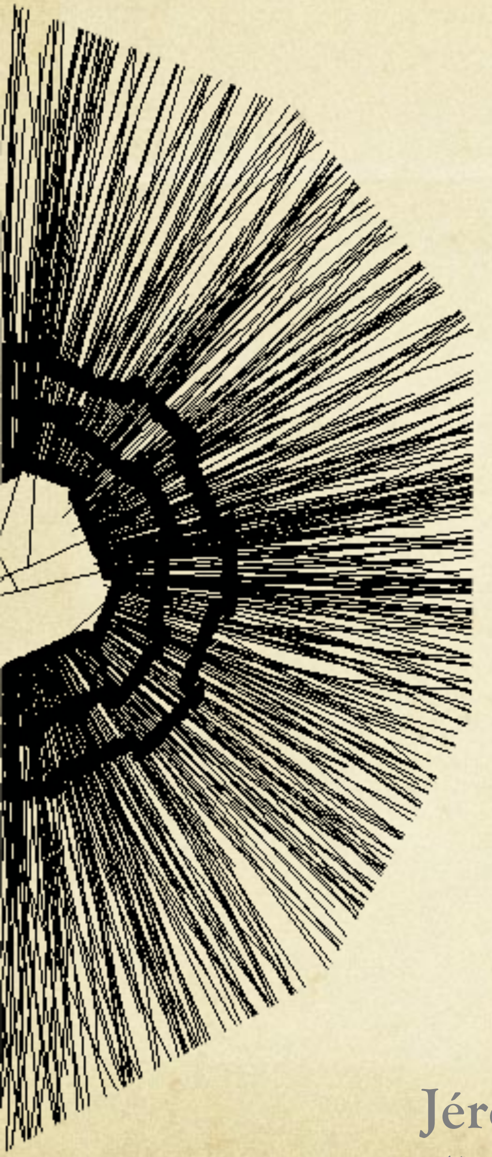

European School of Instrumentation in Particle & Astroparticle Physics

6-7 February 2017, Archamps

Tracking

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Lecture outline

1. Basic concepts
2. Position sensitive detectors

first lecture

3. Standard algorithms

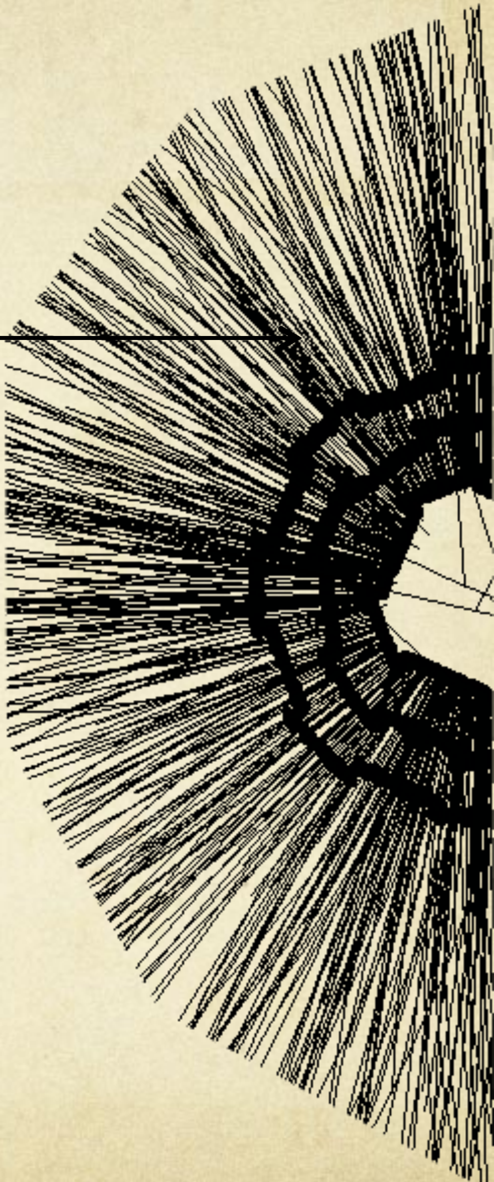
second lecture

4. Advanced algorithms
5. Optimizing a tracking system

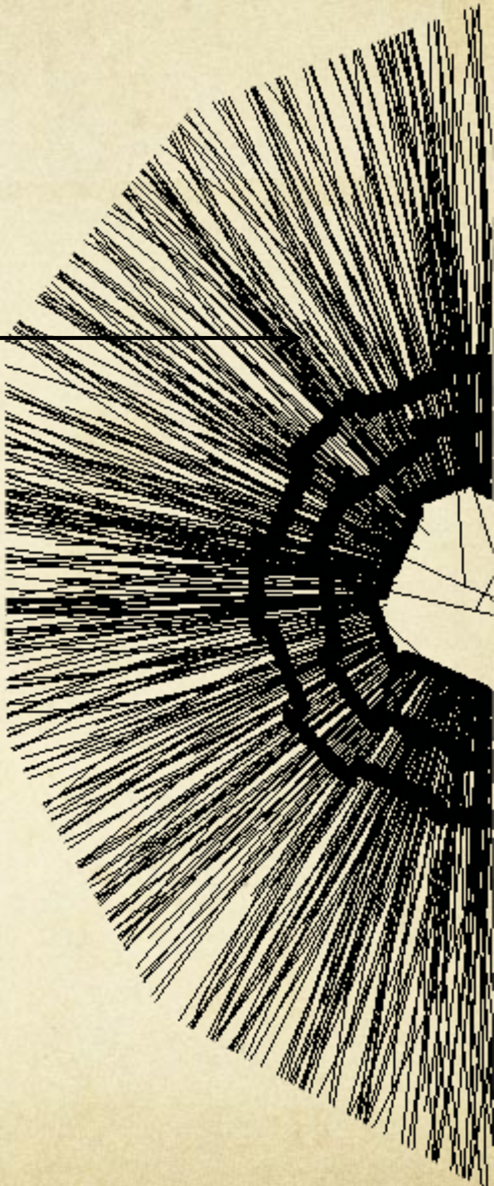
third lecture

practice

6. References



1. Motivations & basic concepts

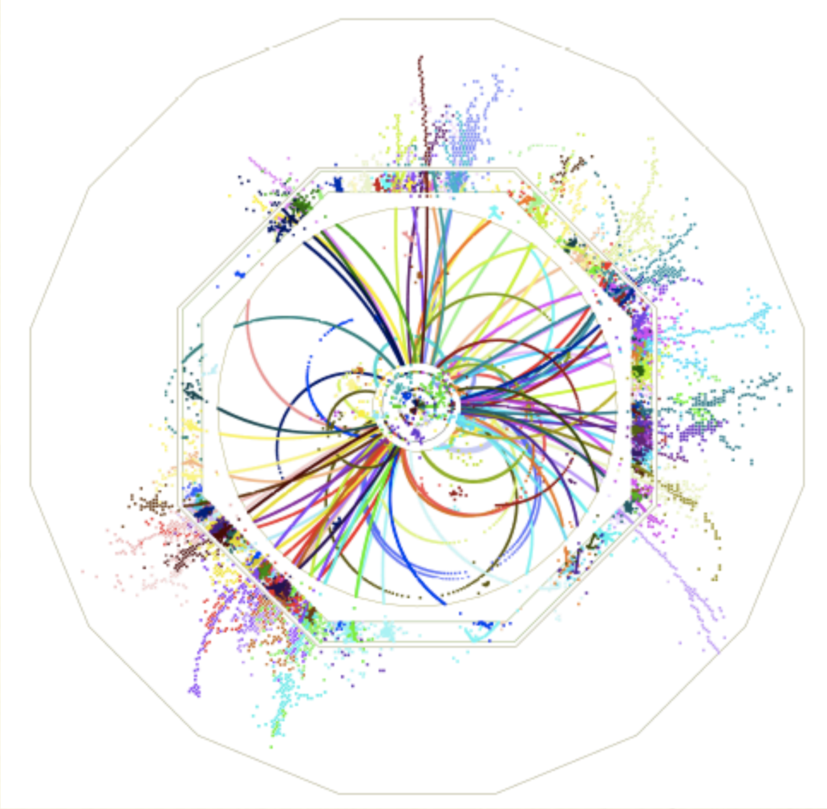


- Motivations
- Types of measurements
- The 2 main tasks
- Environmental considerations
- Figures of merit

Motivations

- Understanding an event
  - Individualize tracks ≃ particles
  - Measure their properties
  - LHC: ~1000 particles per 25 ns "event"

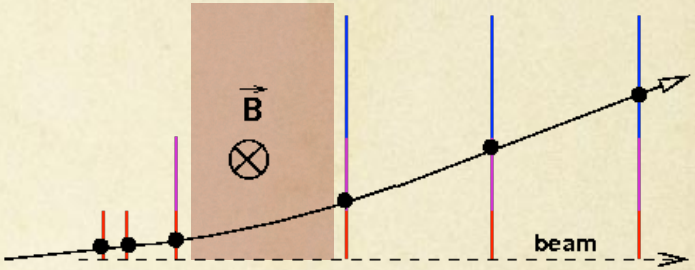
- Track properties
  - Momentum ⇔ curvature in B field
    - Reconstruct invariant masses
    - Contribute to jet energy estimation
  - Energy ⇔ range measurement
    - Limited to low penetrating particle
  - Mass ⇔ dE/dx measurement
  - Origin ⇔ vertexing (connecting track)
    - Identify decays
    - Measure flight distance
  - Extension ⇔ particle flow algorithm (pfa)
    - Association with calorimetric shower



8 jets event (tt-bar h) @ 1 TeV ILC

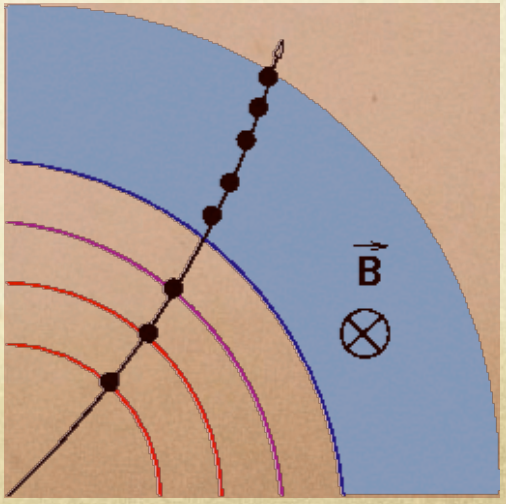
Momentum measurement

- Magnetic field curves trajectories $\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = q\vec{v} \times \vec{B}$
  - Rewritten with position (x) and path length (l) ➞ basic equation: $\frac{\mathrm{d}^2\vec{r}}{\mathrm{d}l^2} \propto \frac{q\vec{B}(\vec{x})}{\|\vec{p}\|}\frac{\mathrm{d}\vec{r}}{\mathrm{d}l}$
  - In B=4T a 10 GeV/c particle will get a sagitta of 1.5 cm @ 1m
- Fixed-target experiments
  - Dipole magnet on a restricted path segment
  - Measurement of deflection (angle variation)



- Collider experiment
  - Barrel-type with axial B over the whole path
  - Measurement of curvature (sagitta)

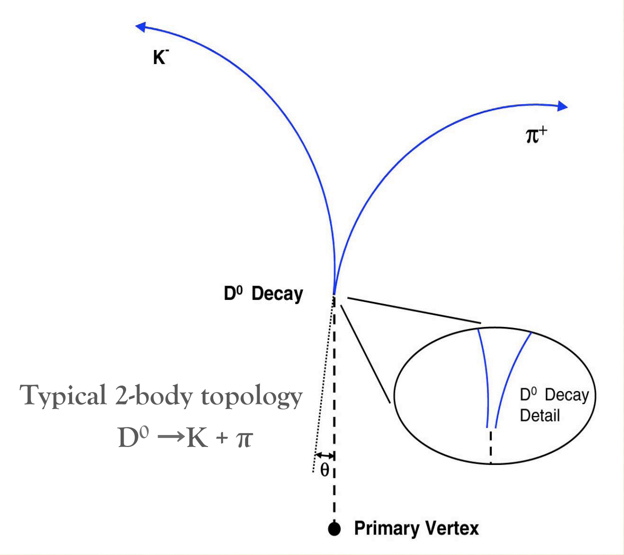
$$\frac{p_T(\mathrm{GeV/c})}{q} = 0.3\cdot B(\mathrm{T})\cdot R(\mathrm{m})$$



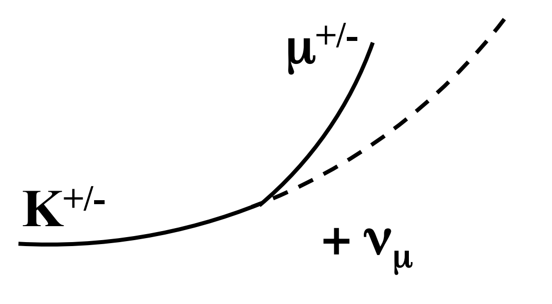
- Other arrangements
  - Toroidal B… not covered
- Two consequences
  - Position sensitive detectors needed
  - Perturbation effects on trajectories limit precision on track parameters

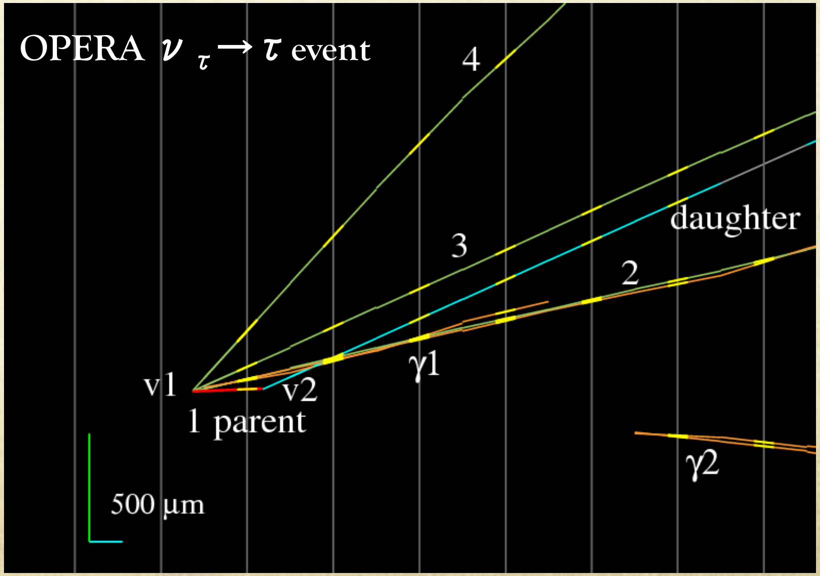
Vertex measurements 1/3

- Identifying through topology
  - ➛ Short-lived weakly decaying particles
    - Charm c τ ~ 120 µm
    - Beauty c τ ~ 470 µm
    - tau, strange/charmed/beauty particle
- Exclusive reconstruction
  - ➛ Decay topology with secondary vertex
  - ➛ Exclusive = all particles associated
- Inclusive “kink” reconstruction
  - ➛ Some particles are invisible (ν)



Typical 2-body topology
$D^0 \rightarrow K + \pi$

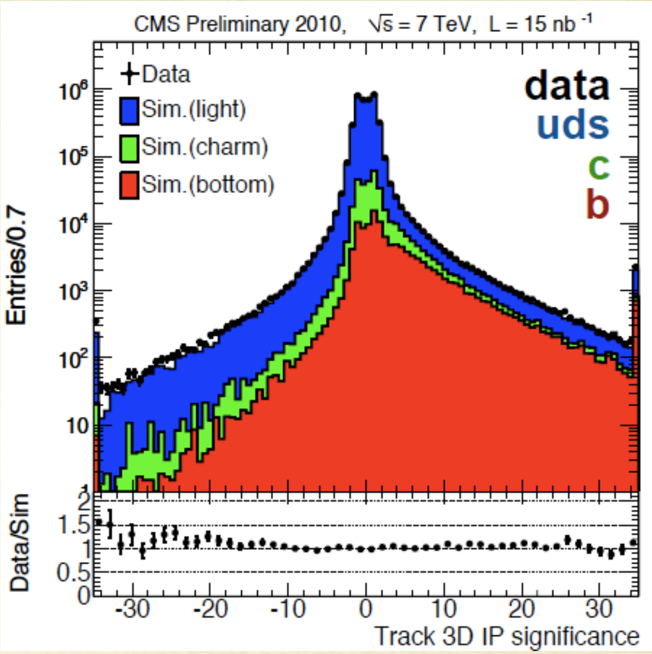




Vertex measurements 2/3

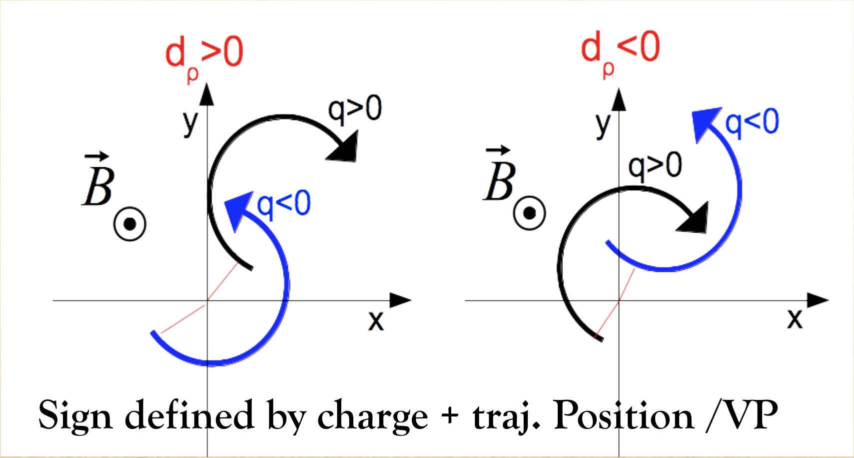
Inclusive reconstruction

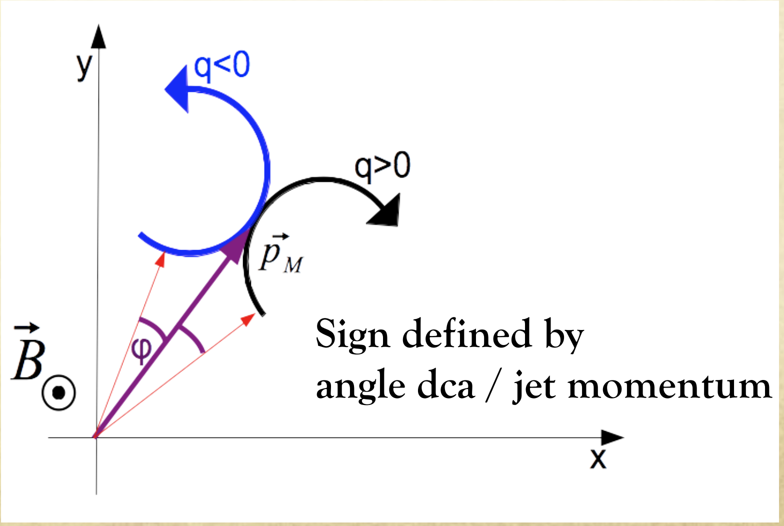
- Selecting parts of the daughter particles = flavor tagging
- based on impact parameter (IP)
- σ_{IP} ~ 20-100 μm requested



Definition of impact parameter (IP)

- Also DCA = distance of closest approach from the trajectory to the primary vertex
- Full 3D or 2D (transverse plane d_ρ) +1D (beam axis)
- Sign extremely useful for flavor-tagging

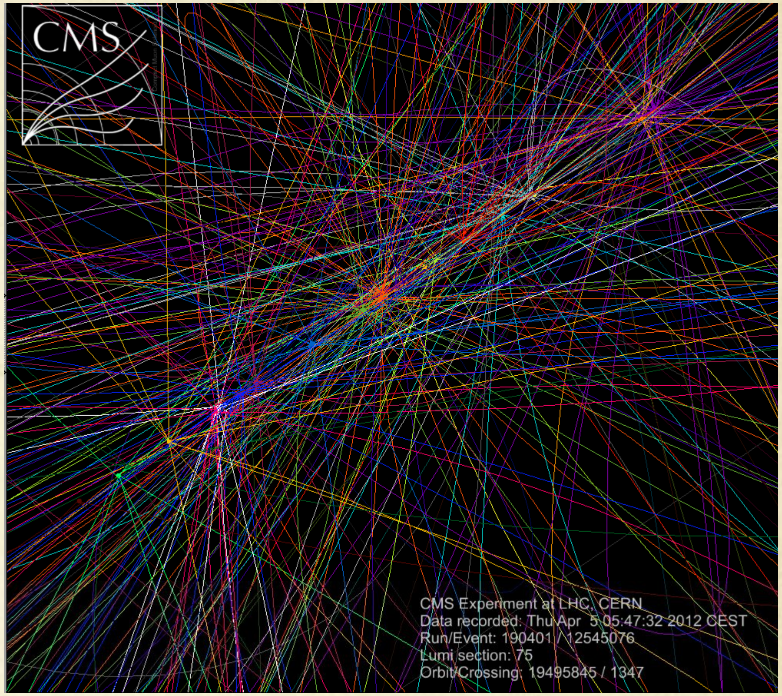




Vertex measurements 3/3

- **Finding the event origin**
  - Where did the collision did occur?

    = Primary vertex
  - (life)Time dependent measurements
    - CP-asymmetries @ B factories (Δz≃60-120 μm)
  - Case of multiple collisions / event
    - >> 10 vertex @ LHC



- **Remarks for collider**
  - Usually no measurement below 1-2 cm / primary vertex
  - Requires extrapolation ➞ expect uncertainties

Energy measurement

- **Usually not a tracker task**
  - CALORIMETERs (see lecture by Isabelle)
  - Indeed calorimeters gather material to stop particles while trackers try to avoid material (multiple scattering)
  - however…calorimetry tries to improve granularity
- **Particle flow algorithm**
  - LHC / ILC
- **Energy evaluation by counting particles**
  - Clearly heretic for calorimetry experts
  - Requires to separate $E_{deposit}$ in dense environment
- **Range measurement for low energy particles**
  - Stack of tracking layers
  - Modern version of nuclear emulsion

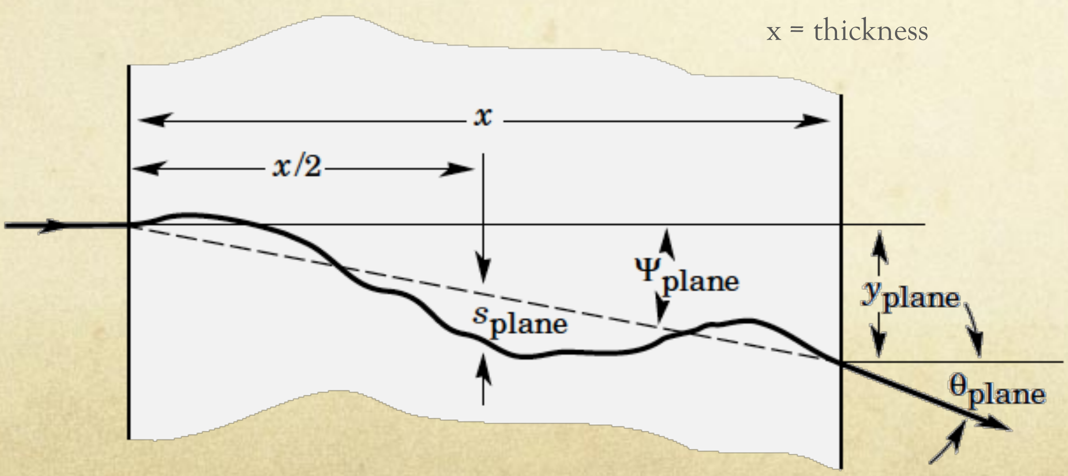
NOT COVERED

Multiple scattering - 1/4

- Reminder on the physics (see other courses)
  - Coulomb scattering mostly on nuclei
  - Molière theory description as a **centered** gaussian process
    - the thinner the material, the less true ➛ large tails

- In-plane description (defined by vectors $\mathbf{p}_{in}$, $\mathbf{p}_{out}$)
  - Corresponds to (ϕ, θ) with $\mathbf{p}_{in} = \mathbf{p}_z$ and $p_{out}^2 = p_{out,z}^2 + p_{out,T}^2$ $\begin{cases} p_{out}\cos\theta \approx p_{out,z} \\ p_{out,T} = p_{out}\sin\theta \approx p_{out}\theta \end{cases}$

$$\sigma_\theta = \frac{13.6\ (\mathrm{MeV/c})}{\beta p} \cdot z \cdot \sqrt{\frac{\text{thickness}}{X_0}} \cdot \left[1 + 0.038\ \ln\left(\frac{\text{thickness}}{X_0}\right)\right] \qquad \left(\text{note: } \phi \in [0, 2\pi] \text{ uniform}\right)$$



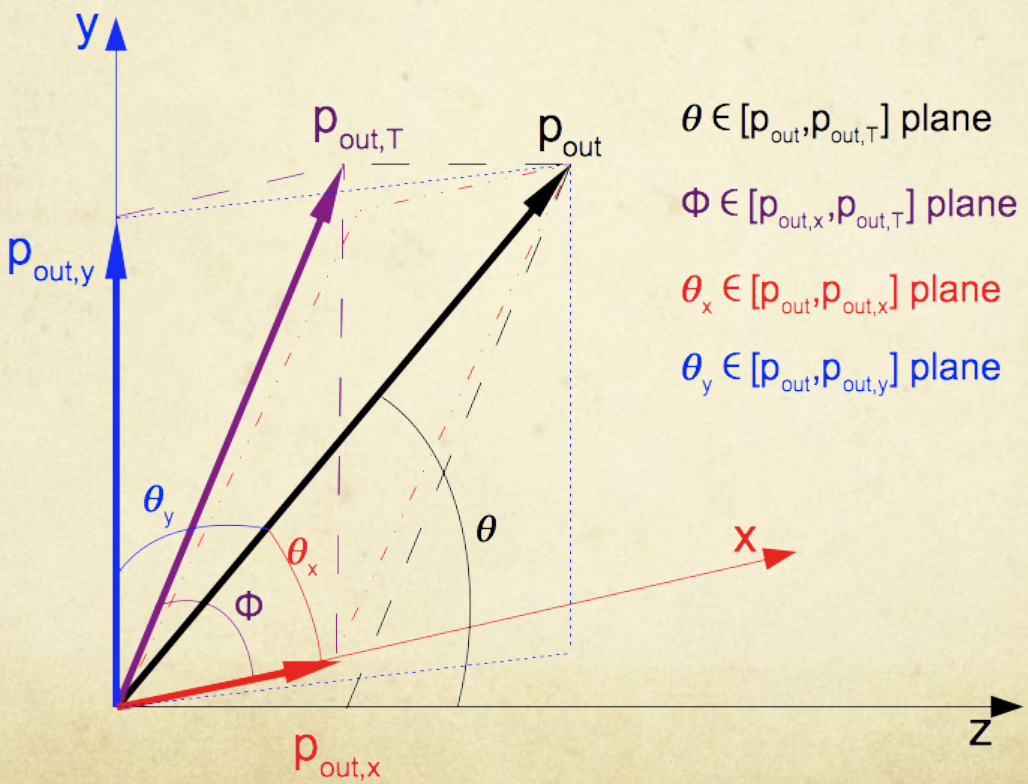
Xo = radiation length

Same definition as in calorimetry

This is accidental

Multiple scattering – 2/4

- In-space description (defined by fixed x/y axes)
  - Corresponds to ($\boldsymbol{\theta}_x$,$\boldsymbol{\theta}_y$) with $p_{out,T}^2 = p_{out,x}^2 + p_{out,y}^2$ $\begin{cases} p_{out}\sin\theta_x \approx p_{out}\theta_x \\ p_{out}\sin\theta_y \approx p_{out}\theta_y \end{cases} \Rightarrow \theta^2 = \theta_x^2 + \theta_y^2$
  - $\boldsymbol{\theta}$x and $\boldsymbol{\theta}$y are independent gaussian processes $\sigma_\theta^2 = \sigma_{\theta x}^2 + \sigma_{\theta y}^2$ and $\sigma_{\theta x} = \sigma_{\theta y} = \dfrac{\sigma_\theta}{\sqrt{2}}$



