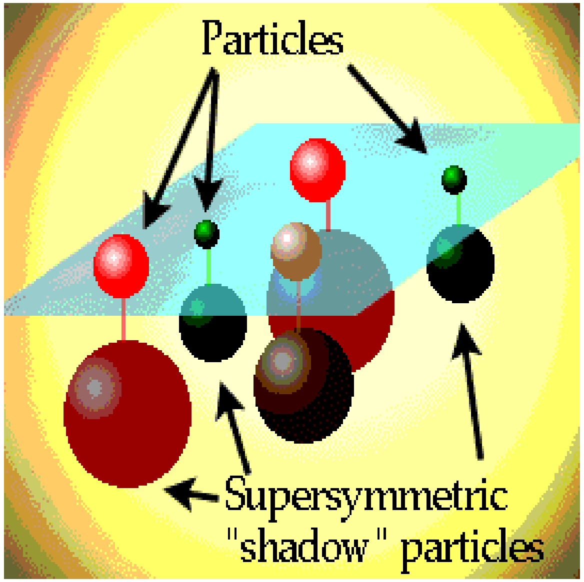
Let's pick-up example of the top sector: If $[f_L - f_R]$ chiral basis:

$$M_{\tilde{t}}^2 = \begin{pmatrix} \tilde{M}_Q^2 + M_T^2 + M_Z^2\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)\cos 2\beta & \boxed{M_T(A_T + \mu\cot\beta)} \\ \boxed{M_T(A_T + \mu\cot\beta)} & \tilde{M}_U^2 + M_T^2 + \frac{2}{3}M_Z^2\sin^2\theta_W\cos 2\beta \end{pmatrix}$$

- $\tilde{M}_Q$: Left squark mass
- $\tilde{M}_U$: Right squark mass
- A_T: Trilinear coupling specific to the top sector
- $M_Q=M_T$: Mass of the SM particle
- μ: Higgs (bilinear) mixing parameter
- β: Higgs vev-specific parameter (see in a couple of slides): Plays a role in the mixing



Chargino sector



MSSM: Chargino sector

Physical states are 2 fermionic mass-eigenstates: Mixtures of charged winos and charged higgsinos, which are SUSY eigenstates

In the charged [wino – higgsino] basis:

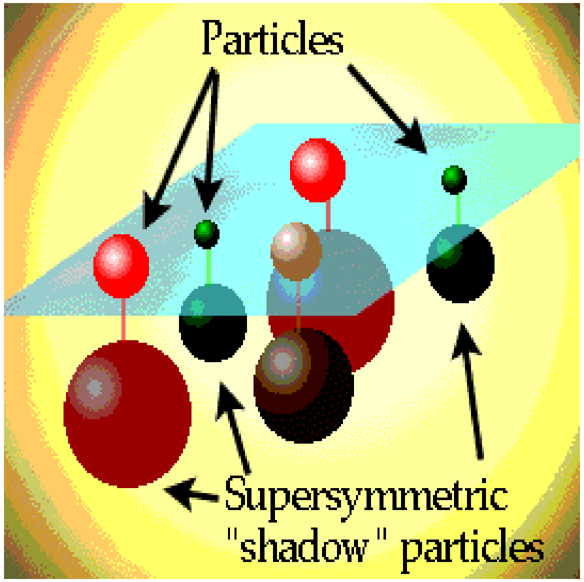
$$M_{\tilde{\chi}^{\pm}} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & -\mu \end{pmatrix}$$

- M_2: Mass of the wino
- μ: Higgs (bilinear) mixing parameter

Comments:

- The more M_2 » 1: The more the charginos are wino-like
- The more μ » 1: The more the charginos are higgsino-like
- β: Not playing a role in mixing

Neutralino sector



MSSM: Neutralino sector

Physical states are 4 fermionic mass-eigenstates: Mixtures of neutral winos w^0 , bino b, and 2 neutral higgsinos, which are SUSY eigenstates

