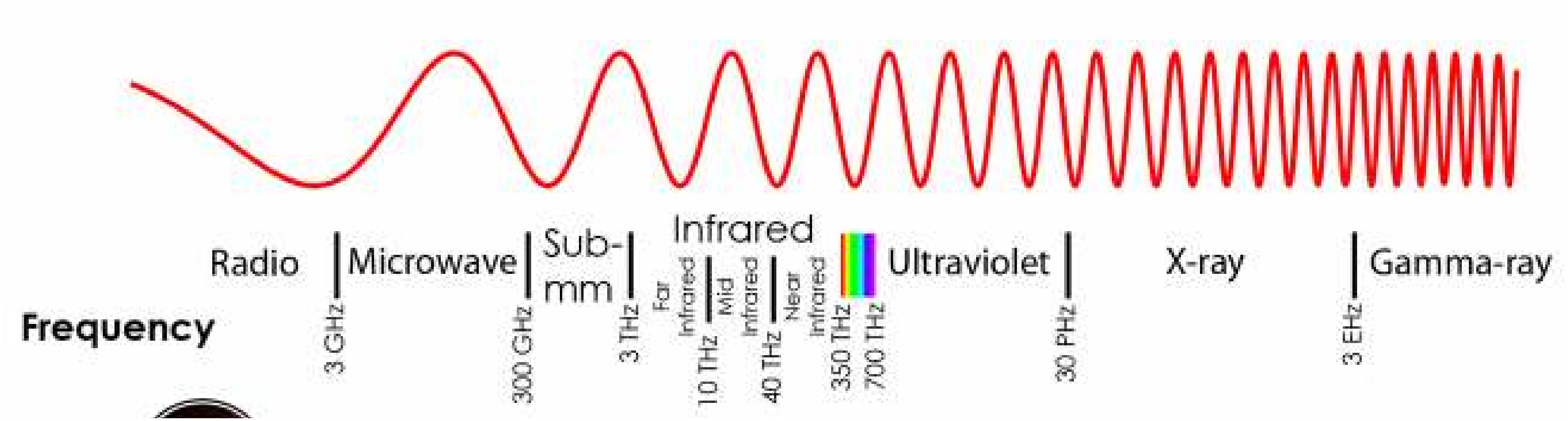
Gravitational Waves Physics and Techniques

