Lecture 3: Violation of the CP symmetry

The previously introduced matrix V (= Cabibbo-Kobayashi-Maskawa, or CKM matrix) is actually responsible for:

- all the processes with flavor violation[7];
- all the (so far observed) processes that violate a discrete symmetry called CP.

The CP operation on a physical object stands for charge conjugation (C) plus a parity transformation (P).

- **C interchanges particles and antiparticles.** So C conservation means that the rate for a process equals the rate for the same process with particles replaced by their respective antiparticles. C is violated in weak interactions,[8] but conserved in e.m. and strong ones.
- **P transforms $\vec{x} \to -\vec{x}$, if $\vec{x}$ is a vector.** On the other hand, pseudovectors, like angular momenta (hence also spins), are left unchanged. Therefore, P exchanges left-handed (LH) with right-handed (RH) particles and viceversa.
- **CP performs the two operations together.** So, in particular it exchanges a right-handed e^- into a left-handed e^+, and a left-handed e^- into a right-handed e^+.

Why CP violation is important. Note that, in the absence of CP violation, LH protons would always balance against RH antiprotons, and viceversa. Therefore, CP violation is required at some point in the history of the universe to generate the observed matter-antimatter imbalance.

As a matter of fact, CP violation is one of the three conditions enunciated by Sakharov in 1967 for matter-antimatter asymmetry to be possible. He was inspired by the experimental discovery of CP violation in the $K^0 - \bar{K}^0$ system, that we now have the instruments to explore.

L3.1 The $K^0 - \bar{K}^0$ system

• The $K^0 - \bar{K}^0$ system is a system of two mesons. As mentioned at the beginning of Lecture 2, mesons are bound states of a quark and an antiquark, bound together by strong interactions. One has:

$$K^0 \sim \begin{pmatrix} d \\ \bar{s} \end{pmatrix}, \qquad \bar{K}^0 \sim \begin{pmatrix} \bar{d} \\ s \end{pmatrix}. \tag{3.1}$$

We have previously drawn the diagram in fig. 9. So the K^0 can become its antiparticle and viceversa. Note that this can only happen via weak interactions (only at the 'loop' level). No other SM interaction is able to generate this kind of process.

• Because the K^0 and the $\bar{K}^0$ oscillate into each other, they are not 'well defined' particles. In fact, a very interesting quantum-mechanics (QM) problem was the one of defining the physical particles associated with the $K^0 - \bar{K}^0$ system.

• A good candidate for a particle is an object with definite mass and certain 'quantum numbers', which express its properties under certain conservation laws. E.g. an electron has a mass of 0.5 MeV, has charge -1 (in units of e), spin $1/2$ (in units of $\hbar$), etc.

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[7] By this we mean either 'oblique' transitions in fig. 4 or FCNCs.

[8] E.g. left-handed e^- and left-handed e^+ don't behave in the same way in the SM.

• In QM, each of a particle's properties is mathematically described by an eigenvalue of a suitable operator, of which the particle 'state' is an eigenstate. For example, for the electron mass:

$$\mathscr{H}\,|e(\vec{p}=0)\rangle \;=\; m_e\,|e(\vec{p}=0)\rangle \tag{3.2}$$

where $\mathscr{H}$ is the Hamiltonian operator: as we know, it is the function expressing the energy of a system. The system is an electron at rest, $|e(\vec{p}=0)\rangle$. So its eigenvalue is the rest energy of the electron, its mass.

• We may then say that a good 'particle' ($\Rightarrow$ definite mass) is an eigenstate of the relevant $\mathscr{H}$.

• How to define good physical states for the $K^0 - \bar{K}^0$ system then? Gell-Mann and Pais (those who solved this problem in 1955) started from observing that C exchanges K^0 and $\bar{K}^0$:

$$\begin{aligned} \mathrm{C}|K^0\rangle &= |\bar{K}^0\rangle \ , \\ \mathrm{C}|\bar{K}^0\rangle &= |K^0\rangle \ . \end{aligned} \tag{3.3}$$

Then, the states defined as

$$|K_\pm\rangle \equiv \frac{|K^0\rangle \pm |\bar{K}^0\rangle}{\sqrt{2}} \tag{3.4}$$

would be states of definite eigenvalue ± 1 under C, respectively, and *maybe* a good basis of physical states.

• However, the relevant Hamiltonian is the weak Hamiltonian (the diagram in fig. 9 arises from weak interactions), and charge conjugation is *not a good conserved property* of weak interactions.