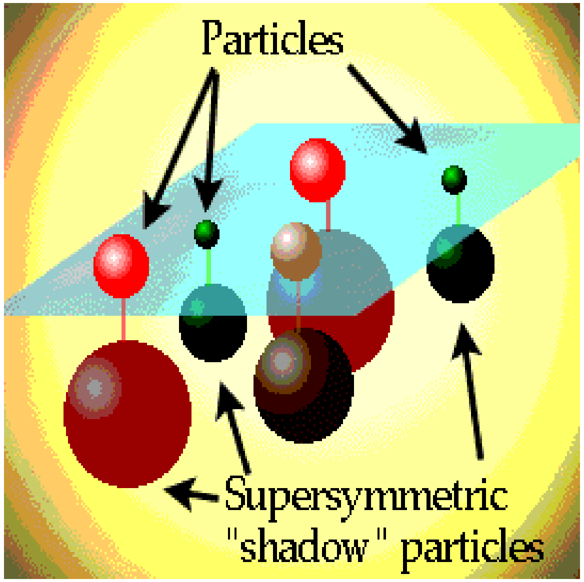
In the charged [b - w^0 - $h^0{}_1$ - $h^0{}_2$] basis:

$$M_{\tilde{\chi}_i^0} = \begin{pmatrix} M_1 & 0 & -M_Z \cos\beta \sin\theta_W & M_Z \sin\beta \sin\theta_W \\ 0 & M_2 & M_Z \cos\beta \cos\theta_W & -M_Z \sin\beta \cos\theta_W \\ -M_Z \cos\beta \sin\theta_W & M_Z \cos\beta \sin\theta_W & 0 & \mu \\ M_Z \sin\beta \sin\theta_W & -M_Z \sin\beta \cos\theta_W & \mu & 0 \end{pmatrix}$$

- M_1: Mass of the bino
- M_2: Mass of the wino
- μ: Higgs (bilinear) mixing parameter

<u>Exercise</u>: Qualitatively gauge the influence of each parameters in the mass-matrix above on the “type” of neutralinos

Higgs sector:
Keeping the most refined for last



MSSM: Higgs sector

<u>2</u> Higgs complex doublets:

$$V_H = \left(|\mu|^2 + m_1^2\right)|H_1|^2 + \left(|\mu|^2 + m_2^2\right)|H_2|^2 - \mu B\epsilon_{ij}\left(H_1^i H_2^j + \text{h.c.}\right)$$
$$+ \frac{g^2 + g'^2}{8}\left(|H_1|^2 - |H_2|^2\right)^2 + \frac{1}{2}g^2 |H_1^* H_2|^2 \quad .$$

8 degrees of freedom – 3 (massive gauge bosons) = 5 physical Higgs fields:
h / H / H$^{\pm}$ / A (CP-odd)

2 VEVs:

$$\langle H_1^0 \rangle \equiv v_1$$
$$\langle H_2^0 \rangle \equiv v_2$$

→ Key MSSM parameter: $\boxed{\tan\beta \equiv \frac{v_2}{v_1}}$

3 parameters to describe the MSSM Higgs sector:

Once $v_{1,2}$ are fixed such that: $\boxed{M_W^2 = \frac{g^2}{2}(v_1^2 + v_2^2)}$

This whole sector is described by (only) 2 other parameters:
→ tanβ
→ M_A: $\boxed{M_A^2 = \frac{2|\mu B|}{\sin 2\beta}}$

MSSM: Higgs sector

Let's look at couplings:

$$Z^\mu Z^\nu h: \quad \boxed{\frac{igM_Z}{\cos\theta_W}} \sin(\beta-\alpha) g^{\mu\nu}$$

$$Z^\mu Z^\nu H: \quad \frac{igM_Z}{\cos\theta_W} \cos(\beta-\alpha) g^{\mu\nu}$$

$$W^\mu W^\nu h: \quad \boxed{igM_W} \sin(\beta-\alpha) g^{\mu\nu}$$

SM couplings

$$\sin(\beta-\alpha) \quad \to 1 \text{ for } M_A \to \infty$$
$$\cos(\beta-\alpha) \quad \to 0 \quad .$$

Similar for coupling to γ & fermions

<u>Exercise</u>: Demonstrate the 2 relations above

It is possible that:

1/ Light h "SM like":

→ Mass: Rather low

→ Br(h -> γγ) ~ Like in SM

2/ {H, H$^\pm$, <u>A</u>} much heavier & degenerate

→ Couplings of lightest Higgs to fermions/γ/W/Z ~ Like in SM

→ Couplings of "additional" Higgs to fermions/γ/W/Z ~ 0

This is called the decoupled regime:

1/ The lightest Higgs field is a) rather light b) behaves *a la* SM

2/ The "new" physical Higgs fields are (much ?) higher in mass

MSSM: Higgs sector

Equation governing lightest Higgs mass:

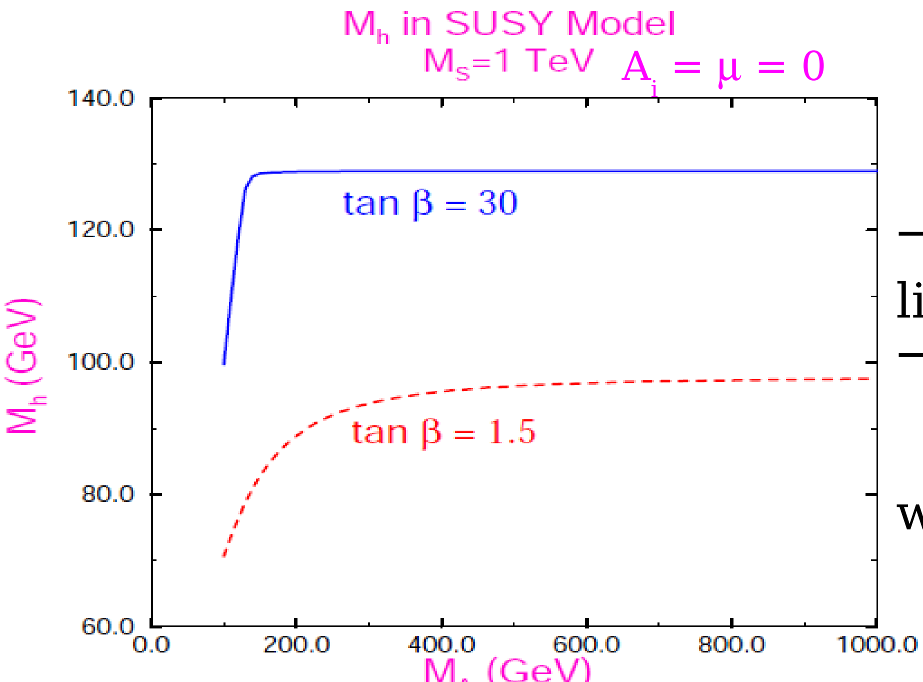
$$M^2_{h,H} = \frac{1}{2}\left\{M_A^2 + M_Z^2 + \frac{\epsilon_h}{\sin^2\beta} \pm \left[\left(M_A^2 - M_Z^2\right)\cos 2\beta + \frac{\epsilon_h}{\sin^2\beta}\right)^2 + \left(M_A^2 + M_Z^2\right)^2 \sin^2 2\beta\right]^{1/2}\right\}$$

with: $\epsilon_h \equiv \frac{3G_F}{\sqrt{2}\pi^2} M_T^4 \log\left(\frac{\tilde{m}^2}{M_T^2}\right)$

Contribution of 1-loop correction only !
Squark masses: Higgs mass particularly sensitive to $\sim t_{1,2}$ system

Upper bound:

$$M_h^2 < M_Z^2 \cos^2 2\beta + \epsilon_h$$



→ The "well-known" M_h < 135 GeV/c^2 limit for any-SUSY lightest Higgs
→ ...is dependent on
→ 2-loop calculations
→ Renormalization calculations
which can evolve...

EXERCISES

1/ Install the SuSpect software on your computer: This one of the only SUSY spectrum calculators with parametrized MSSM (pMSSM) parameters as input: You don't have 124, but 27 parameters to play with ;-)

2/ Just play with different parameters and follow evolution of the generated masses

2i) What are the most sensitive parameters for different types of particles ?