Multiple scattering – 3/4

- **Important remark when combining materials**
  - Total thickness $T = \Sigma T_i$, each material (i) with $X_0(i)$
  - Definition of effective radiation length ➡ $$X_{0,eff} = \frac{\sum T_i \times X_0(i)}{T}$$
  - Consider single gaussian process $$\sigma_{eff} \propto \sqrt{\frac{T}{X_{0,eff}}}$$

  and never do variance addition (which minimize deviation) $$\cancel{\sigma_{eff} = \sqrt{\sum \frac{T_i}{X_0(i)}}}$$

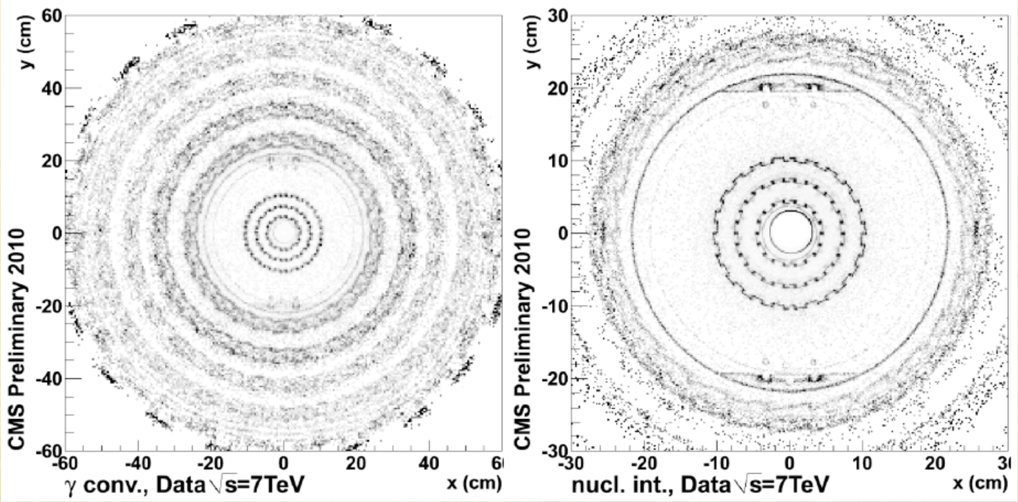
Multiple scattering – 4/4

Impact on tracking algorithm

- The track parameters evolves along the track !
- May drive choice of reconstruction method

Photon conversion

- Alternative definition of radiation length
  probability for a high-energy photon to generate a pair over a path dx: $\text{Prob} = \dfrac{dx}{\frac{9}{7} X_0}$
- $\gamma \rightarrow e^+e^-$ = conversion vertex
- Generate troubles :
  - Additional unwanted tracks
  - Decrease statistics for electromagnetic calorimeter

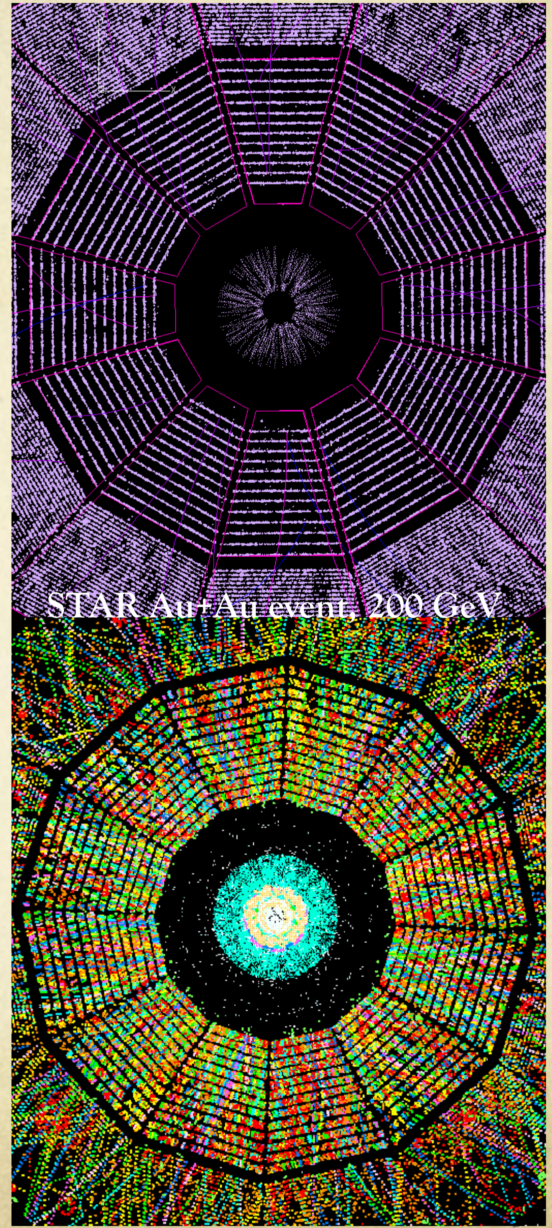


CMS "picture" of material budget through photon conversion vertices (silicon tracker only)

The two main tasks - 1/2

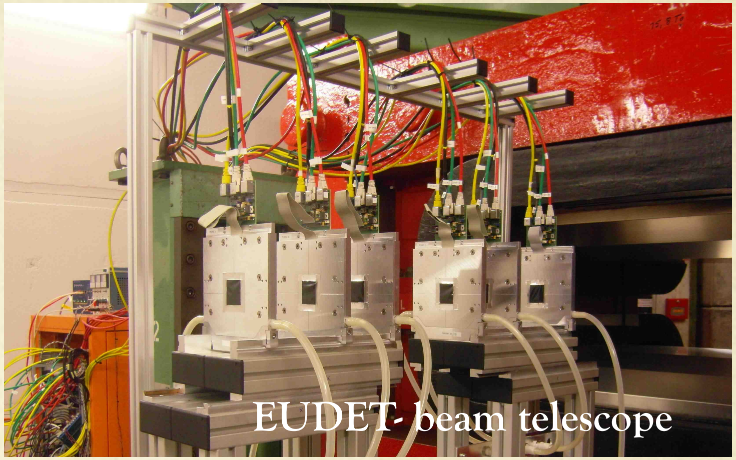
The collider paradigm

- Basic inputs from detectors
  - Succession of 2D or 3D points (or track segments)
    - ➡ Who's who ?
- 2 steps process
  - Step 1: track identification = **finding** = pattern recognition
    - Associating a set of points to a track
  - Step 2: track **fitting**
    - Estimating trajectory parameters ➛ momentum
- Both steps require
  - Track model (signal, background)
  - Knowledge of measurement uncertainties
  - Knowledge of materials traversed (Eloss, mult. scattering)
- Vertexing needs same 2 steps
  - Identifying tracks belonging to same vertex
  - Estimating vertex properties (position + 4-vector)

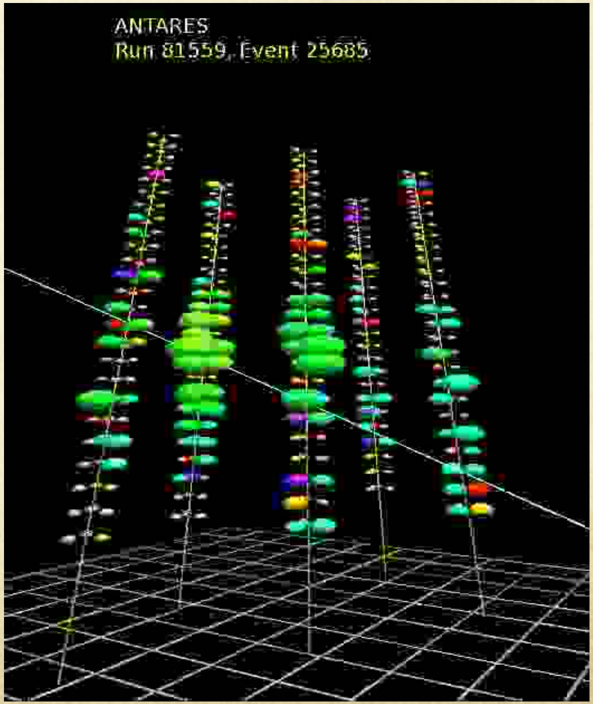


The two main tasks - 2/2

- **Telescope mode**
  - Single particle at a time
    - Sole nuisance = background
  - Trigger from beam
    - Often synchronous
  - Goal = get the incoming direction



- **The astroparticle way**
  - Similar to telescope mode
  - No synchronous timing
  - Ex: deep-water ν telescopes



Environmental conditions – 1/2

- **Life in a real experiment is tough (for detectors of course)**
  - Chasing small cross-sections ➞ large luminosity and/or energy
  - Short interval between beam crossing
    - LHC: 25 ns (and >10 collisions / crossing)
    - CLIC: 5 ns (but not continuous)
  - Large amount of particles (could be > 10^7 part/cm^2/s) ➡ background, radiation
    - makes the finding more complicated
  - Vacuum could be required (space, very low momentum particles (CBM, LHCb))

- **Radiation tolerance**
  - Two types of energy loss
    - Ionizing (generate charges): dose in Gy = 100 Rad
    - Non-ionizing (generate defects in solid): fluence in n_{eq}(1MeV)/cm^2
  - The more inner the detection layer, the harder the radiation (radius^2 effect)
  - Examples for most inner layers:
    - LHC: 10^{15} to <10^{17} n_{eq}(1MeV)/cm^2 with 50 to 1 MGy
    - ILC: <10^{12} n_{eq}(1MeV)/cm^2 with 5 kGy

Environmental conditions – 2/2

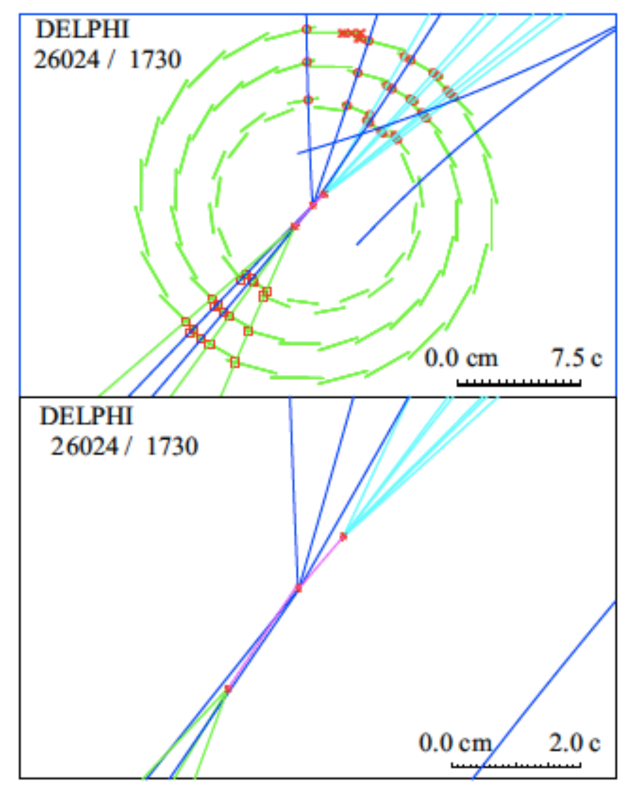
- **Timing consideration**
  - Integration time drives occupancy level (important for finding algorithm)
  - Time resolution offers time-stamping of tracks
    - Tracks in one "acquisition event" could be associated to their proper collision event if several have piled-up
  - Key question = triggered ot not-triggered experiment?

- **Heat concerns**
  - Spatial resolution → segmentation → many channels
    Readout speed → power dissipation/channel
    } Hot cocktail!
  - Efficient cooling techniques exist BUT
    add material budget and may not work everywhere (space)

- **Summary**
  - Tracker technology driven by environmental conditions: hadron colliders (LHC)
  - Tracker technology driven by physics performances: lepton colliders (B factories, ILC),
    heavy-ion colliders (RHIC, LHC)
  - Of course, some intermediate cases: superB factories, CLIC

Figures of Merit

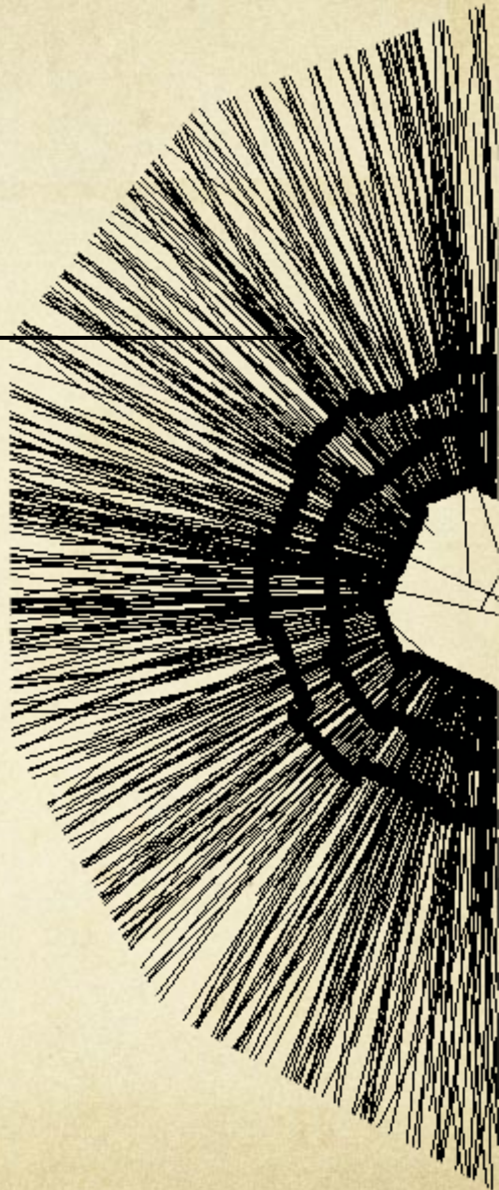
- For detection layer
  - Detection efficiency
    - Mostly driven by Signal/Noise
    - Note: Noise = signal fluctuation ⊕ readout (electronic) noise
  - Intrinsic spatial resolution
    - Driven by segmentation (not only)
    - Useful tracking domain $\sigma < 1\text{mm}$
  - Linearity and resolution on dE/dx
  - Material budget
  - "Speed" (integration time, time resolution, …)



- For detection systems (multi-layers)
  - Two-track resolution
    - Ability to distinguish two nearby trajectories
    - Mostly governed by signal spread / segments
  - Momentum resolution $\frac{\sigma(p)}{p}$
  - Impact parameter resolution
    - Sometimes called "distance of closest approach" to a vertex

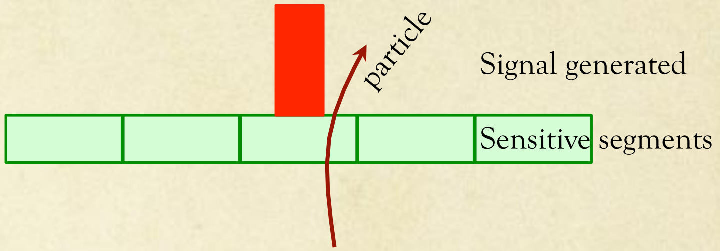
2. Detection technologies

- Intrinsic resolution
- Single layer systems
  - Silicon, gas sensors, scintillator
- Multi-layer systems
  - Drift chamber and TPC
- Tentative simplistic comparison
- Magnets
- Leftovers
- Practical considerations

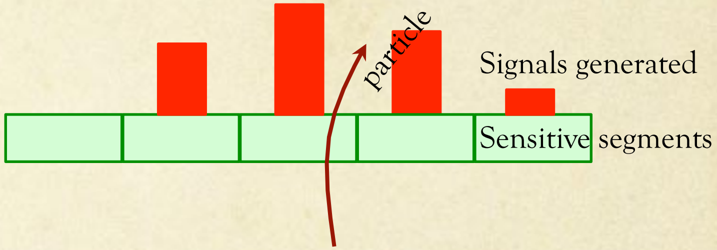


Intrinsic resolution

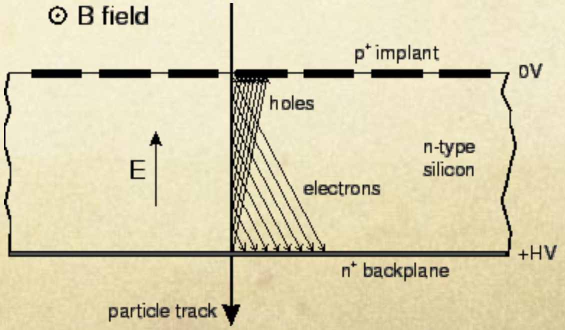
- Position measurement comes from segmentation
  - Pitch
- Digital resolution $$\sigma = \frac{\text{pitch}}{\sqrt{12}}$$



- Improvement from signal sharing
  - Position = charge center of gravity $$\sigma \propto \frac{\text{pitch}}{\text{signal/noise}}$$
  - Effects generated by
    - Secondary charges spread inside volume
    - Inclined tracks (however, resol. limited at large angles)

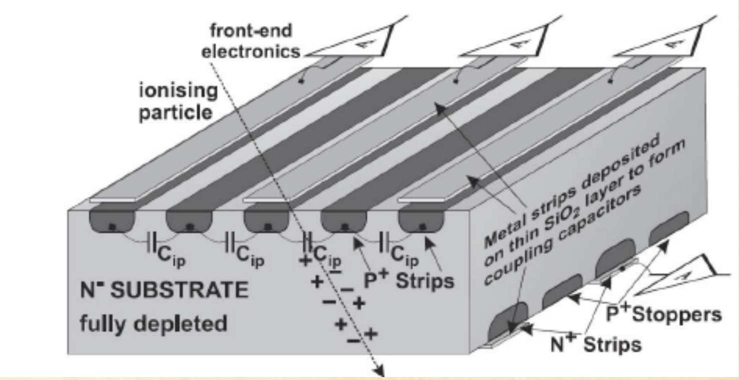


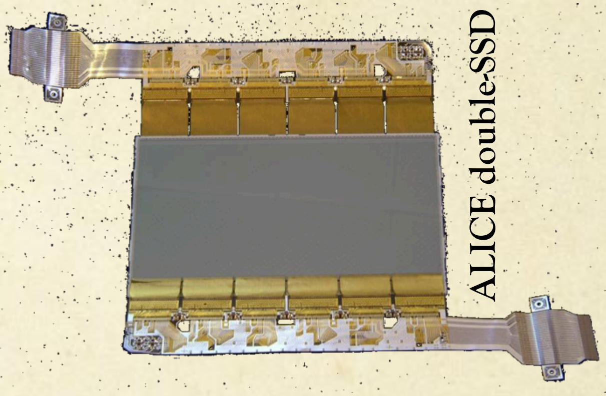
  - Potential optimization of segmentation / sharing
    - Work like signal sampling theory (Fourier transform)
  - Warnings:
    - Lorentz force from B mimic the effect
    - counterproductive / 2-track resolution



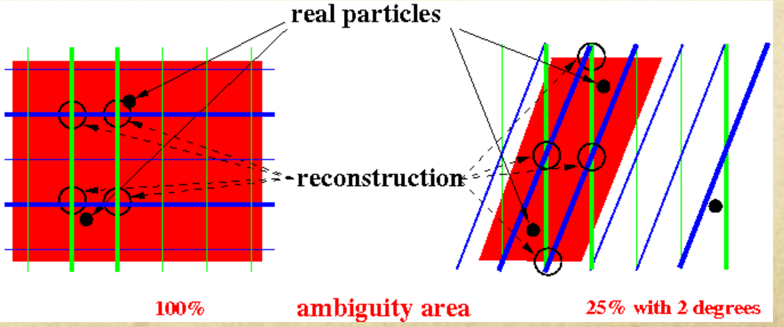
Silicon sensors: strips

- Basic sensitive element
  - E-h pairs are generated by ionization in silicon
    - 3.6 eV needed
    - 300 μm thick Si generates ~22000 charges for MIP BUT beware of Landau fluctuation
  - Collection: P-N junction = diode
    - Full depletion (10 to 0.5 kV) generates a drift field (10^4 V/cm)
    - Collect time ~ 15 ps/μm
- Silicon strip detectors
  - sensor "easily" manufactured with pitch down to ~25 μm
  - 1D if single sided
  - Pseudo-2D if double-sided
    - Stereo-angle useful against ambiguities
  - Difficult to go below 100 μm thickness
  - Speed and radiation hardness: LHC-grade





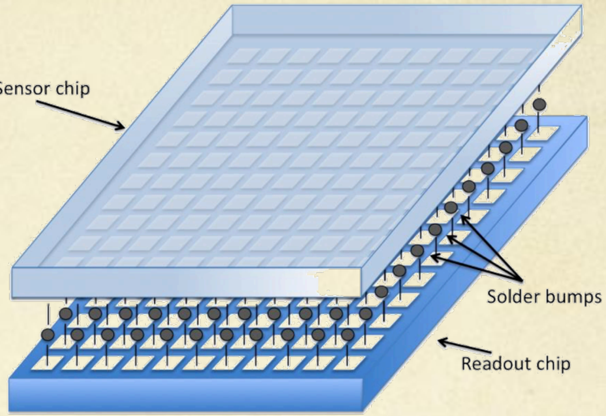
ALICE double-SSD



Silicon sensors: hybrid-pixels

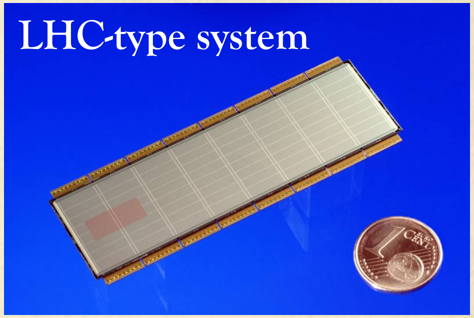
Concept

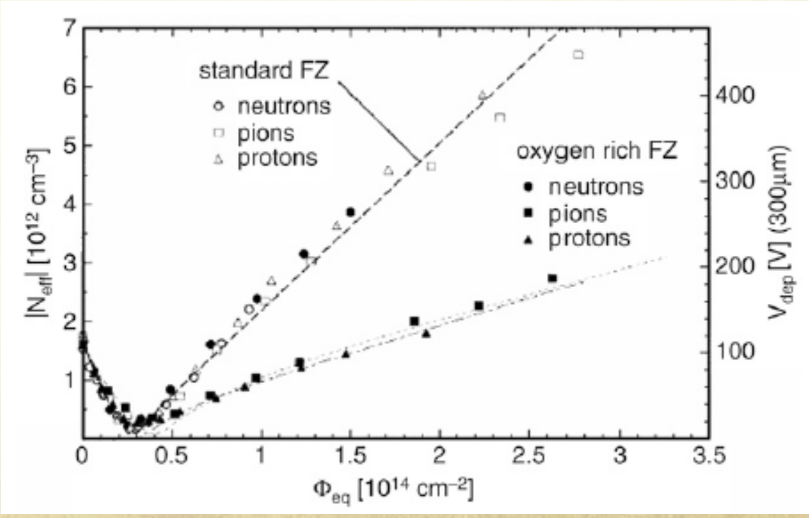
- Strips ⟶ pixels on sensor
- One to one connection from electronic channels to pixels



Performances

- Real 2D detector
  & keep performances of strips
  - Can cope with LHC rate (speed & radiation)
- Pitch size limited by physical connection and #transistors for treatment
  - minimal (today): 50x50 μm^2
    typical: 100x150/400 μm^2
  - spatial resolution about10 µm
- Material budget
  - Minimal(today): 100(sensor)+100(elec.) µm
- Power budget: 10 µW/pixel

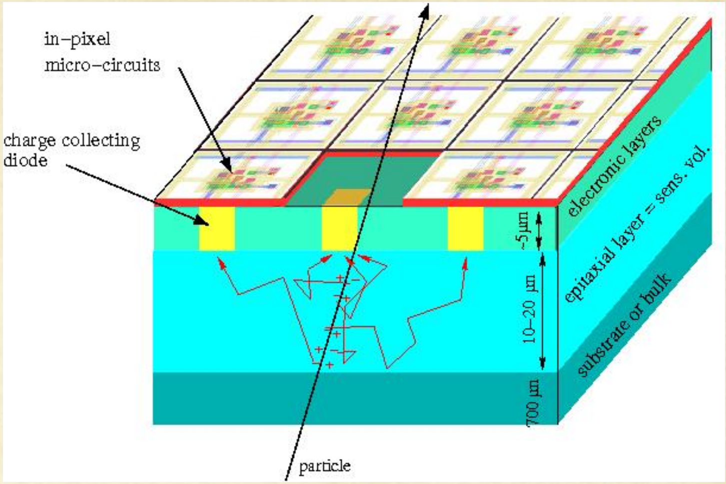




CMOS Pixel Sensor

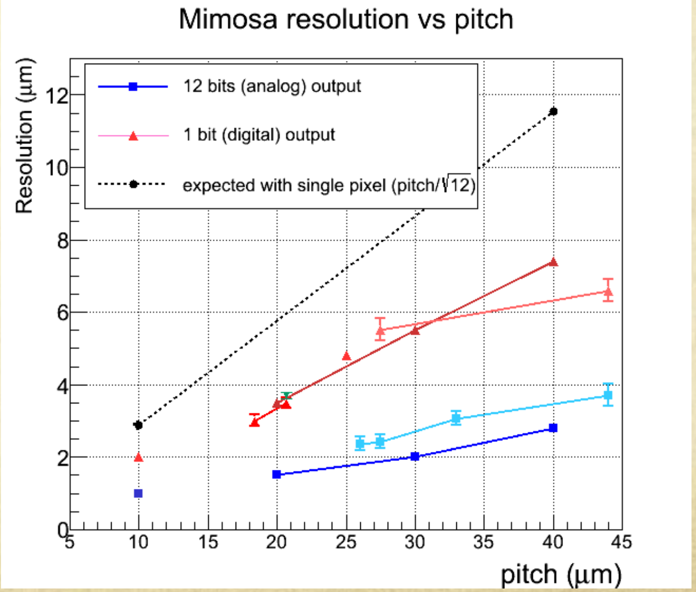
Concept

- Use industrial CMOS process
  - Implement an array of sensing diode
  - Amplify the signal with transistors near the diode
- Gain in granularity: pitch down to ~10 µm
- Gain in sensitive layer thickness ~ 10-20 µm
- BEWARE: full- depletion not systematically available
  - Slow (100 ns) thermal drift
  - Limited non-ionizing rad. tolerance



Performances

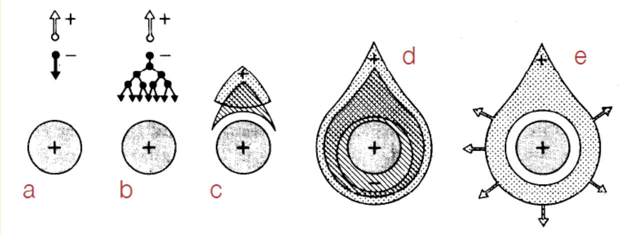
- Spatial resolution 1-10 µm (in 2 dimensions)
- Material budget: ≲ 50 µm
- Power budget: 1-5 µW/pixel
- Integration time ≃5-100 µs demonstrated
  - ~1 µs in development

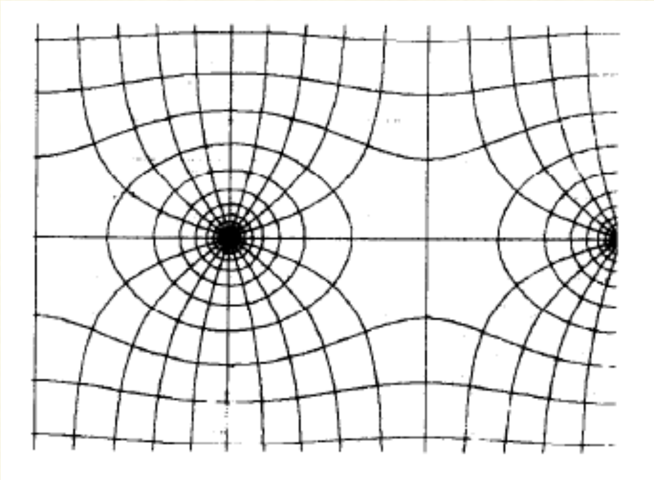


Wire chambers

- Basic sensitive element
  - Metallic wire, 1/r effect generated an avalanche
  - Signal depends on gain (proportional mode) typically 10^4
  - Signal is fast, a few ns

