Solid State Detectors - Physics

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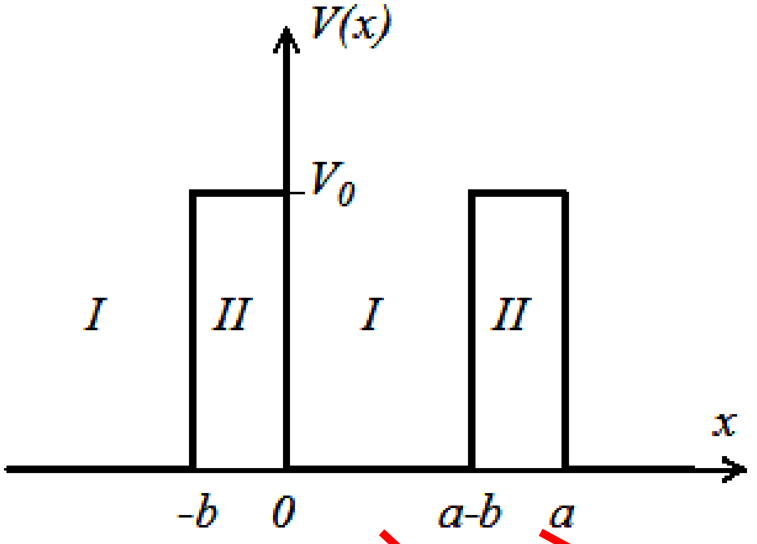
The Kronig-Penney model



$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V[x]\right)\psi = E\psi$$

$$\frac{d^2\psi}{dx^2}+\frac{2m}{\hbar^2}(E-V_0)\psi = 0$$

$$\frac{d^2\psi}{dx^2}+\frac{2m}{\hbar^2}E\psi = 0$$

Bloch's theorem

- Assuming that the solution is a Bloch function

$$\Psi(x) = u(x)e^{ikx}$$

and u (x) has the periodicity of the potential

- Schrödinger's equation becomes:

$$\frac{d^2u_I(x)}{dx^2} + 2ik\frac{du_I(x)}{dx} + (\beta^2 - k^2)u_I(x) = 0 \text{ for } 0 < x < a\text{-}b$$

$$\beta = \frac{\sqrt{2mE}}{\hbar}$$

$$\frac{d^2u_{II}(x)}{dx^2} + 2ik\frac{du_{II}(x)}{dx} - (k^2 + \alpha^2)u_{II}(x) = 0 \text{ for } a\text{-}b < x < a$$

$$\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

Solving…

- Solutions

$$u_I(x) = (A\cos \beta x + B\sin \beta x)e^{-ikx} \text{ for } 0 < x < a\text{-}b$$

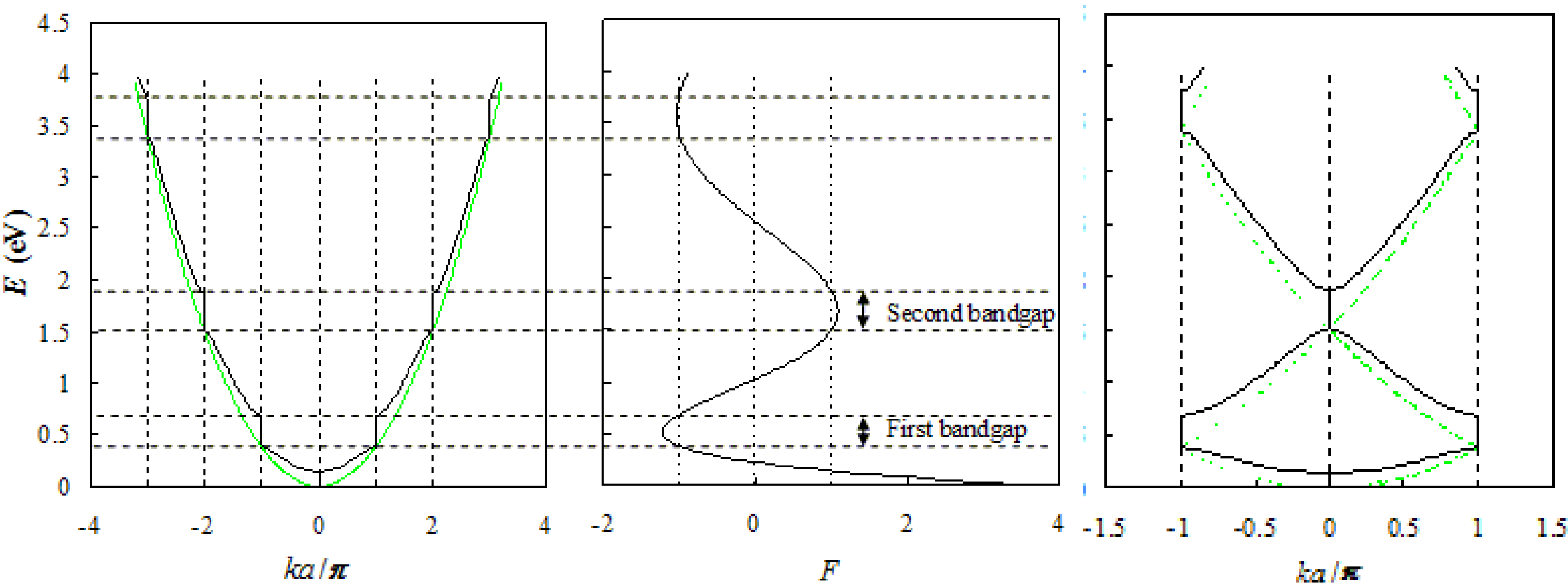
$$u_{II}(x) = (C\cosh \alpha x + D\sin \alpha x)e^{-ikx} \text{ for } a\text{-}b < x < a$$

- By continuity of the functions and derivatives

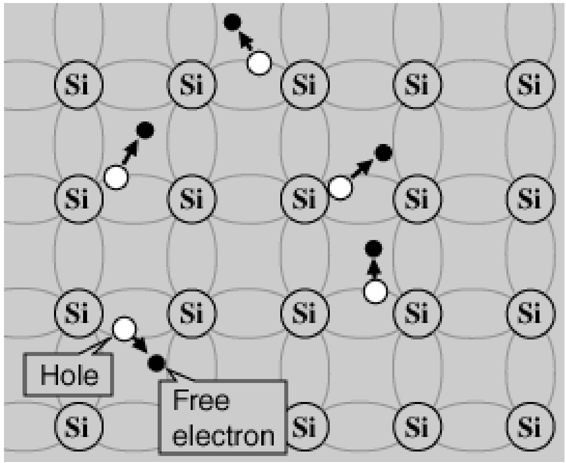
$$\cos ka = F = \frac{\alpha^2 - \beta^2}{2\alpha\beta}\sinh \alpha b \sin \beta(a-b) + \cosh \alpha b \cos \beta(a-b)$$

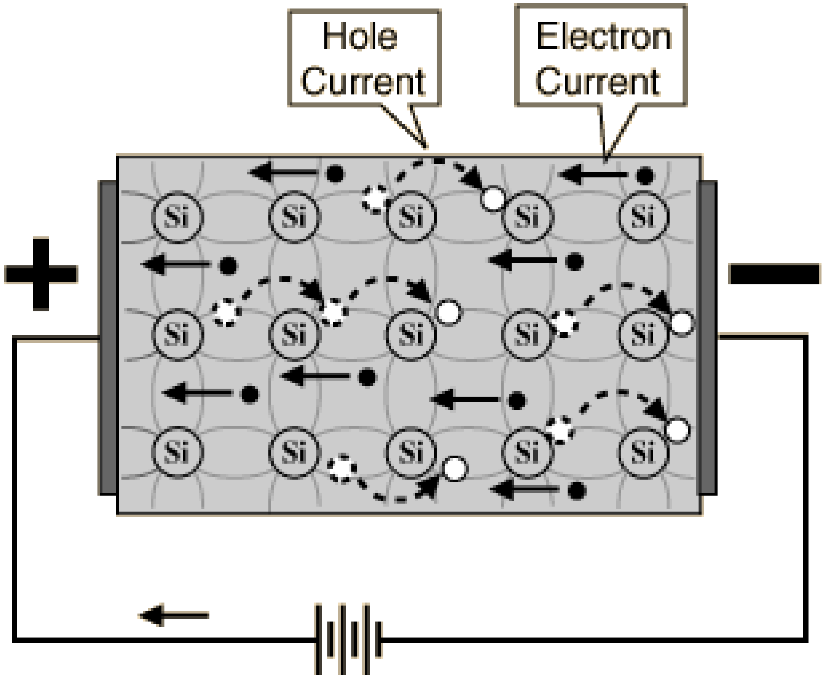
Transcendental equation –> numerical solution