• A better choice for the operator exchanging $K^0 \leftrightarrow \bar{K}^0$ is CP, which is (almost) conserved by weak interactions. Note: since K mesons have zero spin, CP is basically like C. Namely one can always write (by a suitable redefinition of the K^0, $\bar{K}^0$ fields)

$$\begin{aligned} \mathrm{CP}|K^0\rangle &= |\bar{K}^0\rangle \ , \\ \mathrm{CP}|\bar{K}^0\rangle &= |K^0\rangle \ , \end{aligned} \tag{3.5}$$

and define

$$|K_\pm\rangle \equiv \frac{|K^0\rangle \pm |\bar{K}^0\rangle}{\sqrt{2}} \tag{3.6}$$

such that

$$\mathrm{CP}|K_\pm\rangle \equiv \pm|K_\pm\rangle \ . \tag{3.7}$$

The $|K_\pm\rangle$ is thus said to be CP-even and CP-odd, respectively.

• Now, in weak interactions CP is not exact, it is slightly violated. Hence the $|K_\pm\rangle$ are approximate, but not perfect physical eigenstates. We can represent this imperfection by defining

$$\begin{aligned} |K_S\rangle \;&\sim\; |K_+\rangle \;+\; \bar{\epsilon}\,|K_-\rangle \\ |K_L\rangle \;&\sim\; |K_-\rangle \;+\; \bar{\epsilon}\,|K_+\rangle \end{aligned} \tag{3.8}$$

where the $|K_S\rangle$ and $|K_L\rangle$ are the true physical eigenstates. As we send $\bar{\epsilon} \to 0$ the $|K_+\rangle$ and $|K_-\rangle$ coincide with the $|K_S\rangle$ and $|K_L\rangle$, respectively. This $\bar{\epsilon}$ quantifies the amount of CP violation. (The origin of the suffix S and L will soon be clear.)

• We can distinguish the $|K_+\rangle$ and $|K_-\rangle$ components inside the physical states by looking at *decays to states that are CP eigenstates.* E.g.

$$\begin{aligned} |2\pi\rangle &= \text{CP even} \;, \\ |3\pi\rangle &= \text{CP odd} \;. \end{aligned} \tag{3.9}$$

Recalling the CP properties of $|K_+\rangle$ and $|K_-\rangle$ we expect[9]

$$|K_+\rangle \;\to\; |2\pi\rangle \quad \text{but} \quad |K_+\rangle \;\not\to\; |3\pi\rangle \;, \tag{3.10}$$

and conversely

$$|K_-\rangle \;\to\; |3\pi\rangle \quad \text{but} \quad |K_-\rangle \;\not\to\; |2\pi\rangle \;. \tag{3.11}$$

• By using eqs. (3.10) and (3.11) into the definition of the physical states $|K_{S,L}\rangle$, we then expect that

$$\textit{the } |K_S\rangle \textit{ will decay most of the time to } |2\pi\rangle\textit{, and sometimes to } |3\pi\rangle, \tag{3.12}$$

whereas conversely

$$\textit{the } |K_L\rangle \textit{ will decay most of the time to } |3\pi\rangle\textit{, and sometimes to } |2\pi\rangle. \tag{3.13}$$

In these sentences, 'sometimes' quantifies the amount of CP violation.

• Therefore, the ratio

$$\frac{N_{\text{events}}(K_L \to 2\pi)}{N_{\text{events}}(K_S \to 2\pi)} \tag{3.14}$$

will quantify the amount of CP violation. This is indeed the way CP violation was originally discovered for the first time.

L3.2 The discovery of CP violation (1964): the Cronin-Fitch experiment

CP violation was experimentally discovered in 1964 (and awarded the Nobel prize in 1980) in an experiment that measured the rate in eq. (3.14).

• First problem: how to distinguish the K_S from the K_L experimentally? The answer is in eqs. (3.12) and (3.13). The K_S decays most of the time into 2π, whereas the K_L 'has to' decay most of the time into 3π. Therefore, the K_L decay is 'less easy'. In QM decays to fewer bodies are kinematically easier, and this is intuitively understandable. So the K_S is short-lived whereas the K_L is comparably long-lived, and now we understand the S and L subscripts.

• It is now clear how this observation may be used experimentally. This is depicted in fig. 12.

L3.3 More on CP violation within the SM

In connection with fig. 7 we have stated that all flavor-physics phenomena involving quarks arise from the basic interaction in eq. (2.16). We would like to now expand on this statement.

• By tracing back the line of reasoning leading to eq. (2.16), we see that, in order to have a non-trivial CKM matrix, $V \neq 1$, we need off-diagonal entries in the y_u and y_d matrices. In other words

$$y_{u,d} \text{ off-diagonal} \;\Leftarrow\; \text{flavor violation} \;. \tag{3.15}$$

[9] This statement is not strictly true, but the fully correct statement is irrelevant here (and too advanced).