- Gas proportional counters
  - Multi-Wire Proportional Chamber
    - Array of wires
    - 1 or 2D positioning depending on readout
    - Wire spacing (pitch) limited to 1-2 mm
  - Straw or drift tube
    - One wire in One tube
    - Extremely fast (compared to Drift Chamber)
    - Handle high rate
    - Spatial resolution <200 μm
    - Left/right ambiguity

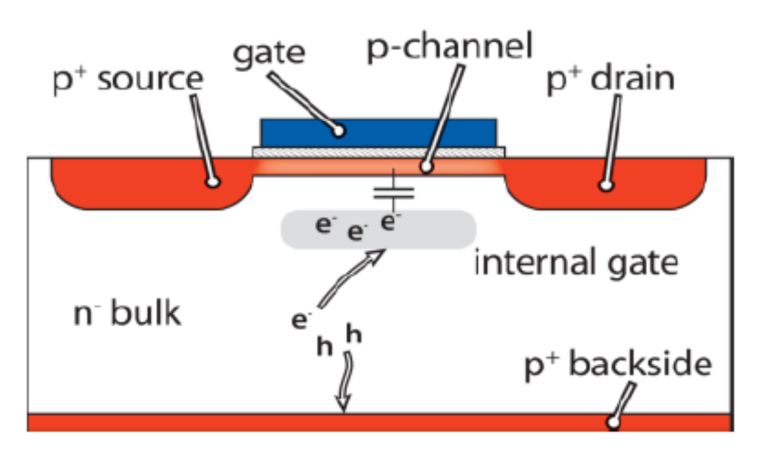




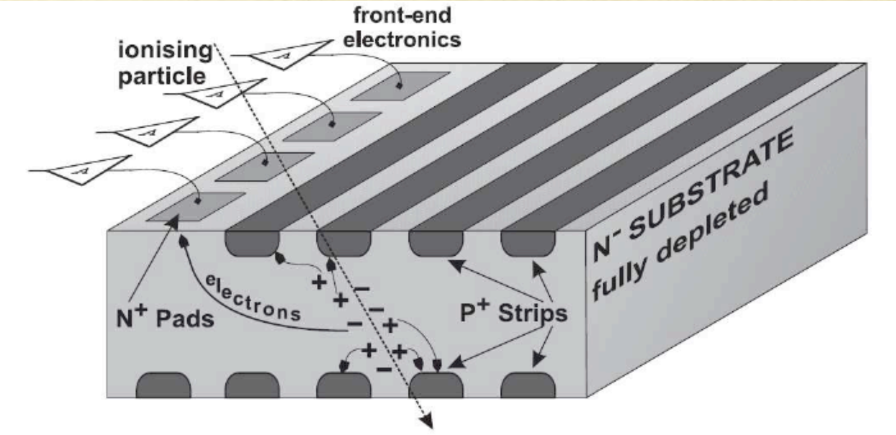
Electric fields line around anode wires

More single-layer pos.sens. detectors

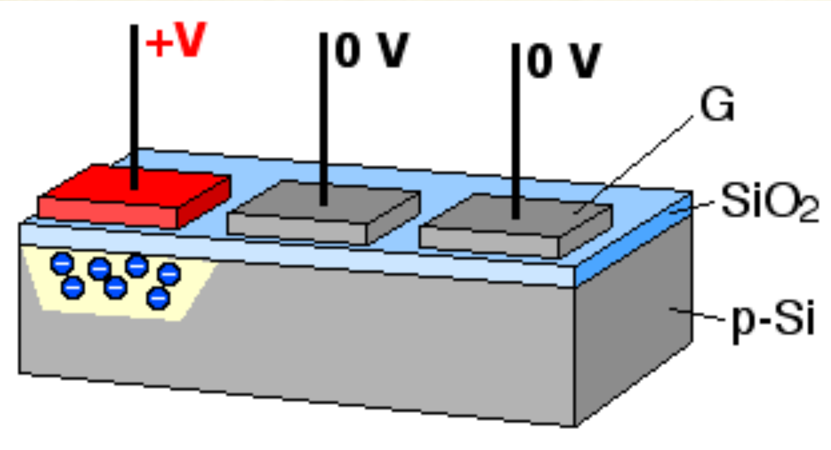
DEPFET



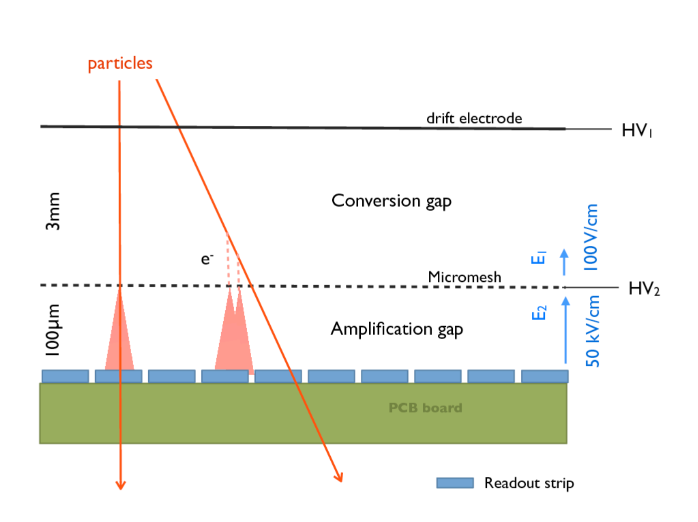
Silicon drift



CCD

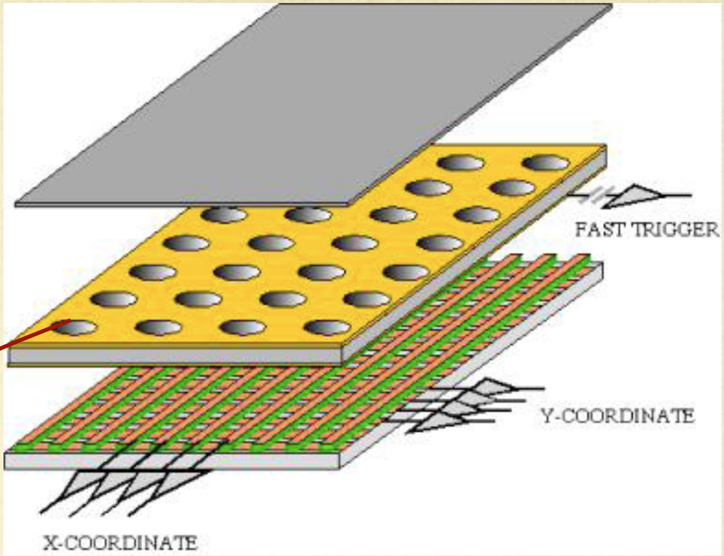


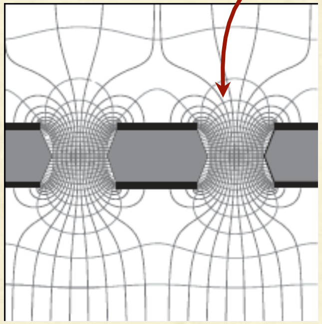
MICROMEGAS

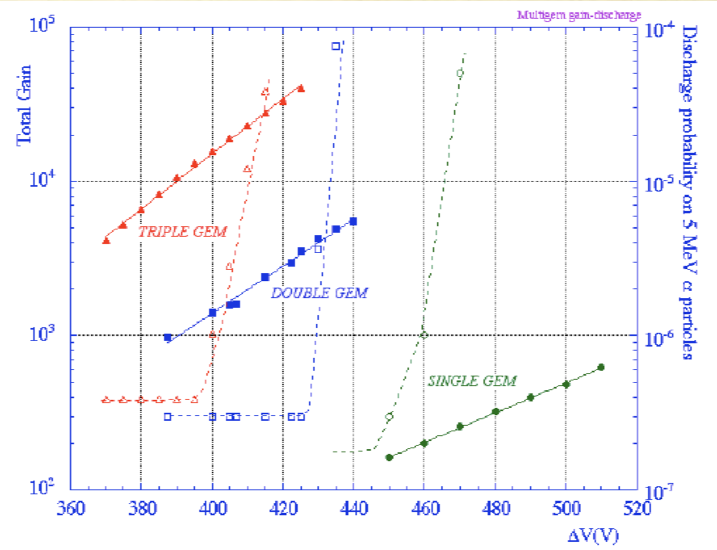


Wire chambers "advanced"

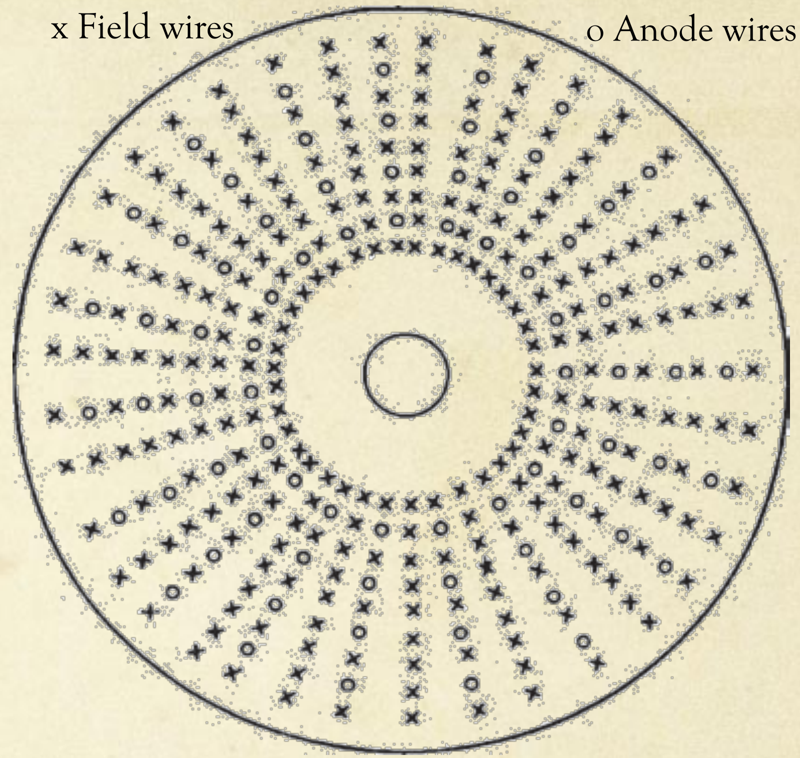
- **Micro-pattern gas multipliers**
  - MSGC
    - Replace wires with lithography micro-structures
    - Smaller anodes pitch 100-200 µm
    - BUT Ageing difficulties due to high voltage and manufacturing not so easy
  - GEM
    - Gain 10^5
    - Hit rate 10^6 Hz/cm2
  - MICROMEGAS
    - Even smaller distance anode-grid
    - Hit rate 10^9 Hz/cm2
  - More development
    - Electron emitting foil working in vacuum!







Drift chambers



Basic principle

- Mix field and anode wires
  - Generate a drift
- Pressurize gas to increase charge velocity (few atm)
- 3D detector
  - 2D from wire position
  - 1D from charge sharing at both ends

Spatial Resolution

- Related to drift path

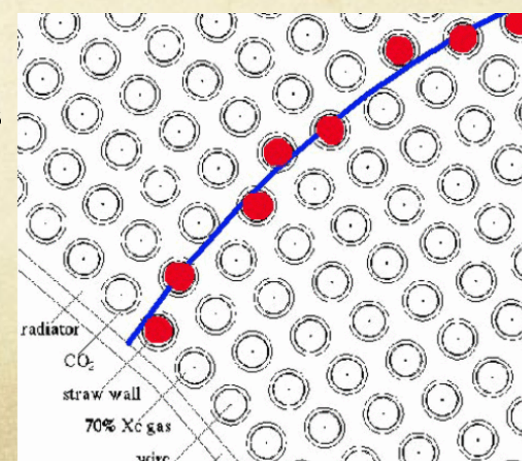
$$\sigma \propto \sqrt{\text{drift length}}$$

- Typically 100-200 μm

Remarks

- Could not go to very small radius

Same principle with straw tubes



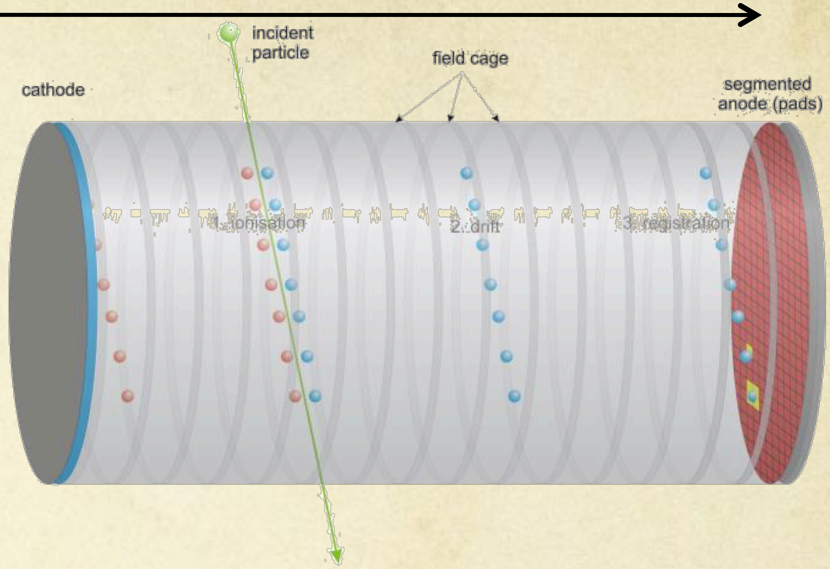
Time Projection Chambers 1/2

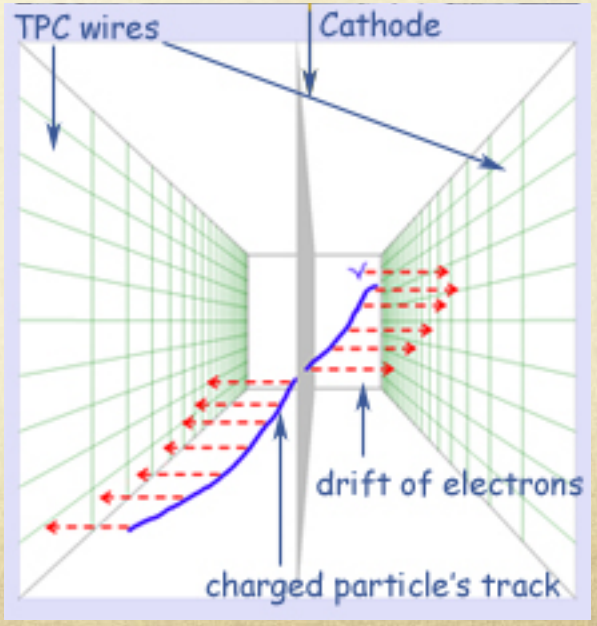
Benefits

- Large volume available
- Multi-task: tracking + Part. Identification

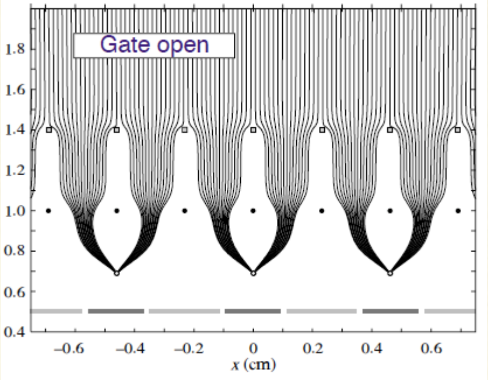
Basic operation principle

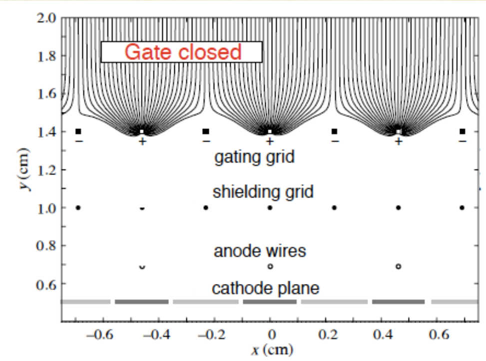
- Gas ionization → charges
- Electric field → charge drift along straight path
- Information collected
  - 2D position of charges at end-cap
  - 3rd dimension from drift time
  - Energy deposited from #charges
- Different shapes:
  - rectangles (ICARUS)
  - Cylinders (colliders)
  - Volumes can be small or very large



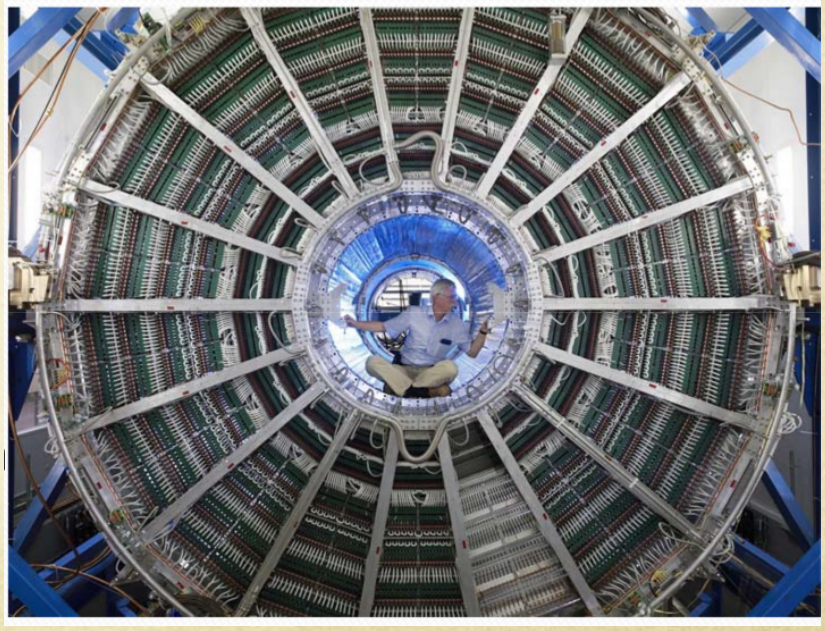


Time Projection Chambers 2/2

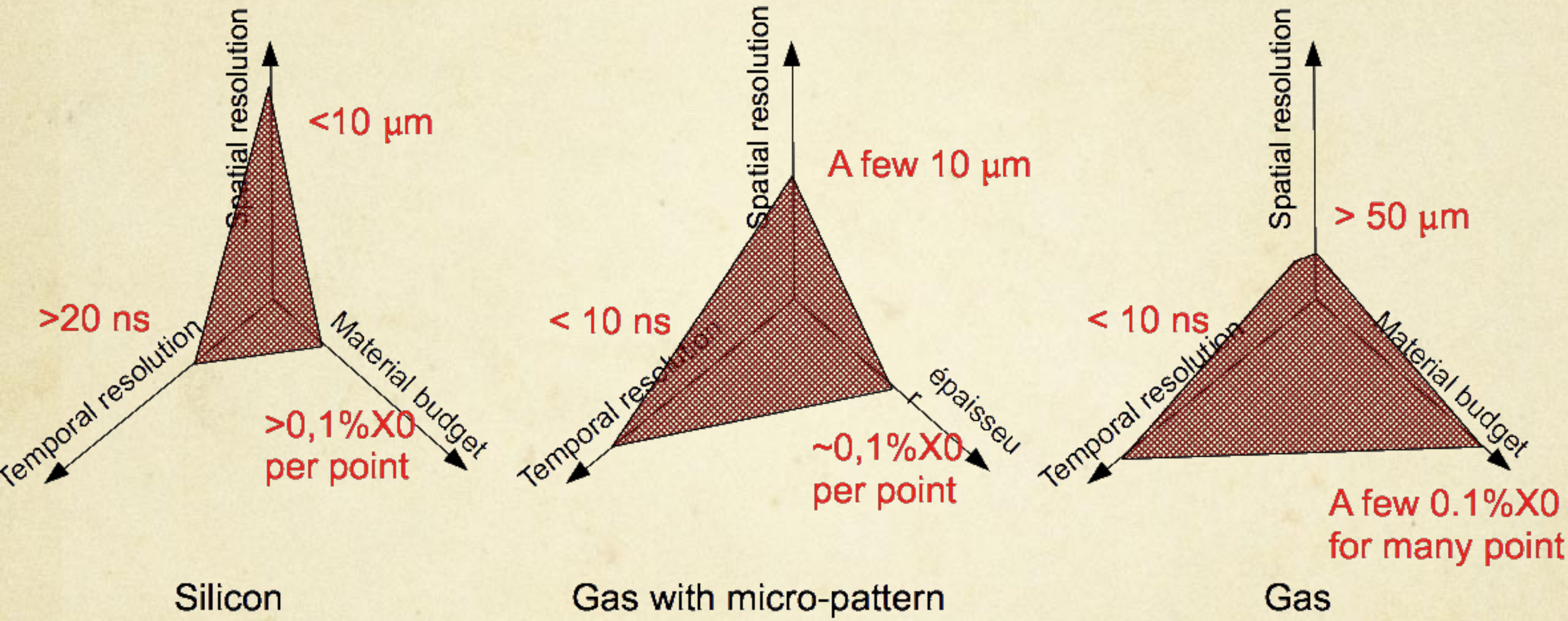




- End cap readout
  - Gas proportional counters
    - Wires+pads, GEM, Micromegas
- Performances
  - Two-track resolution ~ 1cm
  - Transverse spatial resolution ~ 100 - 200 μm
  - Longitudinal spatial resolution ~0.2 - 1 mm
  - Longitudinal drift velocity: 5 to 7 cm/μs
    - ALICE TPC (5m long): 92 μs drift time
  - Pro
    - Nice continuously spaced points along trajectory
    - Minimal multiple scattering (inside the vessel)
  - Cons
    - Limiting usage with respect to collision rate



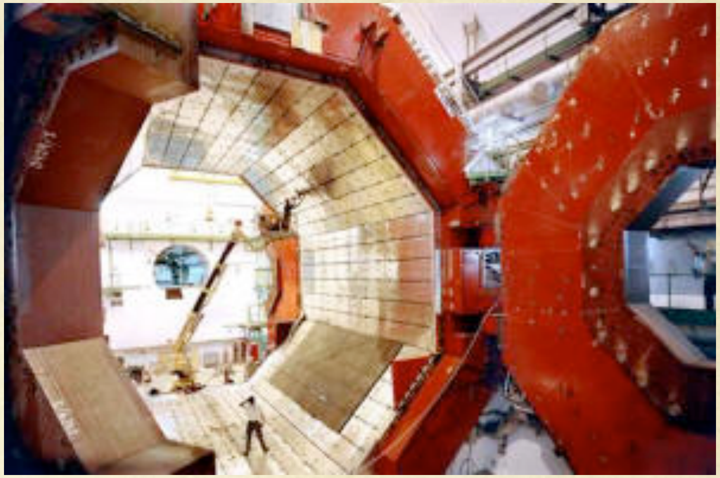
Tentative "simplistic" comparison



Magnets

Solenoid

- Field depends on current I, length L, # turns N
  - on the centerline $$B = \frac{\mu_0 NI}{\sqrt{L^2 + 4R^2}}$$
  - Typically: 1 T needs 4 to 8 kA
    ➛ superconducting metal to limit heat
- Field uniformity needs flux return (iron structure)
  - Mapping is required for fitting (remember B(**x**)?)
  - Usually performed with numerical integration
- Calorimetry outside ➛ limited material ➛ superconducting
- Fringe field calls for compensation



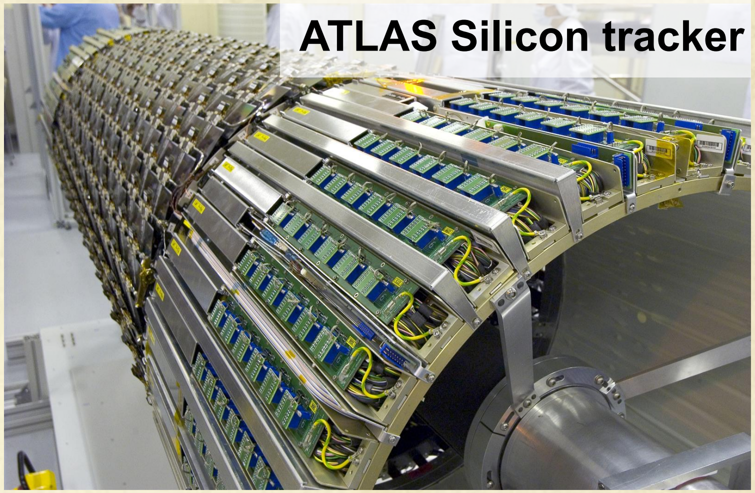
	Field (T)	Radius (m)	Length (m)	Energy (MJ)
ALICE	0.5	6		150
ATLAS	2	2.5	5.3	700
CMS	4	5.9	12.5	2700
ILC	4	3.5	7.5	2000

Supercondiction

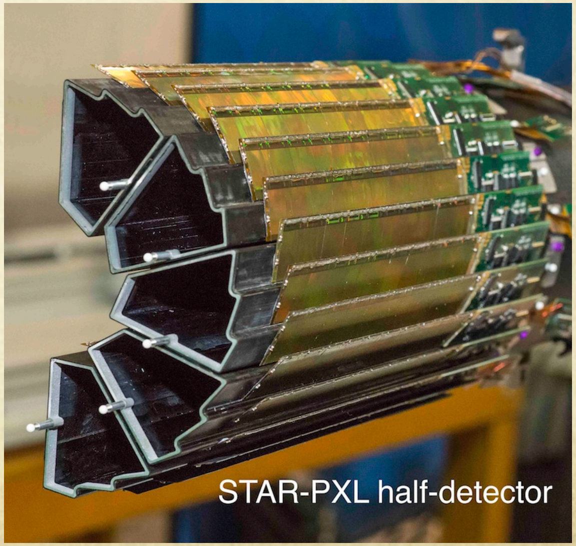
- cryo-operation ➛ quenching possible !
- Magnetic field induces energy: $$E \propto B^2 R^2 L$$
  - Cold mass necessary to dissipate heat in case of quench

Practical considerations

- **From a detection principle to a detector**
  - Build large size or many elements
    - Manufacture infrastructures
    - Characterization capabilities
    - Production monitoring
  - Integration in the experiment
    - Mechanical support
    - Electrical services (powering & data transmission)
    - Cooling (signal treatment dissipates power)
  - Specific to trackers
    - Internal parts of multi-detectors experiment → limited space
    - Material budget is ALWAYS a concern
    - ⇨ trade-offs required



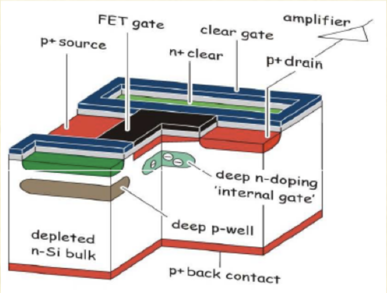
400×10^6 pixels
over 16 cm



Leftovers

- **Silicon drift detectors**
  - Real 2D detectors made of strips
  - 1D is given by drift time
- **Diamond detectors**
  - Could replace silicon for hybrid pixel detectors
  - Very interesting for radiation tolerance
- **Plasma sensor panels**
  - Derived from flat television screen
  - Still in development
- **Charge Coupled Devices (CCD)**
  - Fragile/ radiation tolerance
- **Signal generation**
  ➛ see Ramo's theorem

