Lecture 2: Basics about flavor physics

The example in sec. L1.2 is a metaphor of the way of reasoning in theoretical physics at large, and in flavor physics in particular. In fact, flavor physics makes inherent use of the idea of EFT. The reason is the following: flavor physics is concerned with processes of the kind depicted in fig. 3. In this process, the initial state includes a quark of mass m_i, which interacts in the black box with some other particle (not depicted) to yield a final-state quark of mass m_j *different* than m_i. Flavor violation is exactly the fact that $m_j \neq m_i$. (In general there will be other quarks participating in the process as 'spectators', that namely do not change flavor. These quarks are likewise depicted in the figure.) The mass scales involved

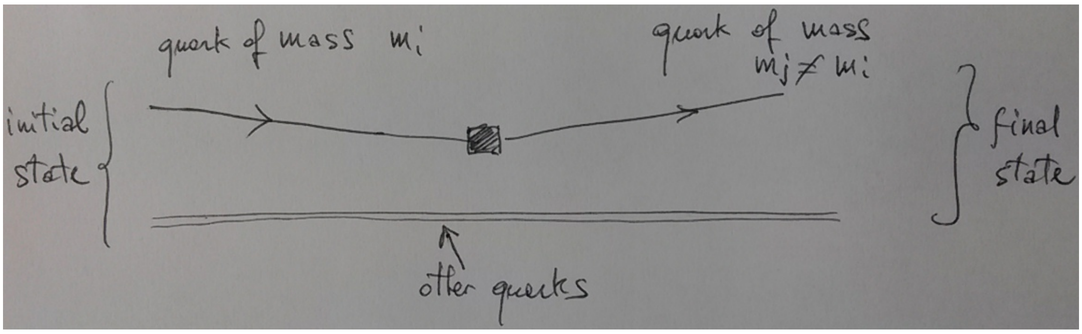

Figure 3: Schematic representation of a flavor-violating process.

can be paralleled to those entering the discussion of Rayleigh scattering. The crucial point is, in particular, that the energy scale m_W involved in the interaction represented by the black box is much larger (i.e. the associated distance m_W^{-1} much shorter) than the energy scale of the external states, of order $m_q = \max(m_i, m_j)$, in the same way as the atom energy scale a_0^{-1} is much larger than the photon energy E_γ. To make predictions in flavor physics it is therefore necessary to use lines of reasoning similar to that used in the previous section, i.e. it is necessary to build an EFT.

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Before going in more detail into flavor-physics interactions, we should address the question **why is flavor physics interesting** at all. The reason is that the initial and final quarks give rise to external states (the simplest ones known as mesons, bound states of a quark and an anti-quark, the binding due to strong interactions) that one can produce copiously at colliders, at relatively cheap costs. Their decay rates can thereby be accurately measured. Many of them can also be accurately calculated by the techniques of EFTs. *One can in this way probe 'indirectly' the high-energy interactions represented by the black box.* Indirectly means that one does not need to directly reach the energy scales (indicated by m_W) involved in the black-box interaction.

The rest of this lecture is devoted to understanding more closely how the process in fig. 3 can arise at all. To make the discussion self-contained, we will first make a short presentation of the elementary constituents of matter, and then of their flavor-violating interactions.

L2.1 Matter constituents and interactions

As mentioned, the process in fig. 3 involves quarks. Quarks and what else? And what are quarks at all? Quarks are among the 'basic' constituents of matter. Basic means here elementary, namely structure-less, or point-like, for what we know. The only matter constituents known so far are quarks and leptons.

Quarks: particles of electric charge $+2/3$ (up-type quarks: up, charm, top) or $-1/3$ (down-type quarks: down, strange, bottom). They also have intrinsic angular momentum, or spin, $1/2$. They are schematized in fig. 4. The different instances of quarks u, d, s, c, b, t are called 'flavors'. Their masses increase in the list u, d, s, c, b, t. The heavier flavors tend to decay to the lighter ones, so only u and d quarks, the lightest, can give rise to stable states.

