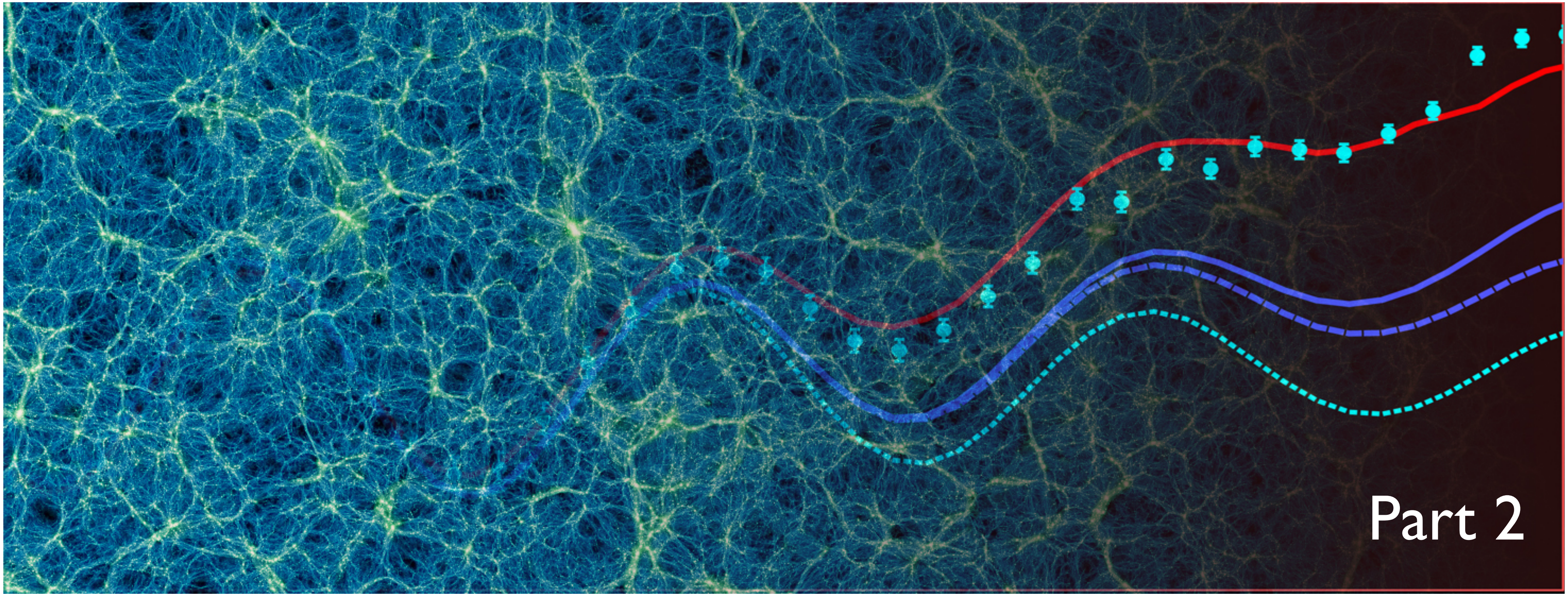
Cosmology and the Large Scale Structure of the Universe



Emiliano Sefusatti
Astronomical Observatory of Brera
National Institute for Astrophysics

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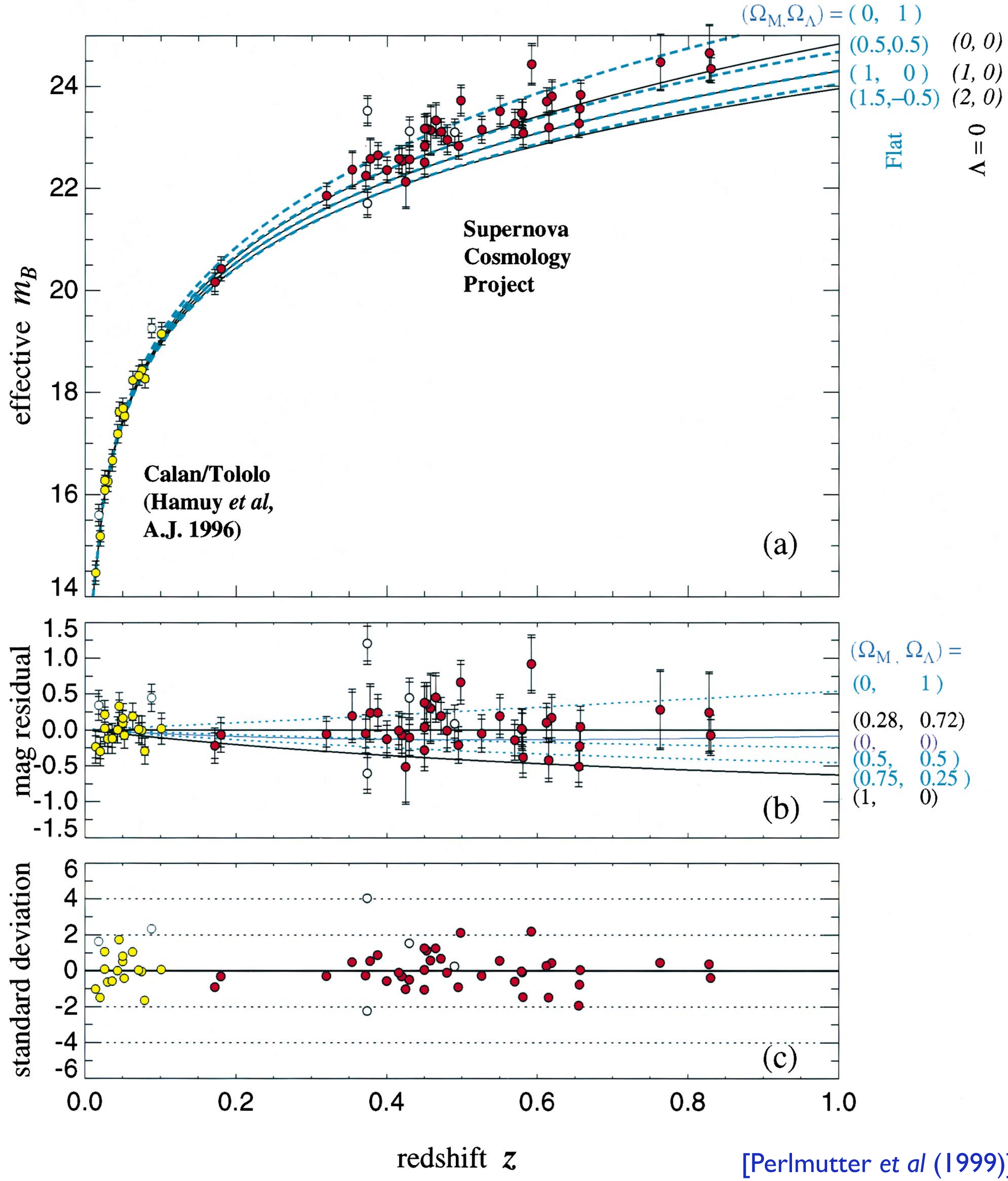
Addendum

$$m - M = 5(\log_{10} D_L - 1)$$

$$D_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$



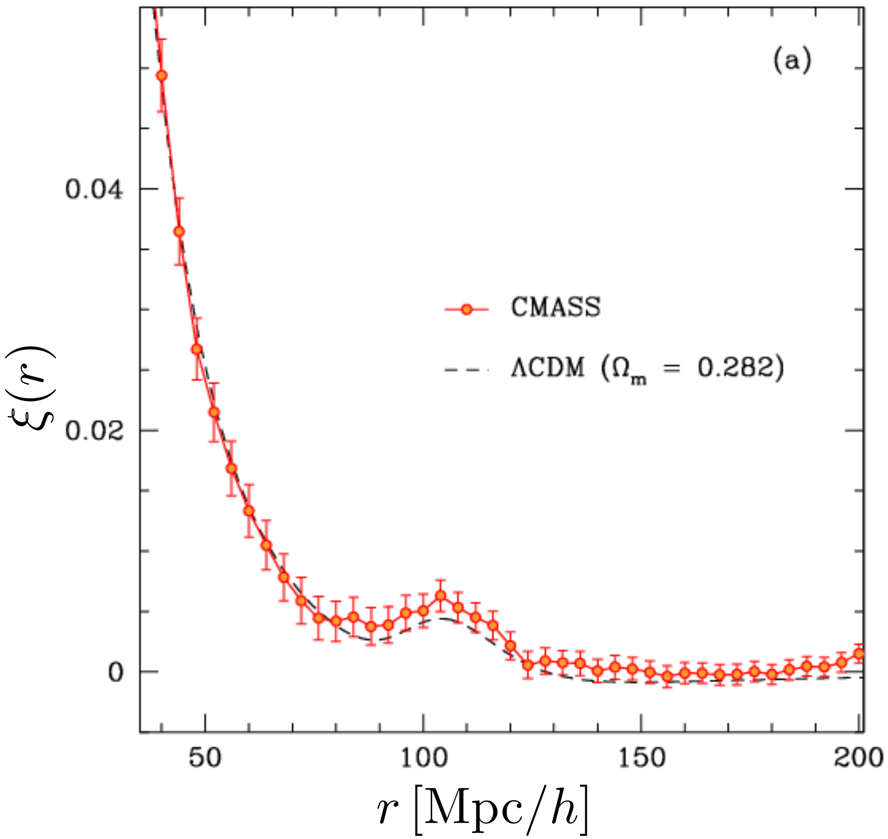
[Perlmutter *et al* (1999)]

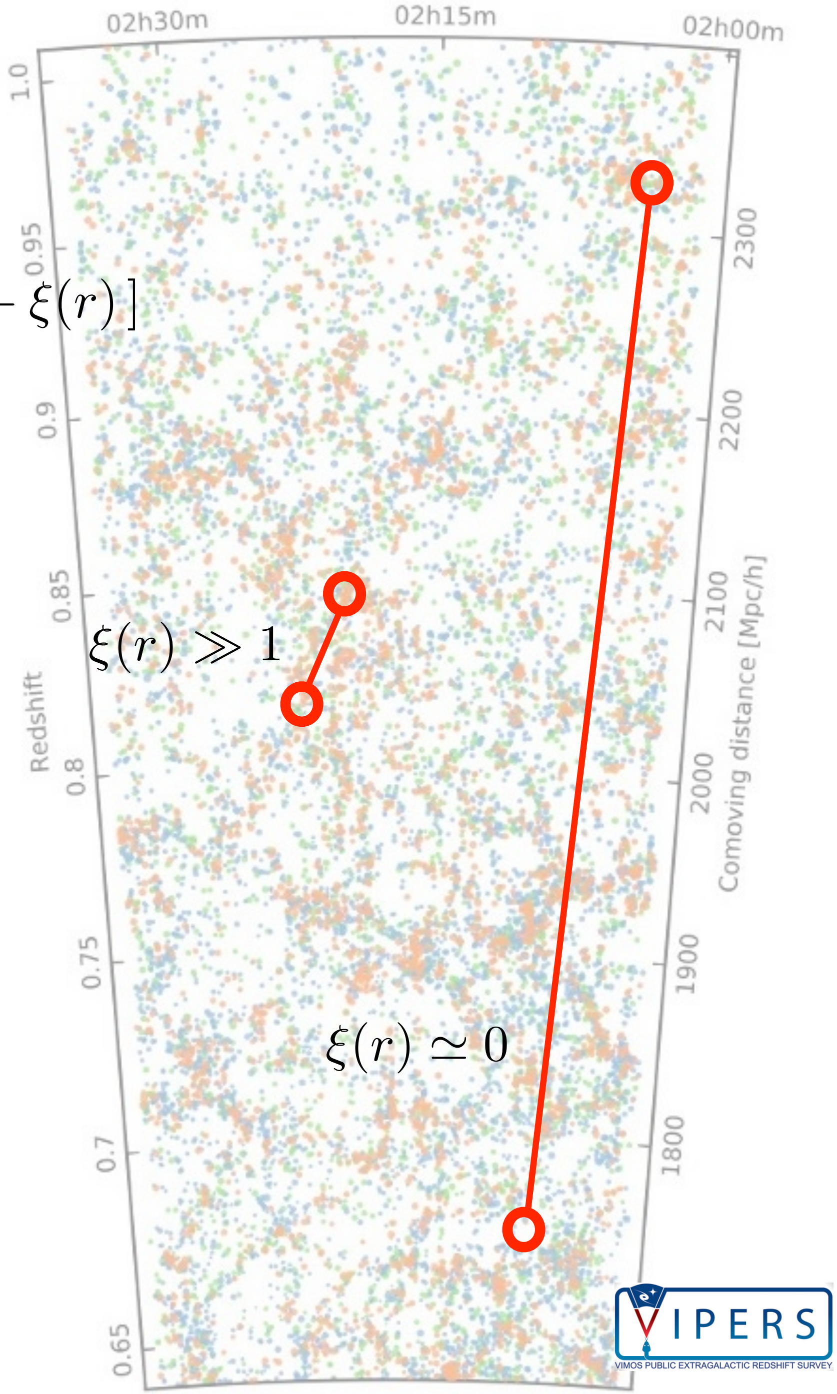
Recap

The galaxy 2-point function is the excess probability of finding two galaxies in the volume elements dV_1 and dV_2

$$dP = dV_1\, dV_2 \,\langle\, n_g(\vec{x}_1)\, n_g(\vec{x}_2)\,\rangle \;= dV_1\, dV_2\, \bar{n}_g^2\; [1 + \xi(r)\,]$$

where $\quad n_g(\vec{x}) \;\equiv\; \bar{n}_g\,[1 + \delta_g(\vec{x})]$



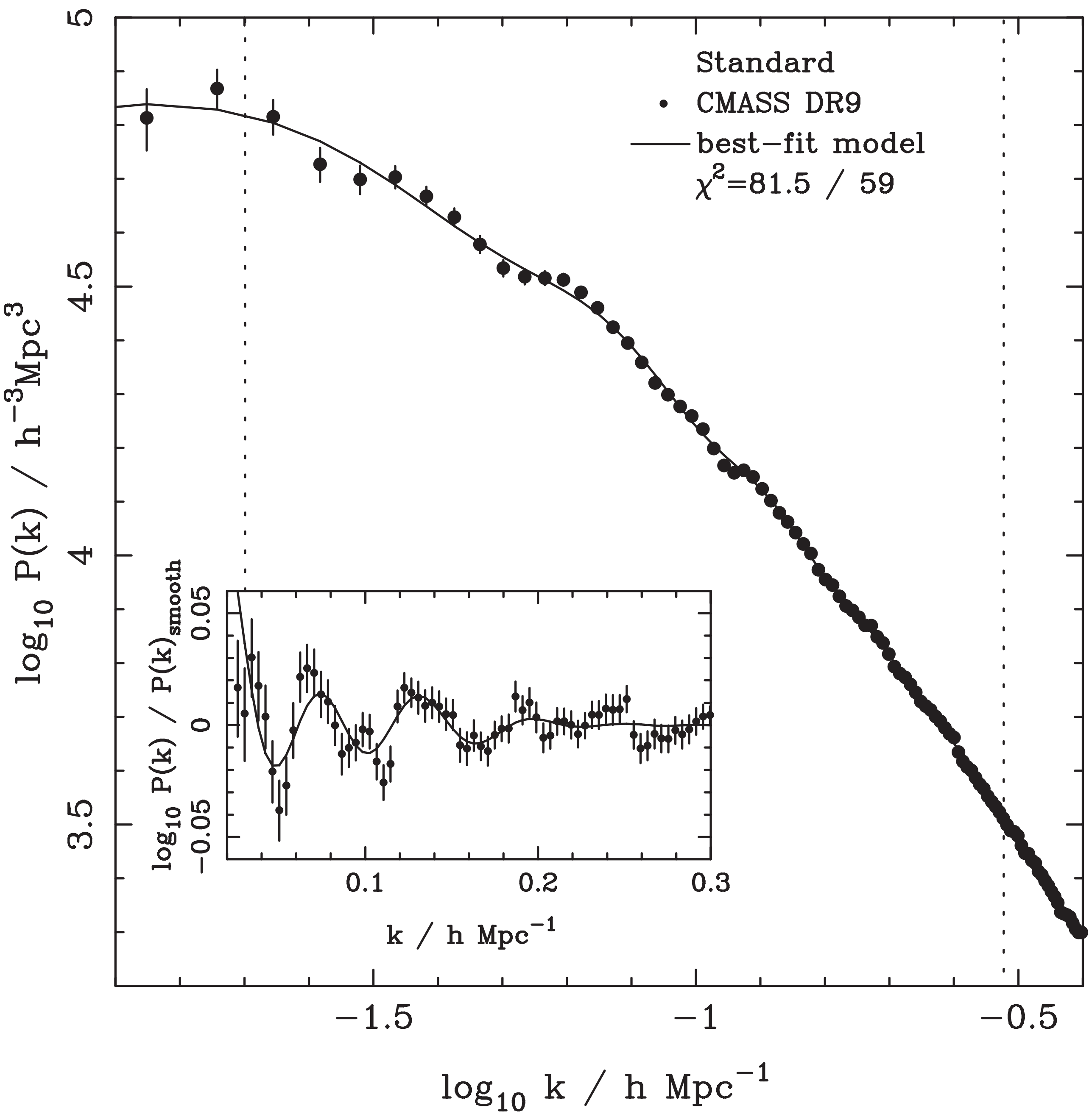


Recap

The power spectrum is a measure of the amplitude of perturbations as a function of scale …
… and the Fourier Transform of the 2PCF

$$P(k) = \int \frac{d^3x}{(2\pi)^3}\, e^{i\,\vec{k}\cdot\vec{x}}\xi(x)$$

The power spectrum is what we want to predict



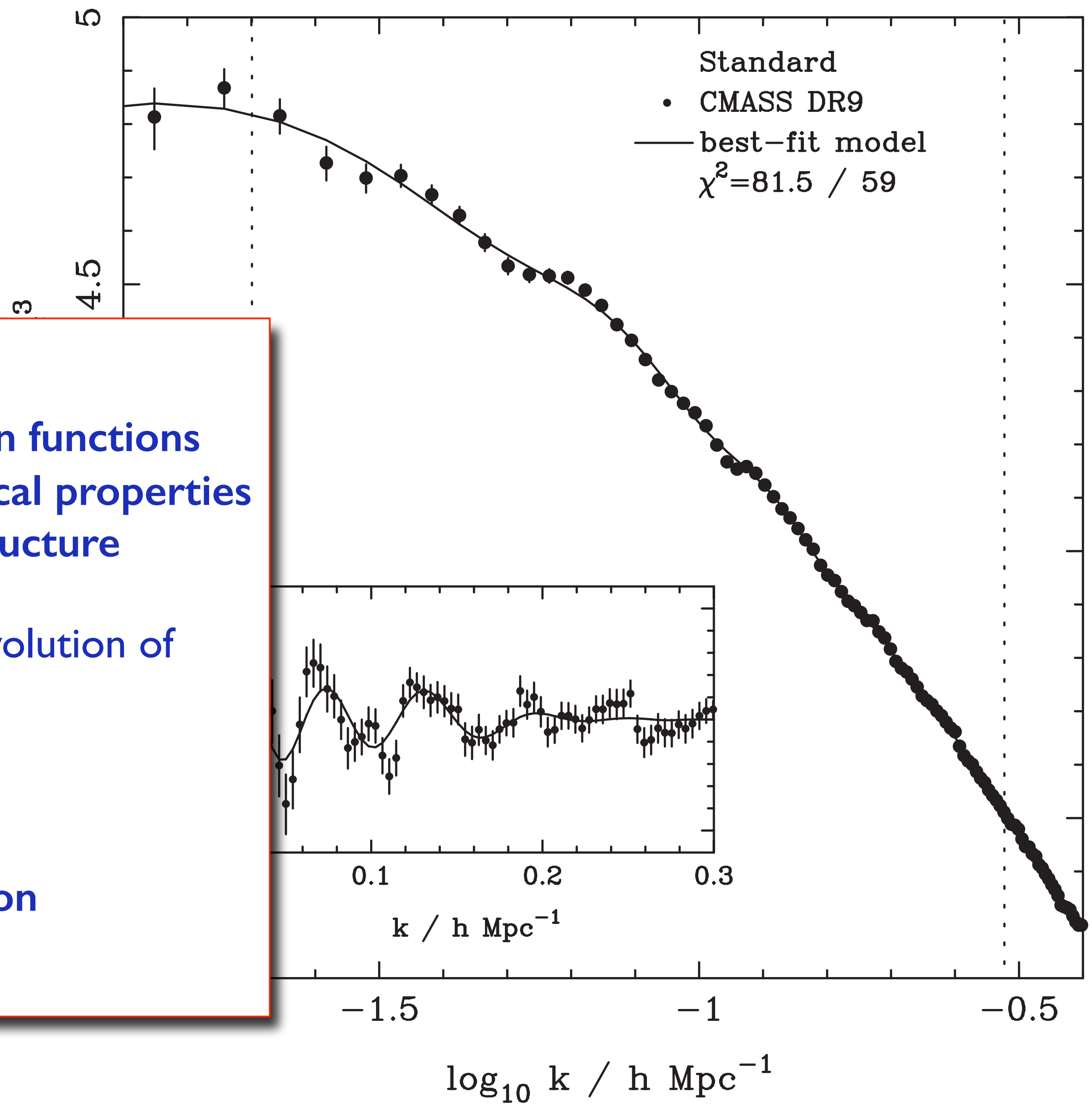
Today's goal

Goal:
predict the correlation functions describing the statistical properties of the Large-Scale Structure

for this we study the evolution of

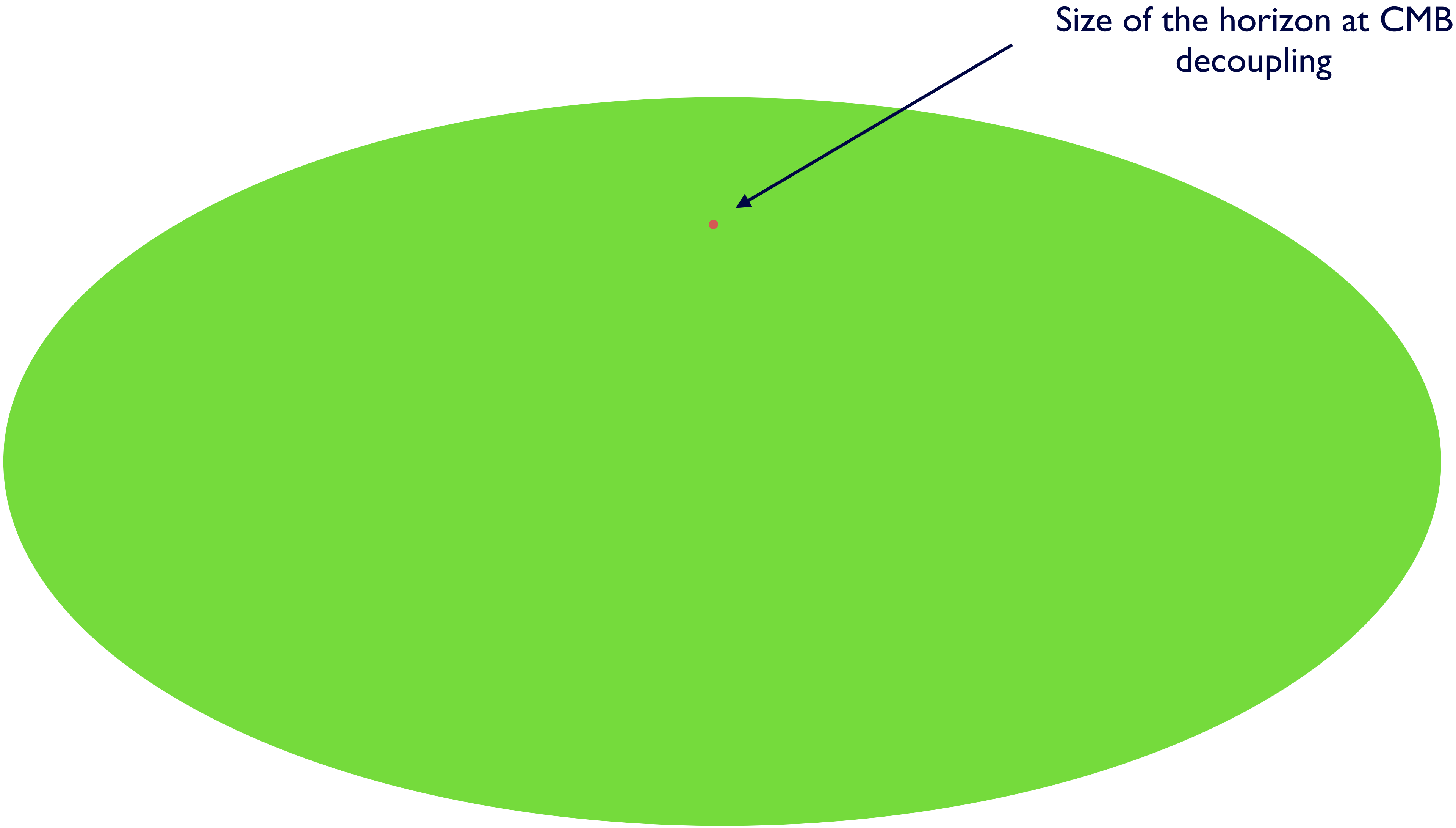
$$\delta_{\vec{k}}(t)$$

We need:
1. **Equations of motion**
2. **Initial conditions**



Evolution of matter perturbation:
Initial Conditions
Part I: Inflation

Big Bang … Problems



The Horizon Problem:

In the CMB we observe a large number ($\sim 10^4$) of causally disconnected patches … all at the same temperature!