Band Structure

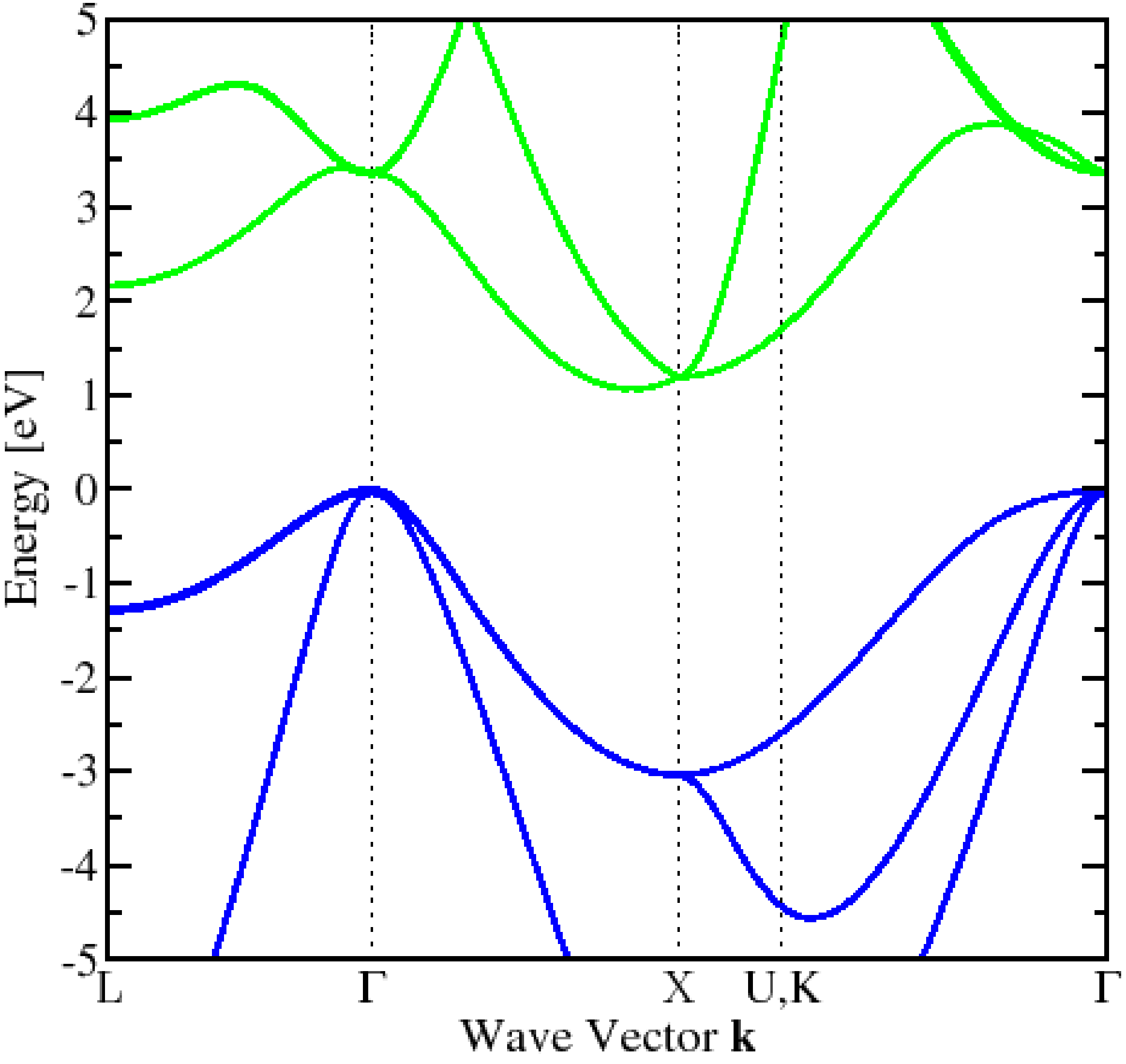


Si Intrinsic Semiconductor





Si Band Structure



Semiconductor BandGap

- Indirect

$$\alpha \propto \frac{(h\nu - E_g + E_p)^2}{\exp(\frac{E_p}{kT}) - 1} + \frac{(h\nu - E_g - E_p)^2}{1 - \exp(-\frac{E_p}{kT})}$$

- Direct

$$\alpha \approx A^* \sqrt{h\nu - E_g}$$

$$A^* = \frac{q^2 x_{vc}^2 (2m_r)^{3/2}}{\lambda_0 \epsilon_0 \hbar^3 n}$$

$$m_r = \frac{m_h^* m_e^*}{m_h^* + m_e^*}$$

Carrier density

- Fermi-dirac distribution (μ in semiconductors is E_F)

$$f(E) = \frac{1}{Exp\left[(E-\mu)/k_b T\right]+1}$$

- Density of sates in the conduction band

$$g(E) = \frac{V}{2\pi^2\hbar^3}\left(2m_e\right)^{3/2}\left(E-E_g\right)^{1/2}$$

- Density of sates in the valence band

$$g(E) = \frac{V}{2\pi^2\hbar^3}\left(2m_h\right)^{3/2}\left(-E\right)^{1/2}$$

Carrier density

- Electron density in the Conduction Band

$$n = \frac{N}{V} = \frac{1}{V}\int_{E_g}^{\infty} f(E)g(E)dE$$

$$= \frac{(2m_e)^{3/2}}{2\pi^2\hbar^3}\int e^{-(E-\mu)/k_BT}\left(E - E_f\right)^{1/2} dE$$

$$= 2\left(\frac{2\pi m_e k_b T}{\hbar^2}\right)^{3/2} e^{-(\mu - E_g)/k_BT}$$

- Similarly, to holes in the Valence Band

$$p = 2\left(\frac{2\pi m_h k_b T}{\hbar^2}\right)^{3/2} e^{-\mu/k_BT}$$

Carrier density

- Finaly

$$np = n_i^2 = N_c N_v e^{-E_g / k_b T}$$

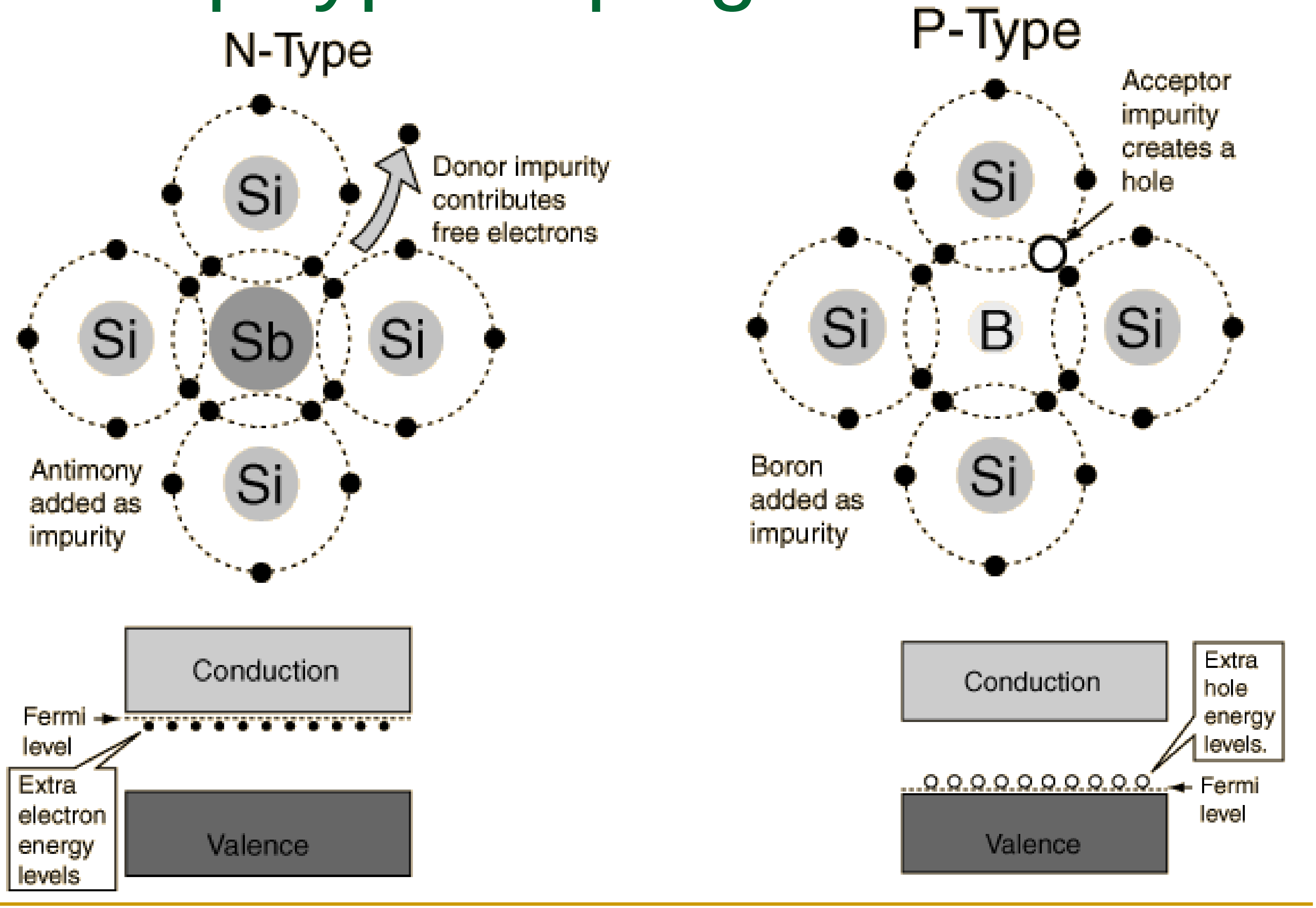
$$N_c = 2\left(\frac{2\pi m_e k_b T}{h^2}\right)^{3/2}$$

