Imaging using ionizing radiations

IV – Positron Emission Tomography

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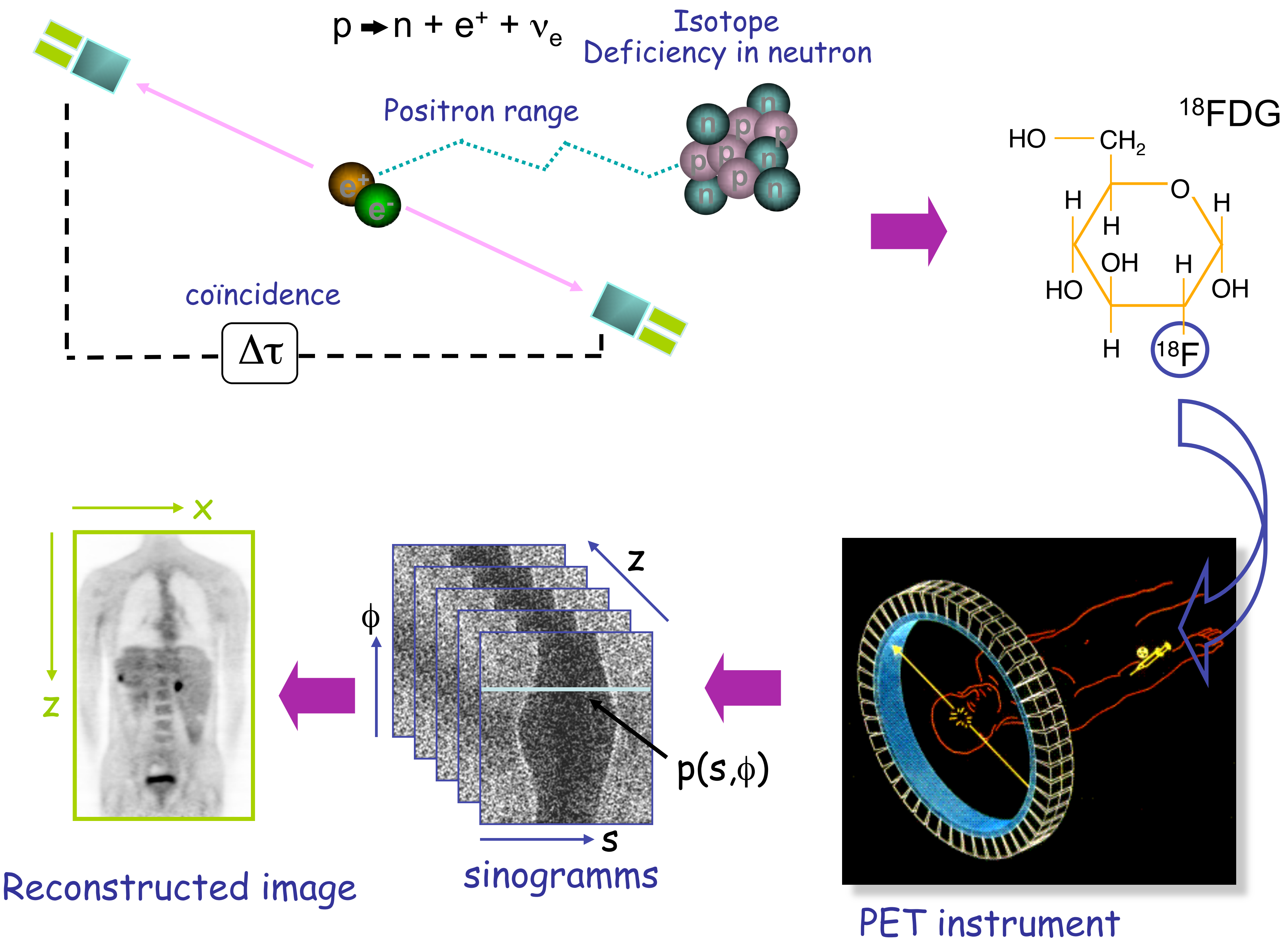
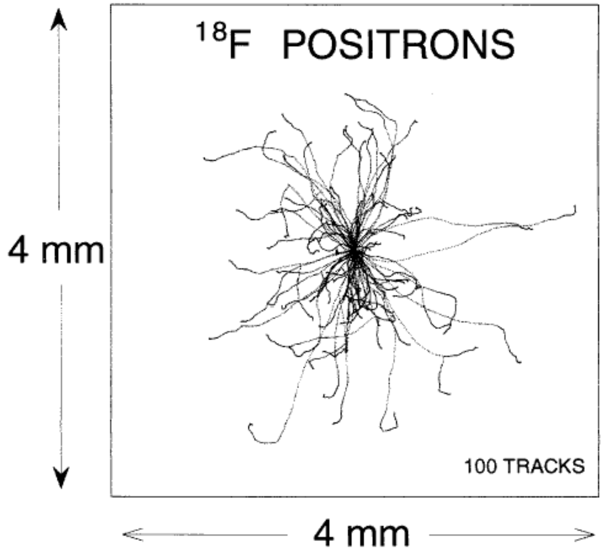
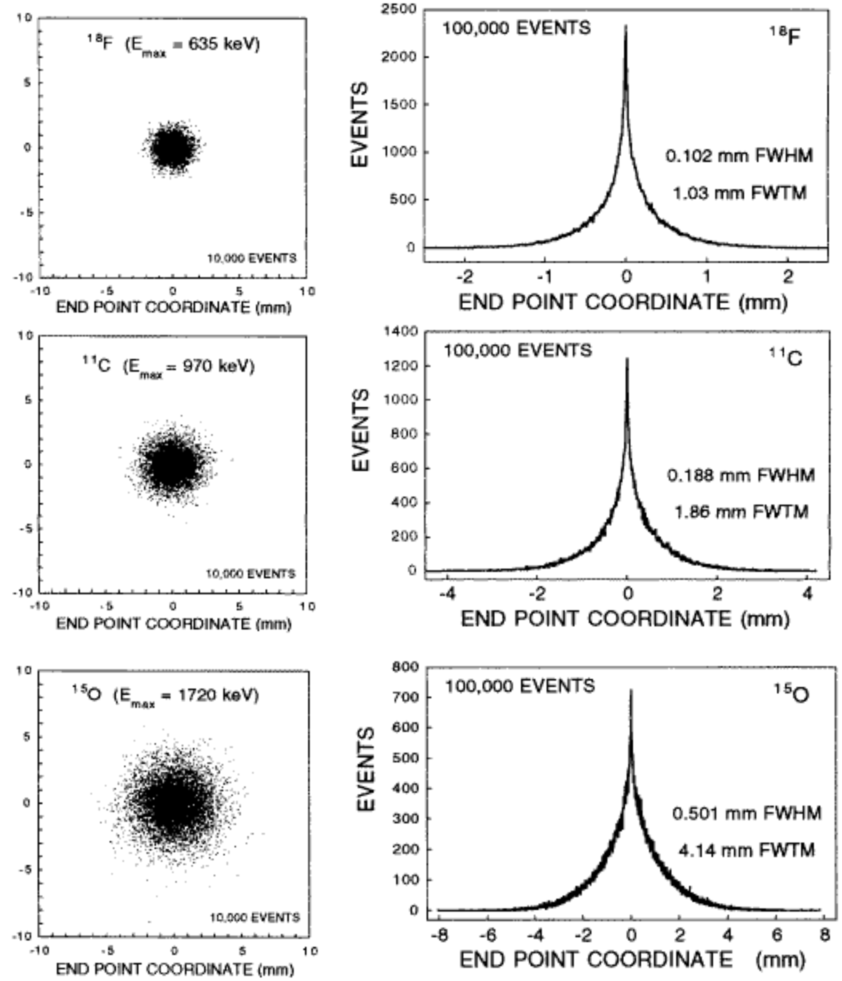
Slides are courtesy of David Brasse



p → n + e+ + νe
Isotope
Deficiency in neutron
Positron range
coïncidence
Δτ
18FDG
HO
CH2
O
H
H
H
OH
H
HO
OH
H
18F
x
z
z
ϕ
p(s,ϕ)
s
Reconstructed image
sinogramms
PET instrument

Positron mean range





$$P(x) = C\,e^{-k_1 x} + (1 - C)\,e^{-k_2 x} \qquad x \geqslant 0$$

	^{18}F	^{11}C	^{13}N	^{15}O
C	0.516	0.488	0.426	0.379
k_1 (mm^{-1})	0.379	0.238	0.202	0.181
k_2 (mm^{-1})	0.031	0.018	0.014	0.009

Phys. Med. Biol. **44** (1999) 781–799.

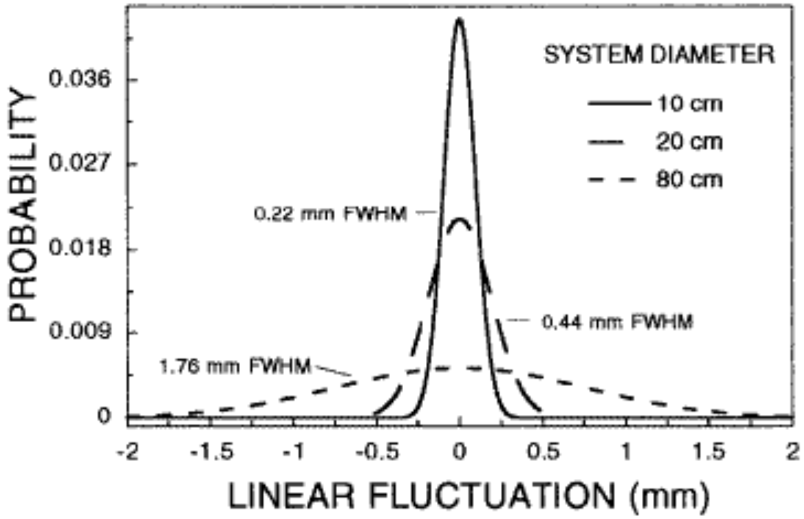
Acolinearity

This effect depends on the energy of the positon

Angular distribution is almost gaussian
With an LTMH around 0,5°

R_{AC} = 0,0022 x D

D = System diameter



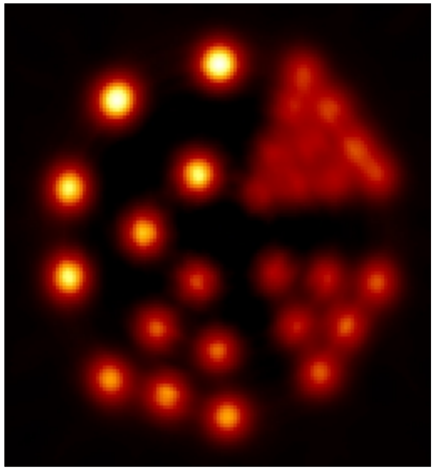
❑ β^+ in a magnetic field

Objective : Limitation of the positon range (flight) in presence of an axial magnetic field – Combination PET/IRM

- **Resolution phantom: Ø 1 mm – 2,5 mm**
- **Simulation of a high resolution animal PET system : 1.8 mm**

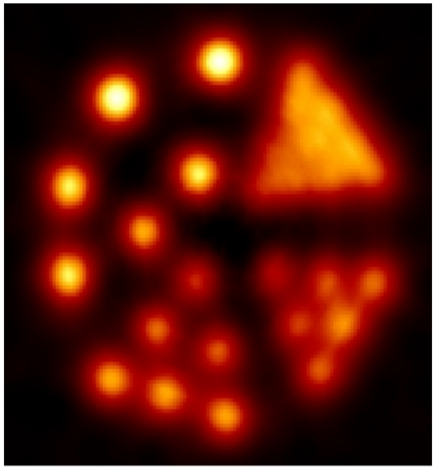
^{18}F

B = 0 Tesla



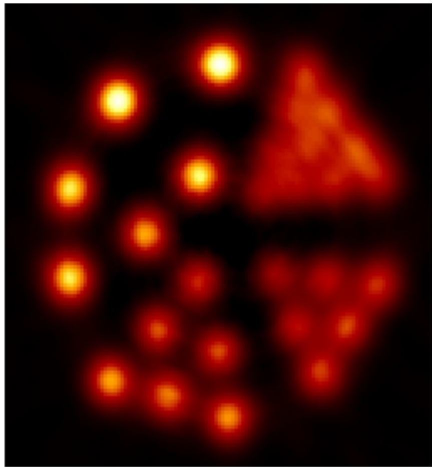
^{15}O

B = 0 Tesla



^{15}O

B = 15 Tesla



S. Jan, SHFJ - CEA

Physical principles: short remindr

Question:
Calculate the minimal energy of a 511 keV photon after undergoing a Compton scatter.
What is the energy of the recoil electron ?

$$E_c(keV) = \frac{E_0}{1+\frac{E_0}{m_e c^2}\left(1-\cos\theta\right)}$$

The minimum energy is obtained when the maximum of energy is transfered to the electron. This happens for $\theta = 180°$

$$E_c(keV) = \frac{511}{2-\cos\theta}$$

$$E_e(keV) = E - E_c = 511\frac{1-\cos\theta}{2-\cos\theta}$$

Numerical application: find the energy of the scattered photon and the recoil electron

$$E_c(keV) = \frac{511}{2-\cos 180°} = 170keV$$

$$E_e = E - E_c = 511 - 170 = 340keV$$

Interaction cross section

$$I(x) = I(0)\exp(-\mu x)$$

$$\mu \approx \mu_{compton} + \mu_{photoelectric}$$

@ 511 keV

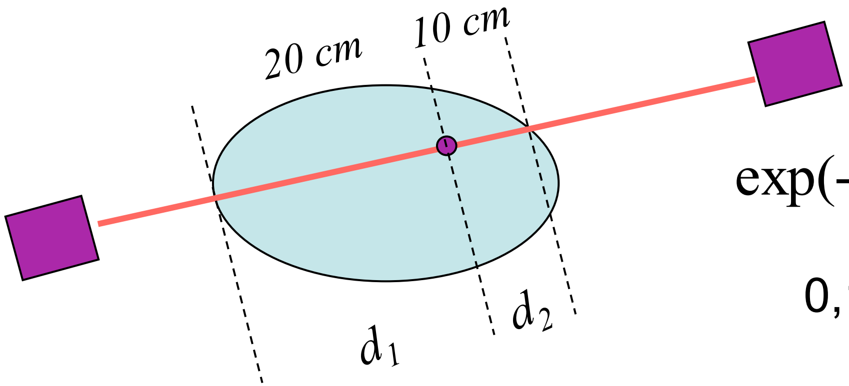
	$\mu_{Compton}$ (cm^{-1})	$\mu_{Photoelectric}$ (cm^{-1})	μ (cm^{-1})
Soft tissues	~0,096	~ 0,00002	~ 0,096
Bone	~ 0,169	~ 0,001	~ 0,17
BGO	0,51	0,40	0,96
Lead	0,76	0,89	1,78
Tungsten	1,31	1,09	2,59

Question:
What is the probability that a 511 keV photon undergoes a Compton scatter @ 7.5 cm inside the brain?
What would be the probability value for a photon coming from the liver @ 20 cm from the body surface?

$$\frac{I(x)}{I(0)} = \exp(-\mu x)$$

$\mu_{Compton} = 0,096\ cm^{-1}$

Brain: x = 7,5 cm; I(x)/I(0) = 0,49 (49 % of photons are unscattered)
Liver: x = 20 cm; I(x)/I(0) = 0,15 (15 % of photons are unscattered)



$$\exp(-\mu d_1) \times \exp(-\mu d_2)$$

0,15 x 0,38 = 0,06
6 % !!!!