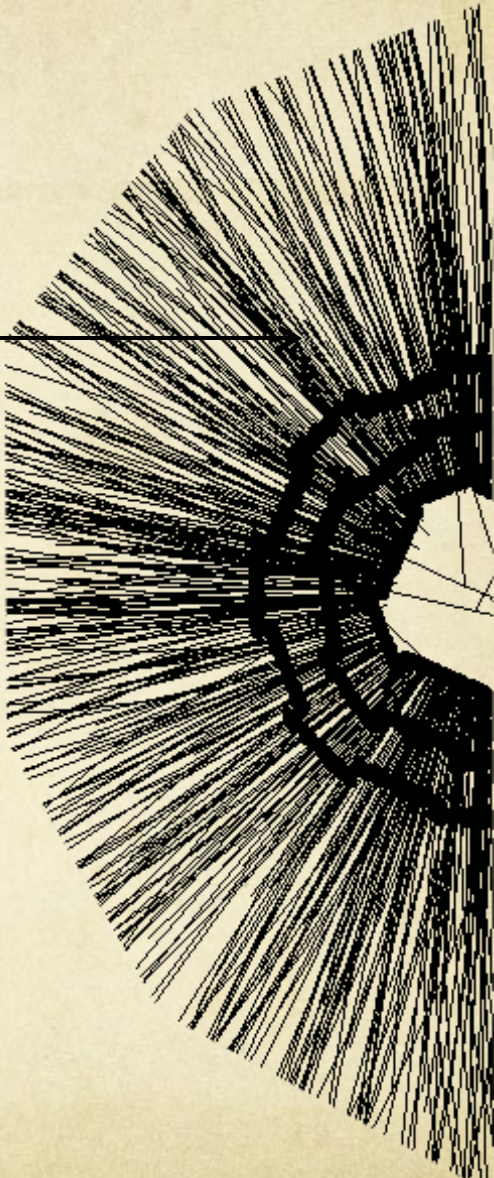
- **DEPFET**
  - Depleted Field Effect Transistor detector
  - Real 2D and partly monolithic



- **Nuclear emulsions**
  - One of the most precise ~ 1µm
  - No timing information ➞ very specific applications
- **Scintillators**
  - Extremely fast (100 ps)
  - Could be arranged like straw tubes
  - But quite thick (X_0 ~ 2 cm)

3. Standard algorithms

- Finders
- First evaluation of momentum resolution
- Fitters
- Alignment



FINDING : 2 strategies

Global methods

- Transform the coordinate space into pattern space
  - "pattern" = parameters used in track model
- Identify the "best" solutions in the new phase space
- Use all points at a time
  - No history effect
- Well adapted to evenly distributed points with same accuracy

Local methods

- Start with a track seed = restricted set of points
  - Could require good accuracy from the beginning
- Then extrapolate to next layer-point
  - And so on…iterative procedure
- "Wrong" solutions discarded at each iteration
- Possibly sensitive to "starting point"
- Well adapted to redundant information

FINDING drives tracking efficiency

Track model

A simple example

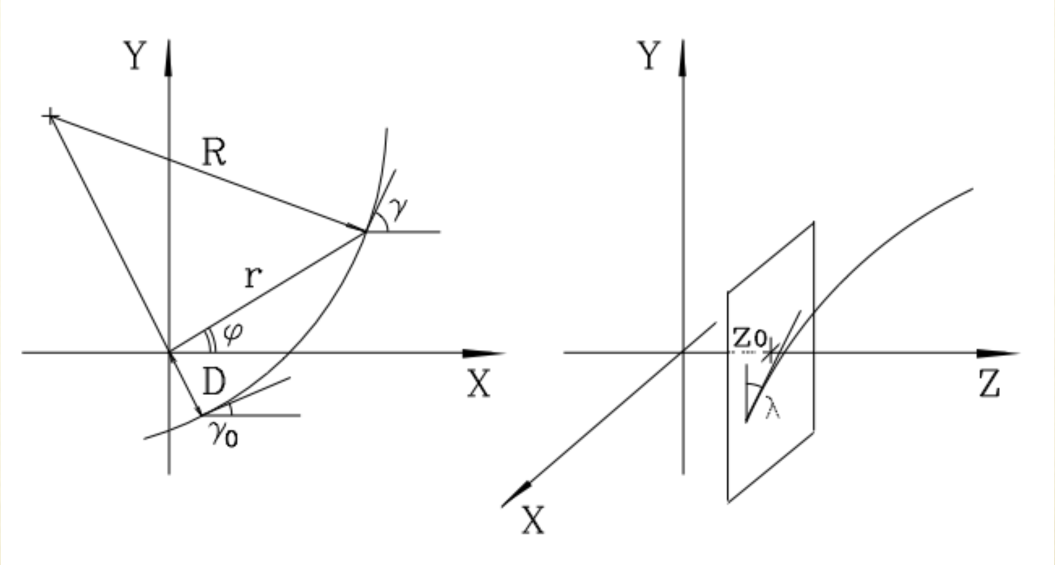
- Straight line in 2D: model is x = a*z + b
- Track parameters (a,b); N measurements x_i at z_i (i=1..N)

A more complex example

- Helix in 3D with magnetic field
- Track parameters (ϕ, *z, D, tan* λ ,C)
- Measurements (ϕ, *z)*

$$\phi(r) = \gamma_0 + \arcsin\frac{Cr + (1+CD)D/r}{1+2CD}$$

$$z(r) = z_0 + \frac{\tan\lambda}{C}\arcsin\left(C\sqrt{\frac{r^2 - D^2}{1+2CD}}\right)$$



Generalization

- Parameters: P-vector **p**
- Measurements: N-vector **c**
- Model: function f ($\mathcal{R}^P \rightarrow \mathcal{R}^N$)

f(**p**) = **c** ↔ propagation

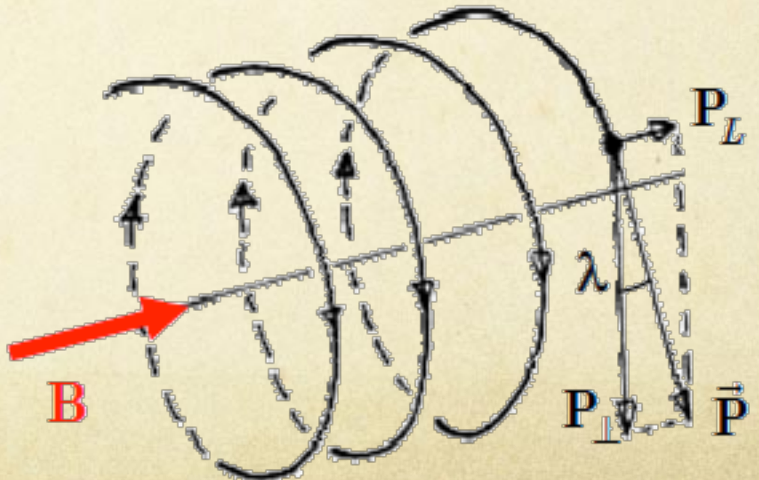
Another view of the helix

- s = track length
- h = sense of rotation
- λ = dip angle
- Pivot point (s=0):
  - position (x_0, y_0, z_0)
  - orientation ϕ_0
-

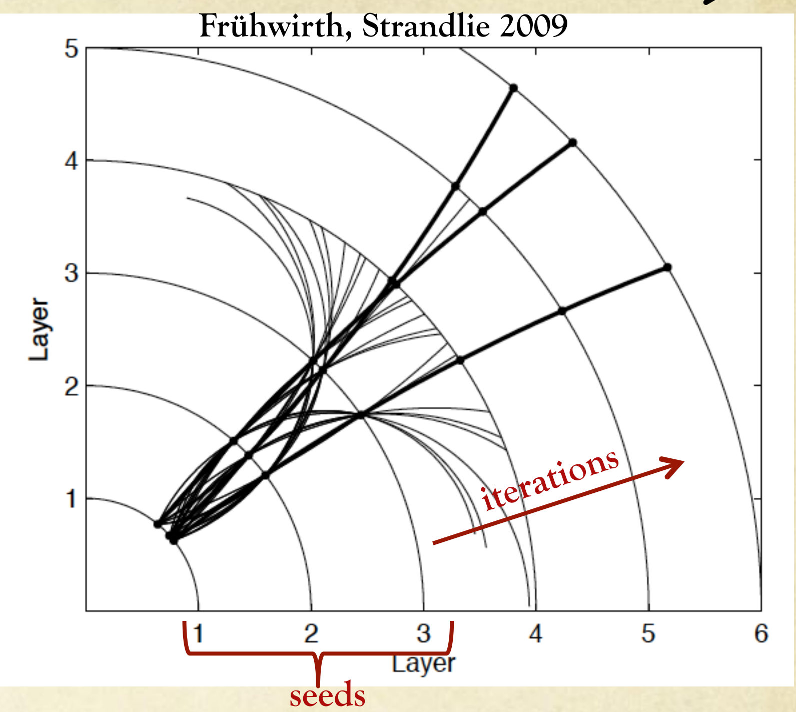
$$x(s) = x_o + R\left[\cos\left(\Phi_o + \frac{hs\cos\lambda}{R}\right) - \cos\Phi_o\right]$$

$$y(s) = y_o + R\left[\sin\left(\Phi_o + \frac{hs\cos\lambda}{R}\right) - \sin\Phi_o\right]$$

$$z(s) = z_o + s\sin\lambda$$



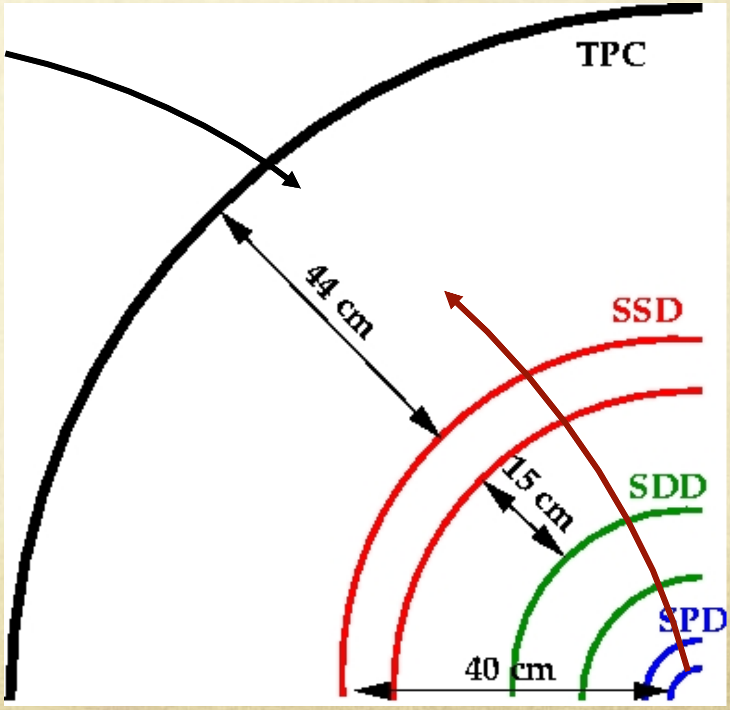
Local method 1/2



- Track seed = initial segment
  - Made of few (2 to 4) points
    - One point could be the expected primary vtx
  - Allows to initialize parameter for track model
  - Choose most precise layers first
    - usually inner layers
  - But if high hit density
    - Start farther from primary interaction @ lowest density
    - Limit mixing points from different tracks
- Extrapolation step
  - Out or inward (=toward primary vtx) onto the next layer
  - Not necessarily very precise, especially **only local model** needed
    - Extrapolation uncertainty $\lesssim$ layer point uncertainty
    - Computation speed important
  - Match (associate) nearest point on the new layer
    - Might skip the layer if point missing
    - Might reject a point: if worst track-fit or if fits better with another track

Local method 2/2

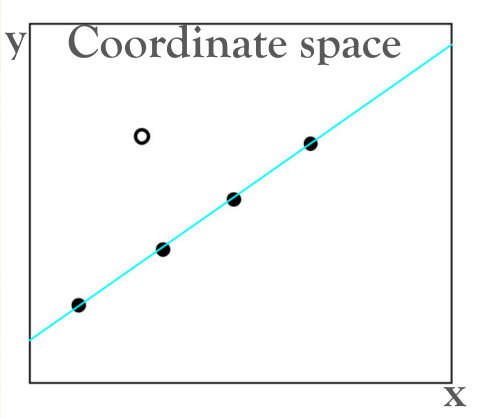
- Variant with track segments
  - First build "tracklets" on natural segments
    - Sub-detectors, or subparts with same resolution
  - Then match segments together
  - Typical application:
    - Segments large tracker (TPC) with vertex detector (Si)
      ➛ layers dedicated to matching

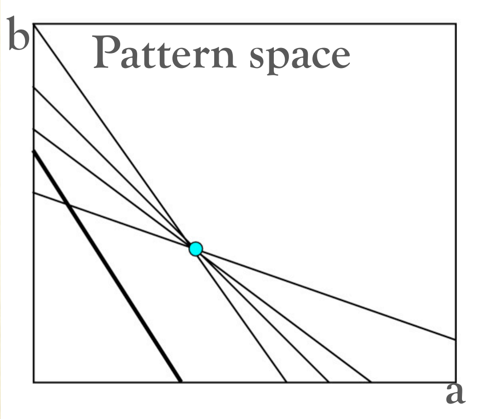


- Variant with track roads
  - Full track model used from start
- Variant with Kalman filter
  - See later

Global methods 1/2

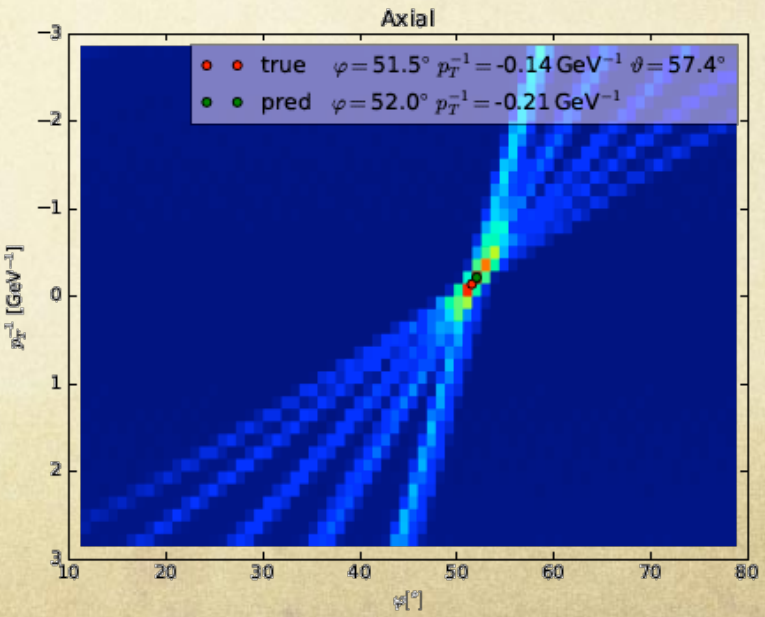
- **Brute force = combinatorial way**
  - Consider all possible combination of points to make a track
  - Keep only those compatible with model
  - Usually too time consuming…
- **Hough transform**
  - Example straight track:
    - Coord. space $y = a*x + b$ ⇔ pattern space $b = y - x*a$
    - Each point (y,x) defines a line in pattern space
    - All lines, from points belonging to same straight-track, cross at same point (a,b)
    - In practice: discretize pattern space and search for maximum
  - Applicable to circle finder
    - needs two parameters as well (r, ϕ of center) if track is assumed to originate from (0,0)
  - More difficult for more than 2 parameters…





Global methods 2/2

- Conformal mapping
  - Helix transverse projection = Circle
    - $(x-a)^2 + (y-b)^2 = r^2$
    - Transform to $u = x/(x^2+y^2)$, $v = y/(x^2+y^2)$
    - Then: $v = -(a/b)\, u + (1/2b)$



FITTING

- Why do we need to fit?
  - Measurement error
  - Multiple scattering error

- Global fit
  - Assume knowledge of:
    - all track points
    - full correlation matrix
      → difficult if $\sigma_{\text{mult. scatt.}} \gtrsim \sigma_{\text{meas.}}$
  - Least square method

- Iterative fit
  - Iterative process:
    - points included in the fit one by one
    - could be merged with finder step
  - Kalman filter

FITTING drives track extrapolation & momentum res.

Least Square Method (LSM)

> "N measurements" means:
> - K points (or layers)
> - D coordinates at each point
> - N = KxD

Linear model hypothesis

- P track parameters **p**, with N measurements **c**

$$\vec{c} = \vec{c}_s + A(\vec{p} - \vec{p}_s) + \vec{\varepsilon}$$

- $\mathbf{p_s}$ = known starting point, **A** = track model NxP matrix,
  $\boldsymbol{\varepsilon}$ = error vector corresponding to **V** = covariance NxN matrix

Sum of squares:

$$\sum \frac{(\text{model} - \text{measure})^2}{\text{uncertainty}^2} \Rightarrow S(\vec{p}) = \left(\vec{c}_s + A(\vec{p} - \vec{p}_s) - \vec{c}\right)^T V^{-1} \left(\vec{c}_s + A(\vec{p} - \vec{p}_s) - \vec{c}\right)$$

Best estimator (minimizing variance)

$$\frac{\mathrm{d}S}{\mathrm{d}\vec{p}}(\underline{\vec{p}}) = 0 \Rightarrow \underline{\vec{p}} = \vec{p}_s + \left(A^T V^{-1} A\right)^{-1} A^T V^{-1} \left(\vec{c} - \vec{c}_s\right)$$

- Variance (= uncertainty) of the estimator:

$$\underline{V_{\vec{p}}} = \left(A^T V^{-1} A\right)^{-1}$$

- Estimator p follows a χ^2 law with N-P degrees of freedom

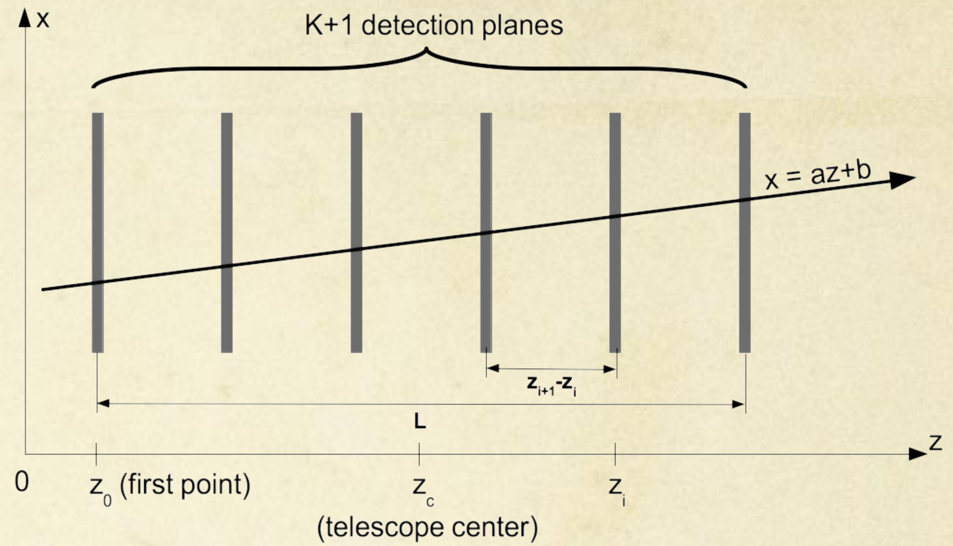
Problem ⇔ inversion of a PxP matrix ($A^T V^{-1} A$)

- But real difficulty could be computing **V** (NxN matrix)
  ⬅ layer correlations if multiple scattering non-negligible if $\sigma_{\text{mult. scatt.}} \gtrsim \sigma_{\text{meas}}$

LSM on straight tracks

Straight line model

- 2D case ➜ D=2 coordinates (z,x)
- 2 parameters: a = slobe, b = intercept at z=0



General case

- K+1 detection planes (i=0…k)
  - located at z_i
  - Spatial resolution σ_i
- Useful definitions

$$S_1 = \sum_{i=0}^{K} \frac{1}{\sigma_i^2} \;,\; S_z = \sum_{i=0}^{K} \frac{z_i}{\sigma_i^2} \;,\; S_{xz} = \sum_{i=0}^{K} \frac{x_i z_i}{\sigma_i^2} \;,\; S_{z^2} = \sum_{i=0}^{K} \frac{z_i^2}{\sigma_i^2}$$

- Solutions $a = \frac{S_1 S_{xz} - S_x S_z}{S_1 S_{z^2} - (S_z)^2}$, $b = \frac{S_x S_{z^2} - S_z S_{xz}}{S_1 S_{z^2} - (S_z)^2}$

- Uncertainties

$$\sigma_a^2 = \frac{S_1}{S_1 S_{z^2} - (S_z)^2} \;,\; \sigma_b^2 = \frac{S_{z^2}}{S_1 S_{z^2} - (S_z)^2}$$

! correction $\mathrm{cov}_{a,b} = \frac{-S_z}{S_1 S_{z^2} - (S_z)^2}$

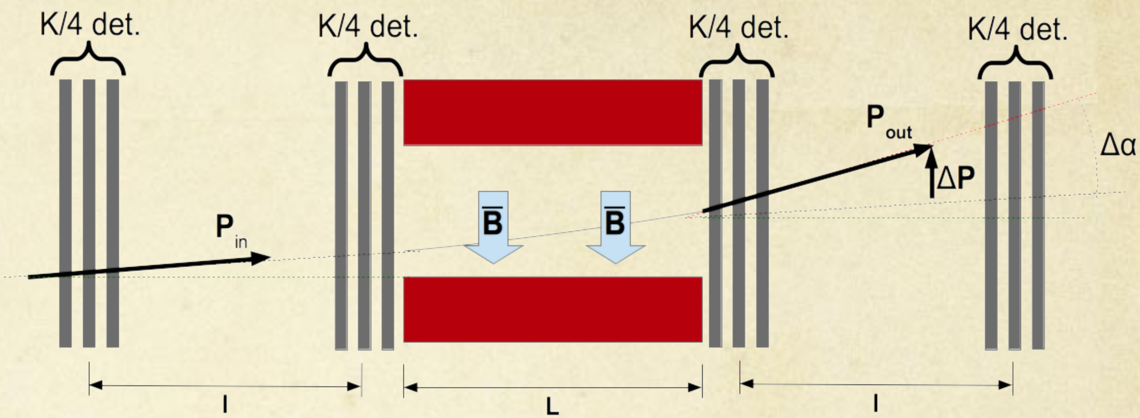
Case of uniformly distributed (N+1) planes

- $z_{i+1} - z_i = L/K$ et $\sigma_i = \sigma \quad \forall i$
- $S_z = 0$ ➜ a,b uncorrelated

$$\sigma_a^2 = \frac{12K}{(K+2)L^2}\frac{\sigma^2}{K+1} \;,\; \sigma_b^2 = \left(1 + 12\frac{K}{K+2}\frac{z_c^2}{L^2}\right)\frac{\sigma^2}{K+1}$$

- Uncertainties :
  - σ_a and σ_b improve with $1/\sqrt{(K+1)}$
  - σ_a and σ_b improve with 1/L
  - σ_b improve with z_c

LSM on fixed target geometry



Hypothesis

- K detectors, each with σ single point accuracy
- Uniform field over L from dipole
  - Trajectory: $\Delta\alpha = \left|\frac{0.3qBL}{p}\right|$
  - Bending: $\Delta p = p\,\Delta\alpha$
- Geometrical arrangement optimized for resolution
  - Angular determination on input and output angle: $\sigma_\alpha^2 = \frac{16\,\sigma^2}{K\,l^2}$

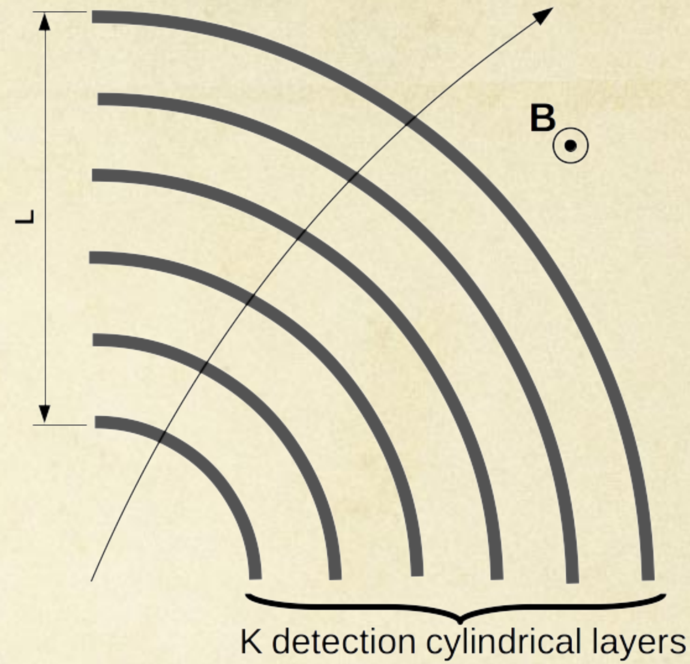
Without multiple scattering

- Uncertainty on momentum

$$\frac{\sigma_p}{p} = \frac{8}{0.3q}\frac{1}{BL}\frac{\sigma}{l\sqrt{K}}\,p$$

LSM on collider geometry

- **Hypothesis**
  - K detectors uniformly distributed each with σ single point accuracy
  - Uniform field over path length L

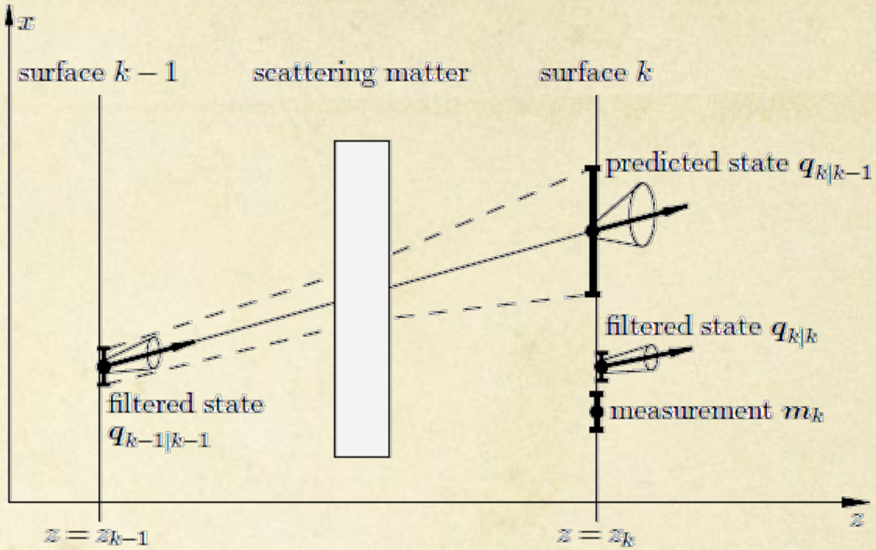


- **Without multiple scattering**
  - Uncertainty on transverse momentum (Glückstern formula)

$$\frac{\sigma_{p_T}}{p_T} = \frac{\sqrt{720}}{0.3q} \frac{1}{BL^2} \frac{\sigma}{\sqrt{K+6}} p_T$$

Kalman filter 1/2

- Dimensions
  - P parameters for track model
  - D "coordinates" measured at each point (usually D<P)
  - K measurement points (# total measures: N = KxD)



- Starting point
  - Initial set of parameters: first measurements
  - With large uncertainties if unknowns

- Iterative method
  - Propagate to next layer = prediction
    - Using the system equation $\vec{p}_k = G\,\vec{p}_{k-1} + \vec{\omega}_k$
    - G = PxP matrix, ω = perturbation associated with covariance PxP matrix V_ω
    - Update the covariance matrix with additional uncertainties (ex: material budget between layers) $V_{k|k-1} = V_{k-1} + V_{\omega_k}$
  - Add new point to update parameters and covariance, using the measure equation $\vec{m}_k = H\,\vec{p}_k + \vec{\varepsilon}_k$
    - H=DxP matrix, ε = measure error associated with diagonal covariance DxD matrix V_m
    - Weighted means of prediction and measurement using variance $\Leftrightarrow \chi^2$ fit
  - Iterate…

$$\boxed{\vec{p}_k = \left(V_{k|k-1}^{-1}\vec{p}_{k|k-1} + H^T V_{m_k}^{-1}\vec{m}_k\right)\cdot\left(V_{k|k-1}^{-1} + H^T V_{m_k}^{-1} H\right)^{-1}}$$