$$N_v = 2\left(\frac{2\pi m_h k_b T}{h^2}\right)^{3/2}$$

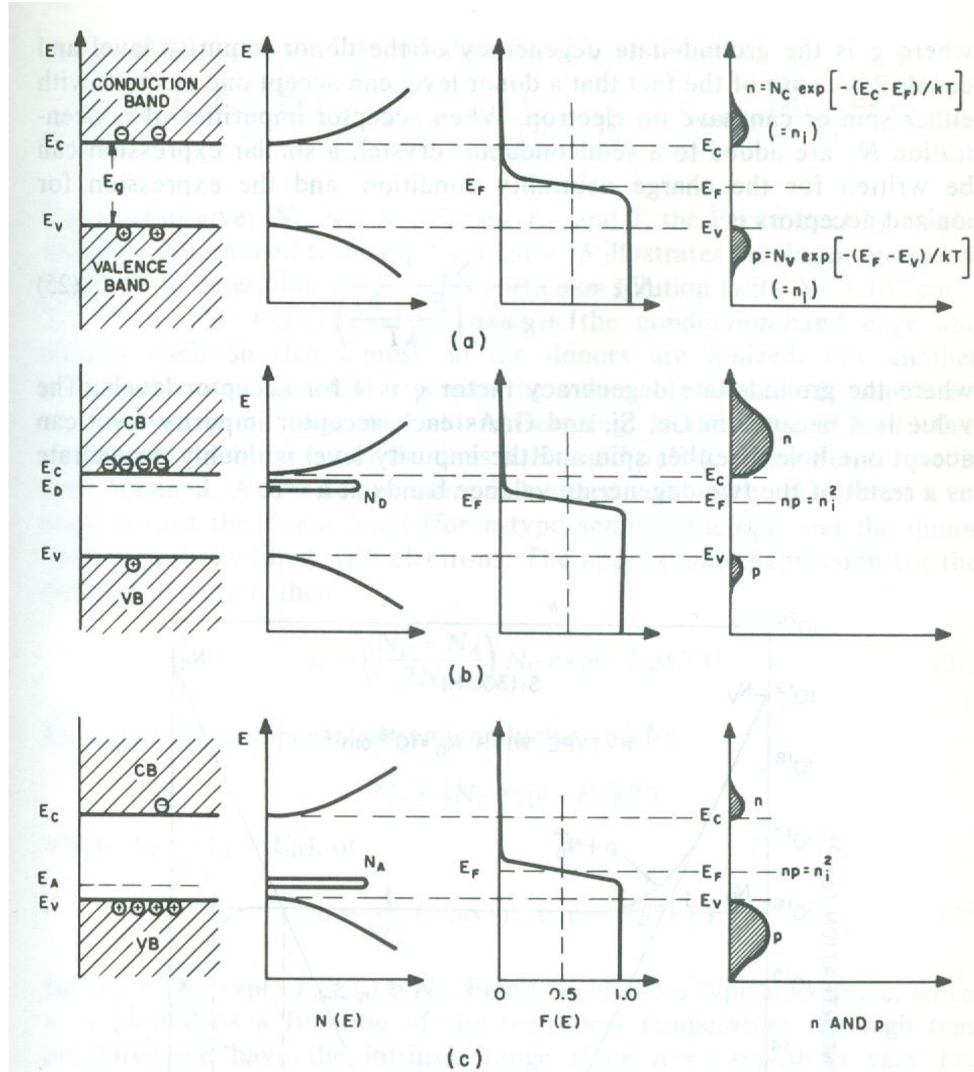
- And since n=p=n_i

$$E_i = \frac{E_C + E_V}{2} + \frac{3}{4} kT \ln\left(\frac{m_h^*}{m_e^*}\right)$$

n- and p-type doping



Density of States



Introducing impurities

- Donors $n = N_D^+ + p$ (neutrality)

$$N_D^+ = N_D\left[1 - \frac{1}{1 + \frac{1}{g}\exp\left(\frac{E_D - E_F}{kT}\right)}\right]$$

$$N_C \exp\left(-\frac{E_C - E_F}{kT}\right) = N_D \frac{1}{1 + 2\exp\left(\frac{E_F - E_D}{kT}\right)} + N_V \exp\left(\frac{E_V - E_F}{kT}\right)$$

- Acceptors

$$N_A^- = \frac{N_A}{1 + g\exp\left(\frac{E_A - E_F}{kT}\right)}$$

Extrinsic Semiconductors

- **n-type** (nondegenerated)

$$n_{no} = \frac{1}{2}\left[(N_D - N_A) + \sqrt{(N_D - N_A)^2 + 4n_i^2}\right]$$

$$\approx N_D \quad \text{if} \quad |N_D - N_A| \gg n_i \quad \text{and} \quad N_D \gg N_A$$

$$p_{no} = n_i^2/n_{no} \simeq n_i^2/N_D$$

$$E_C - E_F = kT \ln\left(\frac{N_C}{N_D}\right) \qquad E_F - E_i = kT \ln\left(\frac{n_{no}}{n_i}\right)$$

- **p-type** (nondegenerated)

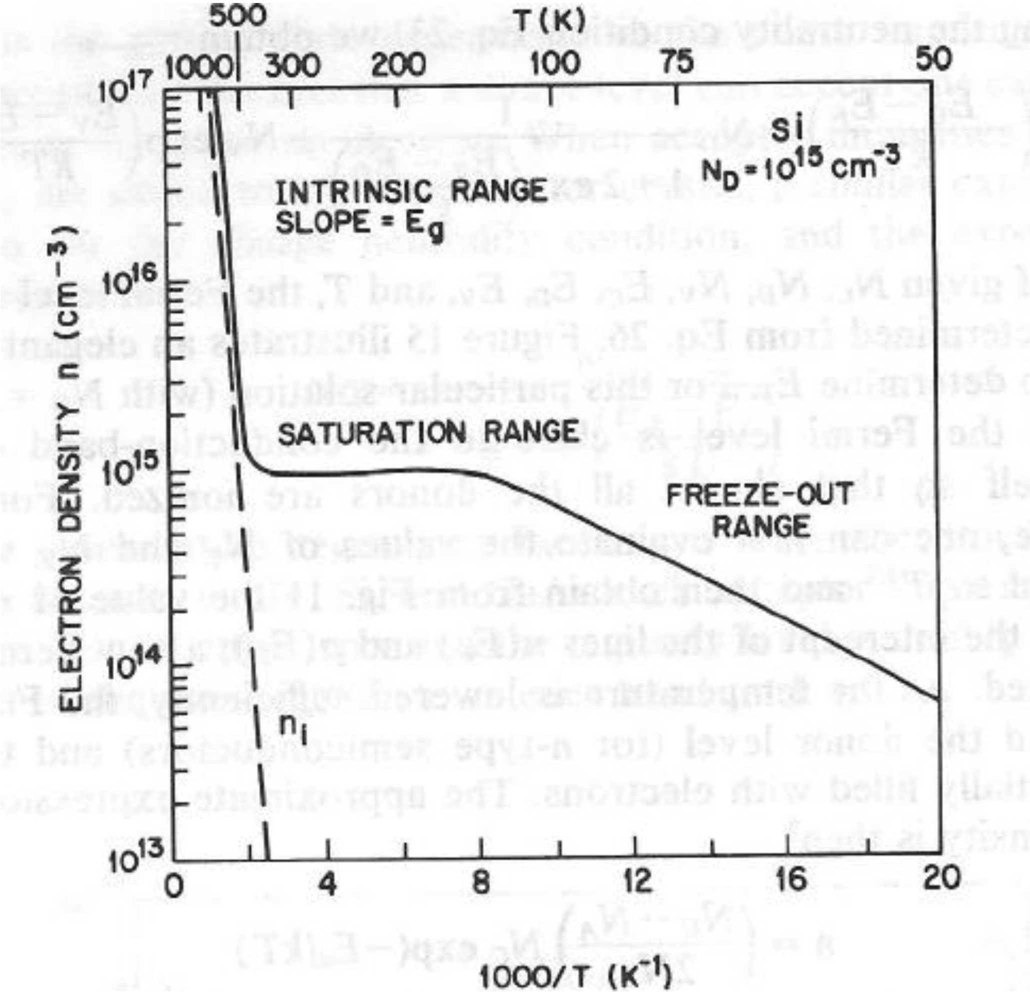
$$p_{po} = \frac{1}{2}\left[(N_A - N_D) + \sqrt{(N_A - N_D)^2 + 4n_i^2}\right]$$

$$\approx N_A \quad \text{if} \quad |N_A - N_D| \gg n_i \quad \text{and} \quad N_A \gg N_D$$

$$n_{po} = n_i^2/p_{po} \simeq n_i^2/N_A$$

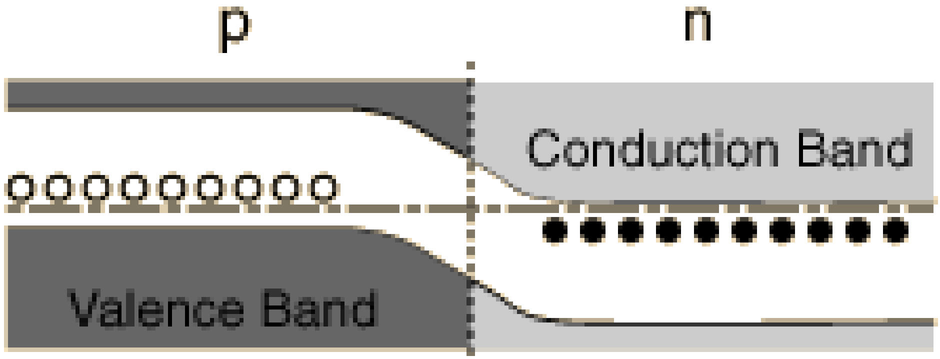
$$E_F - E_V = kT \ln\left(\frac{N_V}{N_A}\right) \qquad E_i - E_F = kT \ln\left(\frac{p_{po}}{n_i}\right)$$

