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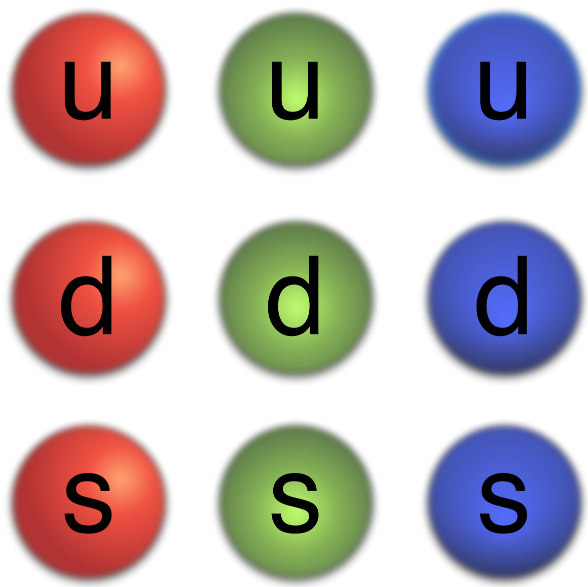
STAG RESEARCH CENTRE

Bulk and shear viscosities in dense quark matter

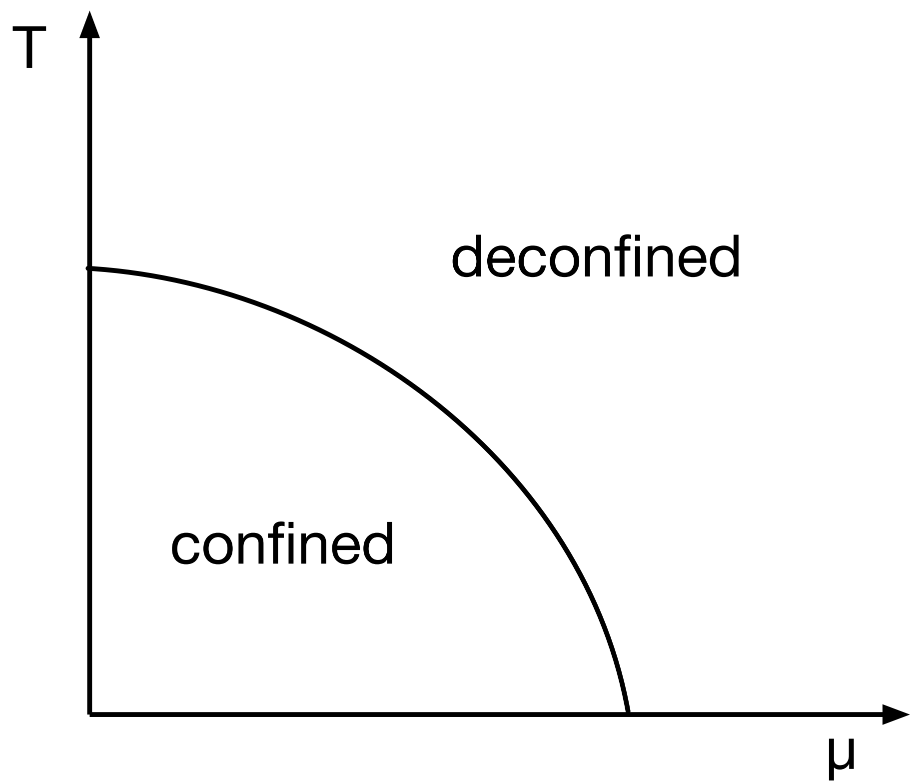


• Outline

- what is dense quark matter?
  - quark matter in the QCD phase diagram
  - unpaired, non-interacting quark matter
  - Cooper pairing in quark matter:
    color-flavor locking and other color superconductors
- transport in quark matter
  - bulk viscosity: definition, physical picture
    & results for various quark matter phases
  - shear viscosity: results for various phases

