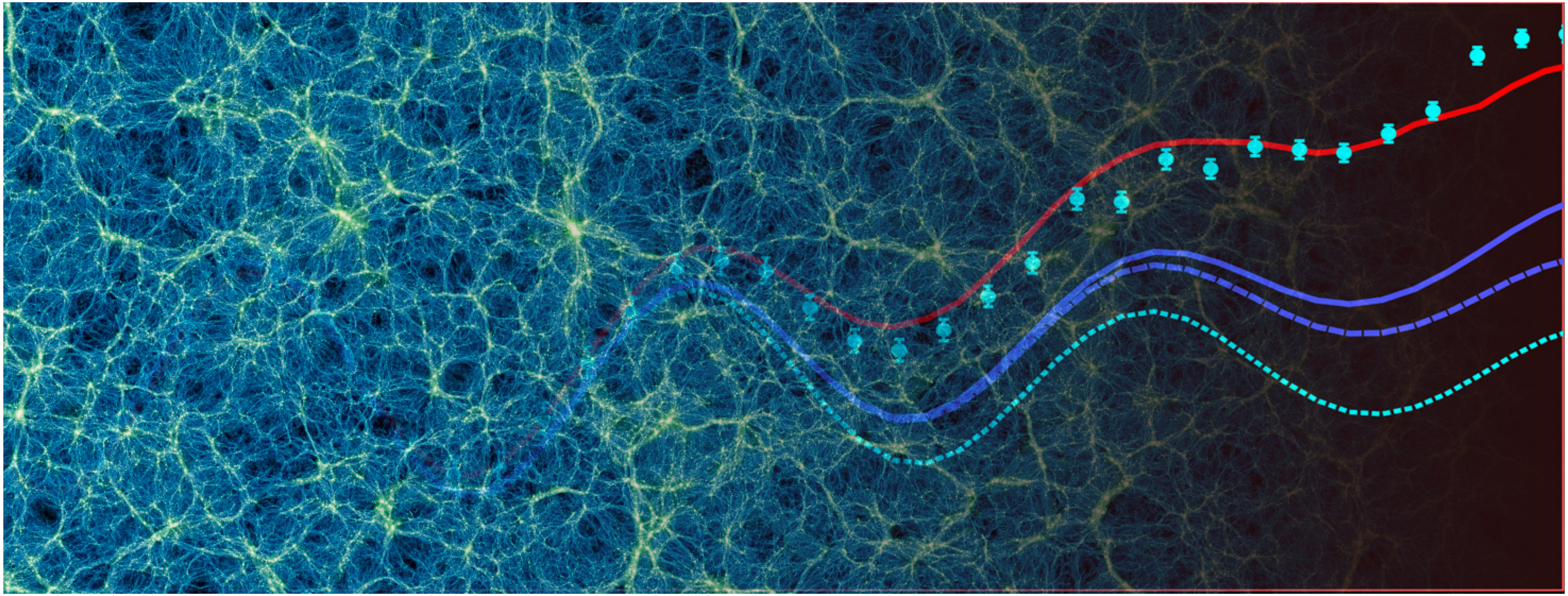
Cosmology and the Large Scale Structure of the Universe



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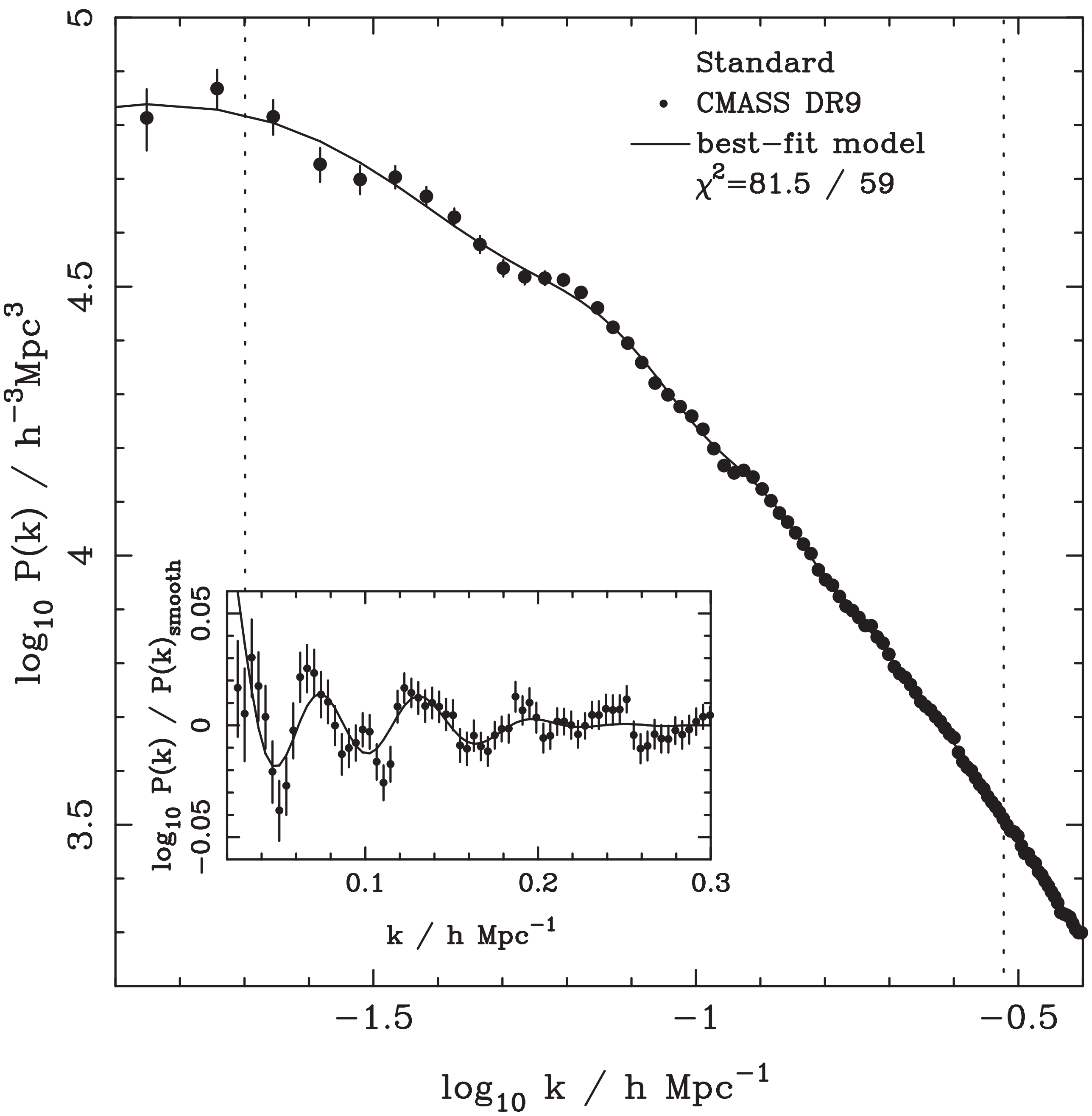
The goal

The galaxy
power spectrum:

what is it and
why it matters



Outlook

Homogeneous cosmology

Density perturbations

The growth of matter perturbations

The power spectrum

Features of the galaxy power spectrum:
BAO & RSD

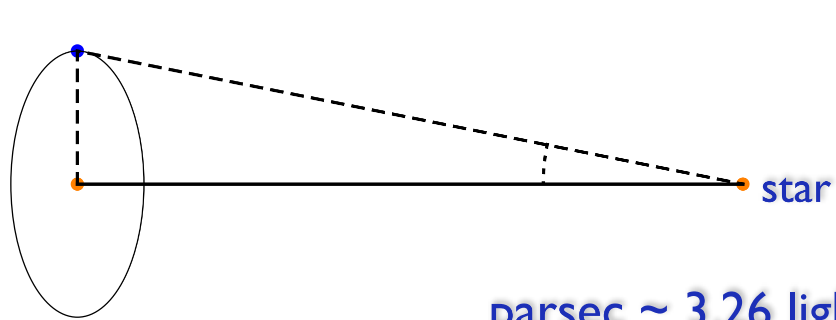
The effect of neutrino mass

The Homogeneous Universe

Distances

Up to the end of the XIX century the observed Universe was essentially our Galaxy

A convenient unit to measure distances to nearby stars is the **parsec**



parsec ~ 3.26 light-years

We are then 8 kpc away from the center of the Milky Way whose size is about 30 kpc

Extragalactic astronomy

Extragalactic objects, however, have already been observed …

e.g. M31 or the Andromeda *Nebula* (as it was known up until the beginning of the XX century)

1924: In the middle of the "Great Debate" of Shapley and Curtis, Hubble recognises it as a galaxy, similar to our own marking the beginning of extragalactic cosmography

The Andromeda *Galaxy* is about 800 kph away from us

The observed Universe at the beginning of the last century was far from being "homogenous"!

An homogeneous and expanding Universe

The idea of a Universe with an homogenous distribution of matter on large scales probably has a theoretical origin …

1916: General Relativity

1922: Friedmann's **expanding Universe** solution (homogeneous & isotropic) of GR equations

1927: Lemaître *predicts* **Hubble's law**

$$v = H_0 \, r \qquad\qquad H_0 \simeq 70 \ \mathrm{Km} \, s^{-1} \, \mathrm{Mpc}^{-1}$$

1929: Hubble *confirms* **Hubble's law**

1931: Lemaître's "primeval atom"

A metric for the Universe

We *assume* **homogeneity** and **isotropy**:
the **Friedmann-Lemaître-Robertson-Walker metric**

$$ds^2 = c^2 dt^2 - \underbrace{a^2(t)}_{\text{scale factor}} \left[\frac{dx^2}{1 - kx^2} + x^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right) \right]$$

scale factor
the Universe is not static!

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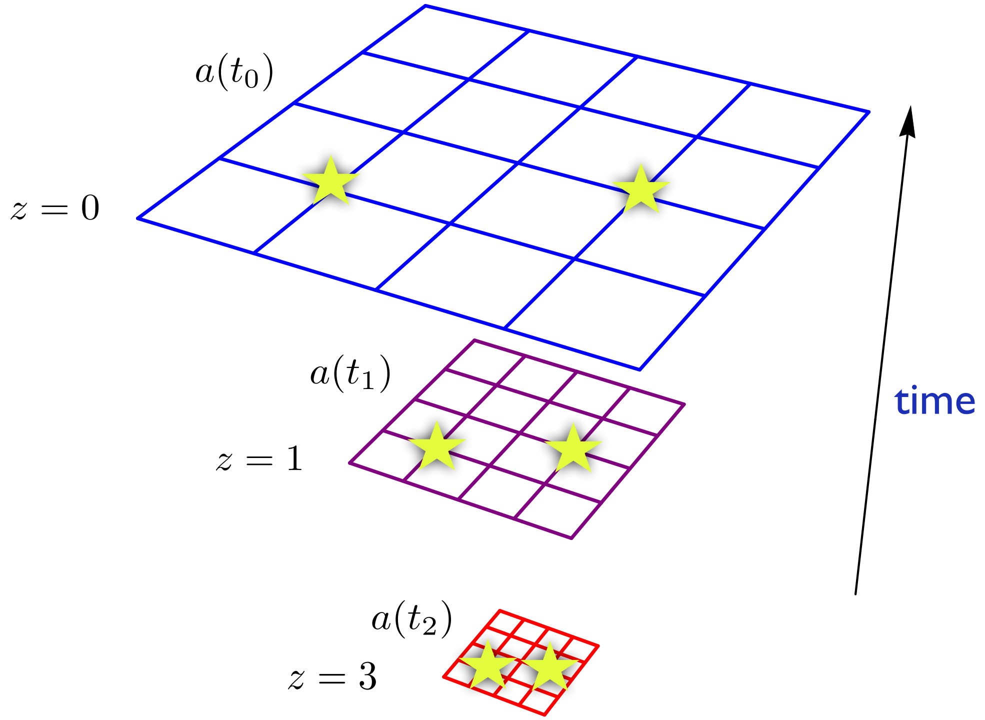
comoving coordinates

$$\vec{r} = a(t)\,\vec{x}$$

physical comoving



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scale factor

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comoving coordinates

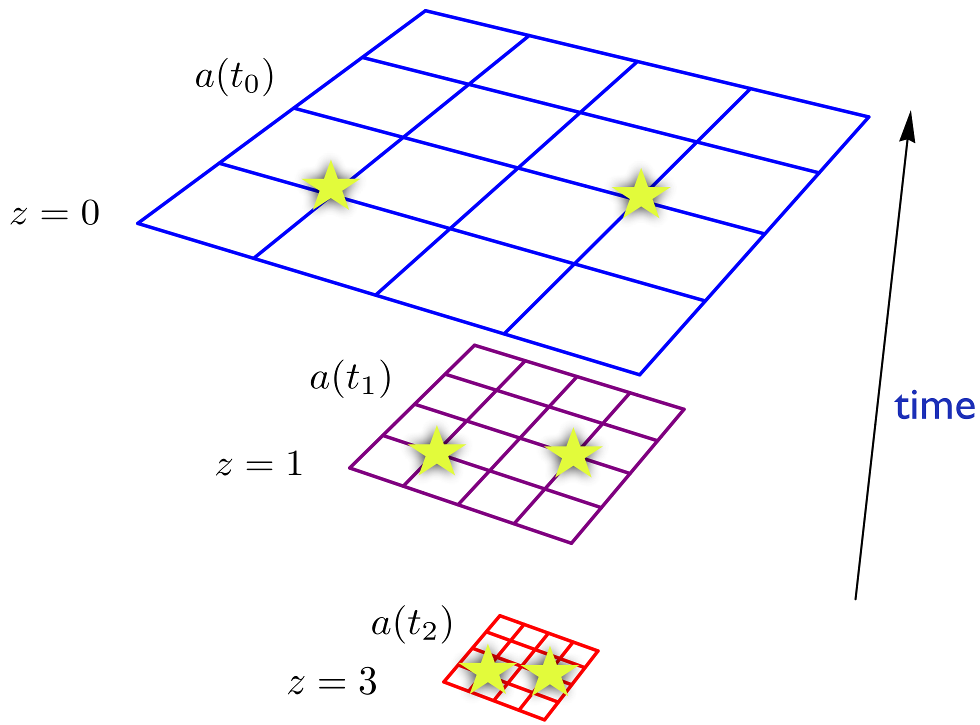
$$\vec{r} = a(t)\, \vec{x}$$

physical comoving

If the two points have constant *comoving* coordinates

$$\frac{dr}{dt} = \dot{a}\, x = \frac{\dot{a}}{a} a\, x \equiv H(t)\, r$$

Hubble's law



Redshift

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the **Friedmann-Lemaître-Robertson-Walker metric**