n-type semiconductor



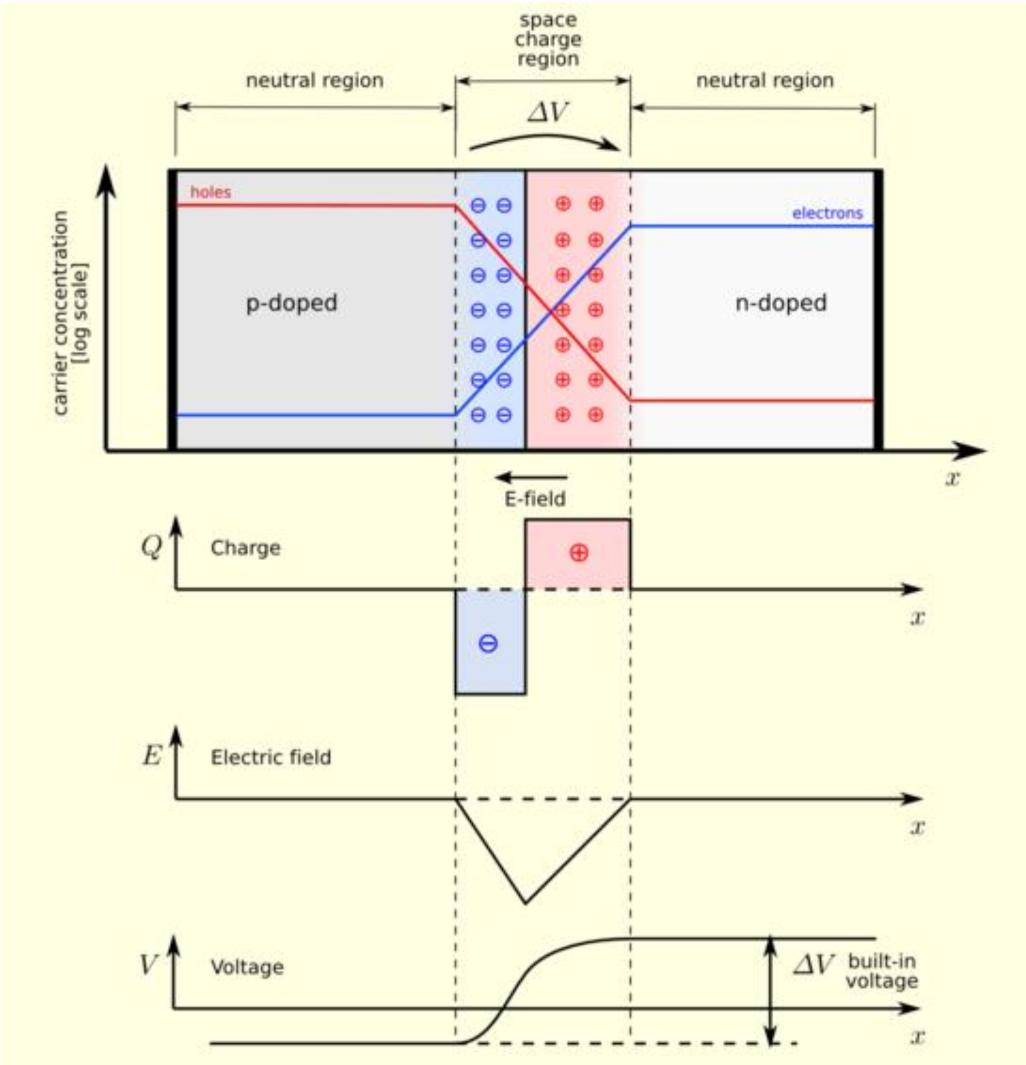
PN Junction

p-n junction



Energy bands at equilibrium

Depletion Region



Charge Neutrality

$$n + N_A^- w_P = p + N_D^+ w_N$$

Assuming:

full ionization,

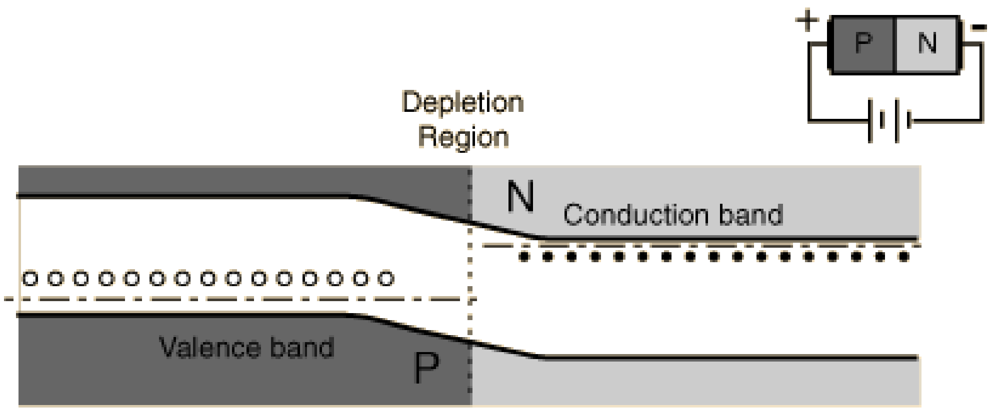
$$n, p << N_D, N_A$$

$$w = w_N + w_P$$

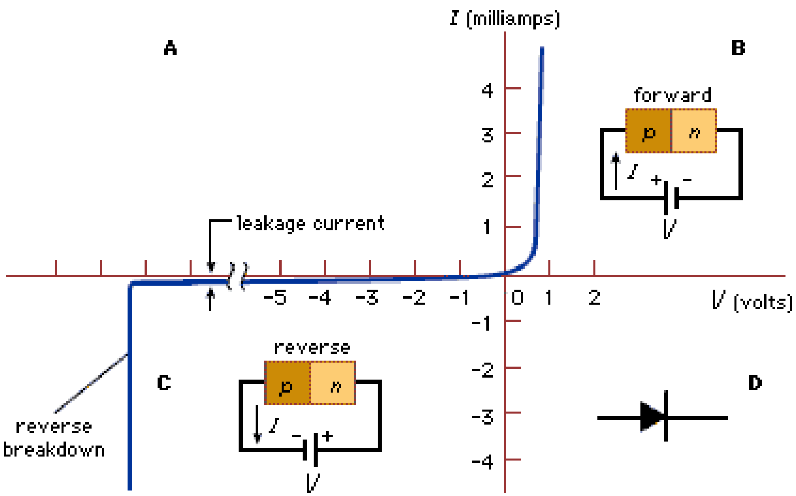
$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{p_0 n_0}\right)$$

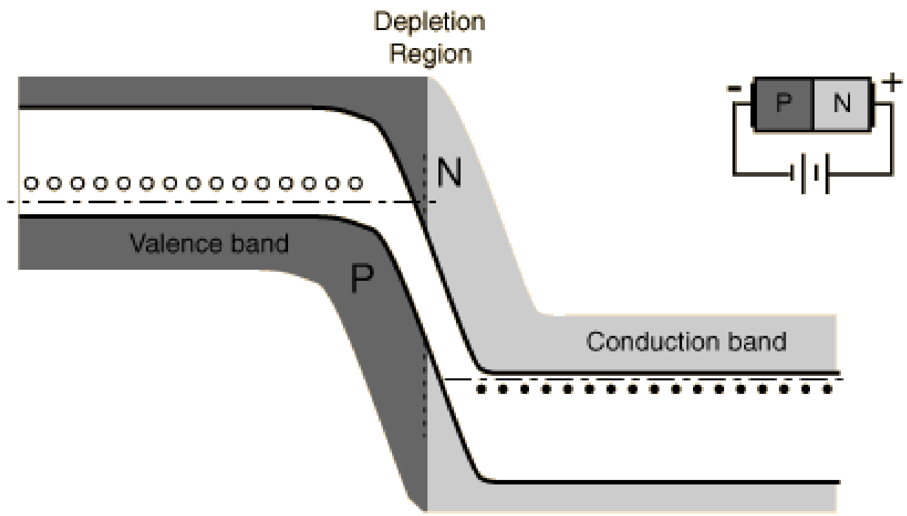
$$W \approx \left[\frac{2K_s\epsilon_0}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_{bi} - V)\right]^{\frac{1}{2}}$$

Forward and Reverse Bias



p-n junction





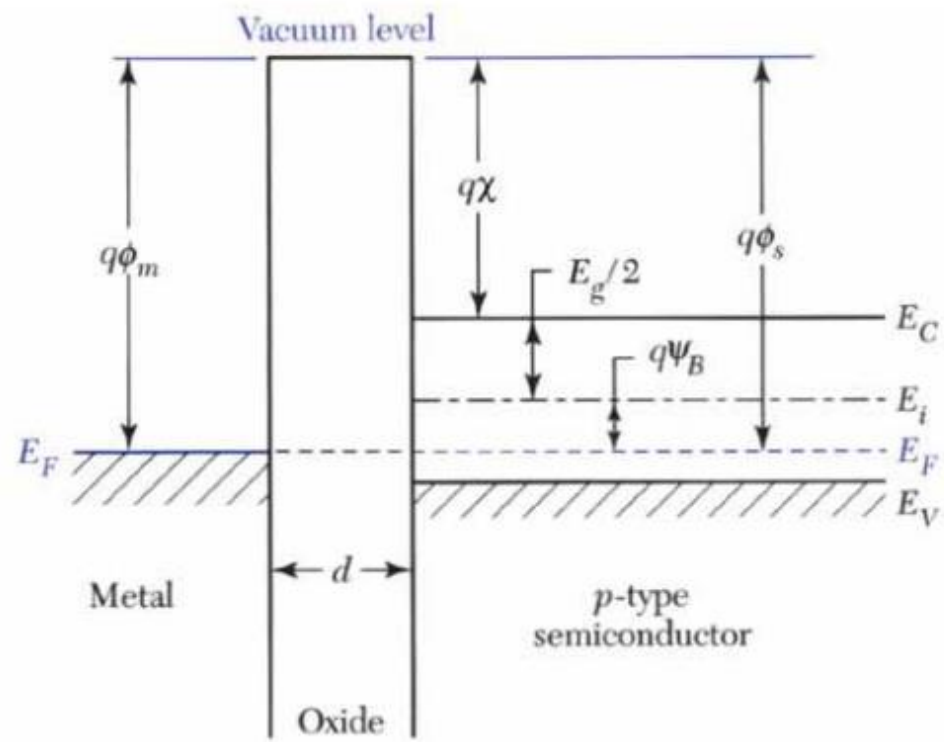
p-n junction

Reverse current in p-n junctions

Contributions to the leakage current under reverse bias:

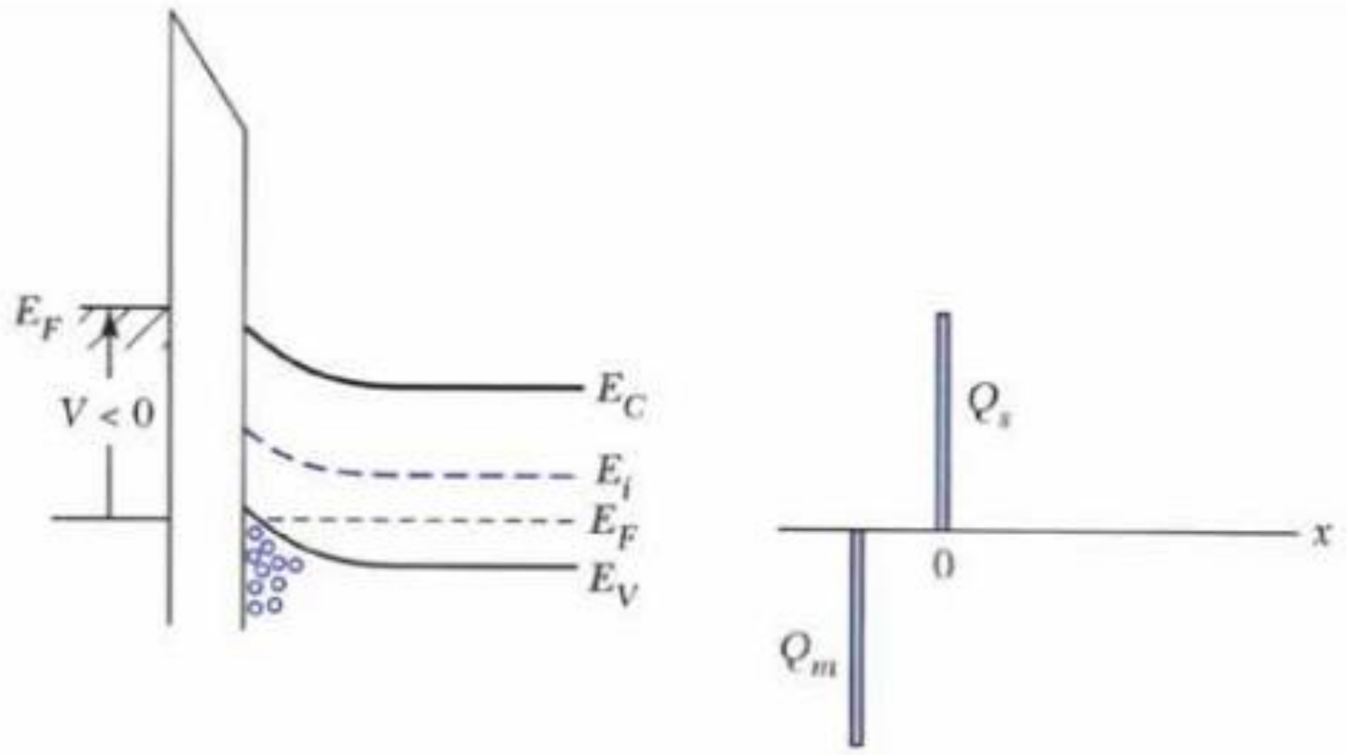
- diffusion current, J_{diff}
- space charge generation current, J_{gen}
- band-to-band tunneling current, J_{tun}
- thermionic emission current (metals), J_{them}
- gate-to-channel leakage (MOS), J_{GOx}

MOS Devices



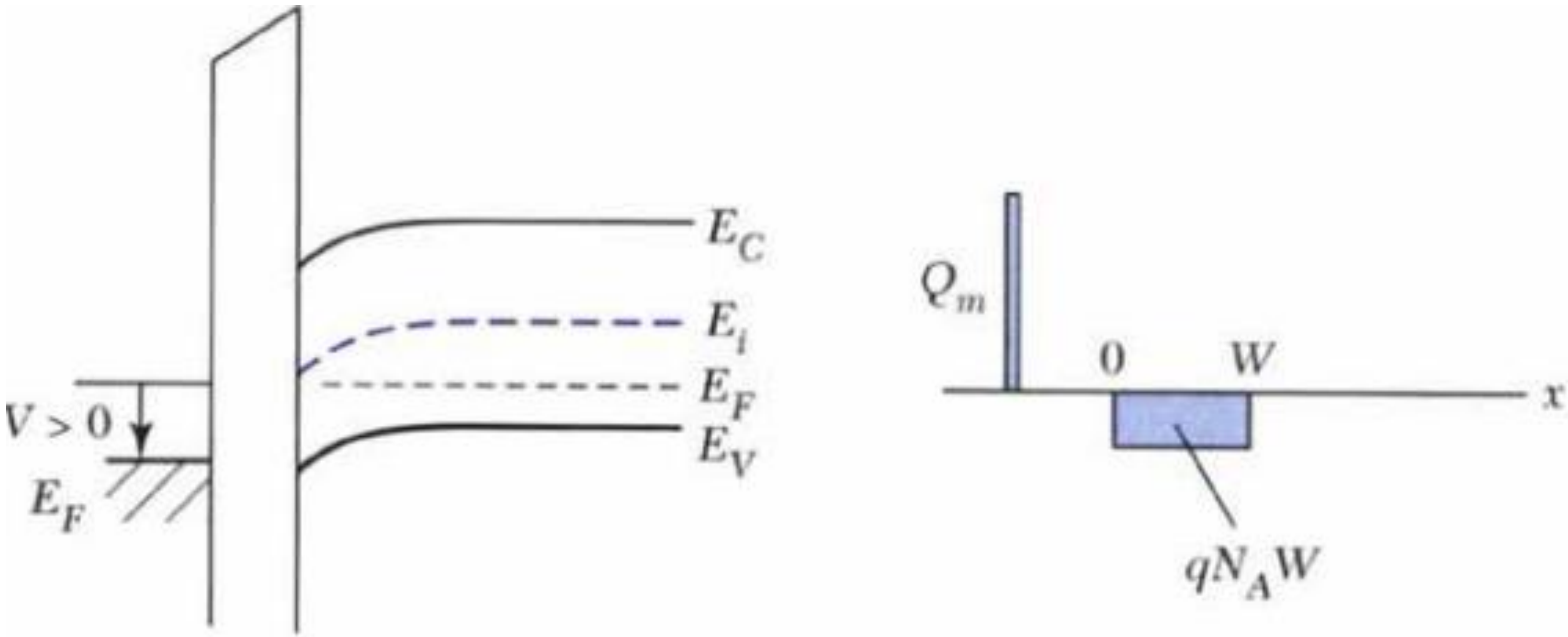
$$q\phi_{ms} \equiv (q\phi_m - q\phi_s) = q\phi_m - \left(q\chi + \frac{E_g}{2} + q\psi_B\right) = 0$$