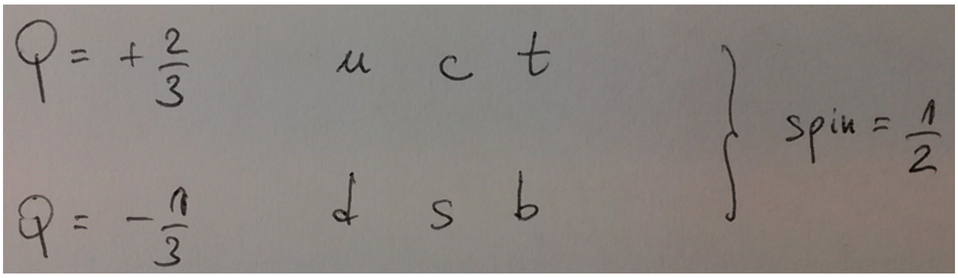

Figure 4: Quarks

For completeness, besides quarks there are leptons, which are entirely analogous to quarks apart from:

- different charge assignments: up-type leptons (neutrinos) have charge 0, down-type leptons (electron e, muon μ, tau τ) have charge -1;
- lepton masses are different than quark masses, and flavor by flavor they are smaller (e.g. $m_e < m_d$ etc.);
- quarks interact strongly, weakly and electromagnetically. Leptons do not interact strongly.

While also leptons have a vast flavor phenomenology, in these lectures we will focus on quarks.

Vector bosons. Strong, weak and electromagnetic interactions are represented by interactions where quarks meet with 'vector bosons', respectively gluons G (strong), massive vectors W, Z (weak) and photons A (e.m.).

Higgs boson. Within our current understanding, any elementary particle that has mass, receives it via its interaction with the Higgs boson. (Neutrinos are a possible exception, but this is irrelevant here.) The Higgs boson is described by a scalar field, the only field that is invariant under space-time transformations.

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In short, quarks have the following interactions:

$$\begin{array}{ll} \text{e.m.} & \bar{Q}\, A\, Q\ , \\ \text{strong} & \bar{Q}\, G\, Q\ , \\ \text{weak} & \bar{Q}\, W\, Q'\ , \bar{Q}\, Z\, Q\ , \\ \text{Higgs} & \bar{Q}\, H\, Q'\ , \end{array} \tag{2.1}$$

where the field $\bar{Q}$ creates a quark or annihilates an anti-quark, and the field Q does the opposite. Note that each interaction with one boson (A, G, W, Z, H) involves two fermions. This is in order to globally conserve charge and angular momentum. *Flavor physics arises from the interplay between the last two classes of interactions:* weak and with the Higgs. Let us see this in more detail.

L2.2 Flavor-violating interactions

Weak and Higgs interactions – Let us write down the quark-W interactions:

$$\mathscr{L}_{qqW} \;\propto\; \bar{U} W^{(-)} D \;+\; \bar{D} W^{(+)} U \;, \tag{2.2}$$

where D denotes either of d, s, b quarks and U either of u, c, t quarks. So for example the first interaction corresponds to the diagram in fig. 5, where an initial d yields a u and a W. The parentheses indicate the electric charges. Similarly, there are quark-quark-Z interactions:

$$\mathscr{L}_{qqZ} \;\propto\; \bar{U} Z^{(0)} U \;+\; \bar{D} Z^{(0)} D \;, \tag{2.3}$$

as well as quark-quark-Higgs interactions. Avoiding unnecessary details,[5] the latter interactions have the basic form

$$\mathscr{L}_{qqH} \;\propto\; \bar{U}_i (y_u)_{ij} H U_j \;+\; \bar{D}_i (y_d)_{ij} H D_j \;, \tag{2.4}$$

where y_u, y_d are two 3×3 complex matrices, and i, j label the flavor: $D_1 = d, U_2 = c$, etc.

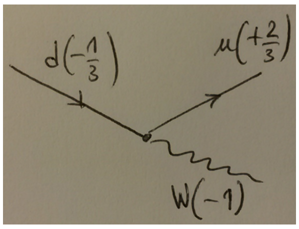

Figure 5: Example of quark-quark-W interaction.

