Introduction to Flavor Physics
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Lecture 1: Introduction to EFTs.

Flavor physics rests on the basic idea of an **effective field theory**, which in turn is one of the most basic guides in modeling physics systems. In this lecture we introduce this idea. We will proceed by examples, introducing the minimal necessary notation only when it's needed.

We will introduce the idea of an EFT by the explicit example of **Rayleigh scattering**, namely elastic scattering of visible light off atoms. Rayleigh scattering explains why the sky is blue. (Disclaimer: technically, what happens in Rayleigh scattering is that the atom gets polarized by the e.m. wave and emits like a dipole. Therefore, one can make an entirely classical derivation of this effect. We want instead to use the tools and the formalism of particle physics. Therefore, we will need a small detour where we will introduce all the basic concepts. The resulting derivation will be much more fun than the classical one.)

L1.1 Introduction

In order to approach the problem using quantum-physics tools, we need some basic concepts:

- **particles**, and how they are mathematically described (= quantum fields),
- **interactions**, namely products of fields satisfying certain requirements. These products of fields appear in 'Lagrangian' functions, akin to the analogous objects describing the dynamics of classical-mechanical systems,
- **relevant interactions**.

The last concept is at the core of the idea of EFTs. It consists in *identifying the characteristic energy or distance scale of a problem*, and accordingly writing down a sensible set of interactions describing that problem, while discarding irrelevant details (= effects at scales widely different than the characteristic one, for example, internal-structure dynamics).

For example, if we are asked to evaluate the amount of heat dissipated by a TGV stopping from full speed, all we need is its kinetic energy $E_{\text{train}} = MV^2/2$, with M its mass and V its average speed. We do not need details about its internal structure, such as the number of people going back to their seats during the braking. In fact, one can estimate both effects using

$$M \simeq 250 \text{ tons} = 2.5 \cdot 10^5 \text{ kg} \quad \text{(train mass)}$$
$$V \simeq 250 \text{ km/h} = \frac{2.5 \cdot 10^5}{3.6 \cdot 10^3} \frac{\text{m}}{\text{s}} = 0.7 \cdot 10^2 \frac{\text{m}}{\text{s}} \quad \text{(average train speed)}$$

$$m \sim 70 \text{ kg} \quad \text{(average human weight)}$$
$$v \sim 5 \text{ km/h} = \frac{5 \cdot 10^3}{3.6 \cdot 10^3} \frac{\text{m}}{\text{s}} = 1.4 \frac{\text{m}}{\text{s}} \quad \text{(average walking speed)}$$

With respect to the train kinetic energy, the ‘internal structure’ effect is a correction of order (we assume $N \sim 20$ for the number of people going back to their seats)

$$\frac{mv^2 \times N}{MV^2} = \frac{0.7 \cdot 10^2 \cdot 1.4^2 \times 20}{2.5 \cdot 10^5 \cdot (0.7 \cdot 10^2)^2} \simeq 2 \times 10^{-6} \ . \tag{1.1}$$

Other checks:

$$N \times E_{\text{human}} \simeq 20 \times \frac{1}{2} \cdot 70 \cdot 1.4^2 \ \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \simeq 1.4 \ kJ$$
$$E_{\text{train}} \simeq \frac{1}{2}(2.5 \cdot 10^5)(0.7 \cdot 10^2)^2 \ \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} = 6 \cdot 10^8 \ J = 600 \ MJ \tag{1.2}$$

which is correct, since for example vehicles energies are in the MJ range.

So, calculating the train kinetic energy including the ‘internal-structure effect’ of people going back to their seats is like giving road-sign distances (usually in km) with mm accuracy: useless for every practical purpose.

In short, the train can be approximated as a ‘material point’. We are very familiar with this approximation in classical mechanics, e.g. it is the same approximation used in describing the motion of the moon around the earth, by namely treating both as point-like objects.

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Here we have encountered the most elementary example of the idea of **separation of scales** when describing a physics problem. The idea of an effective theory is, correspondingly, the idea of *describing phenomena with finite accuracy*, using their characteristic energy or distance scale to identify the relevant interactions.

Let us now put this idea to work in a quantum-physics system, considering a concrete example, where again we will introduce only the minimal necessary formalism as needed.

L1.2 Rayleigh scattering

Problem: the diffusion of light off the atmosphere, and why the sky looks blue.
Let’s analyze the sentence:

- ‘diffusion of light’: photons with wavelength in the visible: 400÷700 nm = $4 \div 7 \times 10^{-7}$ m.
- ‘off the atmosphere’: off atoms. Atom size ~ 1 Å $= 10^{-10}$ m.