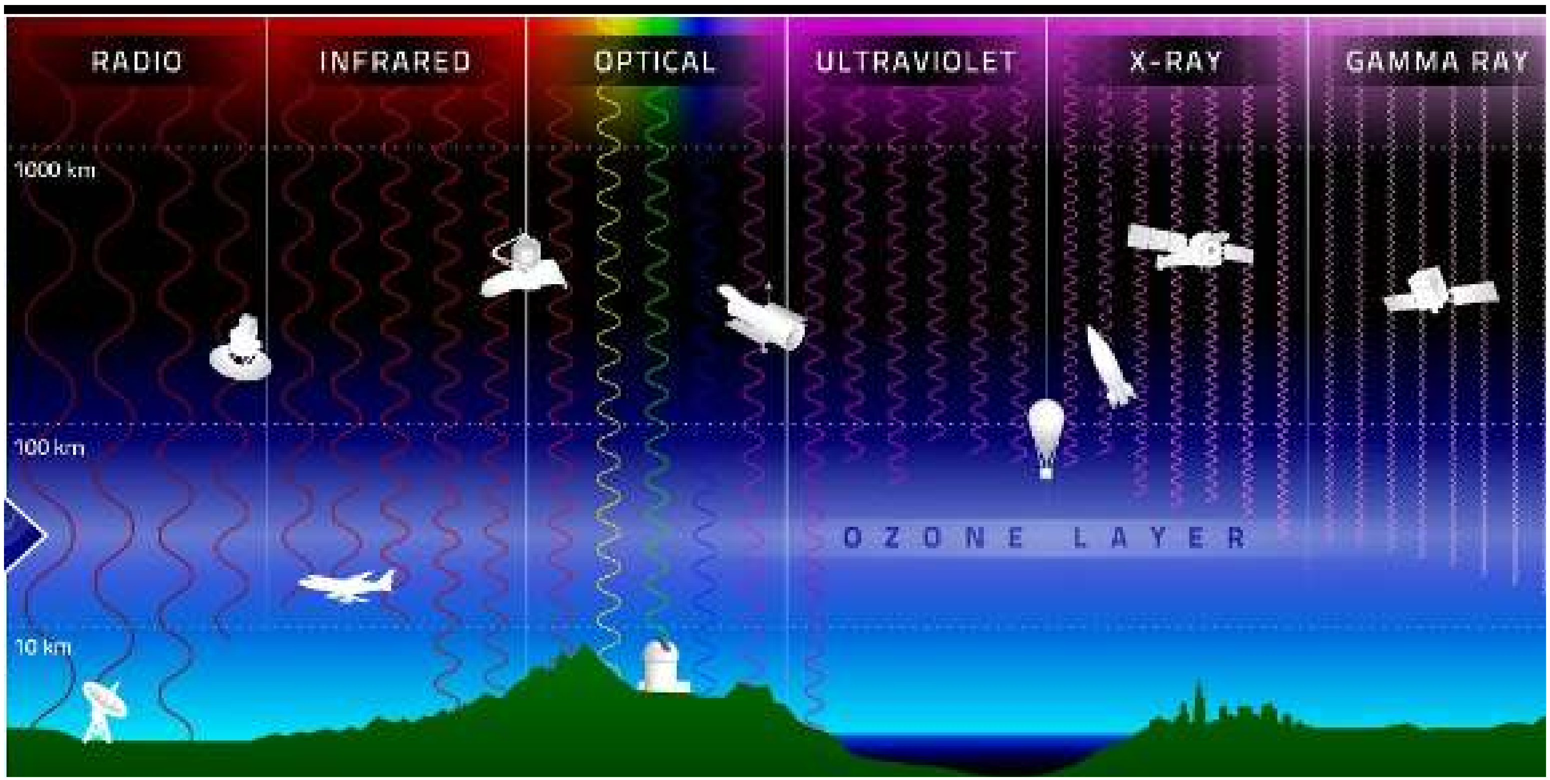
Part I: from GR to GWs

M. Razzano
University of Pisa & INFN-Pisa

IDPASC School – 20-30 June 2017

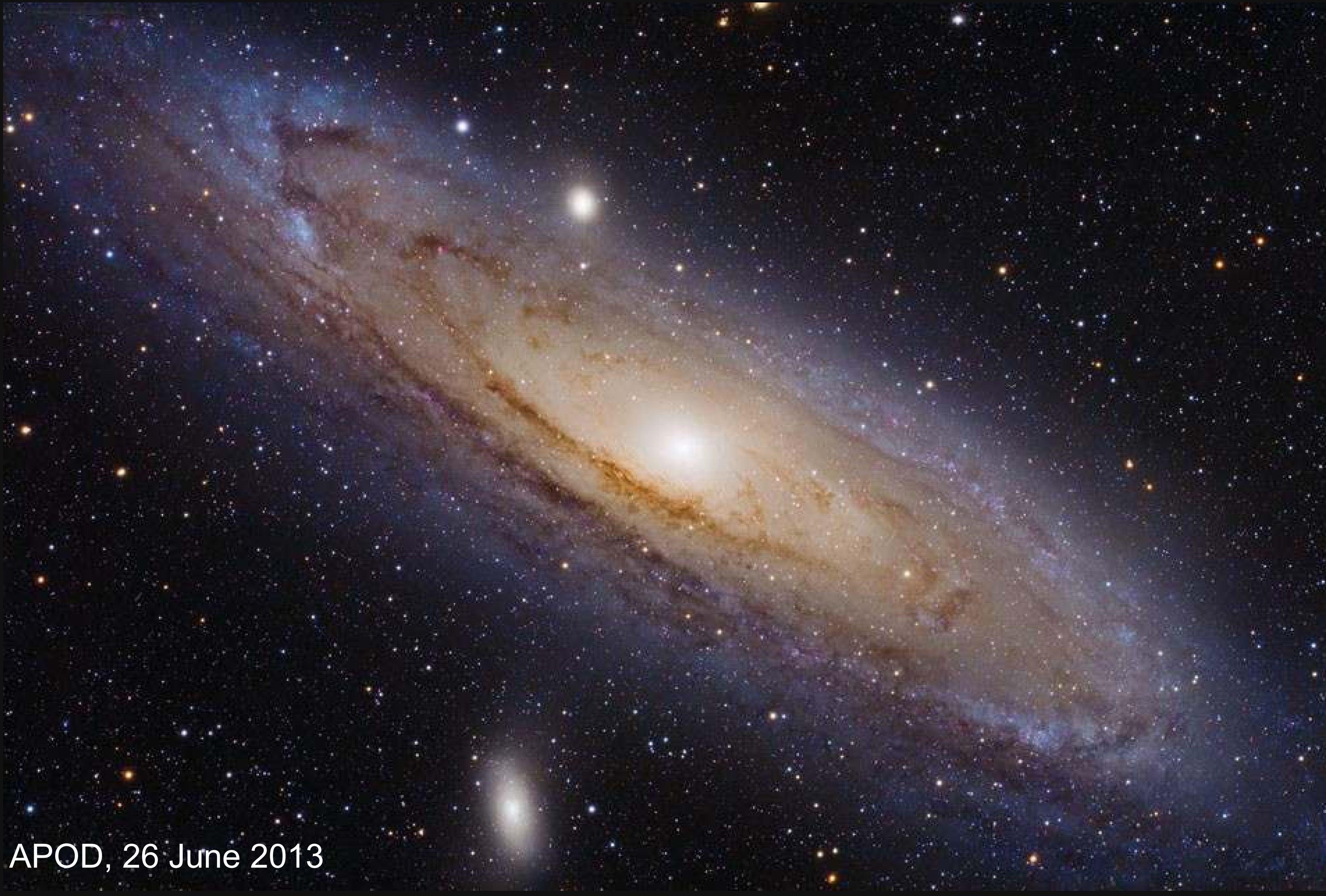
The multiwavelength sky



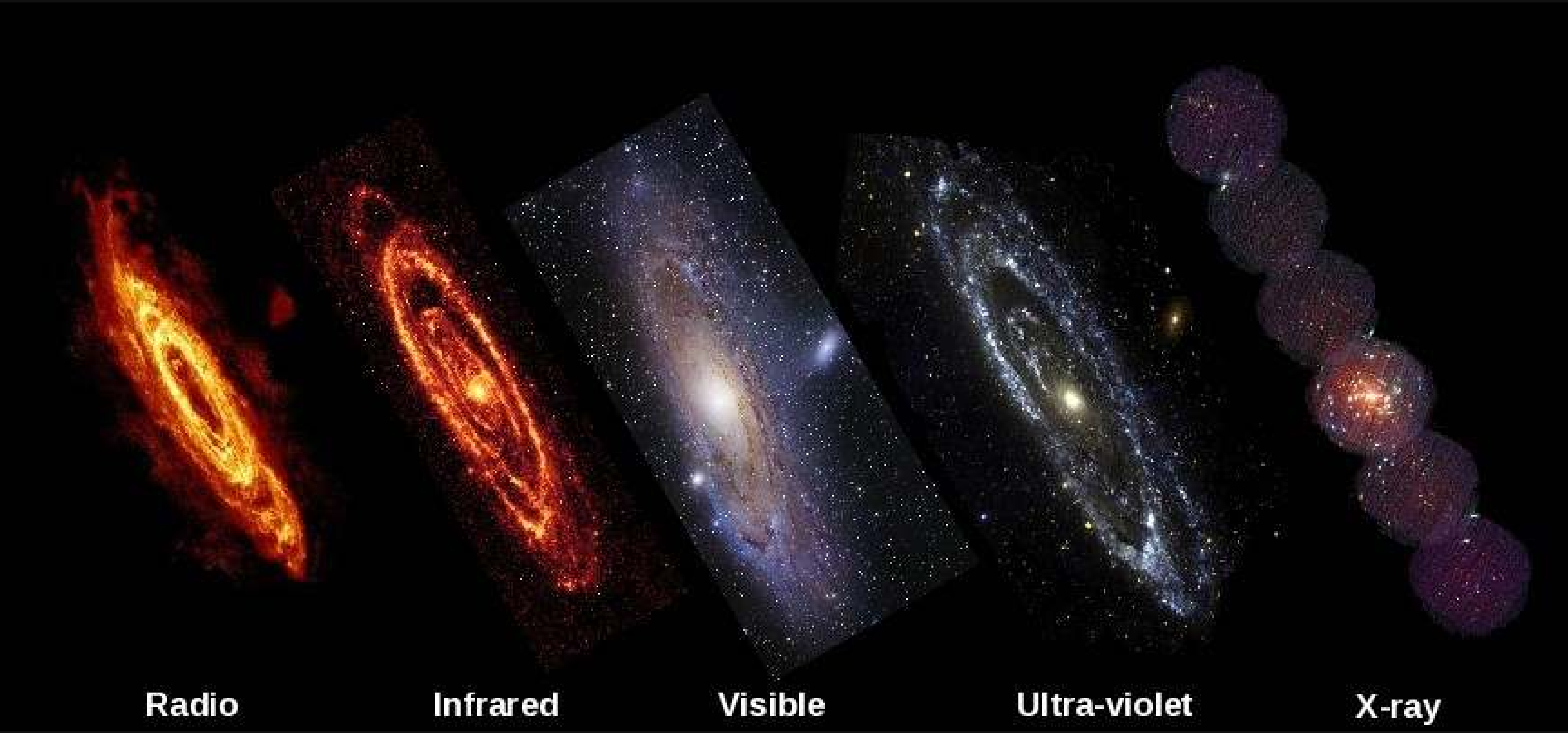


*

M31 (Andromeda Galaxy) in visible...



...and at other wavelengths

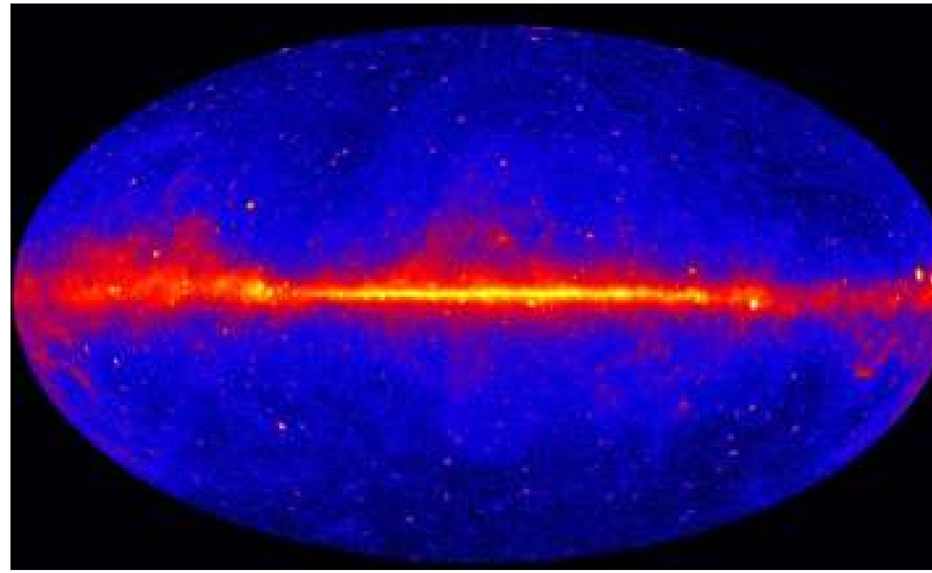


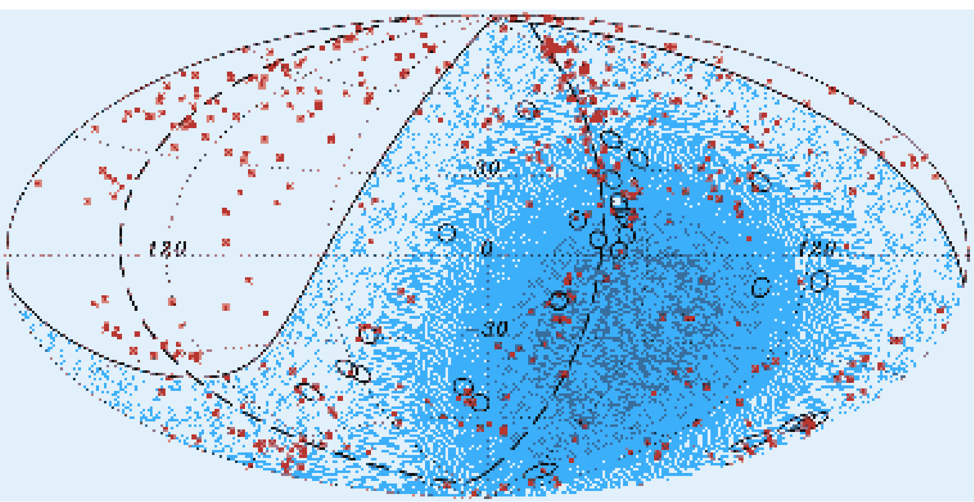
*

The multi-messenger sky today

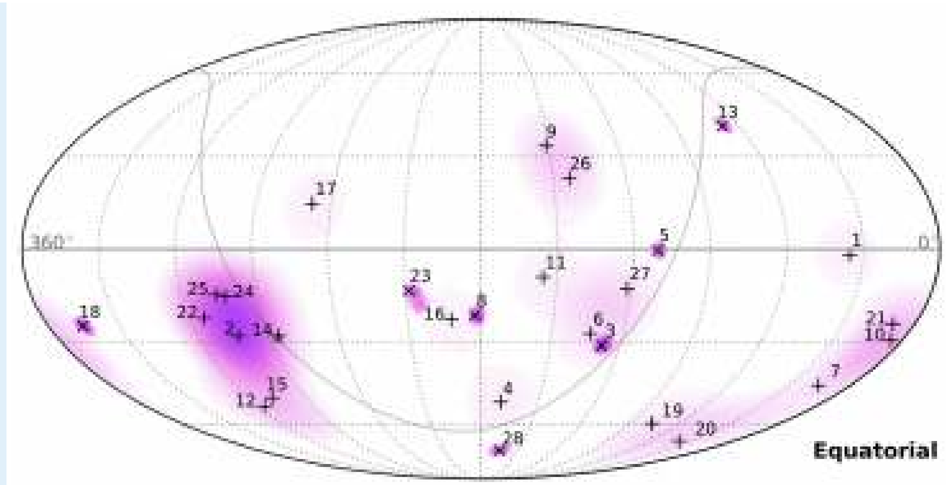
Optical (APOD)

Gamma rays > 0.1 GeV (Fermi-LAT)





Cosmic rays > 57 Eev (Auger, 2007)



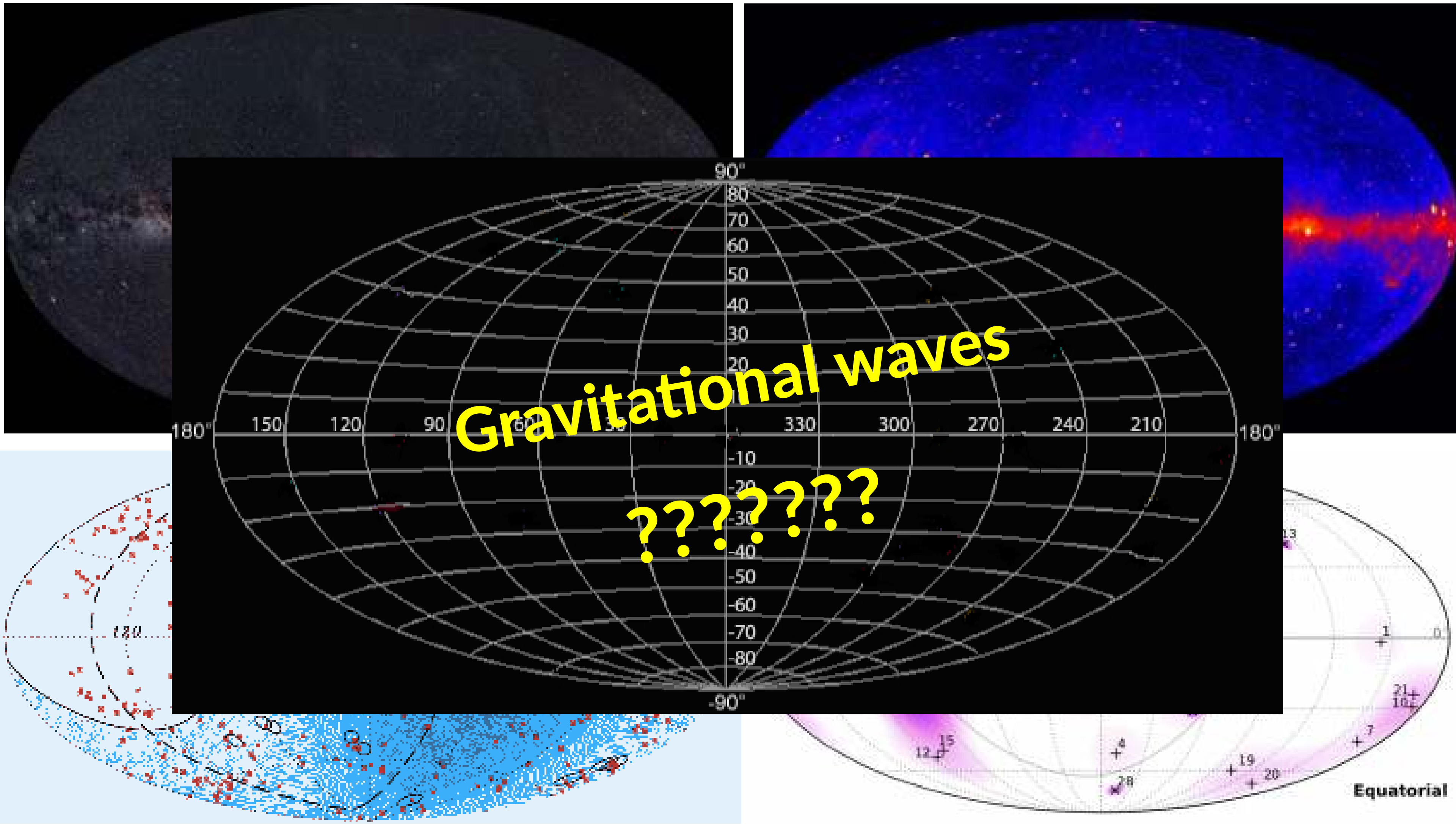
Neutrinos > 30 Tev (Icecube, 2013)