This is the crucial difference with respect to the W, Z interactions in eqs. (2.2)-(2.3): qqW and qqZ interactions weigh equally quarks of different flavors, because of a 'gauge' symmetry similar to that in electromagnetism. On the other hand, no analogous symmetry is known for qqH, so the latter can have the most general couplings y_u and y_d, that weigh differently different combinations of quarks.

Quark masses – As mentioned, quarks receive their masses from their interactions with the Higgs, so mass terms must arise from eq. (2.4). Similarly as in the diagram of fig. 2, quark masses are defined at the Lagrangian level by terms of the kind $\bar{u} m_u u$, $\bar{d} m_d d$, etc.[6] Recalling that the Higgs is a scalar, we know how to obtain these mass terms from the interactions in eq. (2.4):

- Allow the Higgs field to take a non-null energy density even in the vacuum. We can do this only with the Higgs, because it is a scalar (the only fundamental scalar):

  $$H \to v + h \;, \tag{2.5}$$

  with v the vacuum density and h the fluctuations around this density. So, eq. (2.4) contains

  $$\mathscr{L}_{qqm} \;\propto\; \bar{U}_i (y_u)_{ij} v U_j \;+\; \bar{D}_i (y_d)_{ij} v D_j = \bar{U}_i (M_u)_{ij} U_j \;+\; \bar{D}_i (M_d)_{ij} D_j \;. \tag{2.6}$$

- Diagonalize the matrices M_u and M_d. Quark masses will then be:

  $$\hat{M}_{u,d} = \{\text{eigenvalues of } M_{u,d}\} \;, \tag{2.7}$$

  where

  $$\hat{M}_u = \text{diag}(m_u, m_c, m_t) \;, \qquad \hat{M}_d = \text{diag}(m_d, m_s, m_b) \;. \tag{2.8}$$

Diagonalization of $M_{u,d}$ – For any complex matrix M, there exist two unitary matrices X_L and X_R such that

$$X_L^\dagger M X_R = \hat{M} \tag{2.9}$$

with $\hat{M}$ diagonal and real.

[5] Like spin projectors and charge-conjugation operators for the Higgs.

[6] We associated to fig. 2 the non-relativistic kinetic energy term $\phi^* \, (p^2/2m) \, \phi$. Here terms like $\bar{u} m_u u$ are associated to the rest energy of the given particle, namely for the u quark $m_u c^2$, with $c = 1$.

Proof by explicit construction.

- Rewrite $M = HU$, with H Hermitian and U unitary (polar decomposition theorem, like writing a complex number z as $|z|e^{i\arg(z)}$).
- Let then $H = Z^\dagger \hat{H} Z$, with $\hat{H}$ diagonal and Z unitary (this decomposition exists by definition, since H is Hermitian).
- Then eq. (2.9) becomes $X_L^\dagger \underbrace{(Z^\dagger \hat{H} Z U)}_{M} X_R = \hat{M}$. Choosing the two unitary matrices X_L and X_R as $X_L = Z^\dagger$ and $X_R = U^\dagger Z^\dagger$ one sees that $\hat{M} = \hat{H}$.

Using eq. (2.9), we can rewrite the up-quark mass term in eq. (2.6) as (I suppress flavor indices for simplicity here)

$$\bar{U} M_u U = \bar{U} X_L \hat{M}_u X_R^\dagger U \ . \tag{2.10}$$

Therefore, we just need to redefine the up-quark fields as

$$U = X_R U' \ , \qquad \bar{U} = \bar{U}' X_L^\dagger \tag{2.11}$$

and perform analogous transformations for the D fields, and we'll get, for eq. (2.10)

$$\bar{U} M_u U = \bar{U}' \hat{M}_u U' \tag{2.12}$$

where, we recall, $\hat{M}_u$ is the diagonal matrix containing the u, c, t masses as entries (see the first of eqs. (2.8)). The primed quark fields are eigenstates of mass.