Kalman filter 2/2

Forward and backward filters

- Forward estimate of p_k: from 1➛k-1 measurements
- Backward estimate of p_k: from k+1➛K measurements
- Independent estimates ➞combination with weighted mean = smoother step

Computation complexity

- only PxP, DxP or DxD matrices computation (≪NxN)

Mixing with finder

- After propagation step: local finder
- Some points can be discarded if considered as outliers in the fit (use χ^2 value)

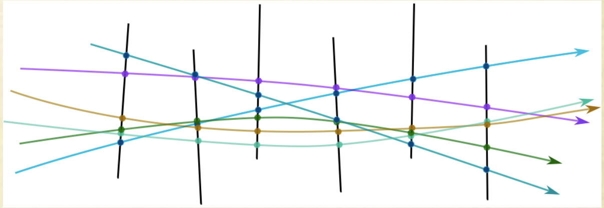
Include exogenous measurements

- Like dE/dx, correlated to momentum
- Additional measurement equation $\vec{m}'_k = H' \vec{p}_k + \vec{\varepsilon}'_k$

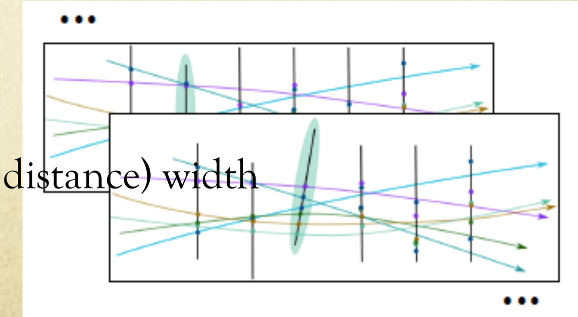
$$\vec{p}_k = \left(V_{k|k-1}^{-1}\vec{p}_{k|k-1} + H^T V_{m_k}^{-1}\vec{m}_k + H'^T V_{m'_k}^{-1}\vec{m}'_k\right)\cdot\left(V_{k|k-1}^{-1} + H^T V_{m_k}^{-1} H + H'^T V_{m'_k}^{-1} H'\right)^{-1}$$

Alignment strategy

- Let's come back to one initial & implicit hypothesis
  - "We know were the point are located."
  - True to the extent we know were the detector is!
  - BUT, mechanical instability (magnetic field, temperature, air flow…) and also drift speed variation (temperature, pressure, field inhomogeneity…) limit our knowledge
  - Periodic determination of positions and deformations needed = alignment
- Methods
  - Track model depends on new "free" parameters, i.e. the alignment
  - Global alignment:
    - Fit the new params. to minimize the overall χ^2 of a set of tracks (Millepede algo.)
    - Beware: many parameters could be involved (few 10^3 can easily be reached)

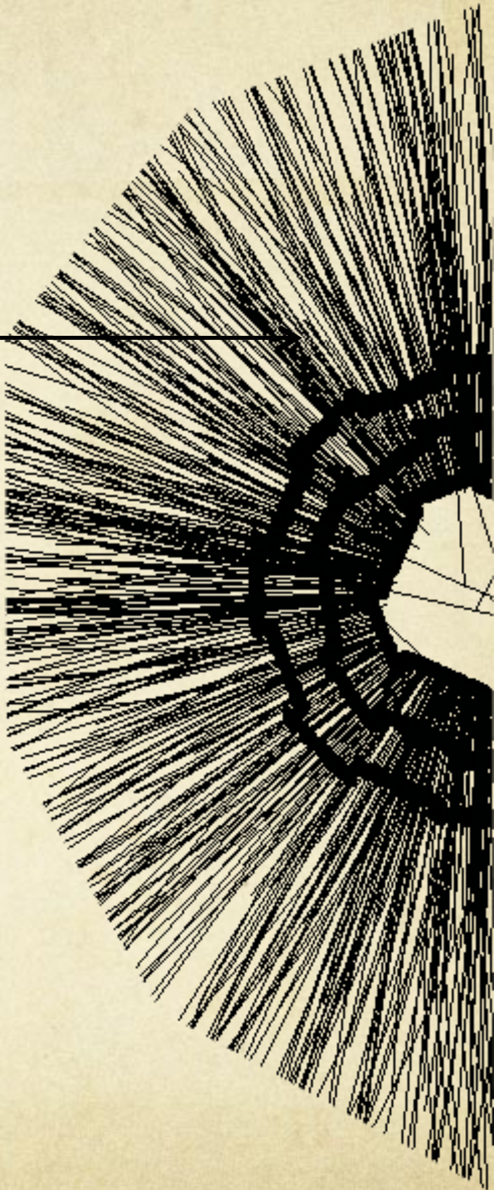


  - Local alignment:
    - Use tracks reconstructed with reference detectors
    - Align other detectors by minimizing the "residual" (track-hit distance) width
  - Use a set of well know tracks and tracking-"friendly" environment to avoid bias



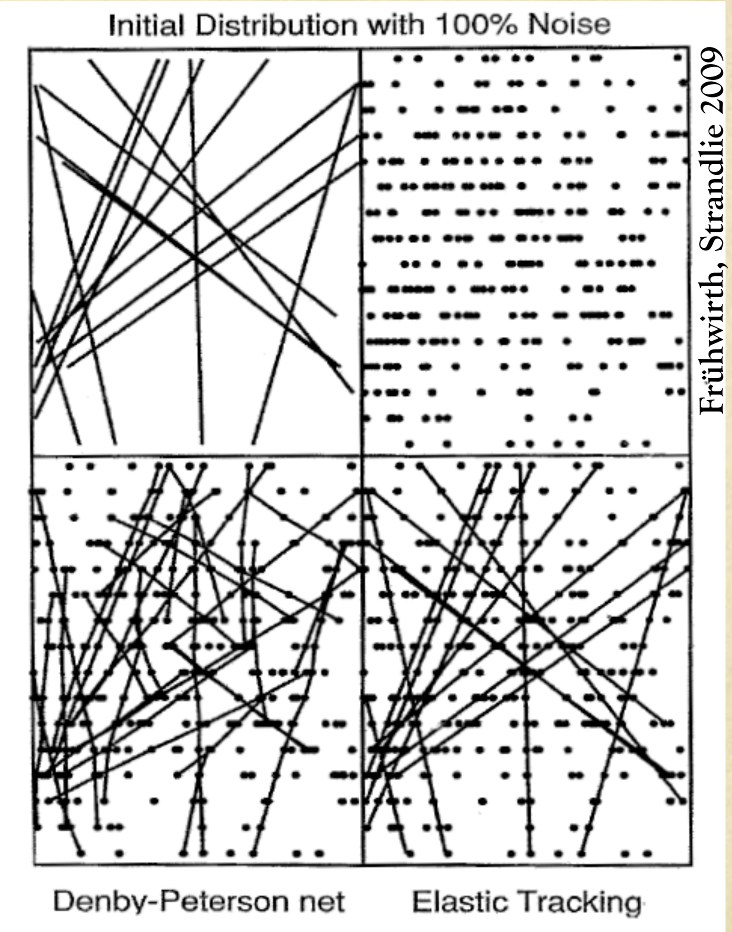
4. Advanced methods

- Why ?
- (Gaussian sum filter: *not treated yet*)
- Neural network
- Cellular automaton



Adaptive methods

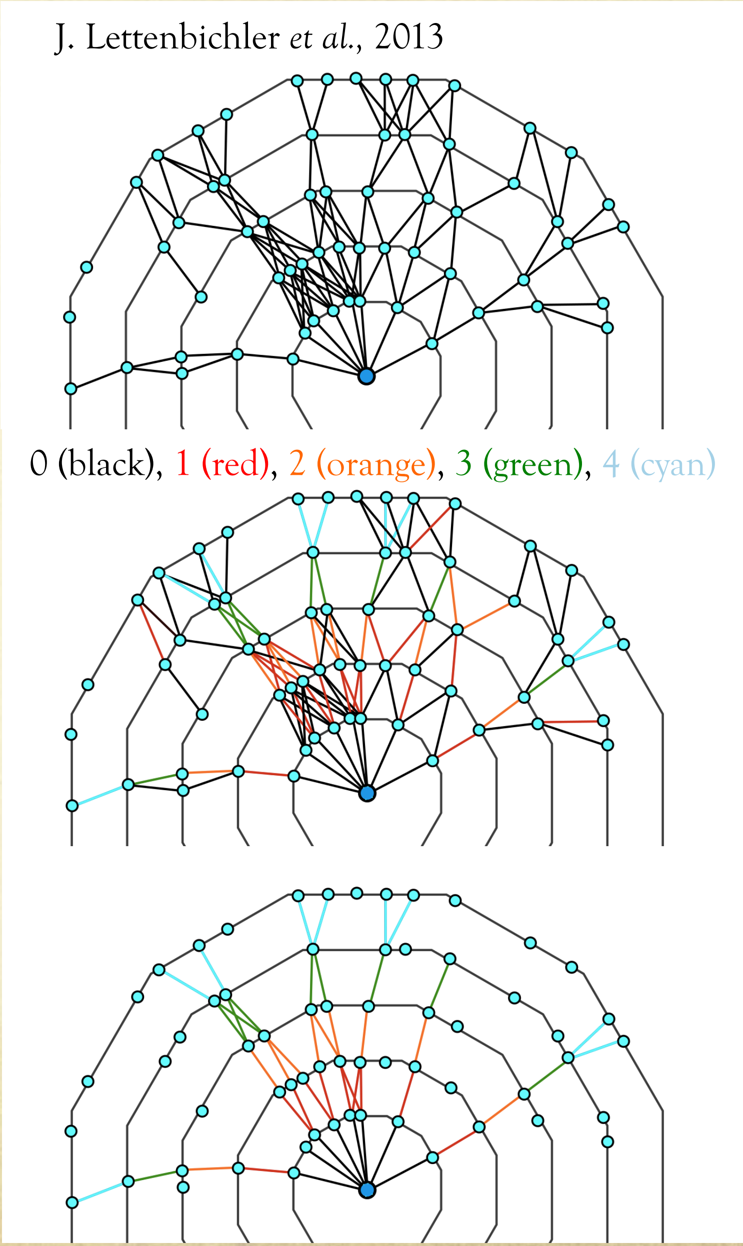
- Shall we do better?
  - Higher track/vertex density, less efficient the classical method
  - Allows for many options and best choice
- Adaptive features
  - Dynamic change of track parameters during finding/fitting
  - Measurements are weighted according to their uncertainty
    - Allows to take into account several "normally excluded" info
  - Many hypothesis are handled simultaneously
    - But their number decrease with iterations (annealing like behavior)
  - Non-linearity
  - Often CPU-time costly (is that still a problem?)
- Examples
  - Neural network, Elastic nets, Gaussian-sum filters, Deterministic annealing



Cellular automaton

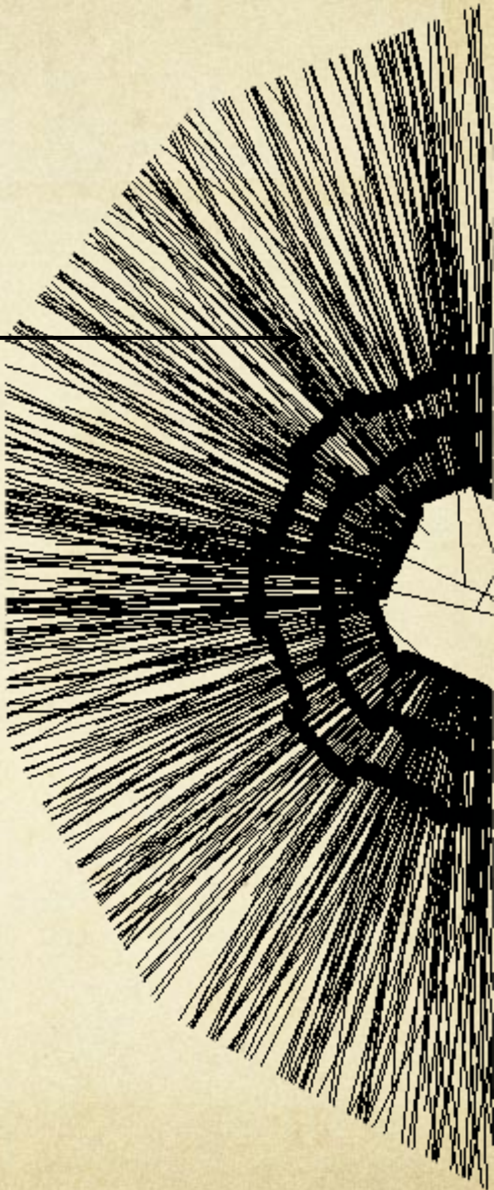
Cellular automaton

- Initialization
  - built any cell (= segment of 2 points)
- Iterative step
  - associate neighbour cells (more inner)
  - Raise "state" with associated cells
  - Kill lowest state cells

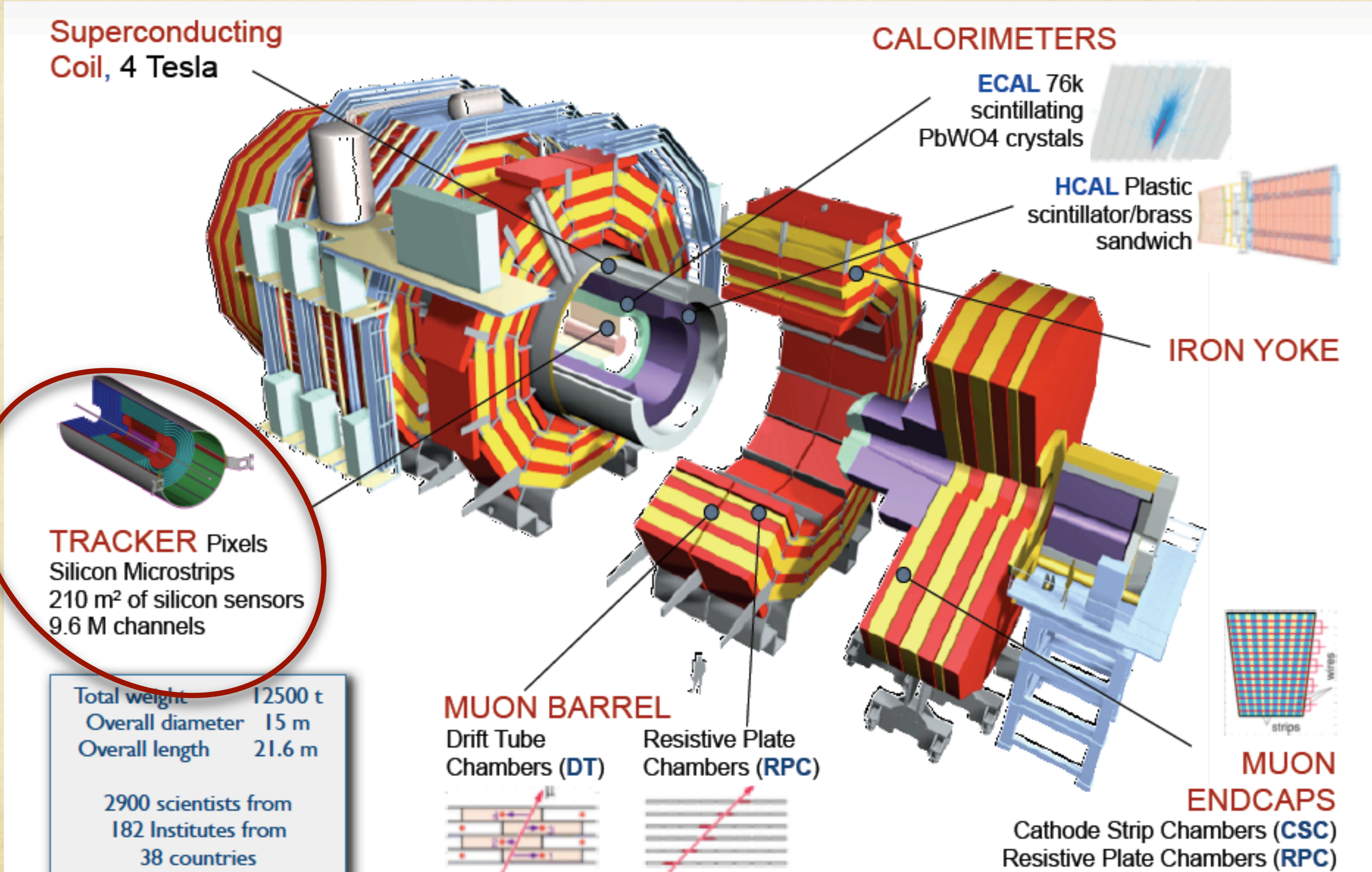


5. Deconstructing some tracking systems

- CMS (colliders)
- AMS, ANTARES (telescopes)



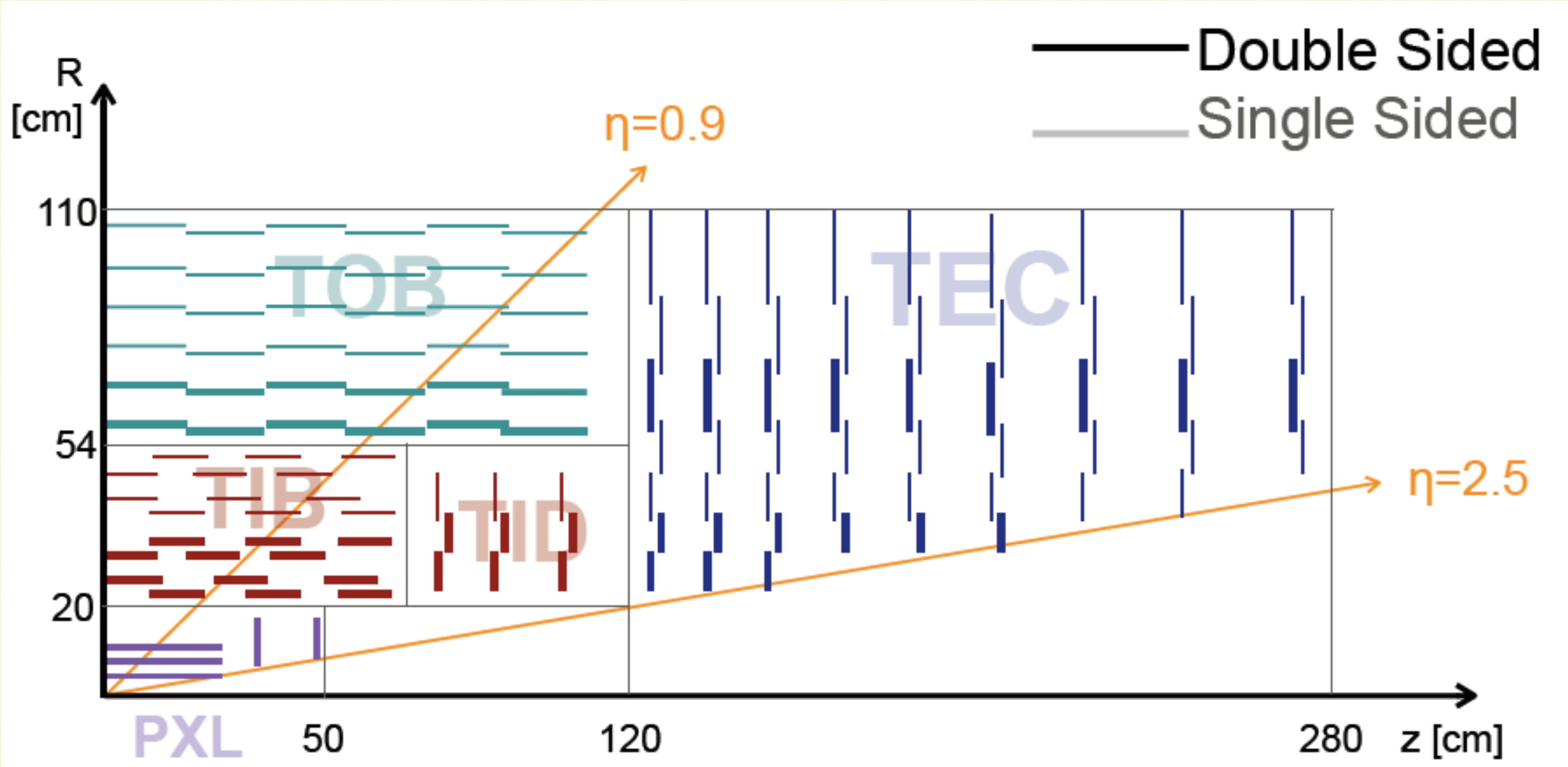
CMS



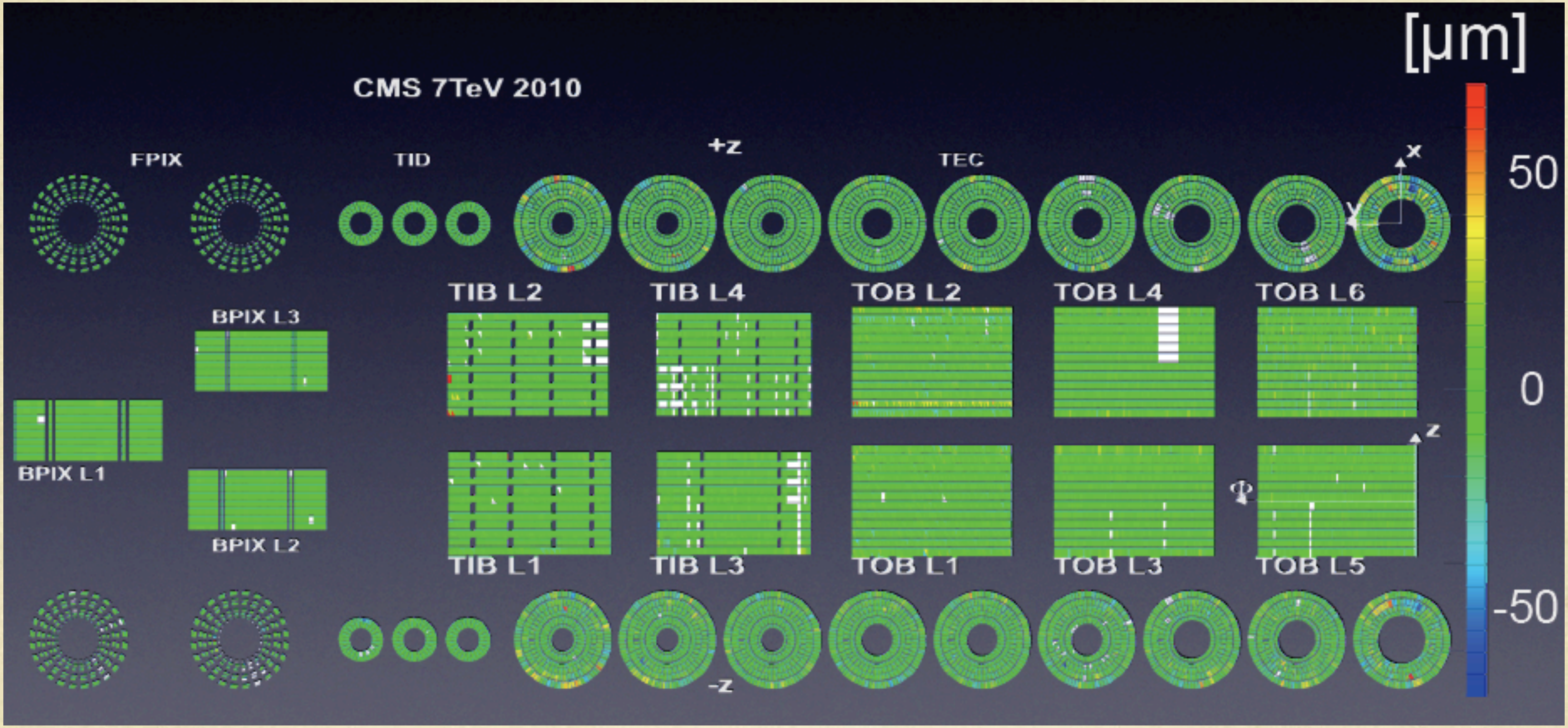
Superconducting Coil, 4 Tesla
CALORIMETERS
ECAL 76k scintillating PbWO4 crystals
HCAL Plastic scintillator/brass sandwich
IRON YOKE
TRACKER Pixels
Silicon Microstrips
210 m² of silicon sensors
9.6 M channels
Total weight 12500 t
Overall diameter 15 m
Overall length 21.6 m
2900 scientists from
182 Institutes from
38 countries
MUON BARREL
Drift Tube Chambers (DT)
Resistive Plate Chambers (RPC)
MUON ENDCAPS
Cathode Strip Chambers (CSC)
Resistive Plate Chambers (RPC)
wires
strips

CMS

- The trackerS



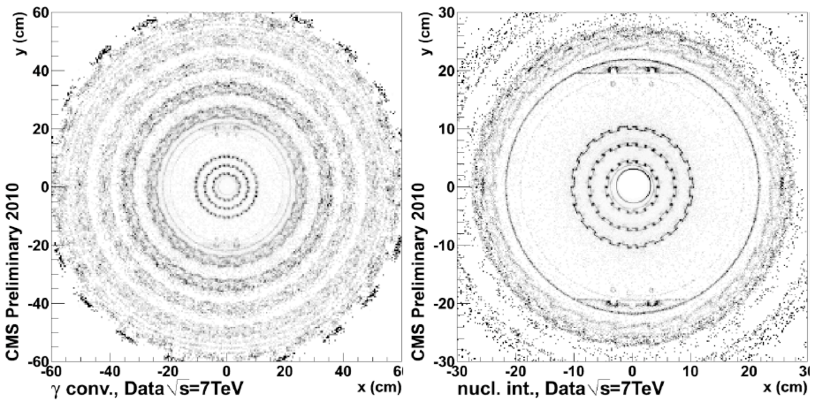
- Alignment residual width



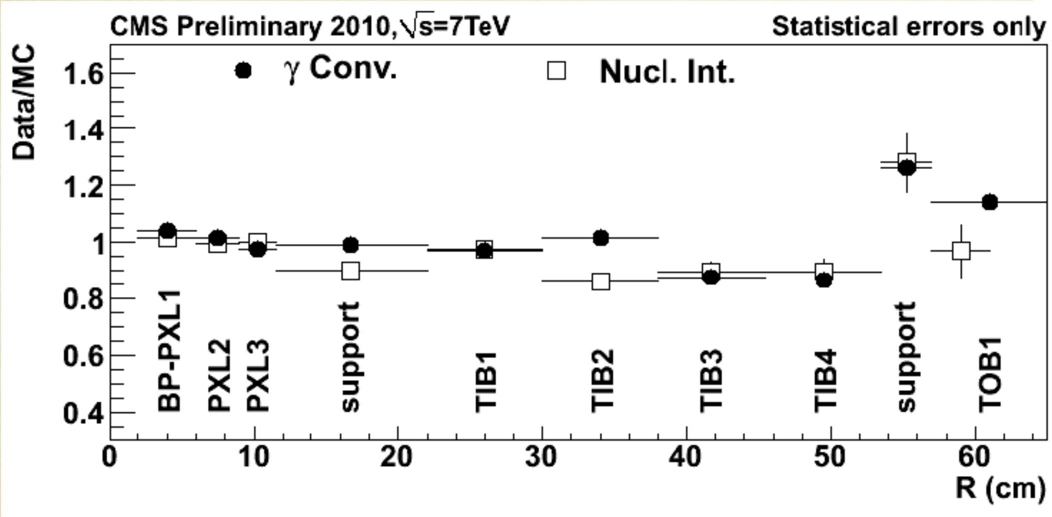
5. Some tracking systems:

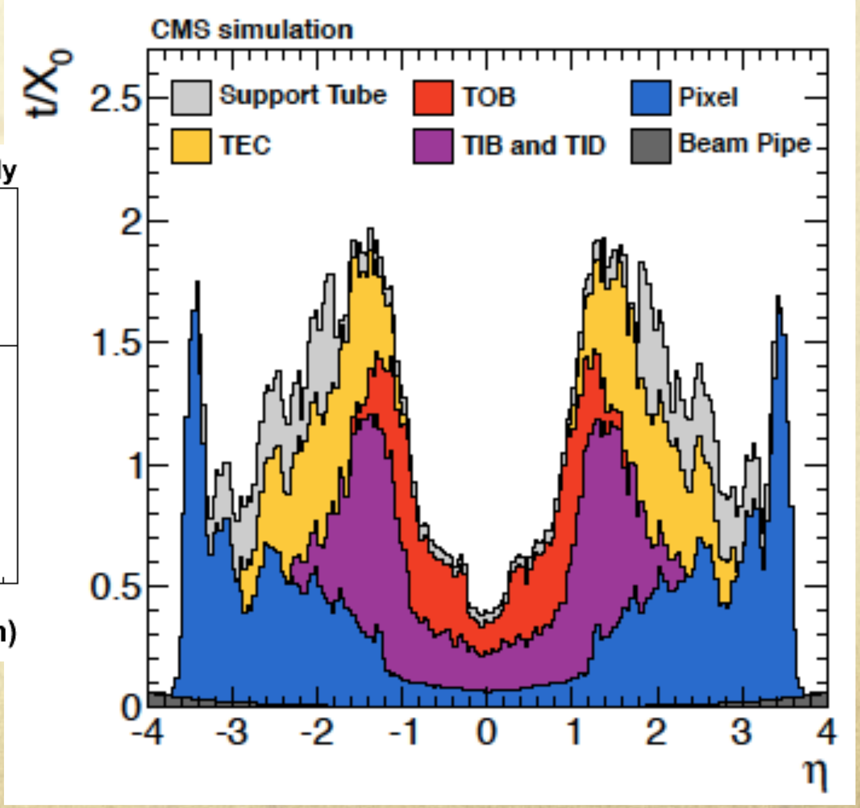
CMS

- Taking a picture of the material budget
  - ➛ Using secondary vertices from $\gamma \rightarrow e^+e^-$



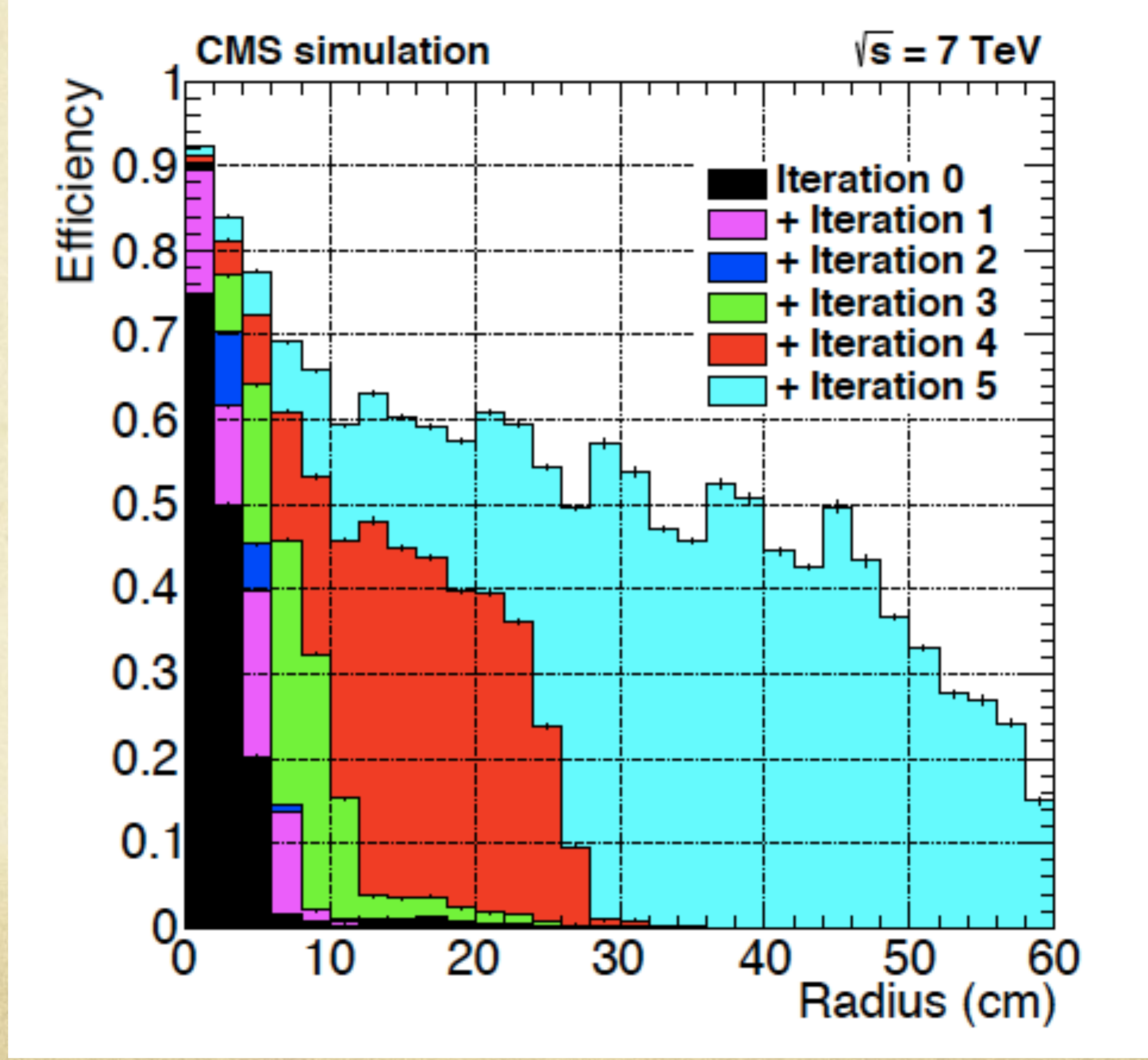
- Measuring it by data/simulation comparison





CMS

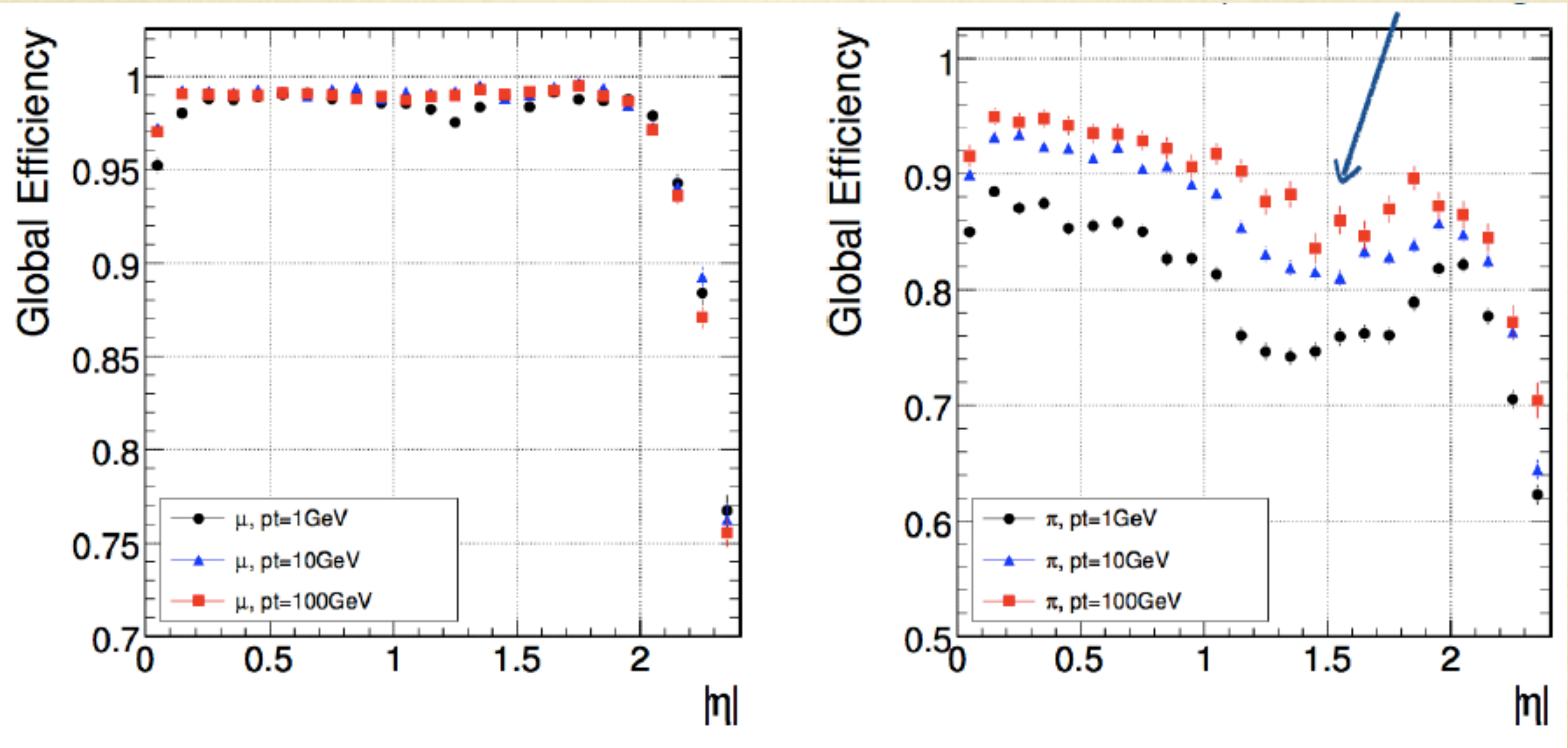
- Tracking algorithm = multi-iteration process



5. Some tracking systems:

CMS

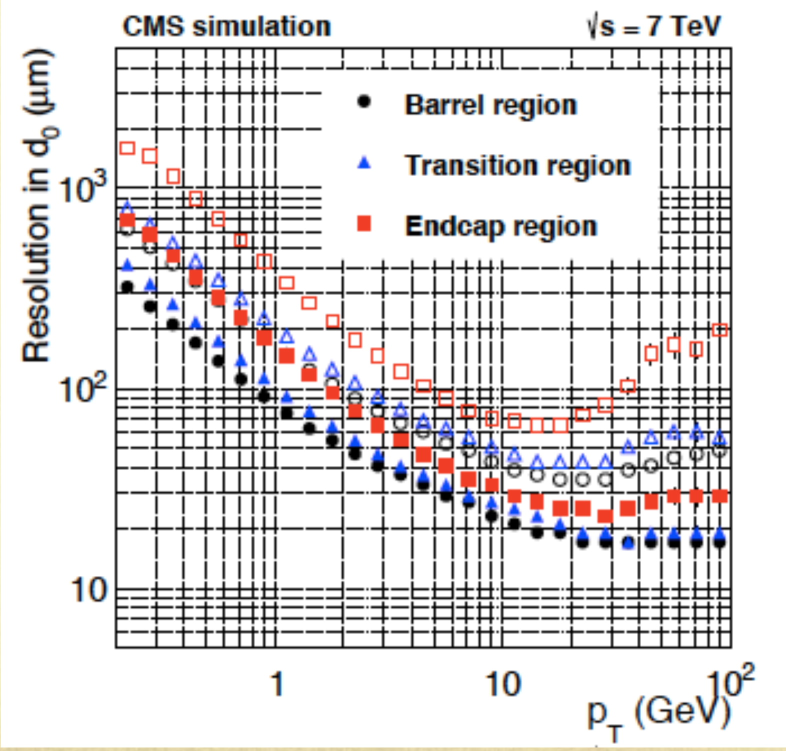
- Tracking efficiency

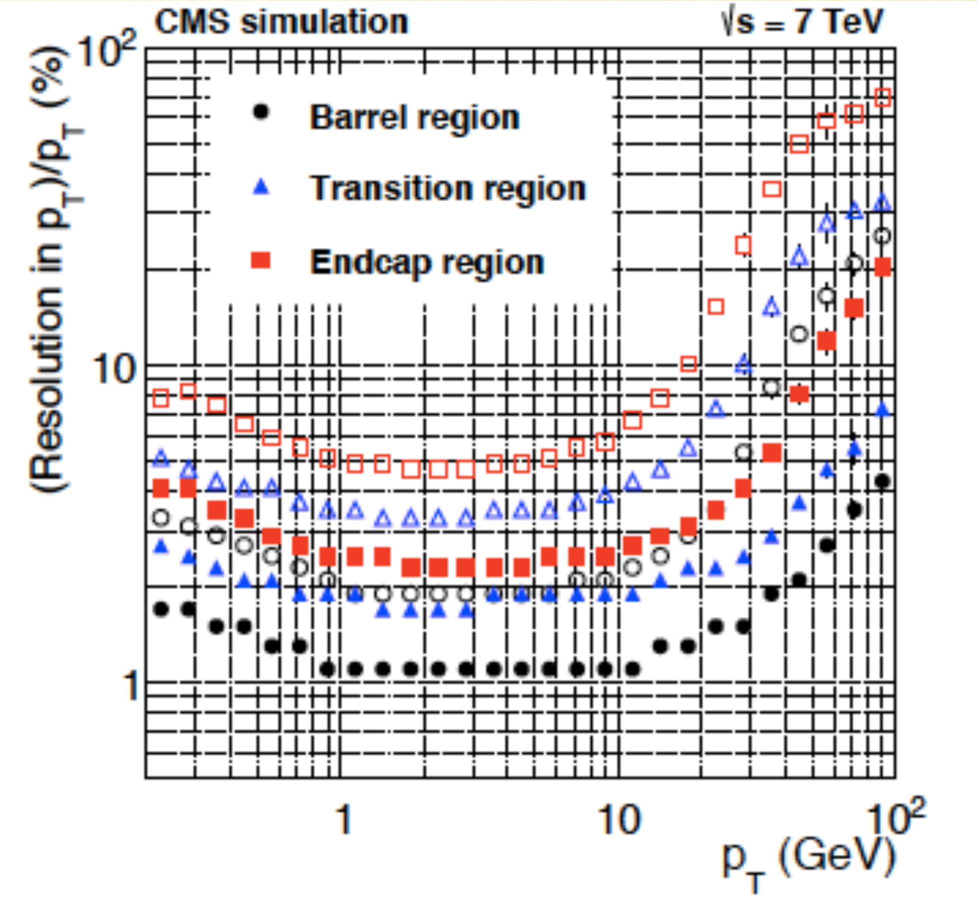


CMS

- Tracking resolution

d0 = transverse impact parameter

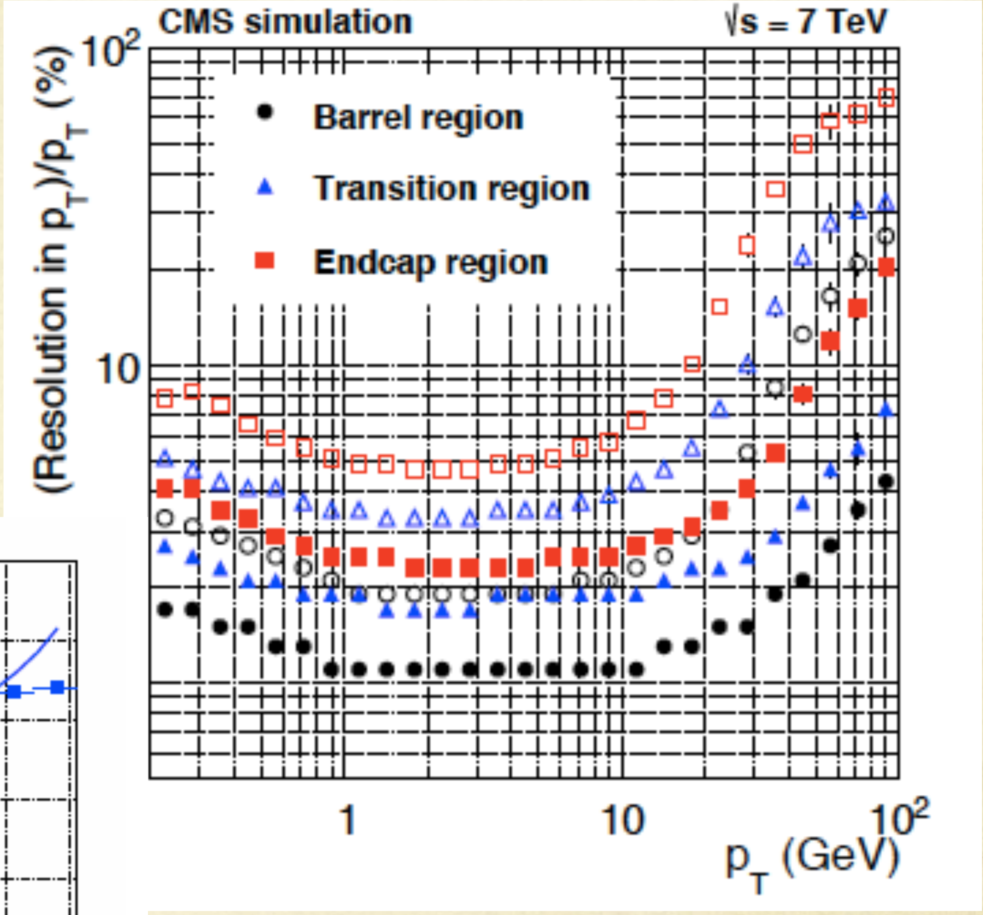




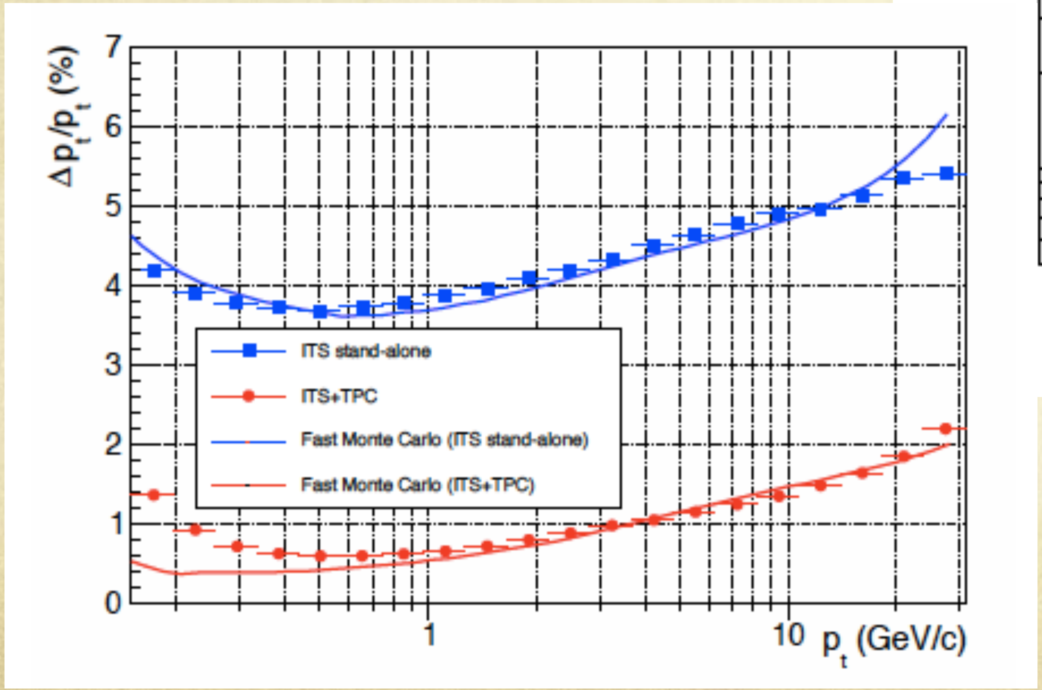
5. Some tracking systems:

CMS

- Tracking resolution

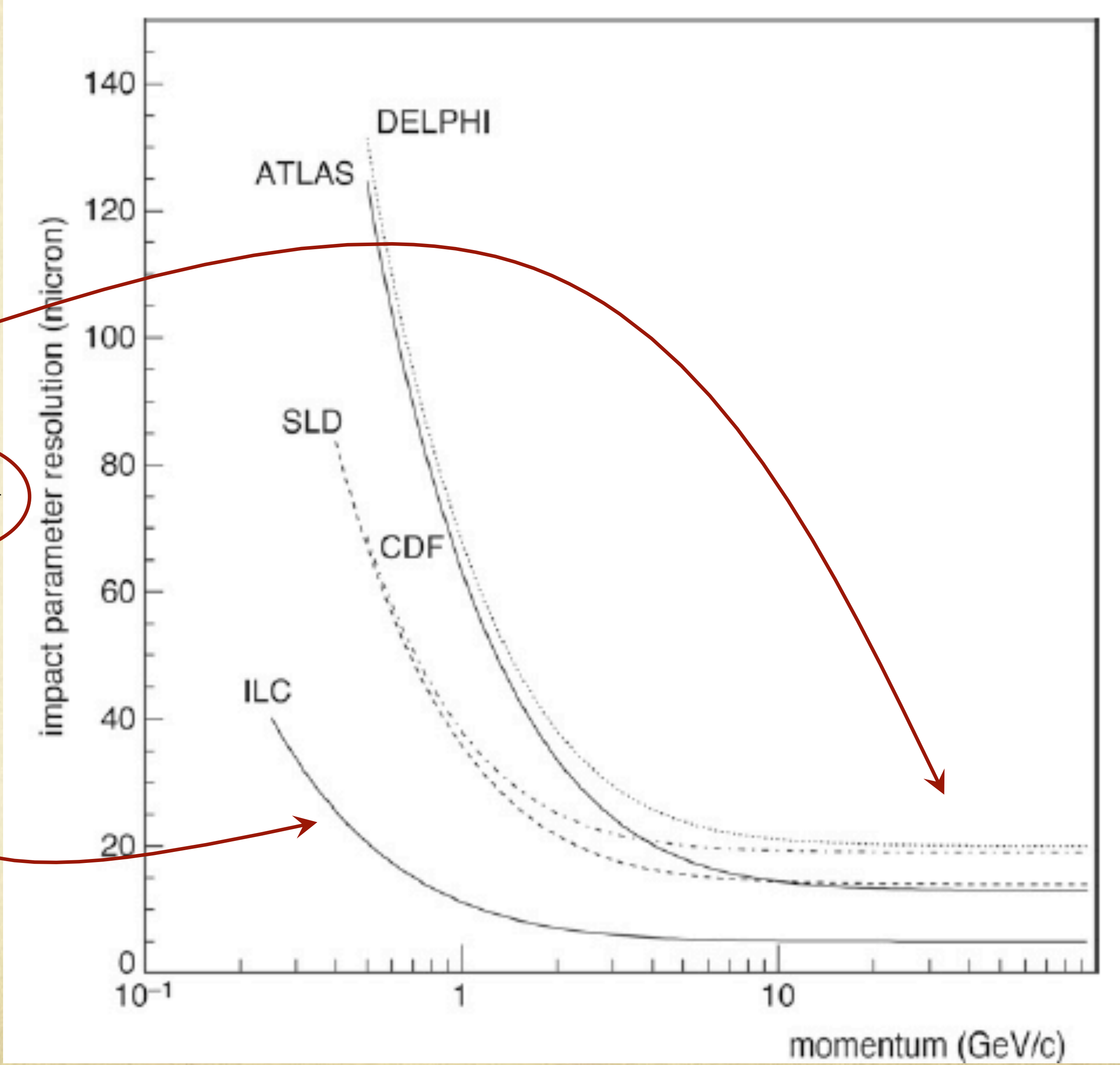


ALICE figure

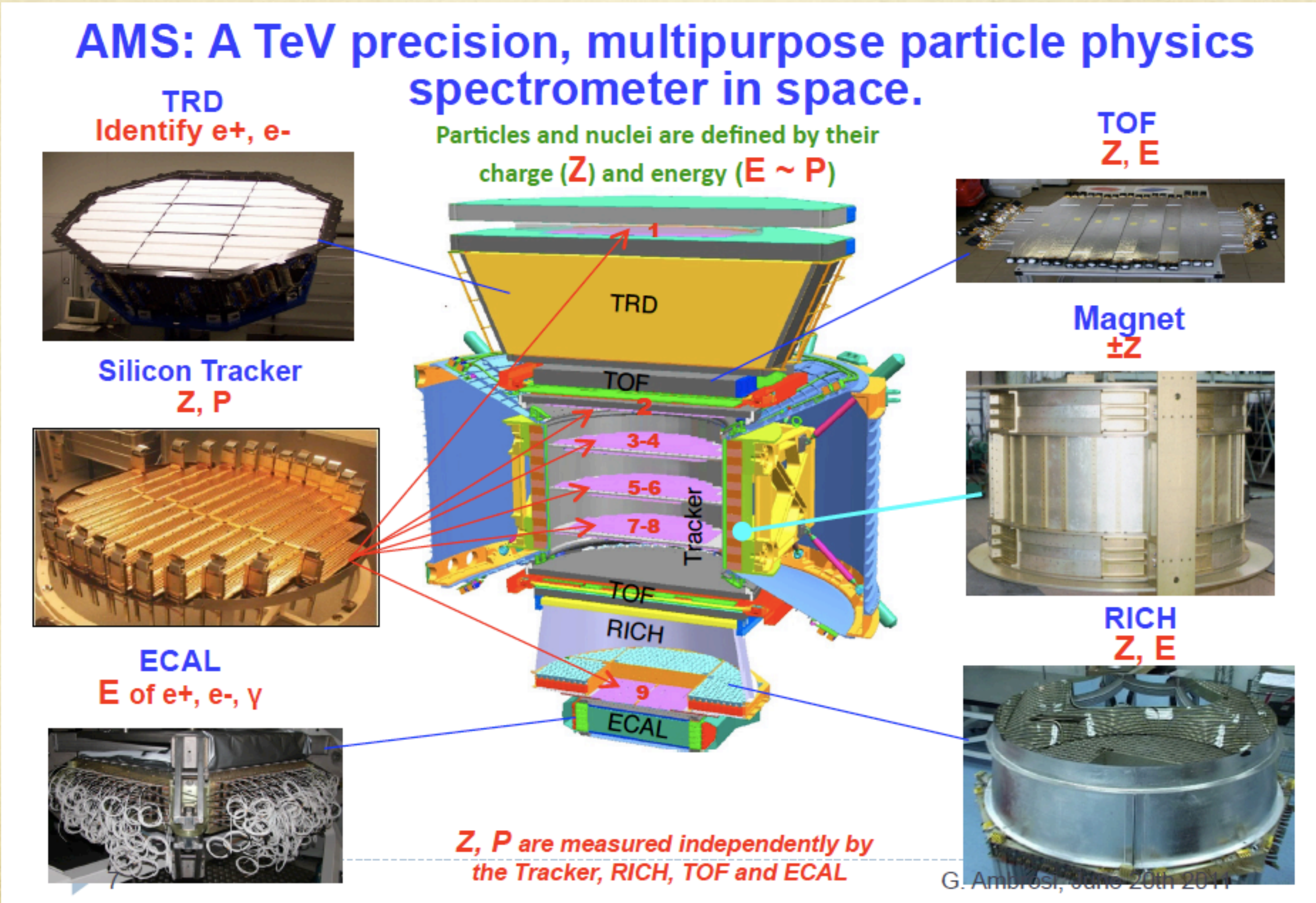


Impact parameter resolution

$$\sigma_{IP} \propto \frac{\sqrt{R_{ext}^2\sigma_{int}^2 + R_{int}^2\sigma_{ext}^2}}{R_{ext} - R_{int}} \oplus \frac{R_{int}\sigma_{\vartheta(ms)}}{p\sin^{3/2}(\theta)}$$