The objective of the detection system is to detect photons coming from the body and that did not scatter.

Need for a dense material
Optimize the rate of photolectric/compton cross sections.

Question:
Calculate the thickness of BGO required to stop 90% of 511 keV photons.

$$\frac{I(x)}{I(0)} = 0{,}1 = \exp(-0{,}96 \times x)$$

$$x = 2{,}4cm$$

Used scintillators

Scintillators	Density (g/cm^3)	Yield (ph/511keV)	Decay (ns)	Refraction index	μ @511 keV (cm^{-1})	μ_{Ph}/μ_C @511 keV
NaI:Tl	3,67	19400	230	1,85	0,34	0,22
BGO	7,13	4200	300	2,15	0,96	0,78
LSO:Ce	7,40	~ 13000	~ 47	1,82	0,88	0,52
GSO:Ce	6,71	~ 4600	~ 56	1,85	0,70	0,35
BaF_2	4,89	700; 4900	0,6; 630	1,56	0,45	0,24
YAP:Ce	5,37	~ 9200	~ 27	1,95	0,46	0,05
$LaBr_3$	5,29	32000	16	1,9	0,45	
$LaCl_3$	3,86	23000	25	1,9	0,36	
LuAP	8,34	5110	18		0,95	
LYSO	7,11	17300	41	1,81	0,83	

Used photodetectors

Photomultiplier
HPD
MCP

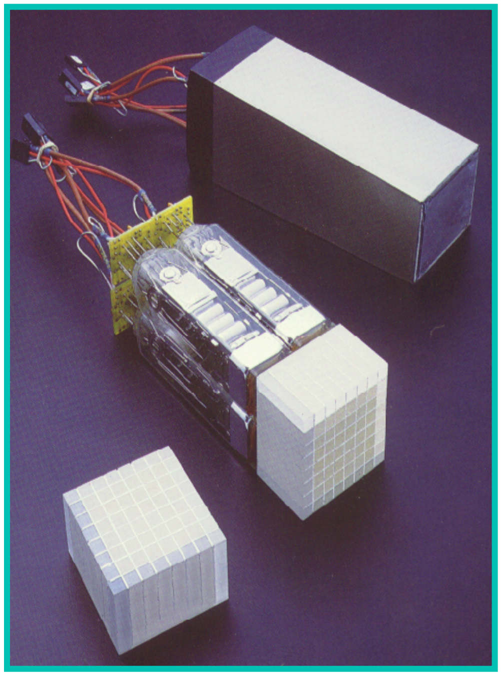
Solid detector
Photodiode
Avalanche photodiode

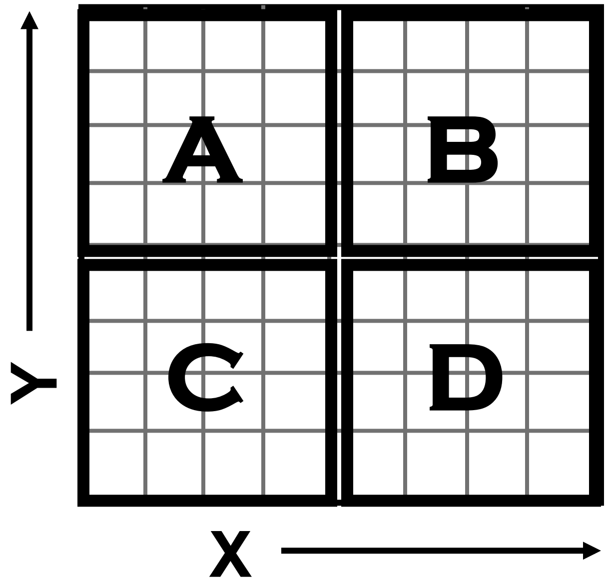
Important parameters to consider:

- *Quantum efficiency*
- *Gain*
- *Signal to noise ratio*
- *Speed*
- *Geometry*
- *Cost/readout channel*

Photodetector/Crystal coupling

« Block detector »

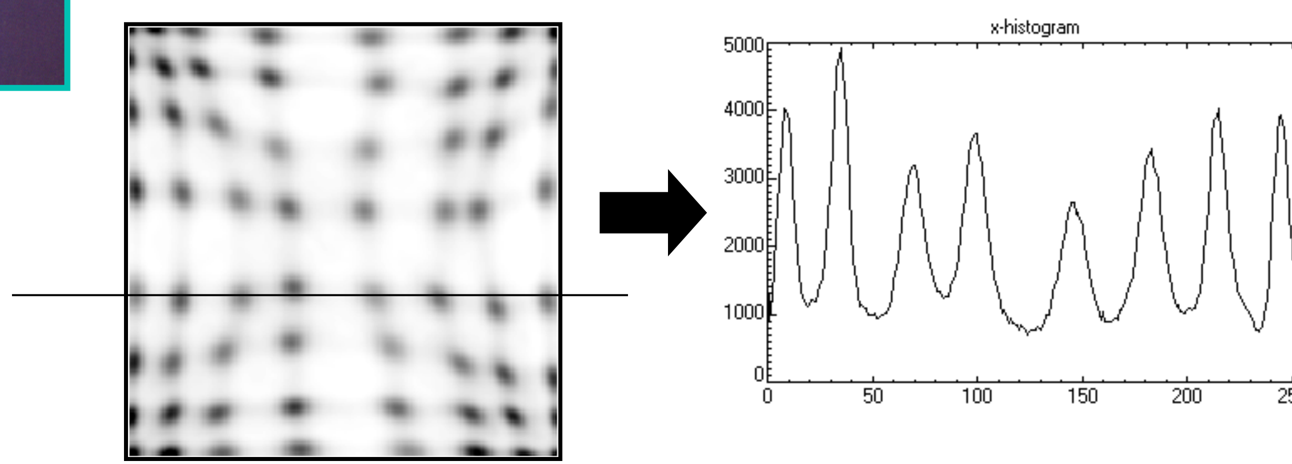




$$X = \frac{(D + B) - (C + A)}{\Sigma}$$

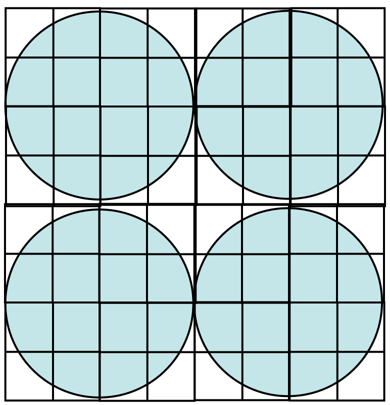
$$Y = \frac{(A + B) - (C + D)}{\Sigma}$$

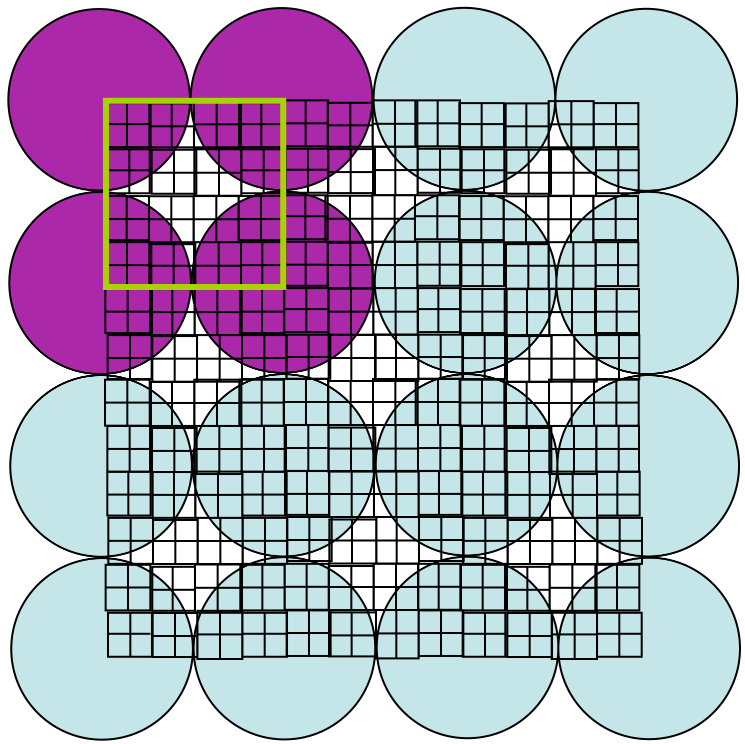
$$\Sigma = A + B + C + D$$



Photodetector/Crystal coupling

Extension of « Block detector »: « Quadrant sharing »





4 PMT for 64 pixels
Pixel size / 2
Cost reducing

Adapted for large detectors
Dead zones edges

Photodetector/Crystal coupling

Continuous detector

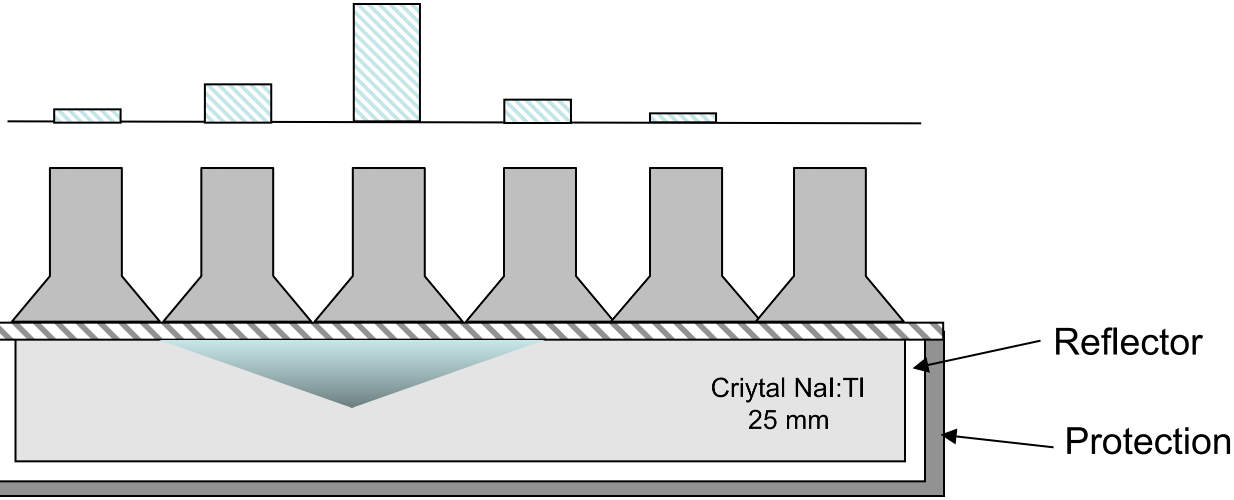
Large FOV: 40-50 cm
Dead zone at edges

Curved detector possible

Barycenter calculation of the interaction position
The spatial resolution depends on the number of detected photons-> NaI:Tl etc…
25 mm = Compromise beteween resolution and efficiency

Thickness	$R_{intrinsèque}$
10 mm	3 mm
25 mm	4-5 mm

Should have the capability of generating many events, unless increasing the dead time …



System configuration

Detection in coïncidence

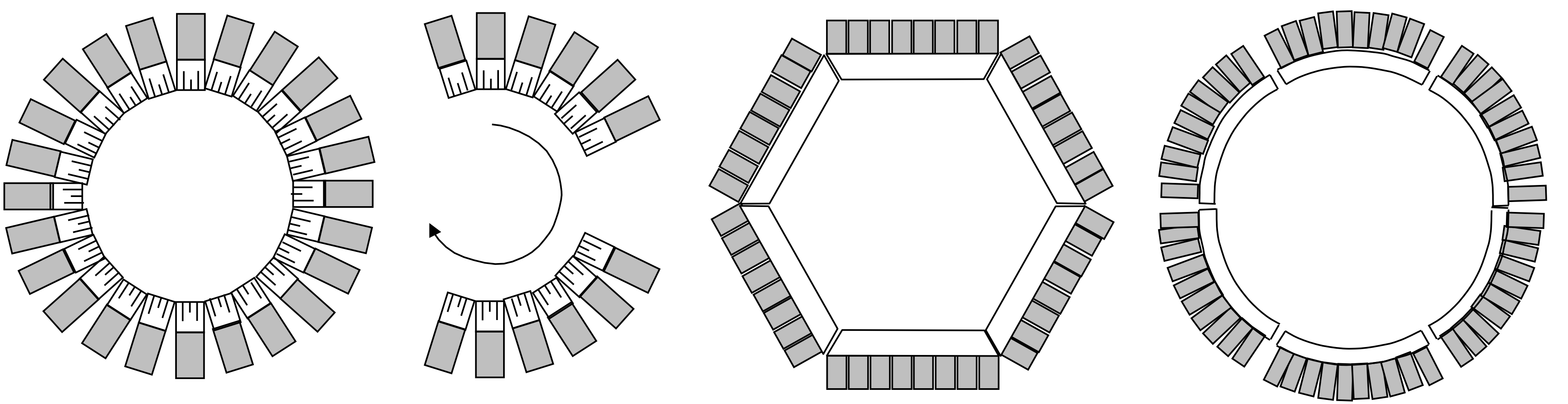
The time resolution of the system depends on