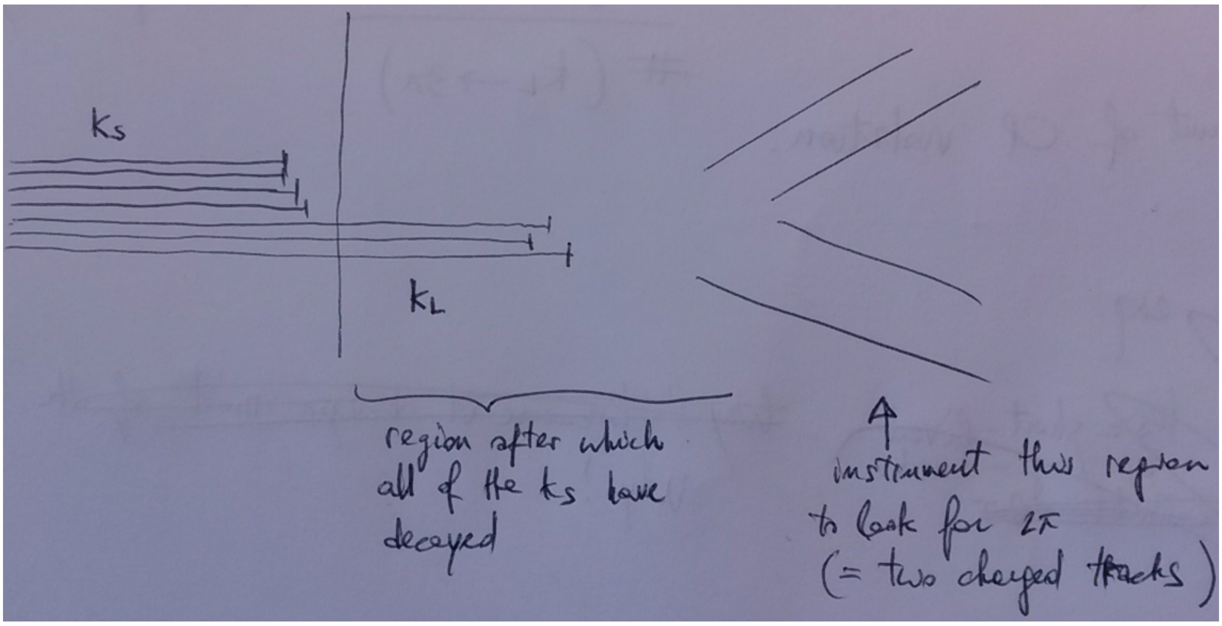

Figure 12: How to measure CP violation at home.

Also all the observed CP violation is, within the SM, caused by the y_u and y_d matrices. In particular[10]

$$y_{u,d} \text{ complex } \Leftarrow \text{ CP violation }. \tag{3.16}$$

• **Unphysical CP violation** – The previous sentence needs actually some qualifications. In fact, in Lecture 2 we have seen that one can redefine quark fields by unitary transformations, as in eq. (2.14). So one may wonder whether phases in y_u and y_d may always be reabsorbed by suitable redefinitions of the quark fields.

Let us make an example. Suppose only the quarks u, d, c, s existed, namely that there were only two generations (see fig. 4). Then, consider the interaction in eq. (2.16) for just two generations, namely with $V = V_{2\times 2}$, and with $U = (u, c)$ and $D = (d, s)$. Let us write the CKM matrix explicitly as

$$V_{2\times 2} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} . \tag{3.17}$$

Then the $\bar{U}D$ combinations present in the interaction in eq. (2.16) can be schematically written as

$$\begin{pmatrix} \bar{u} & \bar{c} \end{pmatrix} \cdot \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \end{pmatrix} . \tag{3.18}$$

Now, even if V_{ud} and V_{us} are complex, their phases can be moved into the definition of the d and s fields, respectively. Therefore, we can take V_{ud} and V_{us} as real. Furthermore, $V_{2\times 2}$ is unitary, namely

$$V^\dagger \cdot V \;=\; \begin{pmatrix} V_{ud} & V_{cd}^* \\ V_{us} & V_{cs}^* \end{pmatrix} \cdot \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} = \mathbb{1}_{2\times 2} \,. \tag{3.19}$$

[10] Showing the last statement requires somewhat too advanced tools for this course, so we will leave this statement without proof (but ask if you feel like).