*

The multi-messenger sky today

Optical (APOD)

Gamma rays > 0.1 GeV (Fermi-LAT)



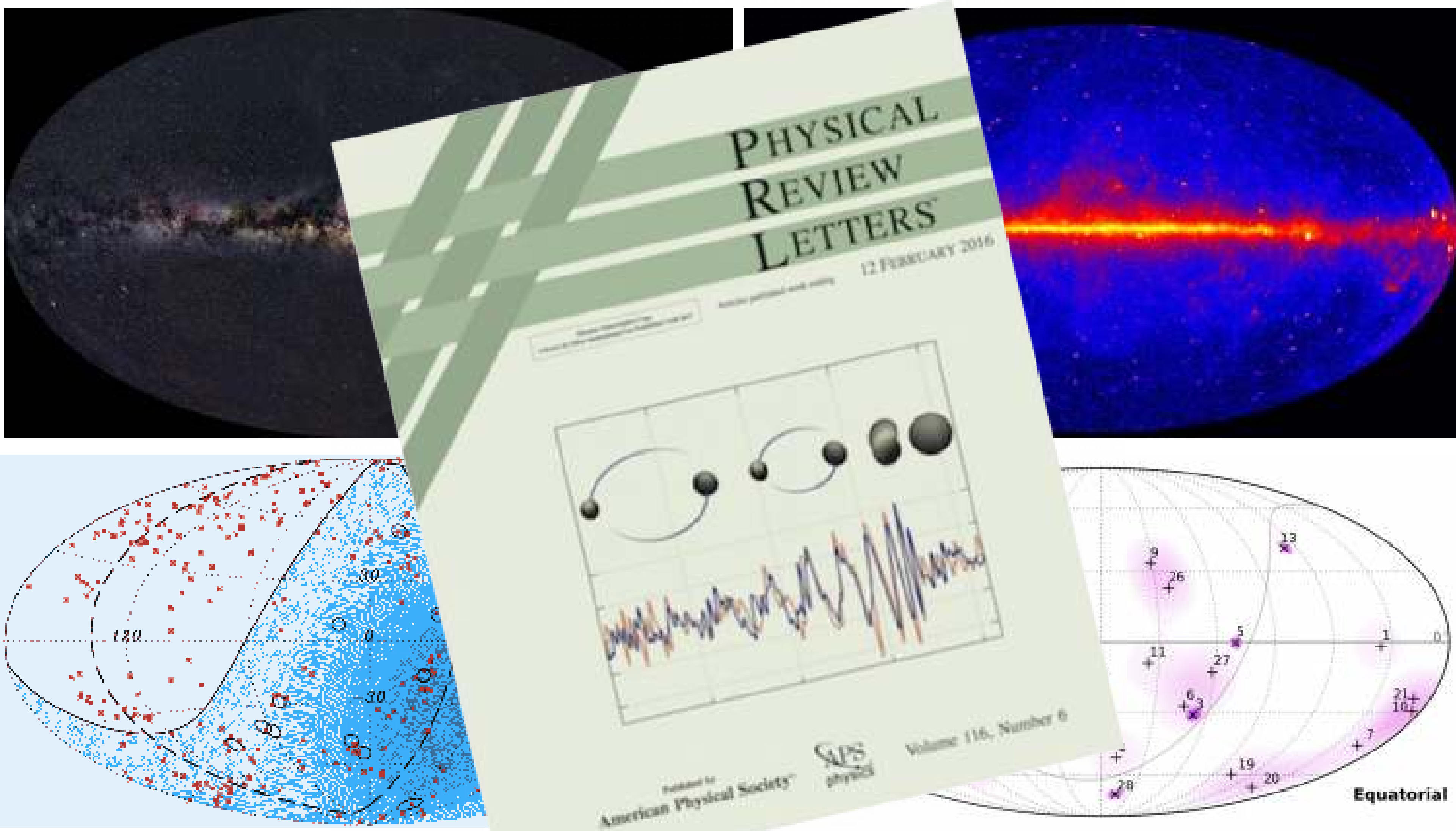
Cosmic rays > 57 Eev (Auger, 2007)

Neutrinos > 30 Tev (Icecube, 2013)

A multi-messenger sky

Optical (APOD)

Gamma rays > 0.1 GeV (Fermi-LAT, 2013)



Cosmic rays > 57 Eev (Auger, …

Neutrinos > 30 Tev (Icecube, 2013)

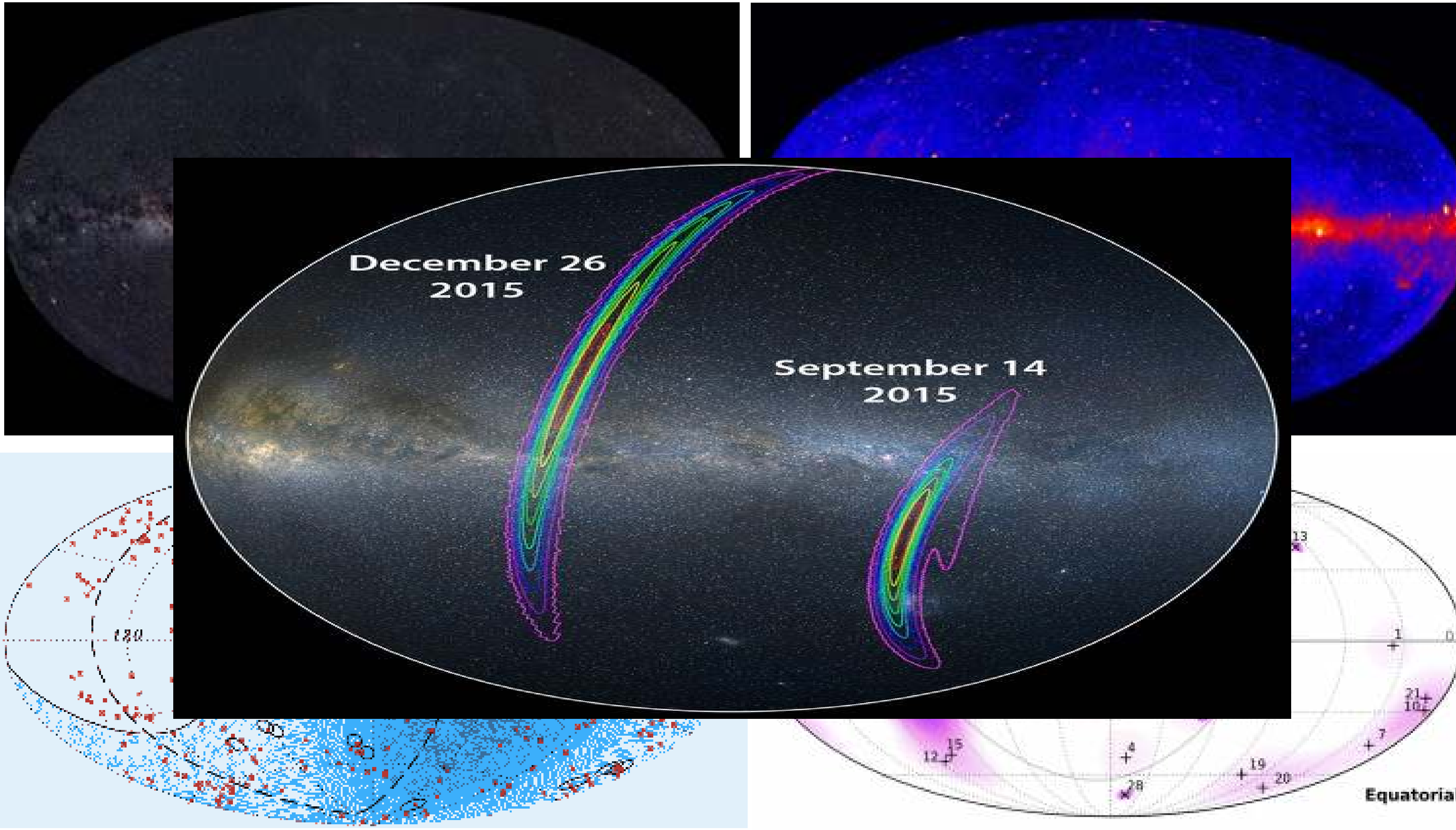
*

M. Razzano

The multi-messenger sky today

Optical (APOD)

Gamma rays > 0.1 GeV (Fermi-LAT, 2013)



Cosmic rays > 57 Eev (Auger, 2007)

Neutrinos > 30 Tev (Icecube, 2013)

M. Razzano

The new frontiers of multimessenger astronomy

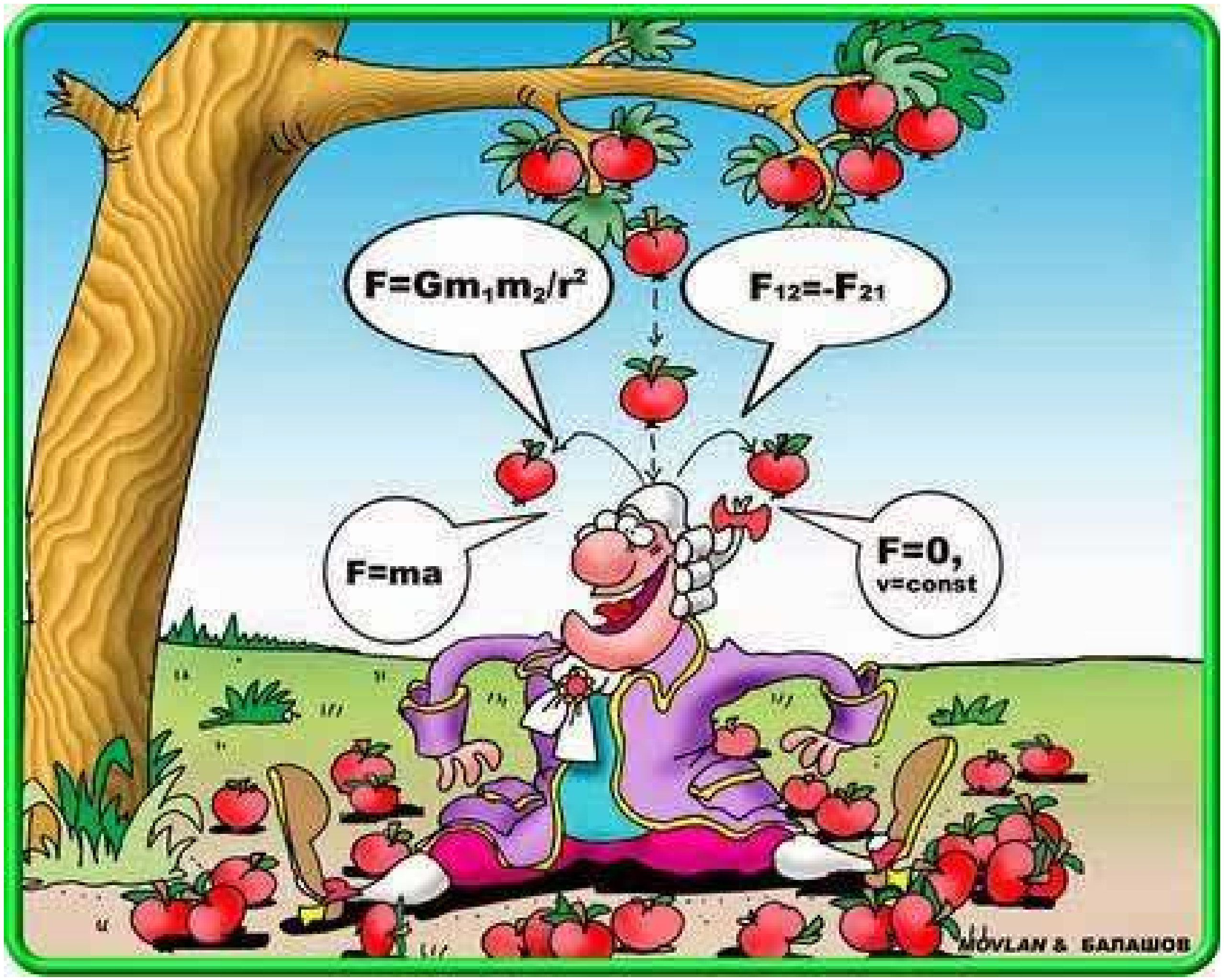
- Complementary information:
  - GW✉ mass distribution
  - EM ✉ emission processes, acceleration mechanisms, environment
  - Neutrinos✉ hadronic/nuclear processes, etc