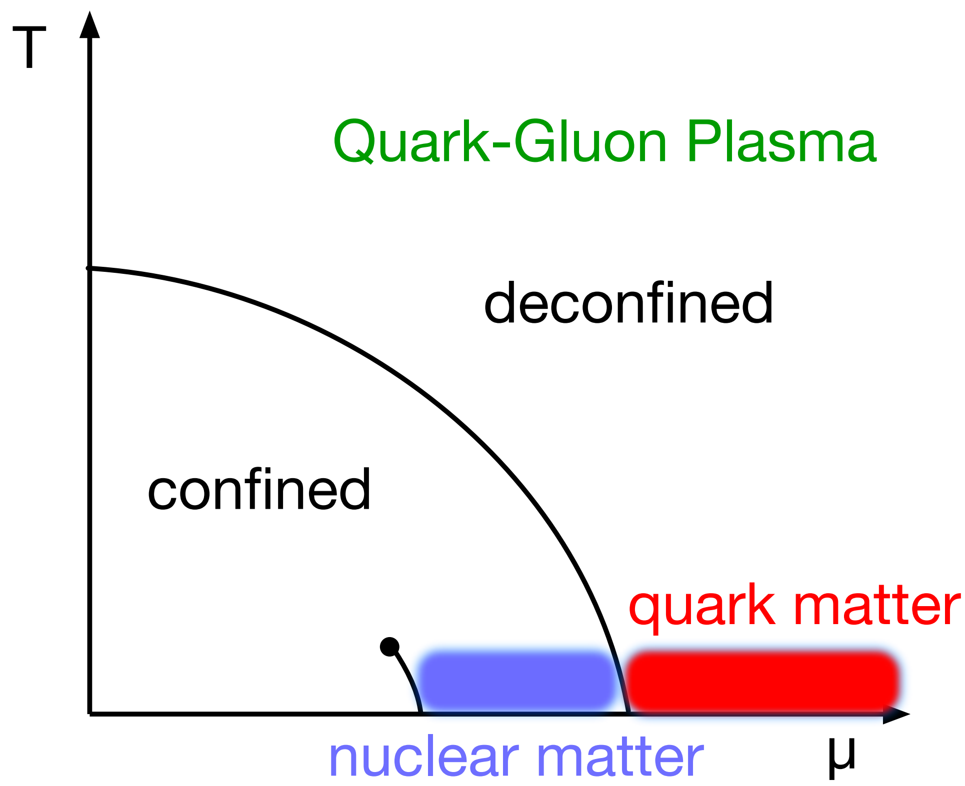
- **Schematically: nuclear matter and quark matter**