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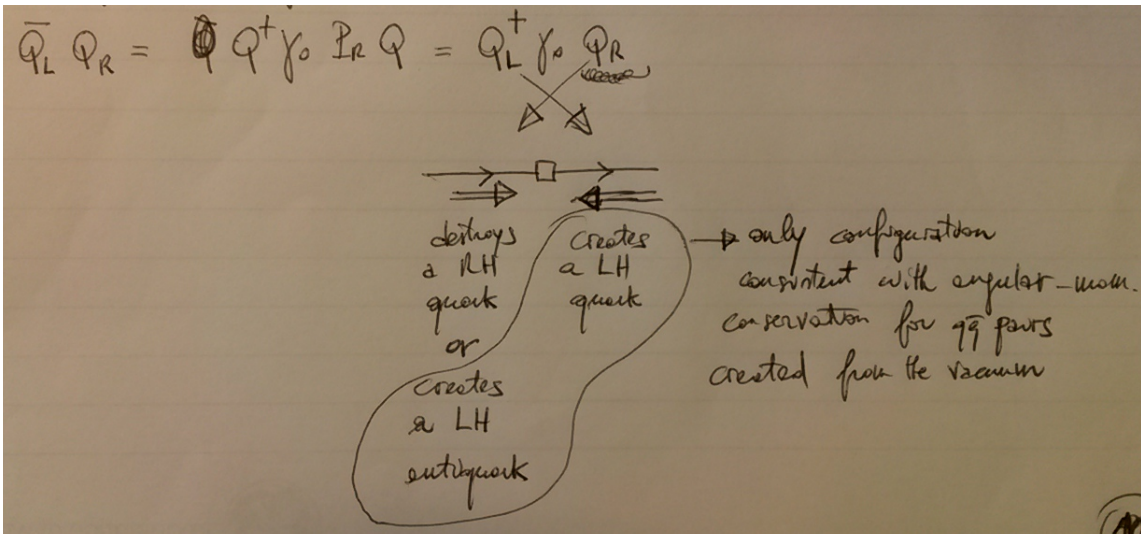

Figure 6: Spin configurations for quark-quark pairs in the interactions (2.6).

The careful reader may wonder whether we are allowed to redefine the fields U and $\bar{U}$ by two independent matrices, as in eq. (2.11). The answer is yes. The reason is the following. Recall that quarks have spin 1/2. This quantum number is such that U and D quarks get a further degree of freedom depending on whether their spin is oriented parallel or antiparallel to their direction of motion. These two possibilities are labelled as R (for right-handed) and L (for left-handed) respectively. The quark fields are then U_R, U_L, D_R, D_L. Each of them is an independent field.

It turns out that the quarks annihilated by U and those created by $\bar{U}$, respectively, in the interaction in eq. (2.6), need to have opposite projections of their spin along their direction of motion. A more in-depth motivation for this statement is depicted in fig. 6.

Flavor-changing interactions, finally –

- The interactions (2.6) are actually (we suppress the explicit indices i, j for simplicity)

$$\bar{U}_L(M_u)U_R \ + \ \bar{D}_L(M_d)D_R \tag{2.13}$$

because they are interactions with a scalar, and the argument given in fig. 6 holds. So, combinations are of the kind LR or RL;

- Conversely, interactions with vector bosons, like the W, Z, A, G will involve $\bar{Q}Q$ pairs in combinations LL or RR (or both).
- To diagonalize M_u, M_d in eq. (2.13), redefine:

$$U_L = X_L^{(u)} U_L' \ \text{ and } \ U_R = X_R^{(u)} U_R' \tag{2.14}$$

and analogous for the $D_{L,R}$ fields, this time with $X_{L,R}^{(d)}$ matrices. Then mass terms will be

$$\bar{U}_L' \underbrace{\left[\left(X_L^{(u)} \right)^\dagger M_u X_R^{(u)} \right]}_{\hat{M}_u} U_R' \ + \ \text{analogous for } D \text{ quarks} \ . \tag{2.15}$$

- What happens to the other interactions in eq. (2.1)? Strong, e.m. and weak interactions with the Z, that are all charge-neutral, involve quarks in the combinations $\bar{U}_L U_L$ or $\bar{U}_R U_R$ or the analogous with the D quarks. The redefinitions in eq. (2.14) will then yield products $(X_L^{(u)})^\dagger X_L^{(u)} = \mathbb{1}$ or $(X_R^{(u)})^\dagger X_R^{(u)} = \mathbb{1}$, hence they will have no effect.
- On the other hand, in quark interactions with the W, one will have

$$\bar{U}_L W^{(-)} D_L = \bar{U}_L' \underbrace{\left[\left(X_L^{(u)} \right)^\dagger X_L^{(d)} \right]}_{V \neq \mathbb{1}!} W^{(-)} D_L' \ . \tag{2.16}$$

The matrix $V \equiv \left(X_L^{(u)} \right)^\dagger X_L^{(d)}$ is a 3×3 unitary matrix, and is in general not diagonal.