- Give a precise (arcmin/arcsecond) localization
  - Localize host galaxy of a merger
  - Identify an EM counterpart with timing signature (e.g. pulsars)
  - EM follow-up is crucial

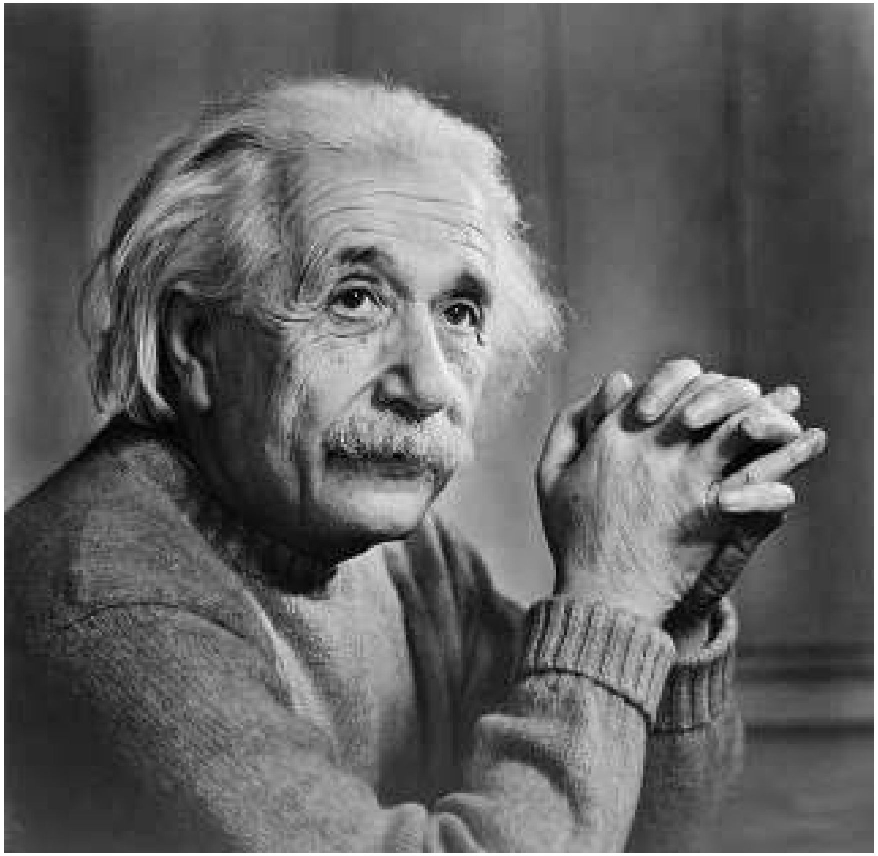
- Provide a more complete insight into the most extreme events in the Universe

When it all started?

- **Not exactly here…..**



...but here



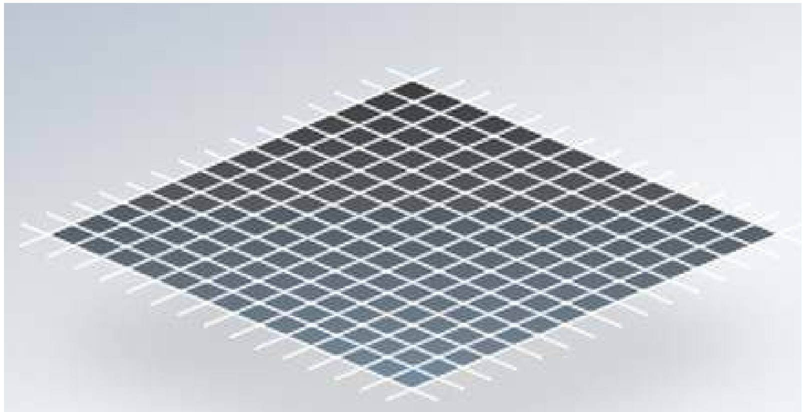
- 1905
  - Special Relativity
- 1915
  - General Relativity (GR)

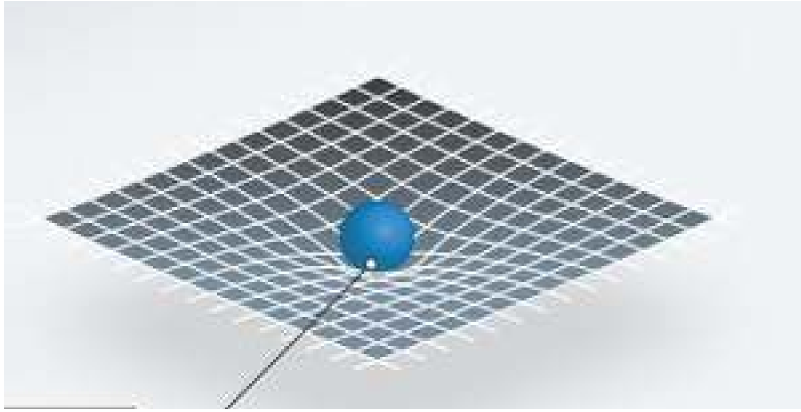
Einstein's gravity

- **Theory of General Relativity**
  - **Gravity as a consequence of the geometry of the spacetime**

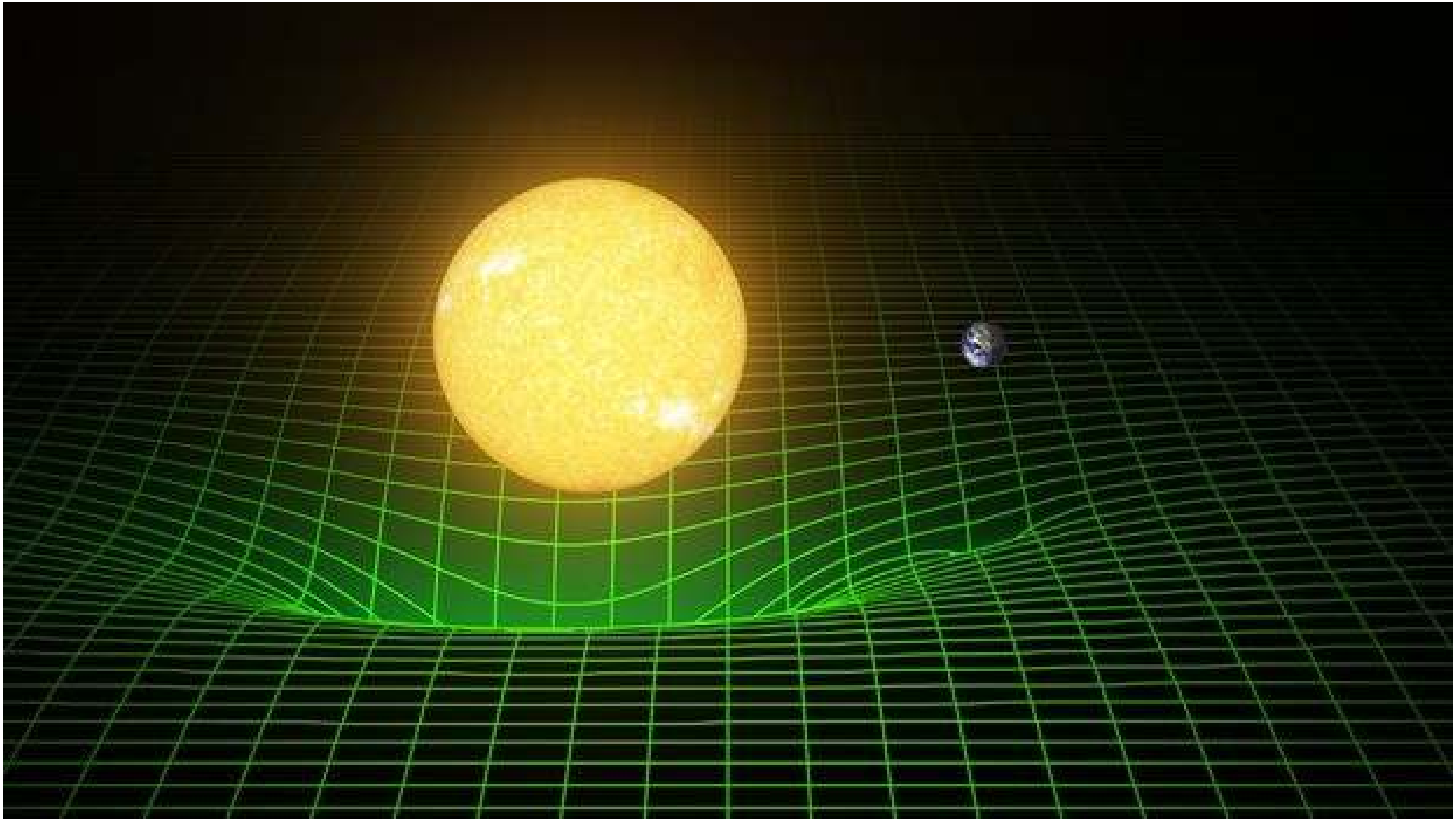
"Spacetime tells matter how to move; matter tells spacetime how to curve"

(J. Wheeler)