  - simplified QCD phase diagram

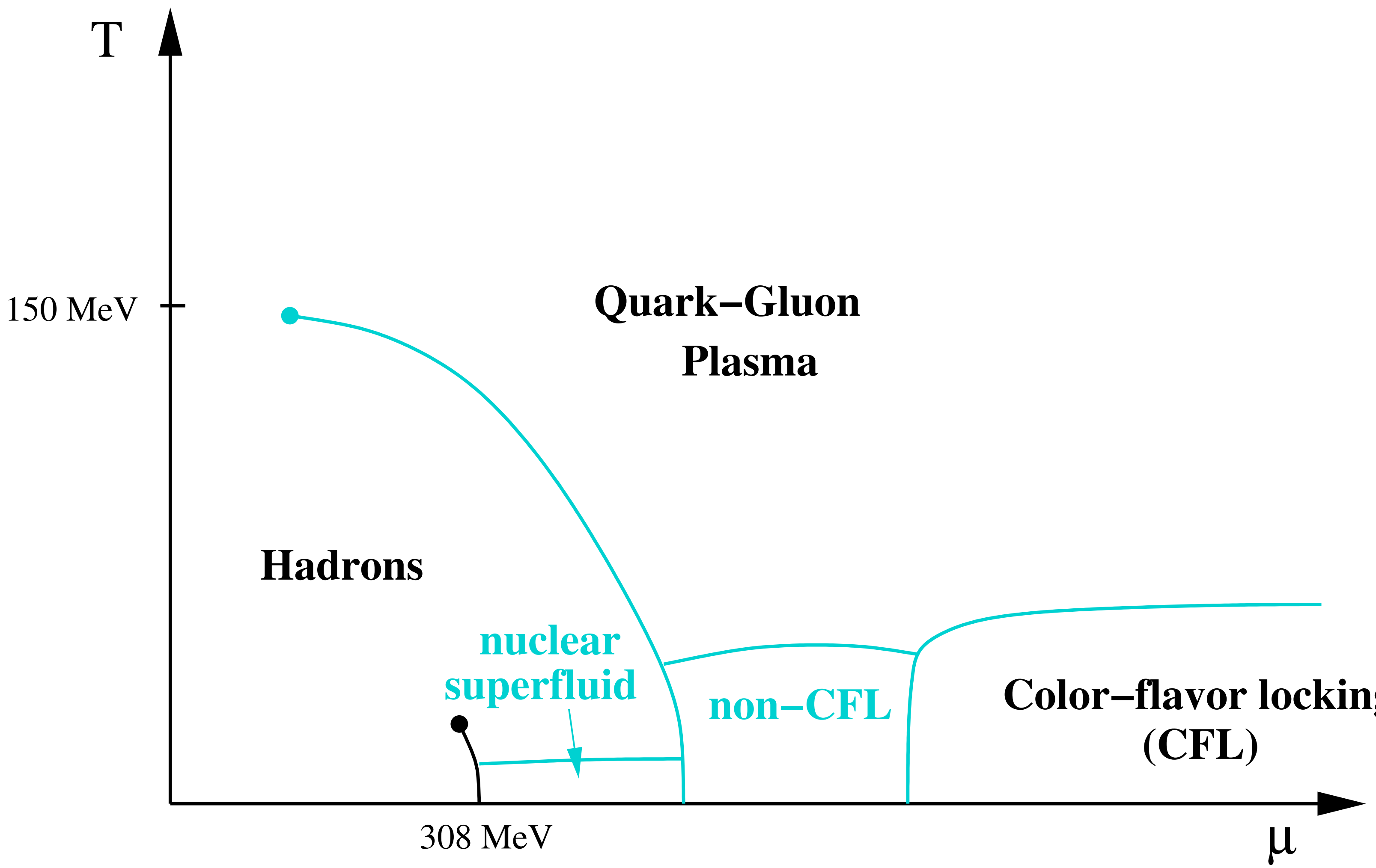


- **Schematically: nuclear matter and quark matter**

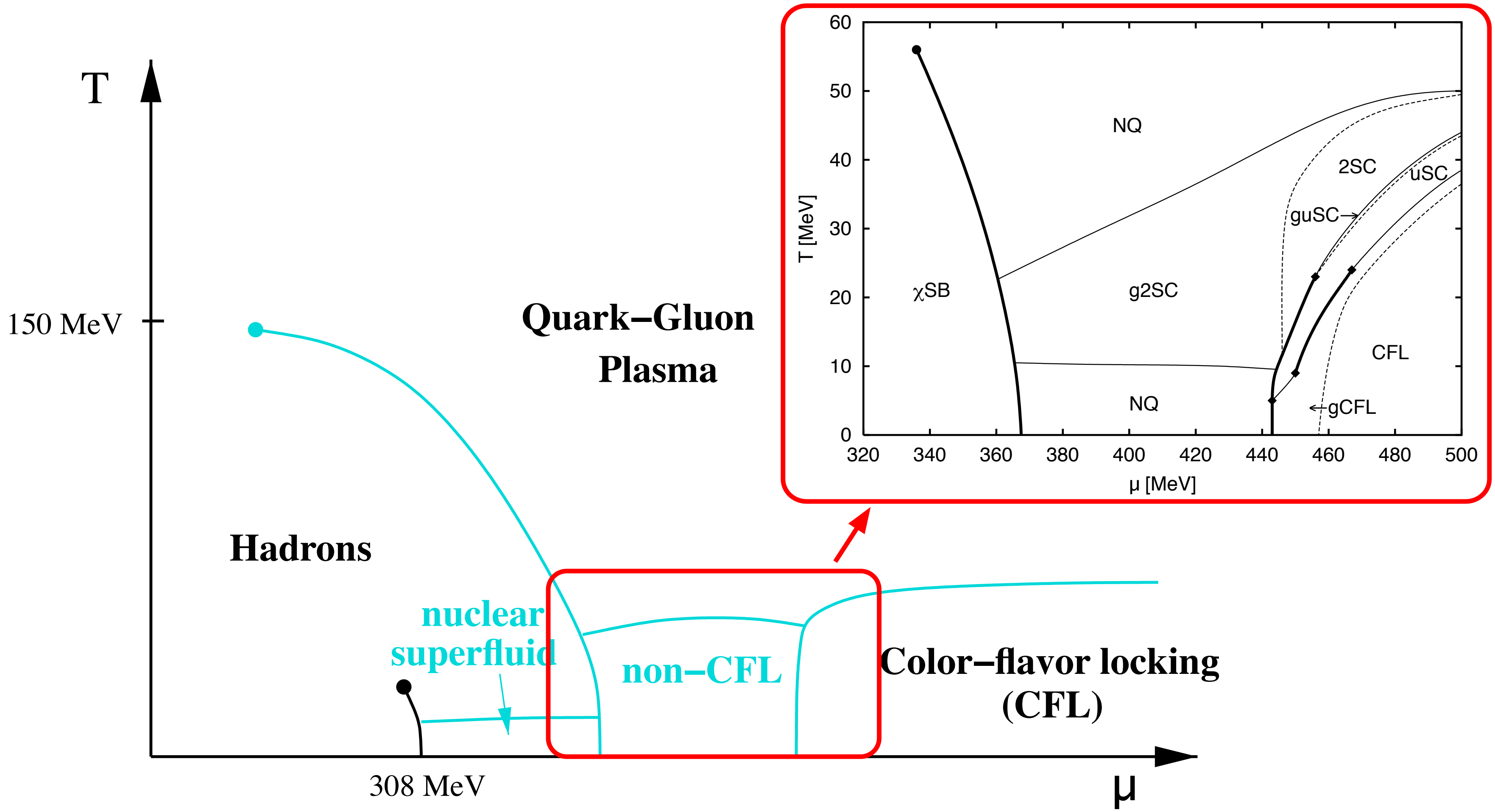
  - simplified QCD phase diagram



- **More detailed view: phases of dense QCD**

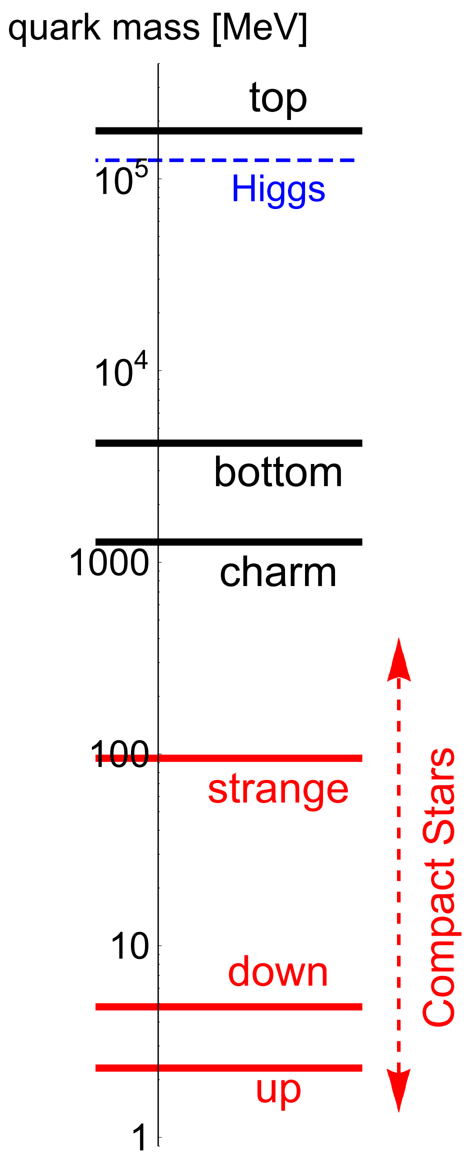


• More detailed view: phases of dense QCD



- details from model calculations (Nambu-Jona-Lasinio model)
  S. B. Ruester, V. Werth, M. Buballa, I. A. Shovkovy and D. H. Rischke, PRD 72, 034004 (2005)

• Three-flavor quark matter



- quark chemical potential in compact stars $300\,\text{MeV} \lesssim \mu \lesssim 500\,\text{MeV}$

> $\Rightarrow$ three-flavor quark matter
> (ignore c,b,t)

- $0 \simeq m_u \simeq m_d \ll \mu$, but m_s not negligible