The atoms are much smaller than the photon wavelength: their internal structure is not resolved, and they can be treated as point-like objects.

Writing down the interaction

In particle-physics, interactions can be visualized (and in fact also mathematically modeled) using the very intuitive tool of Feynman diagrams.

Feynman diagrams: imagine a process, draw its diagrams, use them to write down the mathematical form of the quantum-mechanical (QM) amplitude for the process.

In our case the diagram would be as depicted in fig. 1.

We need to now write down the QM amplitude corresponding to this diagram. How to mathematically represent particles? In particle-physics, particles are described by **fields**. They are functions of the space-time coordinates, describing the *amplitude* of finding that particle in a given point in space-time. From amplitudes, one can calculate the quantum-mechanical probability $\mathcal{P}$ of a given process as $\mathcal{P} = |\text{amplitude}|^2$. The quantum-mechanical probability is then the analogue of a wave intensity = $|\text{wave amplitude}|^2$ in classical wave mechanics.

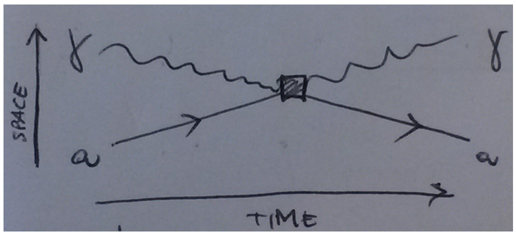

Figure 1: Scattering of photons (γ) off atoms (a).

Why the field viewpoint

Particle-physics processes are understood (and calculated) using *quantum field theories* (QFTs). QFTs put together physics at very *small scales* (necessitating quantum mechanics) and physics at *high energies* (necessitating relativity). So, QFT marries quantum mechanics with special relativity, and is the mathematical framework for relativistic QM. In relativistic processes there is no conservation of the particle number (because energy can turn into mass and viceversa) and one needs to keep track of the event sequence (causality). Hence the necessity of the 'field' viewpoint.

Very basics on Lagrangians and fields

Interactions, like the one in fig. 1, are built out of products of fields. Interactions are terms of the Lagrangian function. The latter, similarly as in classical mechanics, describes the dynamics of the given physical system.

Lagrangian – In classical mechanics, one introduces the Lagrangian as $S = \int \mathcal{L} dt$, with S the action. The dynamics of the system is obtained by the equations of motion that minimize the action. In QFT the integral is performed over space-time coordinates, d^4x, rather than just over time. Hence $S = \int \mathscr{L} d^4x$. The space-time Lagrangian density is a function of the fields describing the system and their first derivatives: $\mathscr{L} = \mathscr{L}(\phi, d\phi/dx^\mu)$, with μ an index labeling the four space-time coordinates. The symbol ϕ denotes collectively the fields, that are just the 'quantized' analogue of waves in classical mechanics, namely space-time distributions of a certain measurable quantity, like charge, or spin.

Fields – In classical mechanics, we can represent fields in coordinate space as the Fourier transform of the corresponding fields, or amplitudes, in momentum space:

$$\phi(x) \propto \int dp \left(a(p) e^{-ixp} + a^*(p) e^{ixp} \right) , \tag{1.3}$$

with $a(p)$ the amplitude for the wave to have momentum p. Quantized fields are 'similar', but for the fact that a are 'quantized', namely they create or absorb one particle with that given momentum.

Note that xp has the dimensions of an action, which is energy × time. Since the 'quantum of action' is a universal constant, $\hbar$, one can choose physics units where $\hbar = 1$ (i.e. it is treated as dimensionless). One can do the same with the speed of light in the vacuum: $c = 1$.[1] In these units $[p] = [E] = \text{mass} = \text{length}^{-1}$, and $[x] = [t] = \text{length}$.

[1] Note that, since $c \simeq 3 \cdot 10^8$ m/s, taking $c = 1$ operatively just means that, if I chose the second as unity