the constant decay of the used crystal

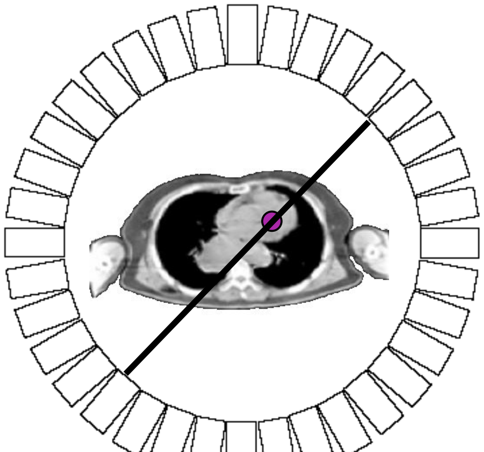
used electronic readout

Typically τ *= 5-6 ns for BGO-NaI:Tl*

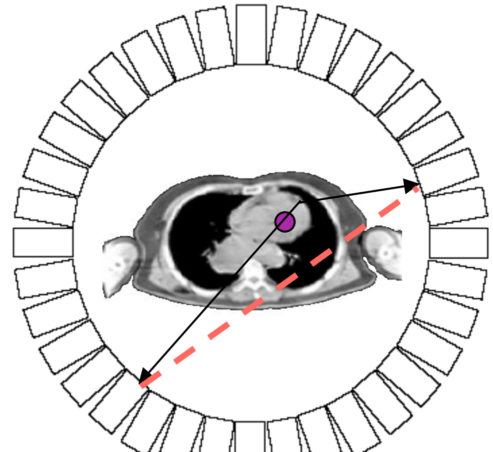
τ *= 2-3 ns for LSO*



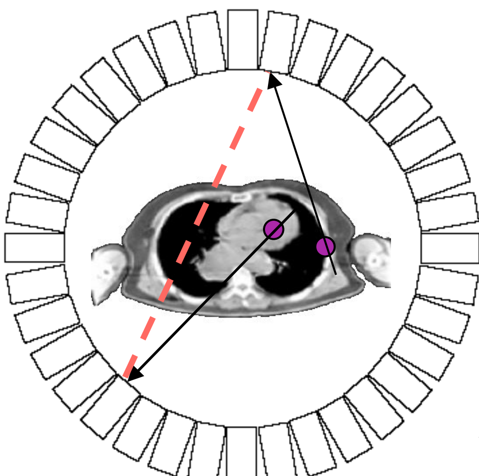
Different registered events



True coïncidences T ~ activité



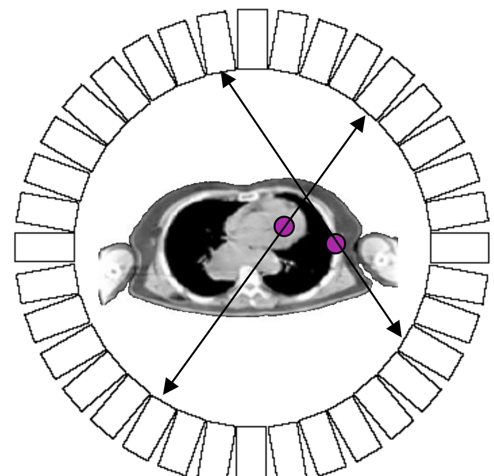
Scattered Coïncidences S ~ f x activité



Random Coïncidences R ~ activity^2

Measurement:
T + S + R

$$N_R = 2\tau N_1 N_2$$



Mutliple Coïncidences

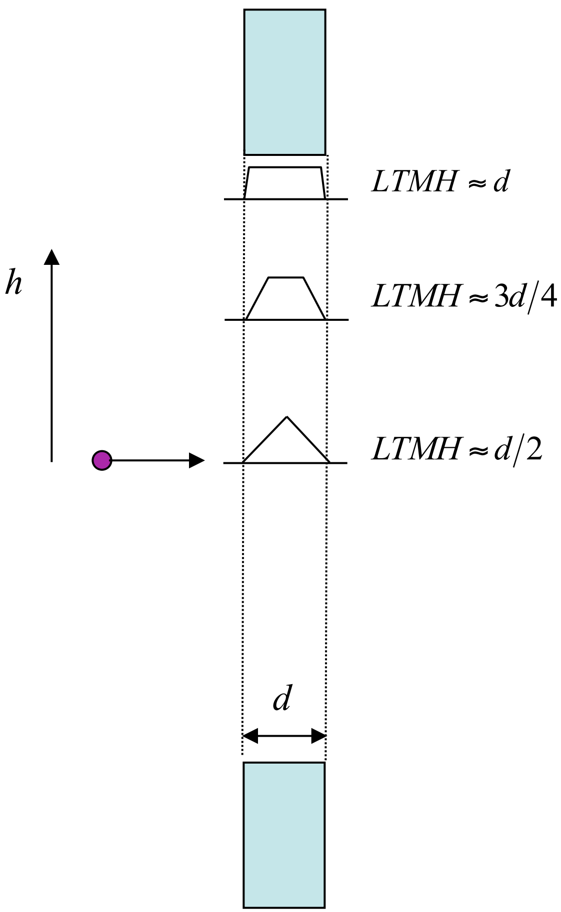
Resolution: Response function

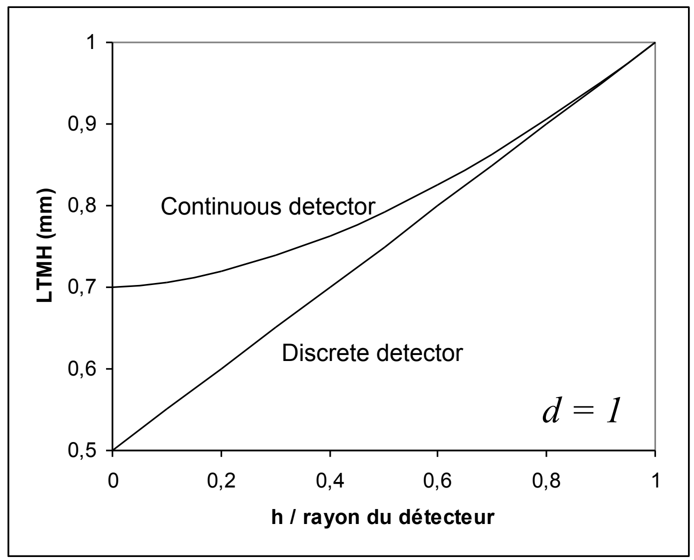
The intrinsic detector resolution can be derived in 2 components

Geometry (Considers a perfect detector)

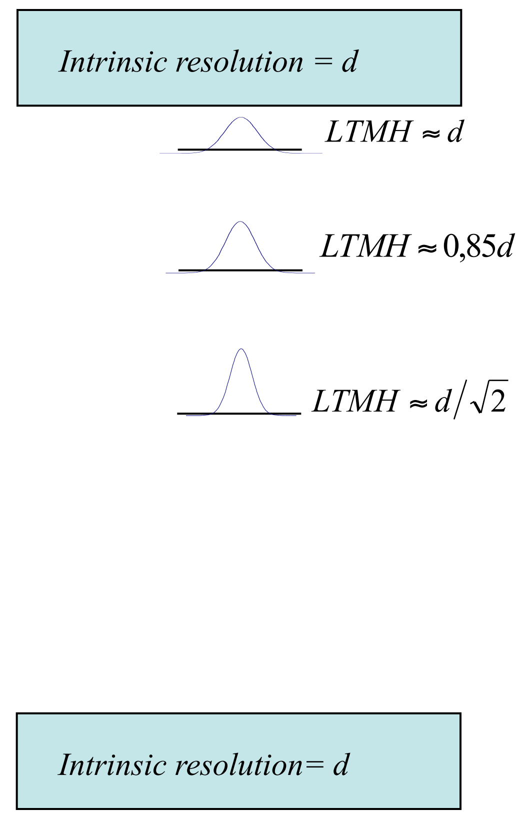
Physics (Physical properties of the detector)

Discrete detector

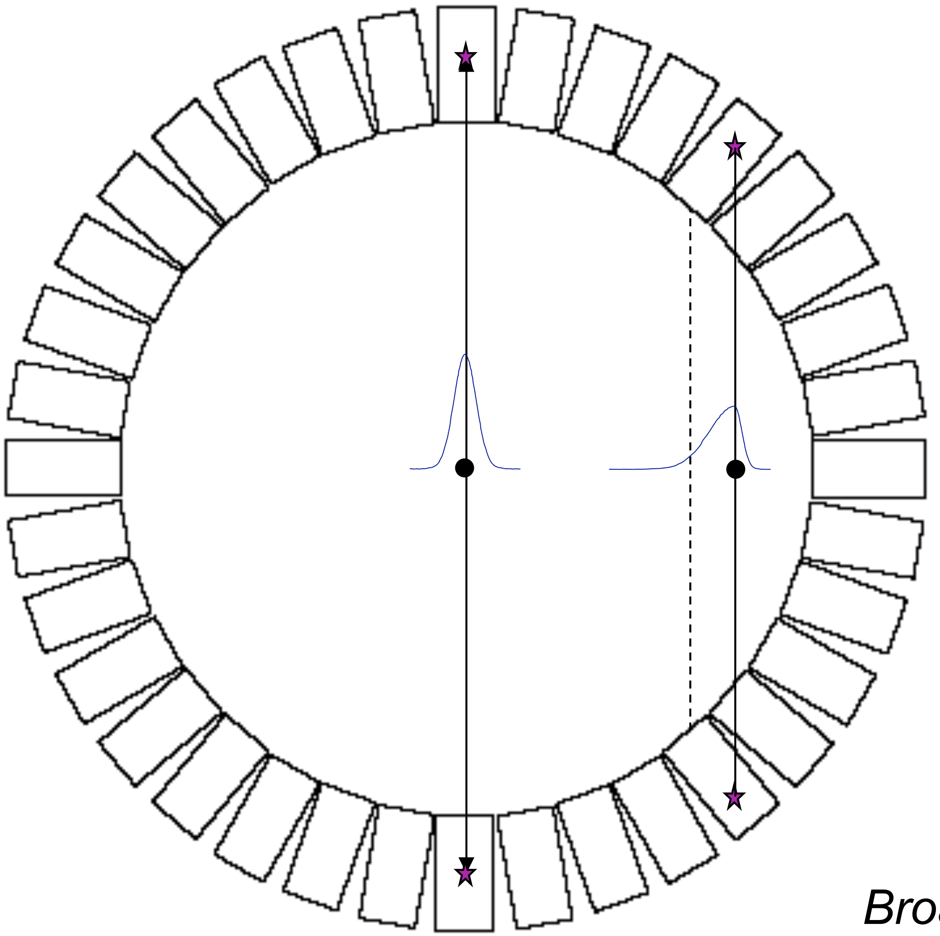




Continuous detector



Parallax problem



Two important consequences

Broadening of the response (Assymetry)

False positionning

The resolution of a detector system
the quadratic sum of the « elementary » resolutions

Positron mean range
Acolinearity
Geometrical factors
Intrinsic spatial resolution (continuous detector)
Physical factors

Question:
Hypothesis: Different responses can be approximated by gaussian functions

Calculate the system resolution for a ^{18}F source centred in the field of view of a clinical scanner which diameter is 80 cm and the size of a detection element is 6 mm

$$R_i = 6mm / 2 = 3mm$$

$$R_{AC} = 0,0022 \times 800 = 1,76mm$$

$$R_{positron} = 0,102mm$$

$$R_{système} = \sqrt{3^2 + 1,76^2 + 0,102^2} = 3,48mm$$

Detection efficiency

The number of registered events is given by

The amount of injected radioactivity
Fraction of radioactivity targeting the Region Of Interest (ROI)
Duration of the exam
Detection efficiency of the PET system

Unit

Cps/Bq/ml

The system efficiency if the product of several factors

Detector efficiency @ 511 keV
Detection solid angle
Source positionning with respect to detector
The width of the energy window
The width of the time window

Detection efficiency

The detection efficiency of an elementary detector is the product of

The probability to detect an incident photon

Times

The fraction of events selected in the energy window

$$\varepsilon = \left(1 - e^{-\mu d}\right) \times \Phi$$

μ = linear attenuation coefficient of the material
d = material thickness

Detection efficiency in coïncidence

$$\varepsilon = \left(1 - e^{-\mu d}\right)^2 \times \Phi^2$$

System geometrical efficiency

The product of

Detection solid angle covered by the detector for a given source position (Ω)

Filling rate (Materials volume / total volume) (φ)

$$\Omega = 4\pi \sin\left[\tan^{-1}\left(A / D\right)\right]$$

Point source centred in a circular system
With diameter D
And axial coverage A

$$\phi = \frac{\text{section} \times \text{height}}{\left(\text{section} + deadzone\right) \times \left(\text{height} + deadzone\right)}$$

Detection efficiency of the system

$$\eta \approx 100 \times \frac{\varepsilon^2 \varphi \Omega}{4\pi}$$

Question:
Calculate the detection efficiency of a point source centred in the field of view of the system.
TEP: simple detector ring of 80 cm diameter composed of detector elements whom dimensions are 4,9 x 6 x 30 mm^3.
Reflector size: 0,25 mm
80% of the events are selected by the energy window

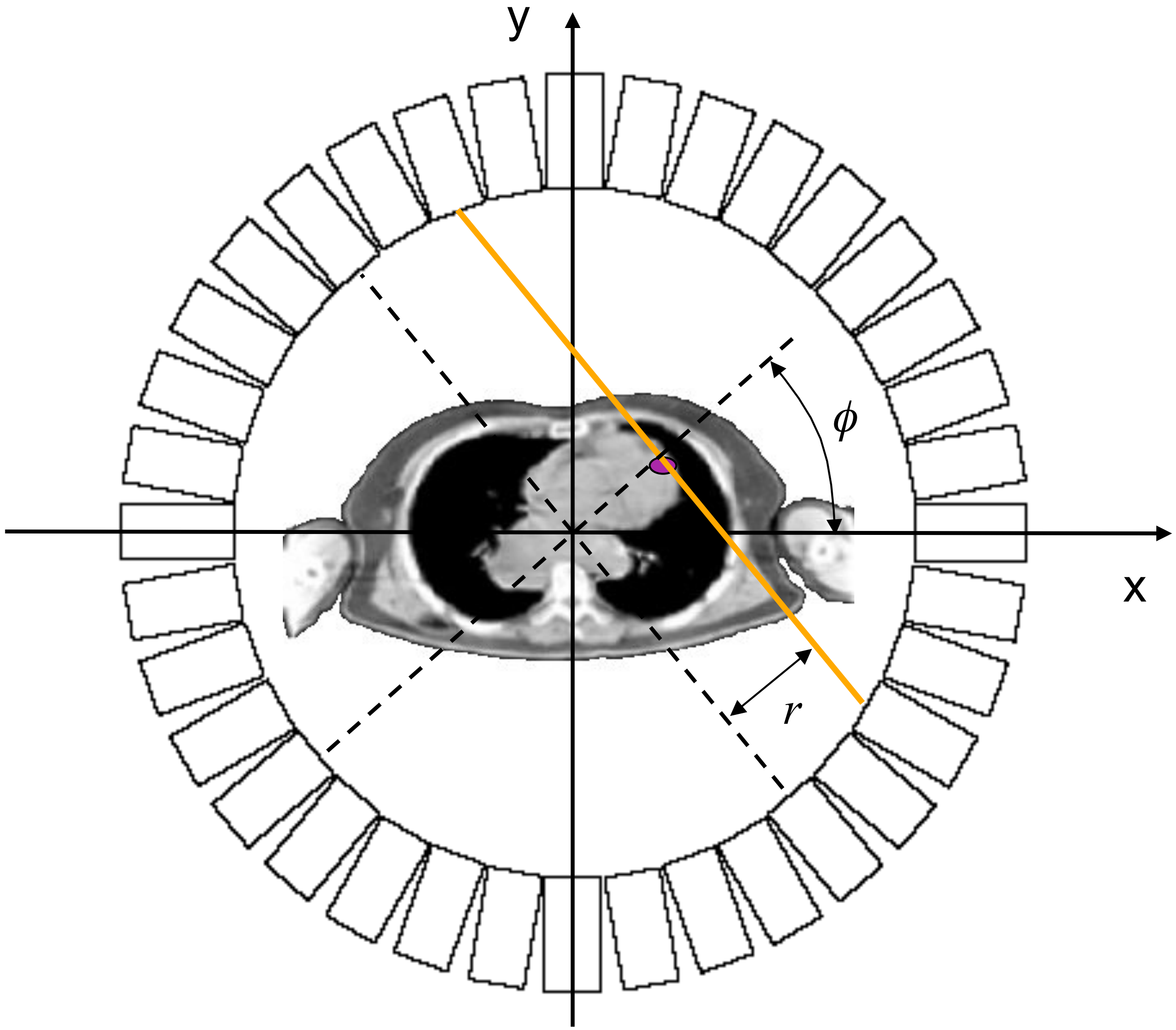
$$\varepsilon = \left(1 - e^{-(0{,}96\times 3)}\right)\times 0{,}8 = 0{,}755$$

$$\Omega = 4\pi \sin\left[\tan^{-1}\left(0{,}6/80\right)\right] = 0{,}094$$

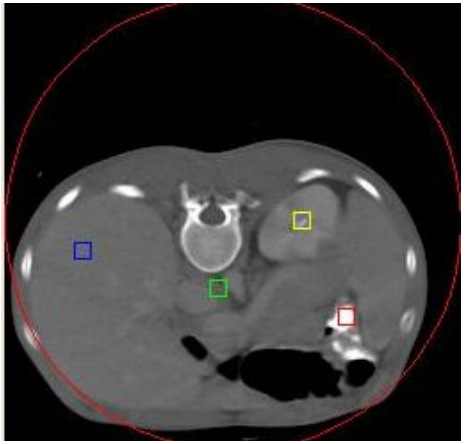
$$\varphi = \left(4{,}4\times 5{,}5\right)/\left(4{,}9\times 6\right) = 0{,}823$$