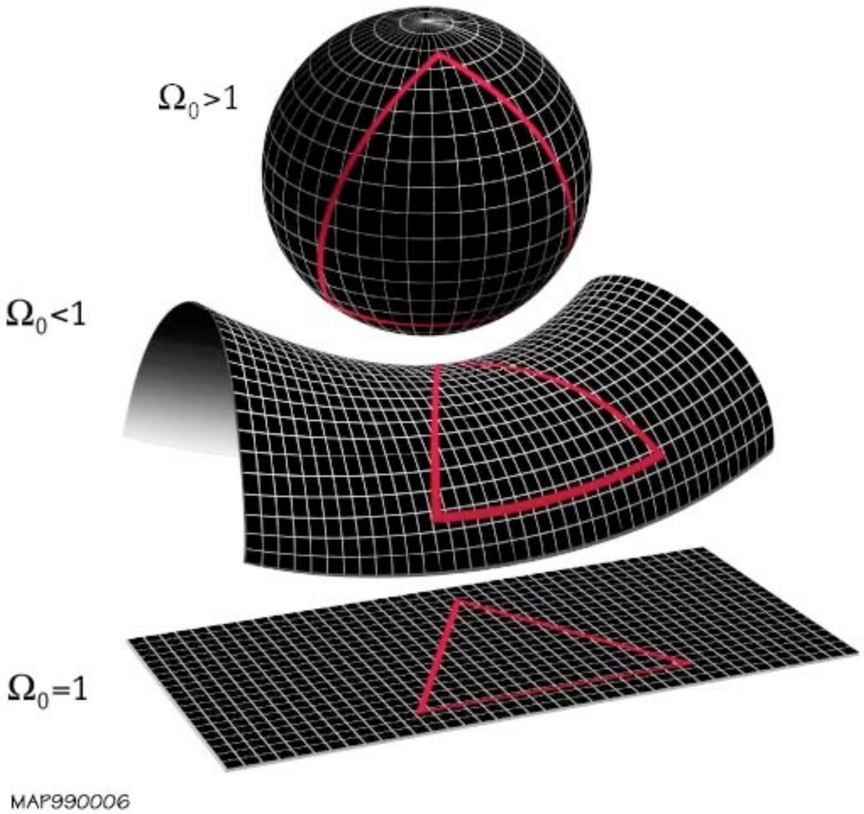
Big Bang … Problems

$$H^2(a) = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2}$$

$$\rho_c \equiv \frac{3H^2}{8\pi G} \qquad \Omega \equiv \frac{\rho}{\rho_c}$$

$$\left|1 - \Omega\right| = \left|\frac{k}{a^2 \rho_c}\right| \simeq 0$$

but … $\sim a^2$

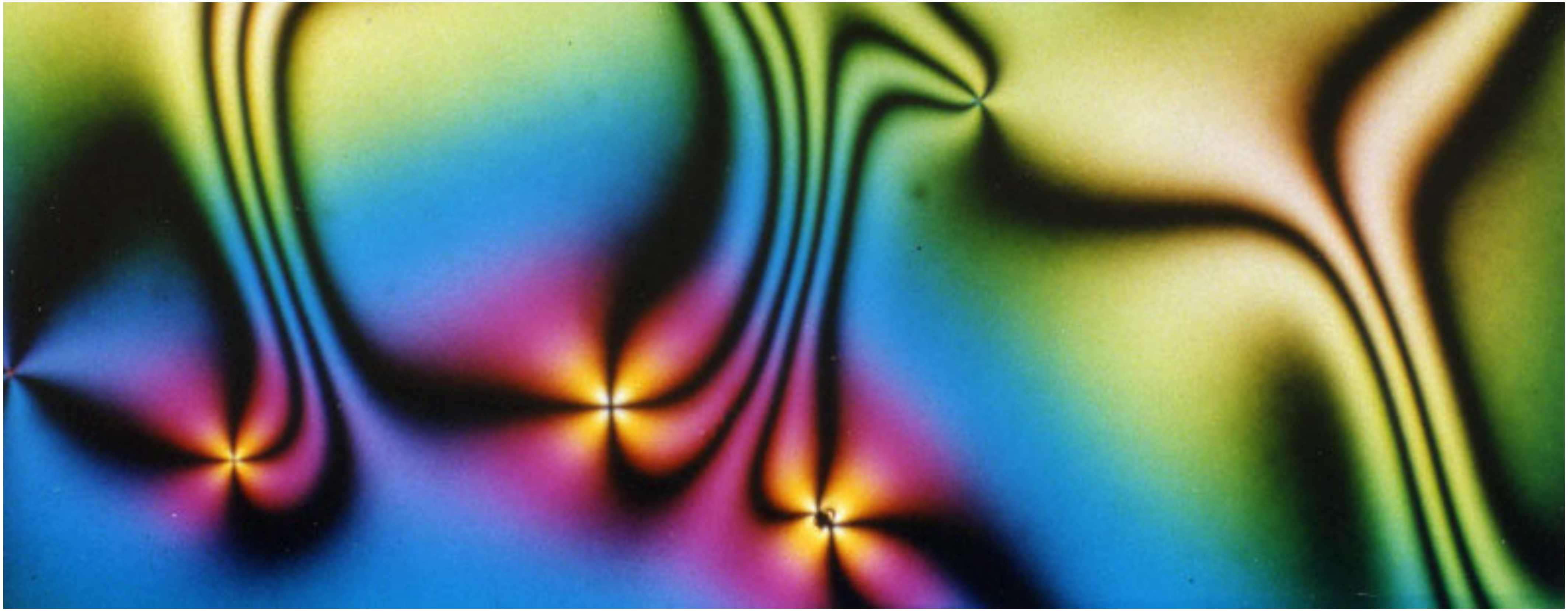


The Flatness Problem:

The curvature contribution at present time is small …
implying that in the Early Universe should be extremely (i.e. **unnaturally**) small!

$$\left|1 - \Omega(t_{BBN})\right| \lesssim 10^{-6}$$

Big Bang … Problems



Unwanted Relics:
Grand Unified Theories (GUTs) predict an overabundance of topological defects (e.g. magnetic monopoles) from phase transitions in the Early Universe … but we don't seen any of such things!

The *inflationary* solution

Horizon, flatness, unwanted relics …

Guth's idea, 1980: *Inflation* can solve *all* these problems at once!

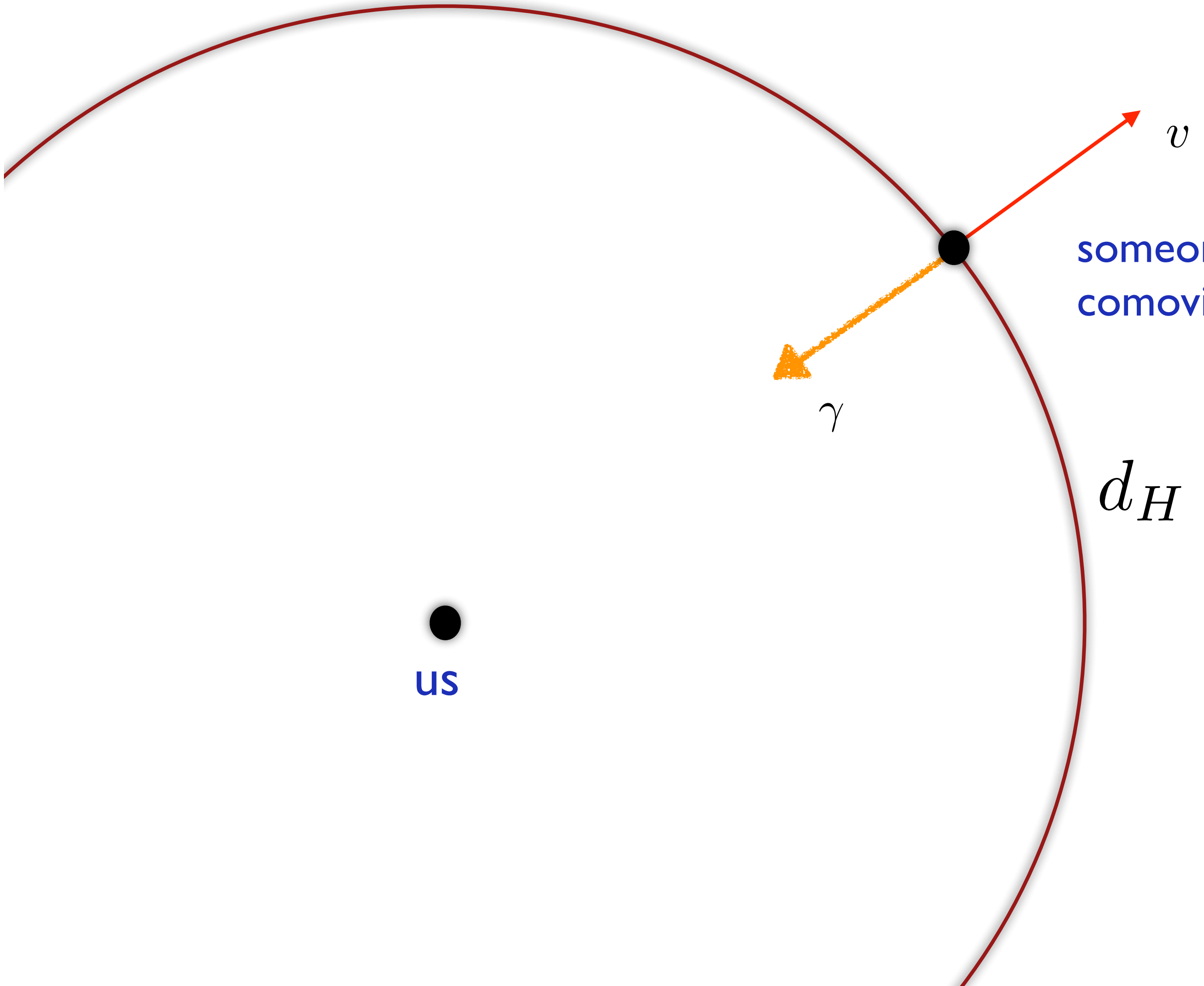
The Universe underwent a
period of *accelerated expansion*
in its early history

NB: this is not a "theory", nor a "model"

The "Hubble horizon"

Hubble's law $v = H\,d$ can also be written like this: $$d = \frac{v}{H}$$

The distance d where the velocity v equals the speed of light is the "**Hubble horizon**"

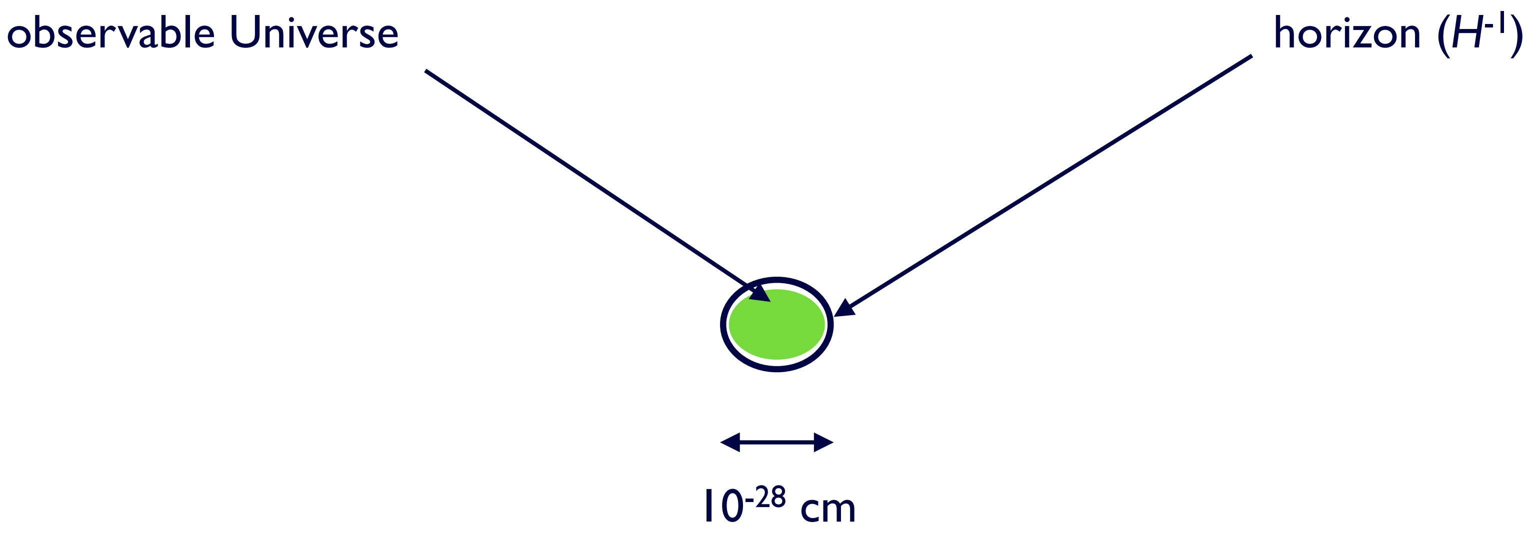


someone else at constant comoving distance from us

$$d_H = \frac{c}{H}$$

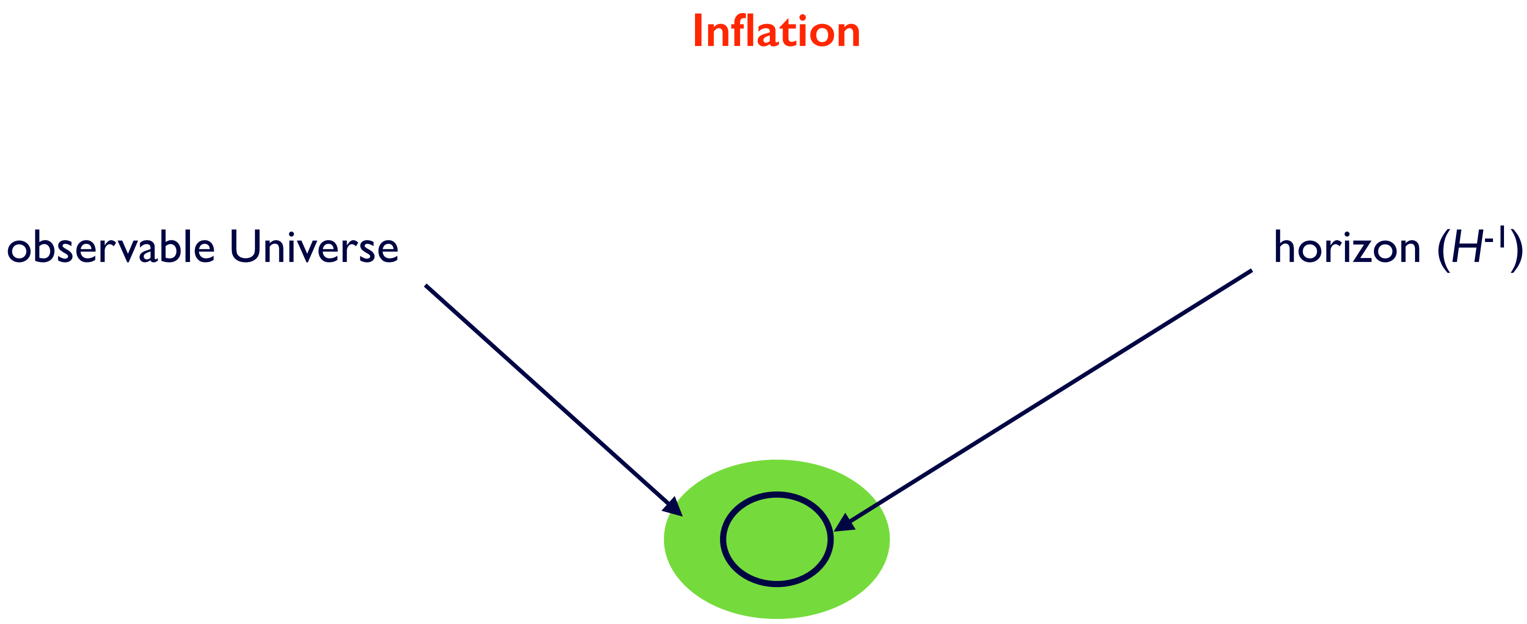
Solving the horizon problem

Inflation



$$v = H\,d$$

Solving the horizon problem



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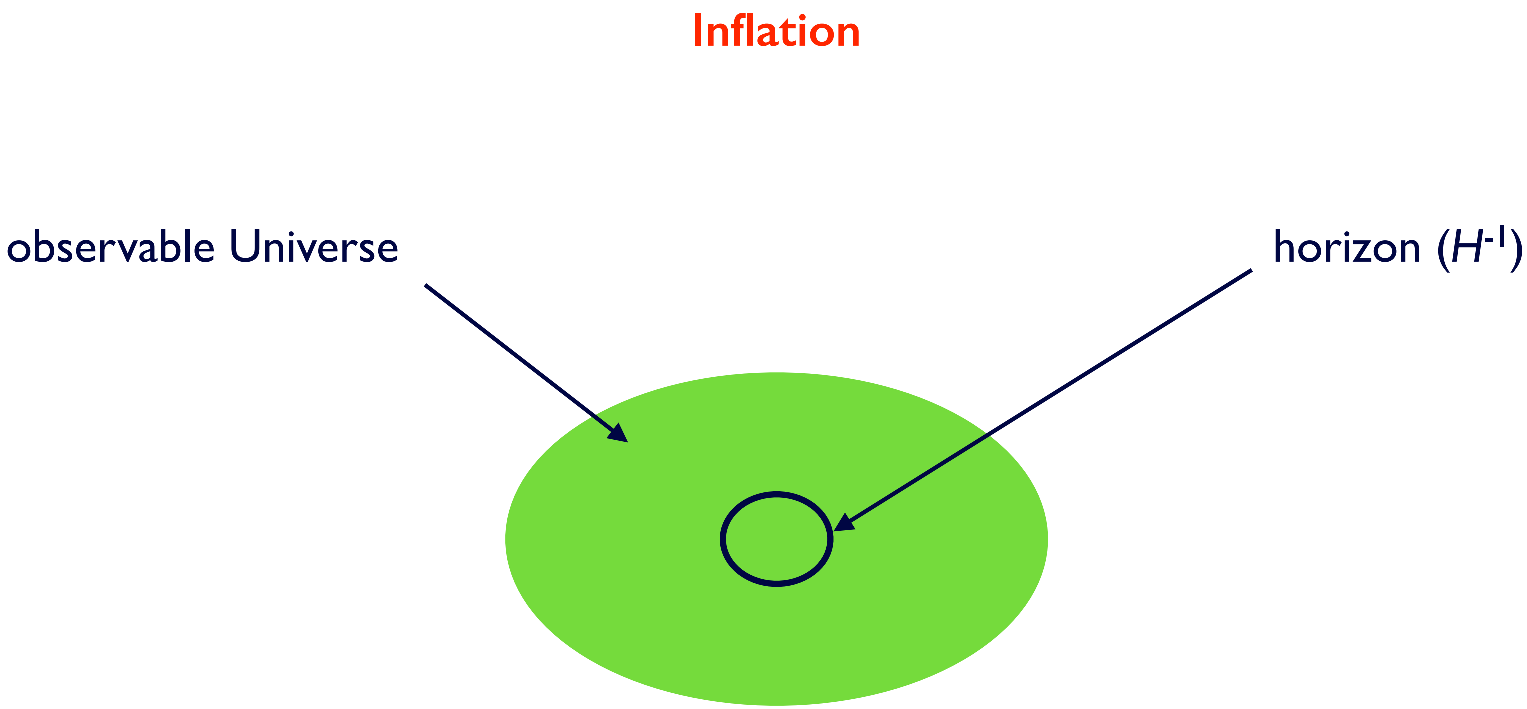

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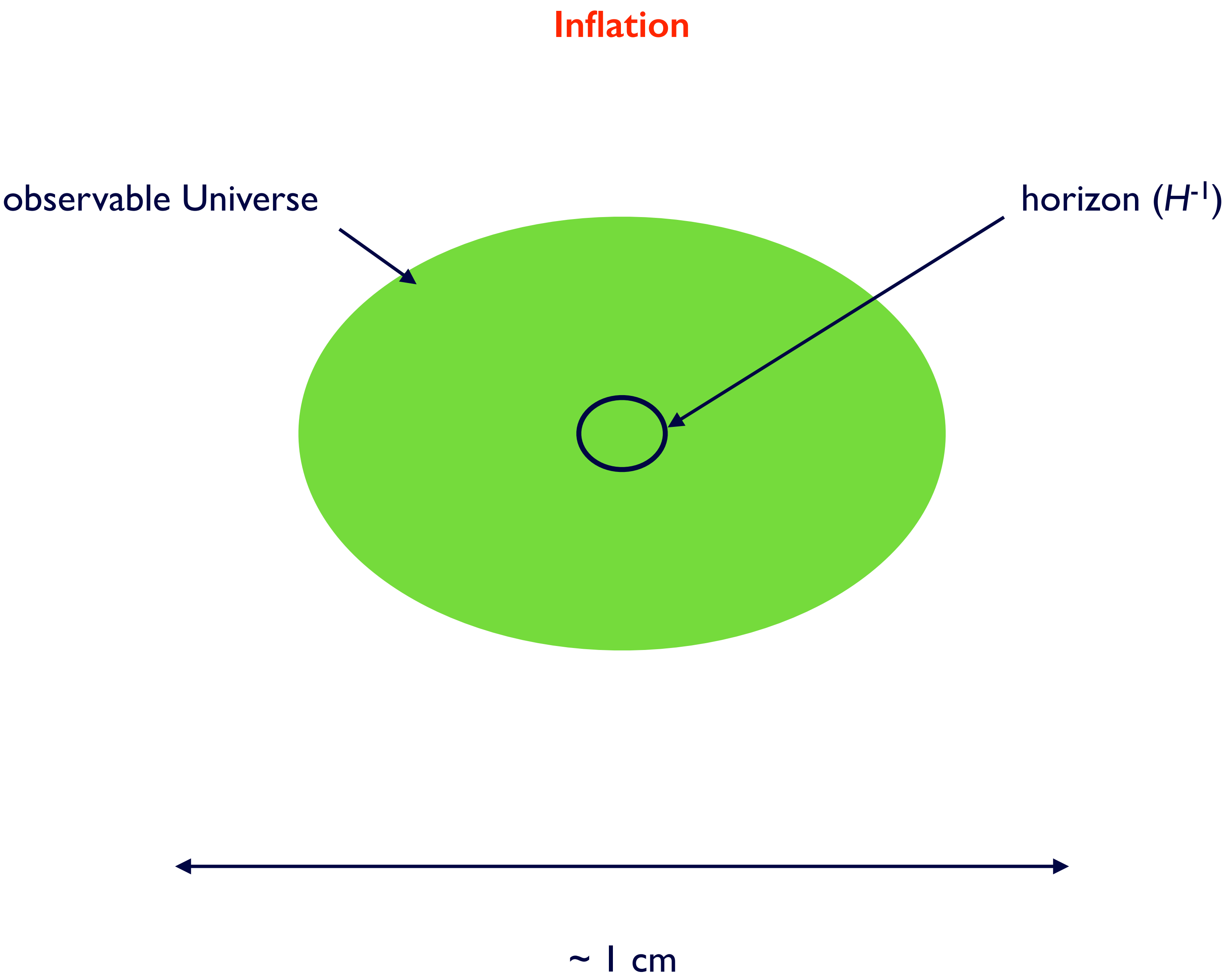

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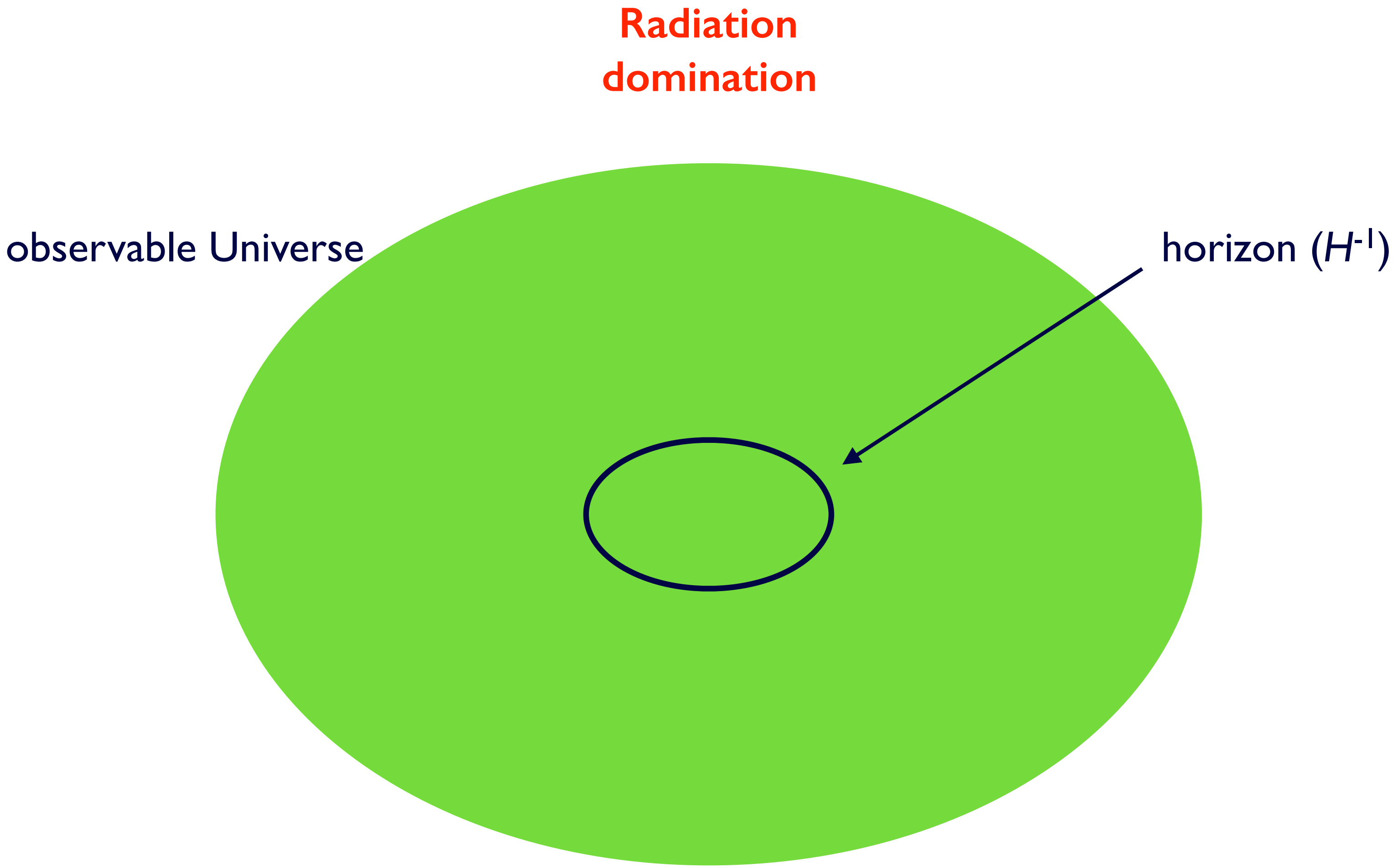