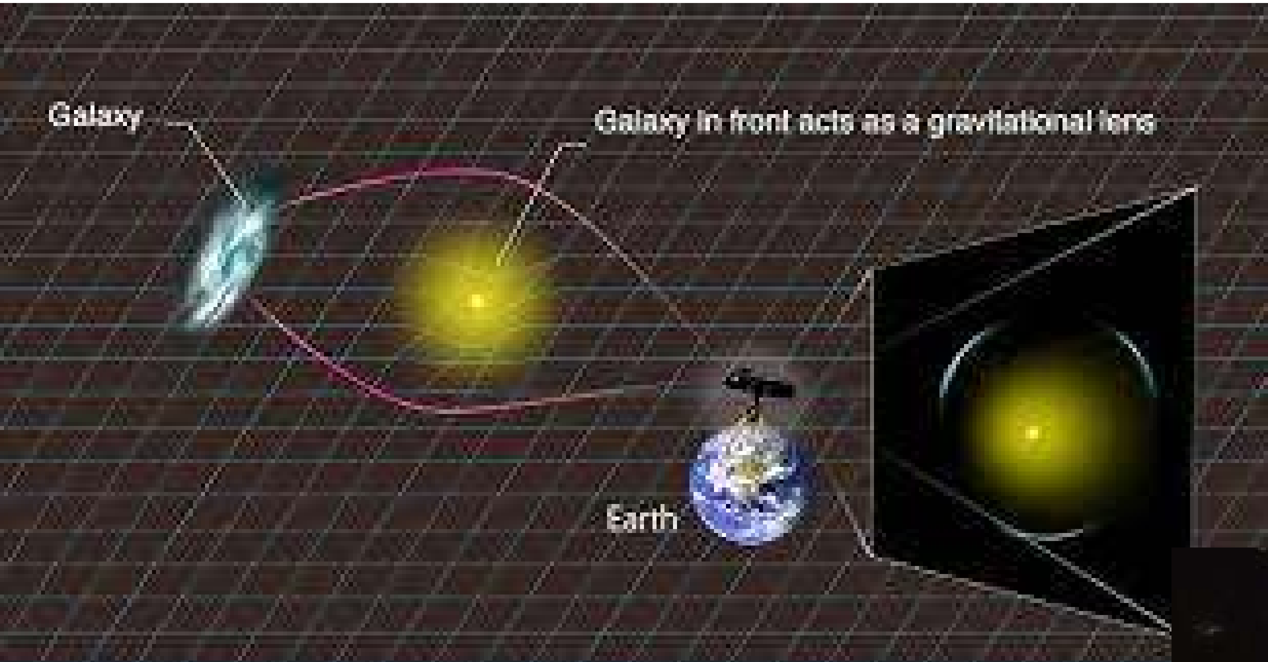


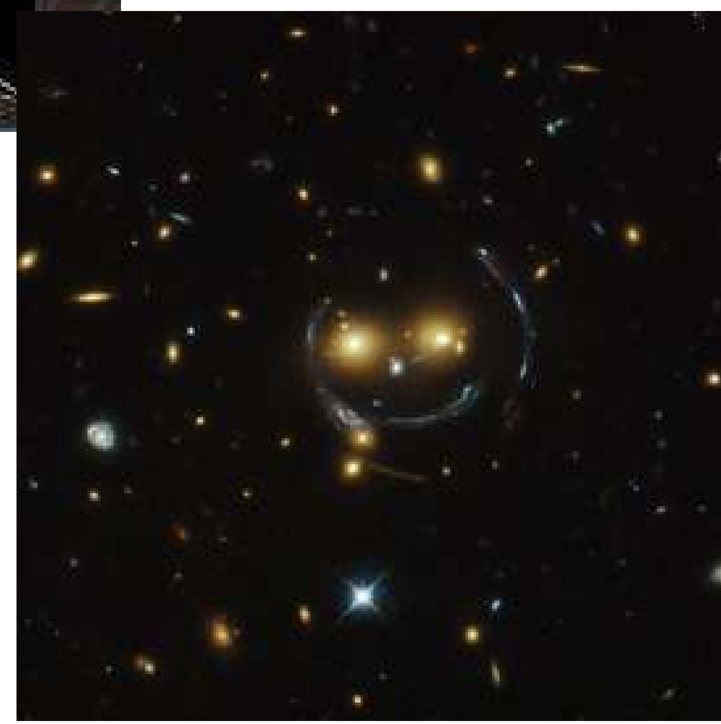


A curved spacetime



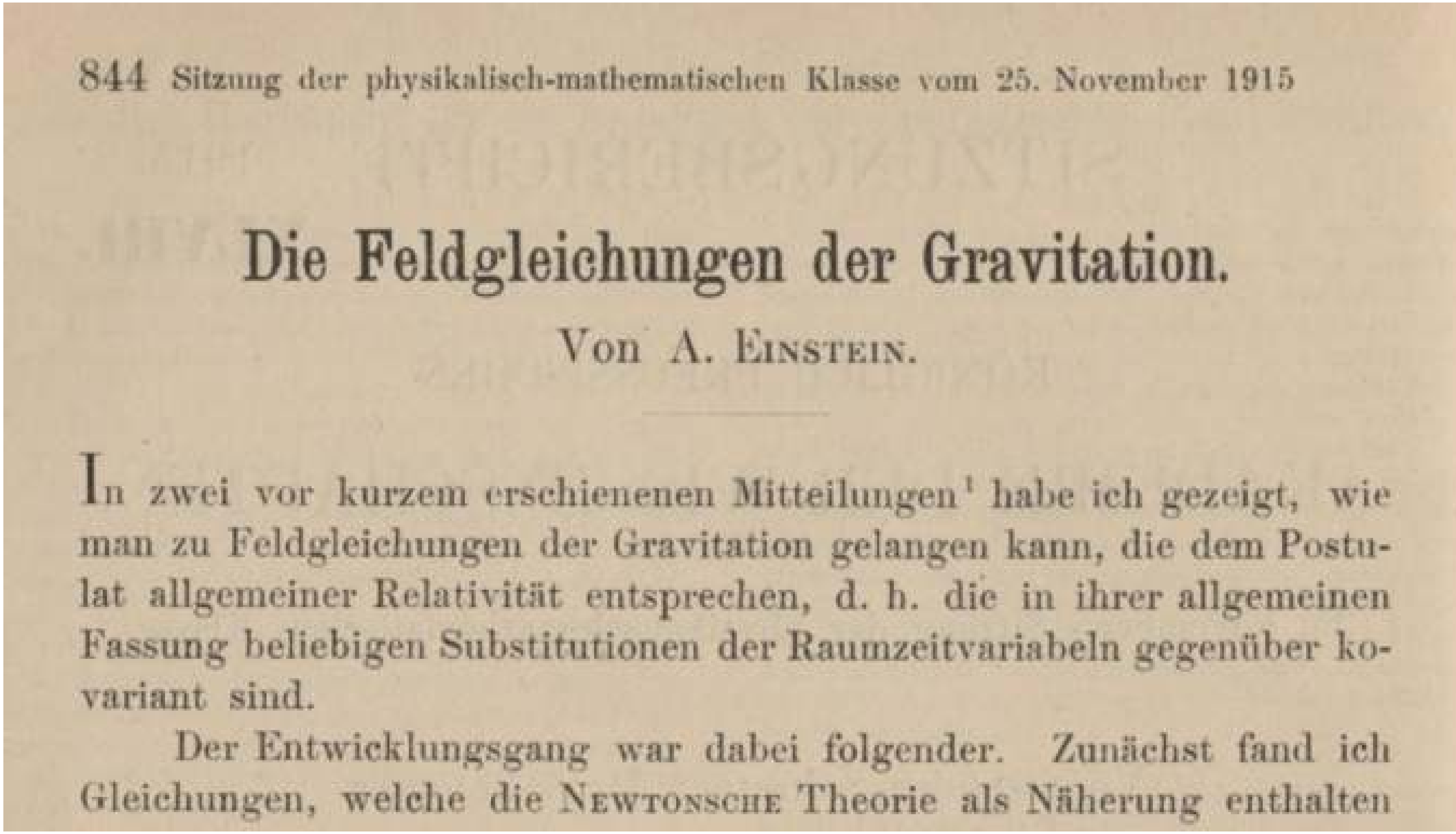
Funny (and useful) effects!





Gravitational lensing

Einstein and GR



844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

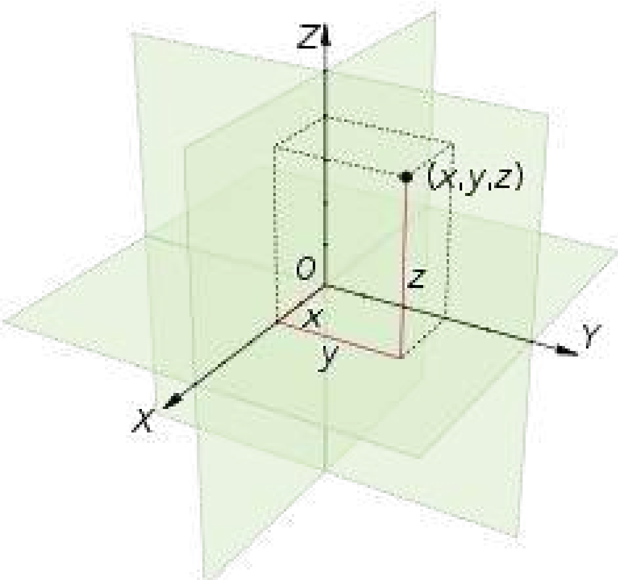
Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten

Credits: Preussische Akademie der Wissenschaften, Sitzungsberichte, 1915

Einstein's field equations

Uyuni Train Cemetery (Bolivia)

What is spacetime?



Set of 3 spatial coordinates + 1 time coordinate

Flat spacetime (Minkowski)
Distance element:

$$ds^2 = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} [\eta_{\mu\nu}]\, dx^\mu dx^\nu$$

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

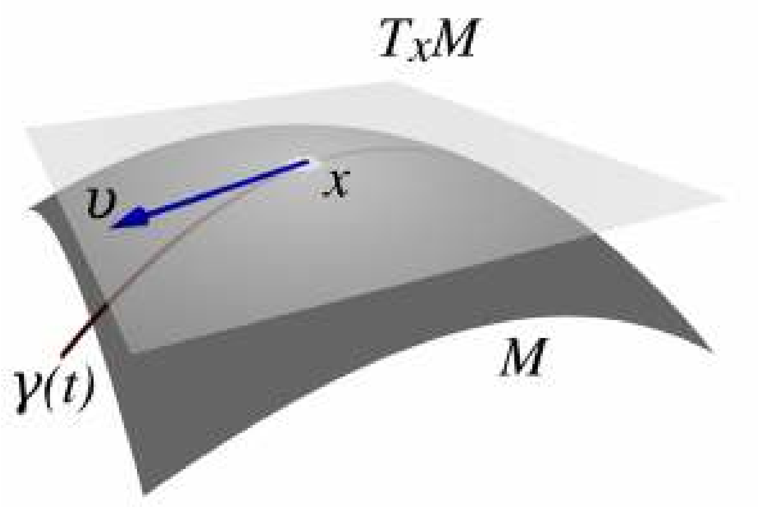
$$\eta_{\mu\nu} = \eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Different conventions (+/-, with/without c)

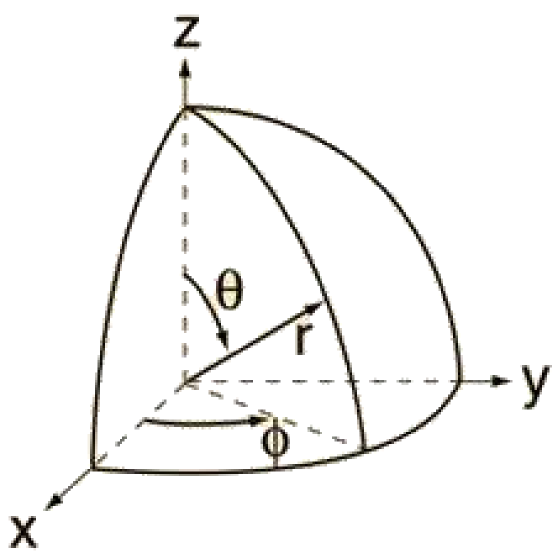
Curved spacetime?