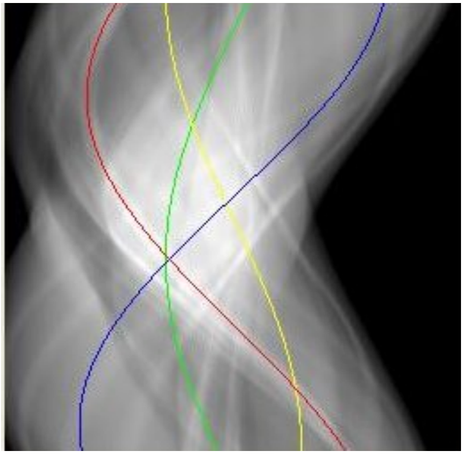
$$\eta = 100\times 0{,}755^2 \times 0{,}094\times 0{,}823 / 4\pi = 0{,}35\%$$

Data representation

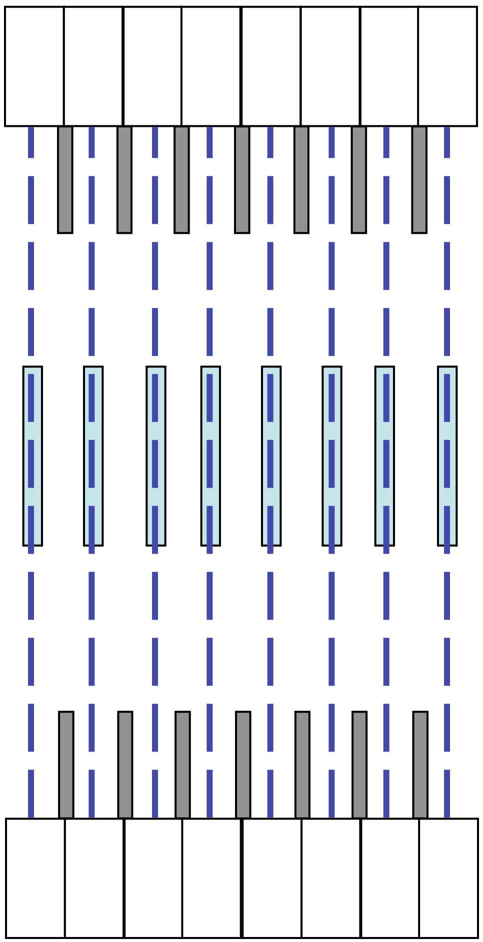


$$r = x\cos\phi + y\sin\phi$$

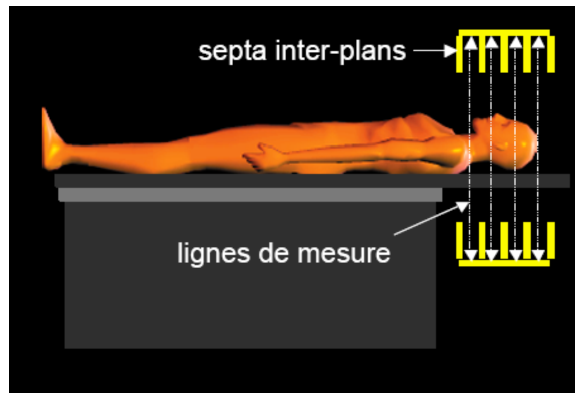




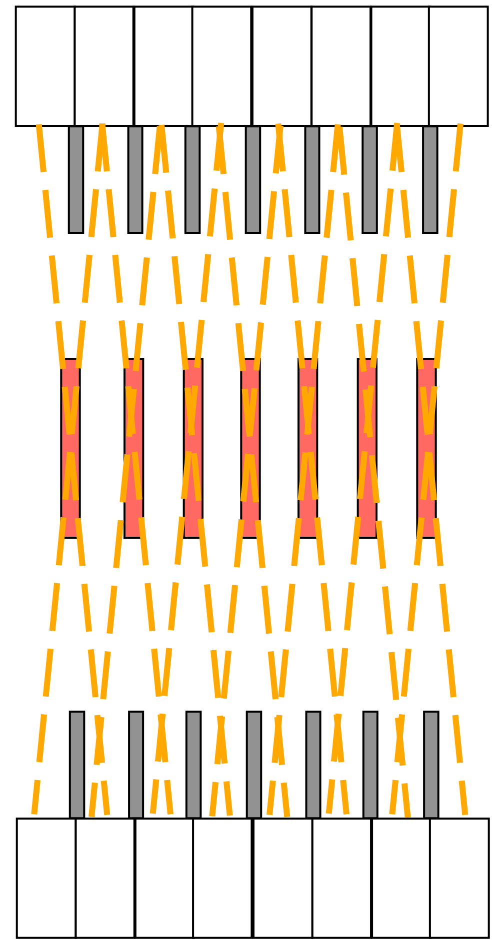
Acquisition 2D



Direct slices

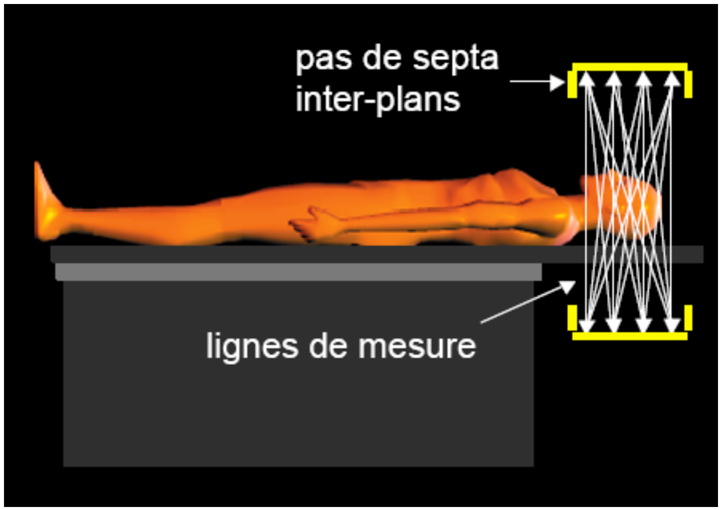


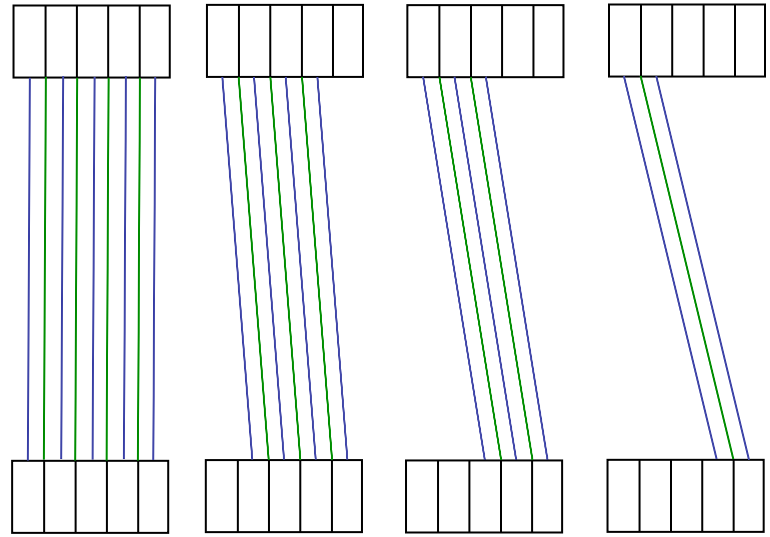
Number of slices
2N-1
with N = number of detectors



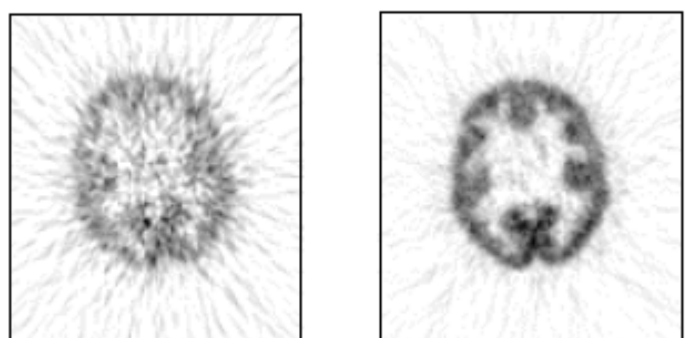
Crossed Slices

Acquisition 3D





[11C]flumazenil images of benzodiazapene receptor distribution



2D PET   3D PET

Question:
Calculate the detection efficiency of a point source centred in the field of view of the system.
TEP: 16 detector ring of 80 cm diameter composed of 4,9 x 6 x 30 mm^3 detector elements.
Reflector size: 0,25 mm
80% of the events are selected by the energy window.
3D Acquisition mode.

$$\varepsilon = \left(1 - e^{-(0,96\times 3)}\right)\times 0,8 = 0,755$$ *unchanged*

$$\Omega = 4\pi \sin\left[\tan^{-1}\left(0,6\times 16/80\right)\right] = 1,50$$

$$\phi = \left(4,4\times 5,5\right)/\left(4,9\times 6\right) = 0,823$$ *uchanged*

$$\eta = 100\times 0,755^2 \times 1,50\times 0,673 / 4\pi = \underline{5,59\%}$$ *To compare with 0,35 %*

Question:
BGO -> LSO

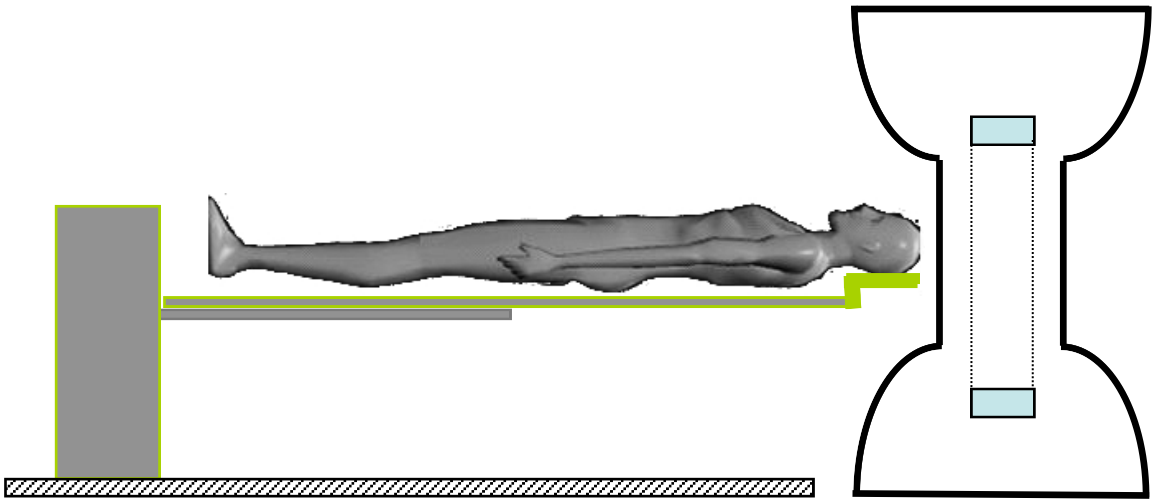
$$\varepsilon = \left(1 - e^{-(0,88\times 3)}\right)\times 0,9 = 0,836$$

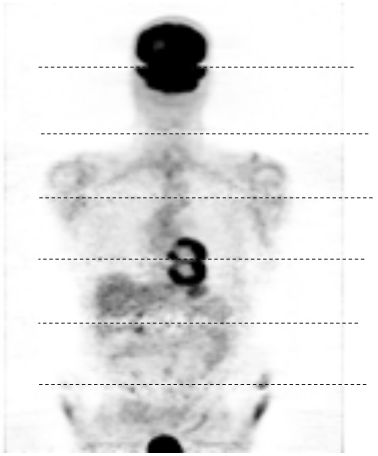
$$\Omega = 4\pi \sin\left[\tan^{-1}\left(0,6\times 16/80\right)\right] = 1,50$$ *unchanged*

$$\varphi = \left(4,4\times 5,5\right)/\left(4,9\times 6\right) = 0,823$$ *unchanged*

$$\eta = 100\times 0,836^2 \times 1,50\times 0,673 / 4\pi = \underline{6,19\%}$$ *To compare with 5,59 %*

Acquisition protocole

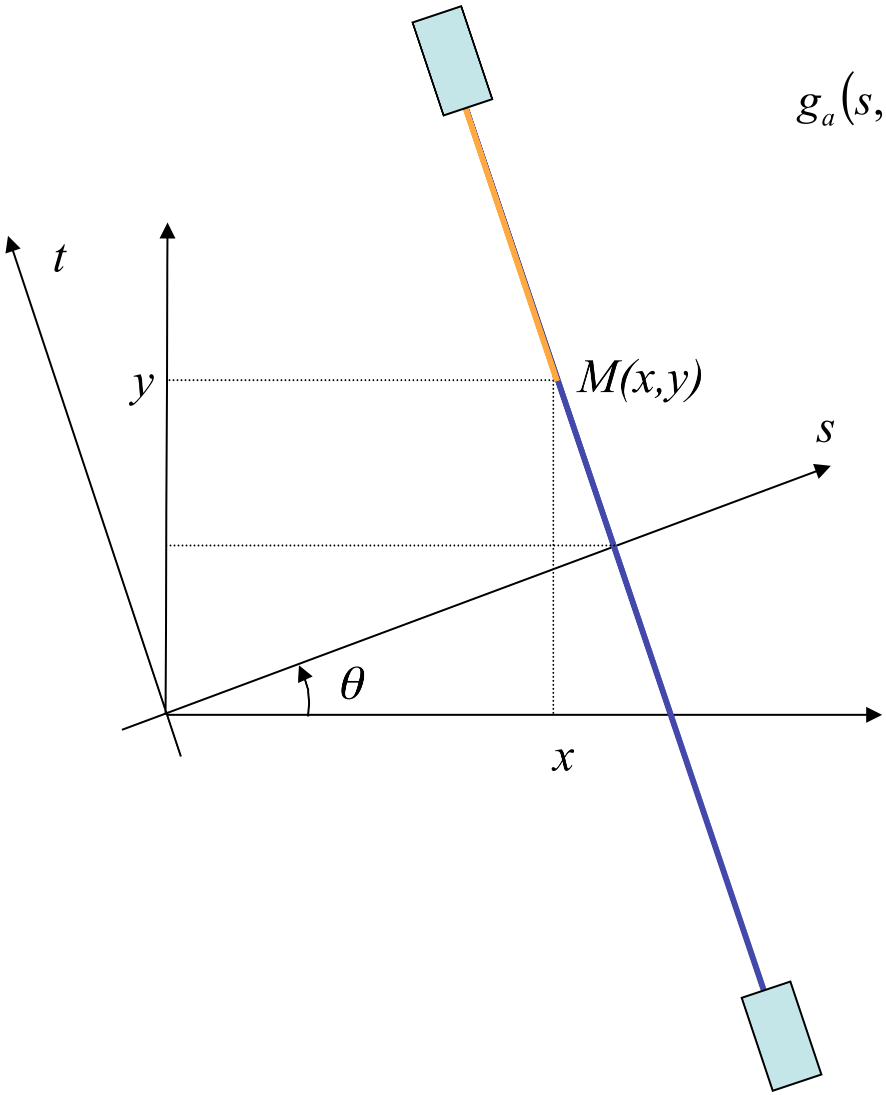




Static or dynamic acquisition in order to cover the whole body.