- remember electric charges:

$$q_u = \frac{2}{3}e\,, \qquad q_d = q_s = -\frac{1}{3}e$$

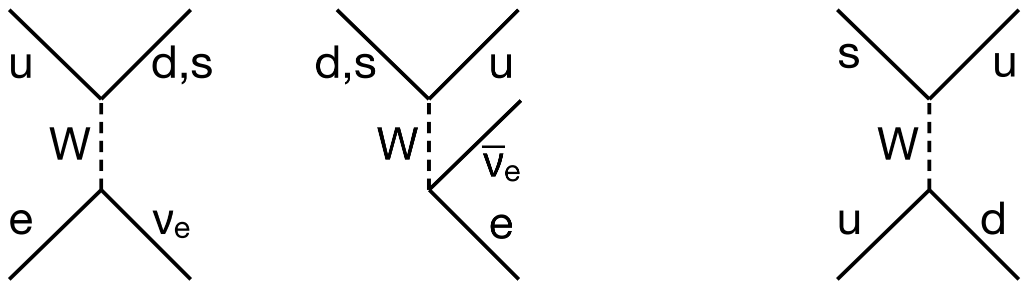
• Noninteracting quark matter (page 1/2)

• pressure

$$\sum_{i=u,d,s,e} P_i = \frac{\mu_u^4}{4\pi^2} + \frac{\mu_d^4}{4\pi^2} + \frac{3}{\pi^2}\int_0^{k_{F,s}} dk\, k^2 \left(\mu_s - \sqrt{k^2+m_s^2}\right) + \frac{\mu_e^4}{12\pi^2}$$

with Fermi momenta $k_{F,u} \simeq \mu_u$, $k_{F,d} \simeq \mu_d$, $k_{F,s} = \sqrt{\mu_s^2 - m_s^2}$

• β-equilibrium



$$\rightarrow \mu_u = \mu - \frac{2}{3}\mu_e, \quad \mu_d = \mu + \frac{1}{3}\mu_e, \quad \mu_s = \mu + \frac{1}{3}\mu_e$$

• electric charge neutrality

$$0 = \sum_{i=u,d,s,e} \frac{\partial P_i}{\partial \mu_e} = -\frac{2}{3}n_u + \frac{1}{3}n_d + \frac{1}{3}n_s + n_e$$