MOS - Accumulation



$$p_p = n_i e^{(E_i - E_F)/kT}.$$

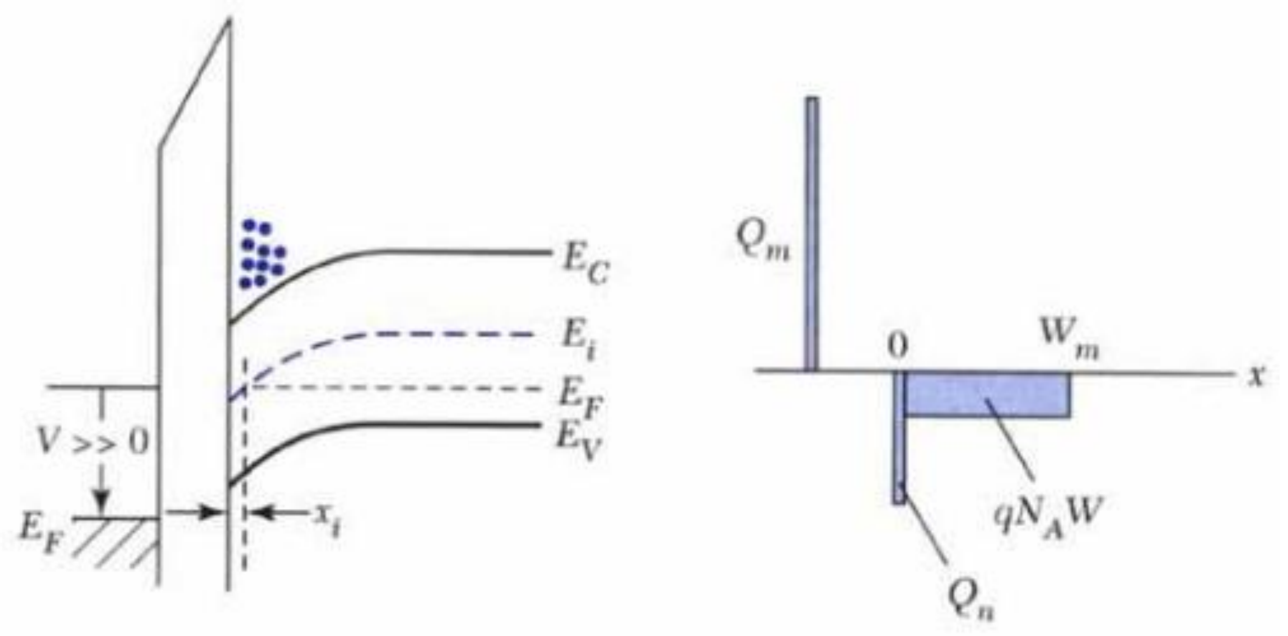
MOS - Depletion



Charge Neutrality (Q on gate): $Q = qN_A w$

$$E_m = Q/A\epsilon_0 = qN_A w/A\epsilon_0,$$

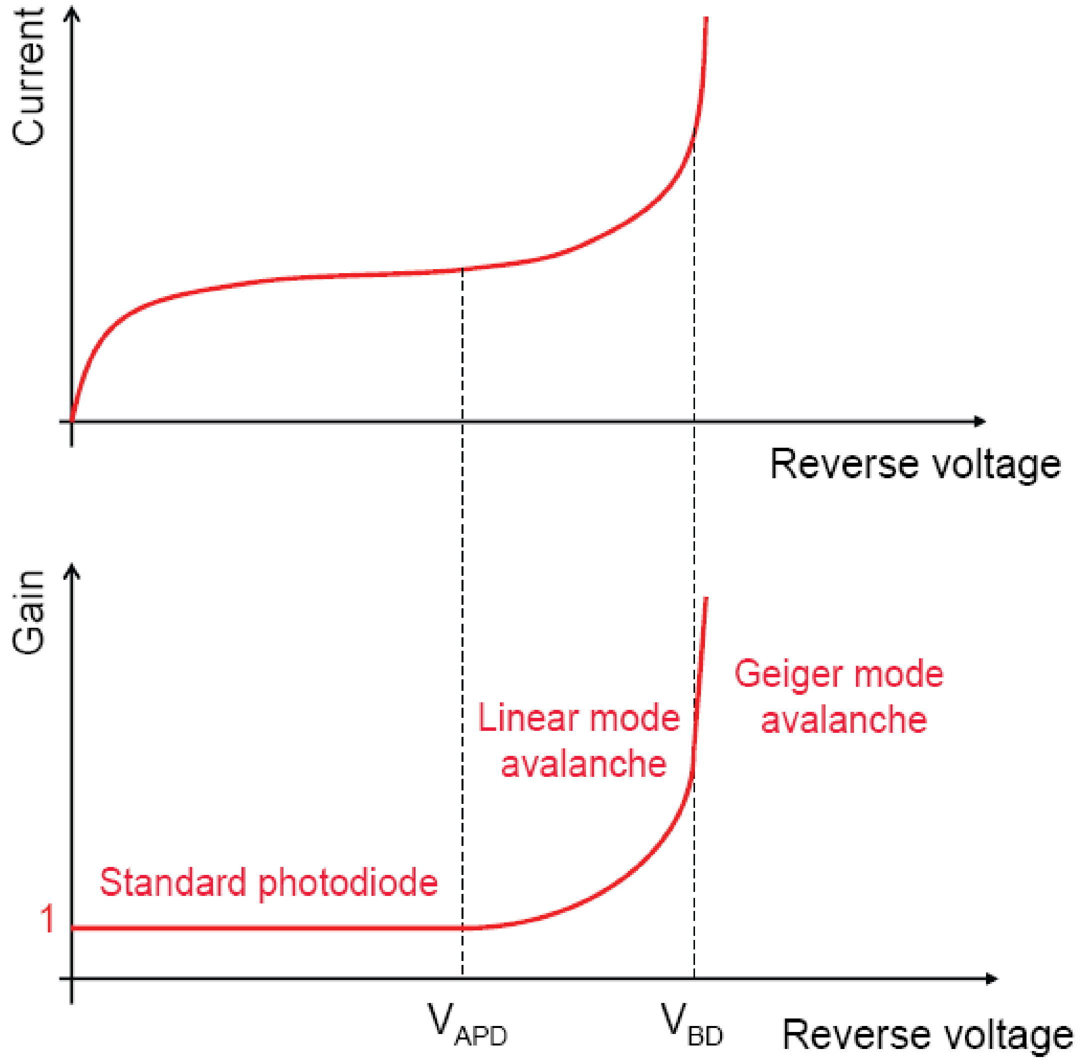
MOS - Inversion



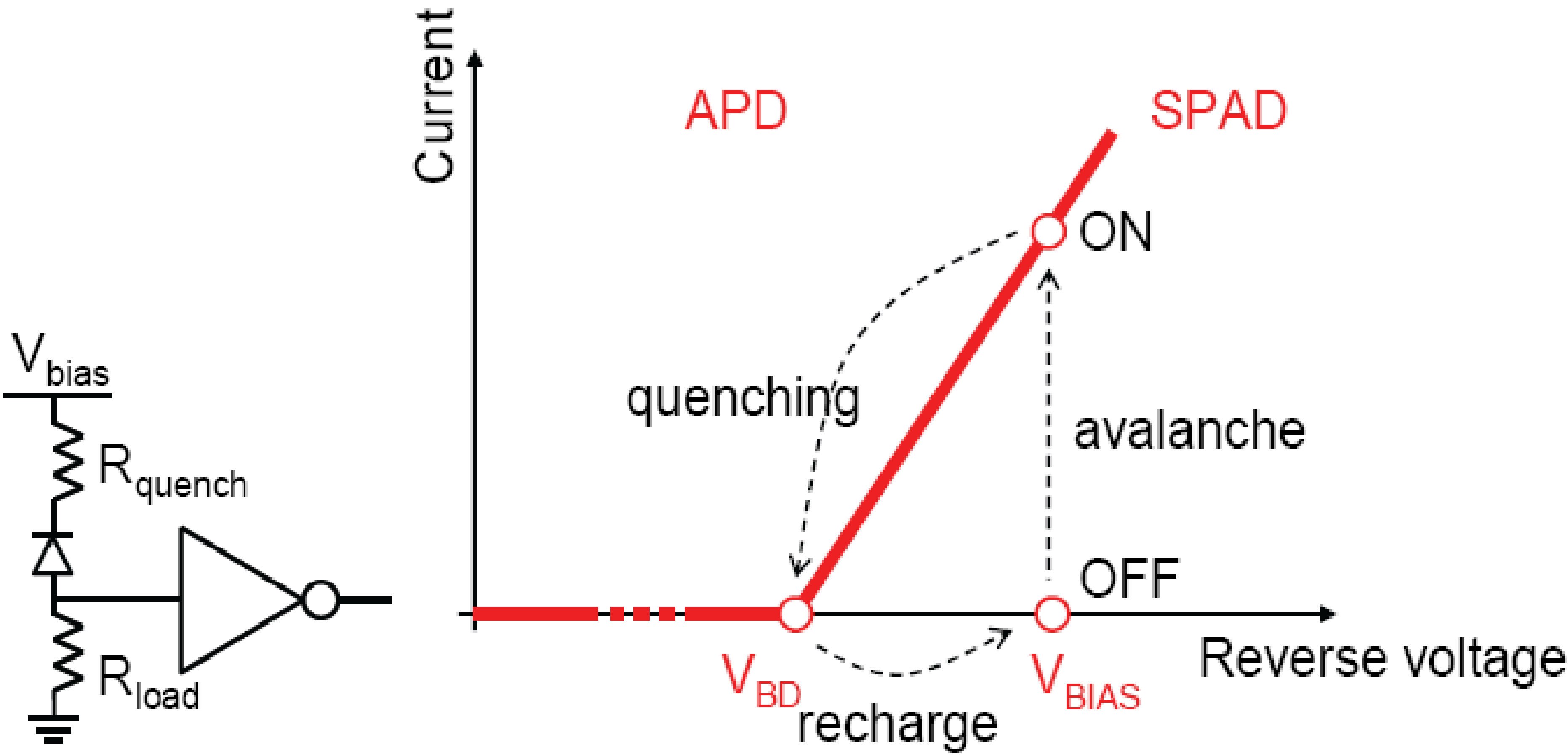
Minority carriers (electrons) accumulate: $n_p = n_i e^{(E_F - E_i)/kT}$

Depletion width at maximum.

Operation principle of APDs

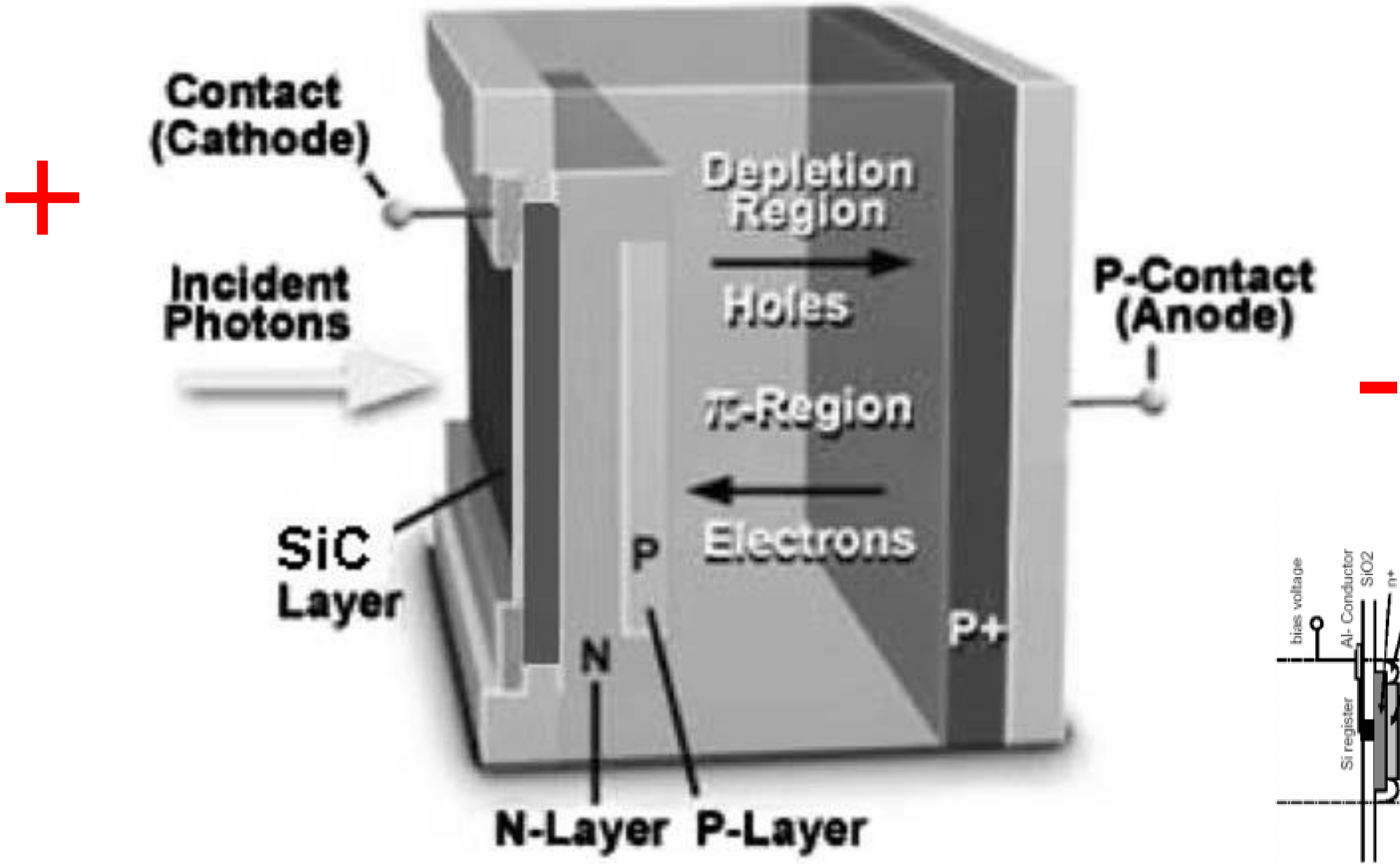


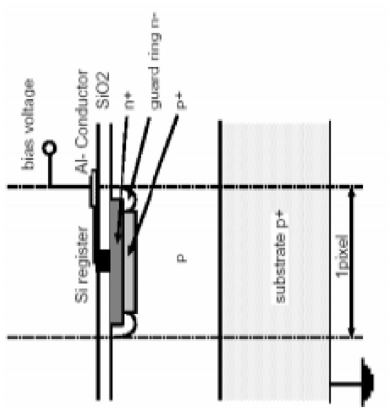
Quenching



Advances in Photodiodes 11, Betta et al.