In curved spacetime, metric tensor depends on the position. We call it g

$$ds^2 = g_{\nu\mu} dx^\mu dx^\nu$$



Simple example: a sphere



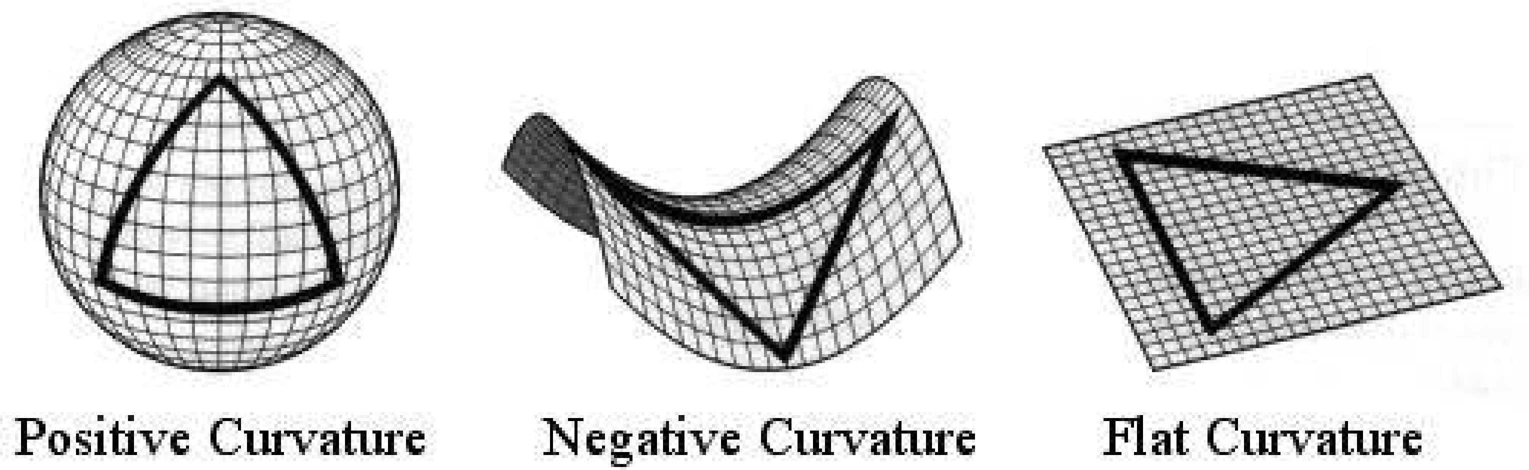
In curved spacetime, metric tensor depends on the position. We call it g

$ds^2 = g_{\nu\mu} dx^\mu dx^\nu$

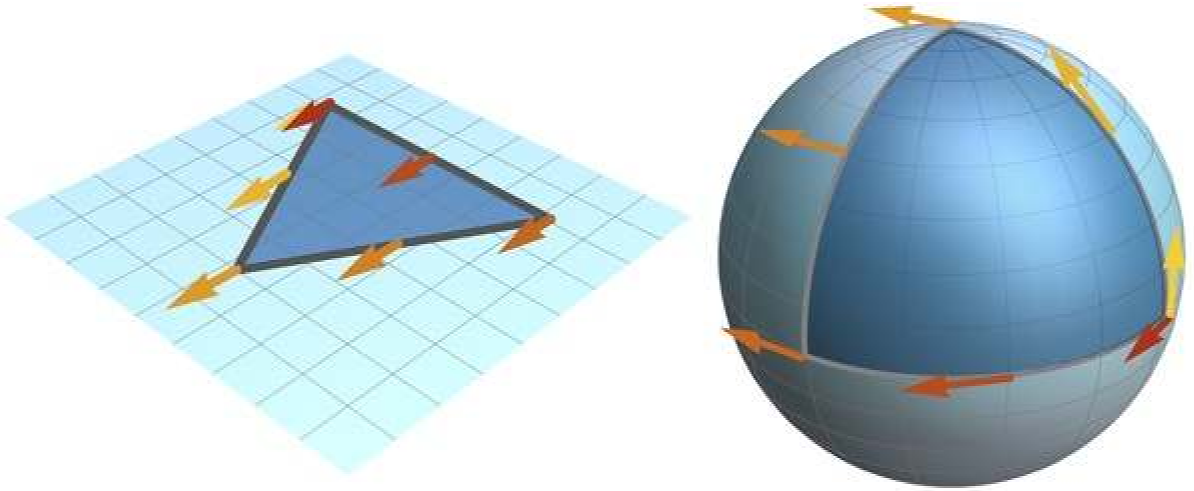
$$x^\mu = (ct, r, \theta, \varphi)$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

Curved surfaces



How to describe spacetime?



Parallel Transport
In flat and curved space

To describe curvature we use:

- Connection coefficients (Christoffel symbols) $\Gamma^{\rho}_{\mu\lambda}$: parallel transport, derivative (covariant)
- Riemann tensor R: curvature

How to describe curvature?

1 - From Cristoffel symbols to Riemann tensor:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}.$$

2 - Riemann curvature ↔ metrics

$$R_{\alpha\beta\mu\nu} = \frac{1}{2}\left[\partial_\mu\partial_\beta g_{\nu\alpha} + \partial_\nu\partial_\alpha g_{\beta\mu} - \partial_\nu\partial_\beta g_{\mu\alpha} - \partial_\mu\partial_\alpha g_{\beta\nu}\right]$$

$$R_{\alpha\beta\mu\nu} = -\frac{1}{2}\left[\partial_\nu\partial_\beta g_{\mu\alpha} + \partial_\mu\partial_\alpha g_{\beta\nu} - \partial_\mu\partial_\beta g_{\nu\alpha} - \partial_\nu\partial_\alpha g_{\beta\mu}\right]$$

$$R_{\alpha\beta\mu\nu} = -\frac{1}{2}\left[\partial_\mu\partial_\alpha g_{\nu\beta} + \partial_\nu\partial_\beta g_{\alpha\mu} - \partial_\nu\partial_\alpha g_{\mu\beta} - \partial_\mu\partial_\beta g_{\alpha\nu}\right]$$

$$R_{\alpha\beta\mu\nu} = -R_{\beta\alpha\mu\nu}$$

Symmetries:

N independent components:

4D → 20
3D → 6
2D → 1
(our well-known "curvature")

3 - Ricci tensor:

$$R_{ij} = R^{k}{}_{ikj}. \xrightarrow{\text{Taking the trace}} R = g^{ij}R_{ij}. \quad \text{(Ricci scalar)}$$

Einstein's field equations

Uyuni Train Cemetery (Bolivia)

Einstein's field equations

Set of 10 equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Geometric part
(aka Einstein's tensor $G_{\mu\nu}$)
=
Geometry of spacetime

Stress-Enery part
(aka momentum-energy tensor)
=
Matter distribution

Einstein's Equations: the geometry

Ricci curvature tensor

Metric tensor

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

Ricci scalar

Einstein's equations: the matter

Gravitational constant

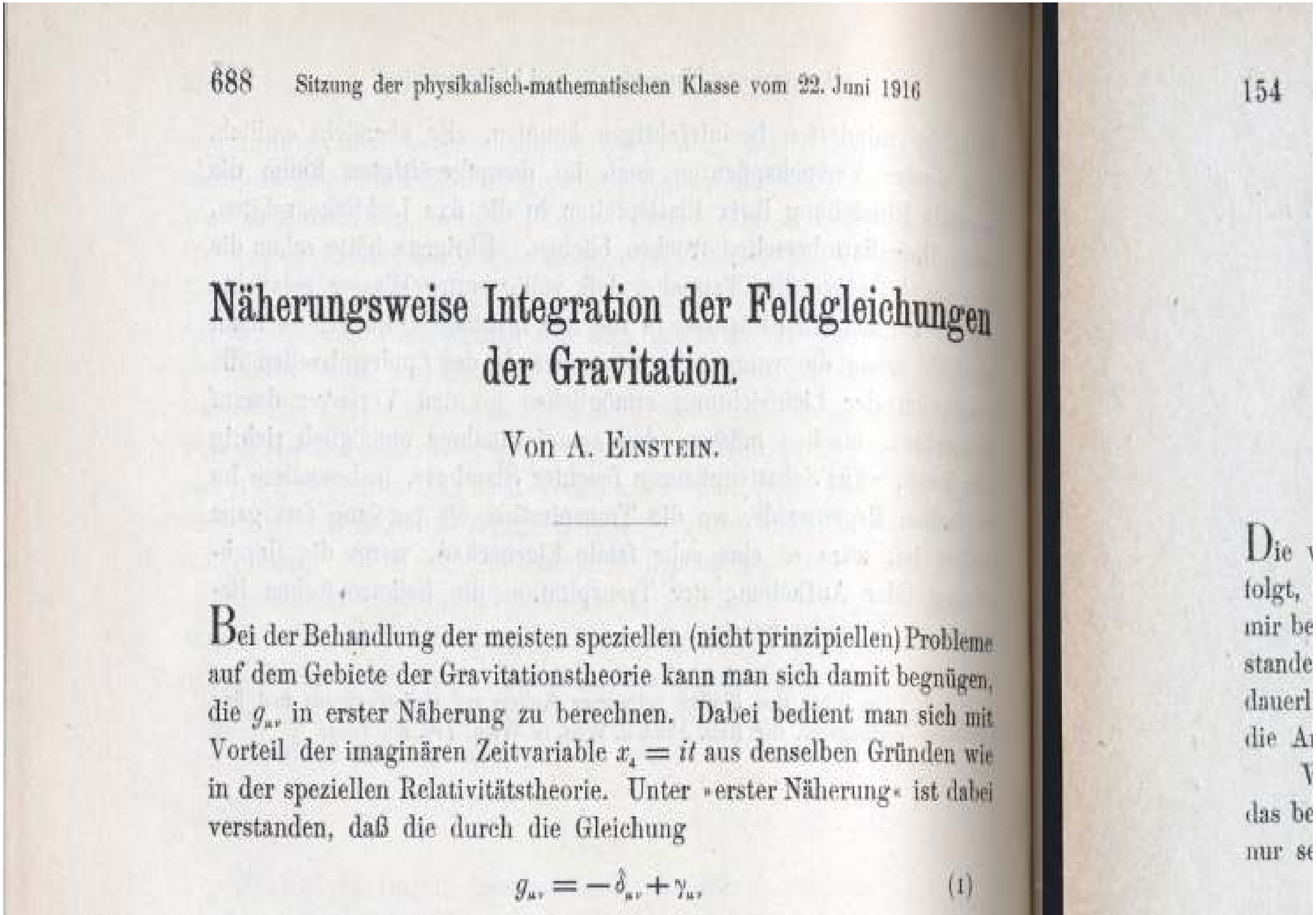
$$\frac{8\pi G}{c^4} T_{\mu\nu}$$

Speed of light

Momentum-energy tensor

Geometric Unit system (G=1, c=1) → $8\pi G_{\mu\nu}$

1916: Gravitational waves



688 Sitzung der physikalisch-mathematischen Klasse vom 22. Juni 1916

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu}, \quad (1)$$

Linearized field equations

Write metrics as

$$g_{\mu v} = \eta_{\mu v} + h_{\mu v}$$

Flat, Minkovski metric

Small perturbation (|h| << 1)

Write (linearized) Riemann tensor as:

$$R^{\mu}_{\nu\alpha\beta} = \frac{1}{2}\eta^{\mu\delta}(h_{\delta\beta,\nu\alpha} - h_{\nu\beta,\delta\alpha} - h_{\delta\alpha,\nu\beta} + h_{\nu\alpha,\delta\beta})$$

Gauge symmetries

1) We adopt the trace-reverse transformation (more compact)

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$$

2) Linearized Riemann tensor invariant under:

$$x^{\mu} \longrightarrow x'^{\mu} = x^{\mu} + \zeta^{\mu}(x)$$

We exploit this Gauge freedom to fix the Lorenz Gauge

$$\sum_{\alpha=0}^{3} \frac{\partial \bar{h}^{\mu\alpha}}{\partial x^{\alpha}} = \bar{h}^{\mu\alpha}{}_{,\alpha} = 0$$

3) → Reduce to 6 independent components of h

We also project on a plane transverse to direction n

→ Reduce to 2 independent components of h

Linearized equations

Putting 1+2+3 we select the **tranverse-traceless Gauge (TT)** (therefore, no longer GR symmetries!)