• Noninteracting quark matter (page 2/2)

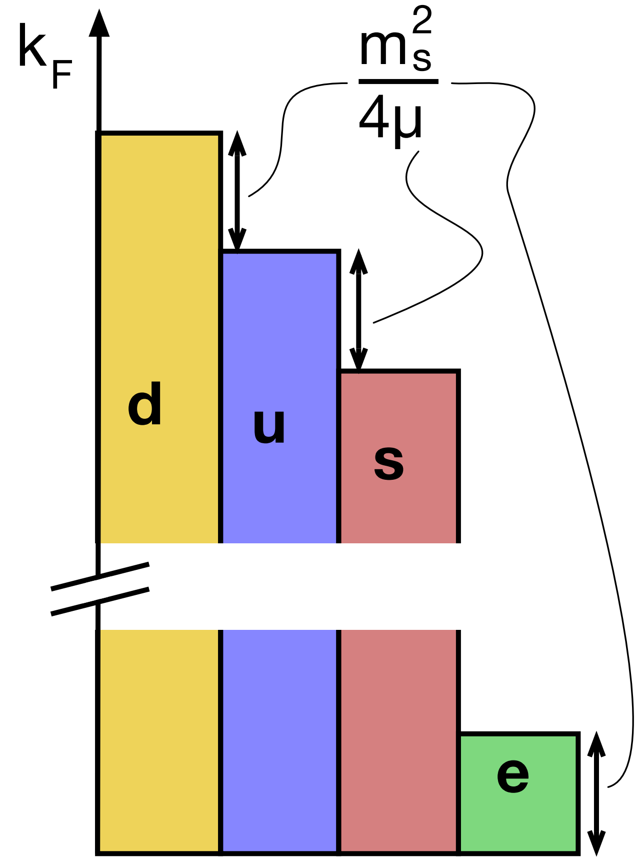
- solve to lowest order in m_s $\Rightarrow$ $\mu_e \simeq \frac{m_s^2}{4\mu}$

- equation of state

$$P \simeq \frac{\epsilon - 4B}{3} - \frac{\mu^2 m_s^2}{2\pi^2}$$

with bag constant B

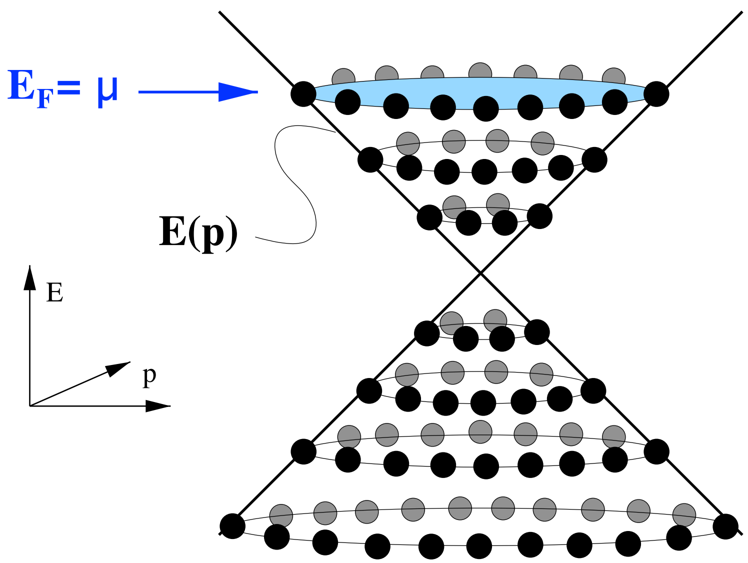
$$P + B = \sum_i P_i\,, \qquad \epsilon = \sum_i \epsilon_i + B$$



- splitting of Fermi surfaces
  → "stressed" Cooper pairing

- large corrections to bag model from perturbative QCD
  A. Kurkela, P. Romatschke, A. Vuorinen, PRD 81, 105021 (2010)

• Cooper pairing of fermions



- free energy $\Omega = E - \mu N$
- no interactions: add fermion at $E = \mu$ without cost
- attractive interaction: add pair with gain
- pairs condense → "Cooper pairing"

This Bardeen-Cooper-Schrieffer (BCS) argument holds for electrons in a metal, ^{3}He atoms, nucleons, quarks, ...