This matrix is known as the Cabibbo-Kobayashi-Maskawa matrix. If it were diagonal, the interaction (2.16) would allow only 'vertical' transitions (see fig. (4)): $d \leftrightarrow u$, $s \leftrightarrow c$ and $b \leftrightarrow t$. Instead, because of its off-diagonal entries, also any 'oblique' transition in this figure is allowed, e.g. $d \leftrightarrow t$ or $s \leftrightarrow u$.

- As a result, $\bar{U} W^{(-)} D$ interactions will be of the form depicted in fig. 7, namely interactions with any down-type quark as initial state and any up-type quark as final state (or viceversa), with coupling strengths ruled by the V-matrix entries.
- *All of flavor-physics phenomena involving quarks arise from the basic interaction in fig. 7.* In the next section we will play with this interaction to build actual flavor processes.

L2.3 Examples of flavor-violating processes

Example 1 – A first example of flavor-physics phenomena that can be built out of the basic interaction in fig. 7 is depicted in fig. 8. In this reaction, an initial-state b-quark undergoes an interaction where two 'virtual' particles, a W and a t quark, are emitted and reabsorbed, and the final-state quark is not a b, but an s. Namely the initial- and final-state down-type

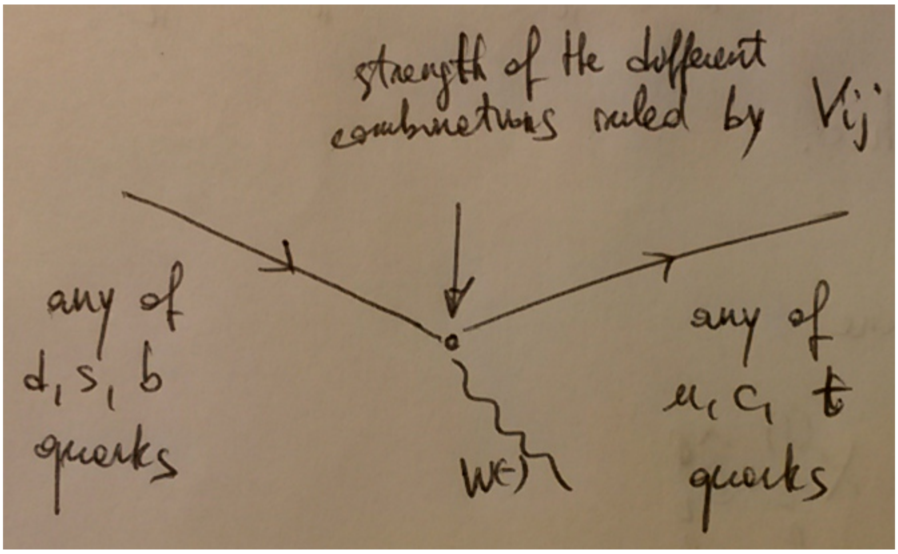

Figure 7: The interaction responsible for all flavor processes involving quarks.