Example of a Geiger-APD design

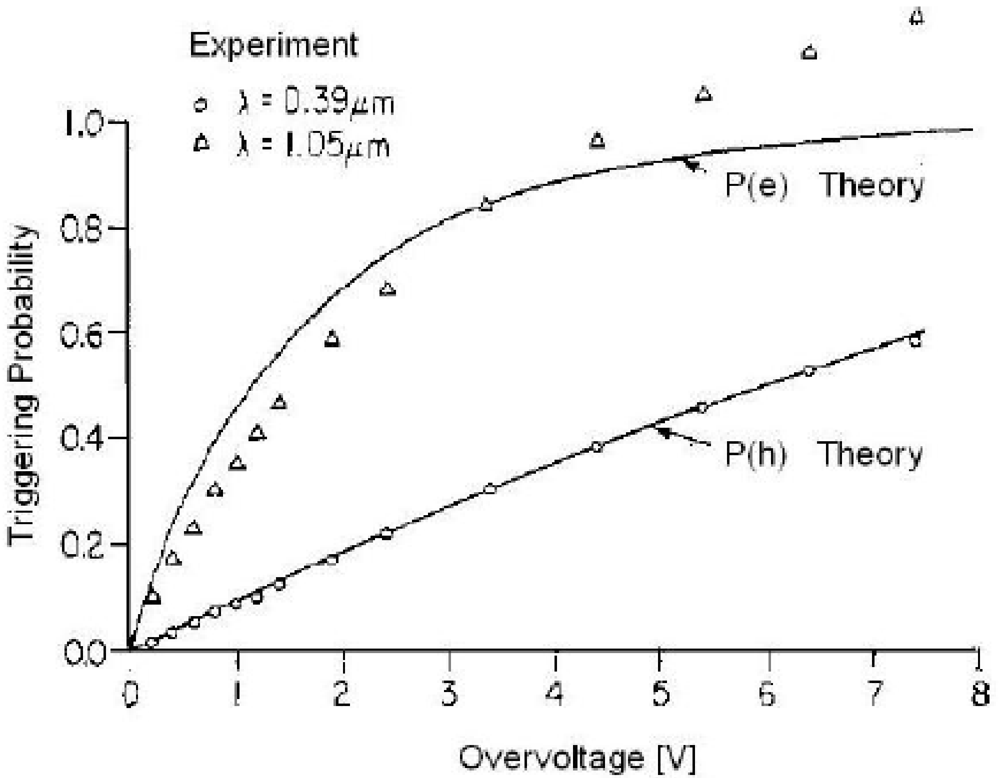




Photon Detection Efficiency

$\text{PDE} = \text{QE}\ \varepsilon\ P_{trigger}$

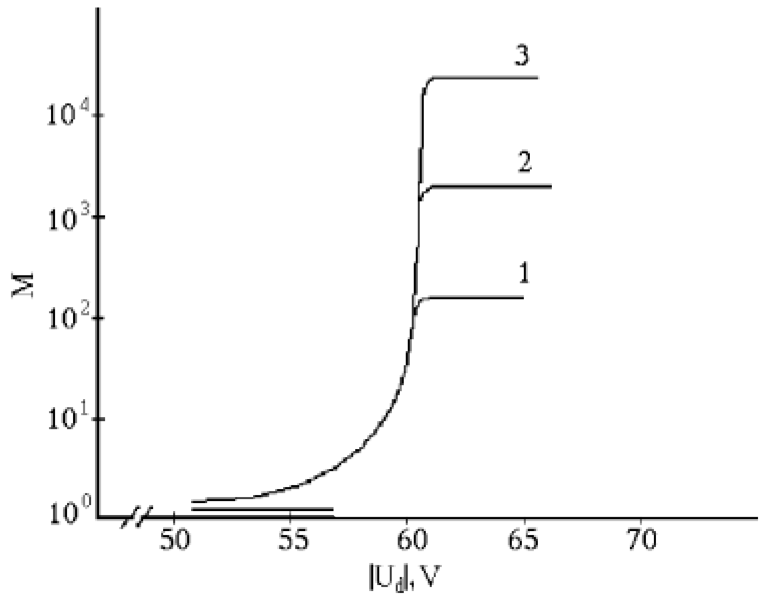
PDE - Photon Detection Efficiency
ε - geometric fill factor
QE – quantum efficiency



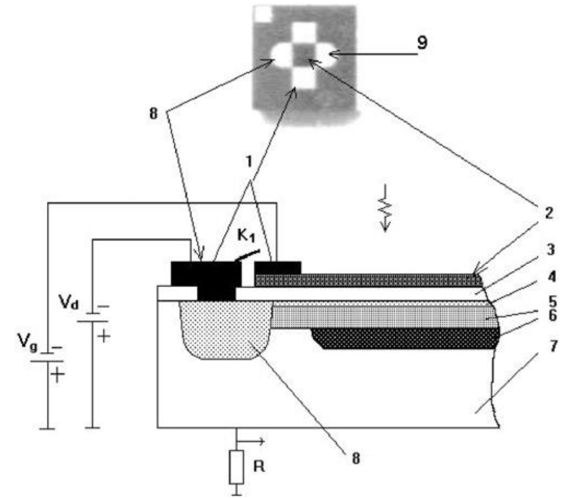
$$QE = \varepsilon_G(1 - R)(1 - e^{-\alpha x}),$$

Renker, Jinst, doi:10.1088/1748-0221/5/01/P01001

Gain



Photocurrent gain as a function of drain potential: 1 — $V_g = -68.5$ V; 2 — $V_g = -69.0$ V; 3 — $V_g = -69.5$;



Structure of the basic element.
1 — thick Al electrodes;
2 — semitransparent Ti gate electrode;
3 — dielectric layer, SiO_2;
4 — p^+–Si surface drift layer;
5 — p–Si layer;
6 — n^+–Si layer;
7 — n–Si wafer;
8 — p^+–Si drain;
9 — p^+–Si source.

$A_i \sim C/q(V-V_b)$

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Quenching

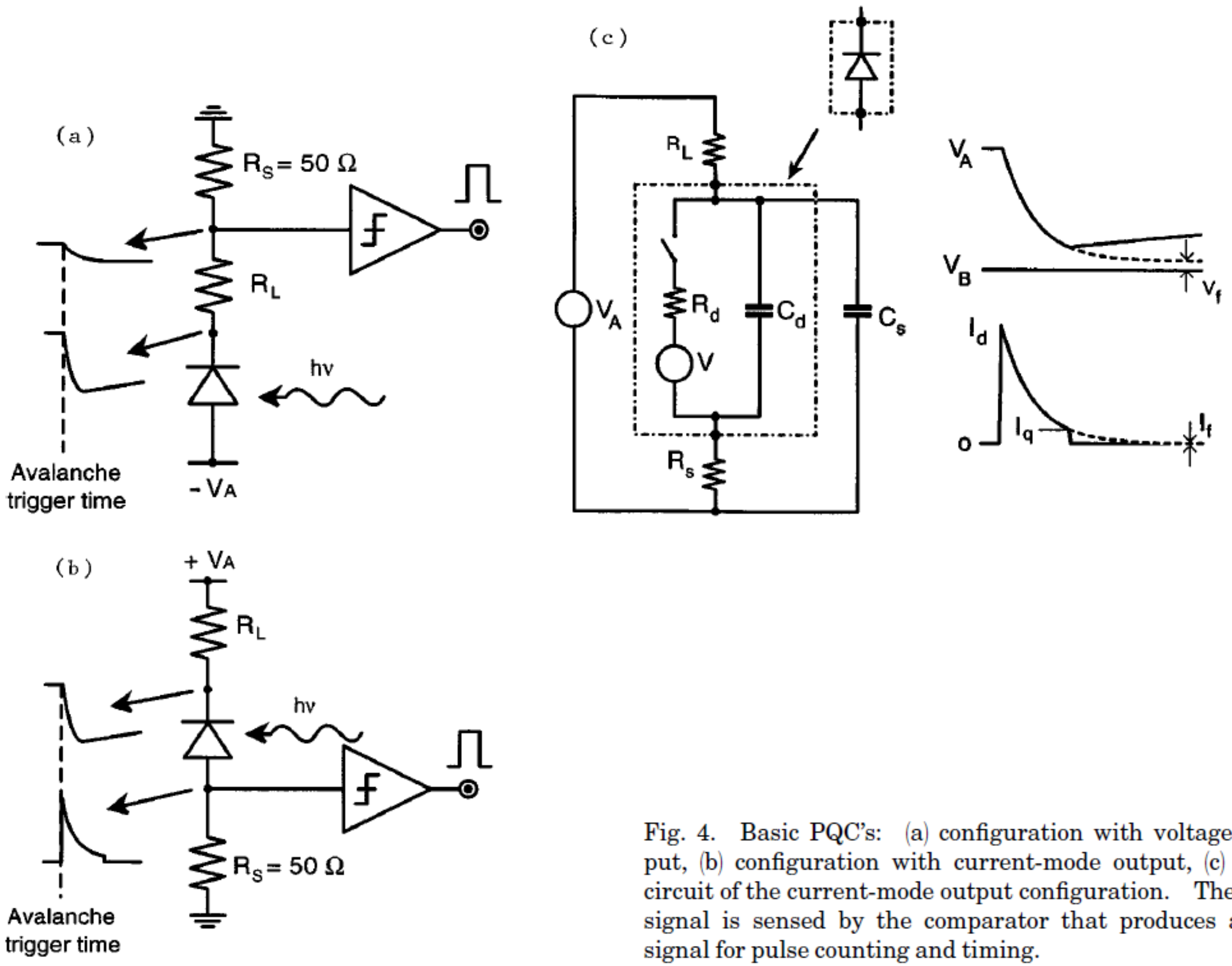

Fig. 4. Basic PQC's: (a) configuration with voltage-mode output, (b) configuration with current-mode output, (c) equivalent circuit of the current-mode output configuration. The avalanche signal is sensed by the comparator that produces a standard signal for pulse counting and timing.