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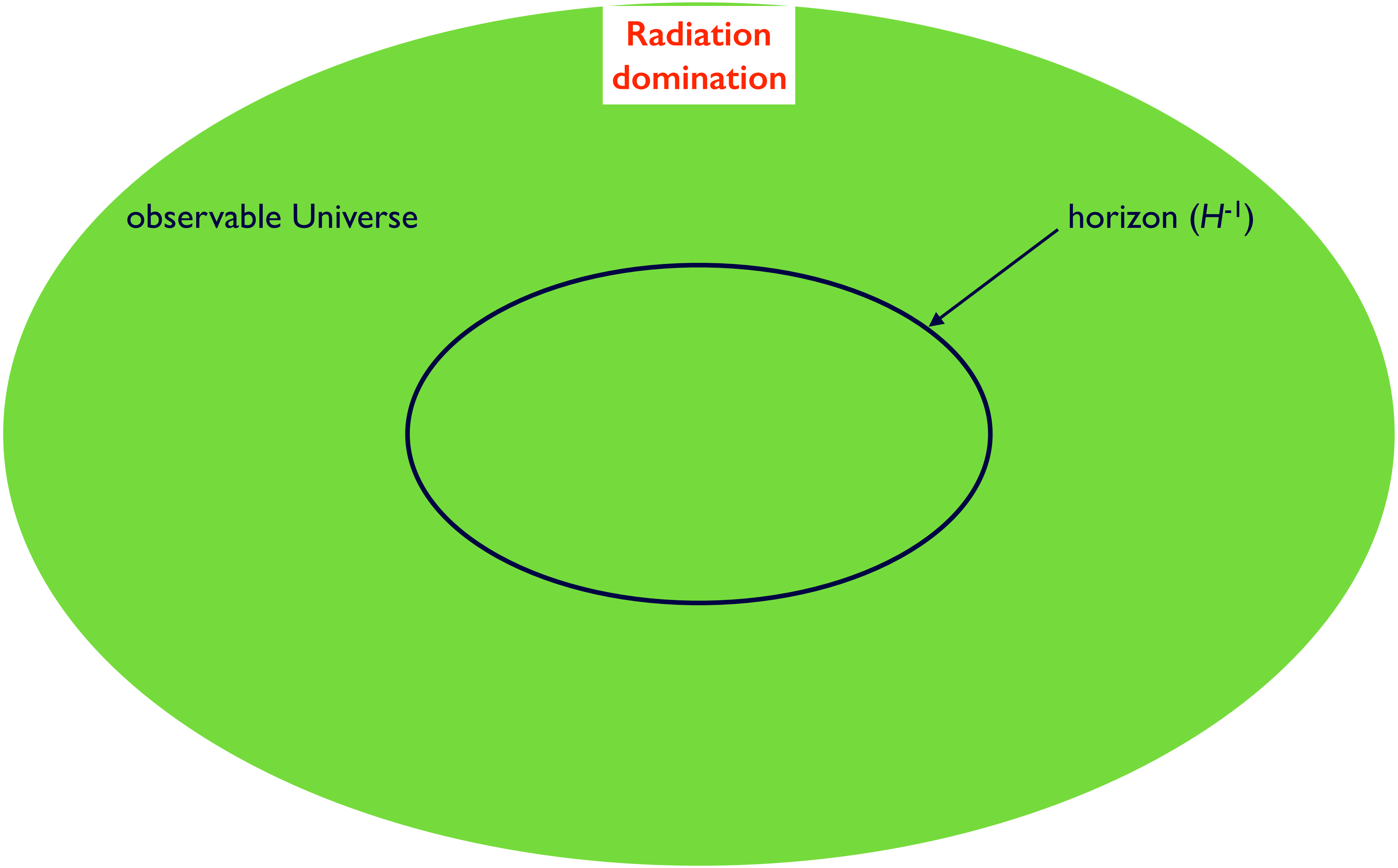

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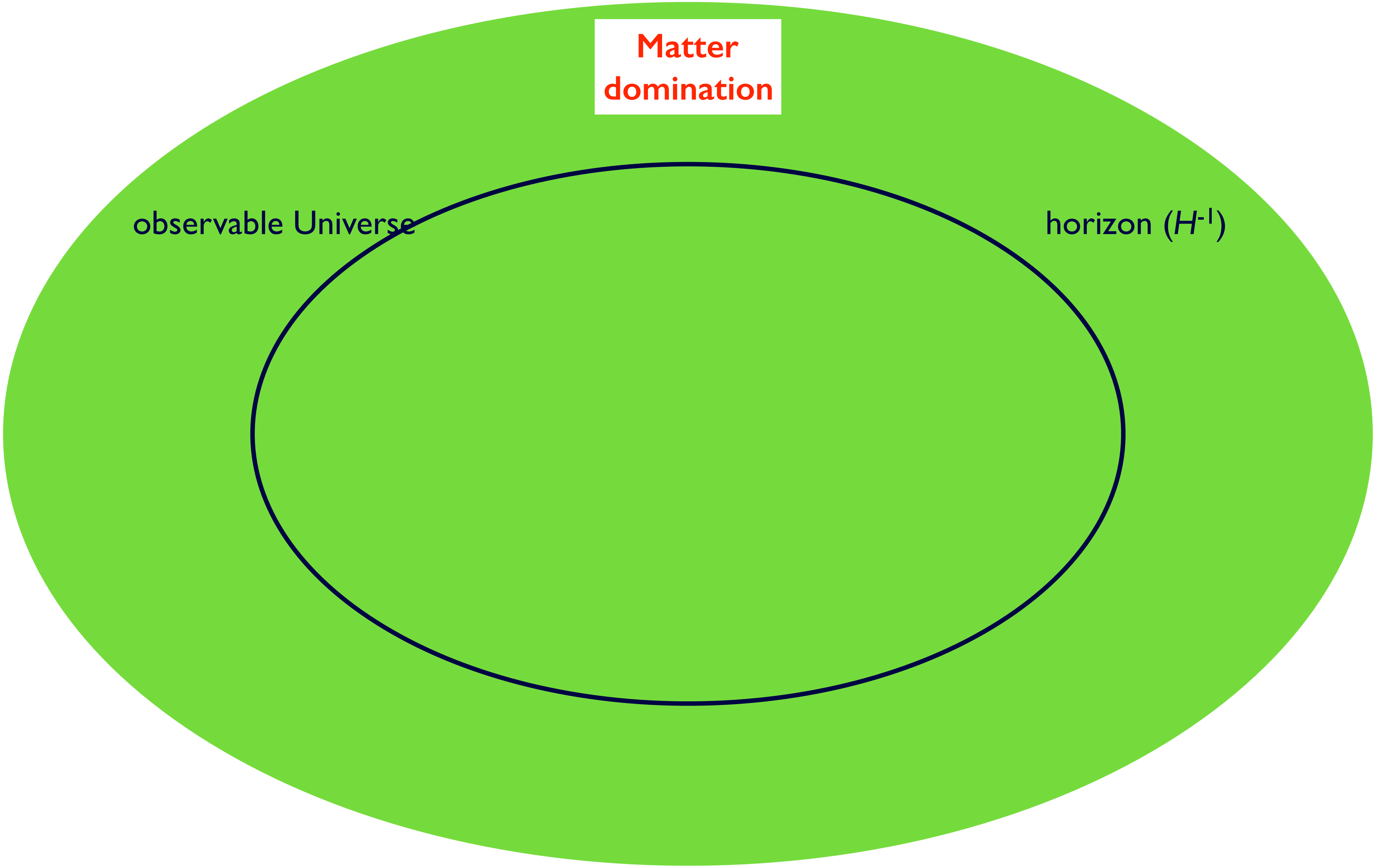

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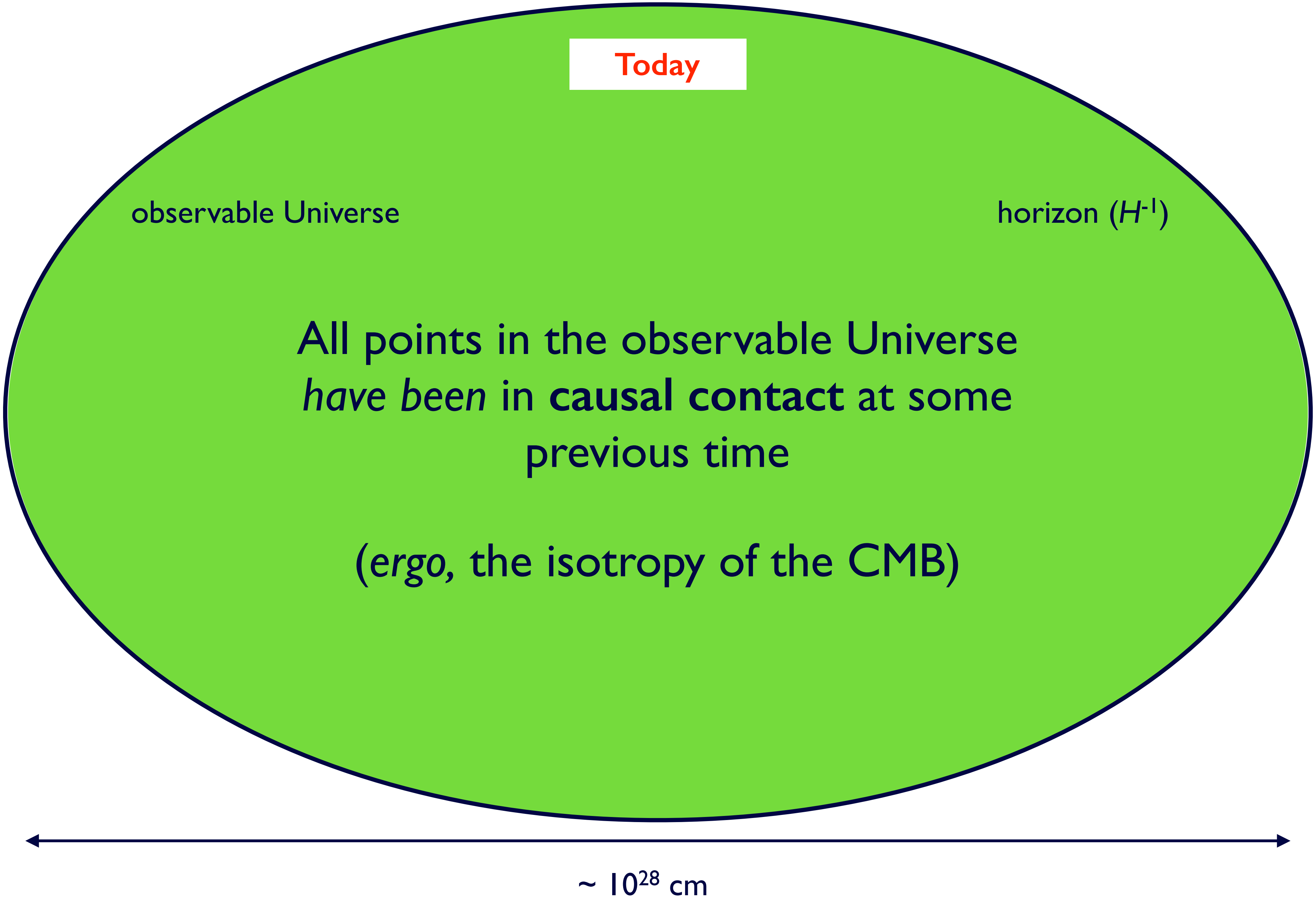
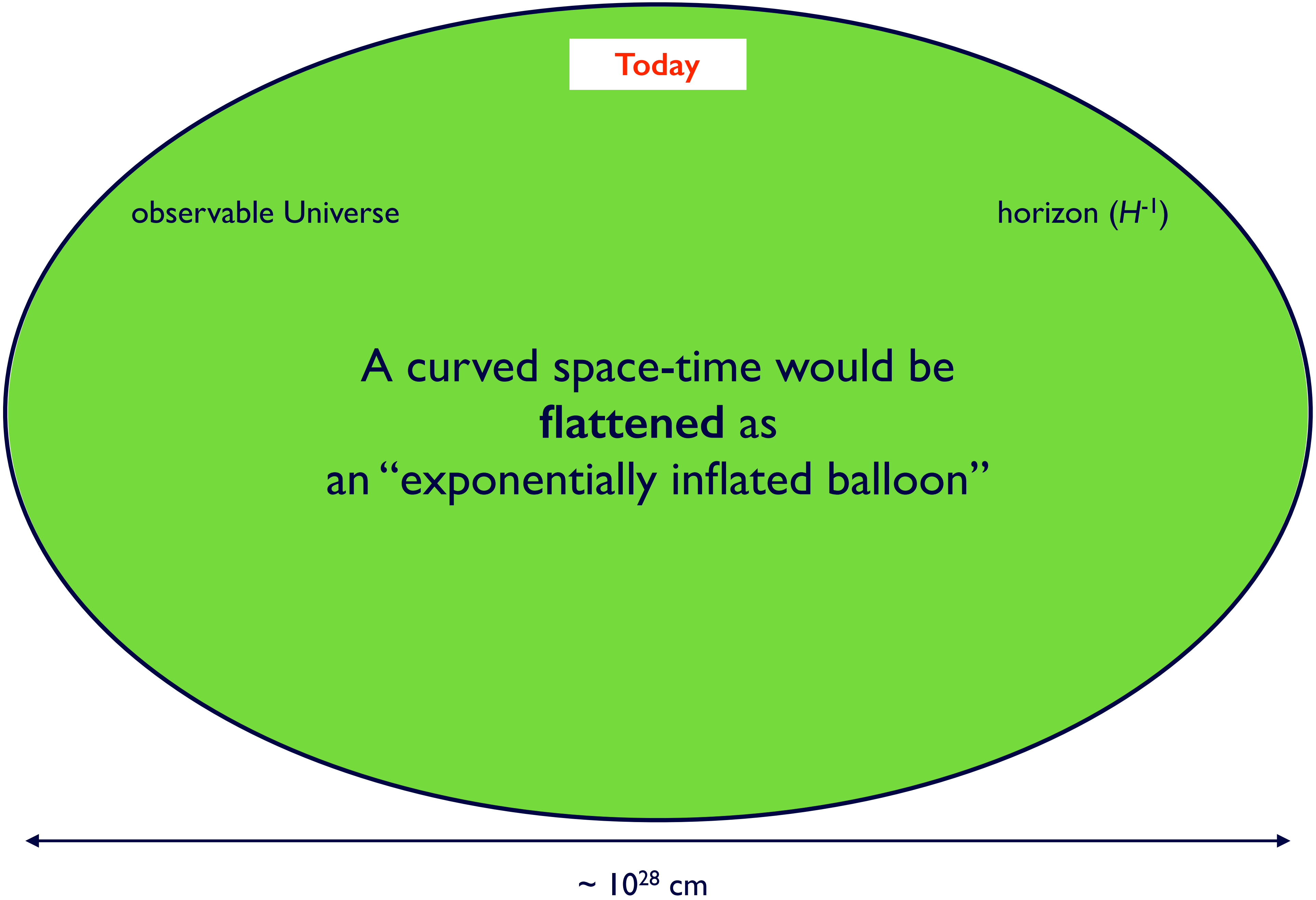
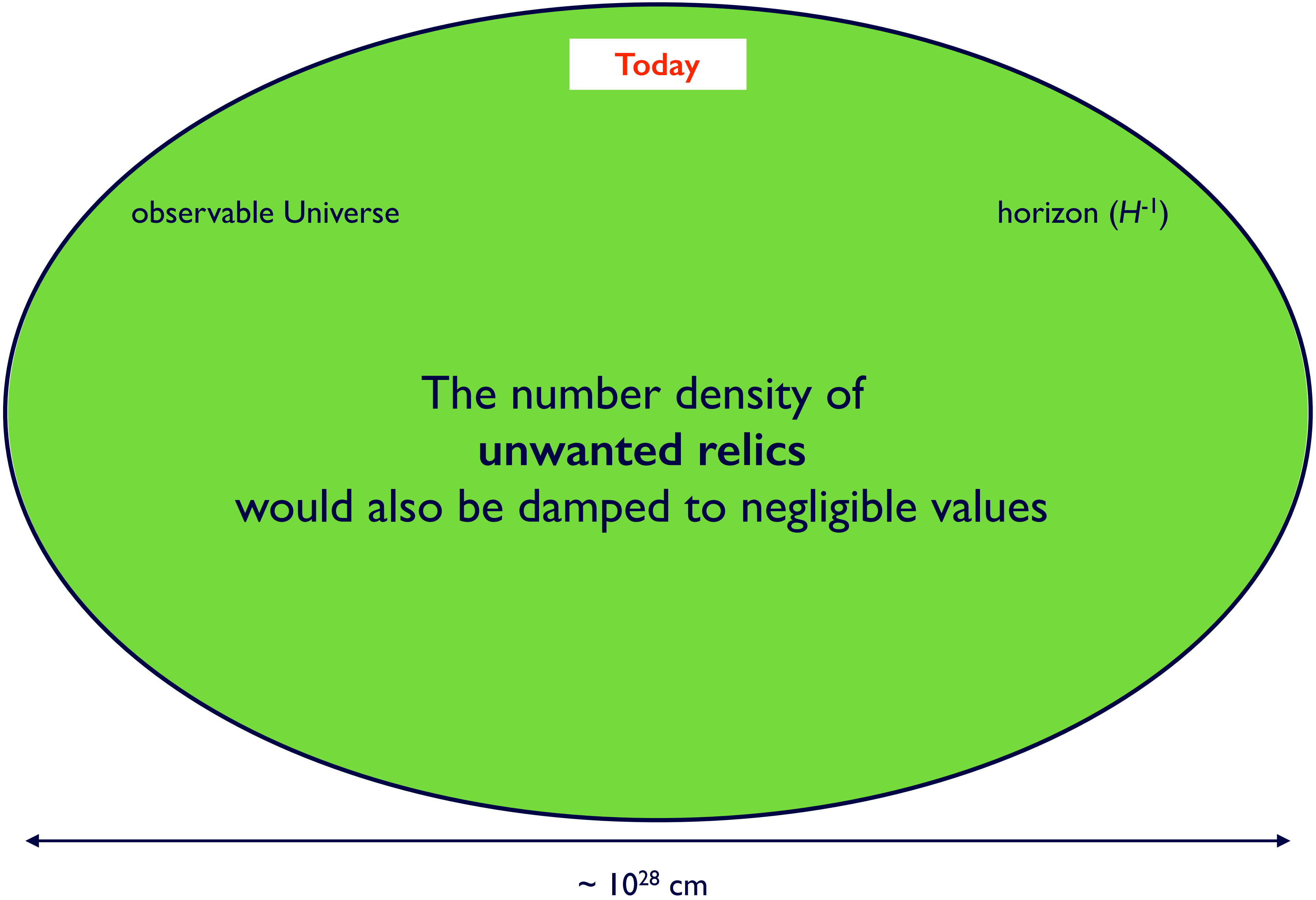


Solving the flatness problem



Getting rid of GUT relics



Energy content

Linde (1982), Albrecht & Steinhardt (1982)

How do we get acceleration?

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot{a}^2}{a^2} = -\frac{4\pi G}{3}(\rho + 3p)$$

We need something with "negative pressure" ...

Slow Roll Inflation

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$$\phi(\vec{x},t) = \phi_0(t) + \delta\phi(\vec{x},t) \qquad V(\phi_0) \longrightarrow \begin{cases} \rho = \frac{1}{2}\dot{\phi_0}^2 + V(\phi_0) \\ p = \frac{1}{2}\dot{\phi_0}^2 - V(\phi_0) \end{cases}$$

A **scalar field** … with some potential

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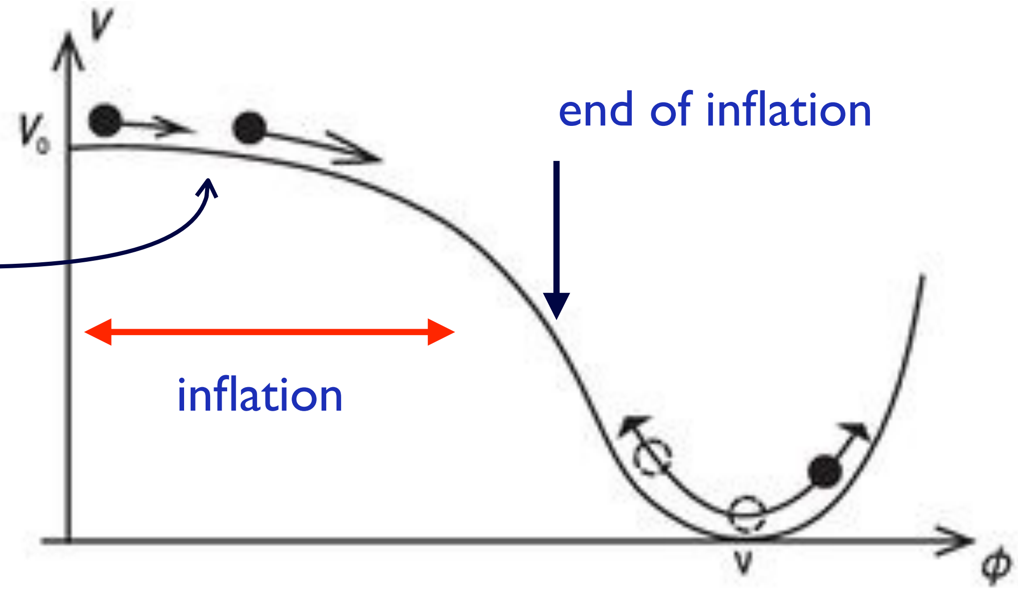
A **scalar field** … with some potential

flat potential

$$V'(\phi) \equiv \frac{dV}{d\phi} \simeq 0$$

$$\rightarrow \quad \frac{1}{2}\dot{\phi}_0^2 \simeq 0$$



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$$\phi(\vec{x}, t) = \phi_0(t) + \delta\phi(\vec{x}, t) \qquad V(\phi_0)$$

$$\begin{cases} \rho \simeq +V(\phi_0) \\ p \simeq -V(\phi_0) \end{cases}$$

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the potential energy dominates over the kinetic one, then

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$$H = \frac{\dot{a}}{a} \simeq \left[\frac{8\pi G}{3}V(\phi_0)\right]^{1/2}$$

~ constant $\longrightarrow$ $a(t) \sim e^{Ht}$

exponential expansion!