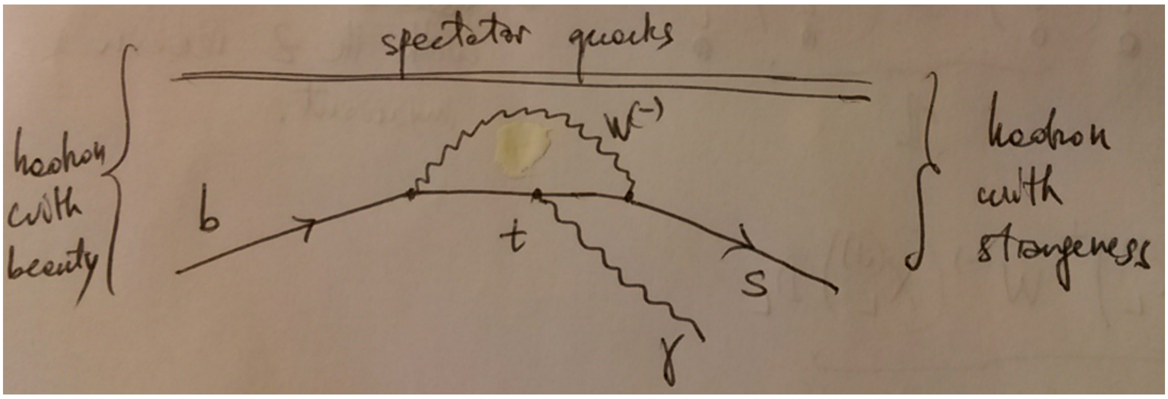

Figure 8: Diagram for the process $b \to s\gamma$.

quarks have different masses, namely flavors. The virtual top (or also the W) can emit a photon, also detected in the final state. The initial b and the final s will form bound states with other 'spectator' quarks, giving rise to hadrons with 'beauty' (containing the quark b) or with 'strangeness' (containing the quark s). An example is $B \to K^* \gamma$, very well measured.

The peculiarity of this kind of processes is that they are *electrically neutral* (in the above example the b and the s have the same charge $-1/3$) but they involve different 'flavors' (i.e. different masses) in the initial and final states. These processes are called **flavor-changing neutral currents** (FCNC). They can occur only at the quantum level (= involve the emission and reabsorption of 'virtual' states, as in fig. 8). In the Standard Model (SM), they are *forbidden at the classical level.* As a result, these processes are generally very rare, and excellent to probe possible effects beyond the SM ones.

Example 2: particle-antiparticle oscillations – Another, even more peculiar flavor phenomenon is depicted in fig. 9. According to this diagram, one can observe a meson containing a $d, \bar{s}$ pair, known as K^0 meson, oscillate into its own anti-particle, containing namely a $\bar{d}, s$ pair, and known as $\bar{K}^0$. Again, the process involves the emission and reabsorption of virtual t and W particles. Such closed diagrams are known as 'loops'. This process will be the topic of lecture 3.

Why these processes are calculable – An important point about processes like in fig.

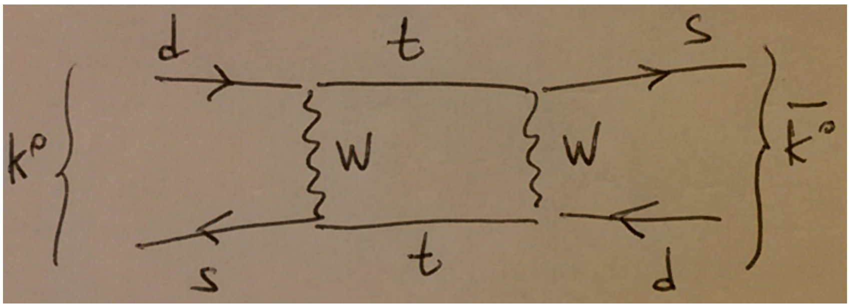

Figure 9: Oscillation of a K^0 meson into its antiparticle, the $\bar{K}^0$.

8 and 9 is the fact that the 'virtual' particles in the loops are much, much heavier than the external particles. For example, in $K^0 - \bar{K}^0$ mixing, one has:

$$m_d = 8 \text{ MeV} , \quad m_s = 100 \text{ MeV} ,$$
$$m_t = 175 \cdot 10^3 \text{ MeV} , \quad m_W = 80 \cdot 10^3 \text{ MeV} , \tag{2.17}$$

where the MeV is a unit of energy, or mass (recall $E = mc^2$, with $c = 1$) convenient in particle physics: 1 MeV $\simeq 2 \times 10^{-30}$ kg!

Therefore, at the energy scale (or 'speed') of the external particles (the d and s, or actually the K^0 and $\bar{K}^0$, which are the observable states), the internal particles t and W are motionless to an excellent approximation (try and compare with the examples of the train or of Rayleigh scattering).