(Note that we have dropped complex conjugation signs in V_{ud} and V_{us} as they are real.) The 12 relation reads $V_{ud}V_{us} + V_{cd}^*V_{cs} = 0$, implying that V_{cd} and V_{cs} must have equal phases, because the product $V_{cd}^*V_{cs}$ must be real. Both these V entries multiply $\bar{c}$ in eq. (3.18), so their common phase can be moved into the definition of the c quark.

So we see that, for two generations, V can always be defined as real, hence there is no physical CP violation.

• **Conditions for physical CP violation** – Let us generalize the previous reasoning to N generations, namely to the case of $2N$ quarks. Within the SM (see fig. 4), $N = 3$.

Recalling that it is unitary, the CKM matrix V has N^2 parameters in the $N \times N$ case. Out of them, $N(N-1)/2$ are Euler angles.[11] The rest, $N^2 - N(N-1)/2 = N(N+1)/2$, are phases. However, not all of them are physical. Generalizing the 2×2 example to N generations, one sees that $2N-1$ phases can be moved into the definition of all but one quark fields. So, for $N = 3$ one will have 6 phases, but one can make 5 quark-field redefinitions, implying one single physical phase. (This argument earned the Nobel prize to Kobayashi and Maskawa in 2008.)

In conclusion

$$\begin{array}{l}\text{CP violation is only possible for } N \geq 3. \\ \text{Within the SM (3 generations) CP violation is due to one single phase.}\end{array} \tag{3.20}$$

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The statement in (3.20) implies that the amount of CP violation predicted by the SM can (in principle) be univocally determined by measuring one single CP-violating quantity. Any other CP-violating quantity can then be predicted. Hence, measuring several CP-violating observables allows to test the SM mechanism of CP violation mentioned above.

As a matter of fact, CP violation has been determined in several other processes after the process in fig. 9. For example, in $B^0 - \bar{B}^0$ oscillations, that are completely analogous to fig. 9, but for the fact that the s quark is replaced by a b quark.

The test of CP violation mentioned above can be visualized in the plots of fig. 13. Each of the bands on these plots represents an observable. If the amounts of CP violation predicted by each observable are compatible with each other, then all the curves should intersect at one point. And it looks they do! This is a very non-trivial test of validity of the SM mechanism of CP violation, to the current level of accuracy of about 15%.

[11] Recall: for 2×2 matrices, there is one rotation, i.e. one such angle; for 3×3 matrices, the angles are 3; for $N \times N$, they are as many as the number of elements above the diagonal, namely $N(N-1)/2$.

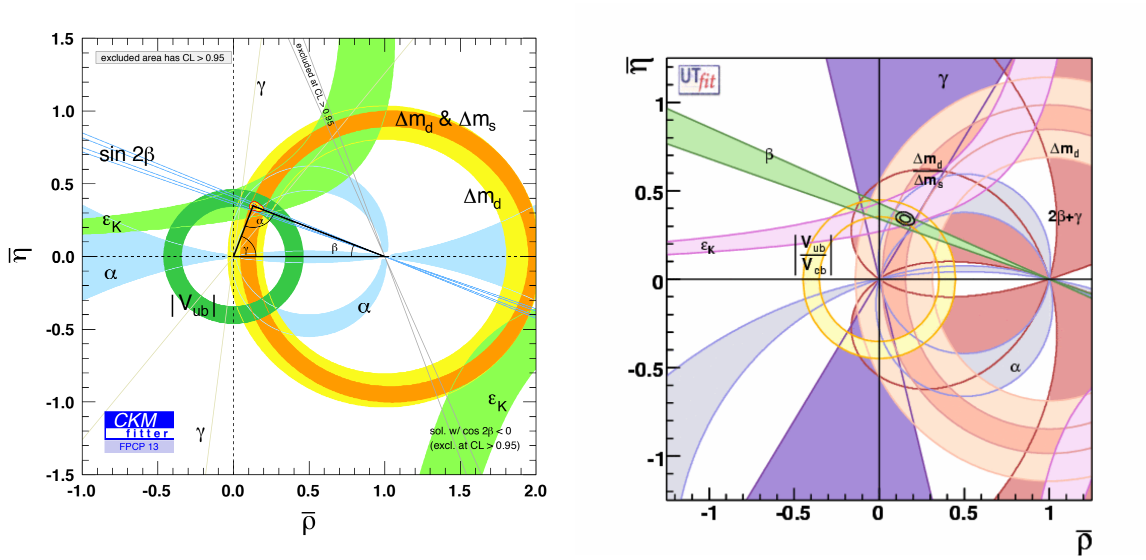

Figure 13: Status of the test of the SM mechanism of CP violation (and of flavor violation in general). Figures taken from `ckmfitter.in2p3.fr` and `utfit.org`, respectively.