SO …

The horizon problem, the flatness problem are solved
while unwanted relics are swept away …

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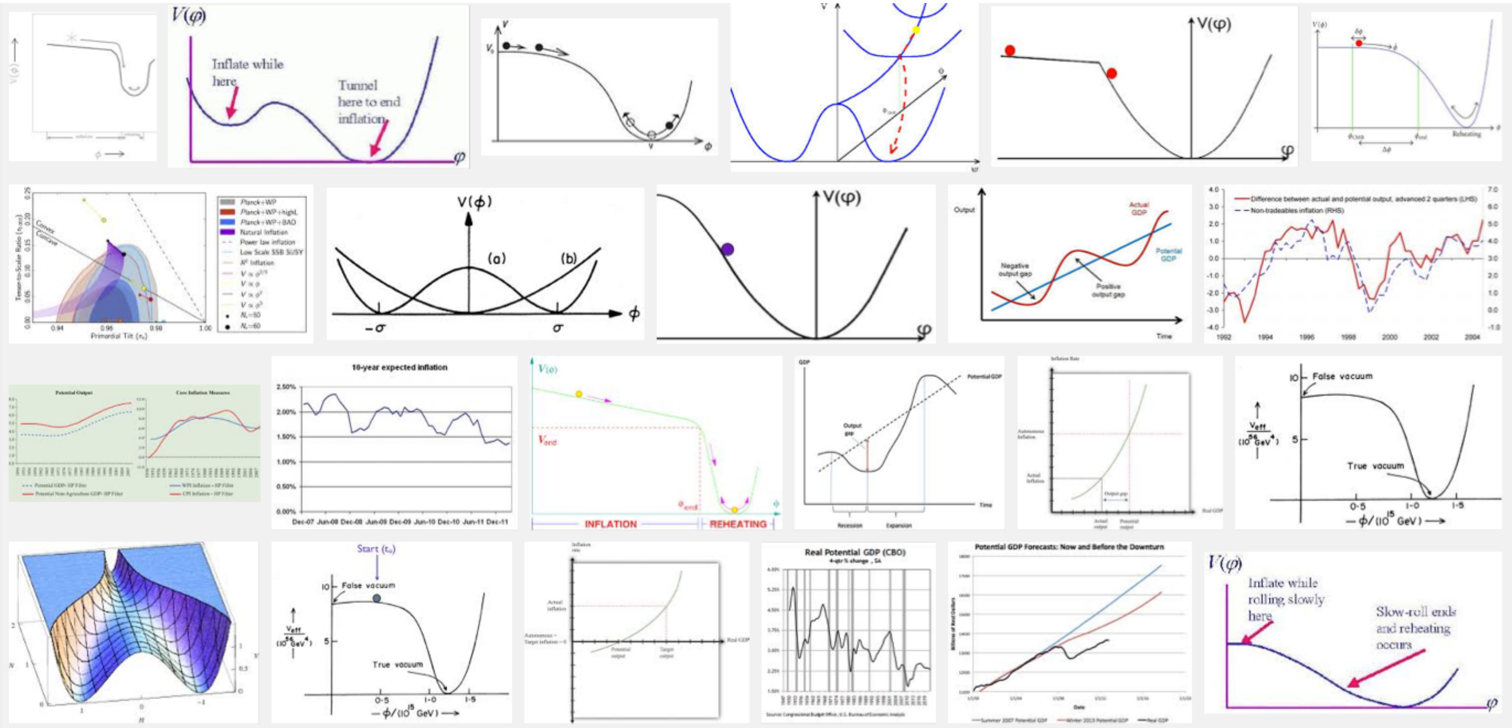
Moreover, there is **plenty of models** implementing slow-roll inflation
(and other varieties of inflation), covering a wide range of energy scales



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Still … Inflation provides **two, crucial, *unrequested* predictions:**

density perturbations **and** *gravitational waves*

Density Perturbations from Inflation

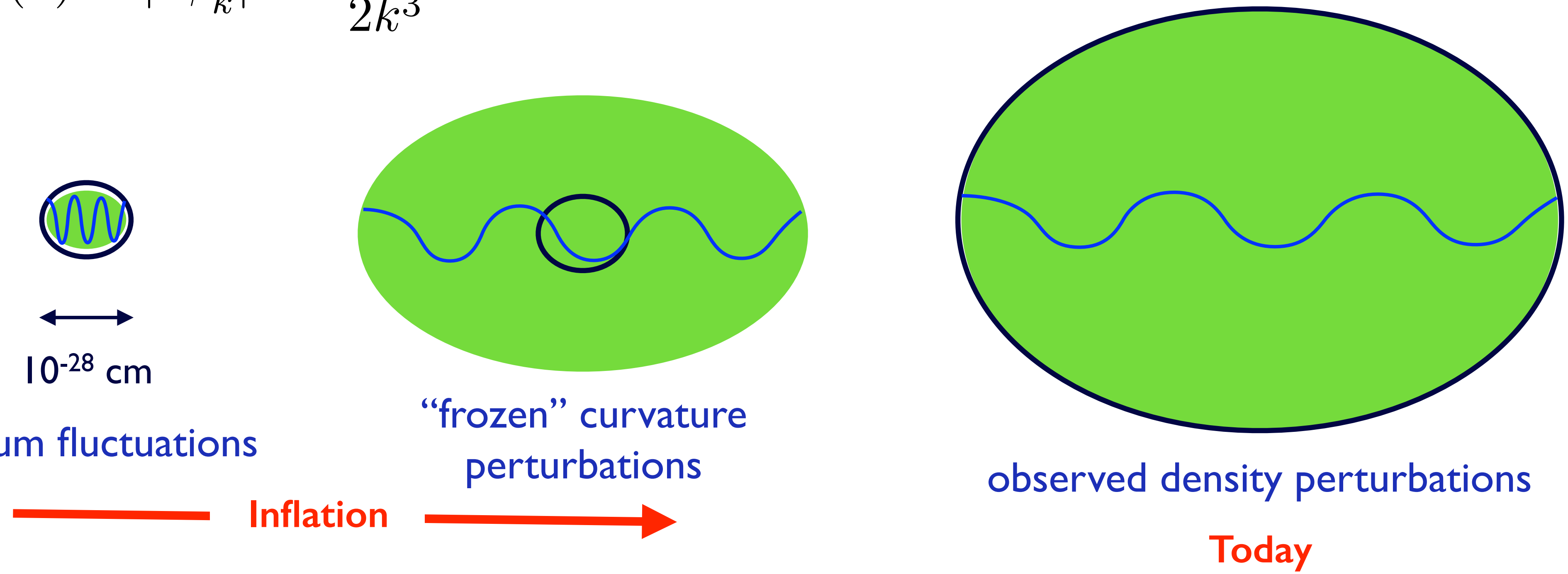
$$\phi(\vec{x},t) = \phi_0(t) + \delta\phi(\vec{x},t)$$ quantum fluctuations of the inflaton

for slow-roll inflation, their equation of motion in Fourier space is

$$\delta\ddot{\phi}_{\vec{k}} + 2aH\delta\dot{\phi}_{\vec{k}} + k^2\delta\phi_{\vec{k}} = 0$$ → **quantum** harmonic oscillator for $k \gtrsim aH$

one can compute the **power spectrum of inflaton** fluctuations

$$P_{\delta\phi}(k) \equiv |\delta\phi_{\vec{k}}|^2 = \frac{H^2}{2k^3}$$



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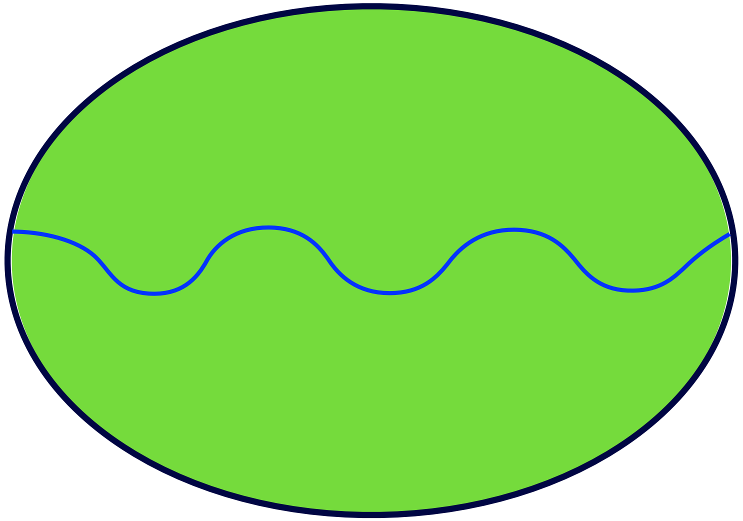
$$P_{\delta\phi}(k) \equiv |\delta\phi_{\vec{k}}|^2 = \frac{H^2}{2k^3}$$

↓

and (after a lot of pain) the **power spectrum of the gravitational potential perturbations** as they enter the horizon again

$$P_\Phi(k) = \frac{2}{9M_p^4}\frac{V^2}{V'^2}\frac{H^2}{k^3}\bigg|_{aH=k}$$

$$M_p = \frac{1}{\sqrt{8\pi G}}$$



observed density perturbations

Today

Density Perturbations from Inflation

Inflation predicts the power spectrum of the perturbations in the gravitational potential (and in the energy density) today!

$$P_\Phi(k) = \frac{2}{9M_p^4}\frac{H^2V^2}{V'^2}\,k^{-4+n_s}\Big|_{aH=k}$$

amplitude

scale-dependence

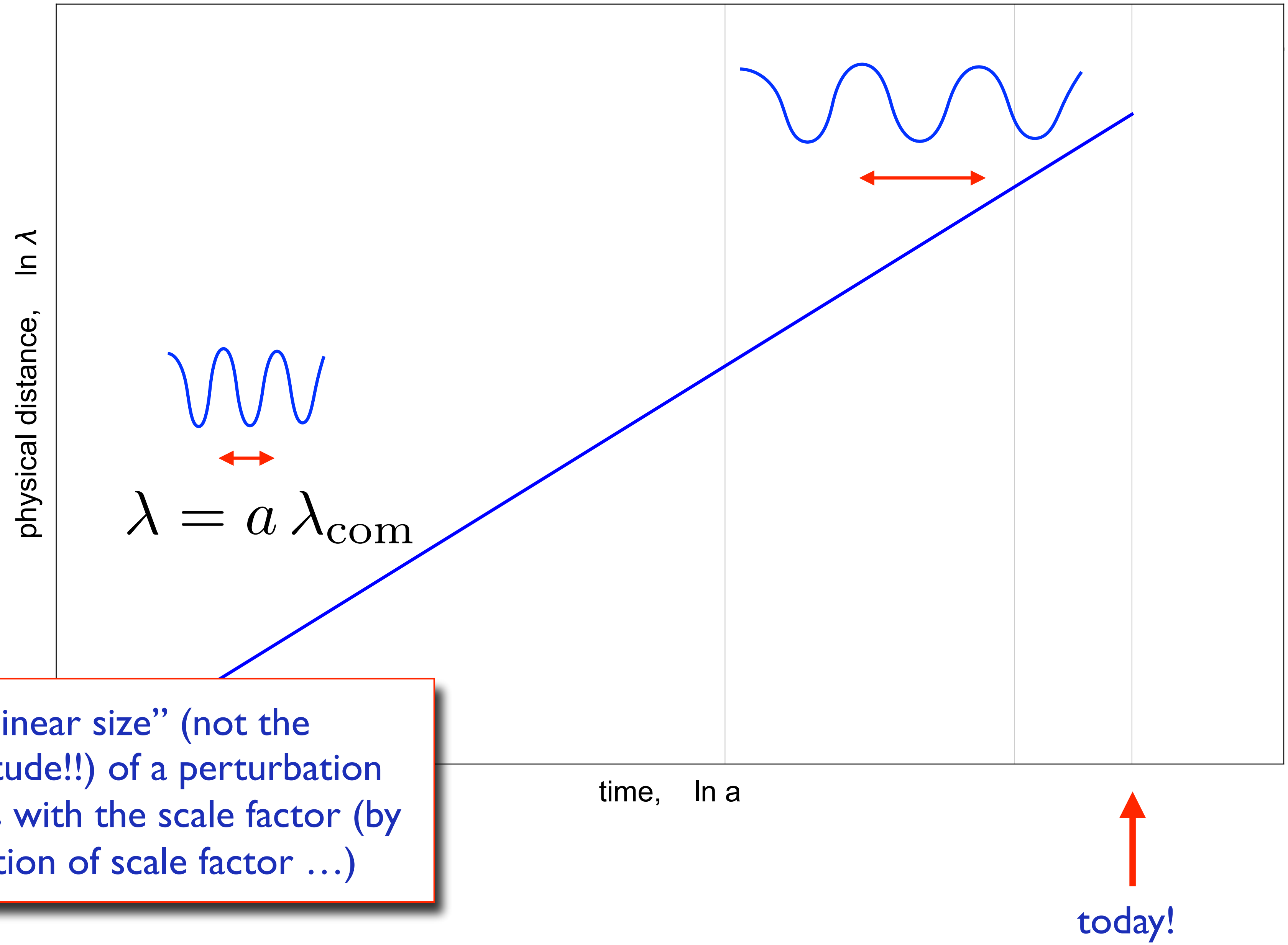
spectral index: $n_s = 1 - 2M_p^2\left(\frac{V'}{V}\right)^2 + 2M_p^2\frac{V''}{V}$

$$\Delta_\Phi(k) \equiv 4\pi k^3 P_\Phi(k) \simeq \text{constant} \simeq (10^{-5})^2$$

Harrison-Zeldovich power spectrum

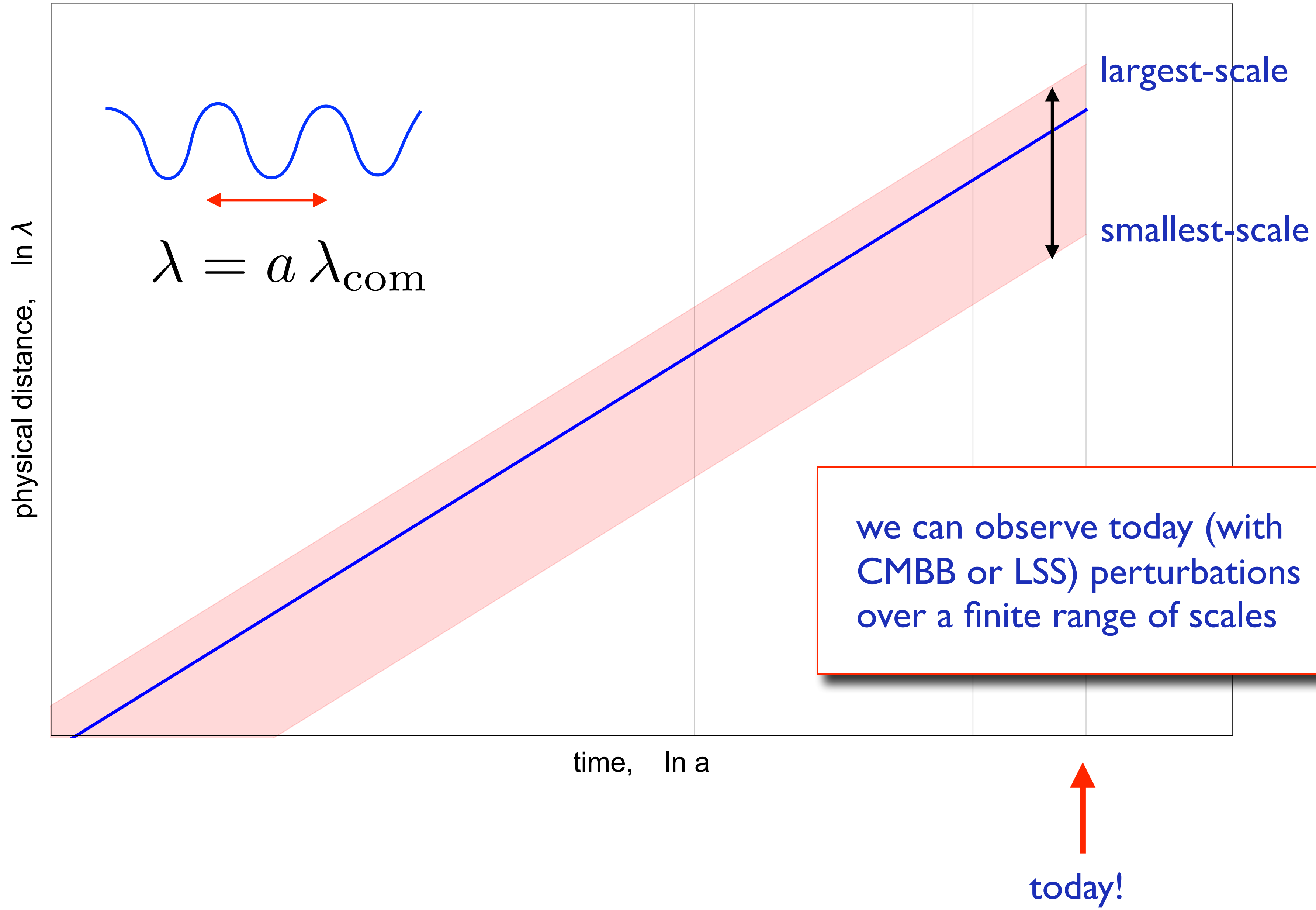
Evolution of matter perturbations:
Initial Conditions
Part 2: From inflation to photon decoupling

The evolution of density perturbations before decoupling



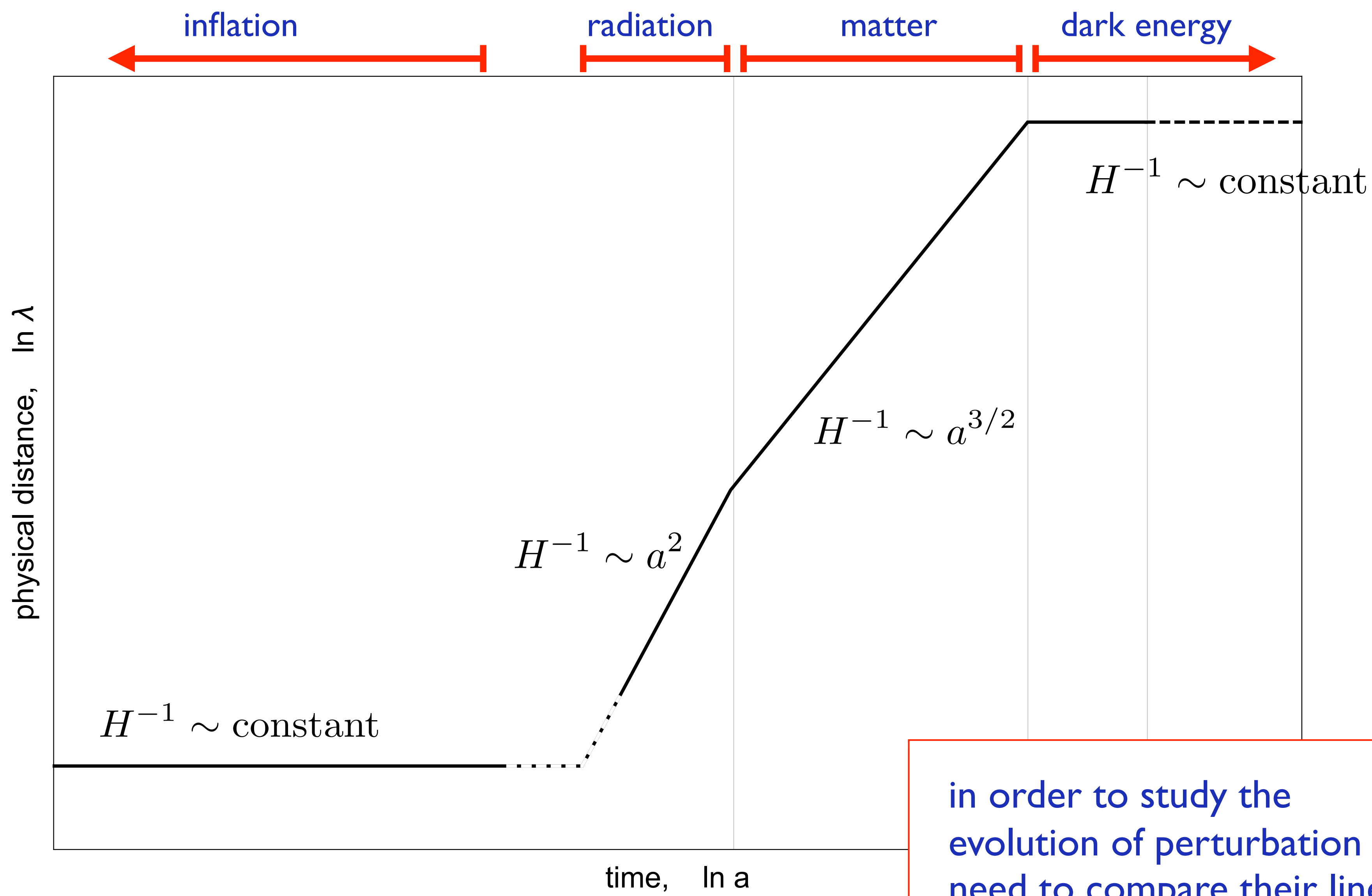
The “linear size” (not the amplitude!!) of a perturbation grows with the scale factor (by definition of scale factor …)

The evolution of density perturbations before decoupling



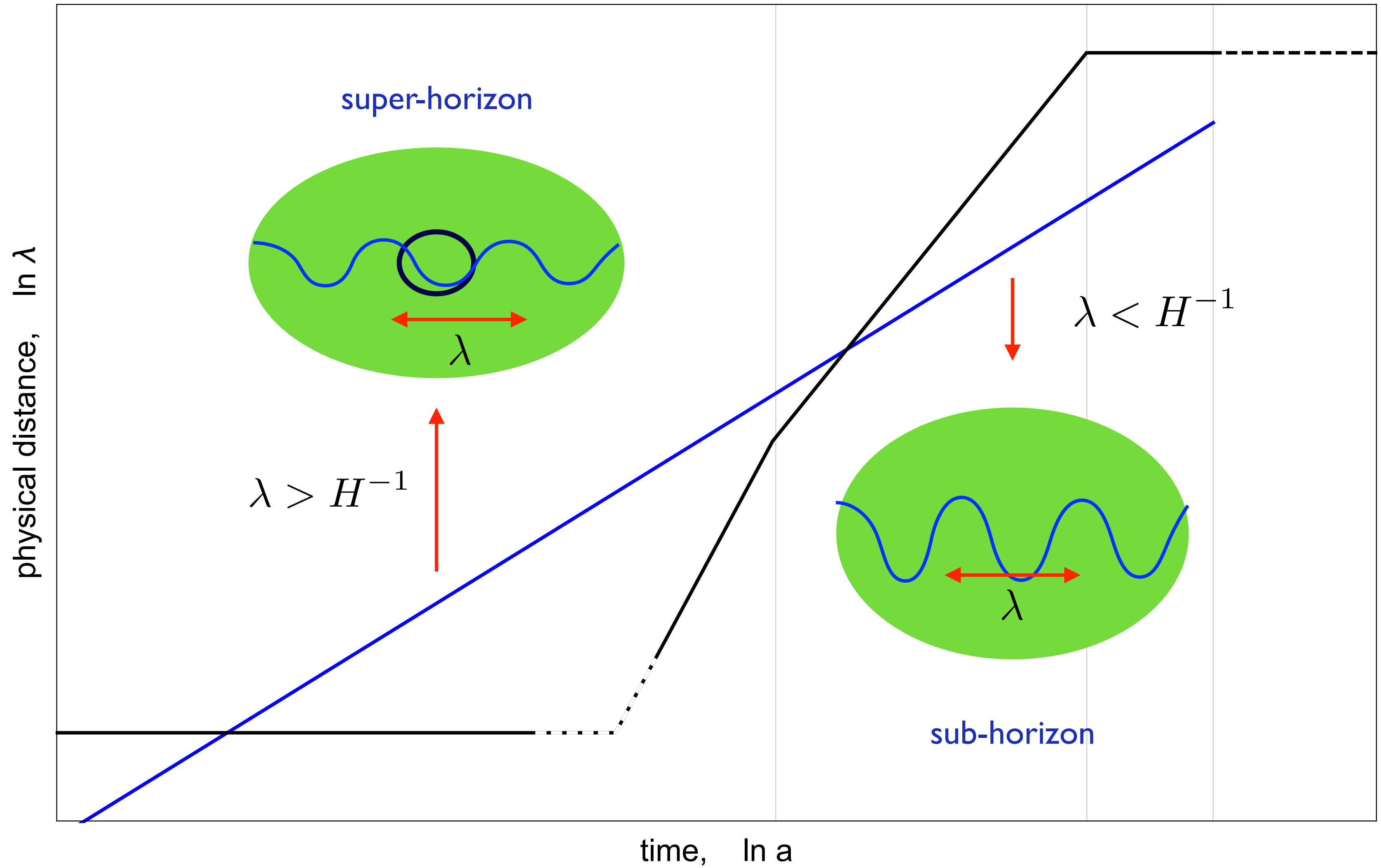
we can observe today (with CMBB or LSS) perturbations over a finite range of scales

The evolution of density perturbations before decoupling



in order to study the evolution of perturbation we need to compare their linear size to the Hubble horizon at any given time

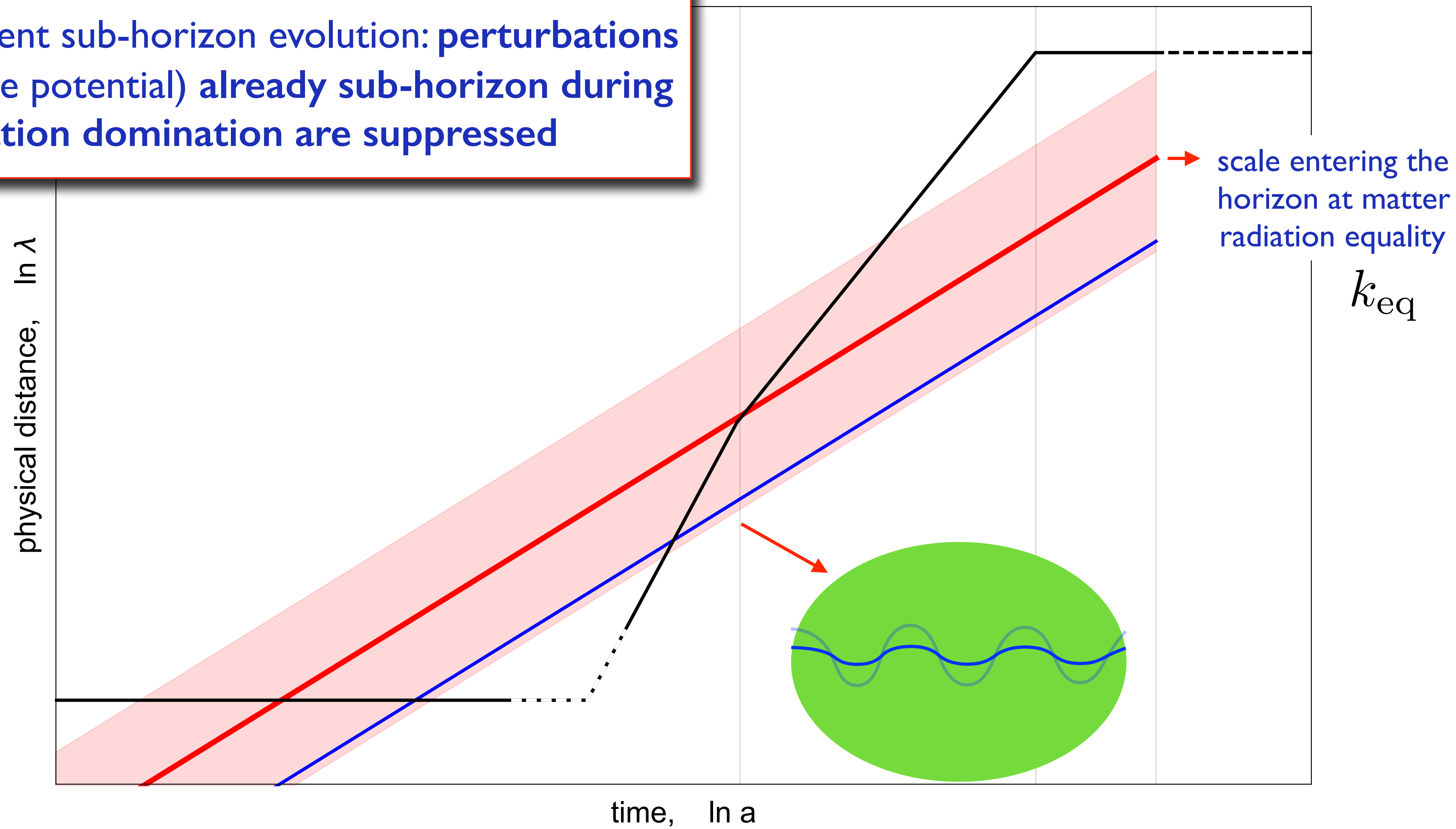
The evolution of density perturbations before decoupling