- **Cooper pairing leads to superfluidity/superconductivity**

superfluidity	**superconductivity**
frictionless "charge" transport through Cooper pair condensate (Bose-Einstein condensate in bosonic system); single fermions "gapped" see below	
spontaneous breaking of global symmetry (Cooper pairs neutral)	spontaneous breaking of local symmetry (Cooper pairs charged)
Goldstone mode ("phonon")	Meissner effect (magnetic screening mass for gauge boson)

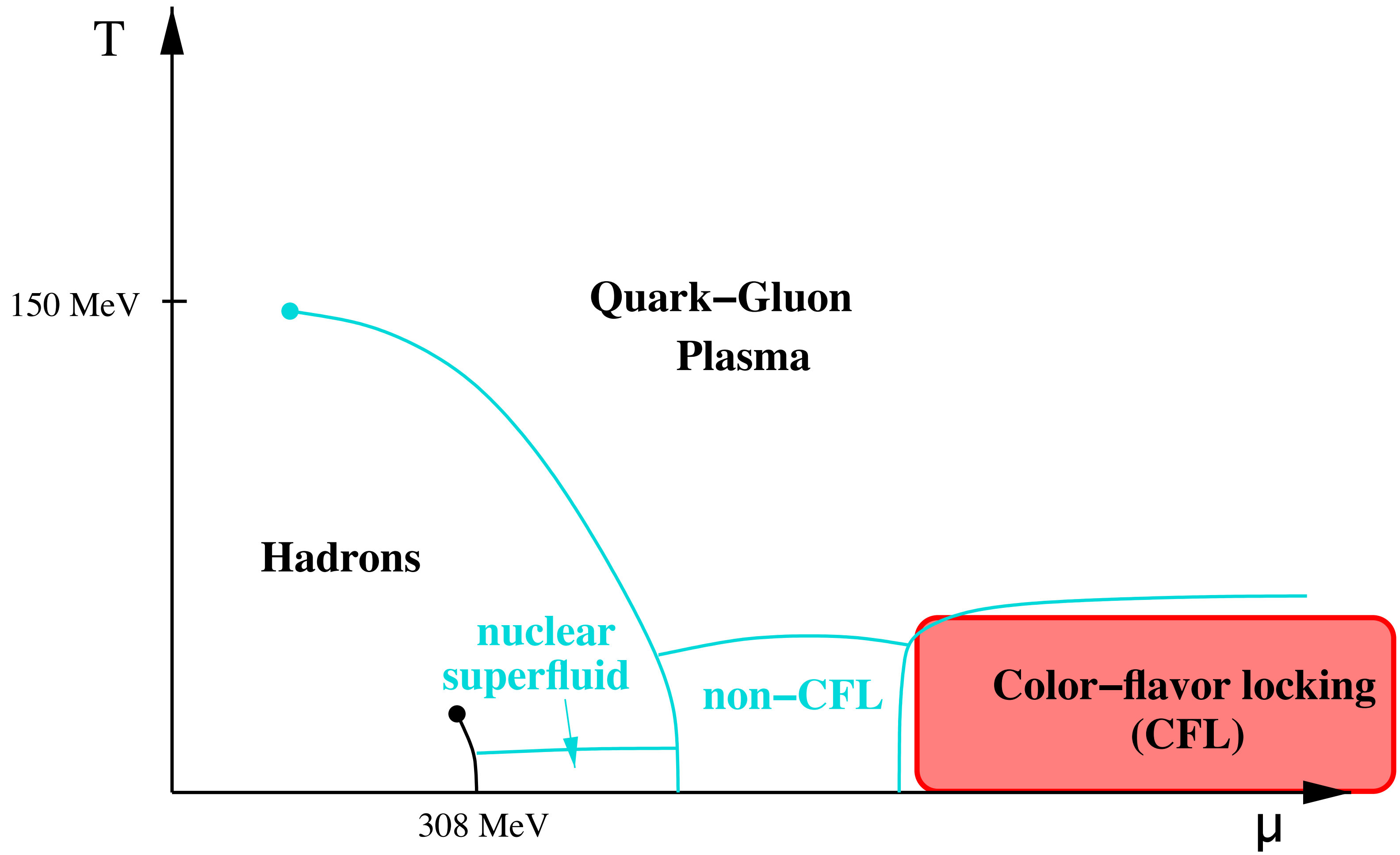
- Cooper pairing of quarks → superfluidity (baryon number charge) and/or electromagnetic superconductivity and/or color superconductivity

• Color-flavor locked (CFL) quark matter

M. Alford, K. Rajagopal, F. Wilczek, NPB 537, 443 (1999)

for a review of CFL and other color superconductors,

see M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, RMP 80, 1455 (2008)



• Cooper pairing of quarks (= color superconductivity)

- one-gluon exchange attractive in antisymmetric antitriplet channel $[\bar{\mathbf{3}}]_c^a$

$$SU(3)_c : \qquad [\mathbf{3}]_c \otimes [\mathbf{3}]_c = [\bar{\mathbf{3}}]_c^a \oplus [\mathbf{6}]_c^s$$

- flavor space

$$SU(3)_f : \qquad [\mathbf{3}]_f \otimes [\mathbf{3}]_f = [\bar{\mathbf{3}}]_f^a \oplus [\mathbf{6}]_f^s$$