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dx^2}{1-kx^2} + x^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

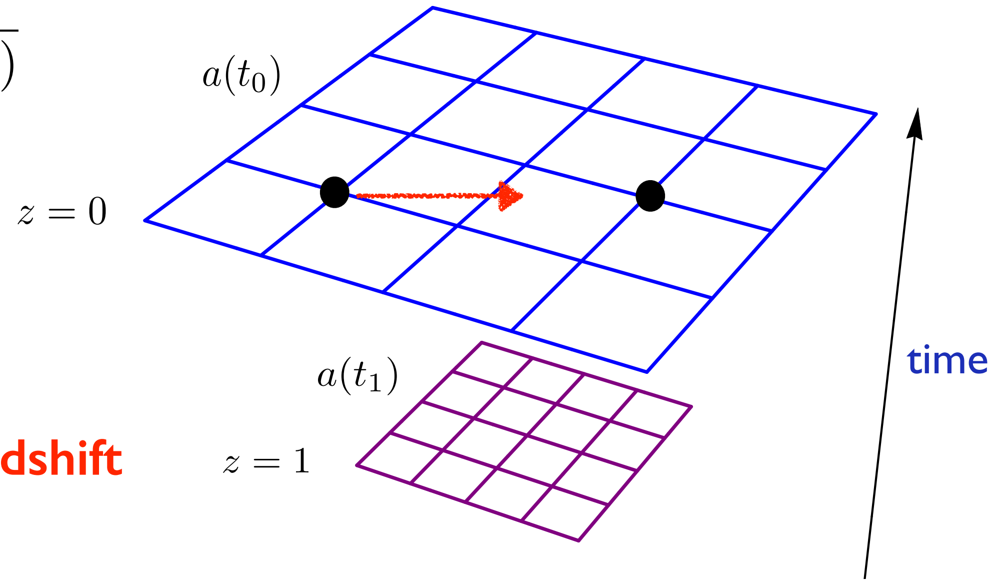
Let's consider light propagation $ds^2 = 0 \quad \rightarrow \quad dt^2 = a^2(t)\dfrac{dx^2}{1-kx^2}$

$$\int_{t_e}^{t_0} \frac{dt}{a(t)} = \int_{x_1}^{x_2} \frac{dx}{\sqrt{1-kx^2}} = \int_{t_e+dt_e}^{t_0+dt_0} \frac{dt}{a(t)}$$

$$\rightarrow \qquad \frac{dt_e}{a(t_e)} = \frac{dt_o}{a(t_o)} \qquad \rightarrow$$

$$\boxed{\frac{dt_o}{dt_e} = \frac{\lambda_o}{\lambda_e} = \frac{\nu_e}{\nu_o} = \frac{a(t_o)}{a(t_e)} \equiv 1+z}$$

redshift



Solving for the expansion

Einstein's equations provide the "equations of motion", given the "content" of the Universe, $T^{\mu\nu}$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

with $T^{\mu\nu}$ subject to the conservation equation

$$\nabla_\mu T^{\mu\nu} = 0$$

Again, a *simple assumption*: the Universe is filled with a **perfect fluid**

$$T^{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

We only need to specify its **density** ρ and **pressure** p

Friedmann's equations

Einstein's equation reduce to **Friedmann's equations** for the scale factor

$$H^2(a) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Given the **fluid equation of state**

$$\rho = w\, p$$

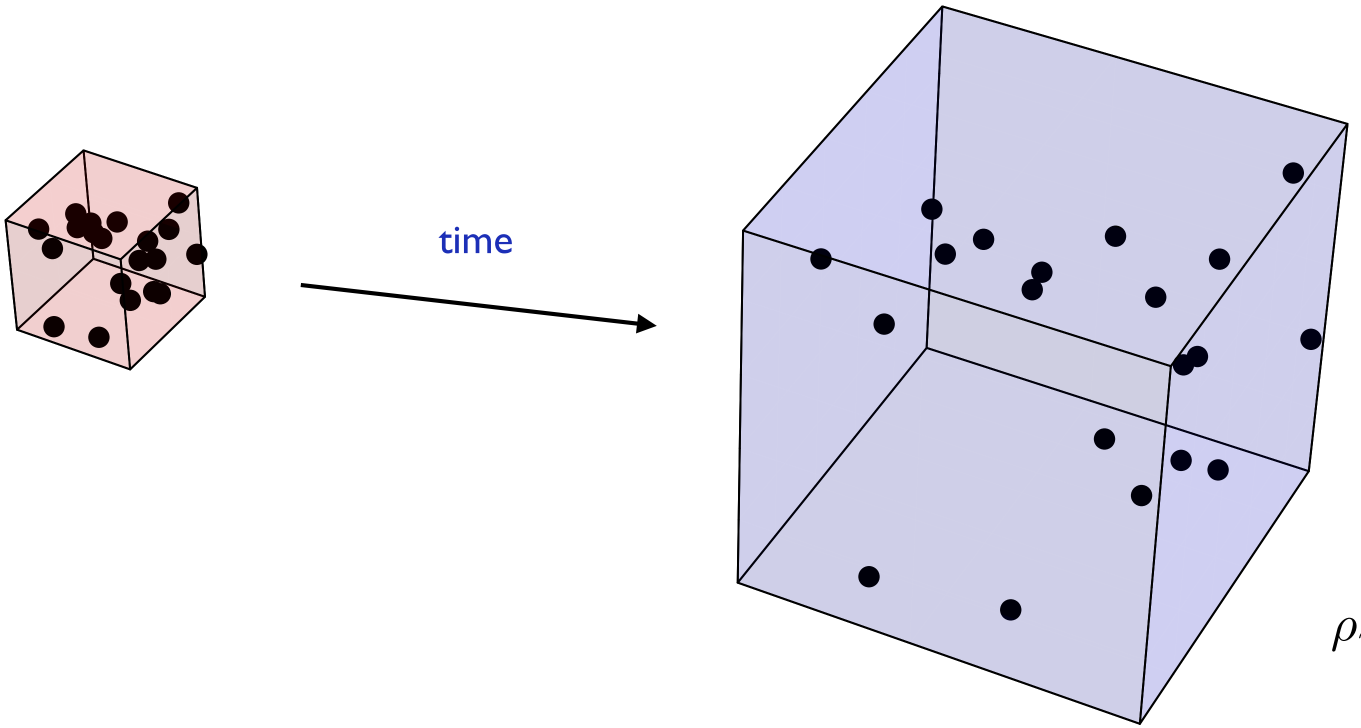
we can find the dependence of the density on the scale, in fact

$$\nabla_\mu T^{\mu\nu} = 0 \quad \rightarrow \quad \frac{\partial \rho}{\partial t} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \quad \rightarrow \quad \boxed{\rho(a) \sim a^{-3(1+w)}}$$

and finally solve for the evolution of the scale factor …

What fluid?

Matter: baryons and *dark matter*



$$\rho_m = m\frac{N}{V}$$

Equation of state:

$$w = 0$$

$$p = 0$$

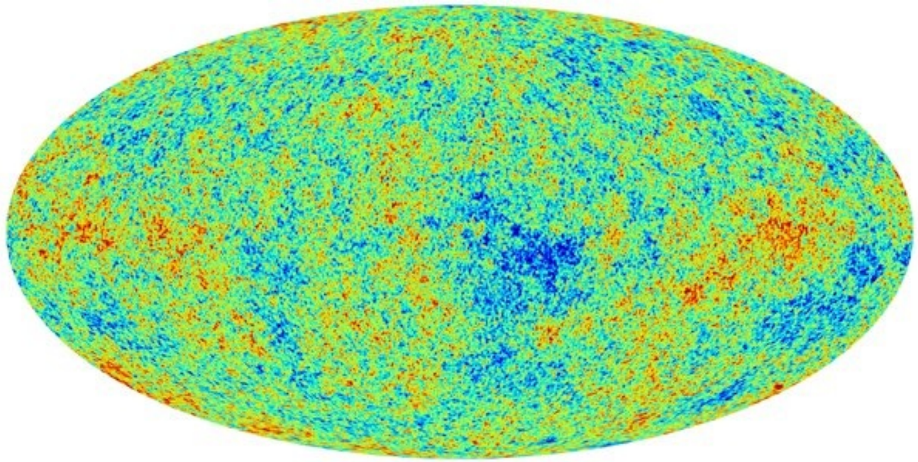
$$\boxed{\rho_m \sim \frac{1}{a^3}}$$

(conservation of matter)

What fluid?

Radiation

In 1964 Penzias & Wilson stumble upon the **Cosmic Microwave Background**



What fluid?

Radiation

In 1964 Penzias & Wilson stumble upon the **Cosmic Microwave Background**

An isotropic,
black-body emission
T_{CMB} = 2.7 K

Equation of state:

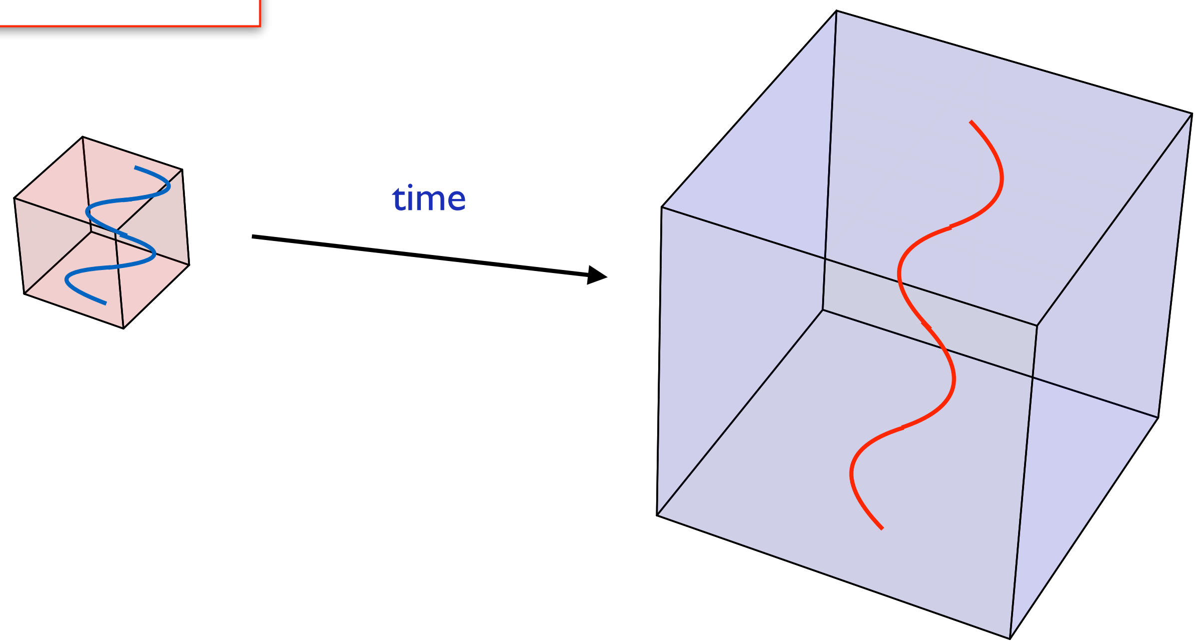
$$w = \frac{1}{3}$$

$$p = \frac{\rho}{3}$$

$$\boxed{\rho_\gamma \sim \frac{1}{a^4}}$$

In addition to the volume expansion, the energy of each photon decreases as

$$\epsilon_\gamma = \frac{h}{\lambda} \sim \frac{1}{a}$$

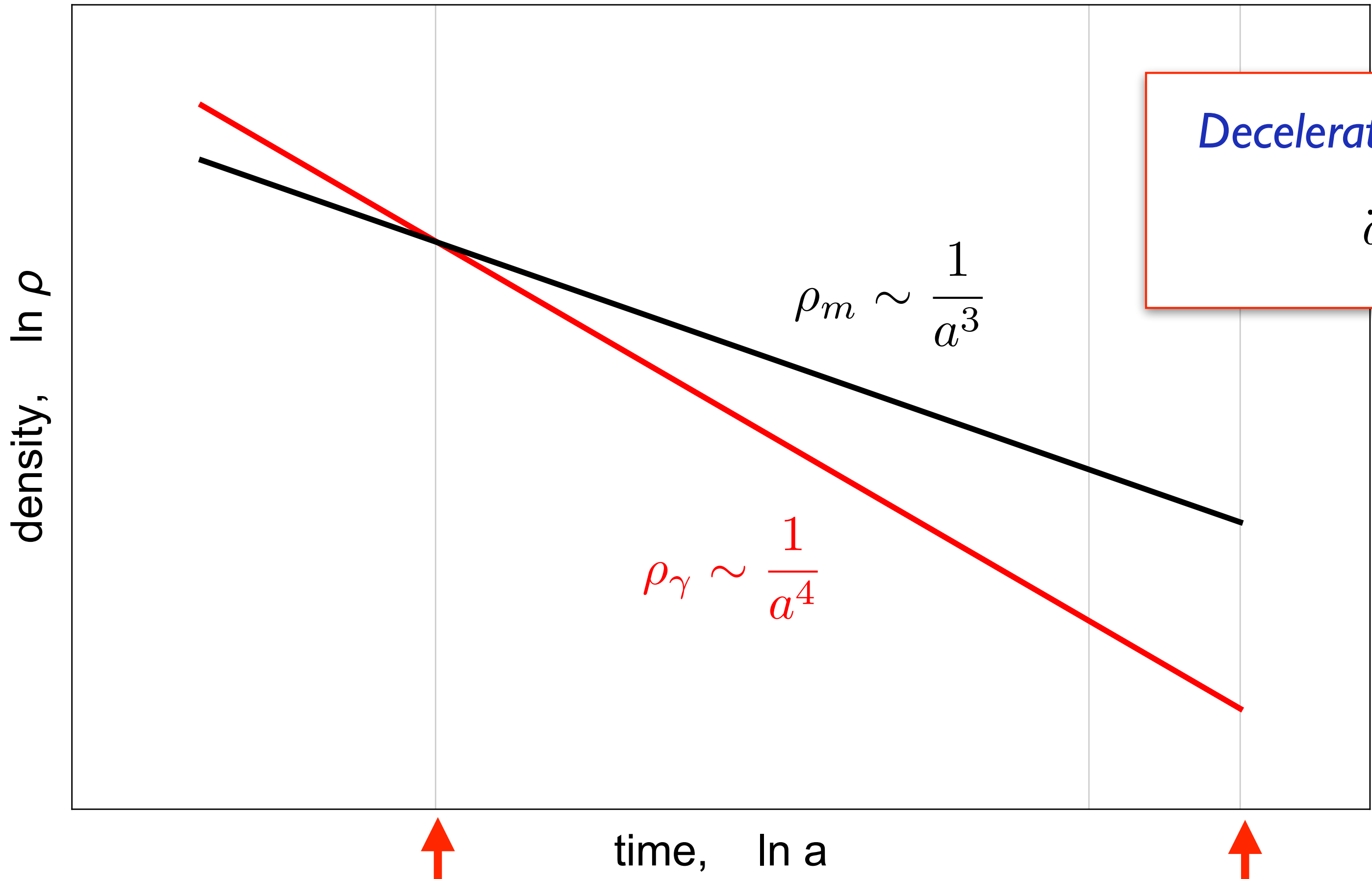


The energy density of the Universe

Extrapolating back in time …

$$\left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left(\rho_m + \rho_\gamma\right) = \frac{8\pi G}{3}\left(\frac{\rho_{m,0}}{a^3} + \frac{\rho_{\gamma,0}}{a^4}\right)$$

radiation domination *matter* domination



Decelerated expansion:

$$\ddot a < 0$$

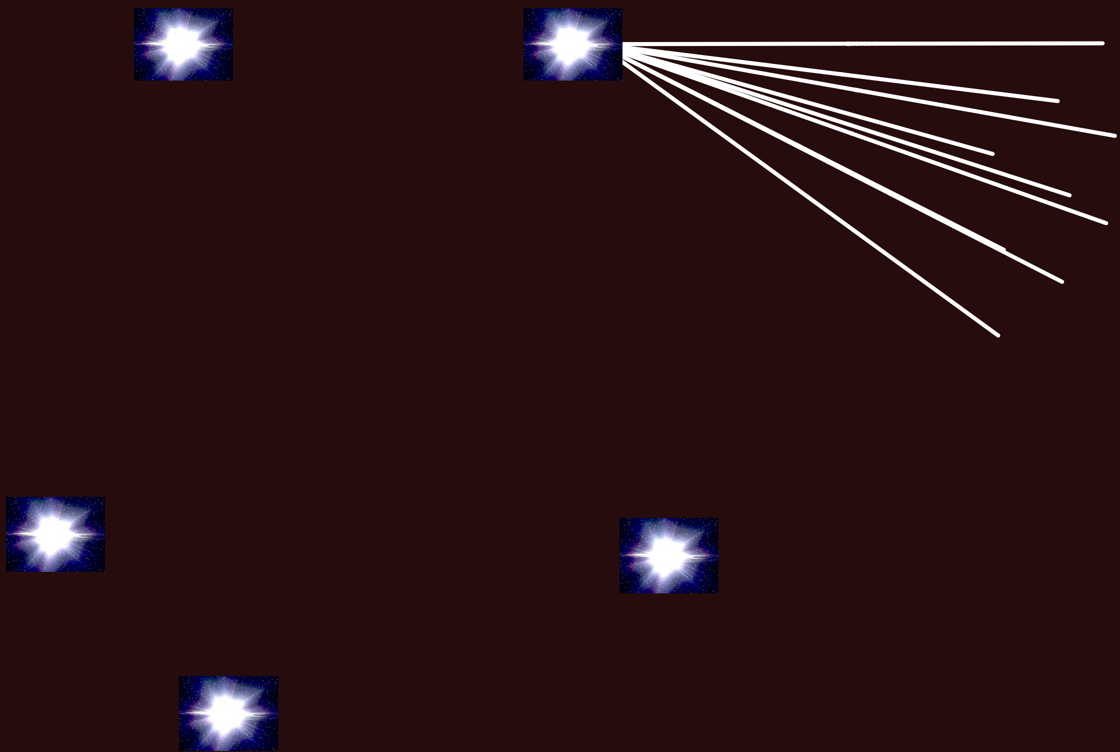
matter-radiation equality

today!

What else could be out there?

This was not the end of the story …

Type IA supernovae have long been recognised as *standard candles* and have been used to reconstruct cosmological distances



they allow us to associate a (luminosity) distance to the observed redshift

$$F = \frac{L}{4\pi D_L^2}$$

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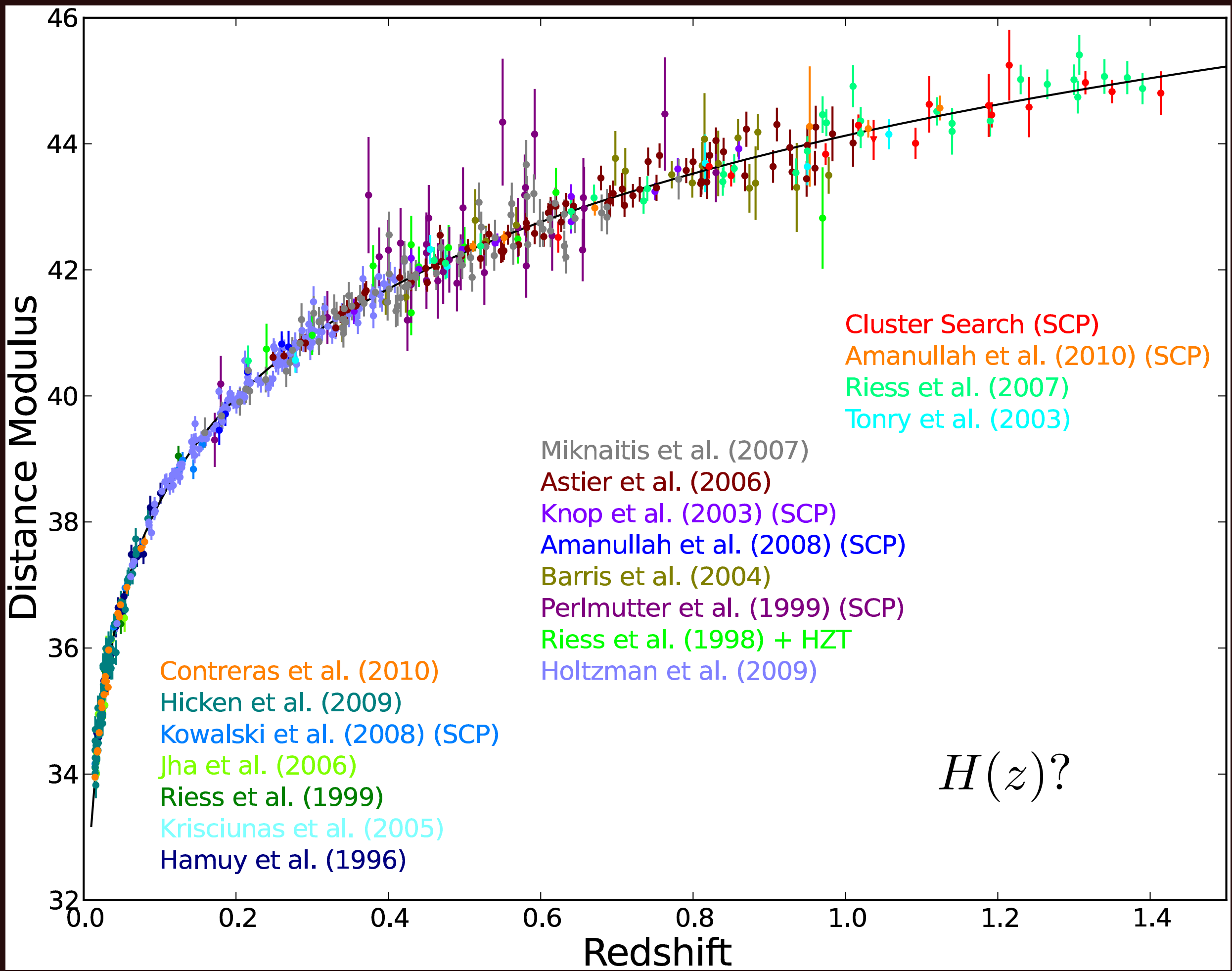

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$$F = \frac{L}{4\pi D_L^2}$$

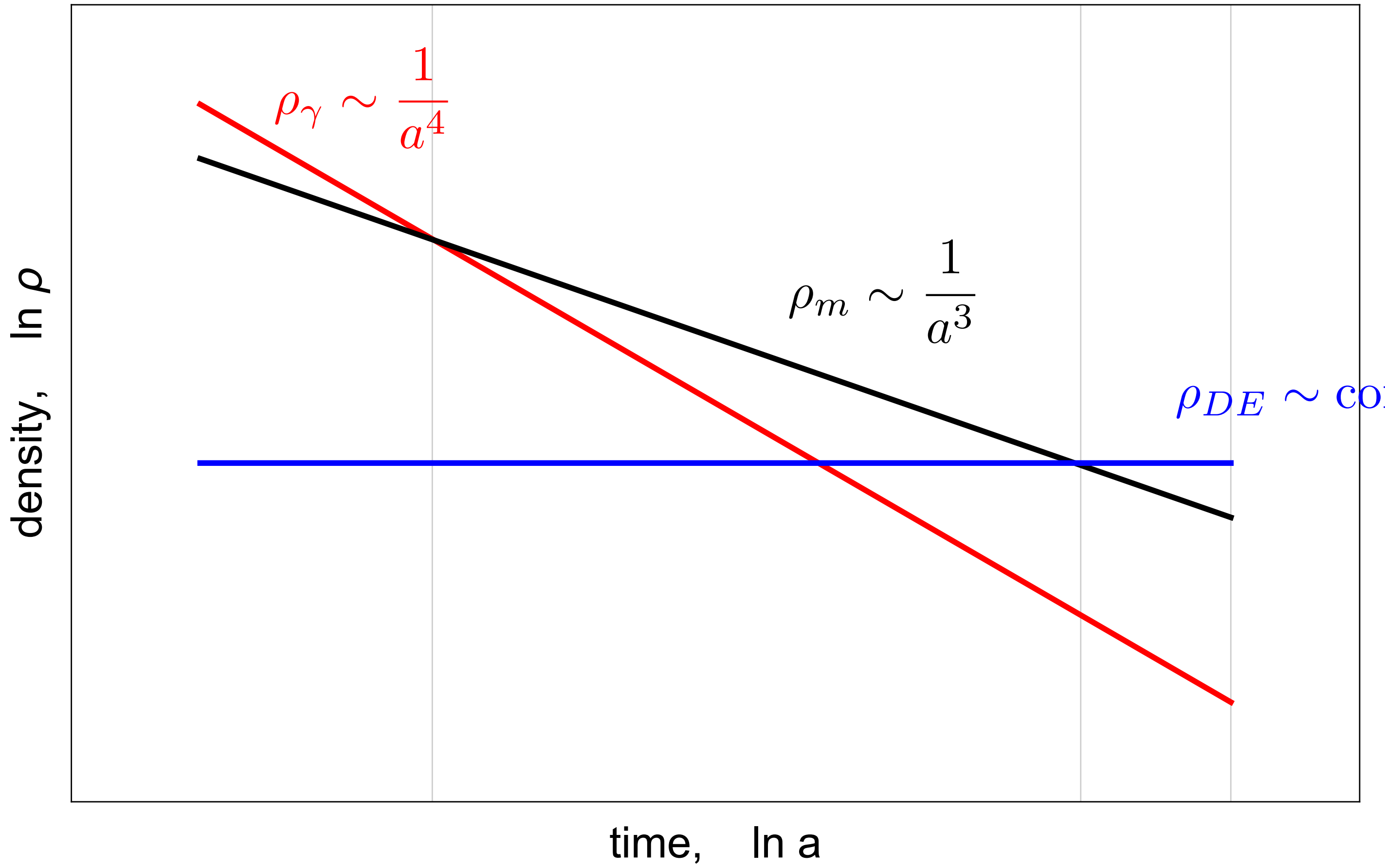
Dark Energy?