Data Correction

Attenuation correction in PET



$$g_a(s,\theta) = \int_{-\infty}^{M} f(x,y)\,dt \cdot e^{-\int_{-\infty}^{M} \mu(x,y)\,dt'} \times \int_{M}^{+\infty} f(x,y)\,dt \cdot e^{-\int_{M}^{+\infty} \mu(x,y)\,dt'}$$

$$g_a(s,\theta) = \int_{-\infty}^{+\infty} f(x,y)\,dt \times e^{-\int_{-\infty}^{+\infty} \mu(x,y)\,dt'}$$

- The attenuation:
  - Do not depend on the localisation of the point of emission in the LOR.
  - Depends only on the attenuation integral *μ(x,y)* different of 0.
  - Depends on the function *μ(x,y)*
    - → required measurement.
  - Example @ 511 keV:
    - Soft tissues: $\mu = 0{,}096\ cm^{-1}$
    - Muscle: $\mu = 0{,}1\ cm^{-1}$
    - Bone: $\mu = 0{,}134\ cm^{-1}$
    - Water: $\mu = 0{,}097 cm^{-1}$

Attenuation artifact

- Loss of an important number of photons
  - Around 17 % of photons pairs emitted in the centre of the brain.
  - Around 5 % of photons emitted in centre of the thorax.
- Bias in quantification.
- Unequal attenuation depending on the depth.
- Error in diagnosis.

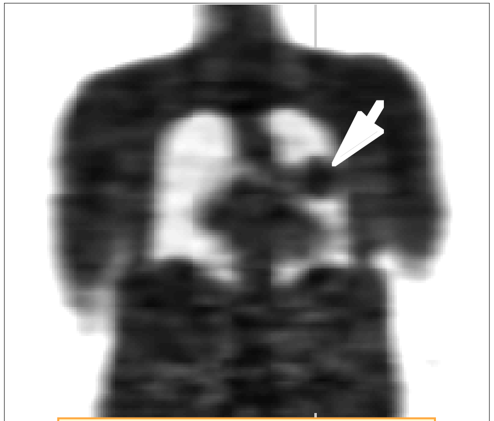
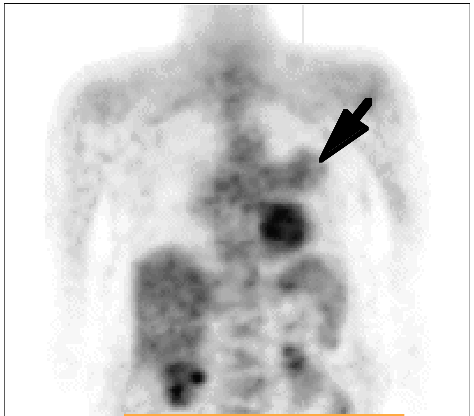


Image de transmission

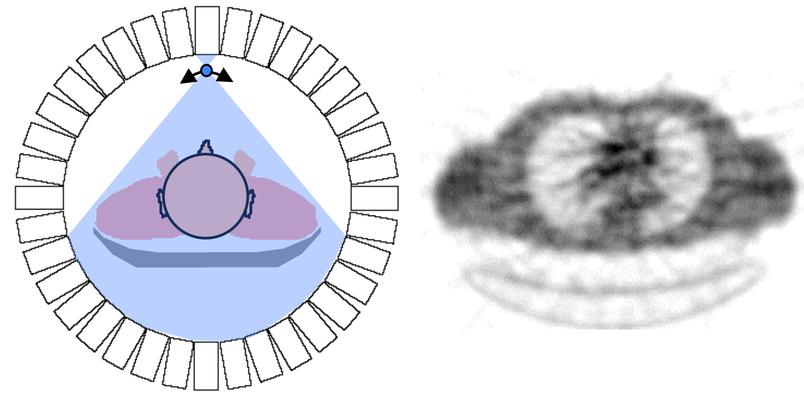
Sans correction



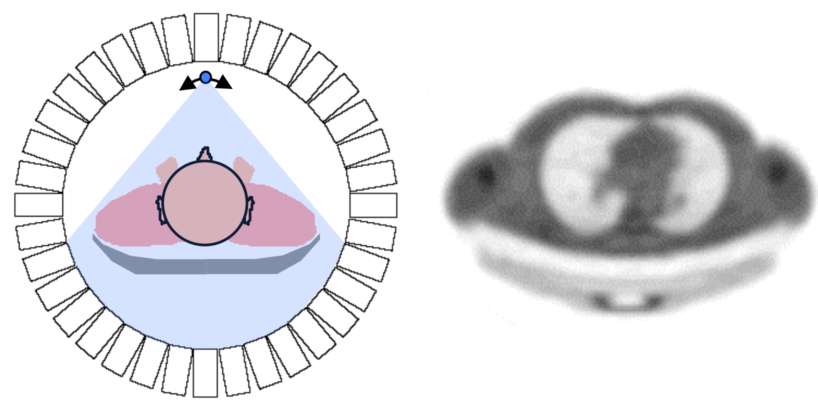
Avec correction

Measurement of the map of the linear attenuation coefficients.

- Source of ^{68}Ge
  - Utilisable for a large period (T = 271 jours)
  - Important dead time for detector near to the source.
  - Acquisition: 15 to 30 min.
  - Noisy image
  - Small bias compared to true value (measured @ 511 keV)



- Source of simple photons (^{137}Cs)
  - Utilisable for a large period (T = 30,2 ans)
  - Acquisition: 5 to 10 min.
  - Biais with respect to true value
  - Mesurement @ 662 keV: requirement of value conversion 662 keV → 511 keV
  - Possible simultaneous Emission/Transmission acquisition



Acquisition protocole

- Projections acquisition in the presence of an object:

$$I(s,\theta) = I_0(s,\theta) \cdot \mathrm{e}^{-\int_{-\infty}^{+\infty} \mu(x,y)\,dt}$$

- Full flux projections acquisition I_0
- Linear attenuation coefficient reconstruction *μ(x,y)* if required.
- Scaling coefficients if required.
- Calculation of attenuation correction factors (ACF):

$$\mathrm{e}^{\int_{-\infty}^{+\infty} \mu(x,y)\,dt} = \frac{I_0(s,\theta)}{I(s,\theta)}$$

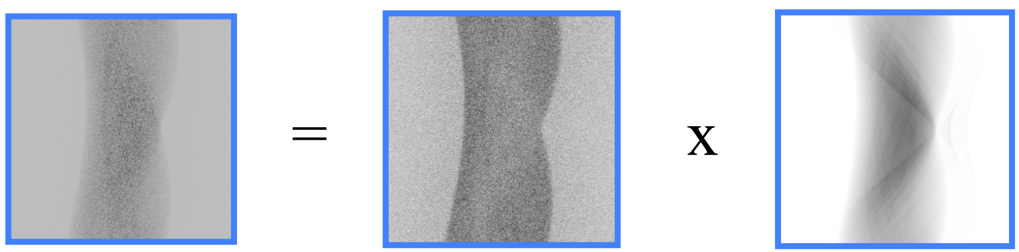
Attenuation correction

Two possible approaches

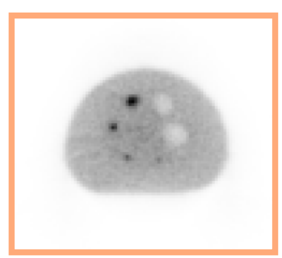
Projections correction

Projections multiplication

$$g_c(s,\theta) = \int_{-\infty}^{+\infty} f(x,y)\,dt \times \mathrm{e}^{-\int_{-\infty}^{+\infty} \mu(x,y)\,dt'} \times ACF$$

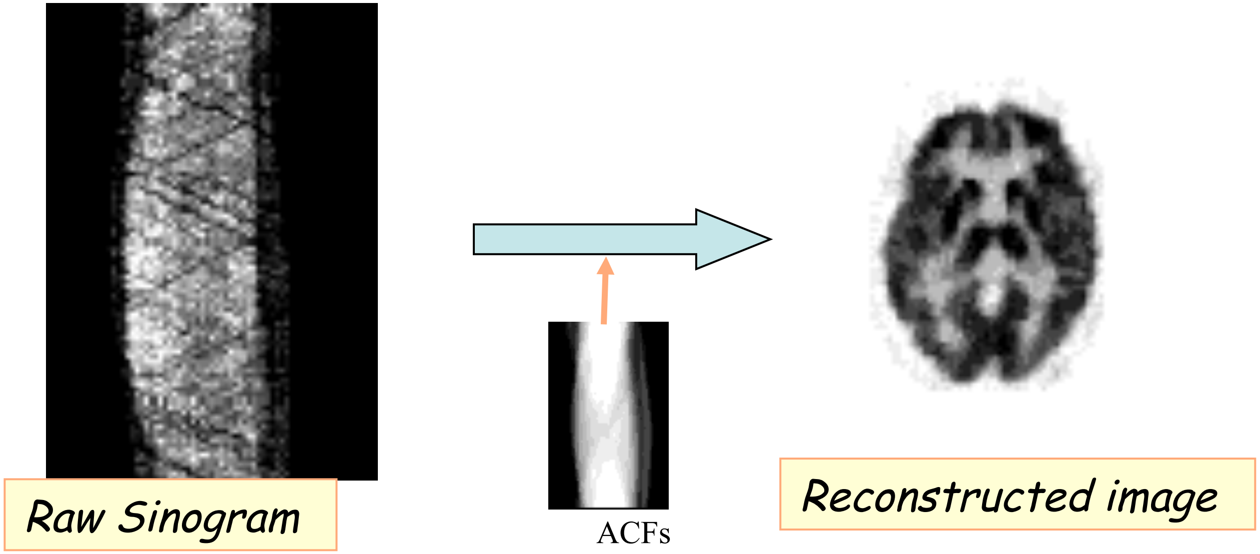


Reconstruction of corrected projections



Correction during reconstruction

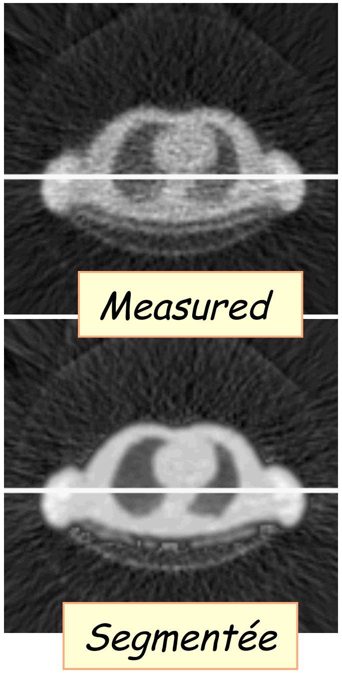
Iterative reconstruction with the modeling of the attenuation into the projector



Problems related to attenuation correction

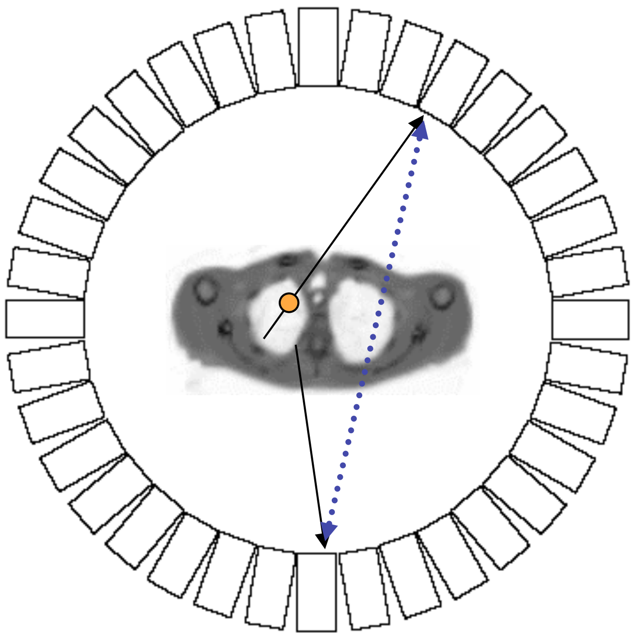
- Patient motion between emission and transmission
  - Data scaling
  - Emission/Transmission simultaneous acquisition

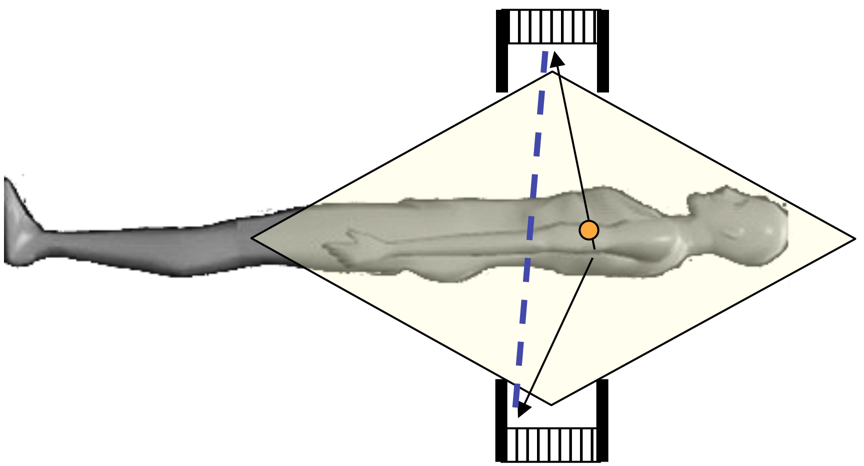
- Noise propagation in images corrected from attenuation
  - Filtering the attenuation map
  - Segmentation of attenuation map
  - Usage of low noise attenuation map



Compton scatter

- In patient ⇨ Mispositioned coïncidence

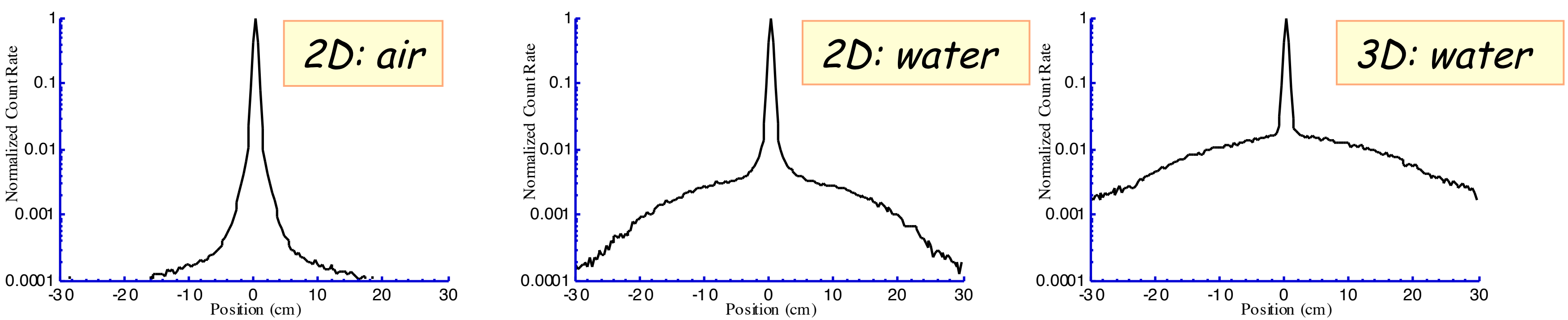




- Dans le cristal ⇨ Détérioration de la résolution intrinsèque
  Rejet d' événements