- order parameter (for spin-0 pairing):

$$\langle \psi_i^\alpha C \gamma_5 \psi_j^\beta \rangle \;\propto\; \epsilon^{\alpha\beta A}\, \epsilon_{ijB}\, \phi_B^A \;\in\; [\bar{\mathbf{3}}]_c^a \otimes [\bar{\mathbf{3}}]_f^a$$

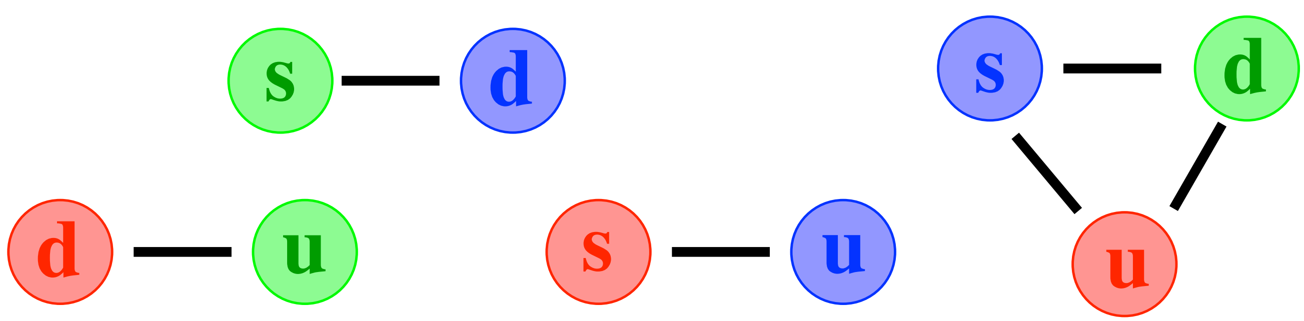
• CFL at asymptotically large density

$$0 \simeq m_s \simeq m_u \simeq m_d \ll \mu \qquad \text{all quark masses negligible}$$

- CFL order parameter

$$\phi^A_B = \delta^A_B \quad \Rightarrow \quad \langle \psi^\alpha_i C\gamma_5 \psi^\beta_j \rangle \propto \epsilon^{\alpha\beta A}\, \epsilon_{ijA}$$

$$\Rightarrow \quad SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$



- **CFL breaks chiral symmetry**

  - usual chiral symmetry breaking: LR pairing $\langle \bar{\psi}_R \psi_L \rangle$
  - CFL: LL, RR pairing $\langle \psi_R \psi_R \rangle$, $\langle \psi_L \psi_L \rangle$, however

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$

  - chiral symmetry broken through "locking" to color
  - octet of pseudo-Goldstone modes K^0, $K^{\pm}$, π^0, ...
    D. T. Son and M. A. Stephanov, PRD 62, 059902 (2000)
    $\rightarrow$ effective theory for CFL just like usual chiral perturbation theory
    P. F. Bedaque and T. Schäfer, NPA 697, 802 (2002)
    $\rightarrow$ important for transport properties

- **CFL is a superfluid**

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$

→ Goldstone mode ϕ
→ vortices in rotating CFL

- **CFL is *not* an electromagnetic superconductor**

$$SU(3)_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset U(1)_{\tilde{Q}}}$$

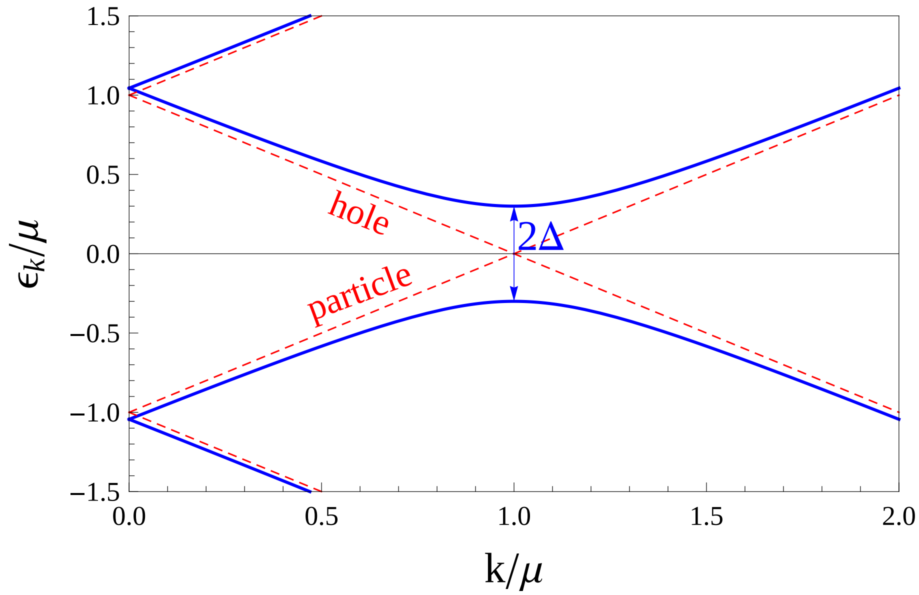
→ Cooper pairs neutral under $\tilde{Q} = Q + \frac{2}{\sqrt{3}} T_8$
→ photon-gluon mixing with (small) mixing angle
$\cos^2\theta = 1 + \mathcal{O}(e^2/g^2)$
(analogous to Weinberg angle in standard model)