In the TT gauge, the Einstein's equations become:

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

where $\Box = -(1/c^2)\partial_t^2 + \nabla^2$

In vacuum (outside a source) $T_{\mu\nu} = 0$

In this case, solutions are plane waves

→ Gravitational waves!

Gravitational waves

In the TT Gauge we have just 2 components.

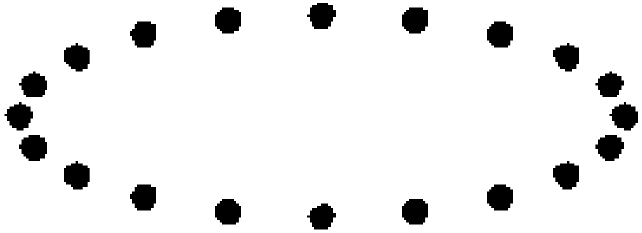
The solutions reads as:

$$h_{\mu\nu}^{TT}(t,z) = \begin{pmatrix} h_+ & h_x & 0 \\ h_x & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}_{\mu\nu} \cos[\omega(t - z/c)]$$

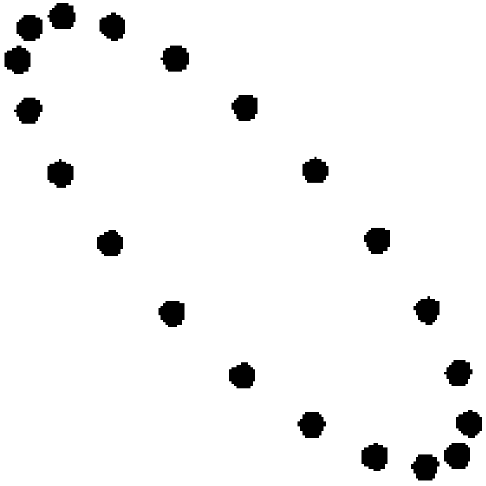
h_+ and h_x are the gravitational wave polarizations

GWs propagate at the speed of light

Gravitational waves Polarizations



Plus (+) polarization



Cross (x) polarization

Sources of Gravitational waves

In the far zone (i.e. far from the source), we can connect the GWs generated from the source, with the mass distribution of the source

In particular, under condition of slow motion and weak field, when we have a non-vanishing quadrupole momentum of the mass distribution, we have GW emission

$$h_{ij}^{TT}(t,z) \simeq \frac{2G}{c^4 r}\ddot{I}_{ij}^{TT}(t-r/c)$$

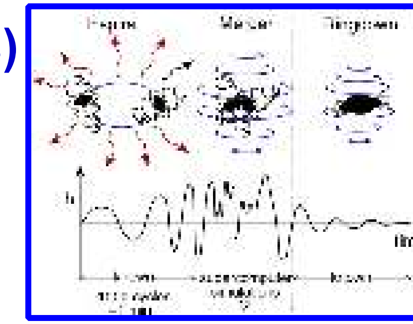
1/r dependence

$G/c^4 \sim 10^{49}\ s^2 g^{-2} cm^{-1}$

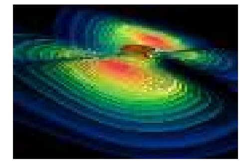
Expected sources detectable by LIGO/Virgo

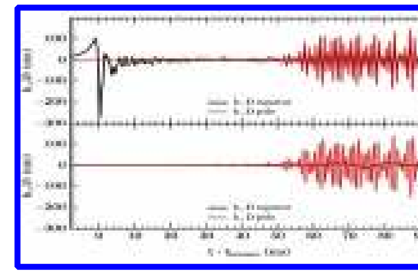
Transients

- **Coalescence of compact binary systems (NSs and/or BHs)**
  - Known waveforms (template banks)
  - $E_{gw} \sim 10^{-2}\ Mc^2$



- **Core-collapse of massive stars**
  - Uncertain waveforms
  - $E_{gw} \sim 10^{-8} - 10^{-4}\ Mc^2$

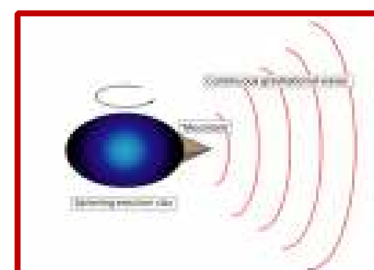




Ott, C. 2009

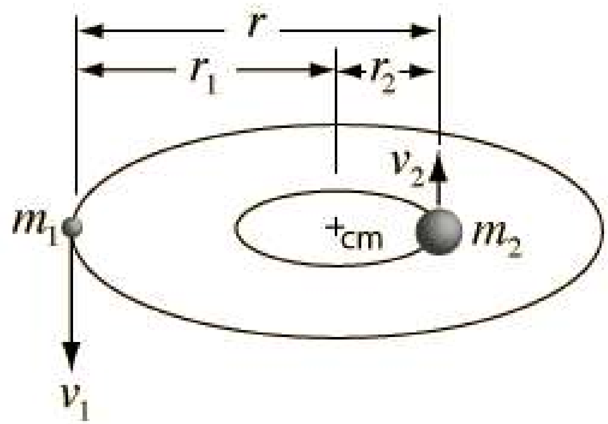
Non transients

- **Rotating neutron stars**
  - Quadrupole emission from star's asymmetry
  - Continuous and Periodic



- **Stochastic background**
  - Superposition of many signals (mergers, cosmological, etc)
  - Low frequency

Binary systems



Working out calculations (a = major semiaxis, μ = reduced mass):

$$\ddot{I}_{11} = -2\mu a^2\omega^2 \cos[2(\omega t + \phi_0)]$$

and the corresponding wave component:

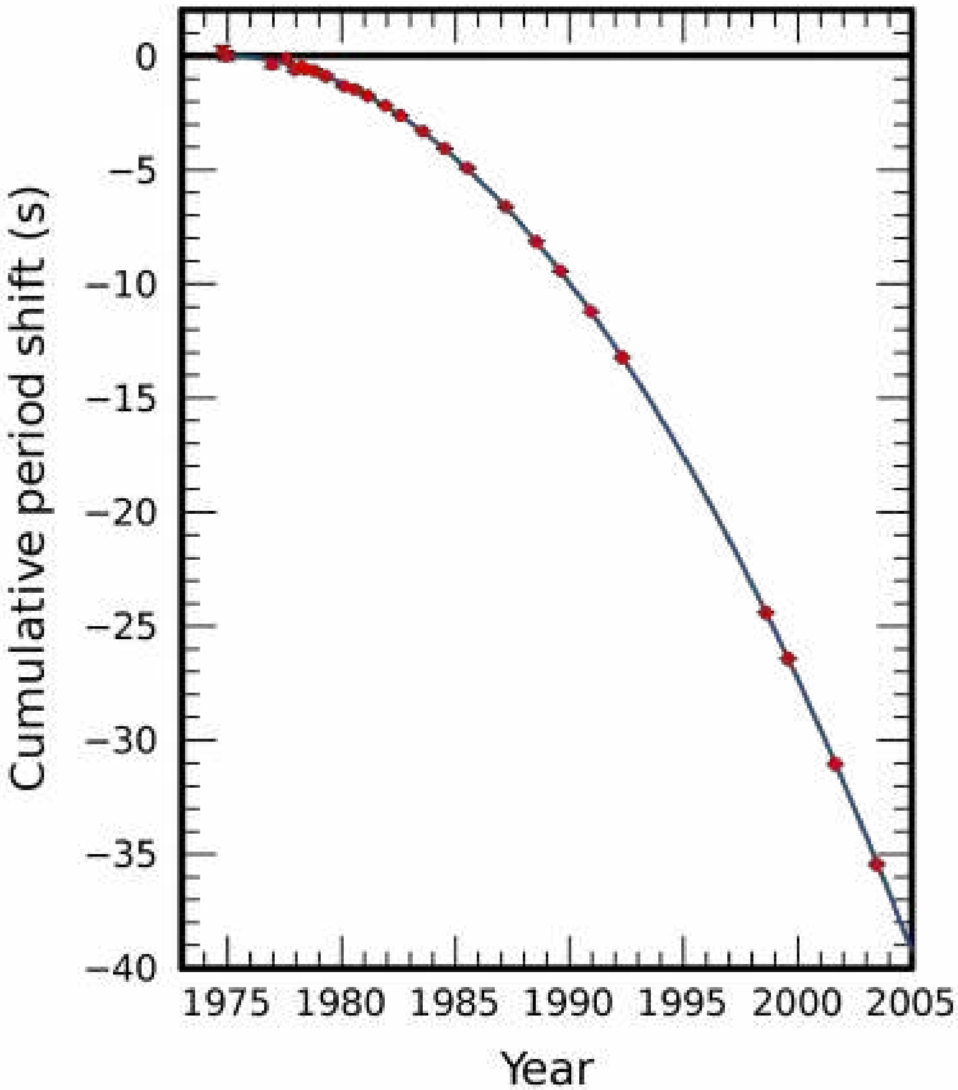
$$h_+^{TT} = h_{11}^{TT} = -\frac{4G\mu a^2\omega^2}{c^4 r}\cos[2(\omega t + \phi_0)]$$

Twice the frequency of the orbit

System loses energy by GWs → orbital contraction!

The case of binary system PSR1916+13

- Discovered in 1978
- Distance 6.4 kpc
- P=7.7 h
- P pulsar 59 ms
- Orbital decay of 3.5 m/yr, or 76.5 us/year
-
- Nobel Prize on Physics 1993: R. A. Hulse e J. H. Taylor



First indirect proof of Gravitational Waves !