Scatter artifact in PET

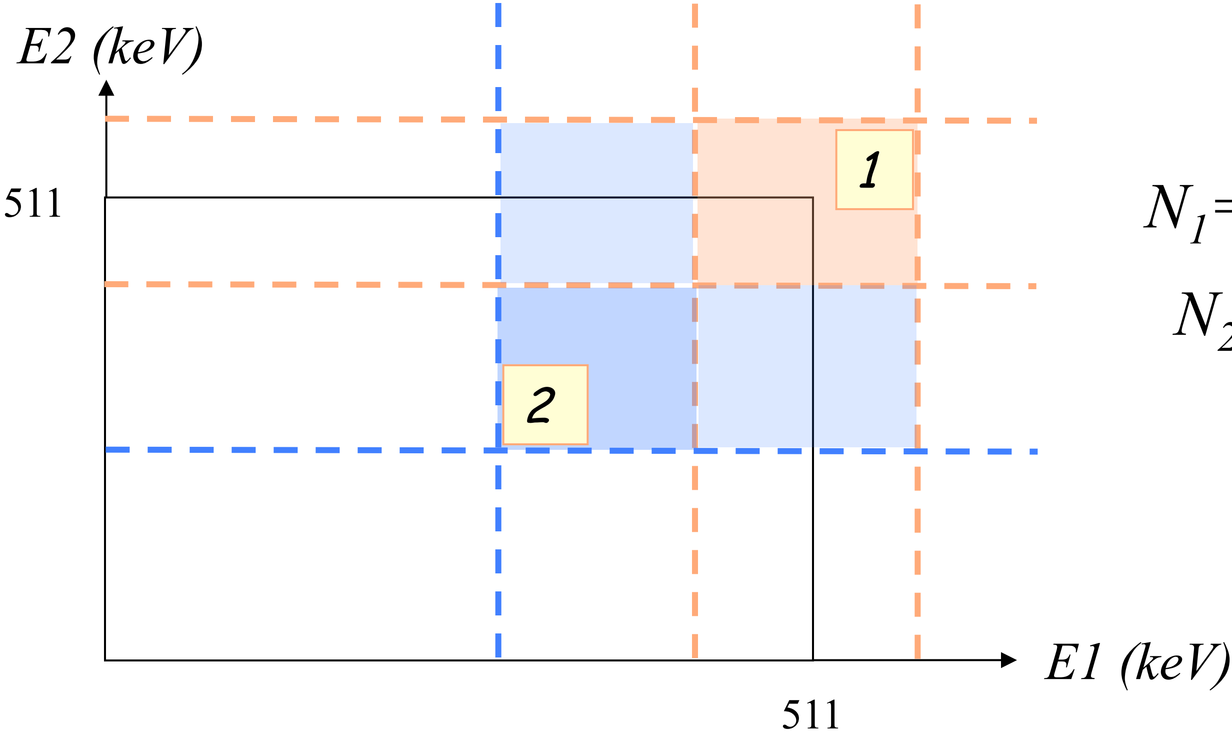
- Mispositioned coïncidences
  - Blurr
  - Loss in image contrast
  - Activity outside of the object
  - Quantitative bias
- More artifact in 3D artifact



Corrections de diffusion en TEP

- Estimation of the number of the scattered photons by energy spectra study:
  - Double energy window
  - True coïncidence estimation
- Estimation of the scattered photons from the projections:
  - Convolution
  - Profile approximation from outside the object
- Estimation of the scatterd by calculating the distribution:
  - Analytical calculation
  - Simulation de Monte Carlo

Double Energy Window (DEW)



$$N_1 = N_{1\_scatterd} + N_{1\_true}$$

$$N_2 = N_{2\_scattered} + N_{2\_true}$$

$$R_{diffuse} = \frac{N_{2\_diffuse}}{N_{1\_diffuse}}$$

$$R_{vrai} = \frac{N_{2\_vrai}}{N_{1\_vrai}}$$

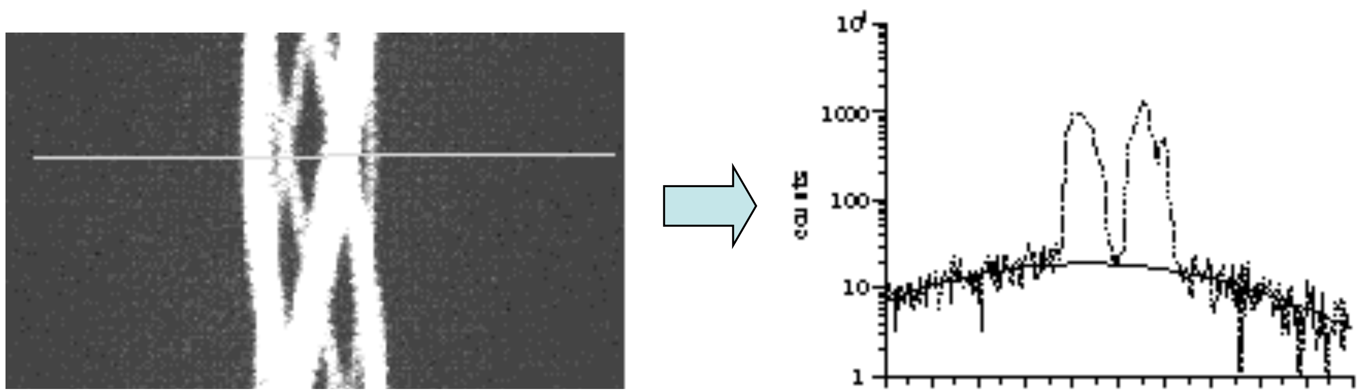
Calibration

$$N_{1\_diffuse} = \left[\frac{N_2}{R_{diffuse} - R_{vrai}}\right] - \left[\frac{N_1 \cdot R_{vrai}}{R_{diffuse} - R_{vrai}}\right]$$

$$N_{1\_vrai} = N_1 - N_{1\_diffuse}$$

Scattered profile approximation

- Hypothesis:
  - Events outside the object
    → scatter distribution
  - Image of scatter = Low frequency image

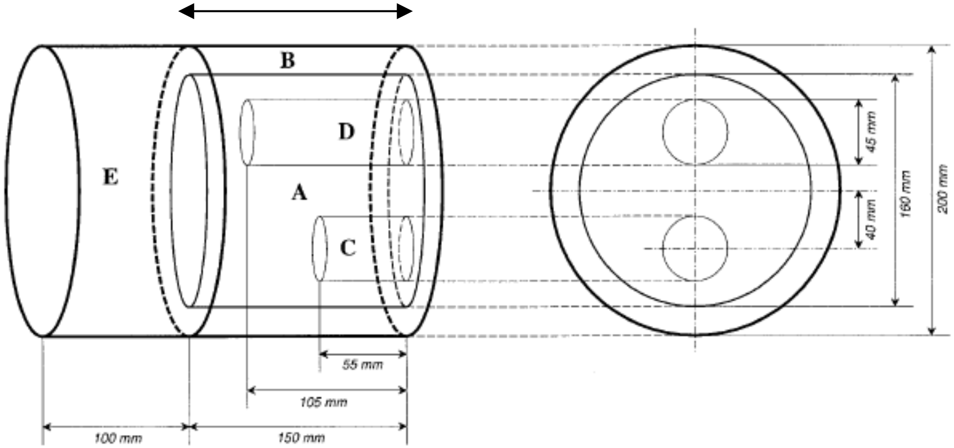


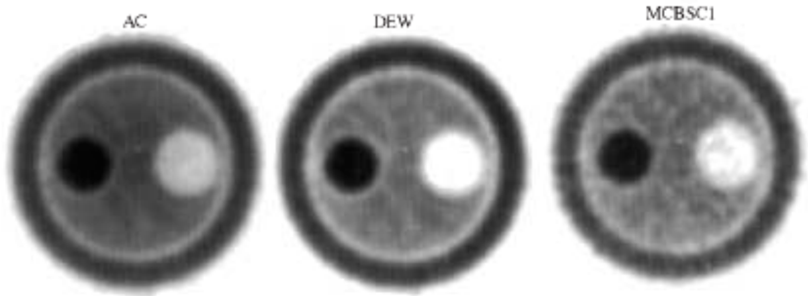
Distribution calculation

- Hypothesis:
  - Known true events distribution
  - Known transmission map
- Algorithm
  - *A* – Analytical calculation or Monte Carlo simulation of the scattered photons distribution.
  - → Scattered sinogram
  - *B* – Sinogram acquired - estimation of scattered sinogram
  - → Sinogram corrected from scatter
  - *C* – Image reconstruction
  - → Estimation of the true events distribution

Possible iteration

3D methods comparison



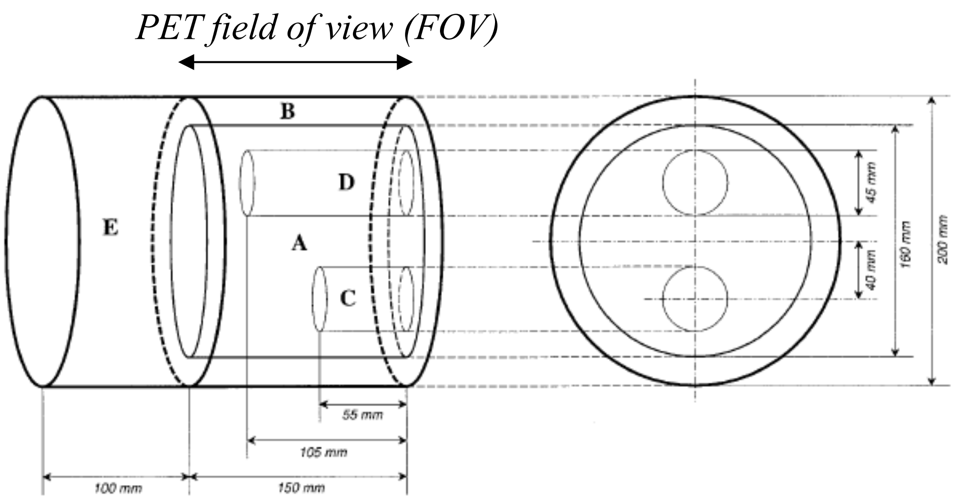


No activity outside the FOV

Figure of merit	Absolute concentration (kBq/ml)				Contrast (%)	SNR	
Case/compartment	B		D		C	A	
Calibration concentration	5.88		4.86		100	–	
AC	7.66±0.28	30%	5.31±0.17	9%	63.82±1.15	21.91±5.17	No correction
DEW	6.05±0.23	3%	4.62±0.18	-5%	91.63±1.84	15.42±3.64	DEW
CVS	6.49±0.30		4.68±0.23		84.11±3.85	18.79±4.54	
SRBSC	6.52±0.30		4.76±0.22		86.26±3.95	19.46±4.72	
MCBSC1	6.51±0.24	11%	4.81±0.21	-1%	81.31±3.93	9.74±2.43	Monte Carlo
MCBSC2	6.55±0.27		4.78±0.15		85.02±1.76	10.32±2.05	

Zaidi et al, Eur J Nucl Med 2000:1813-1826

3D methods comparison



Activity outside of the FOV

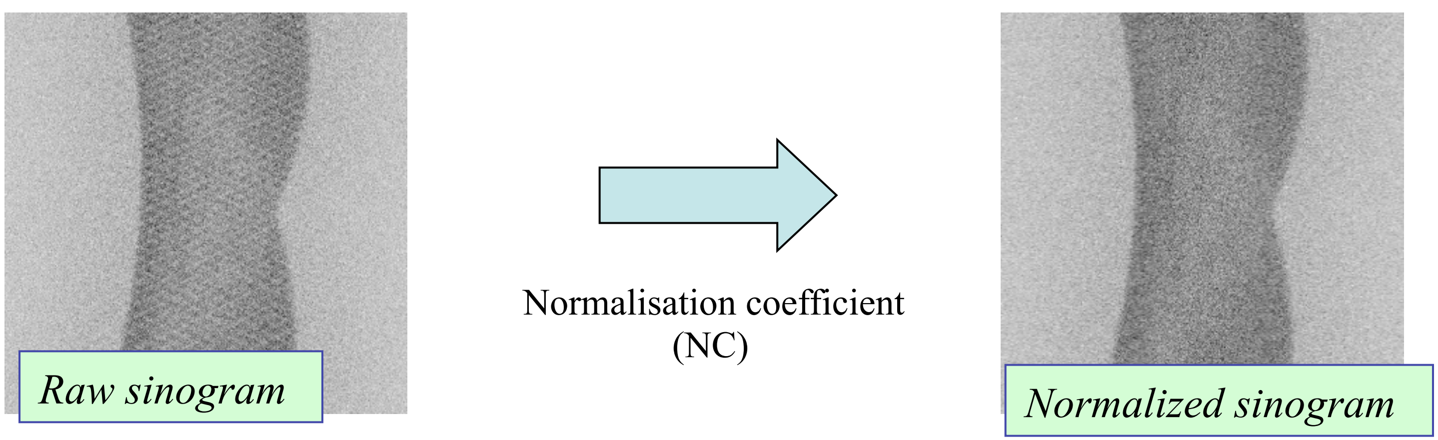
Figure of merit	Absolute concentration (kBq/ml)				Contrast (%)	SNR	
Case/compartment	B		D		C	A	
Calibration concentration	5.88		4.86		10	–	
AC	7.94±0.30	35%	5.47±0.15	13%	64.60±1.08	19.04±4.69	*No correction*
DEW	6.14±0.21	4%	4.61±0.10	-5%	95.74±2.09	12.37±3.97	*DEW*
CVS	6.72±0.32		4.82±0.20		84.90±3.34	16.24±4.33	
SRBSC	6.76±0.32		4.90±0.19		86.78±3.30	16.81±4.60	
MCBSC1	6.62±0.31	13%	4.72±0.24	-3%	86.23±2.64	9.78±3.37	*Monte Carlo*
MCBSC2	6.77±0.24		4.94±0.18		86.33±1.54	9.33±2.33	

Zaidi et al, Eur J Nucl Med 2000:1813-1826

Normalization correction

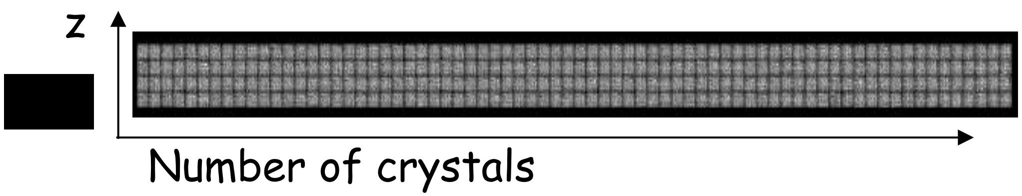
- Required correction:
  - No detection uniformity between different detectors.
  - Geometrical factors (curvature of the detection ring)
  - Dead zone in collected data