Riess et al (1998)
Permutter et al (1999),
Nobel Prize in 2011

The energy density of the Universe

Dark Energy? $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0 \qquad p = w\,\rho \qquad w < -\frac{1}{3}$$

radiation domination — *matter* domination — *dark energy* domination



$w \simeq -1$

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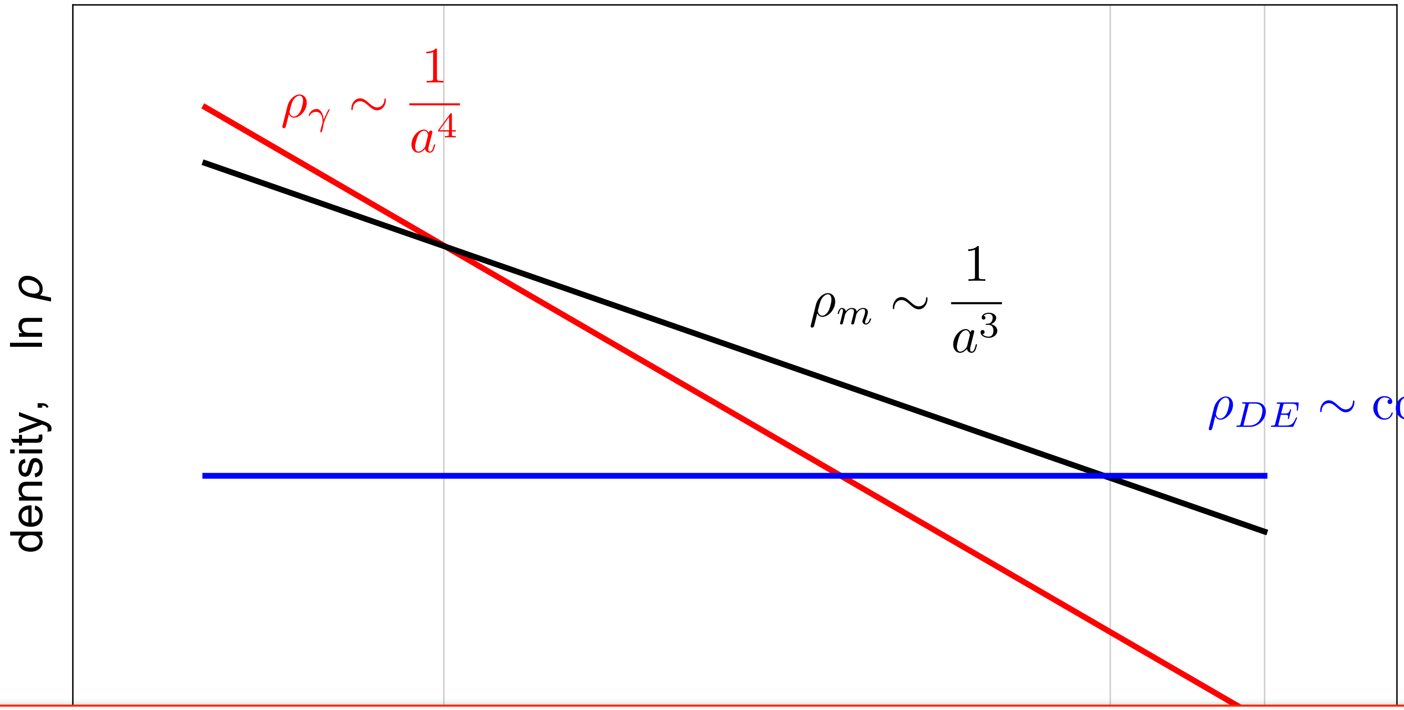
Dark Energy? $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$ $\quad p = w\,\rho \quad$ $w < -\frac{1}{3}$

radiation domination *matter* domination *dark energy* domination



$w \simeq -1$

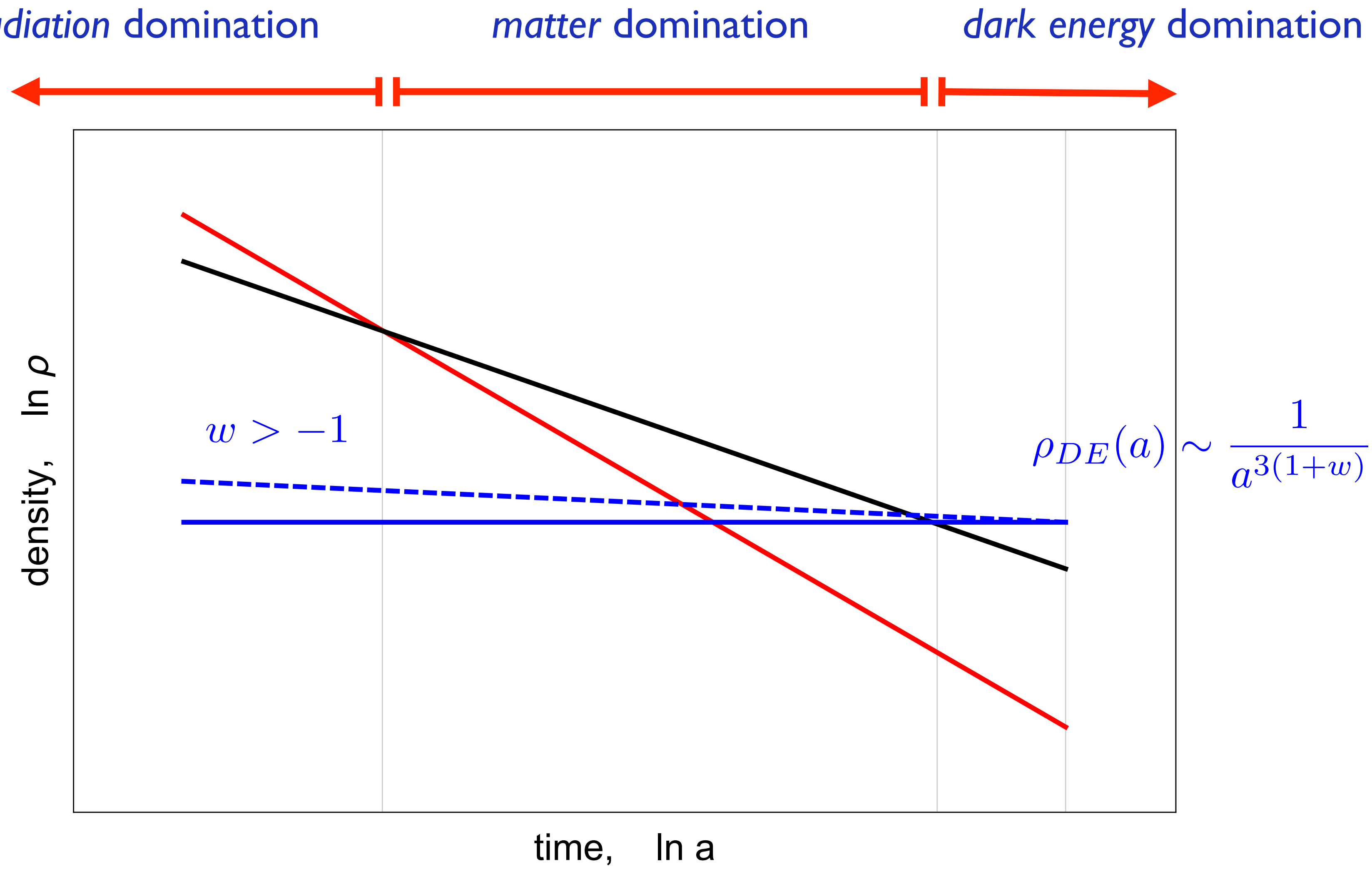
A cosmological constant?

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu} \quad \rightarrow \quad \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left(\rho_m + \rho_\gamma + \frac{\Lambda}{3}\right)$$

$w = -1$

The energy density of the Universe

Dark Energy?



The energy density of the Universe

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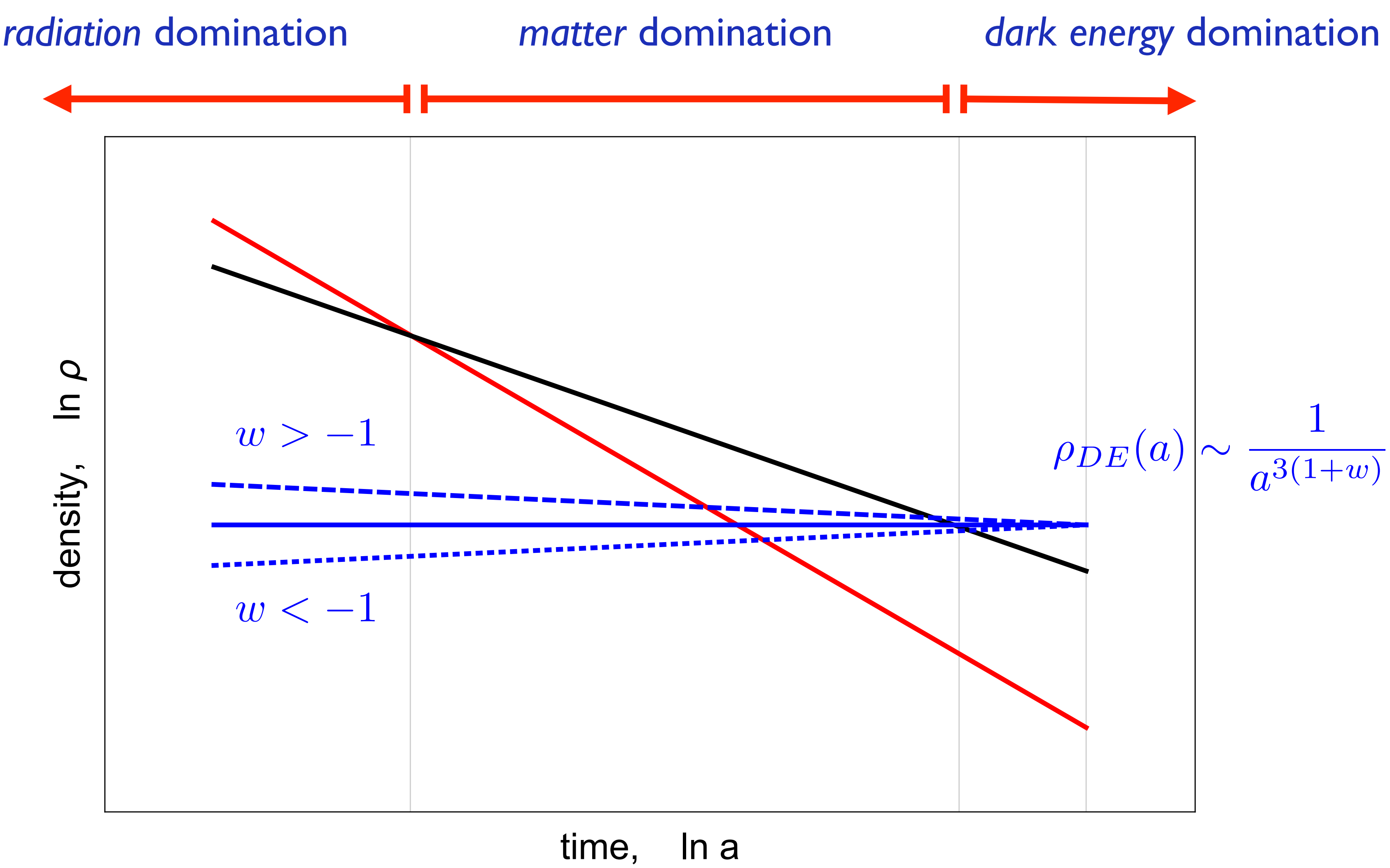
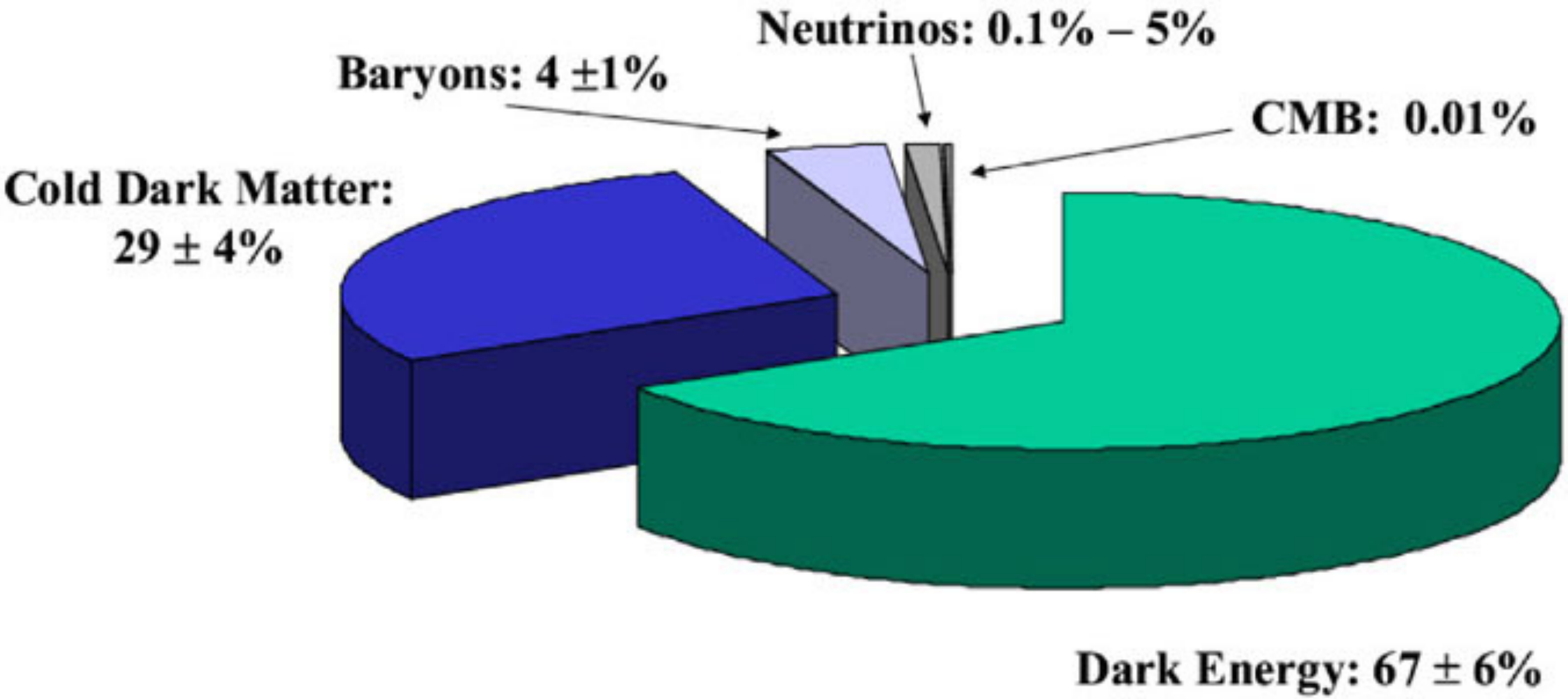