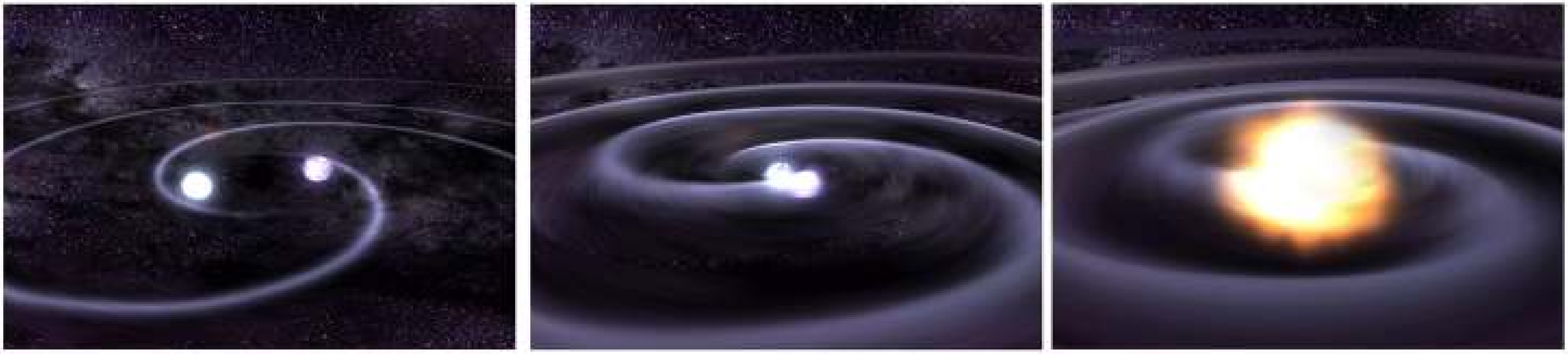
Binary coealescence

Orbital decay → Closer orbit → Faster spin → More GW emission →Larger orbital decay

→Runaway process

$$f_{dot,GW} = k\, f_{GW}^{11/3}$$

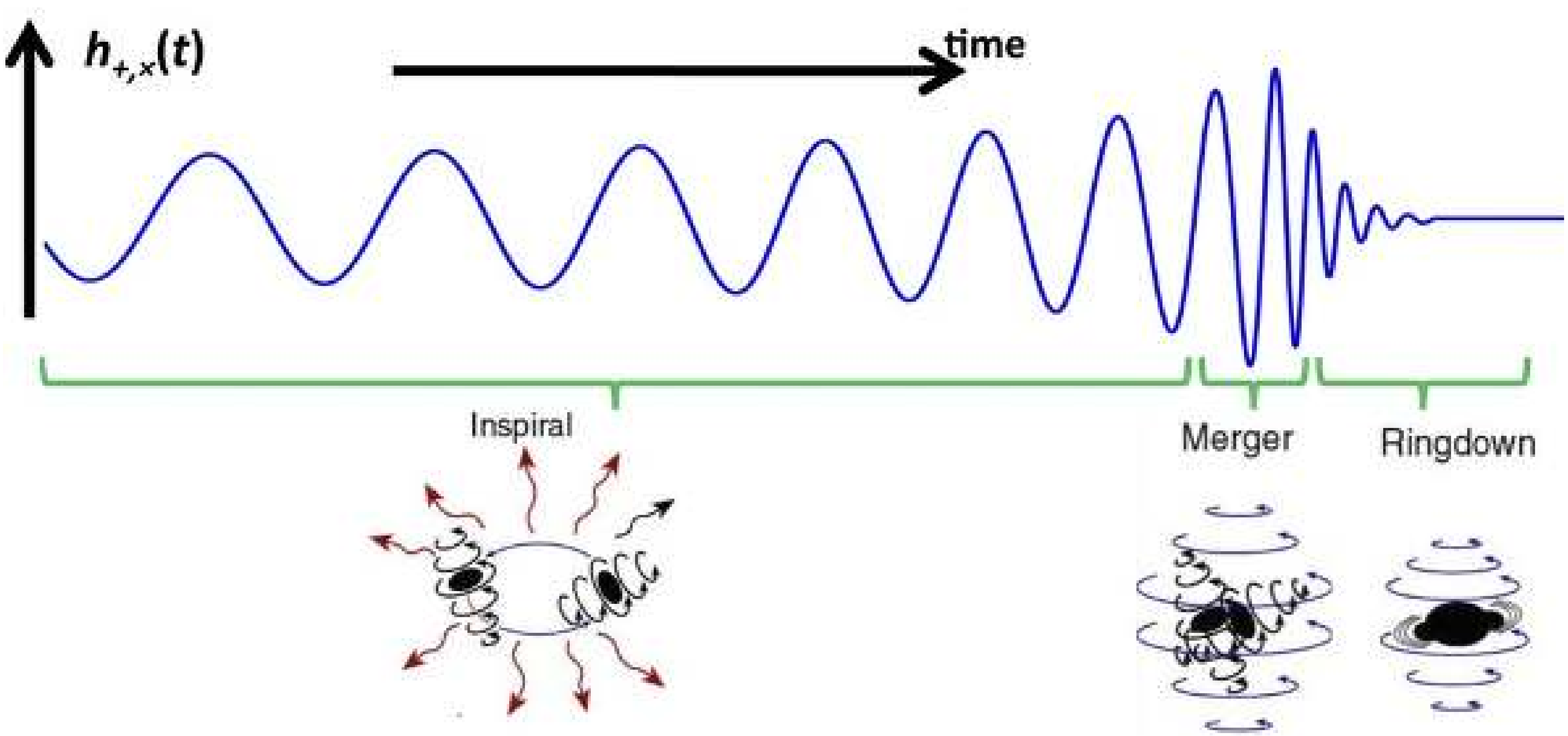
In some time, the 2 object will coalesce and merge
(e.g. PSR B1916+13 in 300 Myr)



Binary coealescence

Closer to the coalescence, GW amplitude and frequency increase → the so-called *chirp*

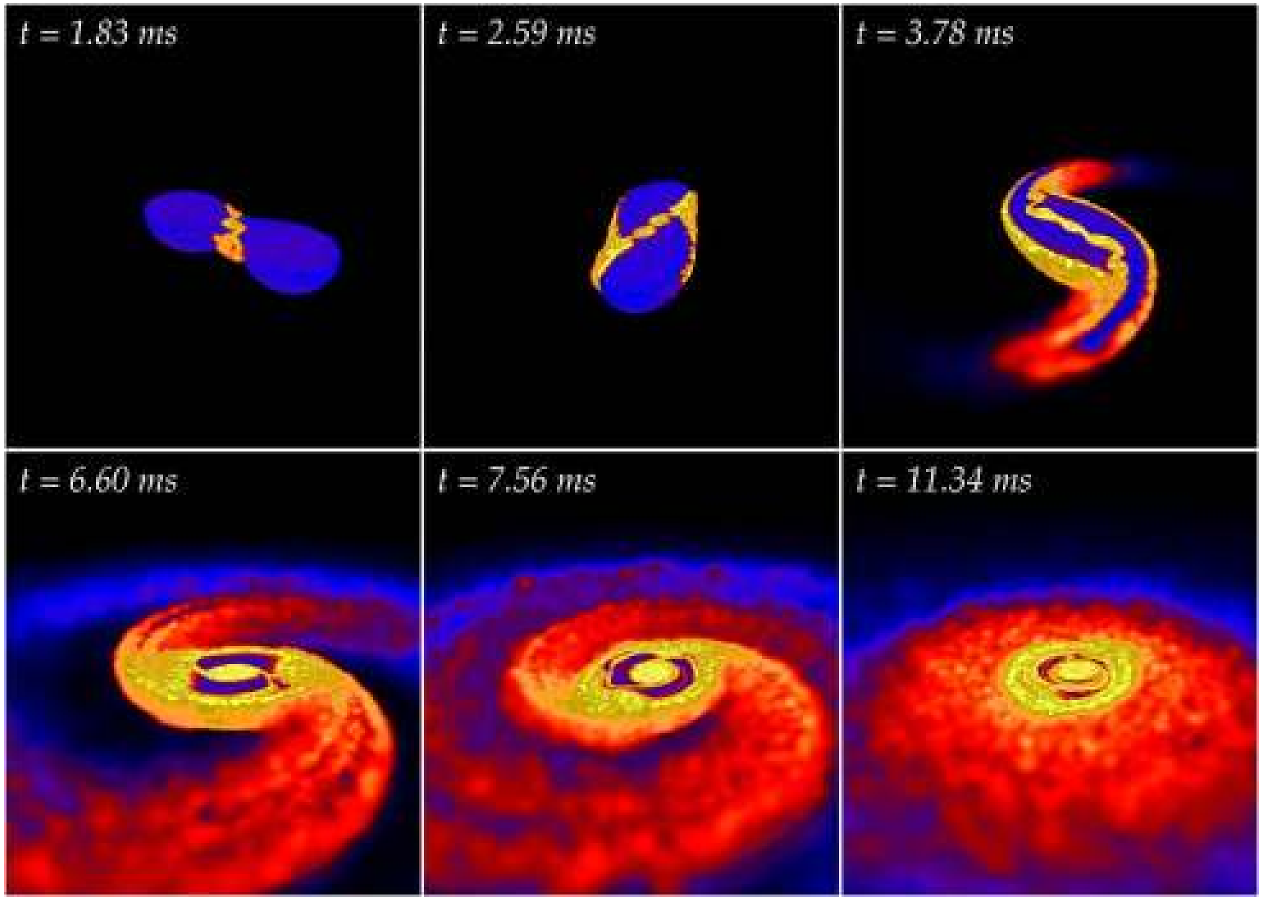
After the merger, the final object (e.g. a BH) will undergo a ringdown phase



M. Favata/SXS/K. Thorne

Binary systems

Interesting binary systems are made by neutron star and/or black holes

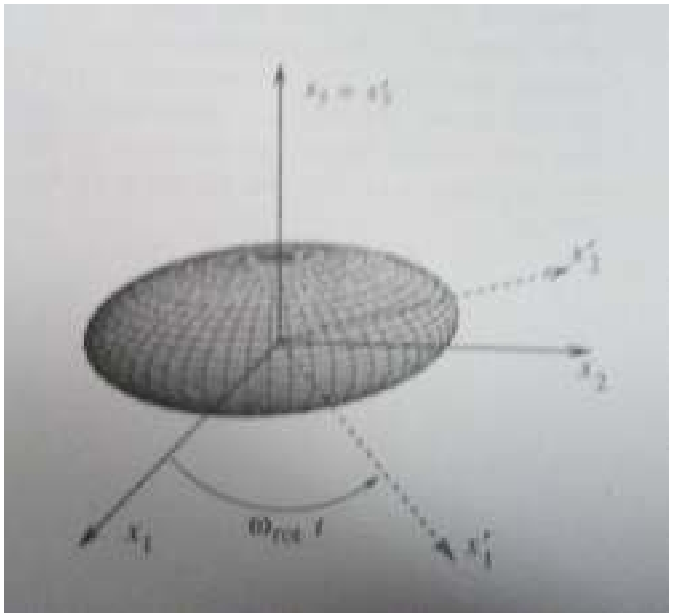


Example of simulations

Credits: E. Ramirez-Ruix (UCSC)

Rotating neutron stars & pulsars

Another interesting case is the rotating, asymmetric neutron star



Quadrupole momentum tensor is proportional to inertia tensor

S' is the rotating frame

We can define the ellipticity as

$$\epsilon = \frac{I_1 - I_2}{I_3} \simeq \frac{b-a}{a} + O(\epsilon^2)$$

Rotating neutron stars & pulsars

We can derive the expression for the h(t):

$f_{GW} = 2\, f_{rot}$

$$h_+ = h_0 \frac{1+\cos^2 \iota}{2} \cos(4\pi f_{rot} t)$$

$$h_x = h_0 \cos\iota \sin(4\pi f_{rot} t)$$

where

$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_3 4 f_{rot}^2}{r} \epsilon$$

Where i indicates the inclination of the system wrt to the line of sight.
A similar coefficient is also valid for binaries

Sources and beyond

Gravitational waves will help us to probe known sources under a new perspective and unveil new, exotic sources

The main question now is:

How do we detect gravitational waves?