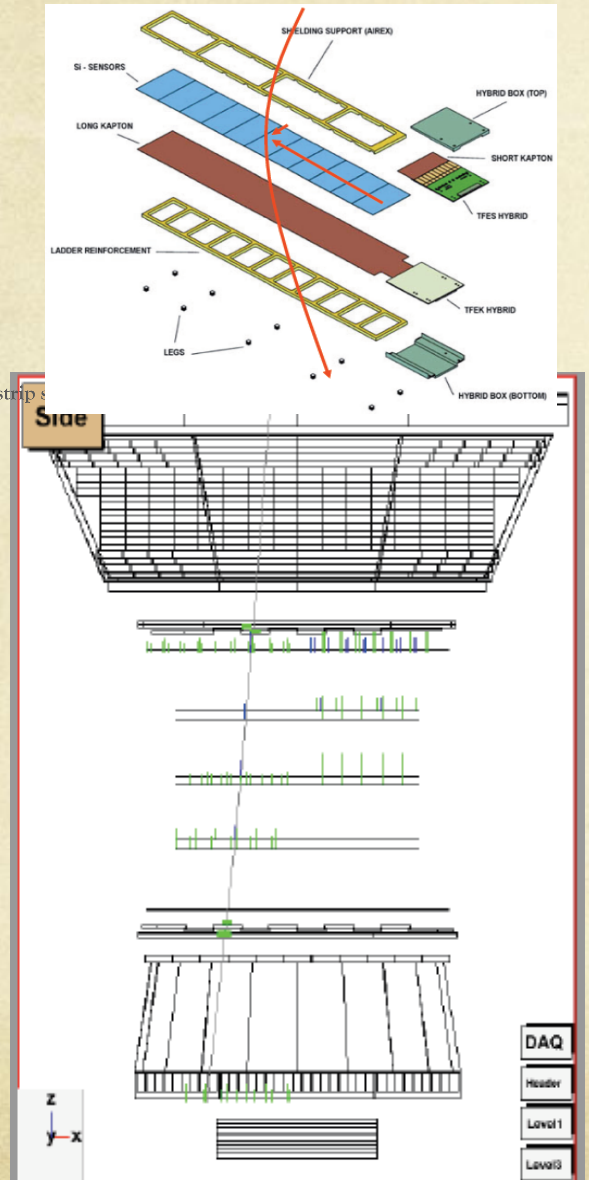
AMS


AMS: A TeV precision, multipurpose particle physics spectrometer in space.
Particles and nuclei are defined by their charge (Z) and energy (E ~ P)
TRD
Identify e+, e-
Silicon Tracker
Z, P
ECAL
E of e+, e-, γ
TOF
Z, E
Magnet
±Z
RICH
Z, E
TRD
TOF
1
2
3-4
5-6
7-8
Tracker
TOF
RICH
9
ECAL
Z, P are measured independently by the Tracker, RICH, TOF and ECAL
G. Ambrosi, June 20th 2011

5. Some tracking systems:

AMS

Silicon strip s



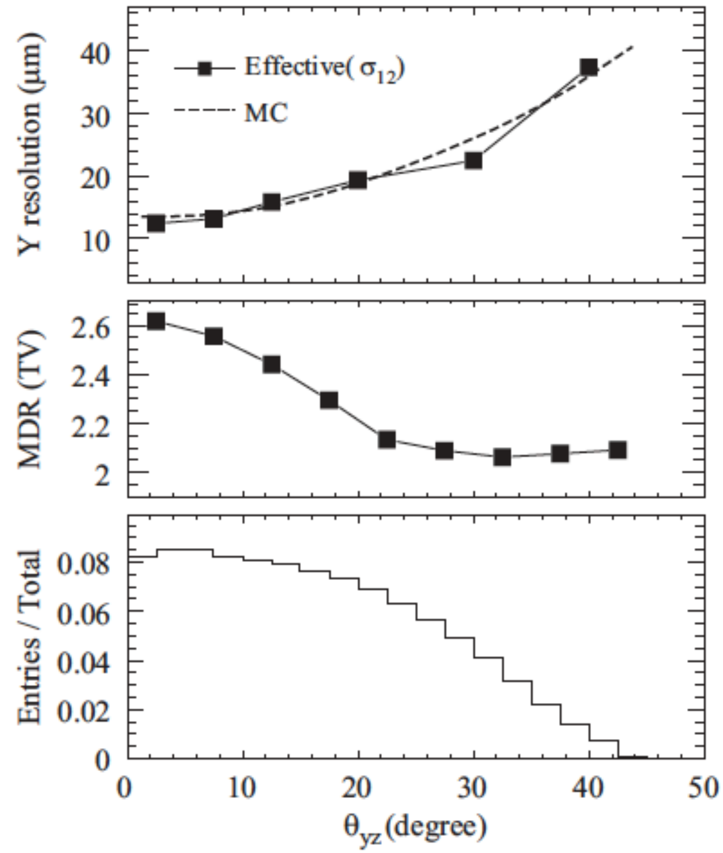
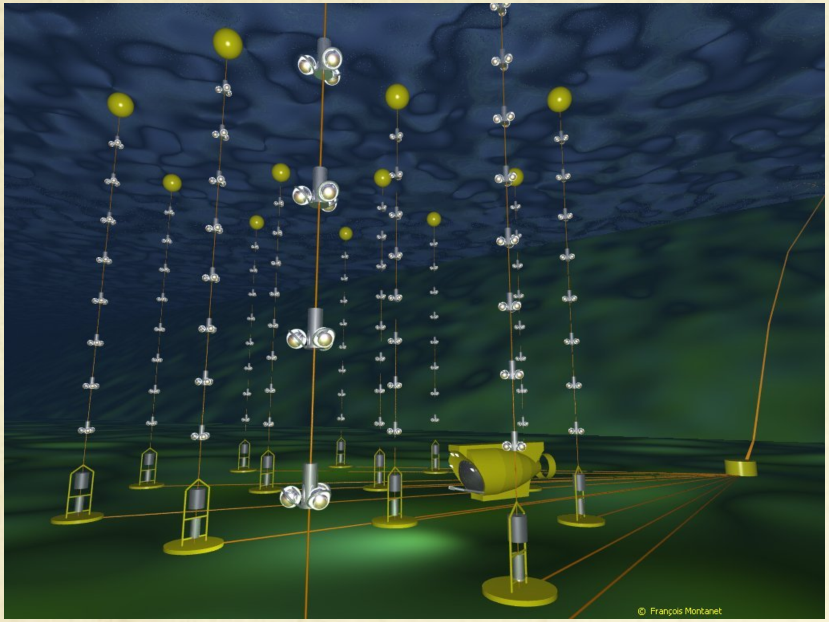
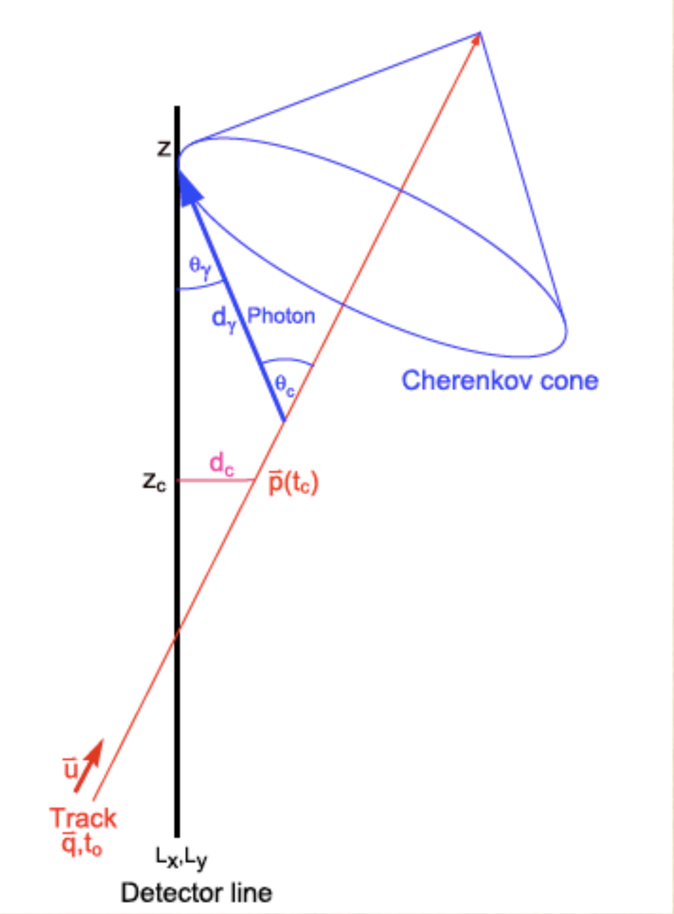


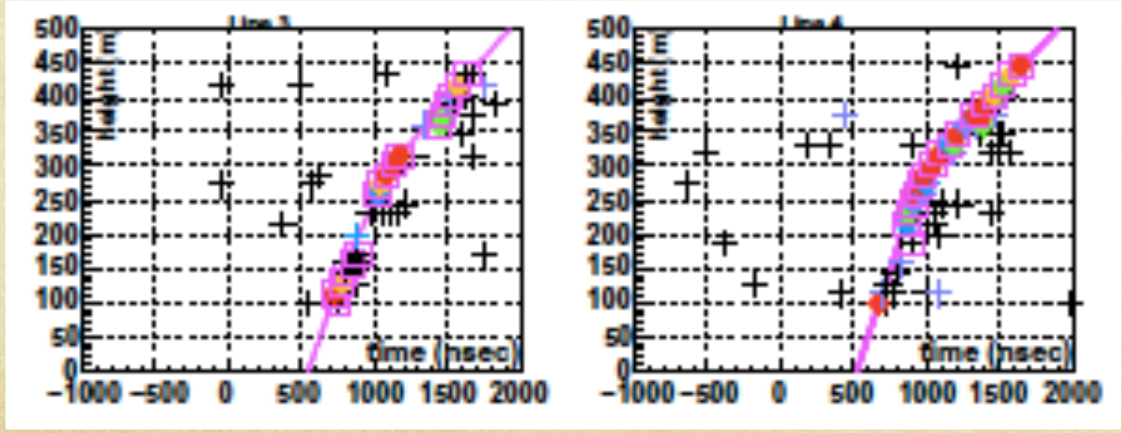
Fig. 5. The effective position resolution (weighted average of two Gaussian widths) in the *y*-coordinate for different inclination angles (top), the Maximum Detectable Rigidity (MDR, 100% rigidity measurement error) as a function of the inclination angle estimated for 1 TV proton incidence with the simulation (middle), and the inclination angle distribution in the geometric acceptance of the tracker (bottom).

5. Some tracking systems:

ANTARES



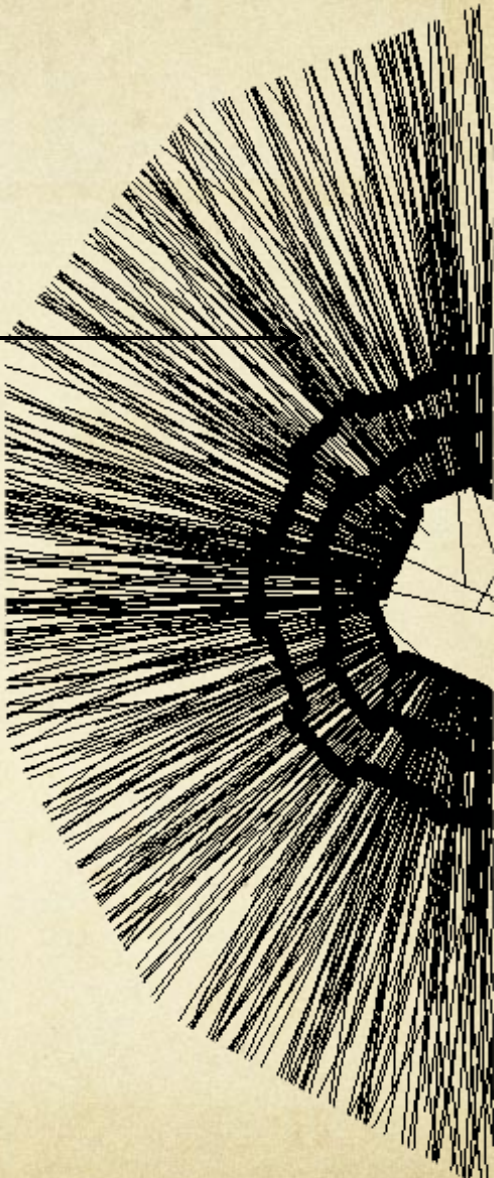




Summary

- **Fundamental characteristics of any tracking & vertexing device:**
  - (efficiency), granularity, material budget, power dissipation, “timing”, radiation tolerance
  - All those figures are intricated: each technology has its own limits
- **Many technologies available**
  - None is adapted to all projects (physics + environment choose, in principle)
  - Developments are ongoing for upgrades & future experiments
    - Goal is to extent limits of each techno. ➞ convergence to a single one?
- **Reconstruction algorithms**
  - Enormous boost (variety and performances) in the last 10 years
  - Each tracking system has its optimal algorithm
- **Development trend**
  - Always higher hit rates call for more data reduction
  - Tracking info in trigger ➞ high quality online tracking/vertexing
- **Link with:**
  - PID: obvious with TPC, TRD, topological reco.
  - Calorimetry: Particle flow algorithm, granular calo. using position sensors

References



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  also consult lhc-detector-**alignment-workshop.web.cern.ch**

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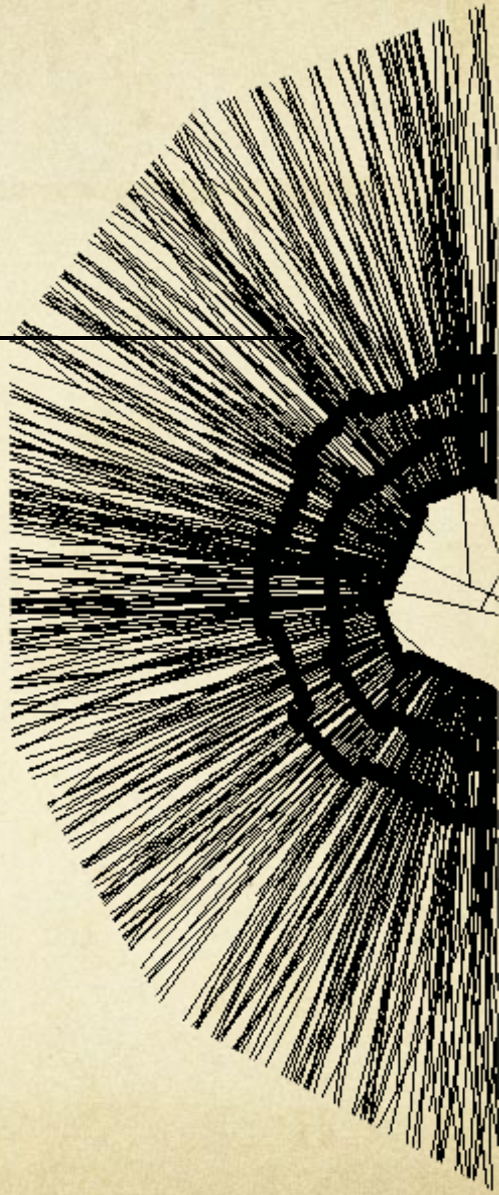
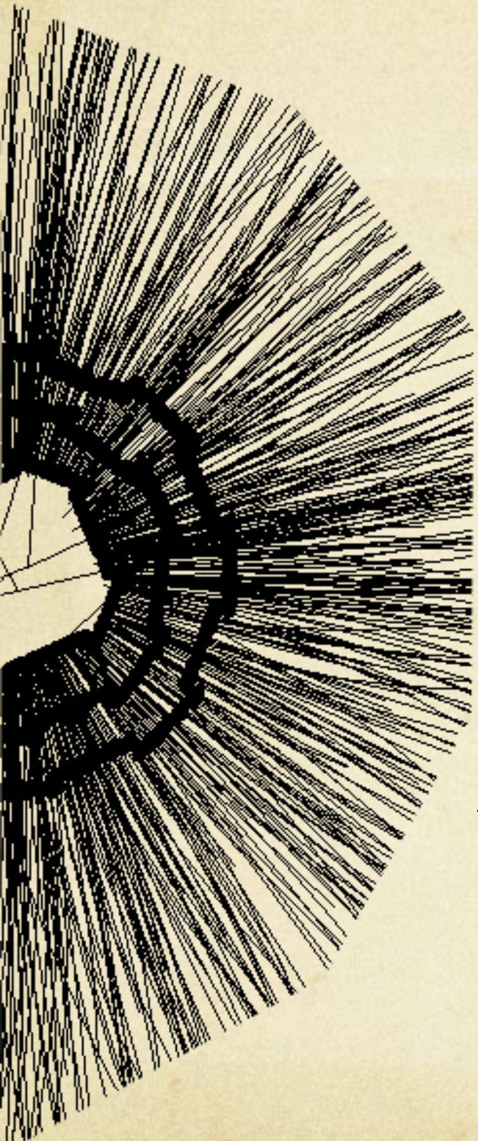
Was not discussed

- Particle interaction with matter
- The readout electronics
- Cooling systems
- The magnets to produce the mandatory magnetic field for momentum measurement
- Vertexing