Quintessence? Modified gravity? …???

The energy density of the Universe

A more detailed budget

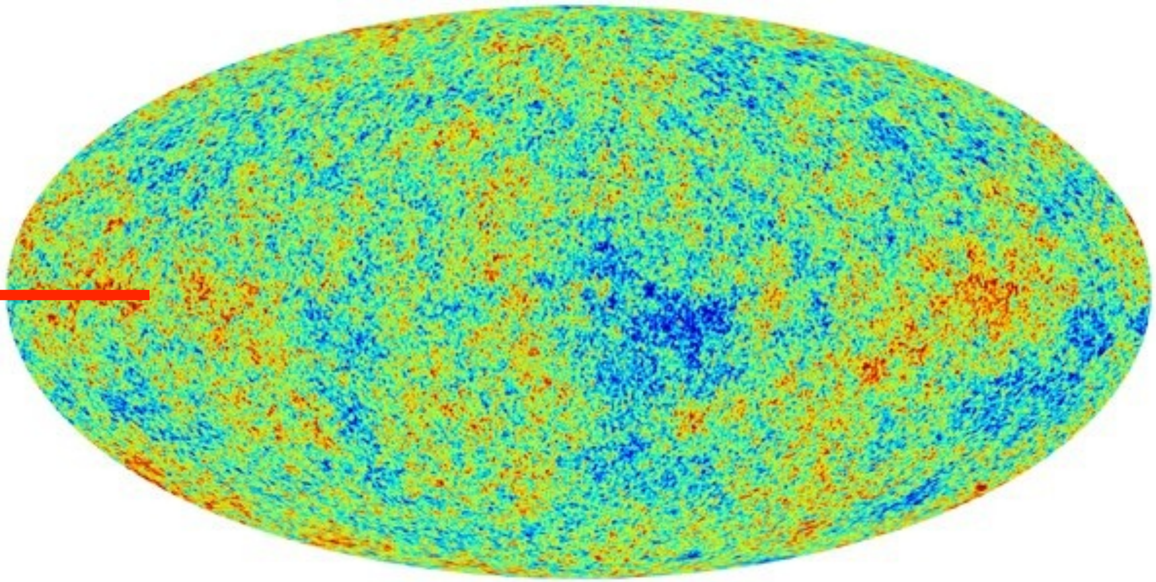


[Courtesy Freedman & Turner (2003)]

Can we learn … *more?*

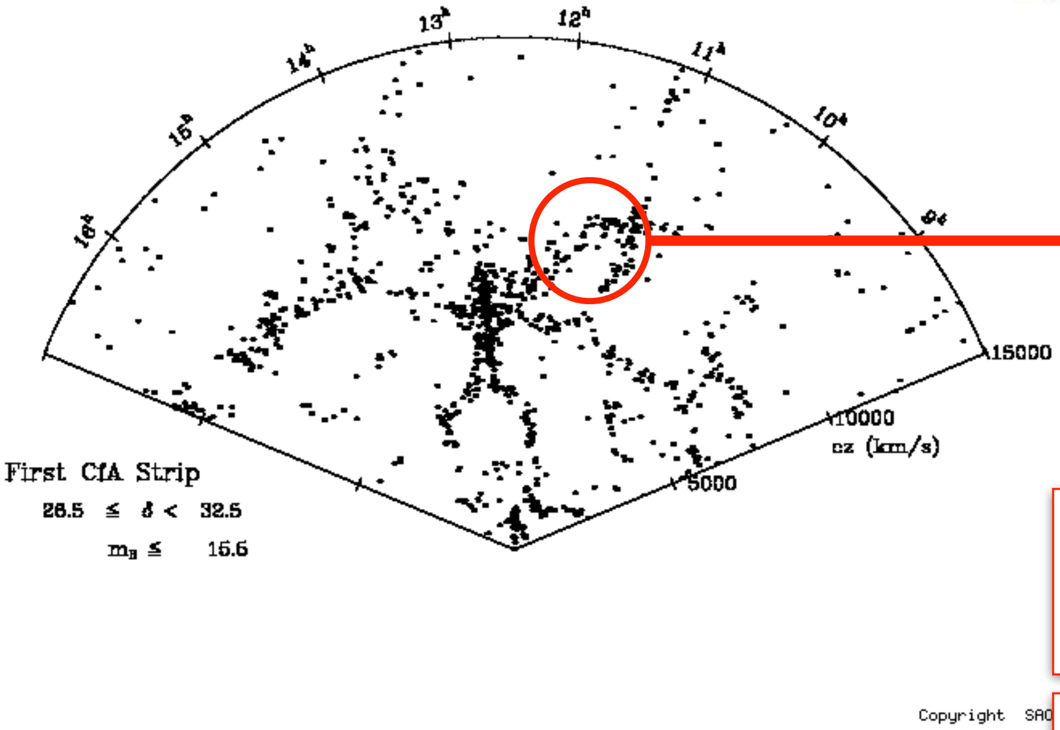
The Perturbed Universe

Cosmological perturbations



CMB temperature fluctuations

$$T(\hat{n})$$



$$n_g(\vec{x})$$

number density of galaxies

We can only study the **statistical properties** of cosmological perturbations

Mathematically, these are **random fields**

Random fields

If ϕ is a **random *variable*** with Probability Distribution Function (PDF) $\mathcal{P}(\phi)$
we can compute:

$$\langle \phi \rangle = \int d\phi \, \mathcal{P}(\phi) \, \phi$$ mean

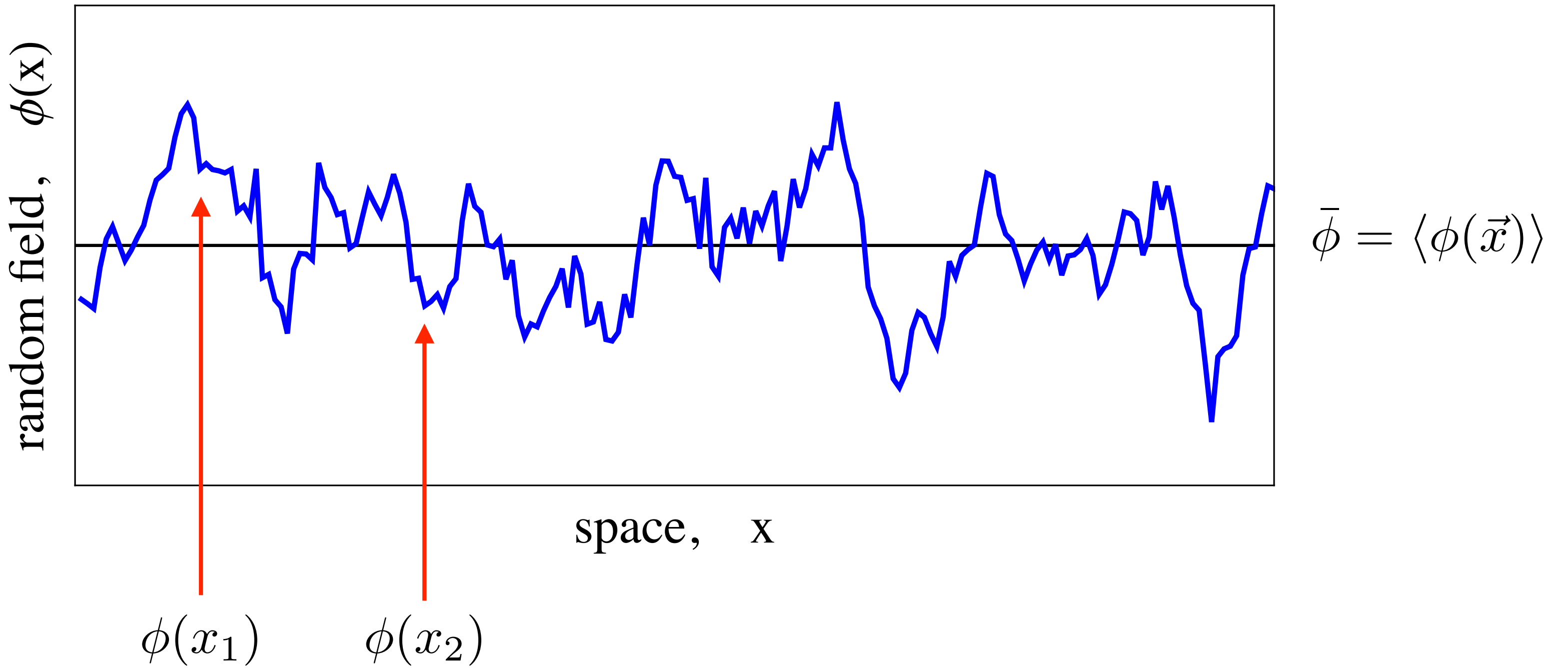
$$\langle \phi^2 \rangle = \int d\phi \, \mathcal{P}(\phi) \, \phi^2$$ 2-nd-order moment

$$\langle \phi^n \rangle = \int d\phi \, \mathcal{P}(\phi) \, \phi^n$$ n-th-order moment

$$\sigma_\phi^2 = \langle \phi^2 \rangle - \langle \phi \rangle^2$$ variance

Random fields

If $\phi(\vec{x})$ is a **random *field*** we can also compute **correlation functions**



two-point function $\quad \langle \phi(x_1)\phi(x_2)\rangle = \langle \phi(x_1)\rangle\langle \phi(x_2)\rangle + \langle \phi(x_1)\phi(x_2)\rangle_c$

three-point function $\quad \langle \phi(x_1)\phi(x_2)\phi(x_3)\rangle = \langle \phi(x_1)\rangle\langle \phi(x_2)\rangle\langle \phi(x_3)\rangle +$

$$+\langle \phi(x_1)\phi(x_2)\rangle_c \langle \phi(x_3)\rangle + \mathrm{perm.} +$$

$$+\langle \phi(x_1)\phi(x_2)\phi(x_3)\rangle_c$$

...

n-point function $\quad \langle \phi(x_1)\phi(x_2)\ldots\phi(x_n)\rangle$

The distribution of galaxies in the Universe

The galaxy number density and its perturbations:

$$n_g(\vec{x}) \equiv \bar{n}_g\,[1 + \delta_g(\vec{x})]$$

$$\delta_g(\vec{x}) \equiv \frac{n_g(\vec{x}) - \bar{n}_g}{\bar{n}_g}$$

$$\bar{n}_g \equiv \frac{N_g^{\rm tot}}{V}$$

mean galaxy number density over the volume V

galaxy **overdensity** or density contrast
(random field!)