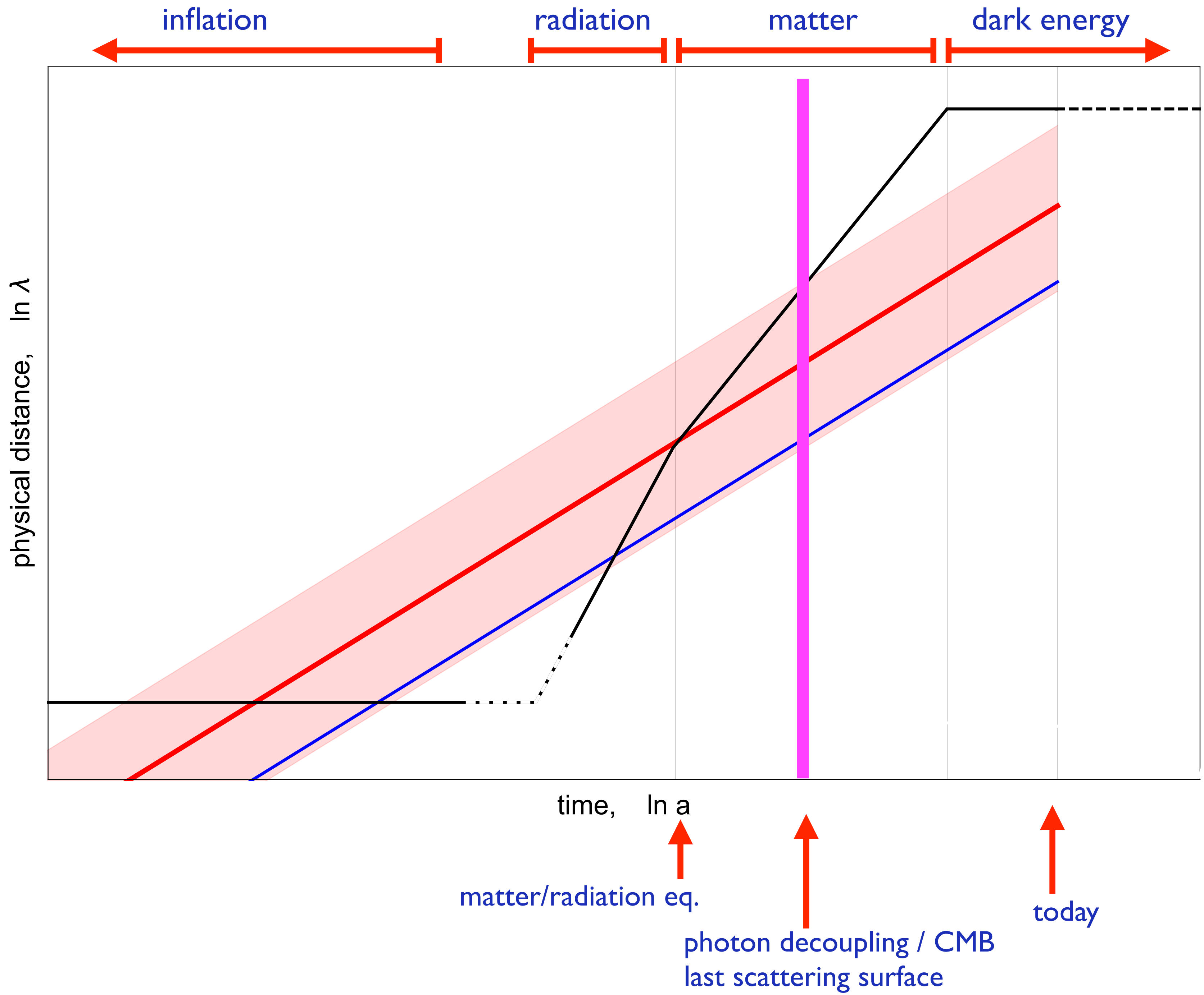
Super-horizon curvature perturbations are "frozen"

The evolution of density perturbations before decoupling

Perturbations at different scales have a different sub-horizon evolution: **perturbations** (in the potential) **already sub-horizon during radiation domination are suppressed**



My initial conditions



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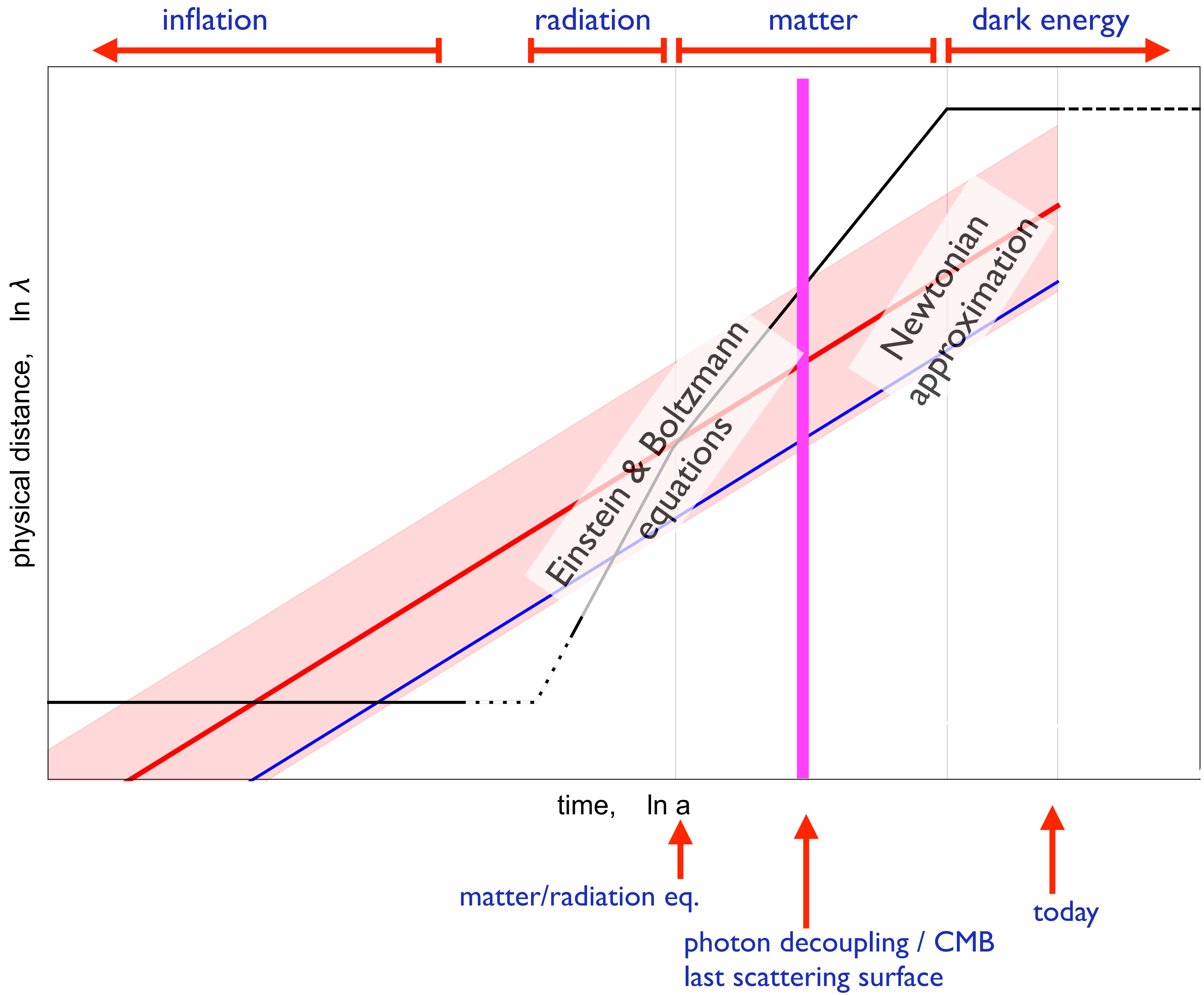
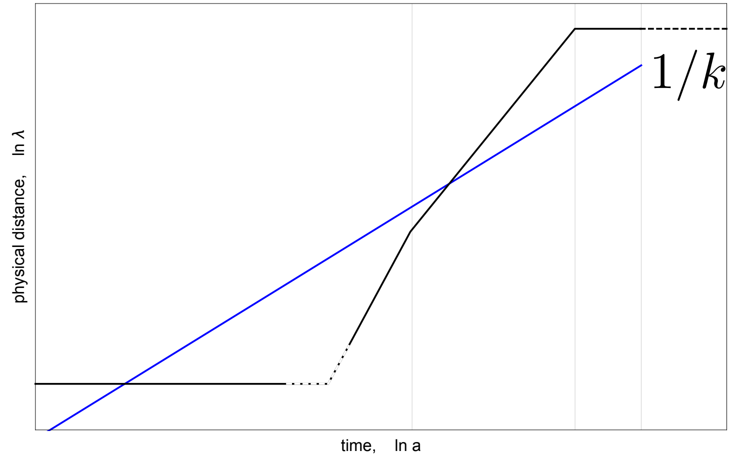


The "initial" matter power spectrum

Let's consider a scale (mode k) that re-enters the horizon during matter domination (that is a large scale today!)



$$\Delta_\Phi(k) \equiv 4\pi k^3 P_\Phi(k) \simeq \text{constant} \qquad P_\Phi(k) \simeq \frac{C}{k^3}$$

To obtain the *matter* power spectrum I should relate matter and gravitational potential perturbations via **Poisson's equation**

$$\nabla^2_r \Phi_{tot} = 4\pi G \rho$$

$$\rho(\vec{r}, t) = \bar{\rho}(t) + \delta\rho(\vec{r}, t)$$

$$\Phi_{tot}(\vec{r}, t) = \bar{\Phi}(\vec{r}, t) + \Phi(\vec{r}, t)$$

background perturbations ???

$$\nabla^2_r \bar{\Phi} = 4\pi G \bar{\rho} \rightarrow \bar{\Phi} = \frac{2\pi G}{3} r^2 \bar{\rho}$$

"Jean's swindle"

$$\boxed{\nabla^2_r \Phi(\vec{r}, t) = 4\pi G\, \delta\rho(\vec{r}, t)} \rightarrow \nabla^2 \Phi(\vec{r}, t) = 4\pi G\, a^2\, \delta\rho(\vec{r}, t)$$

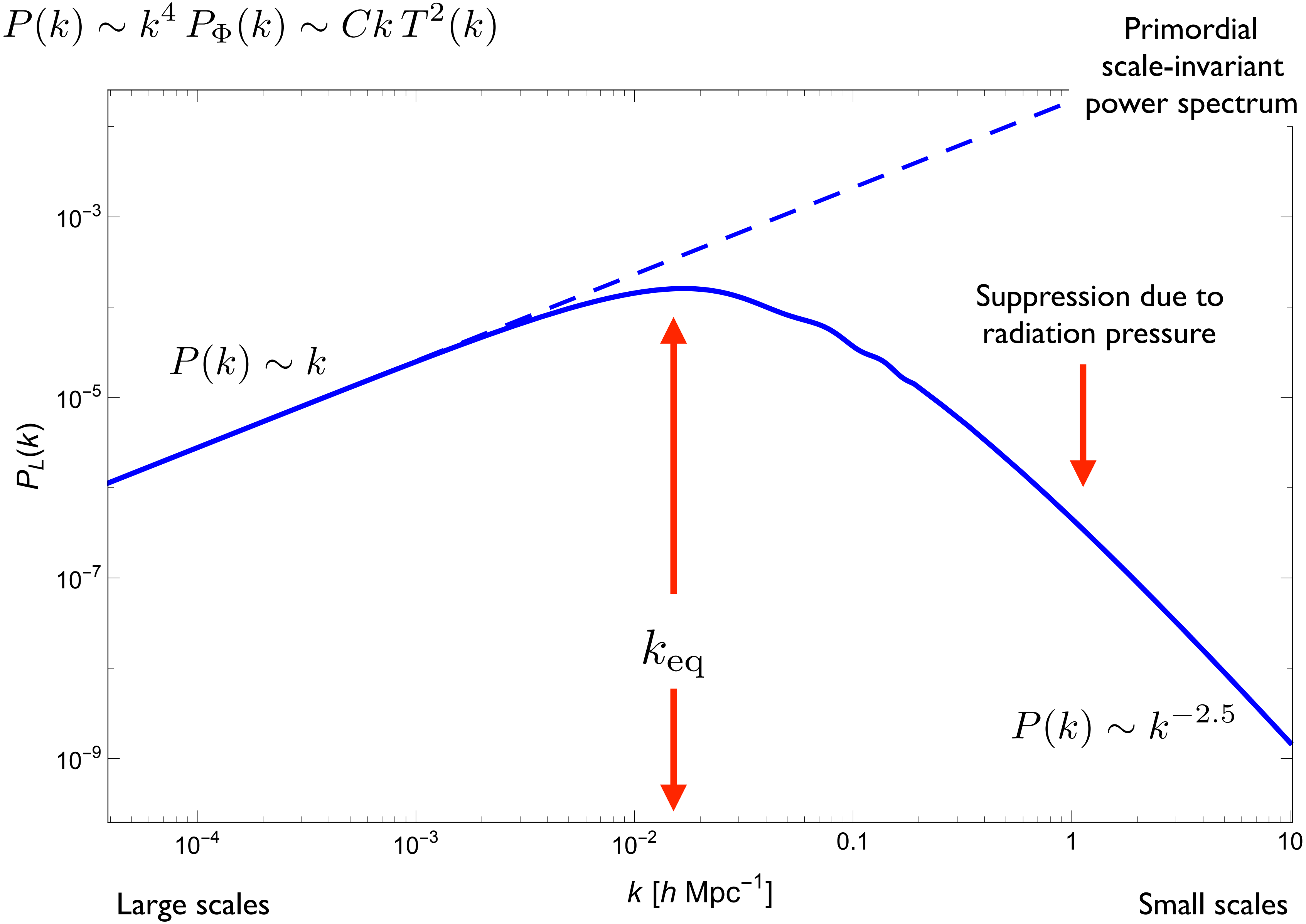
in comoving coordinates $\vec{r} = a(t)\, \vec{x}$

$$\rightarrow -k^2 \Phi_{\vec{k}} = 4\pi G\, a^2\, \bar{\rho} \delta_{\vec{k}} \rightarrow \langle |\delta_{\vec{k}}|^2 \rangle \sim k^4 \langle |\Phi_{\vec{k}}|^2 \rangle \rightarrow$$

The “initial” matter power spectrum

The linear matter power spectrum at $z \simeq 1000$

$$P(k) \sim k^4\, P_\Phi(k) \sim C k\, T^2(k)$$



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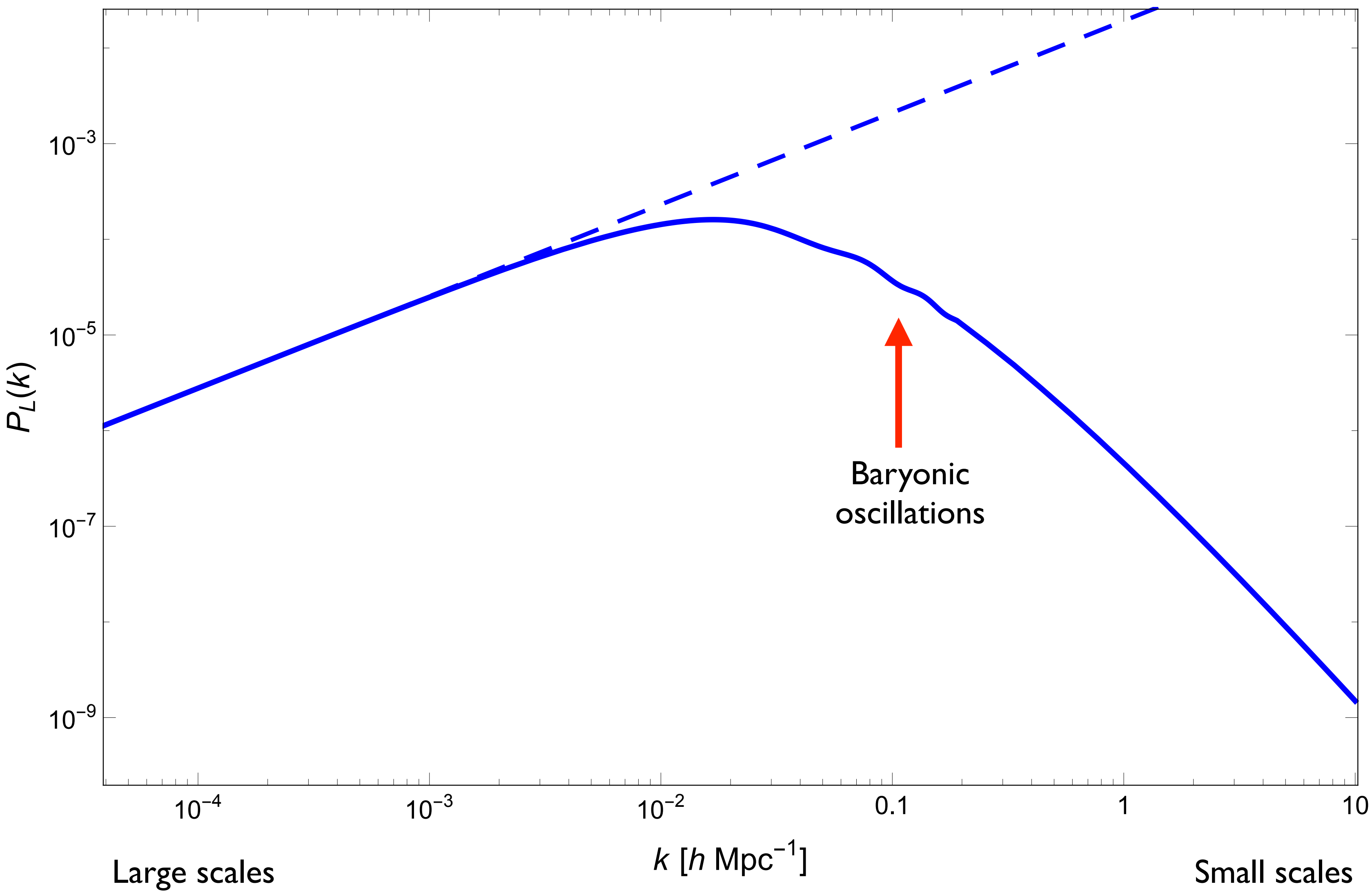

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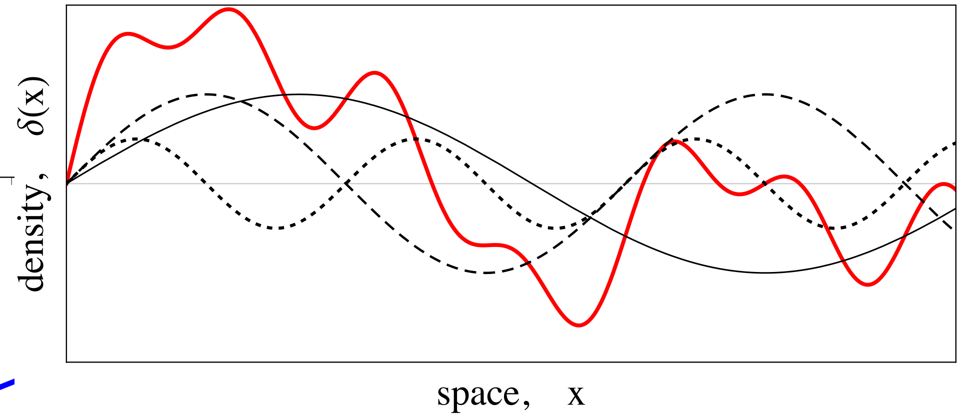
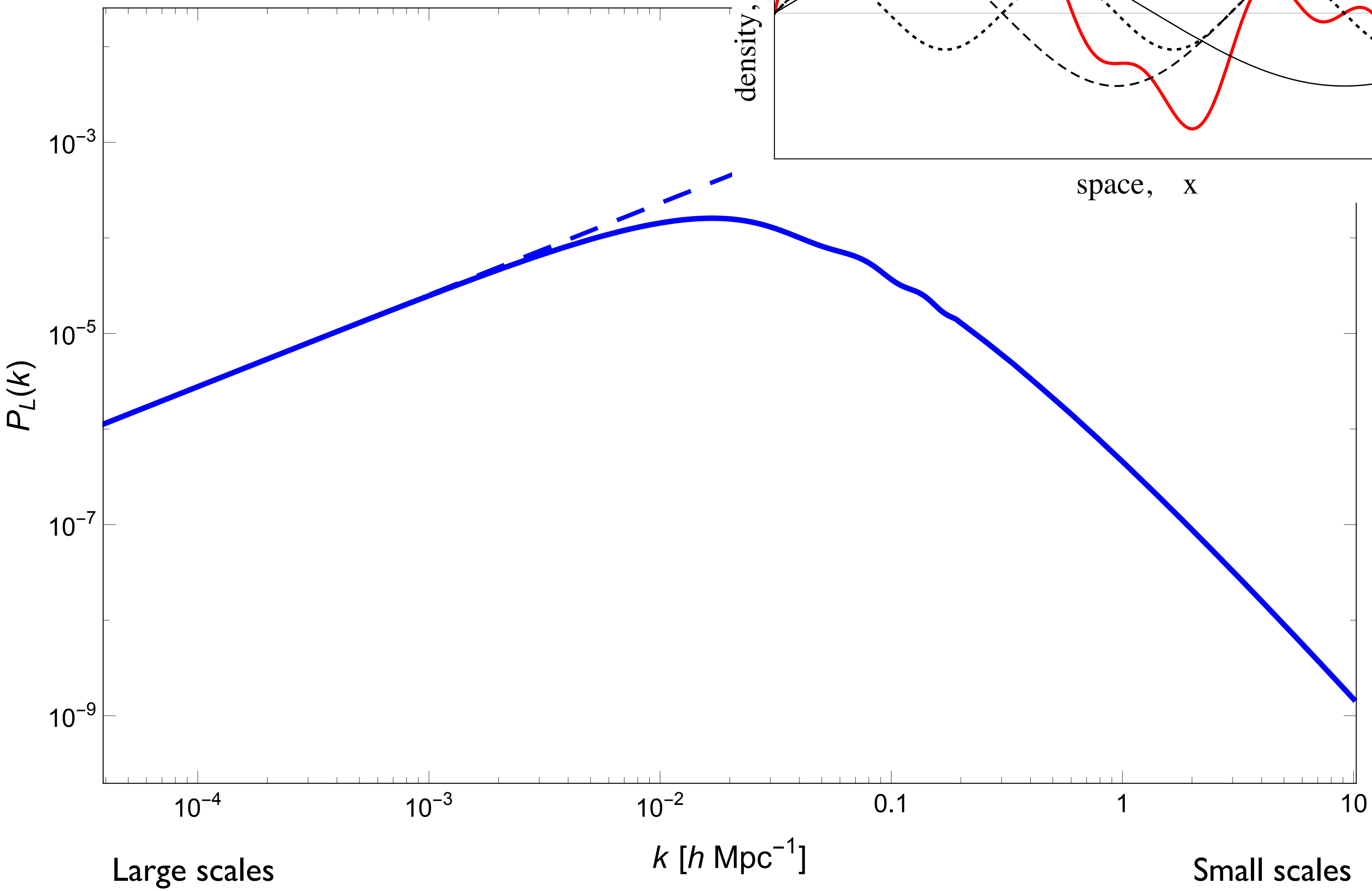




Evolution of matter perturbations: Equations of motion

Evolution of matter perturbations

We will consider now the following approximations for the evolution of matter perturbations:

1. **All matter is cold** (ignore the effects of baryons & neutrinos)

2. **Newtonian approximation:**

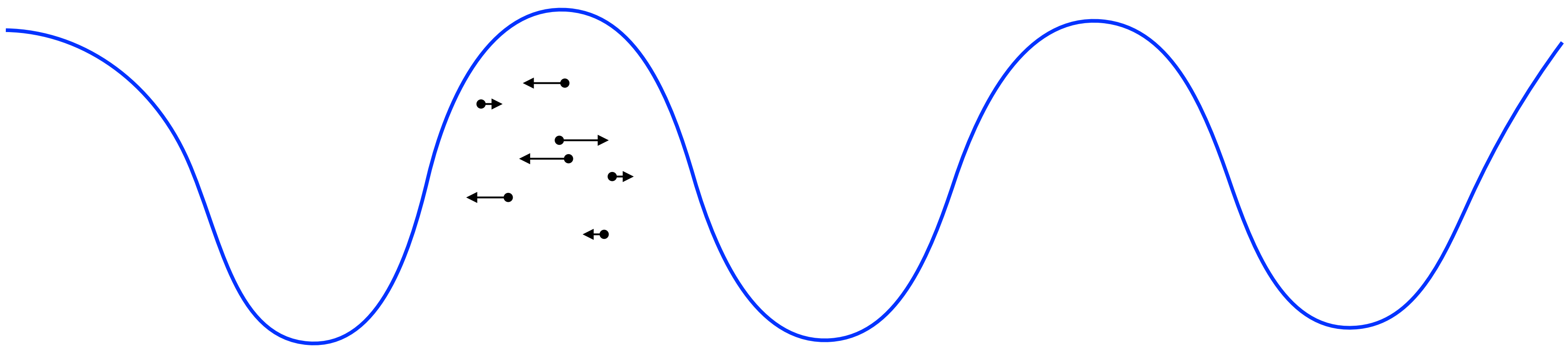
   $k \gg a\,H(a)$ scales much smaller than the horizon