Writing down the interaction

The atom corresponds to a field with zero charge and angular momentum: a scalar field, indicated as a in fig. 1. What about the photon? The photon is a 'quantum' of electromagnetic field. But what combination of the E and B fields is good to represent the photon in our problem? We need to start from the two unsourced Maxwell's equations:

$$\begin{aligned} \vec{\nabla} \cdot \vec{B} &= 0 \, , \\ \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \, , \end{aligned} \tag{1.4}$$

which, as we know, admit the solution

$$\begin{aligned} \vec{B} &= \vec{\nabla} \times \vec{A} \, , \\ \vec{E} &= -\vec{\nabla}\phi - \frac{\partial}{\partial t}\vec{A} \, , \end{aligned} \tag{1.5}$$

with $\vec{A}$ the vector potential and ϕ the scalar potential. Let's look at these two equations. One has for example $E_x = -\partial_x \phi - \partial_0 A_x$, and this makes it natural to identify $\phi \equiv A^0$, the zeroth component of the four-vector

$$A^\mu \equiv \begin{pmatrix} \phi \\ \vec{A} \end{pmatrix} \, . \tag{1.6}$$

So, in relativistic notation[2], the $\vec{E}$ components become

$$E^i = -\partial^0 A^i + \partial^i A^0 = -F^{0i} \, , \tag{1.7}$$

and we are tempted to define the 'e.m. field tensor'

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu \, , \tag{1.8}$$

with $\partial^\mu \equiv \frac{\partial}{\partial x_\mu}$. What do we get for F^{ij}, with $i, j \neq 0$?

$$F^{ij} = \partial^i A^j - \partial^j A^i = -\partial_i A^j + \partial_j A^i = -\epsilon^{ijk} B^k \, , \tag{1.9}$$

where I introduced the antisymmetric symbol ϵ^{ijk}, with $\epsilon^{123} = 1$. The last equality in eq. (1.9) follows by comparison with the first of eqs. (1.5). We therefore see that $F^{\mu\nu}$ bundles together all the components of the electric and the magnetic fields as

$$F^{\mu\nu} \equiv \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ & 0 & -B_z & B_y \\ & & 0 & -B_x \\ & & & 0 \end{pmatrix} . \tag{1.10}$$

Note that the entries below the diagonal are minus the corresponding ones above the diagonal, because $F^{\nu\mu} = -F^{\mu\nu}$, by its definition in eq. (1.8).[3]

of time, then my unity of length would be 10^8 m. In this way measurements of length and measurements of time can be identified with one another, because length = constant × time.

[2] According to this notation, for $i = 1, 2, 3$ A^i denotes the ith component of $+\vec{A}$, whereas A_i denotes minus the same quantity; ∂_i denotes the ith component of $+\partial/\partial\vec{x}$ and ∂^i denotes minus the same quantity. For $i = 0$, the notation is analogous, but for the fact that there are no minus signs around.

[3] It is easy to check (exercise) that the two unsourced Maxwell's equations (1.4) can be written as

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \, , \quad \text{with} \ \ \tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \, , \tag{1.11}$$

Note that the solution in eq. (1.5) is not unique:

$$\begin{aligned} \vec{A}' &= \vec{A} - \vec{\nabla}\alpha(\vec{x}, t) = \vec{A} + i e^{-i\alpha} \vec{\nabla} e^{+i\alpha} , \\ \phi' &= \phi + \frac{\partial \alpha}{\partial t} = \phi - i e^{-i\alpha} \frac{\partial}{\partial t} e^{+i\alpha} , \end{aligned} \tag{1.14}$$

with α an arbitrary function of space-time coordinates, would be equally good solutions of eqs. (1.5). In four-dimensional notation, these 'gauge' transformations become simply

$$A'^{\mu} = A^{\mu} + \partial^{\mu} \alpha . \tag{1.15}$$

In physical, measurable quantities, the dependence on the above gauge function must drop. [Gauge dependence is only a redundancy due to the way we solve Maxwell's equations – through the scalar and vector potentials.]

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Since the atom is electrically neutral, it can only couple to $F^{\mu\nu}$, not to A^{μ} individually. So, the correct building block to describe the photon field is a combination of $F^{\mu\nu}$. This combination must be invariant under space-time transformations (= relativistic invariance). The latter is achieved by the field combination $F^{\mu\nu} F_{\mu\nu}$, with all indices saturated[4]. Quantities with all indices saturated are invariant under space-time transformations in the same way as the scalar product $\vec{u} \cdot \vec{v} \equiv u_i v_i$ ($i = 1, 2, 3$ or x, y, z), with $\vec{u}$ and $\vec{v}$ two spatial vectors, is invariant under space rotations, represented by orthogonal matrices.

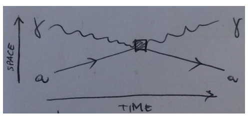
Hence, the atom-photon interaction must be of the form

$$\mathscr{L}_{\rm int} \propto \phi^* \phi \, F^{\mu\nu} F_{\mu\nu} . \tag{1.16}$$

Let us look at fig. 1. In the interaction in eq. (1.16), the field ϕ describes the atom approaching the interaction point (squared box in the figure), the field ϕ^* describes the atom leaving the interaction point, and each of the two powers of $F^{\mu\nu}$ describes the photon field (approaching and respectively leaving the interaction point).