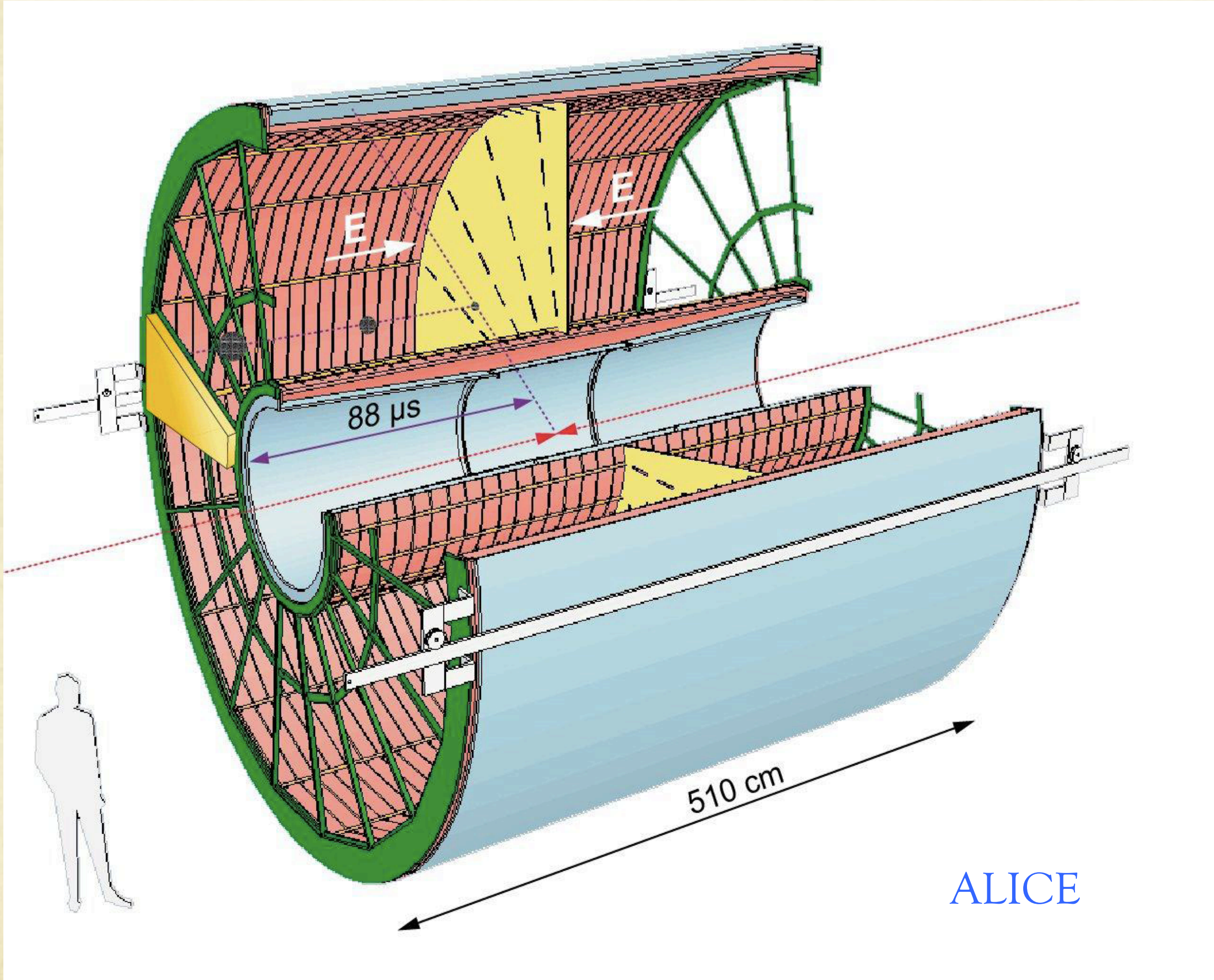




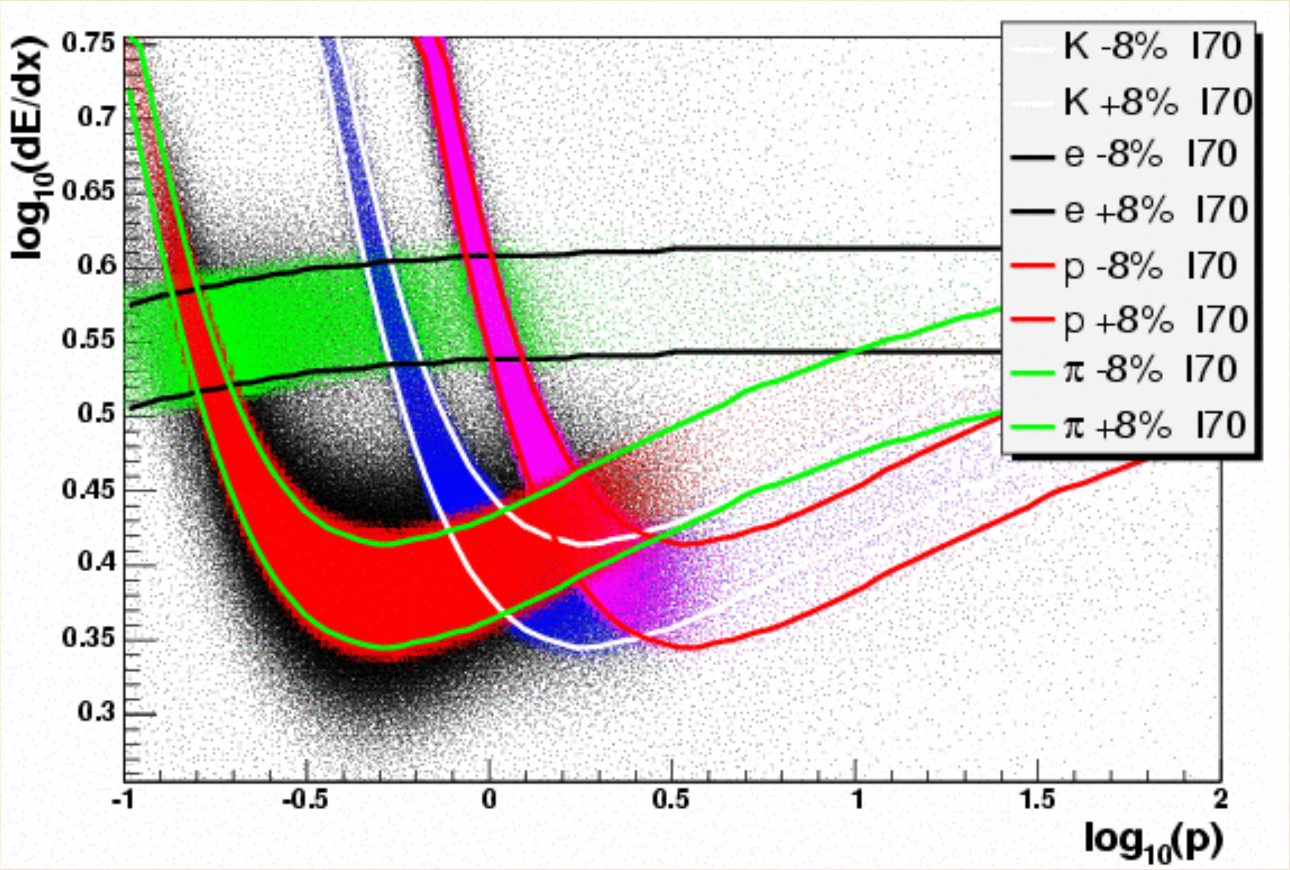
Backups

OPAL drift chamber

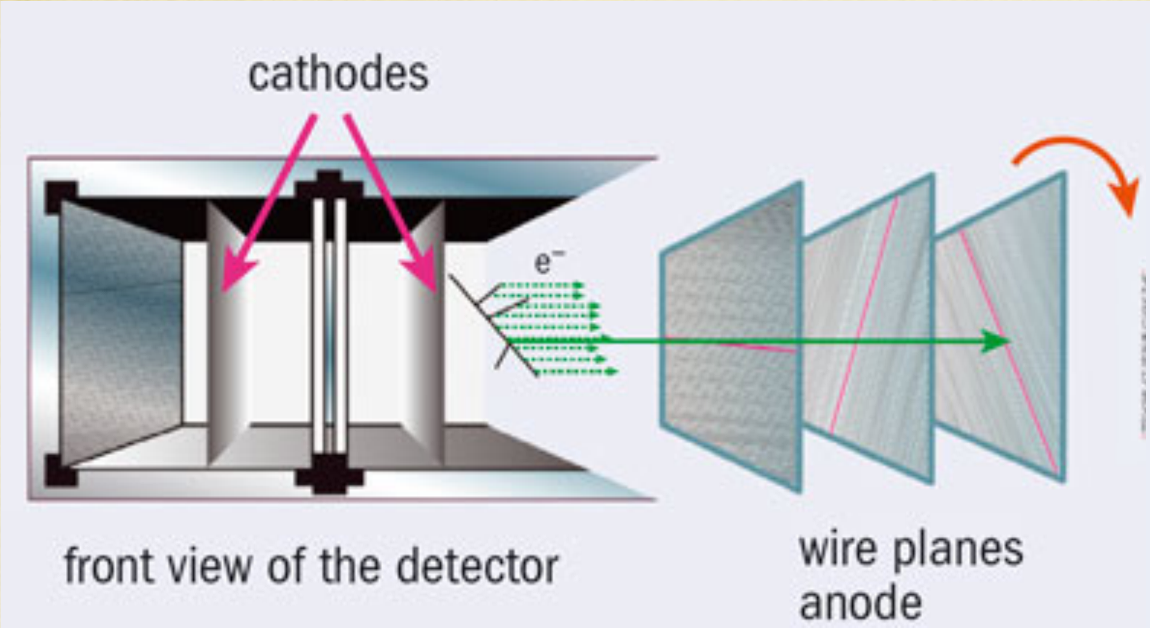
ALICE - TPC

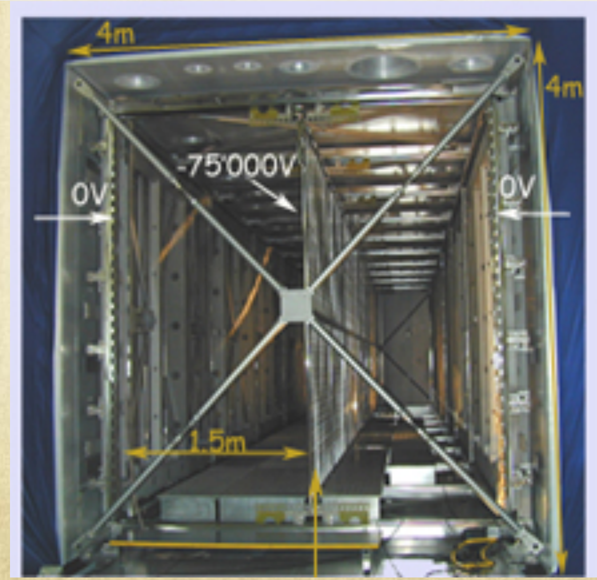


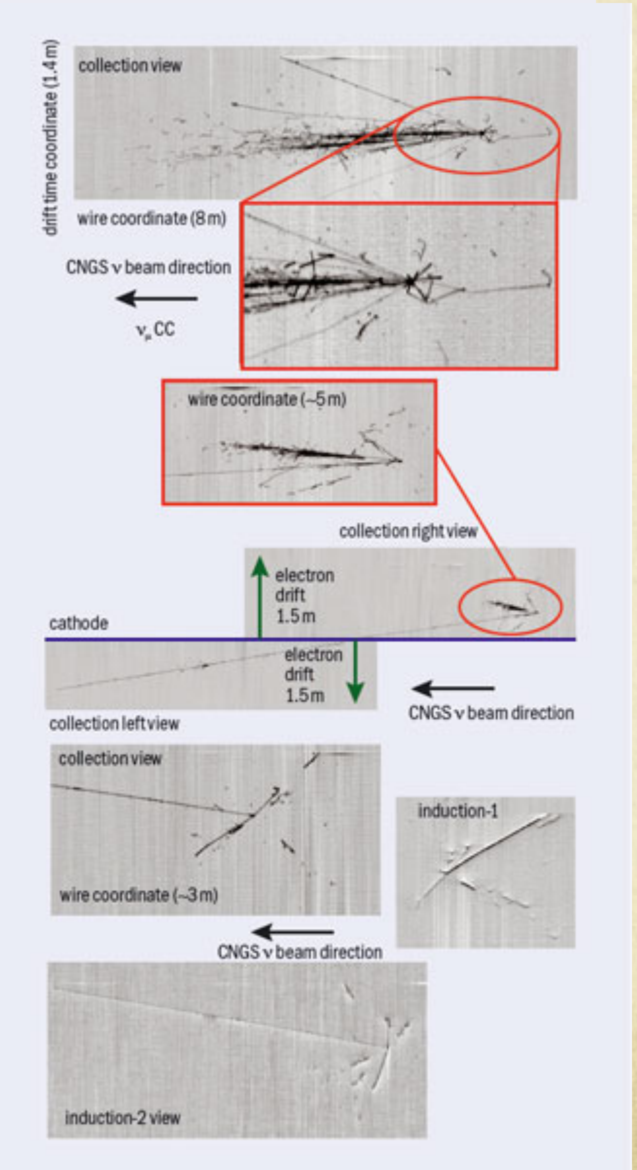
(ALICE) TPC dE/dx



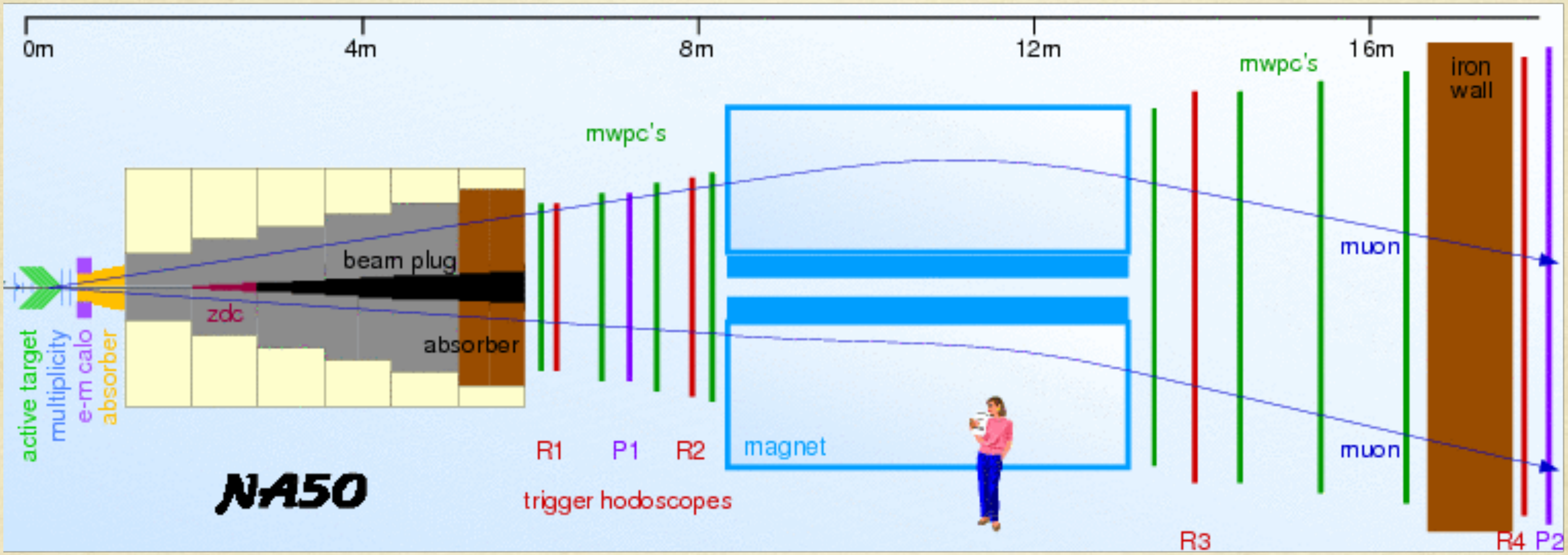
ICARUS - TPC


cathodes
e⁻
front view of the detector
wire planes
anode

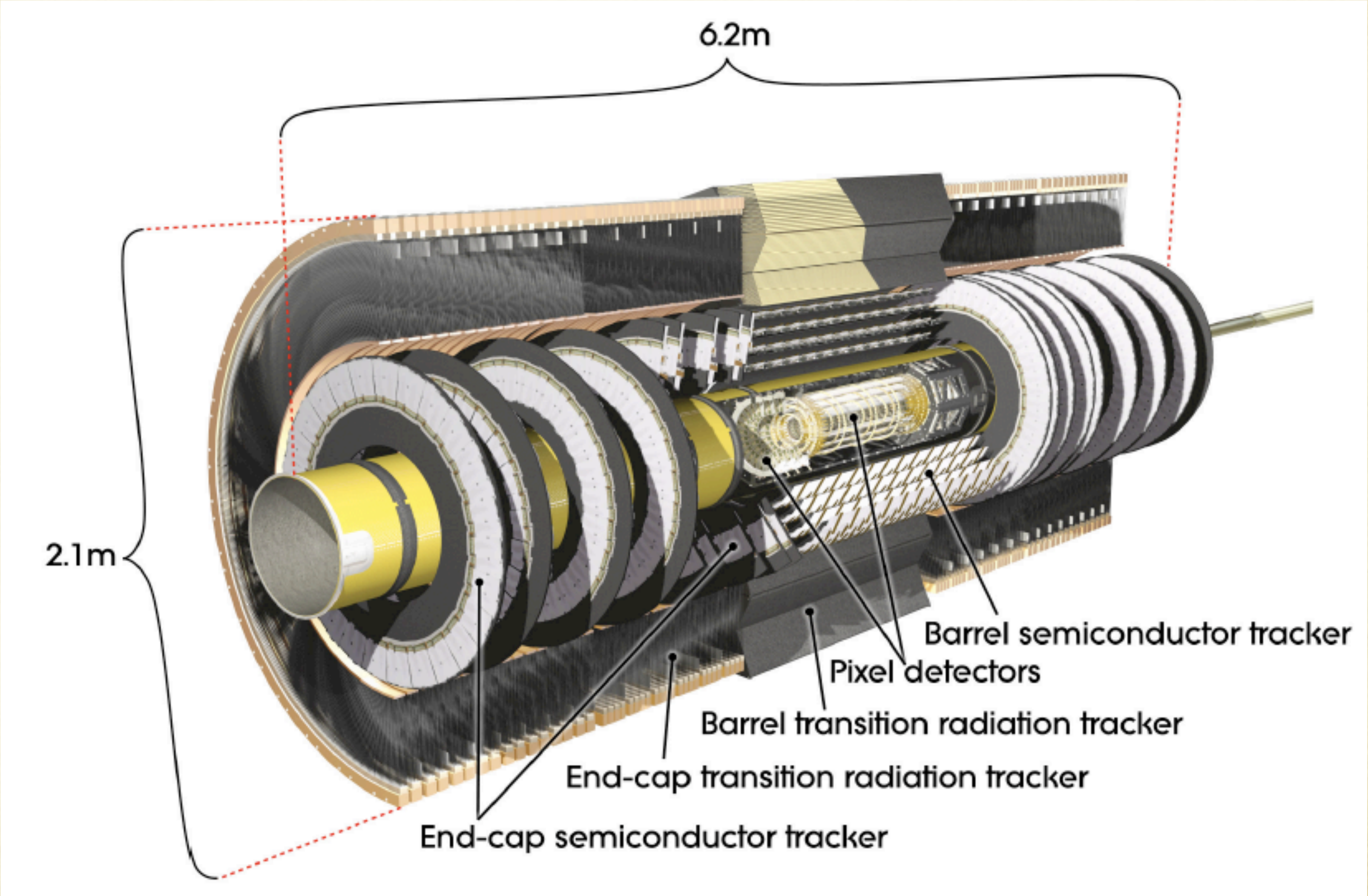

4m
4m
-75 000V
0V
0V
1.5m


collection view
drift time coordinate (1.4 m)
wire coordinate (8 m)
CNGS ν beam direction
ν_μ CC
wire coordinate (~5 m)
collection right view
electron
drift
1.5 m
cathode
electron
drift
1.5 m
CNGS ν beam direction
collection left view
collection view
wire coordinate (~3 m)
induction-1
CNGS ν beam direction
induction-2 view

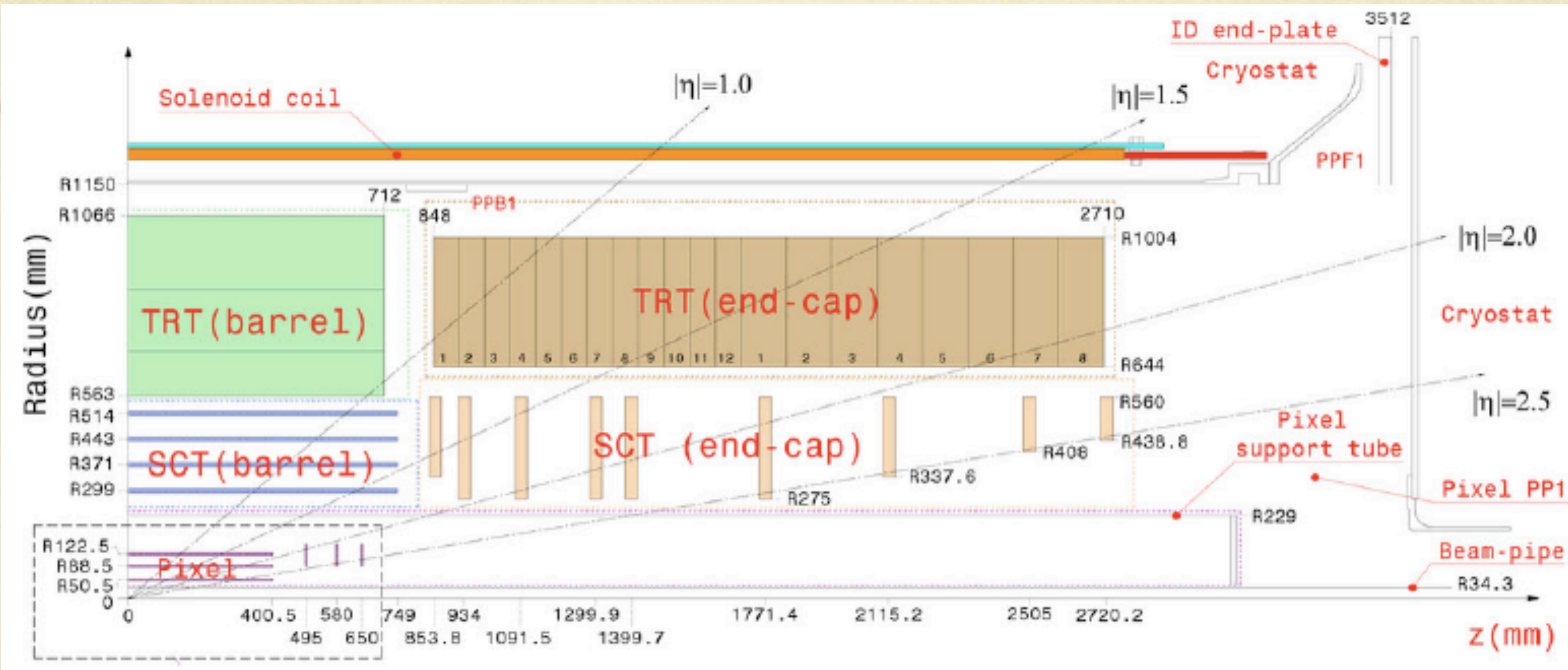
NA-50 fixed target



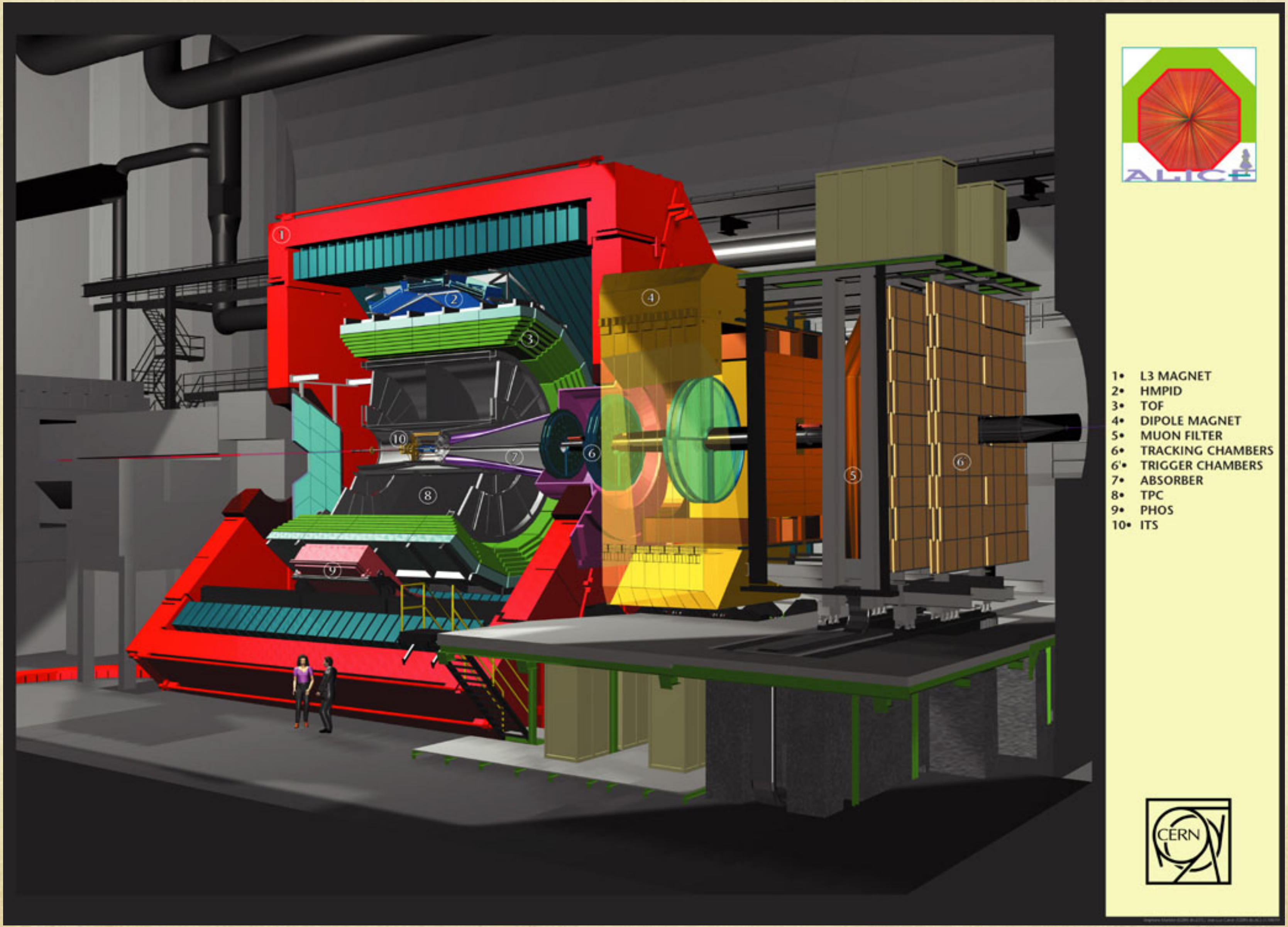
ATLAS tracking setup



ATLAS tracking setup



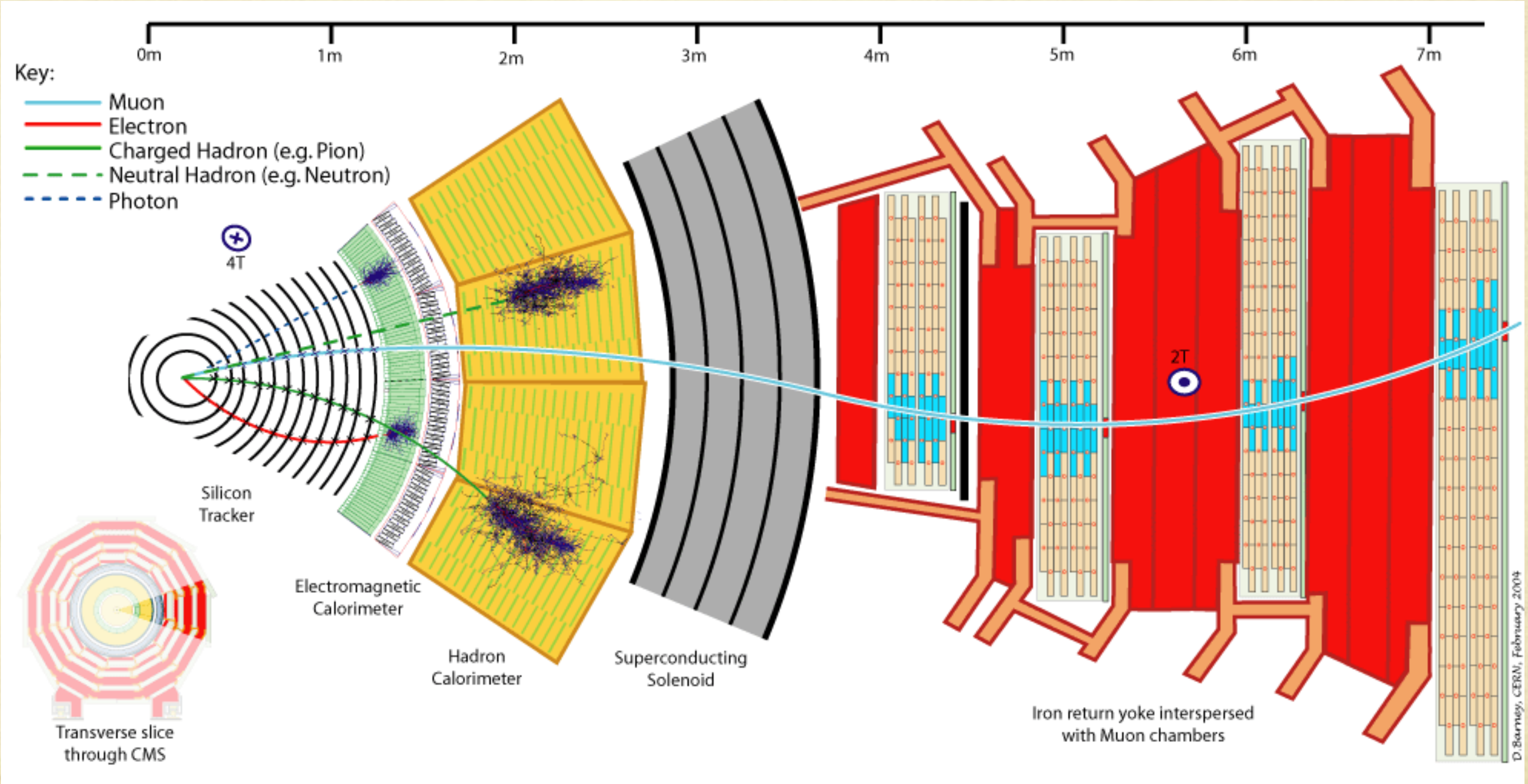
ALICE setup



1• L3 MAGNET
2• HMPID
3• TOF
4• DIPOLE MAGNET
5• MUON FILTER
6• TRACKING CHAMBERS
6'• TRIGGER CHAMBERS
7• ABSORBER
8• TPC
9• PHOS
10• ITS
CERN

Backups:

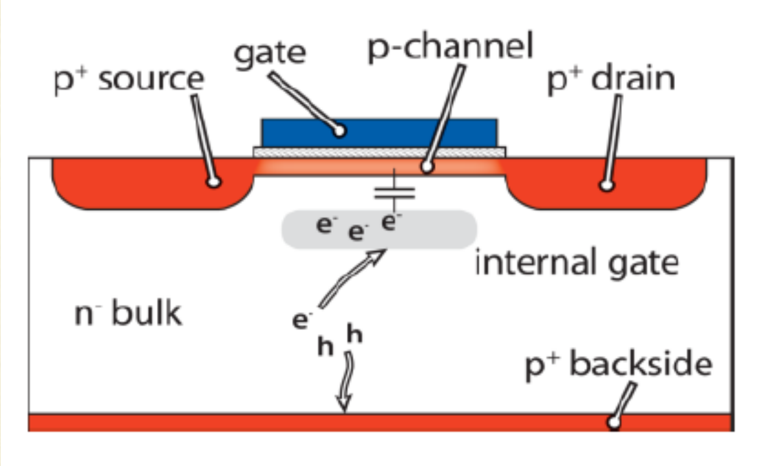
CMS



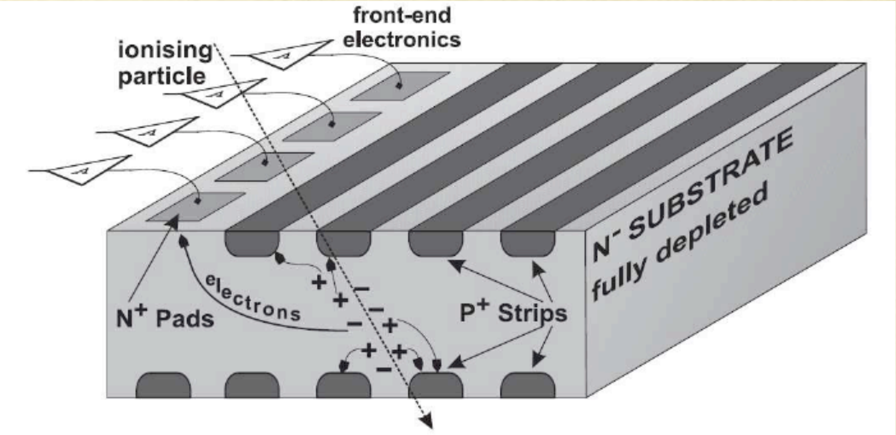
0m
1m
2m
3m
4m
5m
6m
7m
Key:
Muon
Electron
Charged Hadron (e.g. Pion)
Neutral Hadron (e.g. Neutron)
Photon
4T
2T
Silicon Tracker
Electromagnetic Calorimeter
Hadron Calorimeter
Superconducting Solenoid
Iron return yoke interspersed with Muon chambers
Transverse slice through CMS
D.Barney, CERN, February 2004

More position sensitive detectors

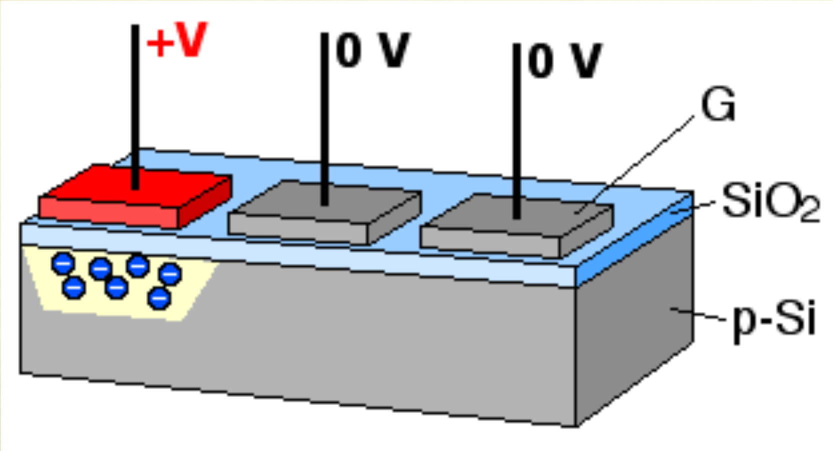
DEPFET



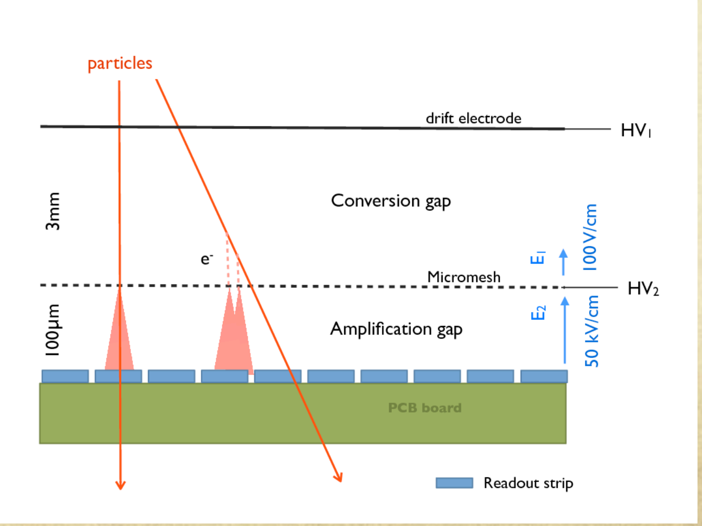
Silicon drift



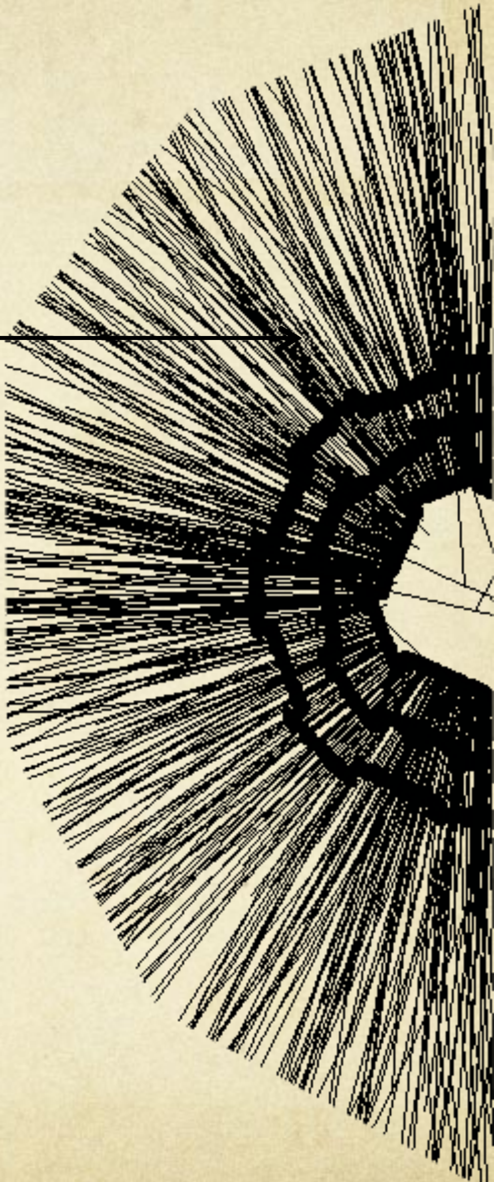
CCD



MICROMEGAS



Was not discussed



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