   $v \ll c$ velocities much smaller than the speed of light

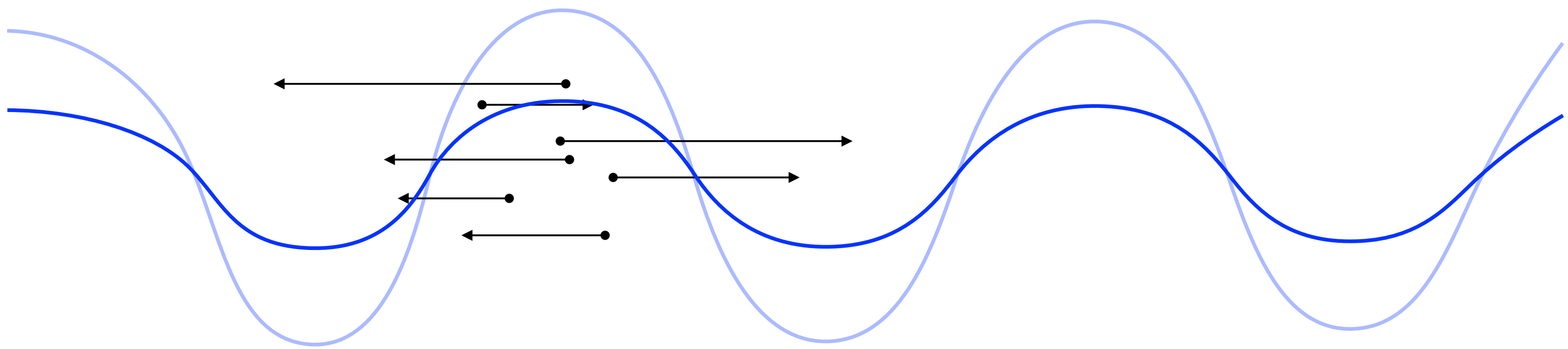
3. **Matter domination** (ignore effects of dark energy at late times)

Evolution of matter perturbations

Cold Dark Matter



Warm Dark Matter



Evolution of matter perturbations

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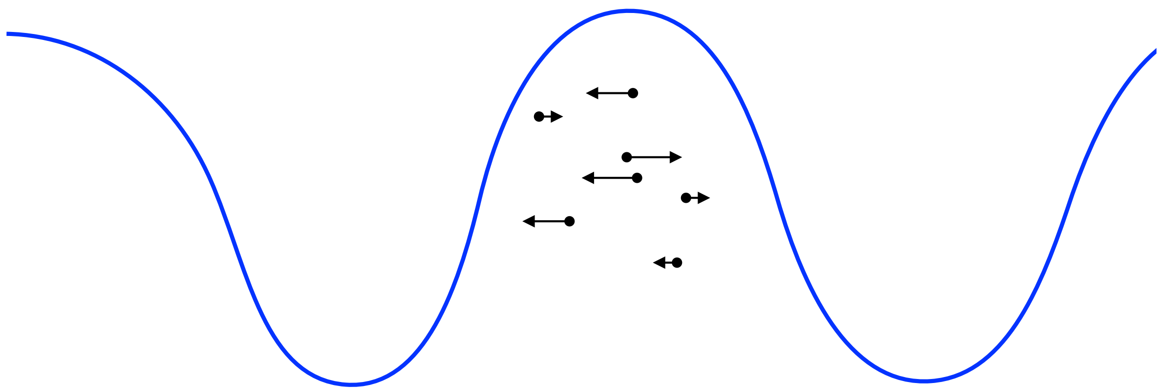

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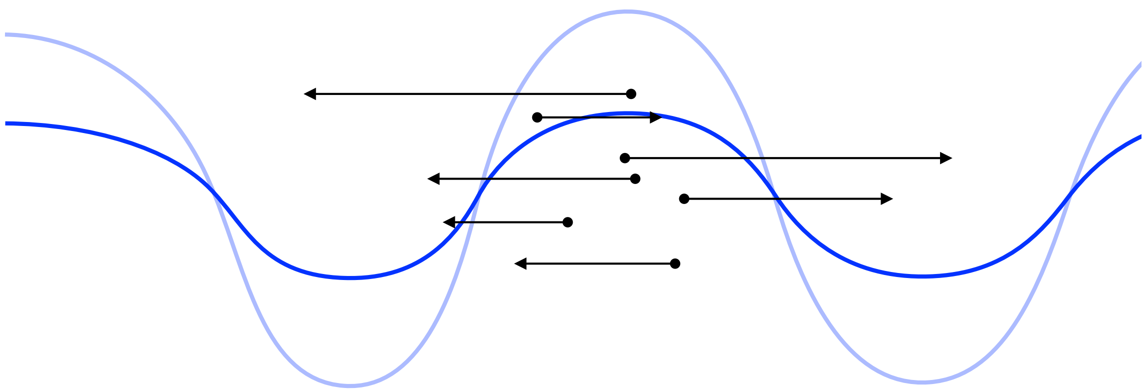
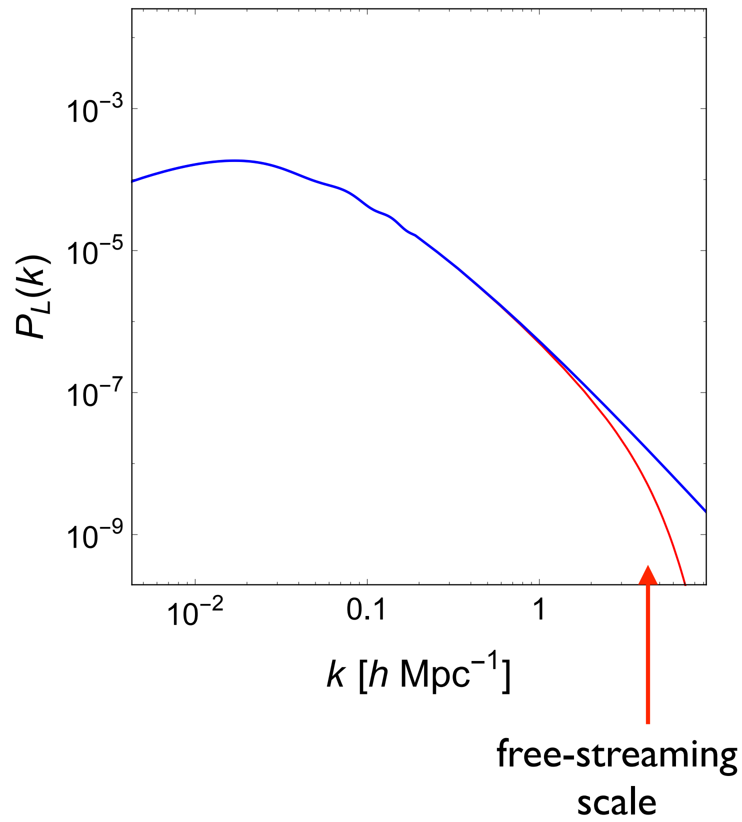




Fluid equations

Assuming **CDM as ideal fluid** we need the following equations:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}_r \cdot (\rho\,\vec{v}) = 0$$

continuity equation
(conservation of mass)

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}_r)\,\vec{v} = -\underbrace{\cancel{\frac{\vec{\nabla} p}{\rho}}}_{\text{pressure term (vanishing for CDM)}} - \underbrace{\vec{\nabla}_r \Phi_{tot}}_{\text{force}}$$

Euler's equation
(conservation of momentum)

$$\nabla_r^2 \Phi_{tot} = 4\pi G \rho$$

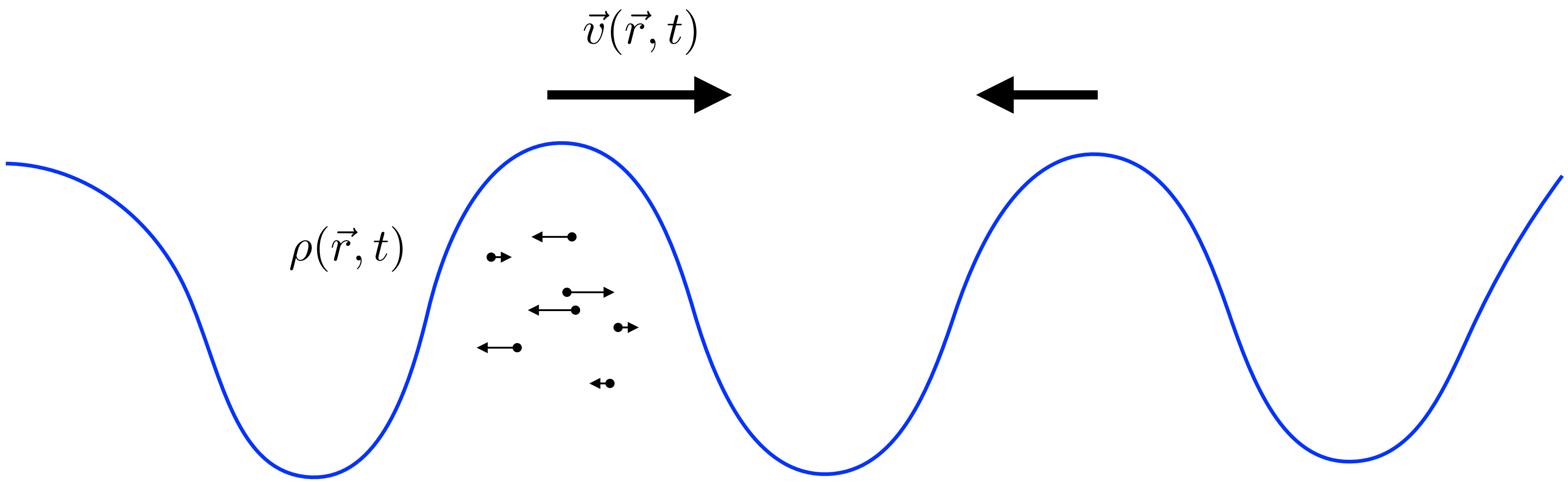
Poisson's equation

3 equations, 3 unknowns: ρ, v and Φ_{tot}

Single-stream approximation

Single-stream approximation

for **Cold** Dark Matter we can ignore the
thermal motion of individual particles,
and study the evolution of **perturbations**



Perturbations

We want the equations of motions for *perturbations* and as a function of comoving coordinates and conformal time

$$d\tau = \frac{dt}{a(t)}$$

For the **matter density** we have

$$\boxed{\rho(\vec{x},\tau) = \bar{\rho}(\tau)[1+\delta(\vec{x},\tau)]}$$

$\delta(\vec{x},\tau)$ **matter perturbations**

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$\delta(\vec{x},\tau)$ **matter perturbations**

For the **matter velocity**, instead we have

$$\vec{r} = a(t)\,\vec{x} \qquad \vec{v} \equiv \frac{d\vec{r}}{dt} = \frac{da}{dt}\vec{x} + a\frac{d\vec{x}}{dt} = H(\tau)\,\vec{r}(\tau) + \vec{u}(\vec{x},\tau) \qquad \mathcal{H} \equiv \frac{1}{a}\frac{da}{d\tau} = a\,H$$

Hubble flow

$$\boxed{\vec{v}(\vec{x},\tau) = \mathcal{H}(\tau)\,\vec{x}(\tau) + \vec{u}(\vec{x},\tau)}$$

$\vec{u}(\vec{x},\tau)$ **peculiar velocities**

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$\vec{u}(\vec{x},\tau)$ **peculiar velocities**

$$\boxed{\Phi_{tot}(\vec{x},\tau) = \bar{\Phi}(\vec{x},\tau) + \Phi(\vec{x},\tau)}$$

$\Phi(\vec{x},\tau)$ **gravitational potential perturbations**

Equations for the perturbations

Assuming **CDM as ideal fluid** we need the following equations:

$$\boxed{\frac{\partial \delta}{\partial \tau} + \vec{\nabla} \cdot [(1+\delta)\, \vec{u}] = 0}$$

continuity equation

$$\boxed{\frac{\partial \vec{u}}{\partial \tau} + \mathcal{H}\vec{u} + (\vec{u} \cdot \vec{\nabla})\, \vec{u} = -\vec{\nabla}\Phi}$$

Euler's equation

$$\nabla^2 \Phi = 4\pi G\, \bar{\rho}\, a^2 \delta$$ but from Friedmann's eq. $$\mathcal{H}^2 = \frac{8\pi G}{3}\, a^2\, \bar{\rho}$$

$$\boxed{\nabla^2 \Phi = \frac{3}{2}\, \mathcal{H}^2 \delta}$$

Poisson's equation

Again: 3 equations, 3 unknowns: δ , $\vec{u}$ and Φ

Equations for the perturbations

Linearizing ...

$$\boxed{\frac{\partial \delta}{\partial \tau} + \vec{\nabla} \cdot \left[(1+\delta)\, \vec{u} \right] = 0}$$

continuity equation

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Euler's equation

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Poisson's equation

Equations for the perturbations

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Poisson's equation

Equations for the perturbations

Linearizing ...

$$\frac{\partial \delta}{\partial \tau} + \vec{\nabla} \cdot \vec{u} = 0$$

continuity equation

$$\vec{\nabla} \cdot \left(\frac{\partial \vec{u}}{\partial \tau} + \mathcal{H} \vec{u} = -\vec{\nabla} \Phi \right)$$

Euler's equation

$$\nabla^2 \Phi = \frac{3}{2} \mathcal{H}^2 \delta$$

Poisson's equation

then introducing the **velocity divergence**

$$\theta(\vec{x}, \tau) \equiv \vec{\nabla} \cdot \vec{u}(\vec{x}, \tau)$$

Linear equations for the perturbations

$$\frac{\partial \delta}{\partial \tau} + \theta = 0$$

continuity equation

$$\frac{\partial \theta}{\partial \tau} + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}^2\,\delta = 0$$

Euler's equation

$$\boxed{\frac{\partial^2 \delta}{\partial \tau^2} + \mathcal{H}\frac{\partial \delta}{\partial \tau} - \frac{3}{2}\mathcal{H}^2\,\delta = 0}$$

2nd order equation

friction gravity

where (for a flat, matter-dominated Universe) $\mathcal{H} = \frac{1}{a}\frac{da}{d\tau} = \frac{2}{\tau}$

Linear growth of perturbations

$$\frac{\partial^2 \delta_{\vec{k}}}{\partial \tau^2} + \mathcal{H}\frac{\partial \delta_{\vec{k}}}{\partial \tau} - \frac{3}{2}\mathcal{H}^2\,\delta_{\vec{k}} = 0$$

2nd order equation in Fourier space

Linear growth of perturbations

$$\frac{\partial^2 \delta_{\vec{k}}}{\partial \tau^2} + \mathcal{H} \frac{\partial \delta_{\vec{k}}}{\partial \tau} - \frac{3}{2} \mathcal{H}^2 \, \delta_{\vec{k}} = 0$$

2nd order equation in Fourier space

Look for a separable solution like $\delta_{\vec{k}}(\tau) = D(\tau) \, A_{\vec{k}}$ $\quad D(\tau)$ **growth factor**

$$\rightarrow \begin{cases} D_+(a) \sim a & \text{growing mode} \\ D_-(a) \sim a^{-3/2} & \text{decaying mode} \end{cases}$$

$$\boxed{\begin{aligned} \delta_{\vec{k}}(a) &= A_{\vec{k}} \, a + B_{\vec{k}} \, a^{-3/2} \\ \theta_{\vec{k}}(a) &= -\frac{\partial \delta_{\vec{k}}}{\partial \tau} = -\mathcal{H} \left(A_{\vec{k}} \, a - \frac{3}{2} \, B_{\vec{k}} \, a^{-3/2} \right) \end{aligned}}$$

Linear growth of perturbations

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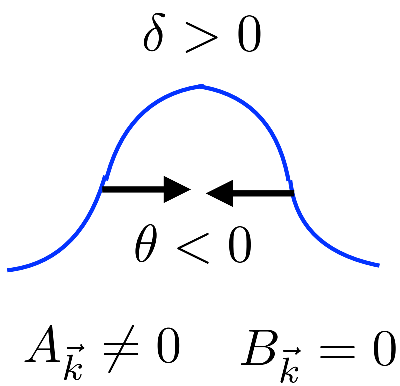
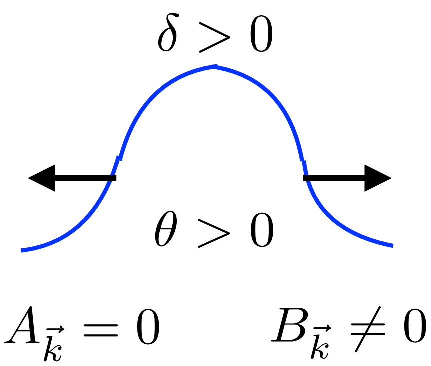
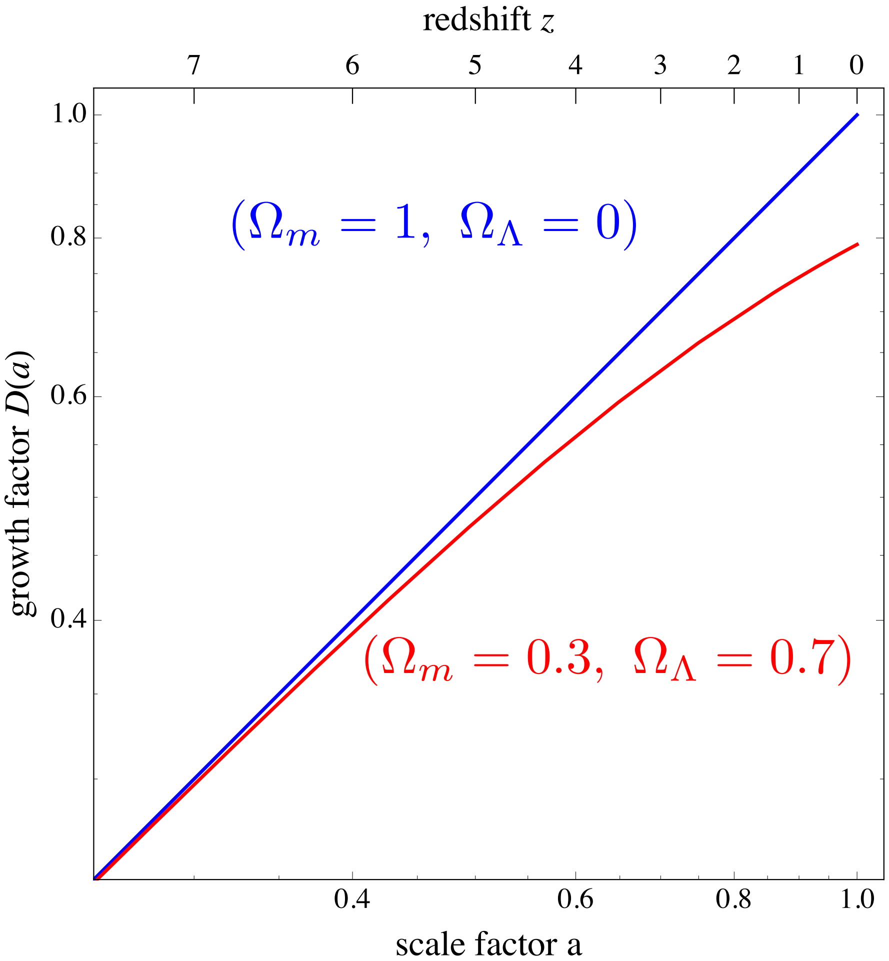
growing mode