N.B. $\langle \delta_g(\vec{x}) \rangle \equiv 0$

$\delta_g(\vec{x}) \geq -1$

galaxy number density
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galaxy number density
(random field!)

Similarly, for the matter density we have

$$\rho(\vec{x},t) = \bar{\rho}(t)\left[1+\delta(\vec{x},t)\right]$$

$$\delta(\vec{x},t) \equiv \frac{\rho(\vec{x},t)-\bar{\rho}(t)}{\bar{\rho}(t)}$$

mean matter density

matter **overdensity**

The galaxy two-point correlation function

What is the probability of finding two galaxies in the volume elements dV_1 and dV_2 ?

$$dP = dV_1\, dV_2 \,\langle\, n_g(\vec{x}_1)\, n_g(\vec{x}_2)\,\rangle$$

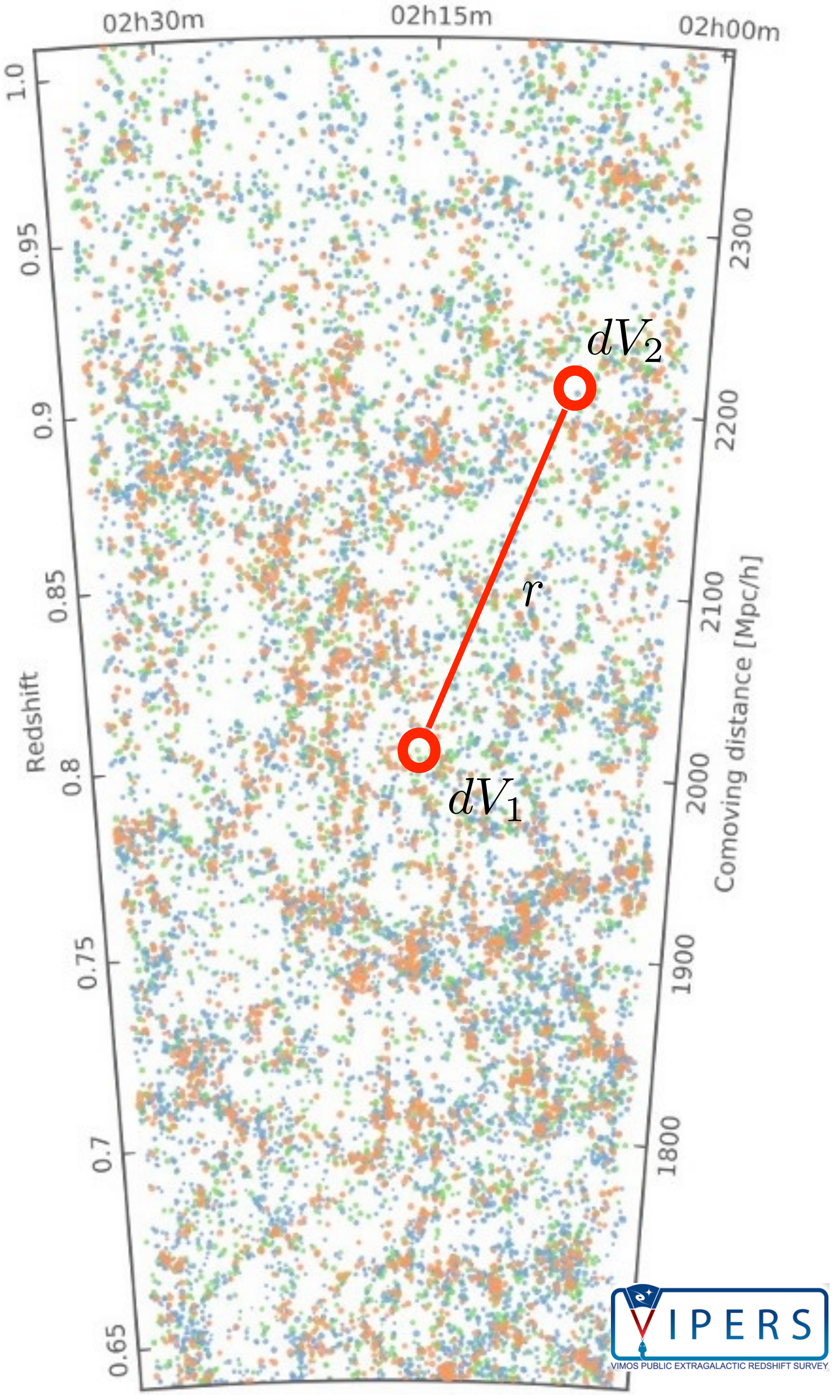
$$= dV_1\, dV_2\, \bar{n}_g^2\, [1 + \langle\, \delta_g(\vec{x}_1)\, \delta_g(\vec{x}_2)\,\rangle]$$

excess probability

We now make the assumption of *statistical* homogeneity and isotropy

$$\xi(|\vec{x}_1 - \vec{x}_2|) \;\equiv\; \langle\, \delta_g(\vec{x}_1)\, \delta_g(\vec{x}_2)\,\rangle$$

the two-point correlation function $\xi(r)$
only depends on the distance $r = |\vec{x}_1 - \vec{x}_2|$
between the two points



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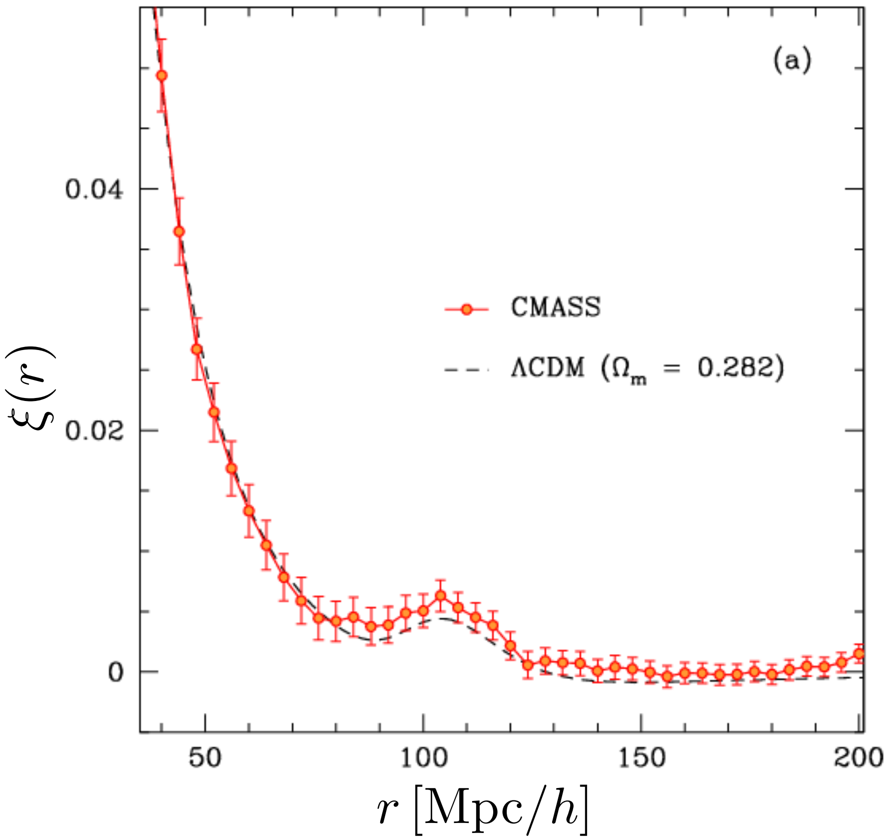
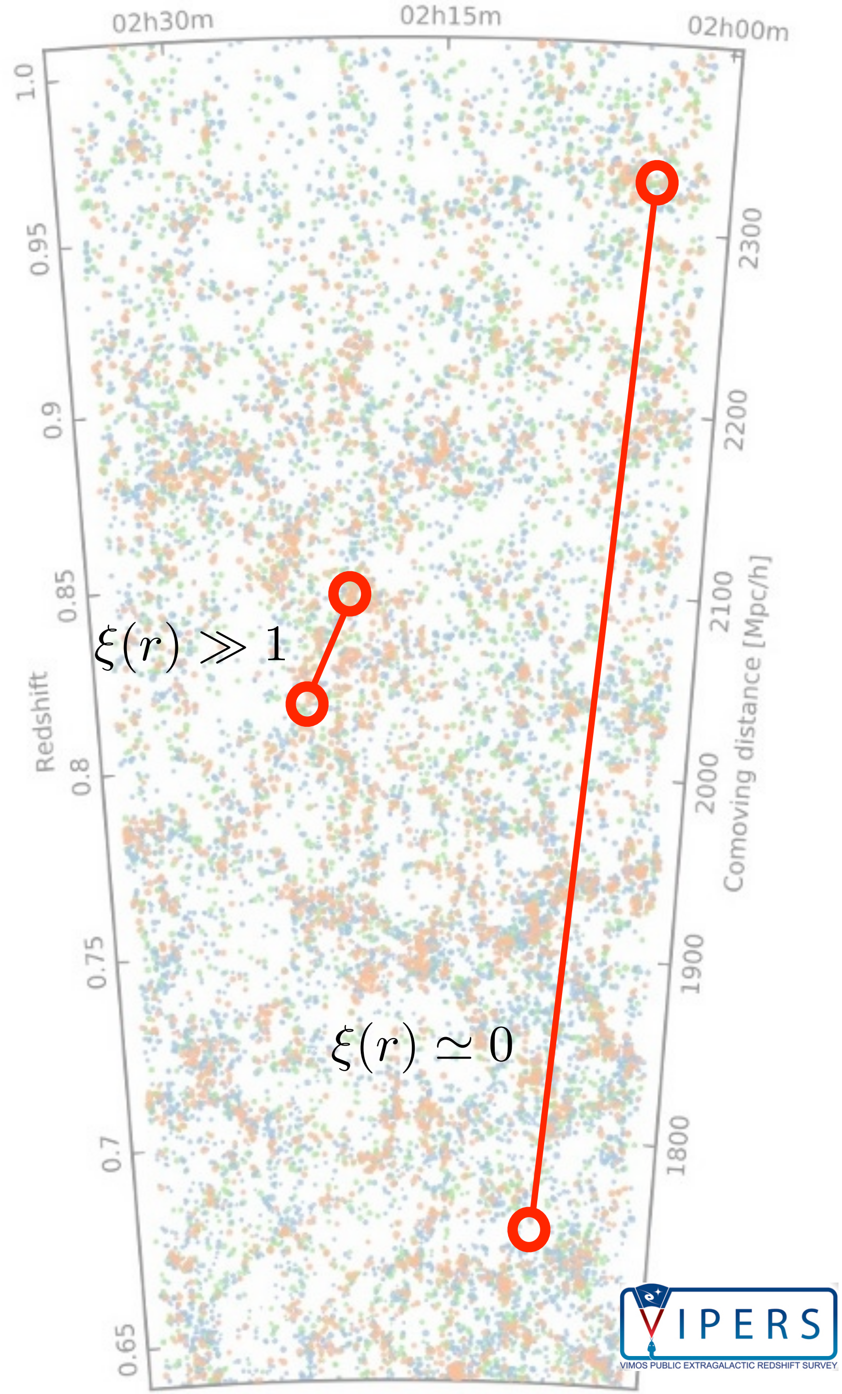




The galaxy three-point correlation function

Similarly I can ask the probability of finding three galaxies in the volume elements dV_1 , dV_2 and dV_3

$$dP = dV_1 dV_2 dV_3 \langle n_g(\vec{x}_1) n_g(\vec{x}_2) n_g(\vec{x}_3)\rangle$$