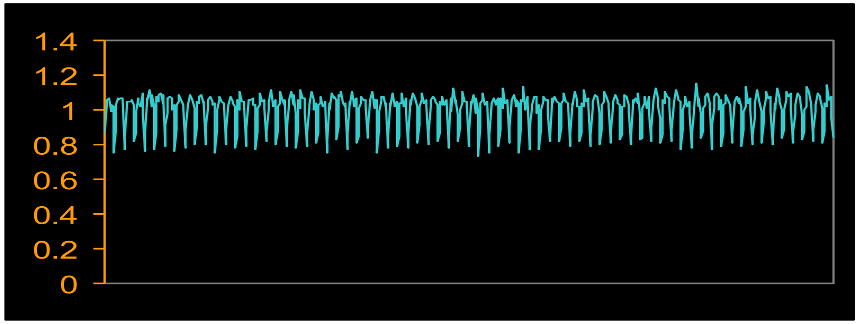
Each system line response should have the same detection efficiency

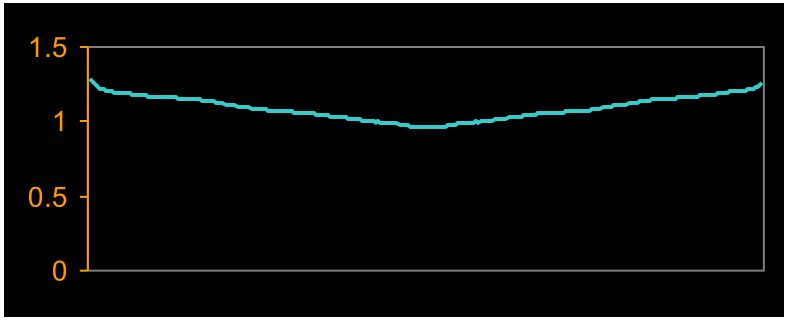


Normalisation coefficients

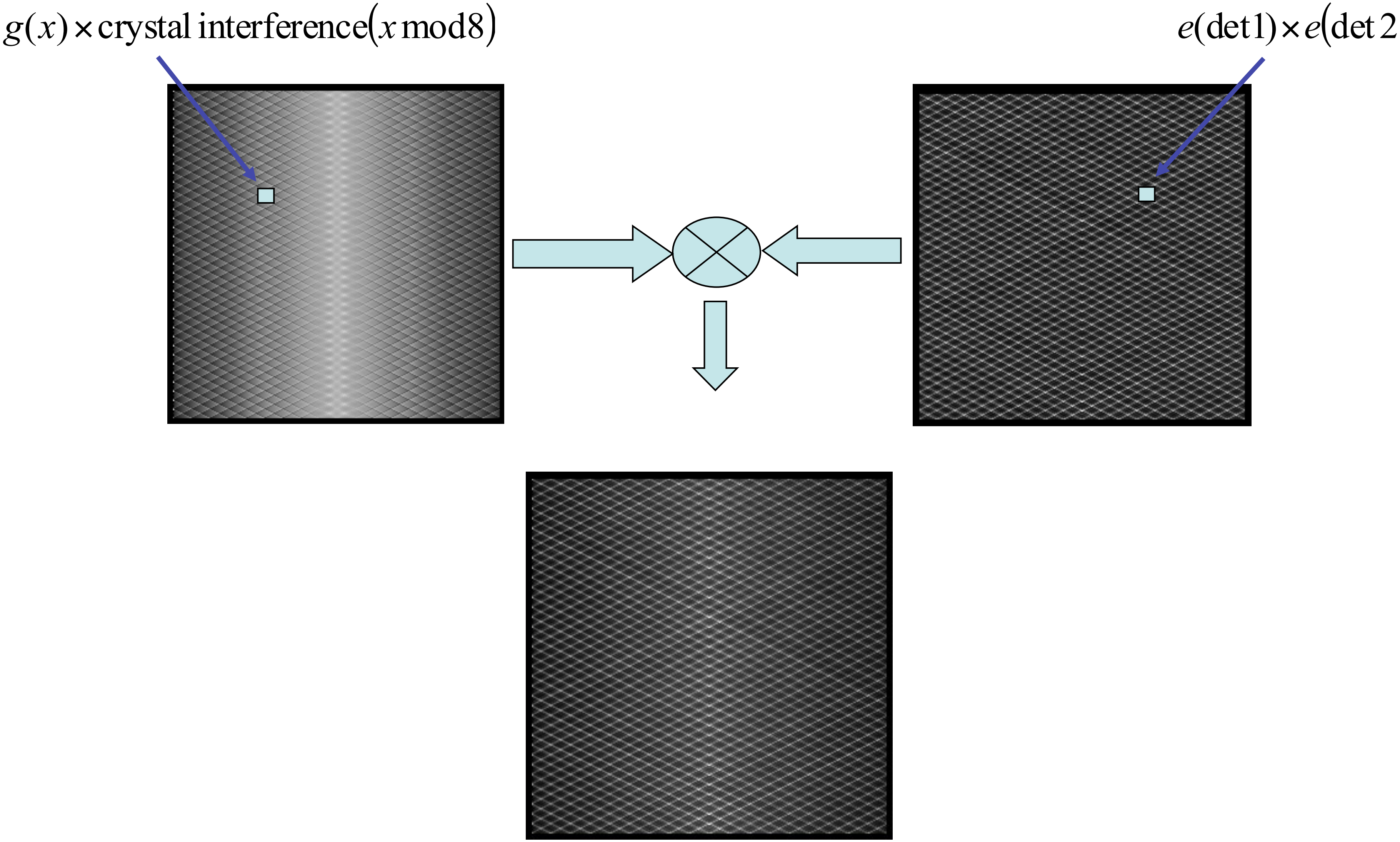
- Full flux acquisition
  - All lines of response are irradiated by the same source.
  - Require a high number of counts (important statistics).
- Modeling
  - NC = Crystal efficiency x Geometrical factor



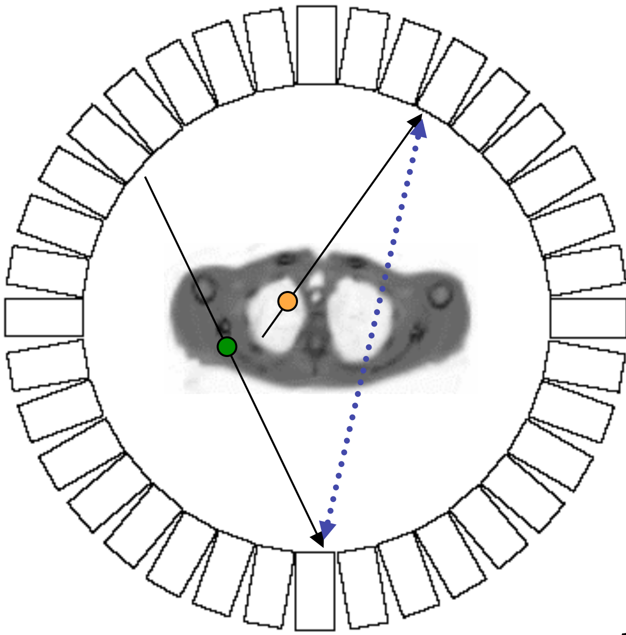


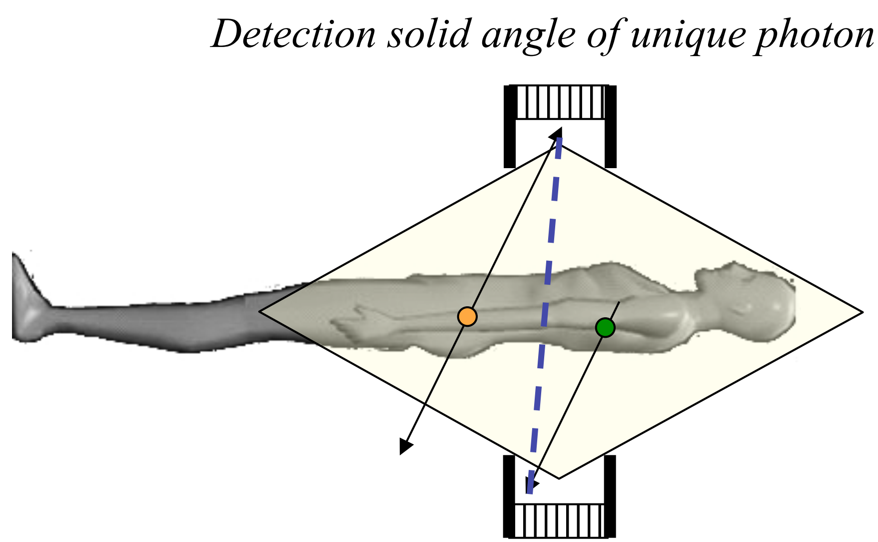


Obtaining the coefficients



Random coïncidences in PET



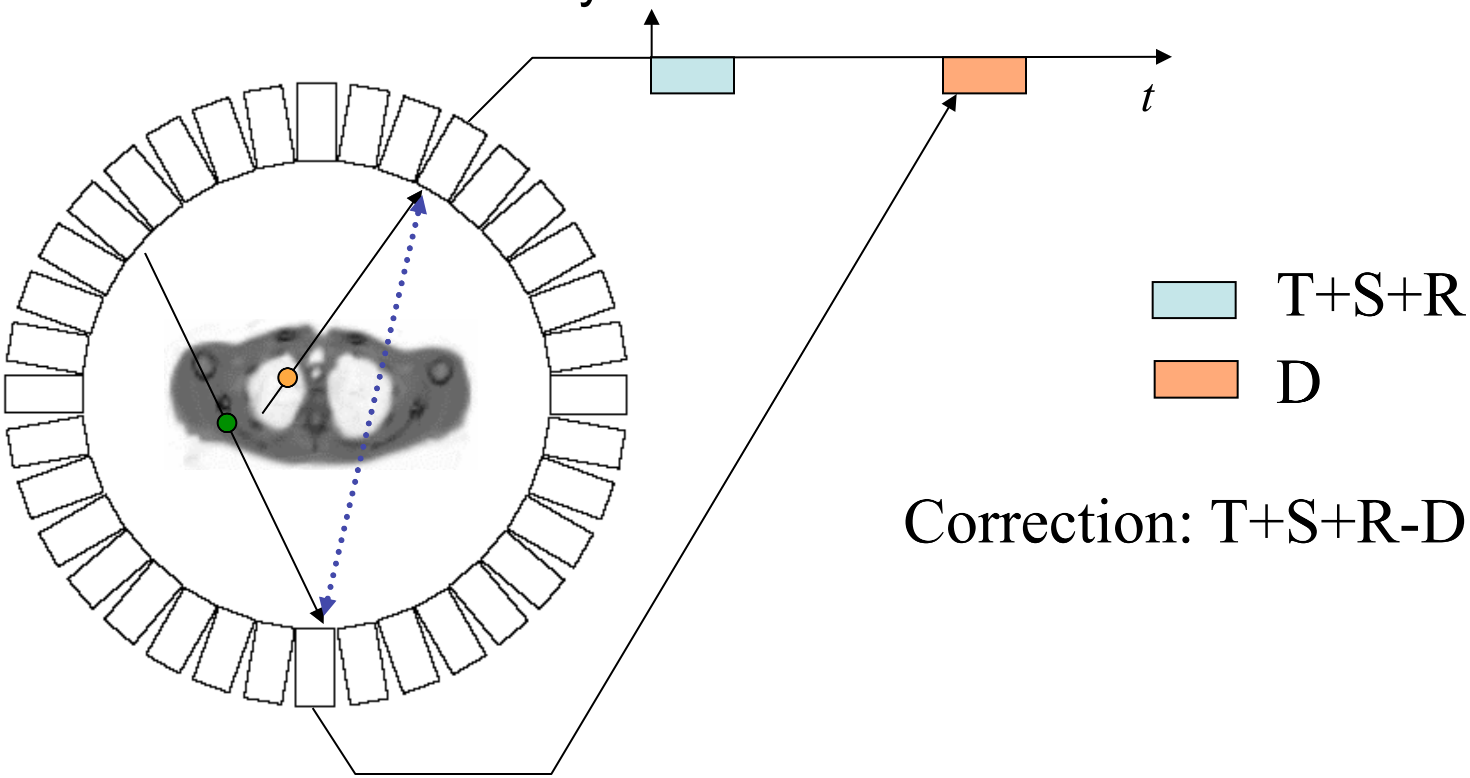


$$R_{ij} = 2\tau S_i S_j$$

- Random coïncidences depend on:
  - Used time window
- Consequences of the random coïncidences:
  - Bad localisaton
  - A quantitatif bias

Correction of random coïncidences

- Measurement of delayed coïncidences
  - Utilisation of a delayed window



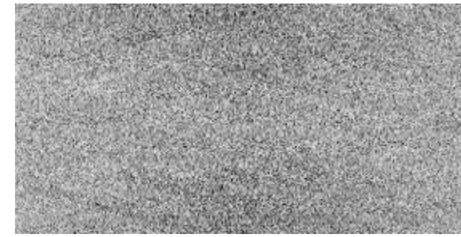
Correction: T+S+R-D

Correction of random coïncidences

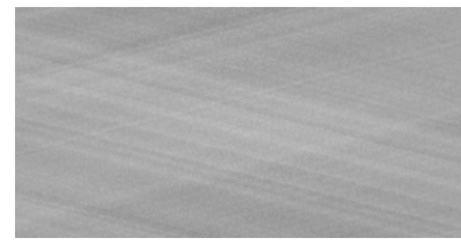
- Variance reduction
  - Smoothing of the random coïncidences sinogram delayed with an appropriate methods.
  - Estimation of the sinogram of random coïncidences fortuites from the measurement of simple photons.