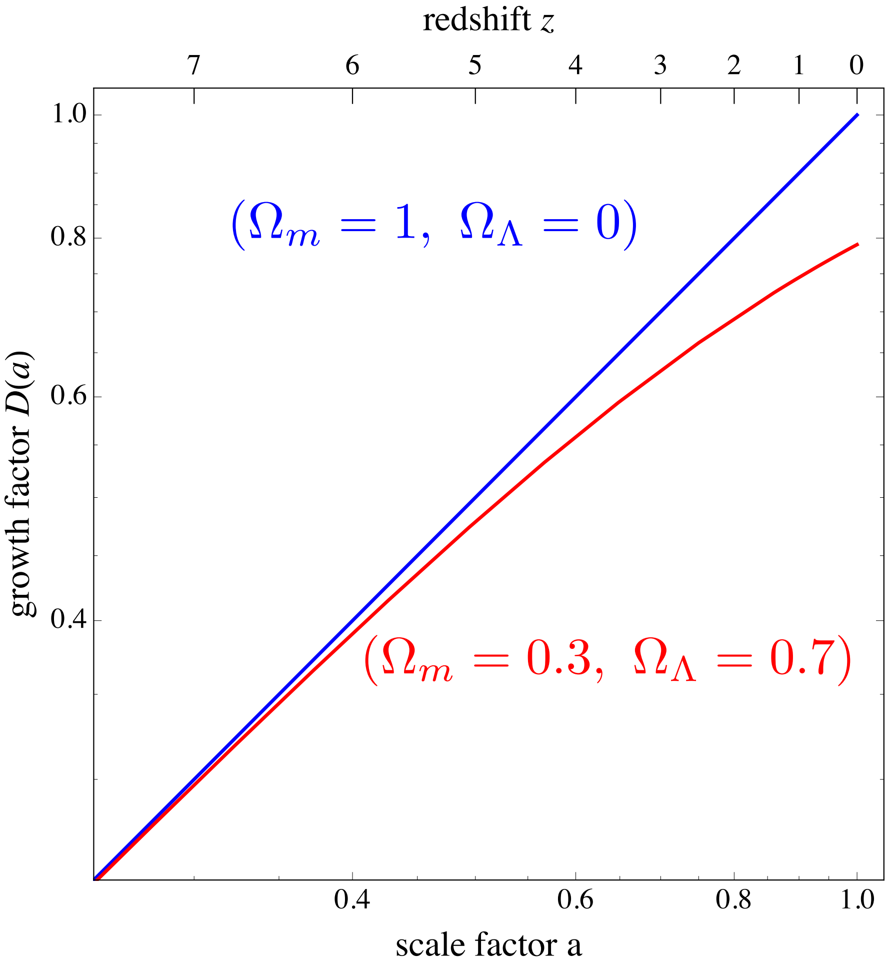


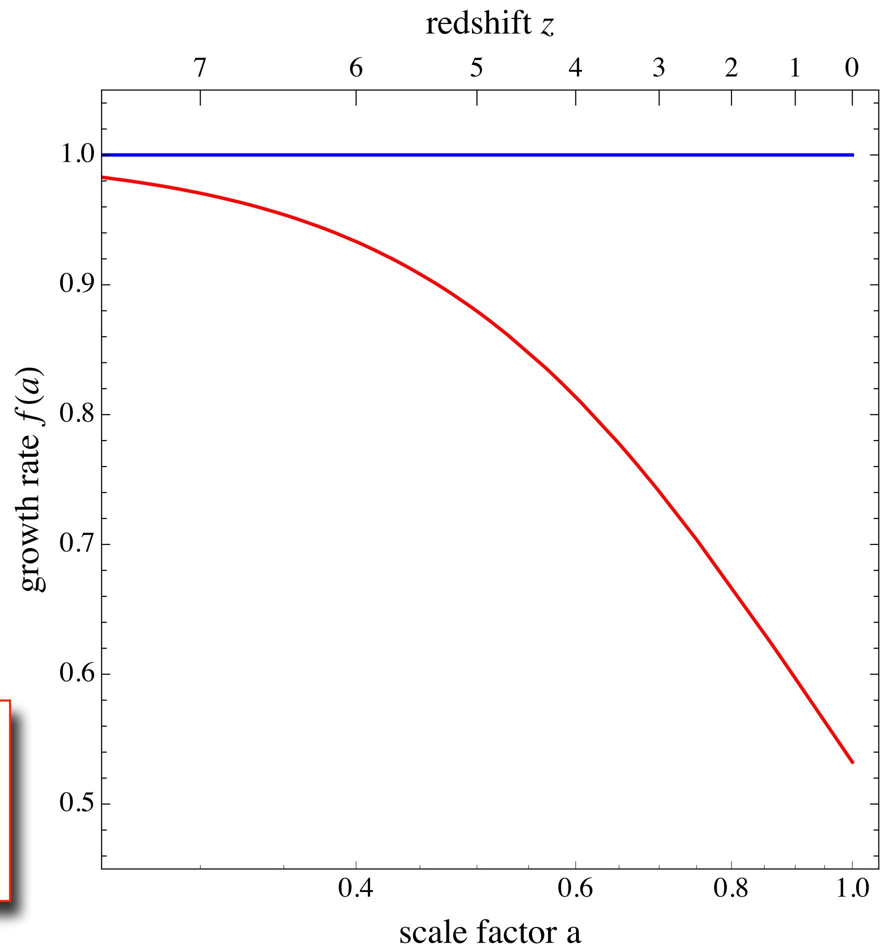
decaying mode

Linear growth in a ΛCDM cosmology



Linear growth in a flat, ΛCDM cosmology

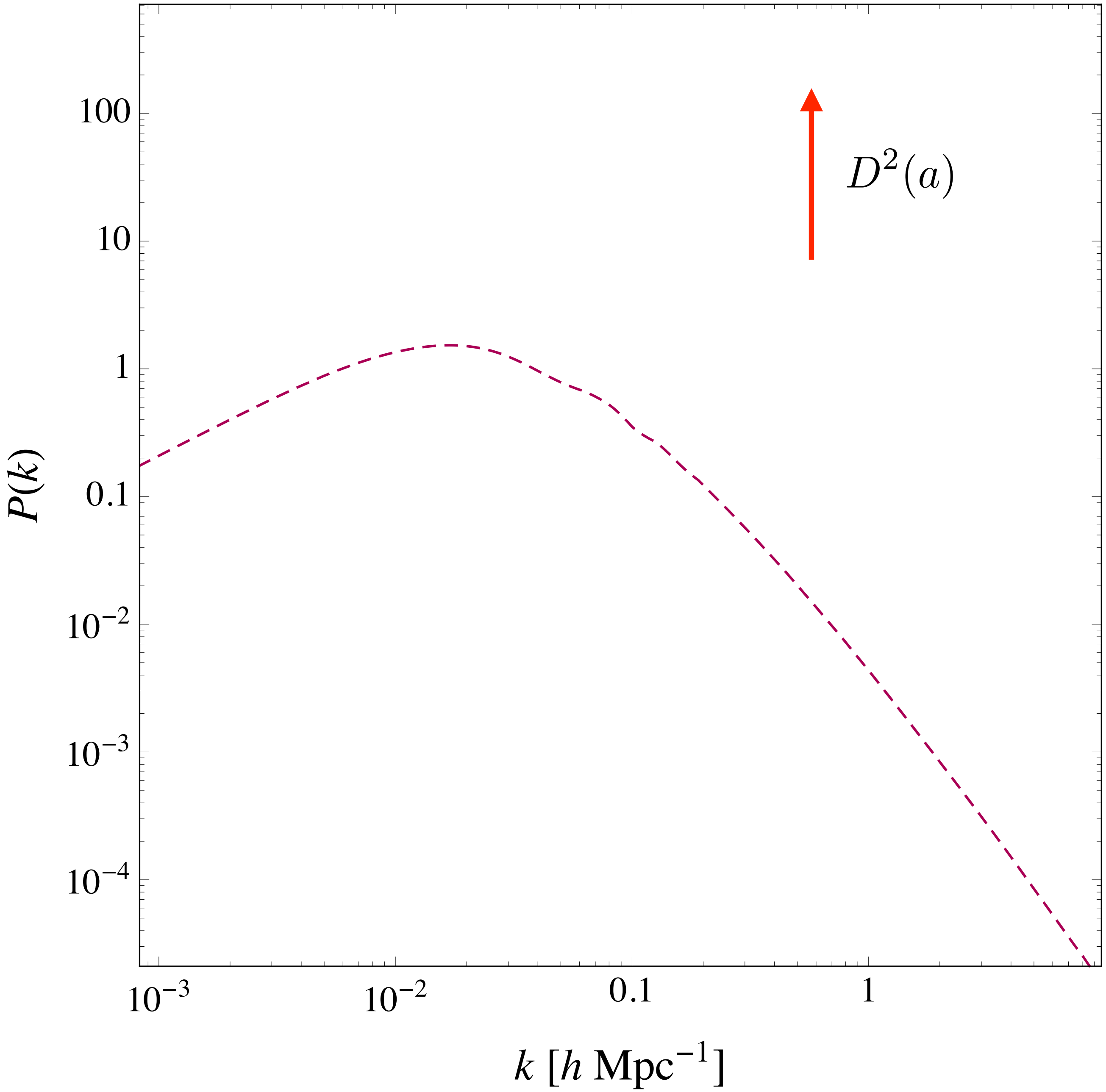




growth *rate*

$$f \equiv \frac{d \ln D}{d \ln a} \simeq \Omega_m^{0.55}(z)$$

Linear vs Nonlinear evolution

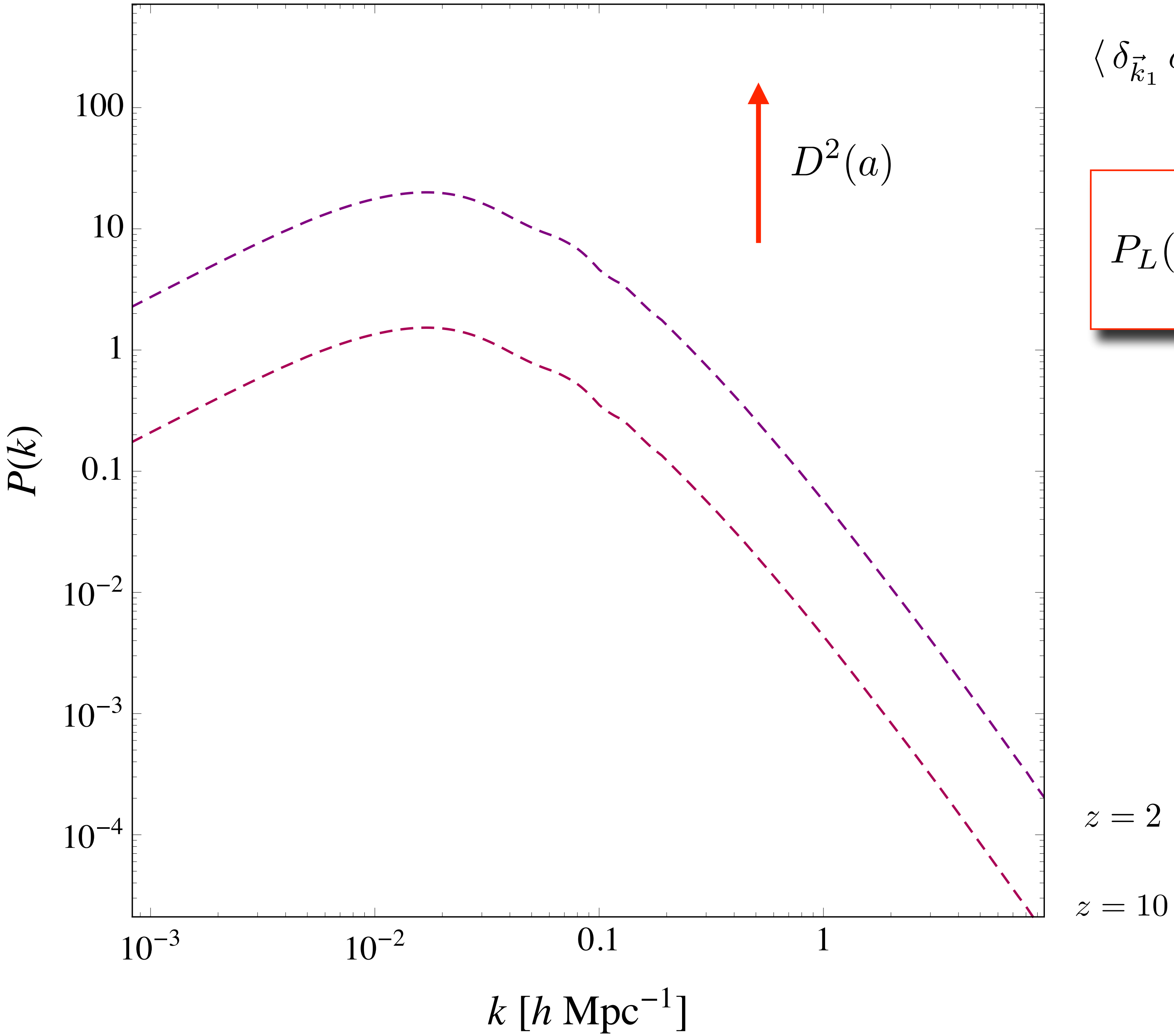


$$\langle\, \delta_{\vec{k}_1}\, \delta_{\vec{k}_2} \rangle = \delta_D(\vec{k}_1 + \vec{k}_2)\, P(k_1)$$

$$\boxed{P_L(k,a) = D^2(a)\, P_0(k)}$$

$z = 10$

Linear vs Nonlinear evolution



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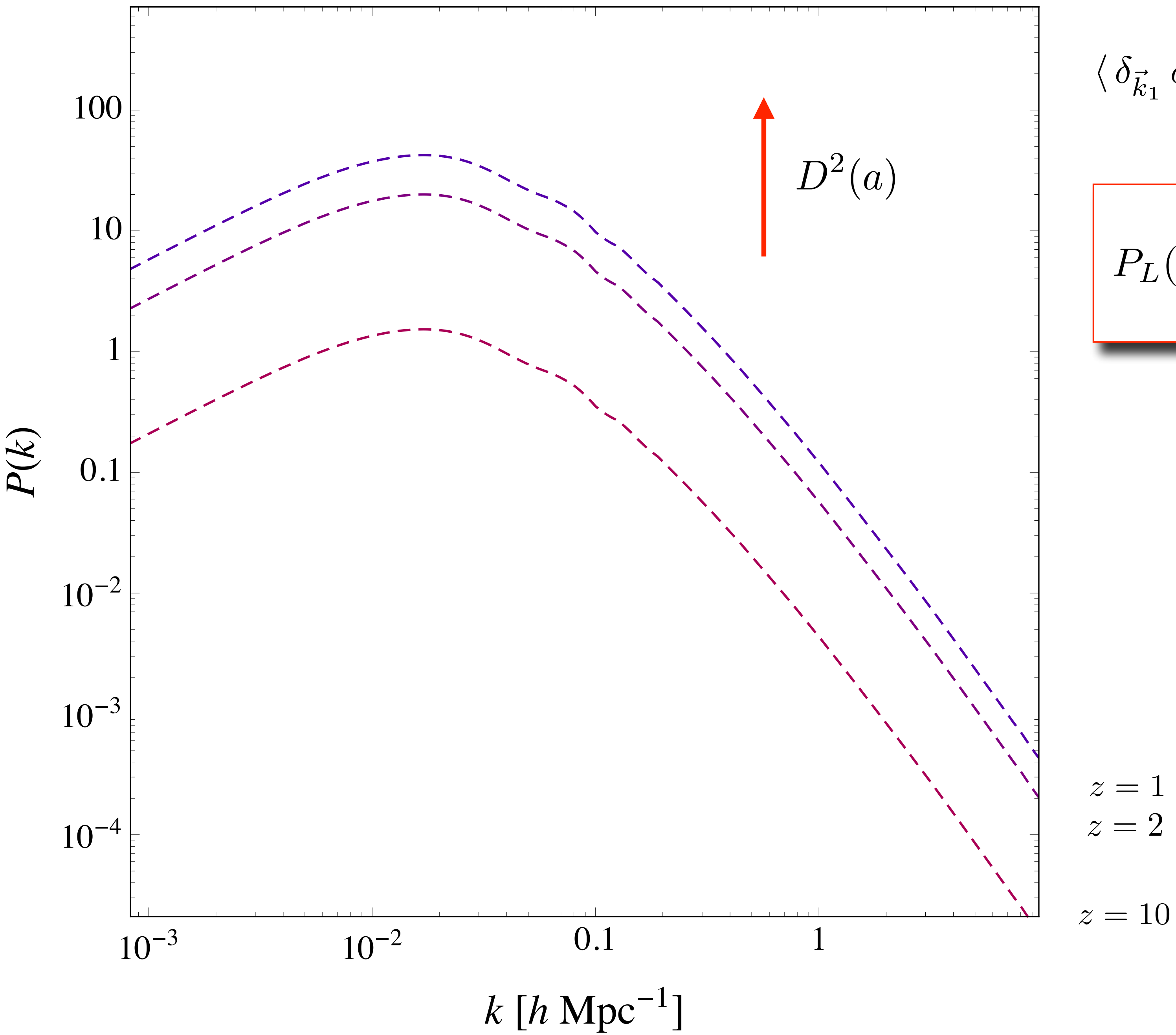

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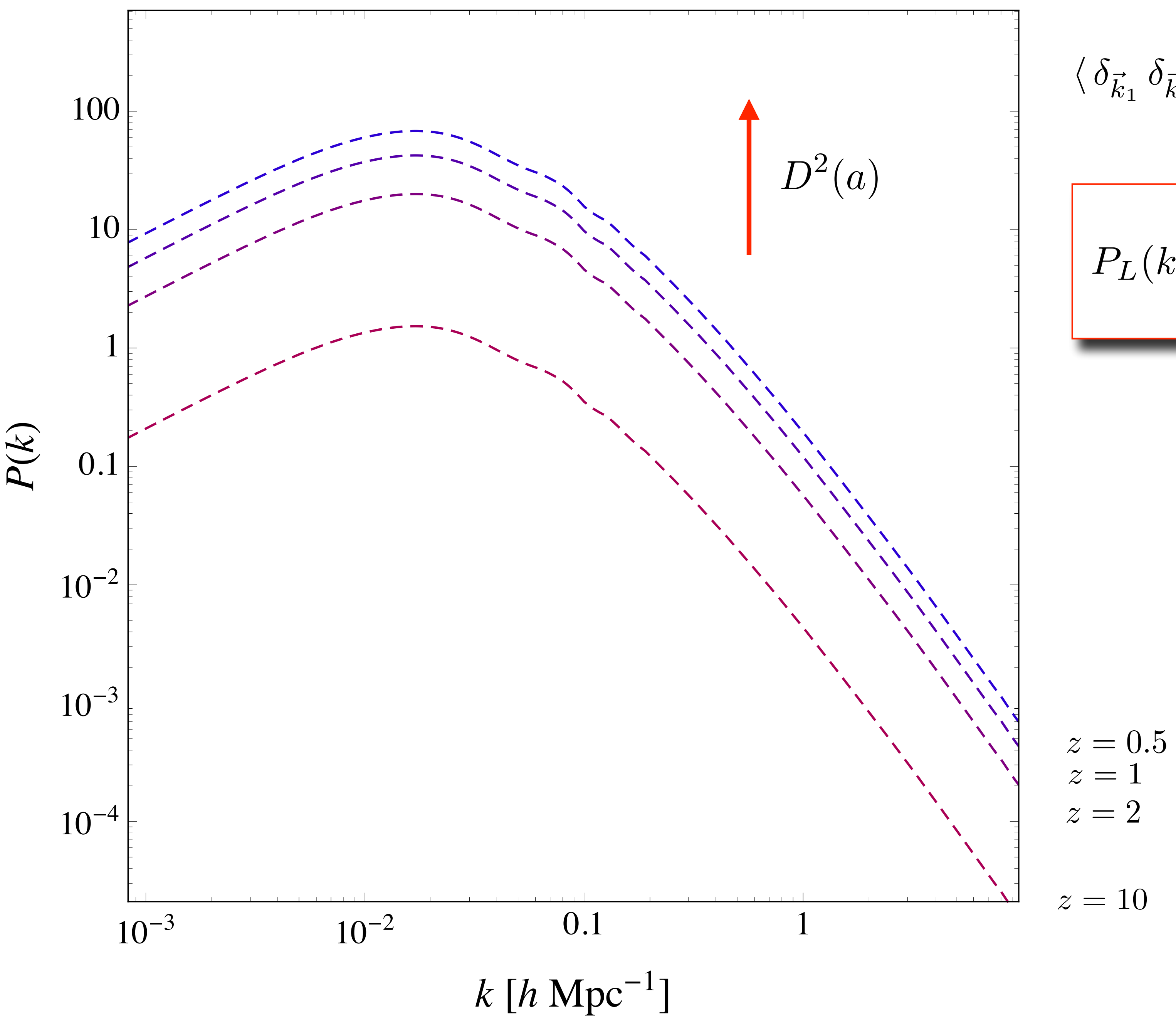

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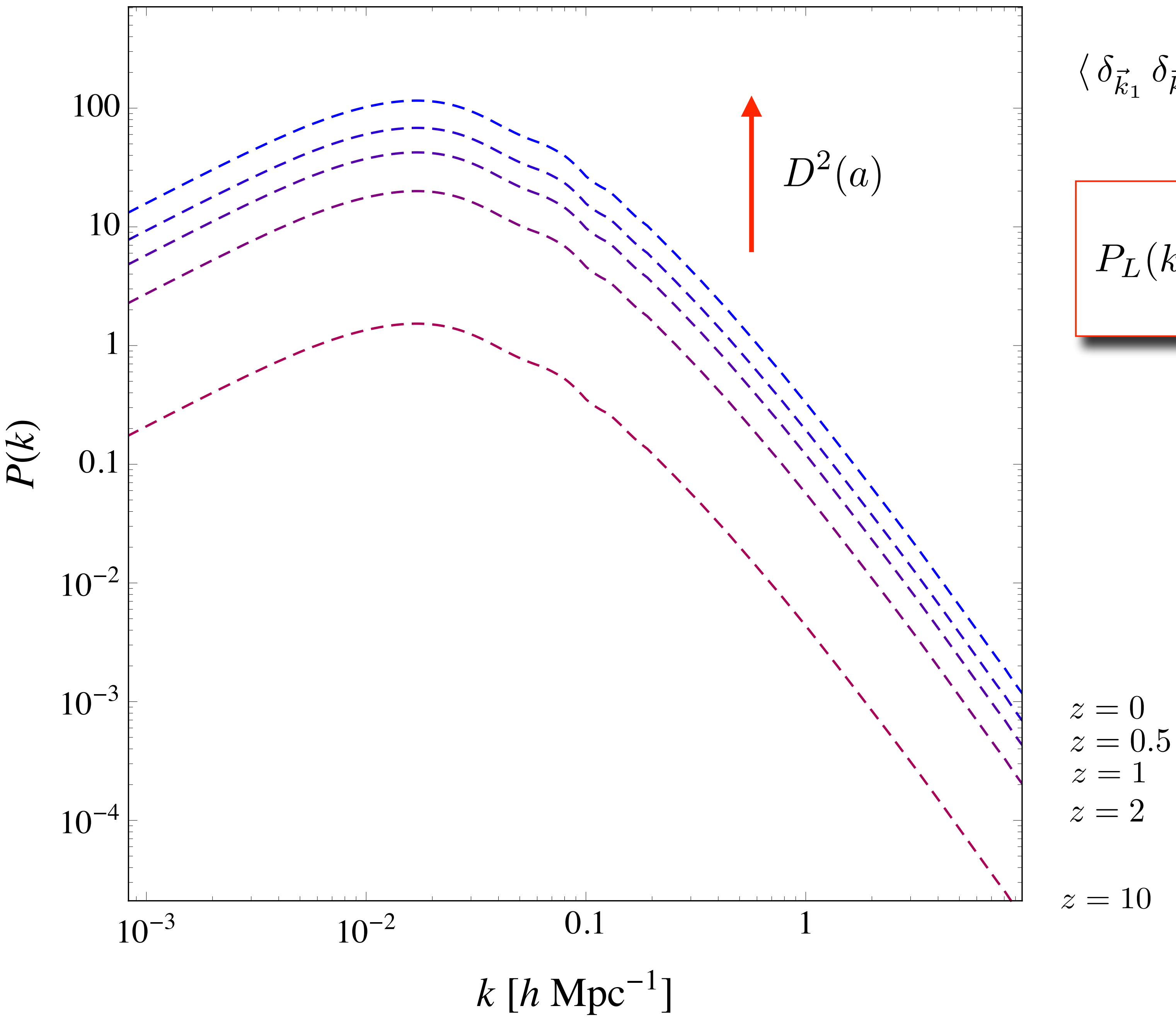

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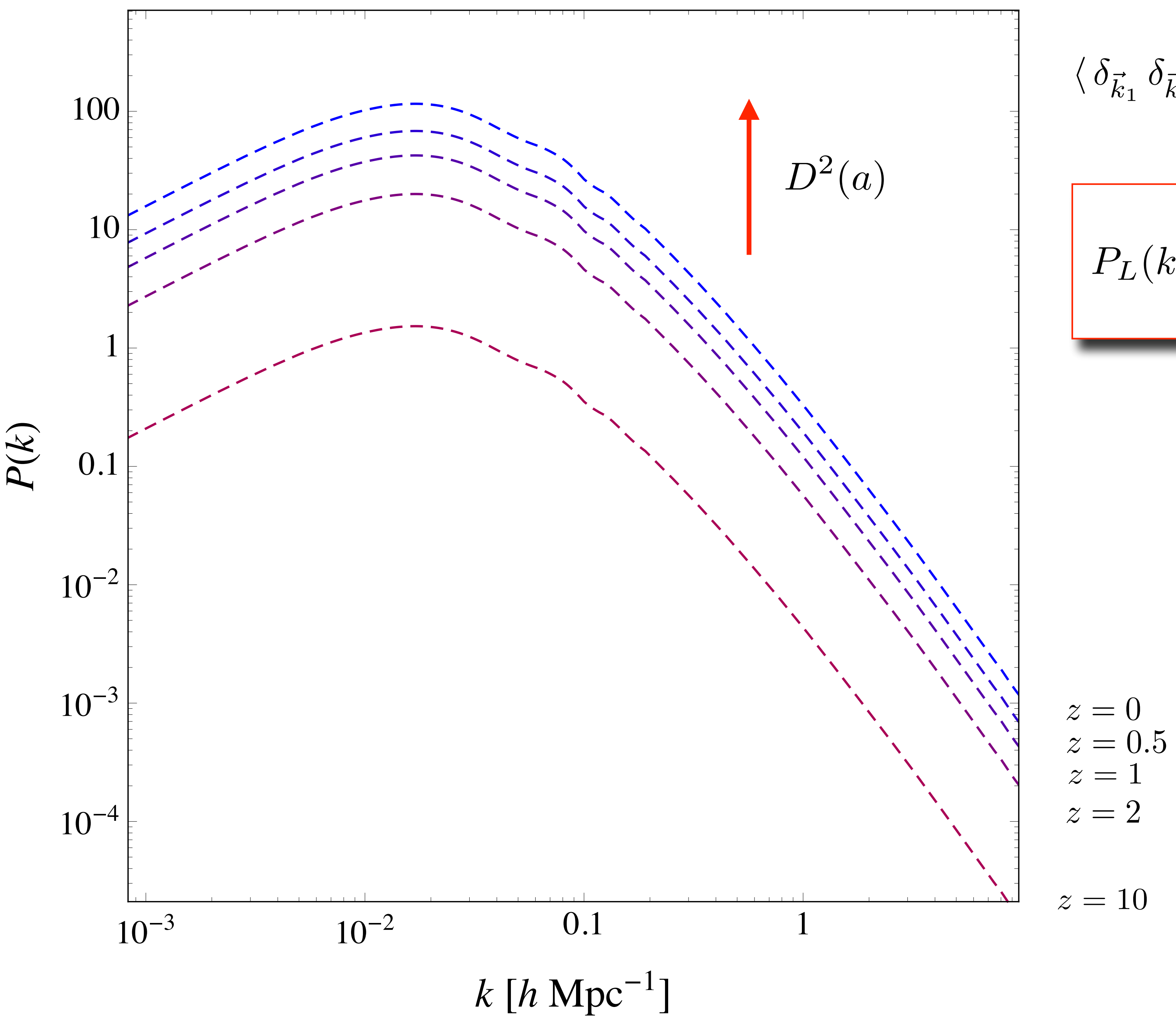

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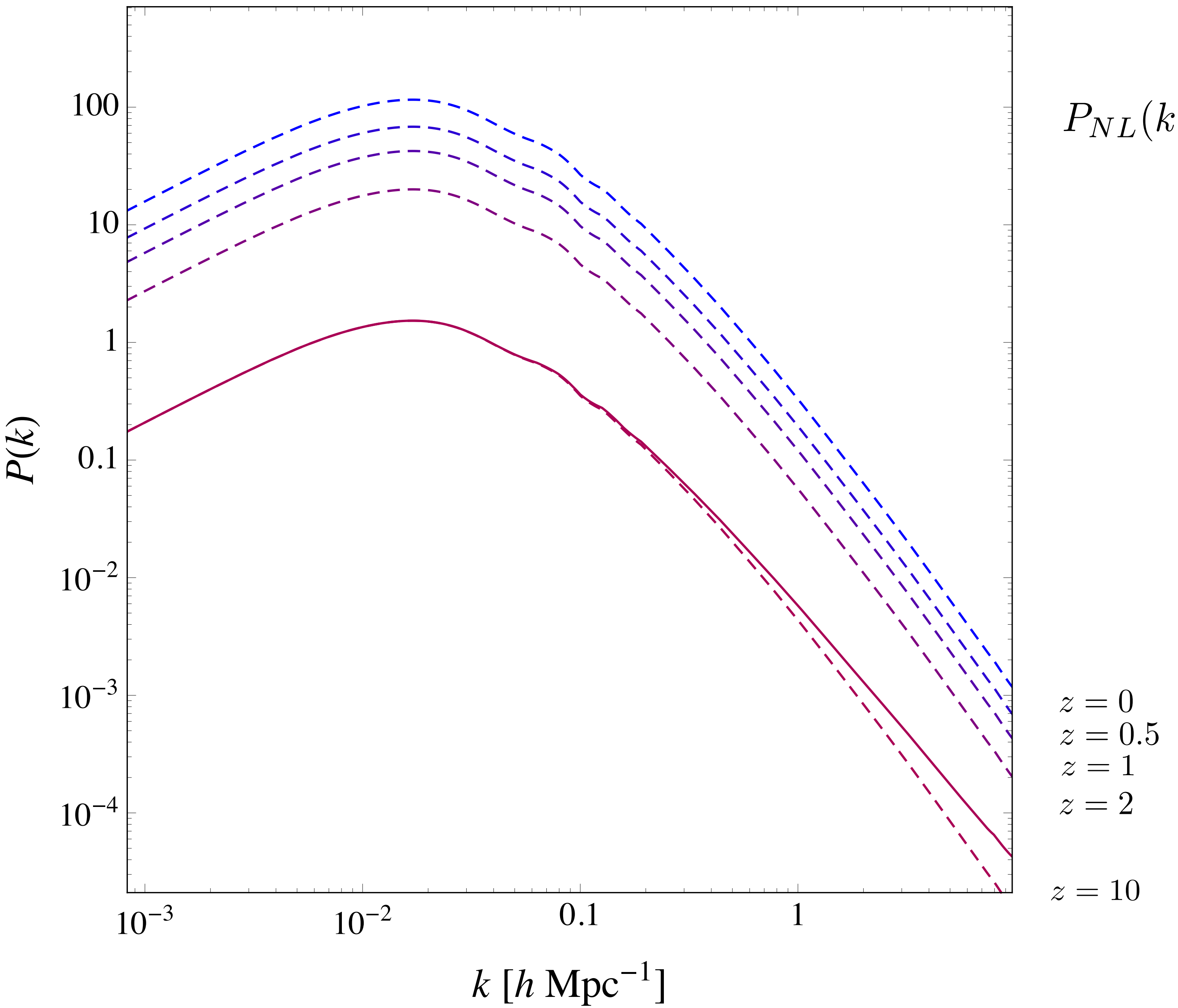
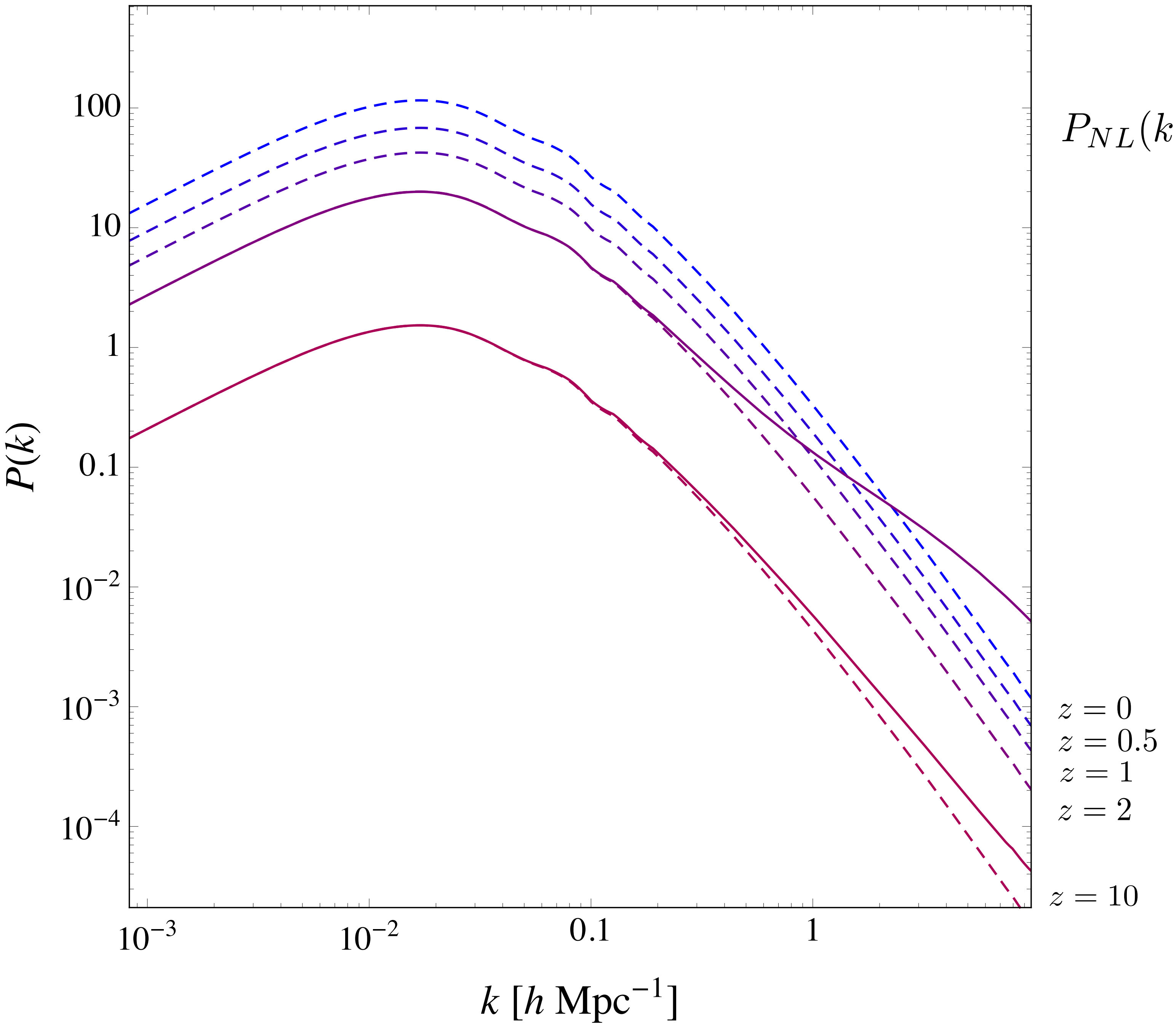


$$P_{NL}(k) = P_L(k) + \Delta P_{NL}(k)$$

nonlinear corrections!