As a consequence, the 'box' loop in fig. 9 can be approximated with a point. The plausibility of this situation is depicted in fig. 10.

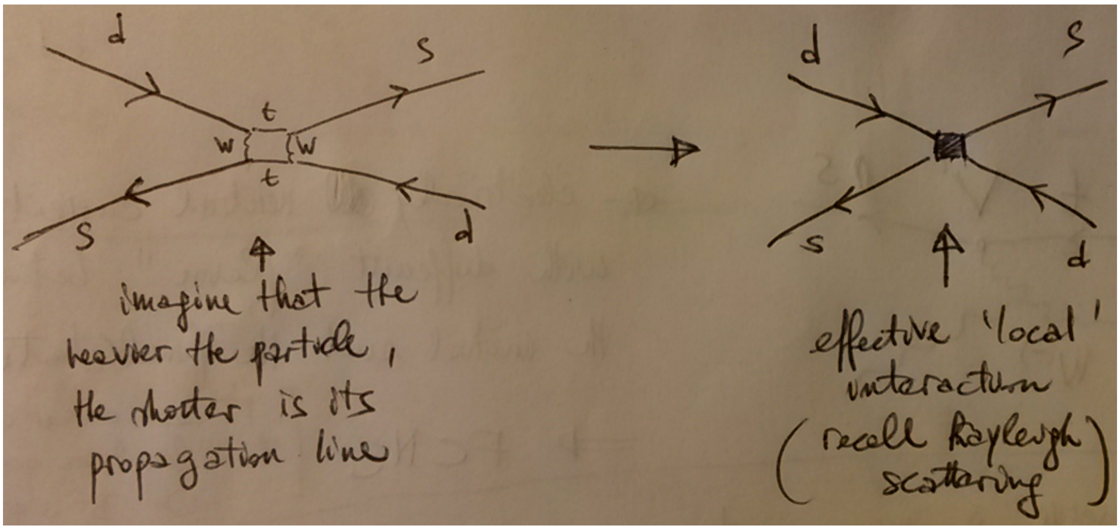

Figure 10: Modeling $K^0 - \bar{K}^0$ oscillations as a point-like interaction.

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Take-home message – Flavor processes are calculable *if very massive dynamics in the loops can be bundled in 'local' or point-like interactions.* The overall strength of these interactions can in these cases be either calculated, or measured from experimental processes.

On the other hand, non-local loops are very, very difficult objects to calculate in quantum field theory.

Fermi theory – The above reasoning is exactly the one that was originally followed by Enrico Fermi, who pioneered the understanding of weak interactions.

Example: Fermi's theory of β decay (1933, as he was professor at Rome University). To understand the radioactive reaction $n \to pe^-\bar{\nu}$, he wrote down an 'effective' interaction with 4 fermions at the vertex, exactly as the rightmost diagram in fig. 10. Thereafter, he determined the overall coupling strength from data. We modernly understand the Fermi interaction as depicted in fig. 11.

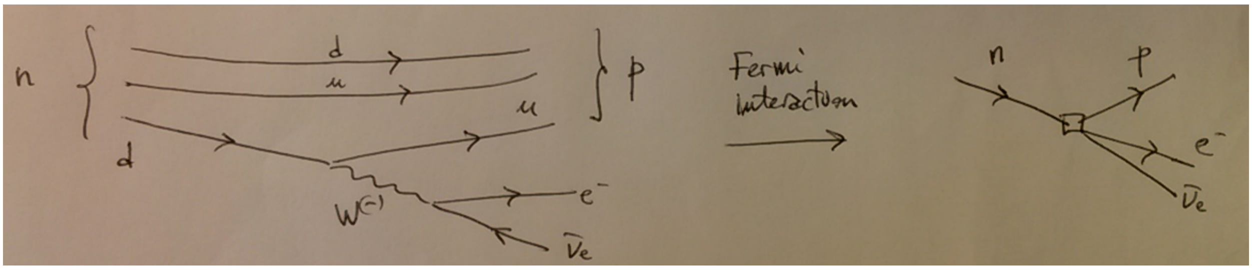

Figure 11: The β-decay of a neutron within the Fermi theory.