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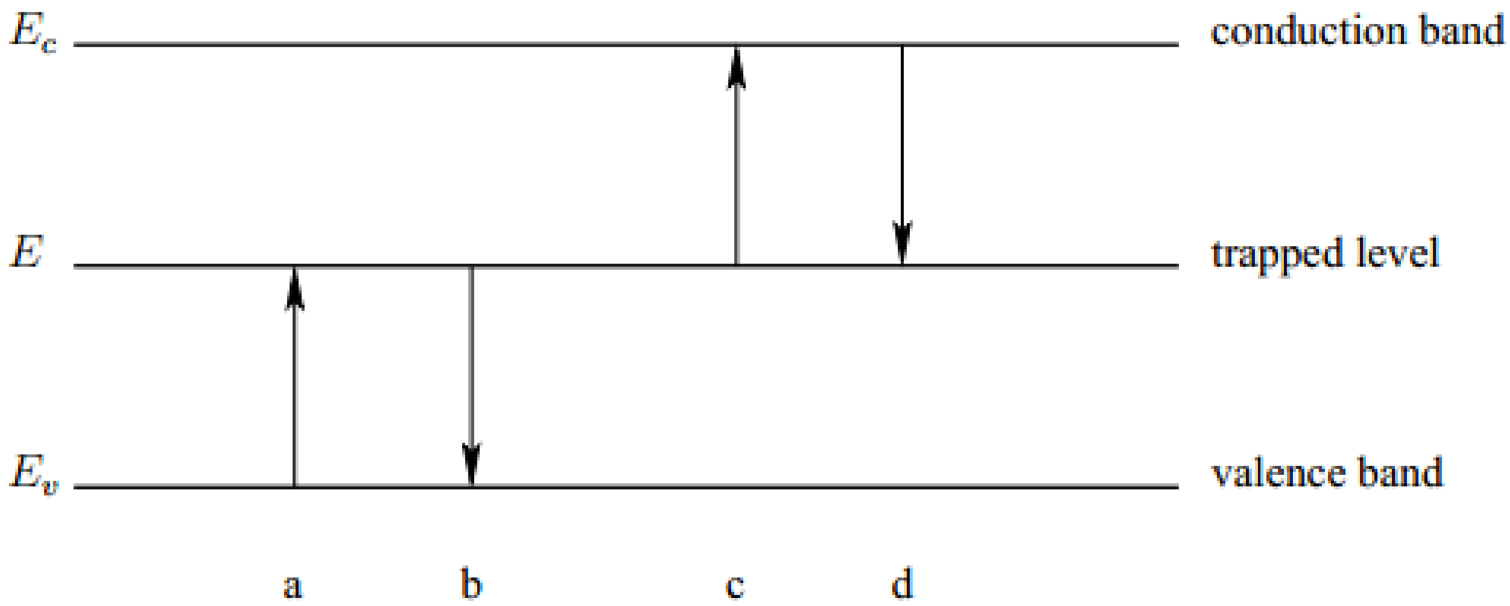
Dark Counts

- Main Causes:
  - thermally generated e-h pairs
  - diffusion from neutral regions
  - band to band tunnelling
  - release from charge traps (see also afterpulsing).
- 100kHz to several MHz per mm^2 at 25
- factor 2 reduction of the dark counts every 8–10 ºC temperature decrease.
- minimized by improving fabrication processes to reduce the number of generation-recombination centers (GR center), the impurities and crystal defects, which give rise to the Shockley-Read-Hall GR.

Shockley-Read-Hall GR

- Trap levels within the forbidden band caused by crystal impurities facilitates GR since the jump can be split into two parts



(a) hole emission (an electron jumps from the valence band to the trapped level),

(b) hole capture (an electron moves from an occupied trap to the valence band, a hole disappears),

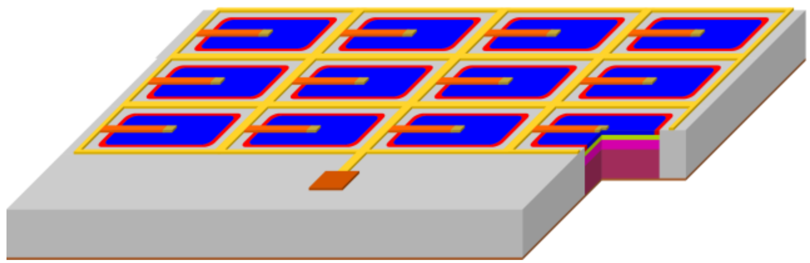
(c) electron emission (an electron jumps from trapped level to the conduction band),

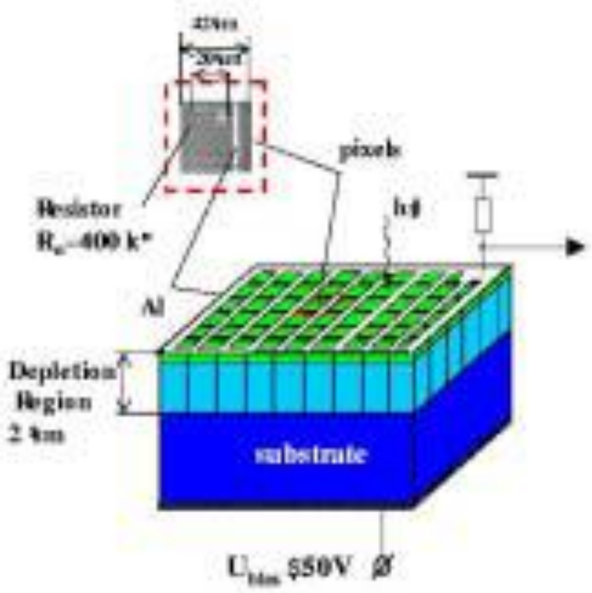
(d) electron capture (an electron moves from the conduction band to an unoccupied trap).

SiPM

G-APDs can exhibit excellent single photon sensitivity in a wide spectral range with very short signal rise times but they lack proportional response to multi-photon events.

SiPM - multipixel G-APDs





~1000 pixels in small area (~1mm x 1mm)

Silicon photomultiplier

- Gain
  - $M \sim 10^6$
  - Stable w/ T

The change of the temperature and bias voltage needed for a gain variation of 1%

Photodetector	ΔT	$\Delta V/V$
APD EG&G C30626E[a]	0.15°	$0.4V/400V = 10^{-3}$
APD Hamamatsu S5345[a]	0.3°	$0.04V/300V = 1.5 \times 10^{-4}$
SiPM ($M = 2 \times 10^6$)	2.5°	$0.05V/50V = 10^{-3}$

[a] For the gain $M = 100$ [5].

- Dark counts
- Optical crosstalk (~ 3 photons above E_g)
- Afterpulses
  - Charge traps results in generation-recombination (GR) centres.
  - High avalanche current increases the probability to fill the traps with carriers (deep traps ~ longer lifetimes).

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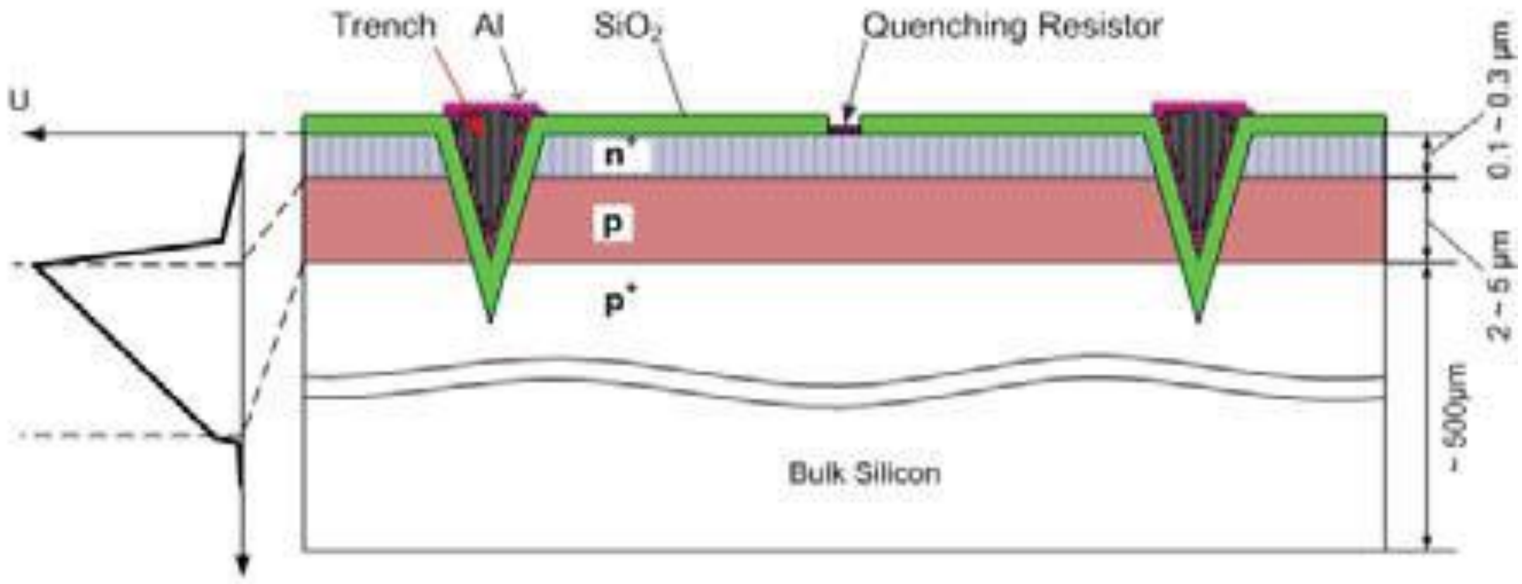
Trench separation of micro-cells



D. McNally and V. Golovin, Nuclear Instruments and Methods in Physics Research A (2008)