Linear vs Nonlinear evolution



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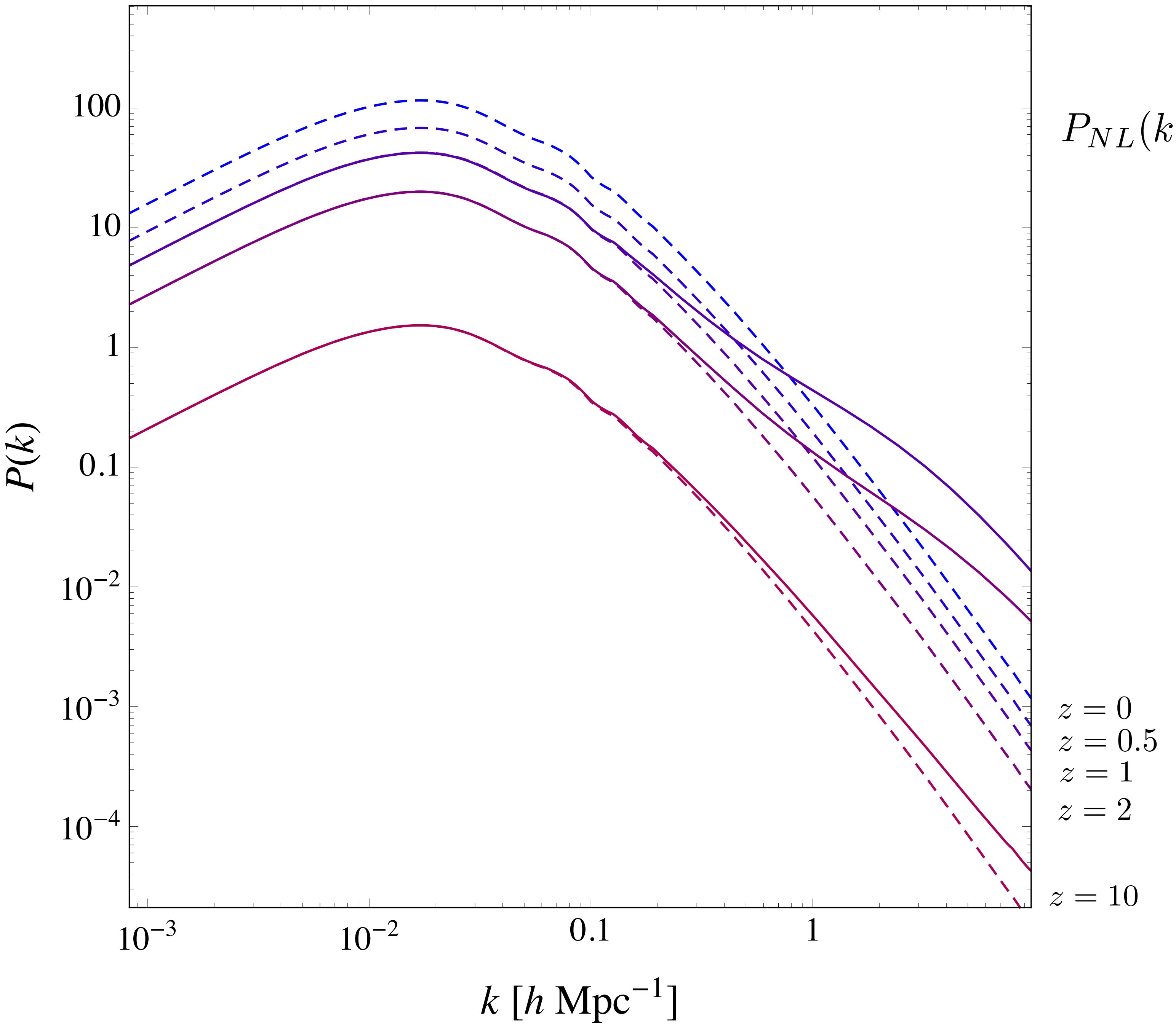
Linear vs Nonlinear evolution



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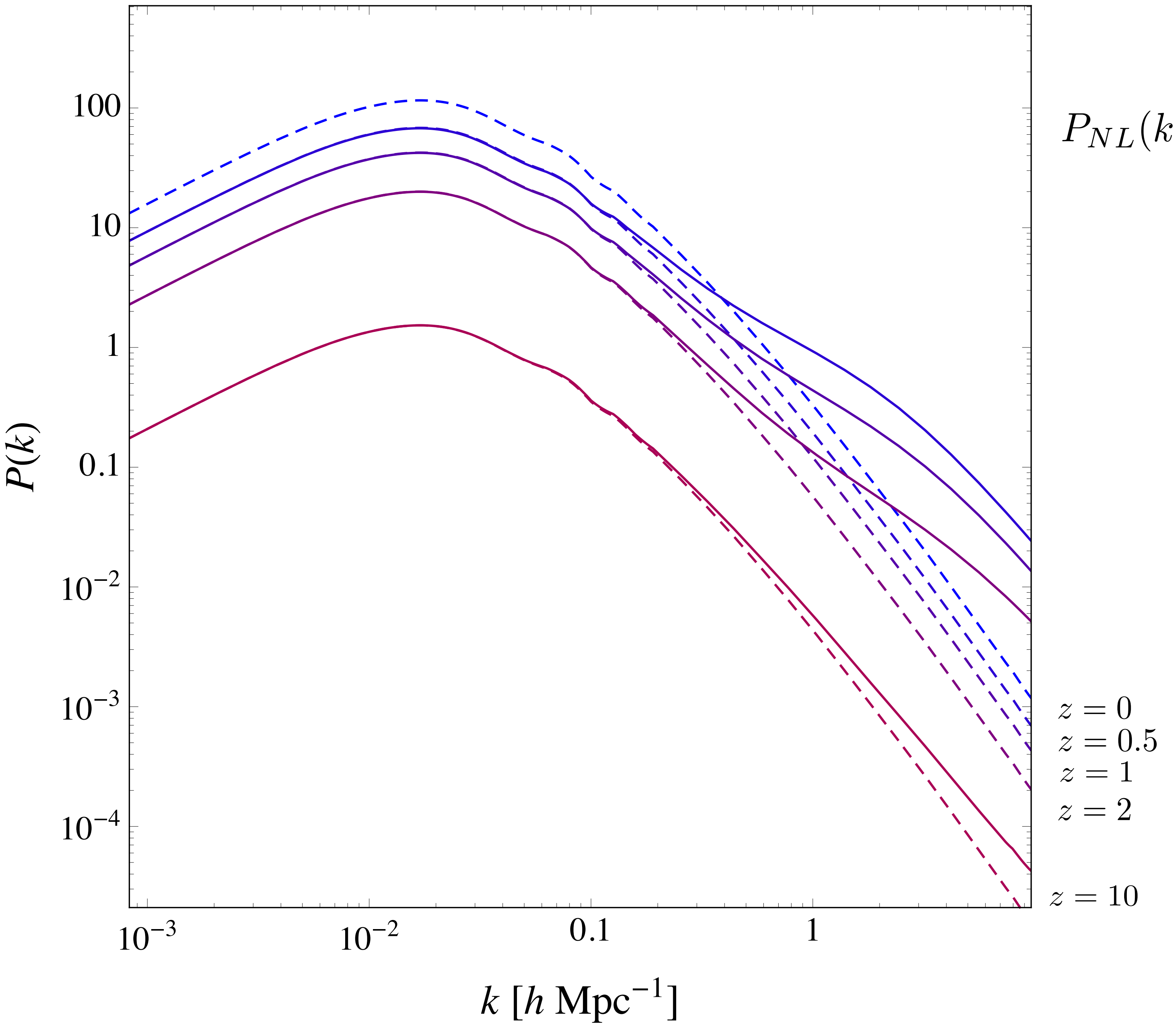

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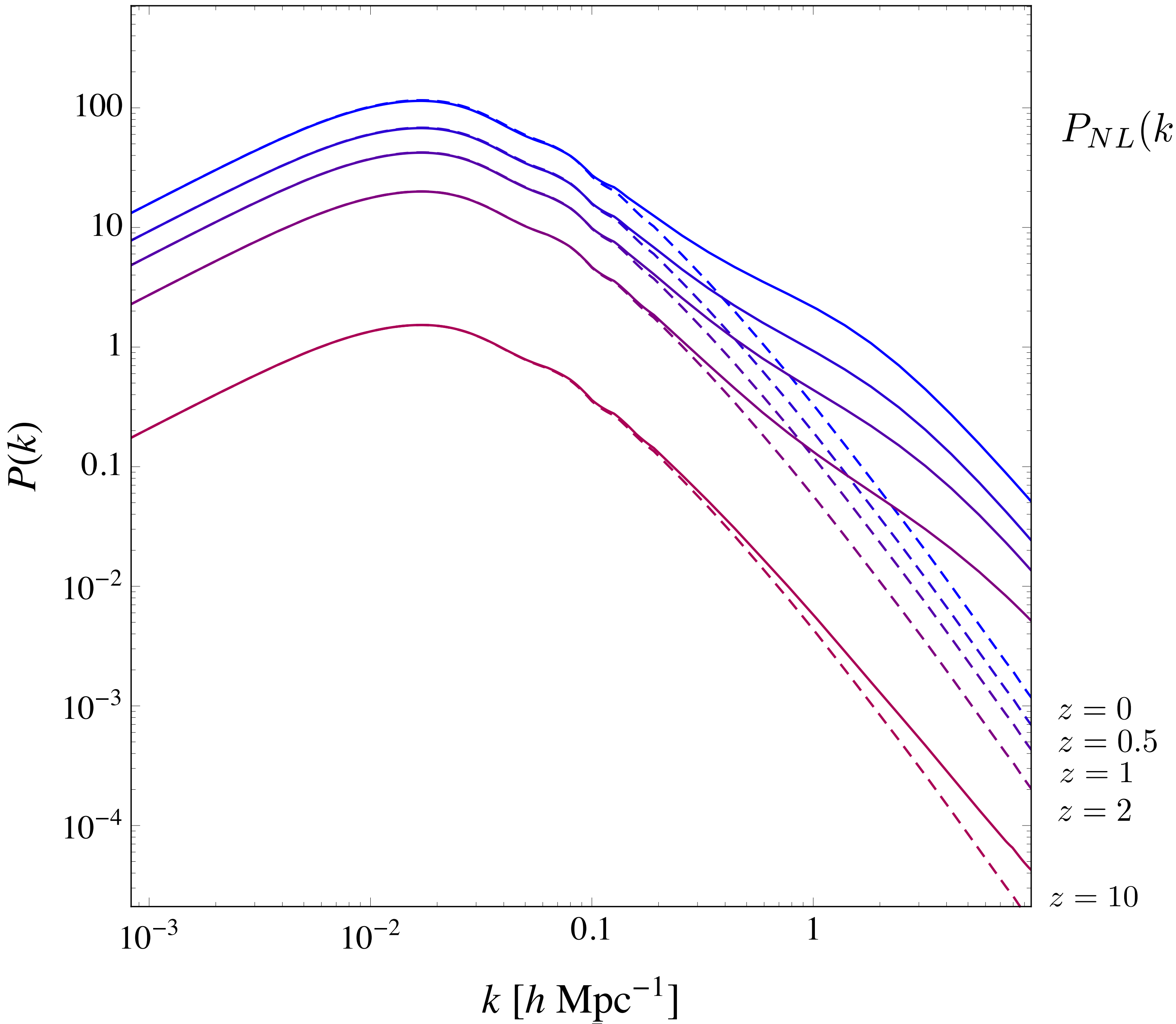

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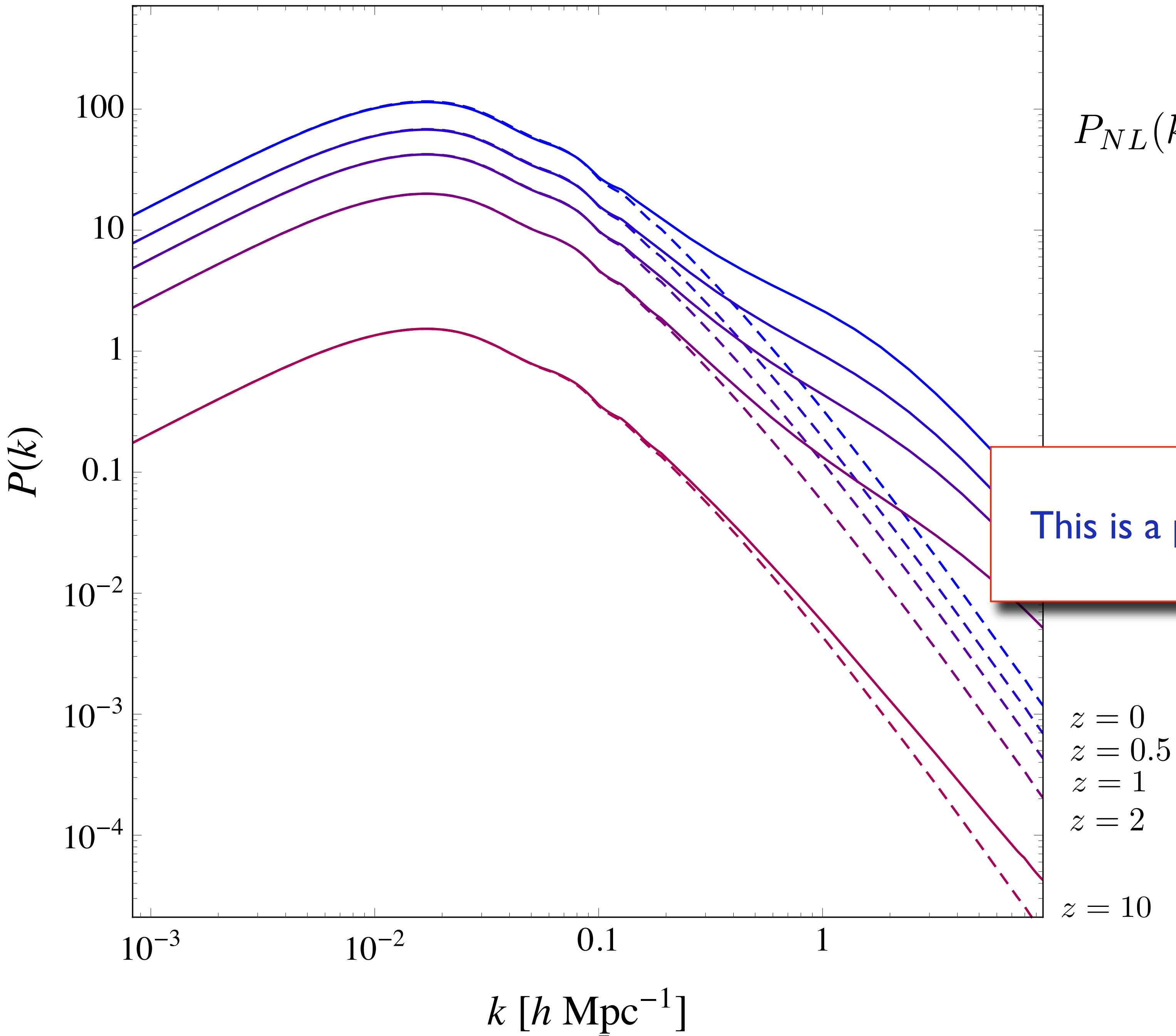

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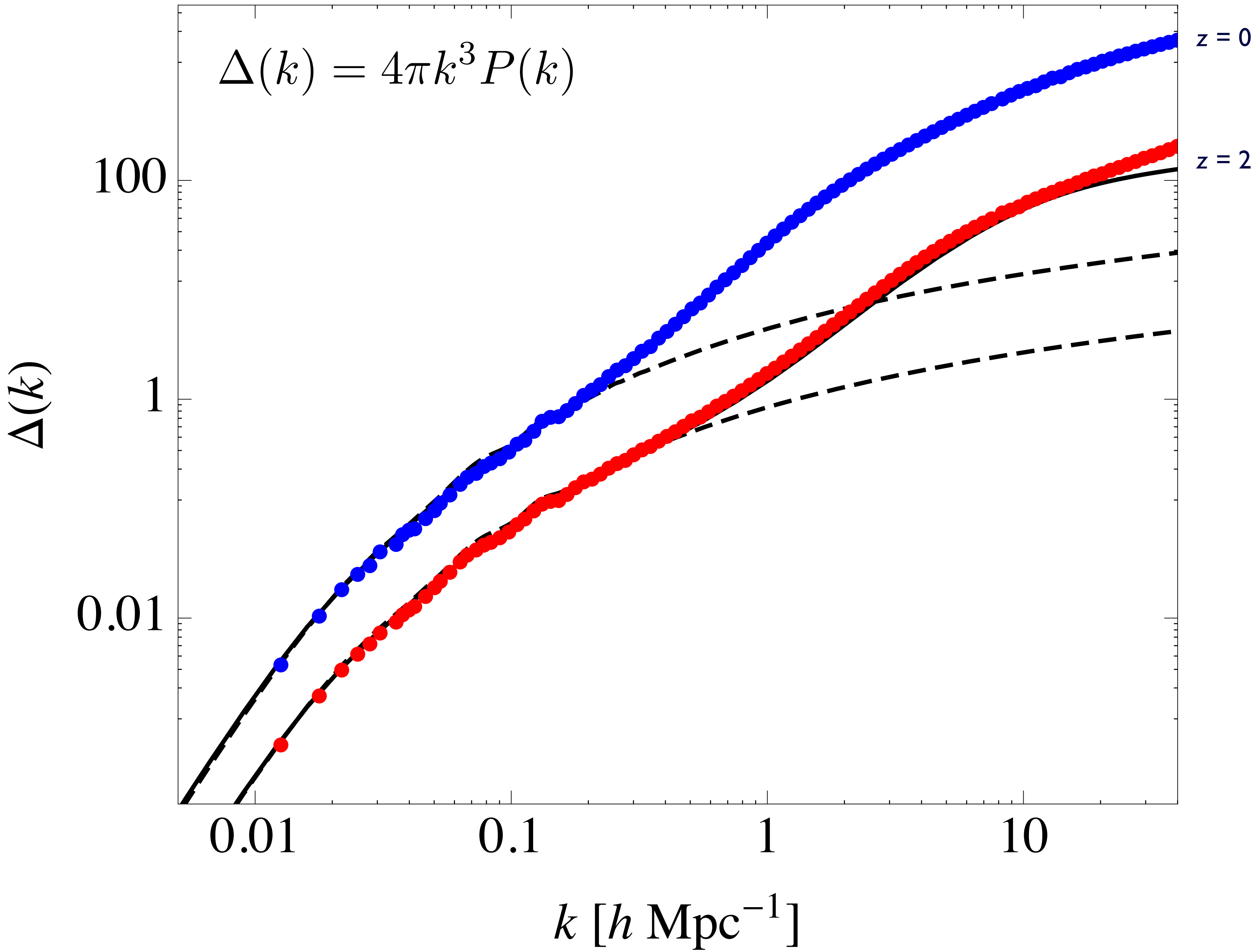
nonlinear corrections!

This is a proof of **Dark Matter**!

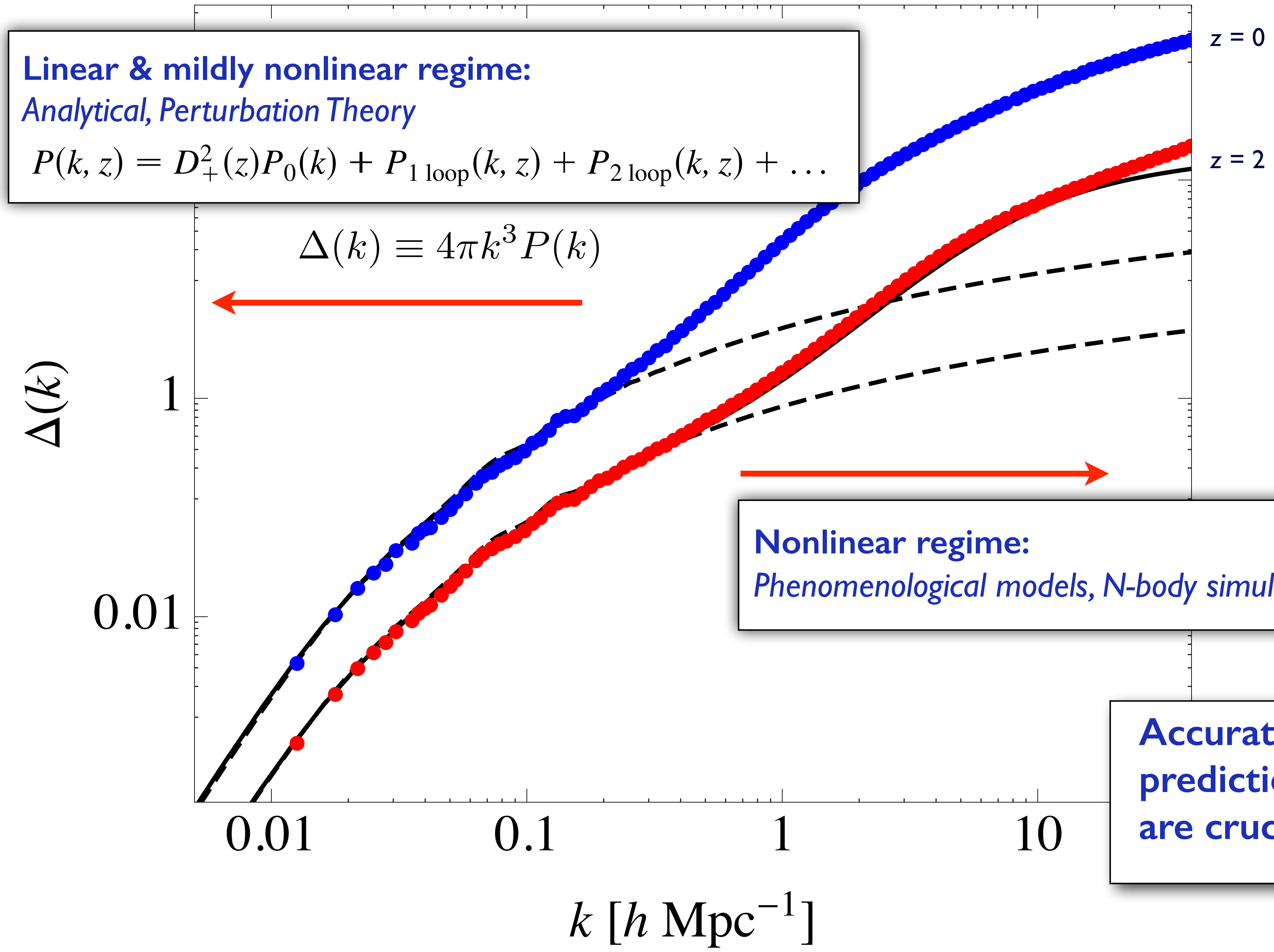
The growth of matter perturbations



The growth of matter perturbations

Linear & mildly nonlinear regime:
Analytical, Perturbation Theory

$$P(k,z) = D_+^2(z)P_0(k) + P_{1\,\mathrm{loop}}(k,z) + P_{2\,\mathrm{loop}}(k,z) + \ldots$$



Nonlinear regime:
Phenomenological models, N-body simulations

Accurate predictions are crucial!