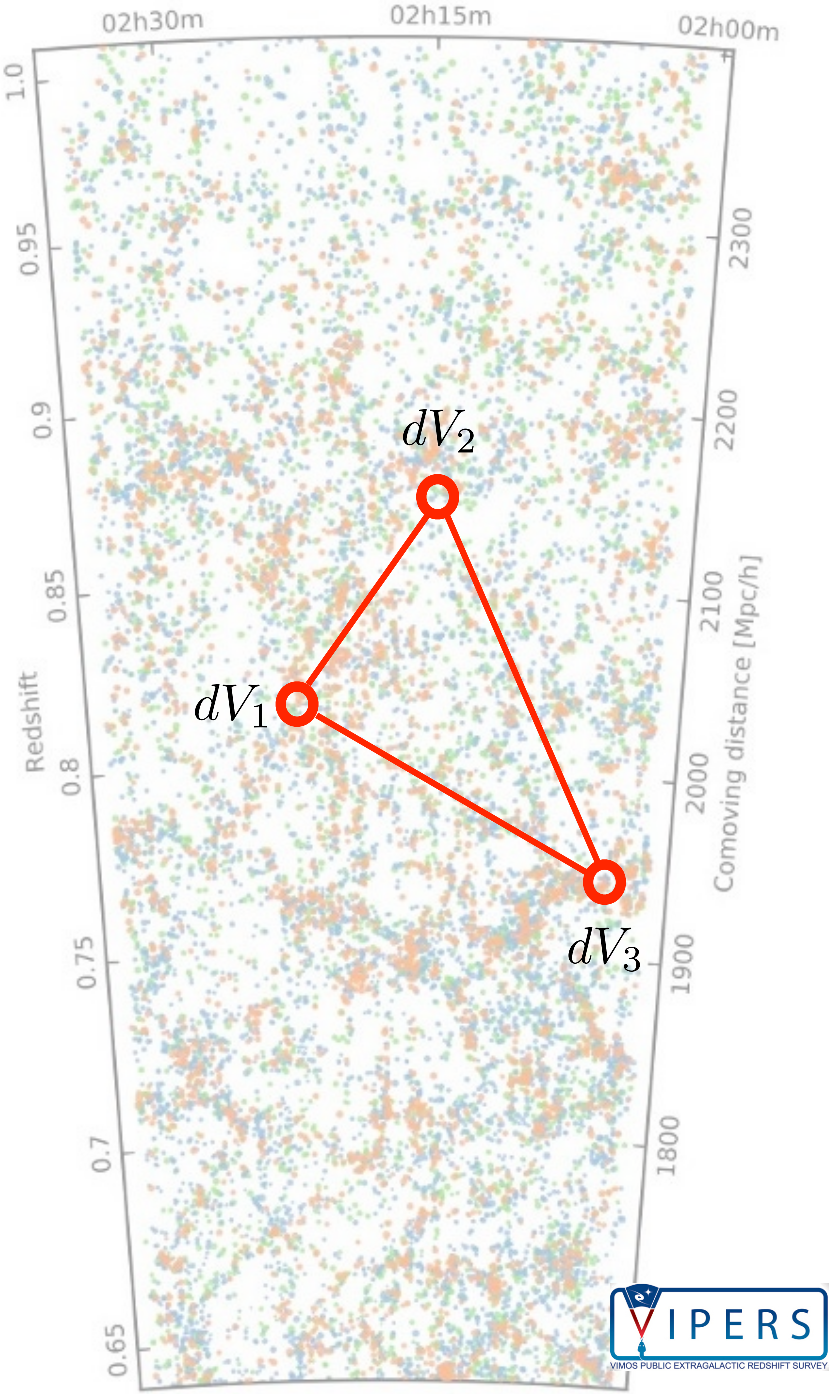
$$= dV_1\, dV_2\, dV_3\, \bar{n}_g^3\, [1 +$$

$$+ \xi(r_{12}) \,+\, \xi(r_{13}) \,+\, \xi(r_{23}) + \zeta(r_{12}, r_{13}, r_{23})]$$

excess probability

$$\zeta(r_{12}, r_{13}, r_{23}) \equiv \langle \delta_g(\vec{x}_1)\delta_g(\vec{x}_2)\delta_g(\vec{x}_3)\rangle$$

the 3-point correlation function represents the (excess) probability to find 3 galaxies forming a triangle of a given shape and size

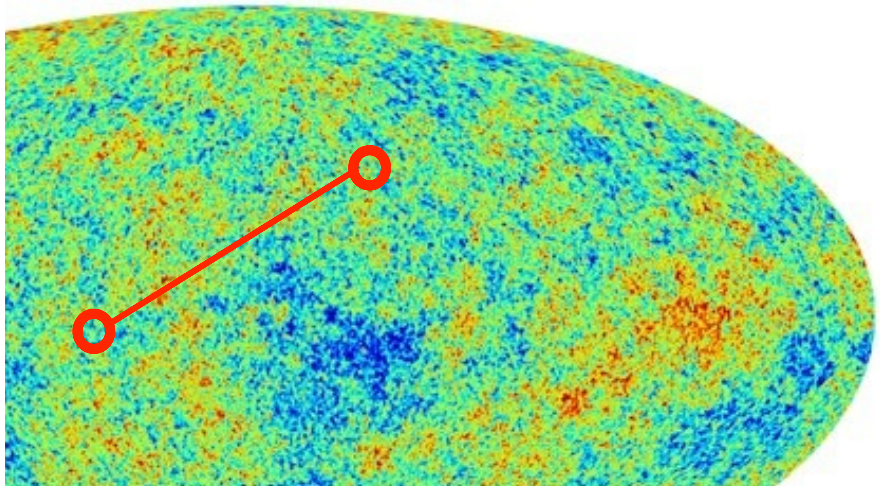


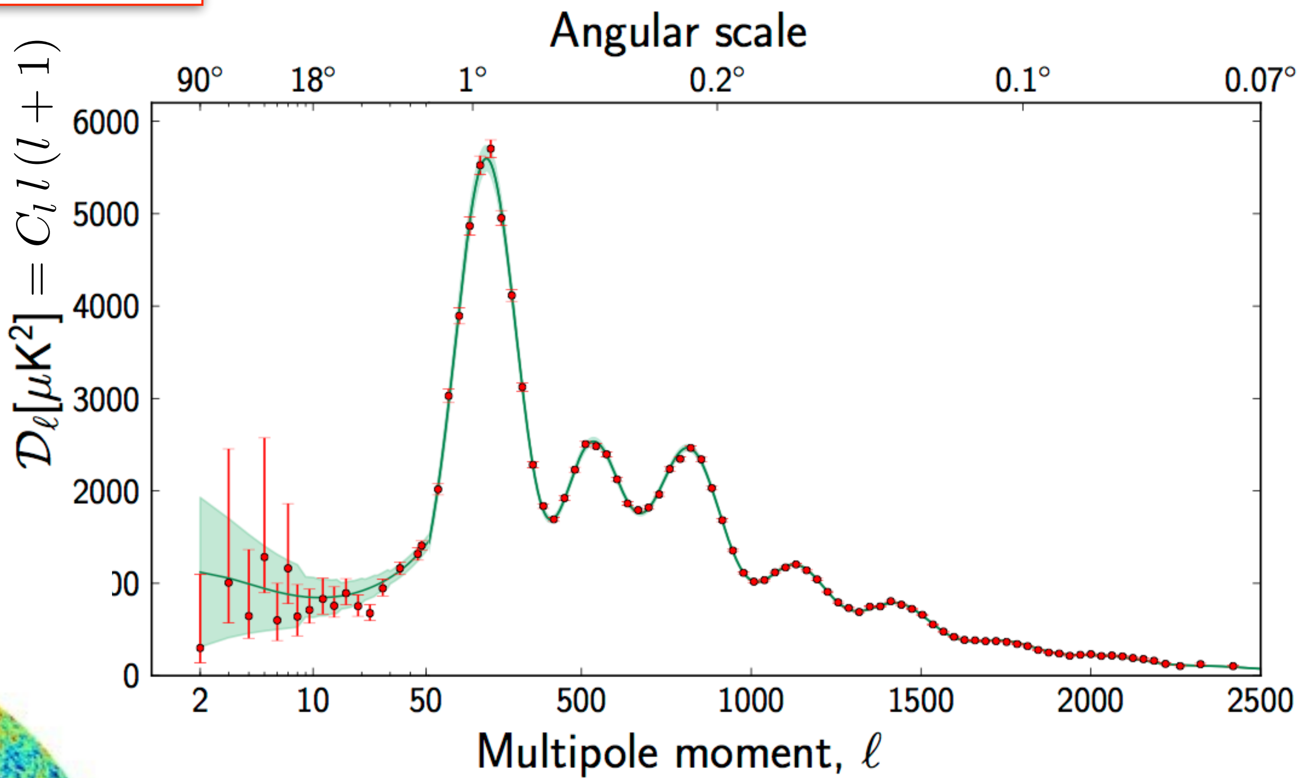
Gaussian and non-Gaussian random fields

The statistical properties of a Gaussian random field are completely characterised by its 2-point correlation function. All higher-order, *connected* correlation functions are vanishing

$$\delta_T(\hat{n}) \equiv \frac{T(\hat{n}) - \bar{T}}{\bar{T}}$$

$$\mathcal{P}[\delta_T(\hat{n})] = \frac{1}{\sqrt{2\pi\sigma_T^2}} e^{-\frac{1}{2}\frac{\delta_T^2}{\sigma_T^2}}$$



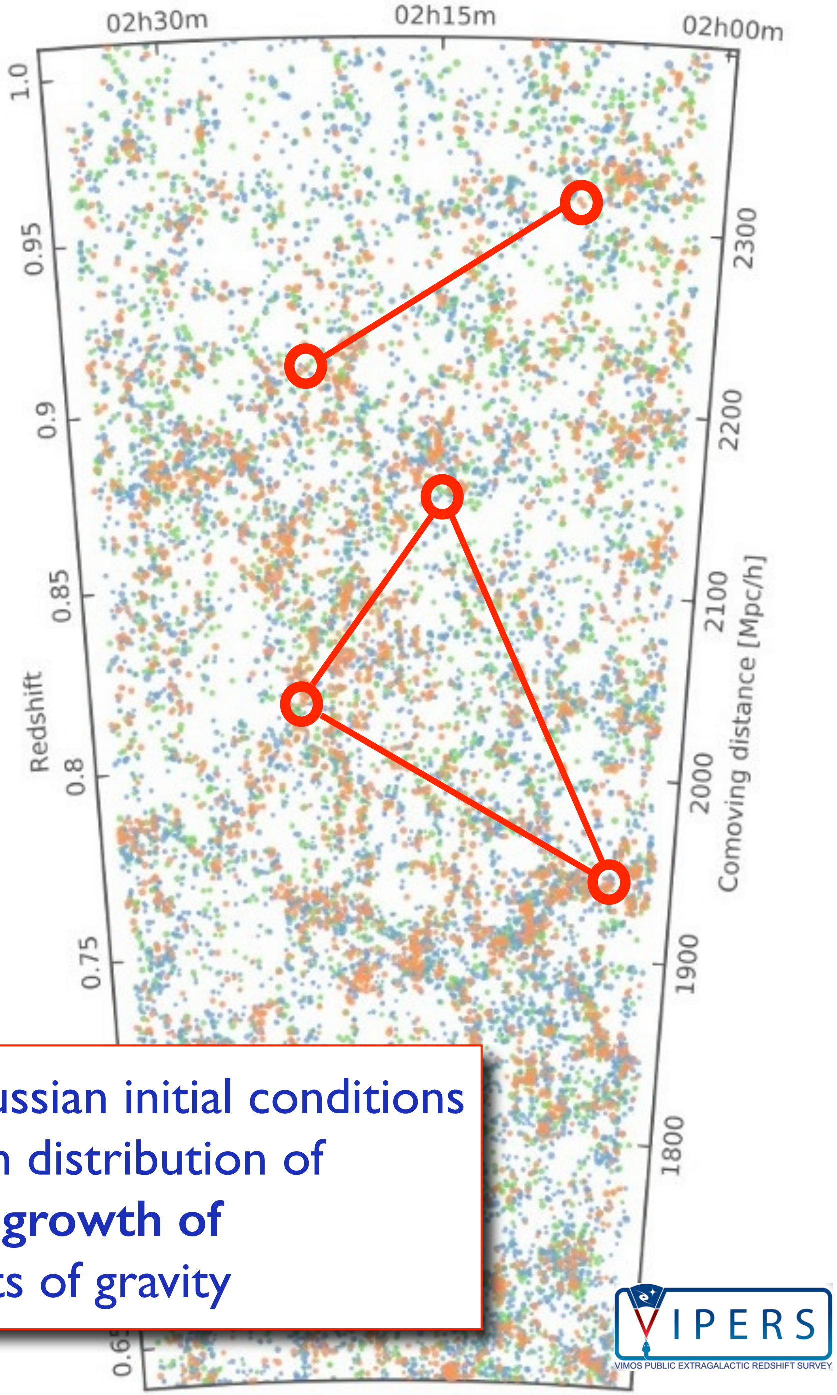


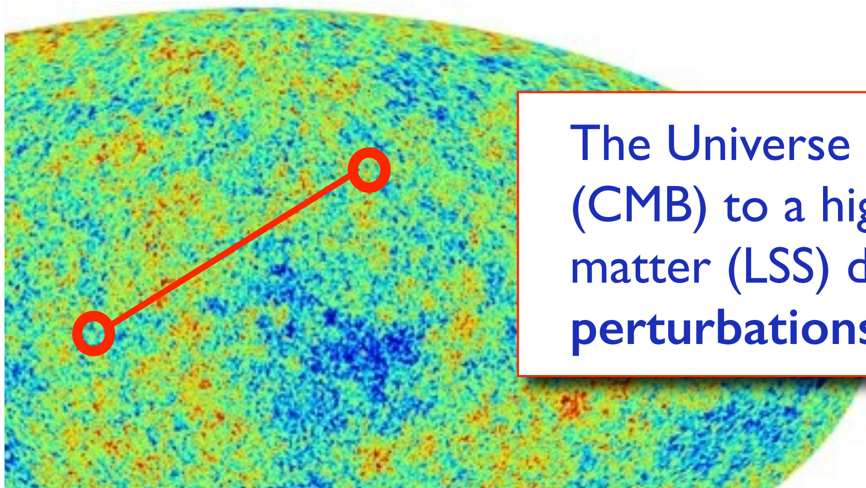
Perturbations in the CMB are one of the best examples of Gaussian random field!

Gaussian and non-Gaussian random fields

The statistical properties of a Gaussian random field are completely characterised by its 2-point correlation function. All higher-order, *connected* correlation functions are vanishing

all other random fields are non-Gaussian!