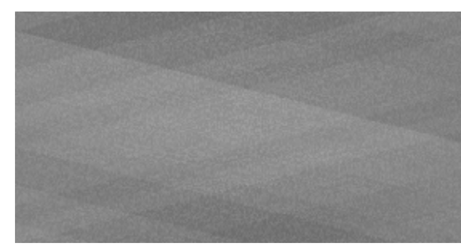
$$R_{ij} = 2\tau S_i \varepsilon_i S_j \varepsilon_j$$

Simples Normalisation



Noisy Sinogram





Question:
5 minutes whole body acquisition
Rate of measured coïncidences = 50 000 cps
Rate of random coïncidences = 20 000 cps

Acquisition in 3D mode with 10^6 LORs

Compare the two correction methods (noisy and not noisy) in terms of % $\Delta N_{true}/N_{true}$ per LOR

Hypothesis:
Scattered coïncidences are negligeables
True and random coïncidences are distributed uniformly on all the LORs

Total number of coïncidences per LOR: 50000 x 300 / 10^6 = 15
Number of random coïncidences per LOR: 20000 x 300 / 10^6 = 6

Number of true coïncidences per LOR: 15 – 6 = 9

Noisy method

$$\Delta N_{vraie} = \sqrt{N_{vraie} + 2 \times N_{fortuit}} = \sqrt{9+12} = 4,58$$

$$100 \times \frac{\Delta N_{vraie}}{N_{vraie}} = 50,9\%$$

No Noisy method

$$\Delta N_{vraie} = \sqrt{N_{vraie} + N_{fortuit}} = \sqrt{9+6} = 3,87$$

$$100 \times \frac{\Delta N_{vraie}}{N_{vraie}} = 43\%$$

Quality image estimation

- How to estimate the image quality?
  - Based on the concepts of:
    - *Sensibility: detection of true positives*
    - *Specificity: détection de false positives*
    - *→ Receiver Operating Charactéristics: represents the probability of detecting a true positive as function of detecting a false positive.*
    - *Difficult set up*

- Estimate the signal to noise ratio in the image: Noise Equivalent Count Rate

Signal to noise ratio in the image

- $$SNR = k\frac{t_e}{\sqrt{VAR_e}}$$

  Number of true coïncidences (t_e)

  Pixel variance (VAR_e)

- A line of response registers the following signal:

$$\left(t_p + s_p + r_p\right) - \left(s_p + r_p\right)$$

True coïncidences (t_p)

Scattered Coïncidences (s_p)

Random coïncidences (r_p)

After corrections

Variance calculation

The variance of an image uncorrected from attenuation and dead time:

Weighting factor

$$VAR = \sum_m w_m \left(t_p + s_p + r_p\right)$$

$$VAR = \sum_m w_m t_p \left(1 + \alpha_{sp} + \alpha_{rp}\right)$$

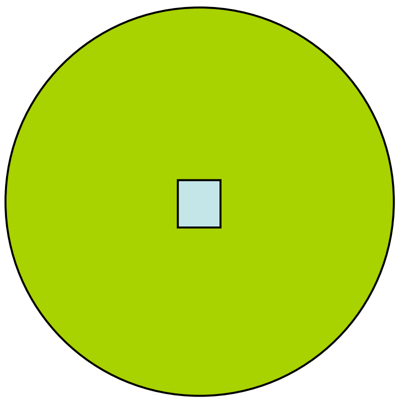
Number of projections

Hypothesis: Scattered and random coïncidences correction does not increase the signal

Correction factor

After dead time and attenuation correction: $VAR_e = ac^2 \cdot VAR$

Simplifying the problem



Cylindre corrected
From attenuation and dead time

Analyse the pixel in the centre of the reconstructed image

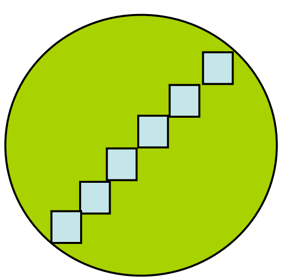
Number of true coïncidences
Per lime of response

Cylindre diameter

Number of true coïncidences
In the pixel

$$t_p = \frac{D \cdot t_e}{d \cdot m \cdot ac}$$

Pixel size



Number of coïncidences in a pixel

Number of true coïncidences
In the whole image without correction

$$t_e = T \cdot moy(ac) \cdot \frac{d^2}{\frac{\pi}{4} D^2}$$

$$VAR_e = ac^2 \cdot VAR$$

$$SNR = k \frac{t_e}{\sqrt{VAR_e}}$$

$$t_e = T \cdot moy(ac) \cdot \frac{d^2}{\frac{\pi}{4} D^2}$$

$$SNR = k . \sqrt{\frac{4}{\pi}} \left(\frac{d}{D} \right)^{\frac{3}{2}} \sqrt{\frac{moy(ac)}{ac}} \sqrt{\frac{T}{1 + \alpha_{sp} + \alpha_{rp}}}$$

Noise Equivalent Count

$$SNR = k.\sqrt{\frac{4}{\pi}}\left(\frac{d}{D}\right)^{\frac{3}{2}}\sqrt{\frac{moy(ac)}{ac}}\sqrt{NEC}$$

$$NEC \approx SNR^2$$

True coïncidences in the image

$$NEC = \frac{T}{1 + \alpha_{sp} + \alpha_{rp}}$$

Fraction for a line of response

Reminder
Hypothesis: Corrections of random and scattered coïncidences do not introduce noise

In presence of noise,

	Signal	*Variance*
Random coïncidences:	$t_p + s_p + r_p - r_p$	$t_p + s_p + (1+\beta) f_{FOV} r_p$
Scattered coïncidences:	$t_p + s_p + r_p - r_p - \alpha_{cp}(t_p + s_p)$	$t_p + s_p + \beta r_p + \alpha_{cp}^2 (t_p + s_p + (1+\beta) f_{FOV} r_p)$ $(1+\alpha_{cp}^2)(t_p + s_p + (1+\beta) f_{FOV} r_p)$

Noise Equivalent Count

$$NEC = \frac{T}{1 + \alpha_{sp} + \alpha_{rp}}$$

$$NEC = \frac{T}{\left(1 + \alpha_{cp}^2\right)\left(t_p + s_p + \left(1 + \beta\right) f_{FOV} r_p\right)}$$

The X_i are approximations of X_p in the image

$$NEC = \frac{T^2}{\left(1 + \alpha_{ci}^2\right)\left(T + S_i + \left(1 + \beta\right) f_{FOV} R_i\right)}$$

The more often, $\alpha_{ci} = 0$ because the scattered correction does not introduce noise

$$NEC = \frac{T^2}{\left(T + S_i + \left(1 + \beta\right) f_{FOV} R_i\right)}$$

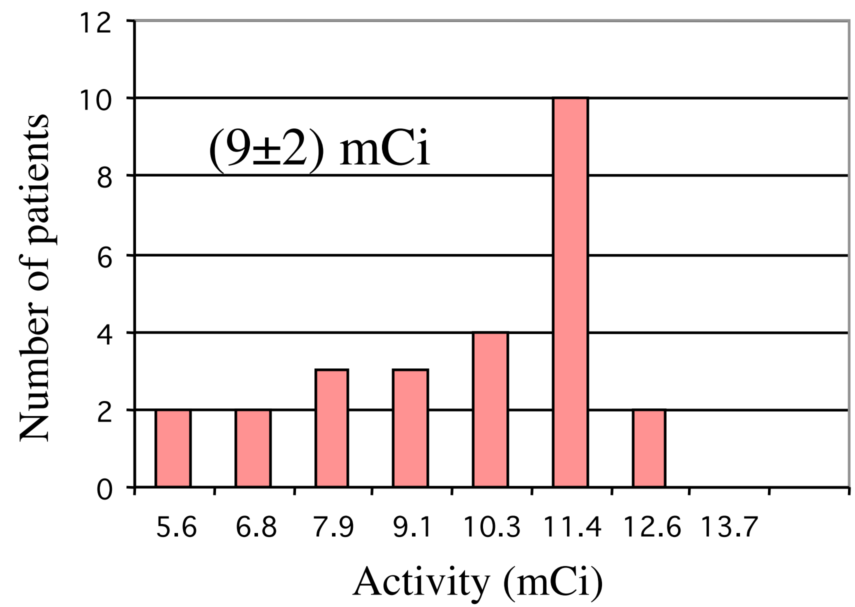
Application example: Random correction methods comparison

TEP HR+

Detector size (mm)	4.0 x 4.1 x 30
No. of slices	63
Slice width (mm)	2.4
Ring diameter (cm)	82.7
Axial FOV (cm)	15.2
Max. axial angle	10.2°
Mode	2D/3D

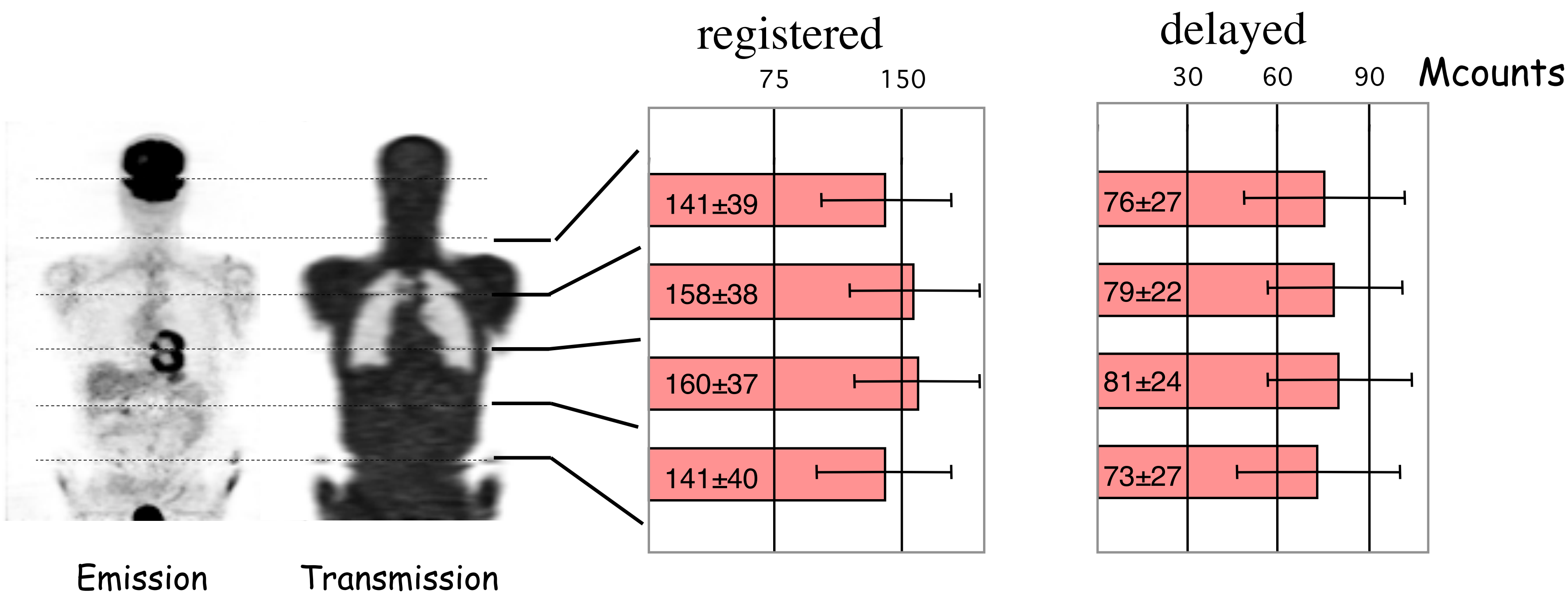
23 patients (13m, 10f)
(170 ± 10) cm
(76 ± 16) kg

Dose distribution,
1h agter injection de [^{18}F]-FDG



Methods comparison

Count rate per bed position

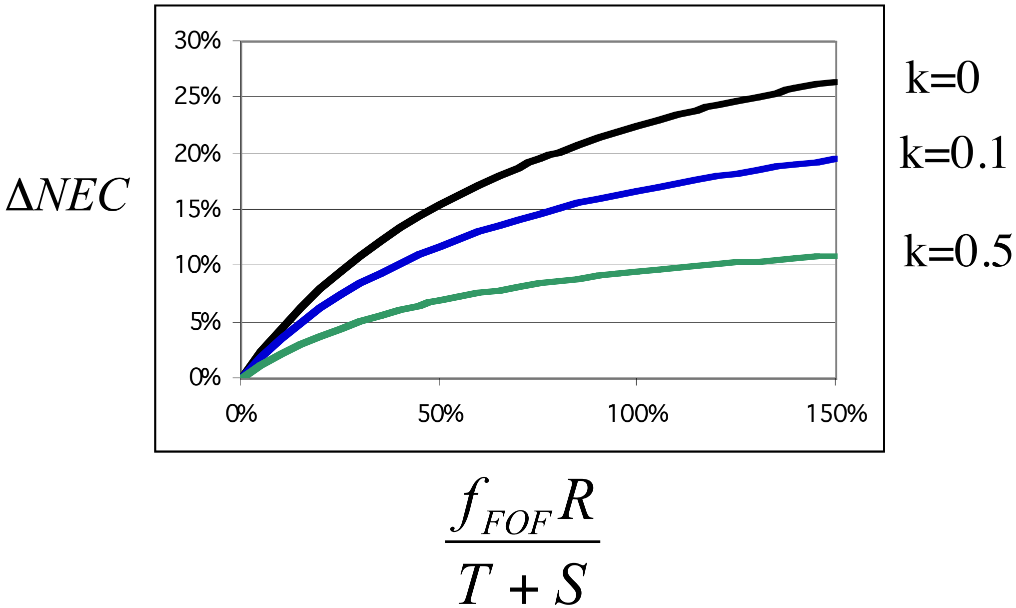


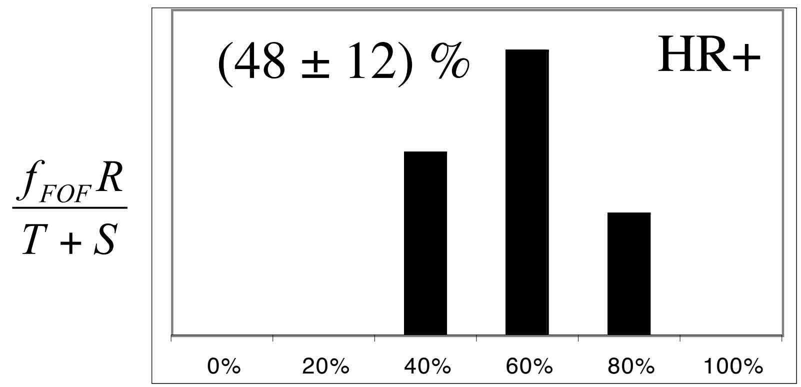
Methods comparison

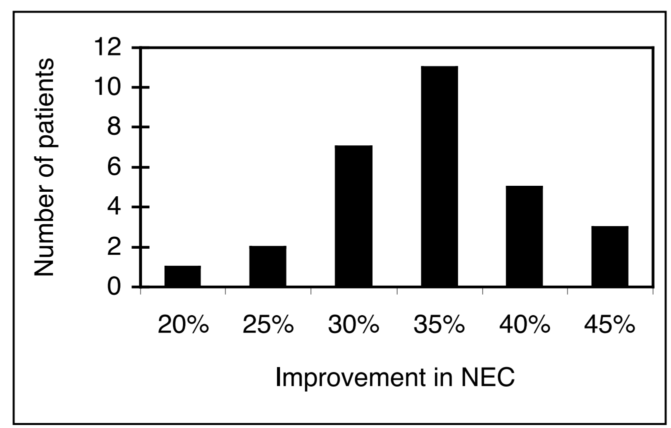
Relative improvement of NEC:

$$\Delta \mathrm{NEC} = 100 \cdot \frac{\mathrm{NEC}_{\mathrm{Noiseless}} - \mathrm{NEC}_{\mathrm{Online}}}{\mathrm{NEC}_{\mathrm{Online}}}$$

$$\Delta NEC(k) = \sqrt{1 + \frac{1-k}{1 + k + \frac{T+S}{f_{FOV} R}}} - 1$$







In summary

TABLE 1
Methods for Estimating Random Coincidences

Method	Comments
Delayed coincidences	Accurate. Higher noise (Eq. 1). Lowest processing requirements.
Smoothed delayed coincidences	Accurate. Lower noise ($0 \leq k \ll 1$, Eq. 1). Higher processing requirements.
Calculated from single photon rates	Potential for bias if scanner is not properly calibrated. Lower noise ($0 \leq k \ll 1$, Eq. 1). Low processing requirements.

Variance reduction used with high $\frac{f_{FOF}R}{T+S}$

Crystal choice