The QM probability $\mathcal{P}$ for the atom-photon scattering is then

$$\mathcal{P} \propto \left| \quad \right|^2 . \tag{1.17}$$



Now, to understand why the sky is blue, we need to work out the dependence of the scattering probability in eq. (1.17) on the photon energy. This is easy to work out. Recall from

and that the two sourced Maxwell's equations

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} , \\ \vec{\nabla} \times \vec{B} &= \frac{\vec{j}}{\epsilon_0} + \frac{\partial \vec{E}}{\partial t} , \end{aligned} \tag{1.12}$$

become

$$\partial_{\mu} F^{\mu\nu} = j^{\nu} / \epsilon_0 , \quad \text{with} \ \ j^{\mu} \equiv \begin{pmatrix} \rho \\ \vec{j} \end{pmatrix} . \tag{1.13}$$

[4] $F^{\mu\nu} \tilde{F}_{\mu\nu}$ is an equally good field combination. For simplicity we will however drop this term in our discussion, as this term leads to the very same conclusions.

definition (1.8) that $F^{\mu\nu}$ involves derivatives of the four-vector A^μ. The latter obeys a plane-wave representation entirely similar to eq. (1.3). Therefore:

$$F^{\mu\nu} \propto \partial^\mu e^{ipx} \propto E_\gamma \; . \tag{1.18}$$

As a consequence, the atom-photon diffusion probability will depend on E_γ^4, namely photons with higher energy (close to the blu color) will be diffused much more than those with lower energy (close to the red). This is why the sky looks blue.

Embellishments

In eq. (1.16) we have written a proportionality relation, we have namely omitted to specify the coupling strength. We can actually work it out very simply by just finding its mass dimensions. To this end, let us first rewrite $\mathscr{L}_{\rm int}$ in eq. (1.16) making explicit its coupling strength C

$$\mathscr{L}_{\rm int} = C \phi^* \phi \, F^{\mu\nu} F_{\mu\nu} \; . \tag{1.19}$$

We know that, since the action is dimensionless (with $\hbar = 1$), $[\mathscr{L}] = \text{mass}^4$. Furthermore, $[F^{\mu\nu}] = \text{mass}^2$. What about $[\phi]$? We can work out the a dimension from the Feynman diagram representing the free propagation of the atom, depicted in fig. 2. We know that the atom propagates at non-relativistic speeds, so the diagram in fig. 2 must be of the form $\mathscr{L}_{\rm prop} = \phi^* \, (p^2/2m) \, \phi$, where again ϕ and ϕ^* represent the atom at the beginning and at the end of the propagation path. Since $[\mathscr{L}_{\rm prop}] = \text{mass}^4$ and $[p^2/2m] = \text{mass}$, it follows that $[\phi] = \text{mass}^{3/2}$. Now we have all the ingredients to work out the mass dimension of ϕ. We know that $[C][\phi]^2[F^{\mu\nu}]^2 = \text{mass}^4$, and that $[\phi]^2[F^{\mu\nu}]^2 = \text{mass}^7$. It follows that $[C] = \text{mass}^{-3} = \text{length}^3$. The only length scale in the problem (apart from the photon wavelength, present in $F^{\mu\nu}$) is the atom size a_0, of the order of 10^{-10} m. So the final behavior of the diffusion probability is

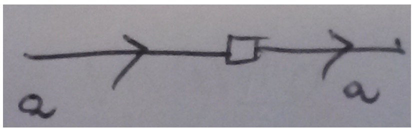


Figure 2: Diagram for the free propagation of an atom a.

$$\mathcal{P} \sim a_0^6 E_\gamma^4 \tag{1.20}$$

which turns out to be right!

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To recapitulate:

- By general considerations of symmetry and mass dimensions we got the correct photon-energy dependence of the diffusion probability.
- By a simple dimensional argument we also got right the overall normalization: a_0^6. That the scattering probability grows as a power of the atom size is what one expects on the basis of geometrical considerations.
- The whole argument is fully consistent for photon energies much smaller than the inverse atom size a_0^{-1}, which is our case, since we wanted to describe photons in the visible.

- Note that, if the photon wavelength had on the other hand been comparable with the atom size, the photon would have been able to resolve the internal constituents of the atom itself. In this case, new scales (those of the internal atom constituents) would have entered the game, and the question would have arisen, which of these scales determines the size of the coupling C. The answer is that, unless forbidden by dynamical or symmetry reasons, all of these scales contribute to C.