The Universe evolves from Gaussian initial conditions (CMB) to a highly non-Gaussian distribution of matter (LSS) due to **nonlinear growth of perturbations** under the effects of gravity

Ergodic hypothesis

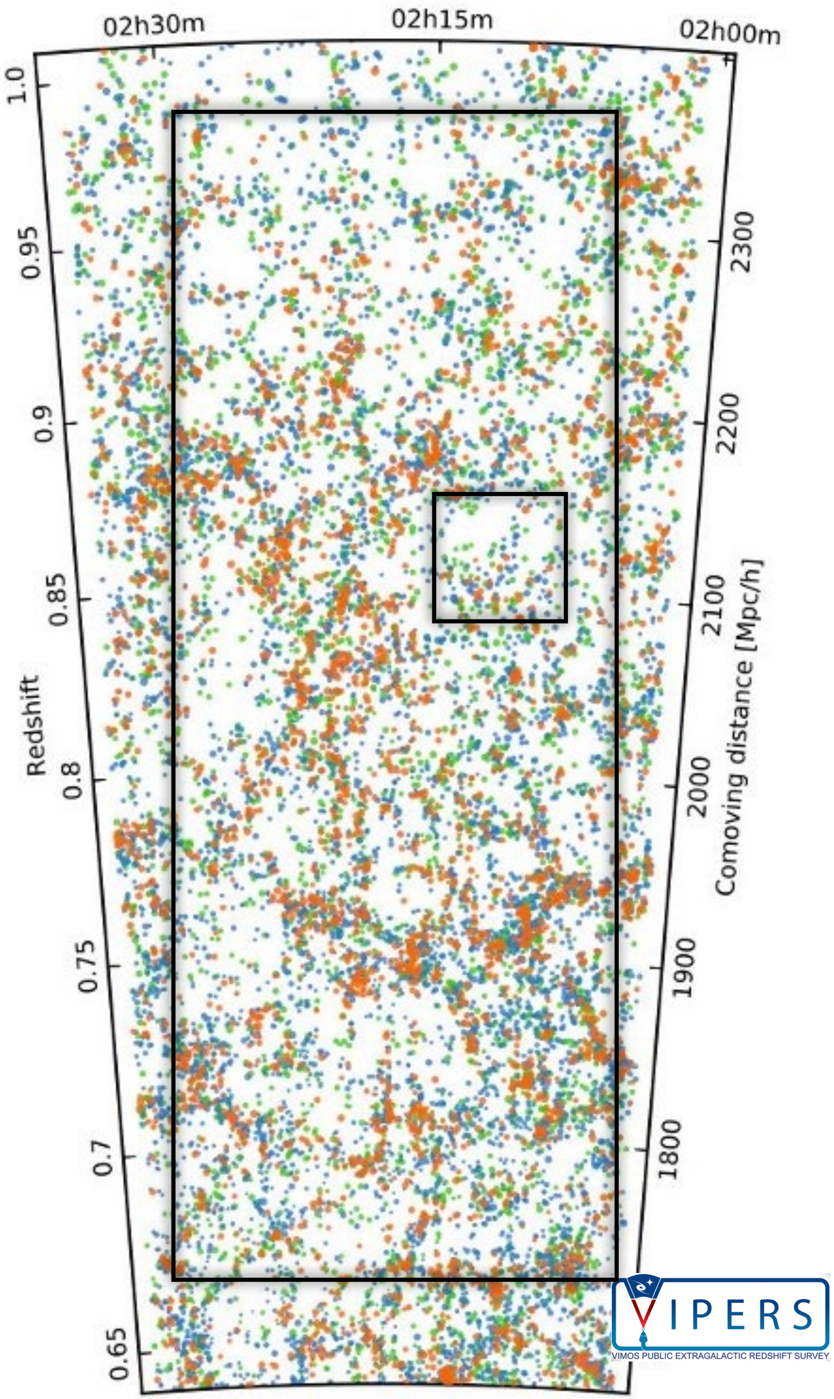
Expectation values, in principle, are to be intended as *ensemble averages*, i.e. averages over many “realisations of the Universe” …

… but we only have one Universe!

We have to assume the **ergodic hypothesis**: **ensemble averages are equal to spatial averages**

$$\langle \phi(\vec{x}) \rangle \equiv \int d\phi \, \phi \, \mathcal{P}(\phi) = \frac{1}{V} \int_V d^3x \, \phi(\vec{x})$$

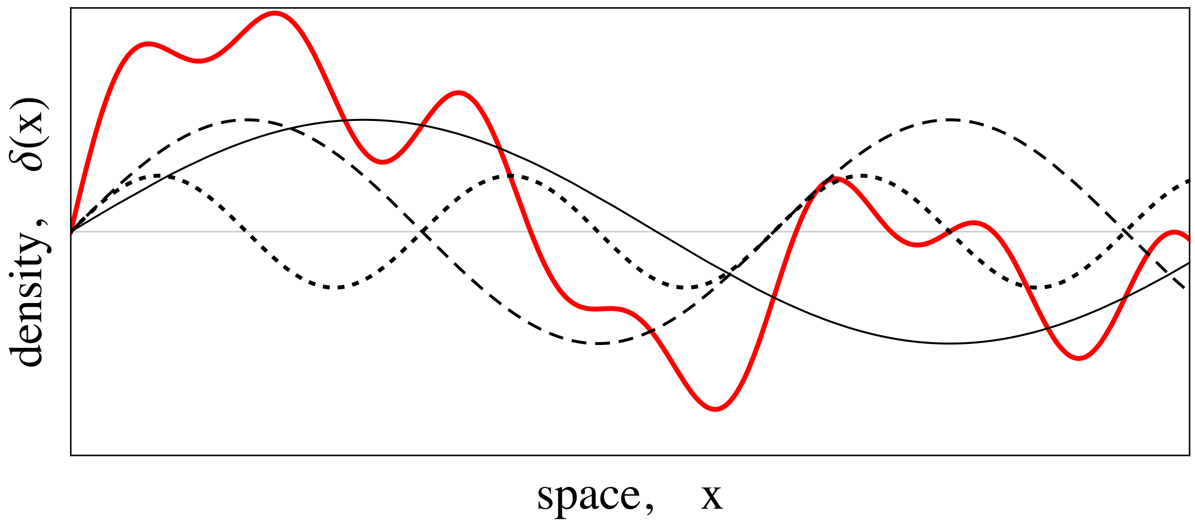
We should make sure, however, that the observed volume correspond to a “fair sample” of the Universe



Fourier space

Theoretical predictions for the matter correlation functions are performed in **Fourier space**

Fourier analysis naturally separates perturbations at different scales:



$$\delta_{\vec{k}} = \int \frac{d^3x}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} \delta(\vec{x})$$

$$\delta(\vec{x}) = \int d^3k e^{i\vec{k}\cdot\vec{x}} \delta_{\vec{k}}$$

- Since $\delta(\vec{x})$ is a random field $\delta_{\vec{k}}$ is also a random field
- Since $\delta(\vec{x})$ is real $\delta^*_{\vec{k}} = \delta_{-\vec{k}}$

Fourier space: correlation functions

The 2-point function in Fourier space: the **power spectrum**

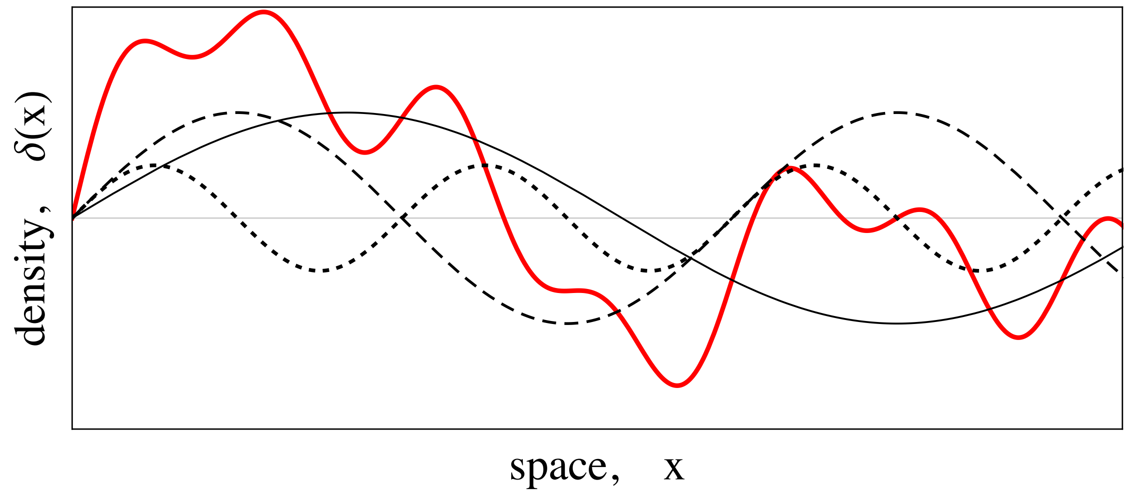
$$\langle \delta_{\vec{k}_1} \, \delta_{\vec{k}_2} \rangle = \delta_D(\vec{k}_1 + \vec{k}_2) \, P(k_1)$$

homogeneity & isotropy

$$P(k) = \int \frac{d^3x}{(2\pi)^3} \, e^{i\,\vec{k}\cdot\vec{x}} \xi(x)$$

The power spectrum is the *Fourier Transform* of the 2-point correlation function

The power spectrum is a measure of the amplitude of perturbations as a function of scale



Fourier space: correlation functions

Higher-order correlation functions:

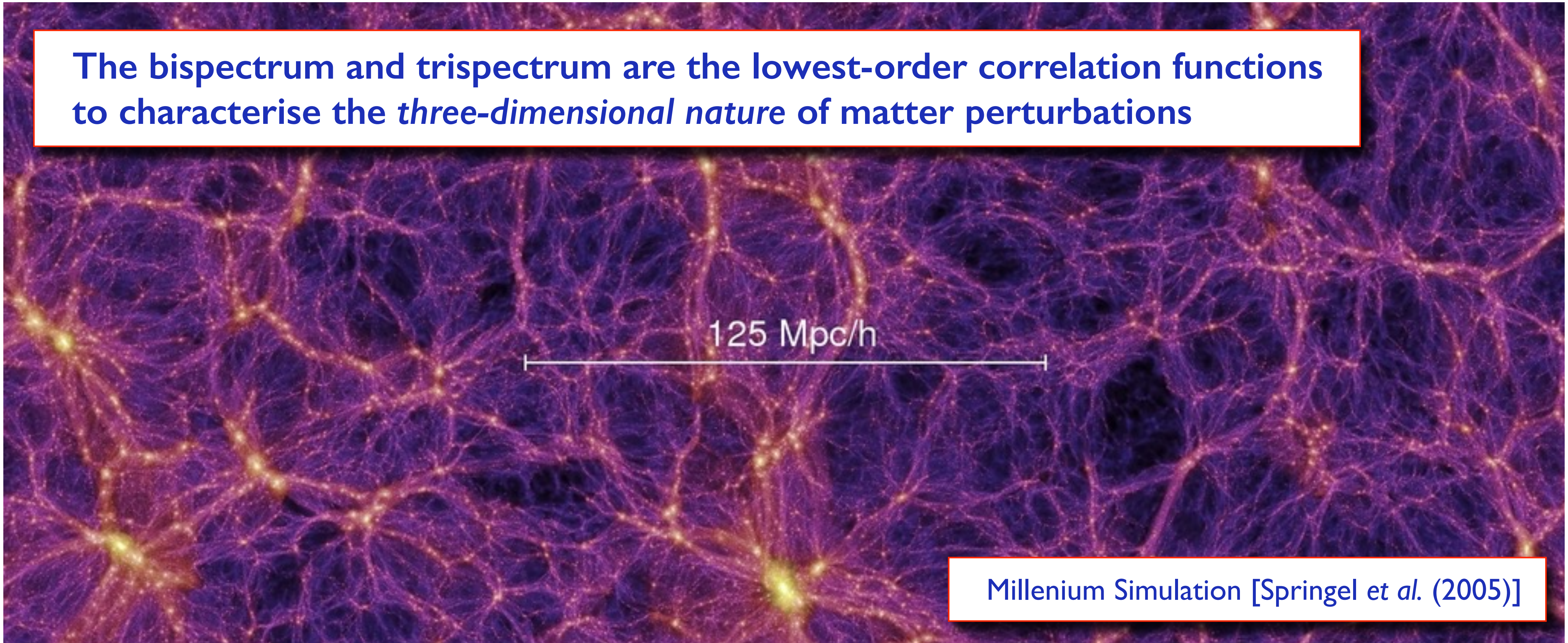
$$\langle \delta_{\vec{k}_1} \delta_{\vec{k}_2} \delta_{\vec{k}_3} \rangle \equiv \delta_D(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\, B(k_1, k_2, k_3)$$

the **bispectrum**

$$\langle \delta_{\vec{k}_1} \delta_{\vec{k}_2} \delta_{\vec{k}_3} \delta_{\vec{k}_4} \rangle \equiv \delta_D(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)\, T(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)$$

the **trispectrum**

The bispectrum and trispectrum are the lowest-order correlation functions to characterise the *three-dimensional nature* of matter perturbations



Millenium Simulation [Springel *et al.* (2005)]