2ii) Once you get an idea for 2i): For a set of frozen parameters, produce plots showing evolution of the physical masses, say , as function of pMSSM parameters

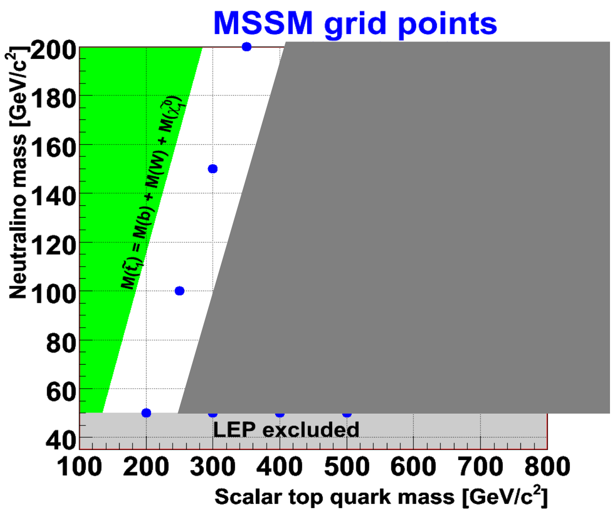
For 2i) & 2ii), let's pick-up:

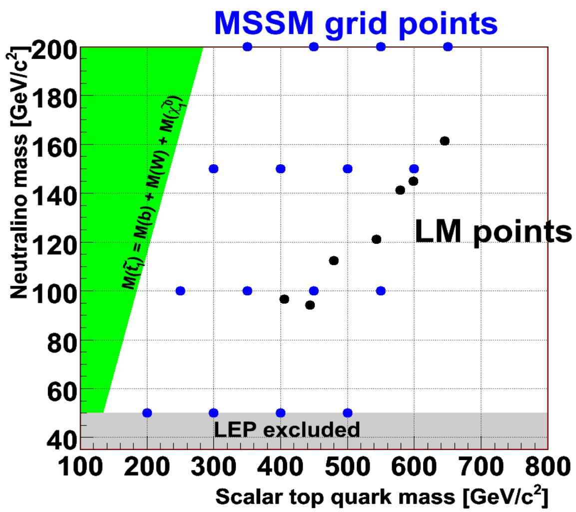
→ The lightest neutralino
→ The chargino
→ The lightest stop and stau
→ The lighest Higgs

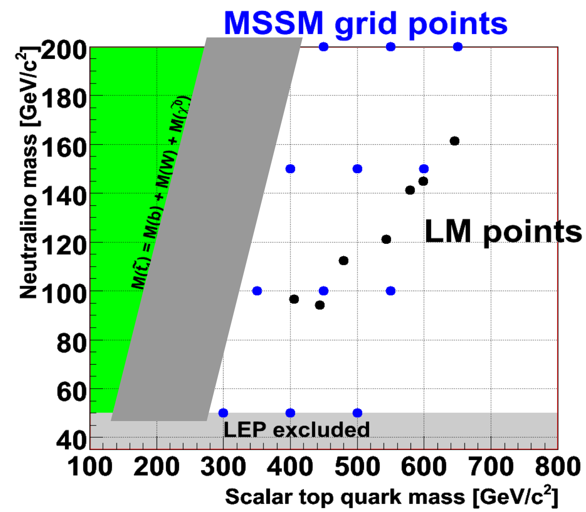
3/ Once your fingers are well warmed-up with pMSSM, produce the points on the following page :-D

Stop decays: Different diagrams for different domains

$\tilde{t}_1 \to b\, W^+\, \tilde{\chi}_1^0$	$\tilde{t}_1 \to b\, \tilde{\chi}_1^+$	$\tilde{t}_1 \to t\, \tilde{\chi}_1^0$







Conditions:

$b+W+\tilde{\chi}_1^0 < \tilde{t}_1$ $\quad$ $b+W+\tilde{\chi}_1^0 < \tilde{t}_1$ $\quad$ $t+\tilde{\chi}_1^0 < \tilde{t}_1$

$\tilde{t}_1 < t+\tilde{\chi}_1^0$: $\quad$ $W+\tilde{\chi}_1^0 < \tilde{\chi}^+_1 < \tilde{t}_1 - b$

Close $\tilde{t}_1 \to t+\tilde{\chi}_1^0$ $\quad$ ← Not exclusive: Will co-exist →

"Dominance" conditions:

$\tilde{t}_1 < \tilde{\chi}^+_1 + b$: $\quad$ $t+\tilde{\chi}_1^0 < \tilde{\chi}^+_1 + b$:

Make $\tilde{\chi}^+_1$ virtual $\quad$ Privilege vs $b\,\tilde{\chi}_1^+$