- **Low-energy modes in CFL are Goldstone bosons**

  - all quarks are gapped

$$\epsilon_k^e = \pm\sqrt{(\mu - ek)^2 + \Delta^2} \quad (8-\text{fold})$$

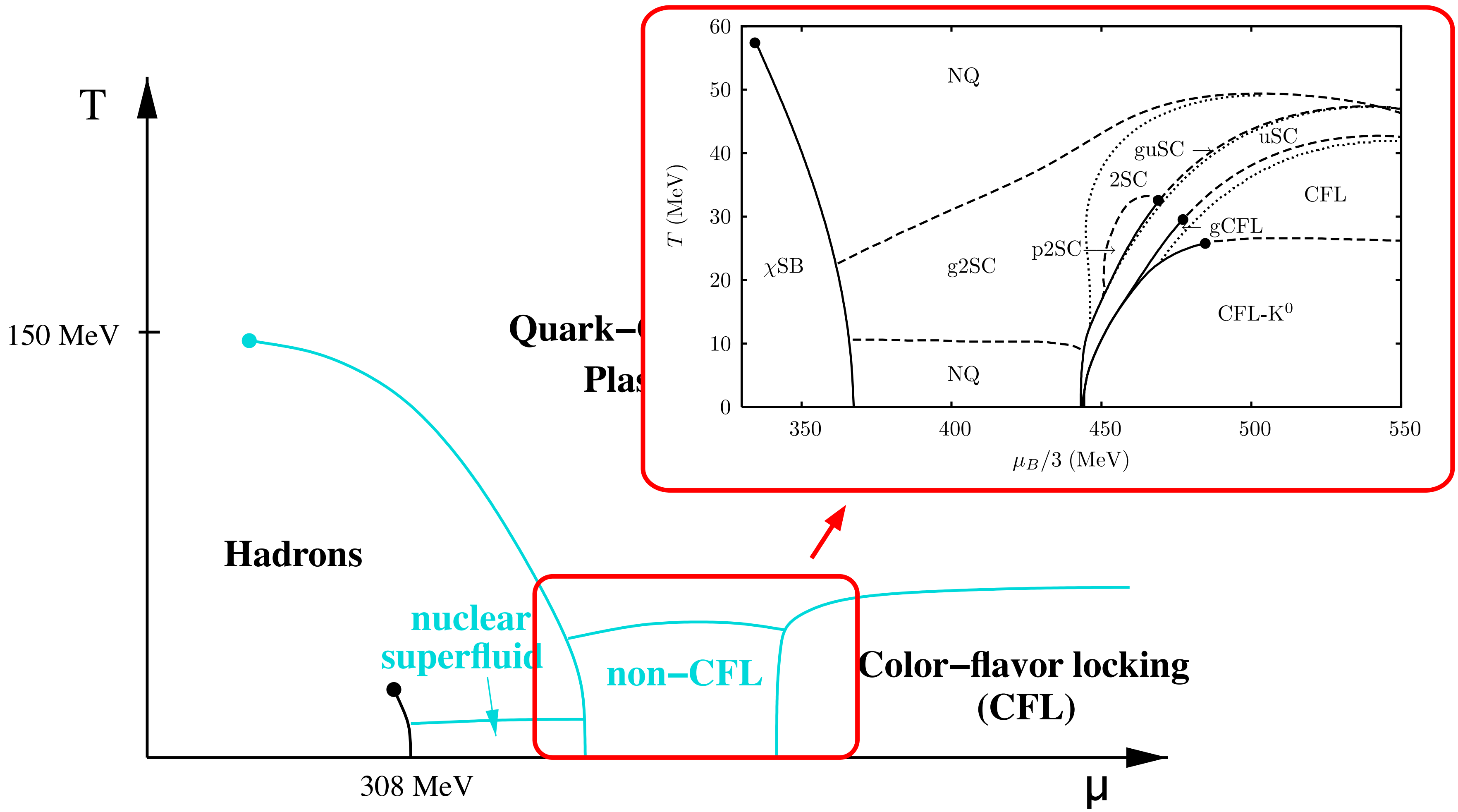
$$\epsilon_k^e = \pm\sqrt{(\mu - ek)^2 + 4\Delta^2} \quad (1-\text{fold})$$



  - $\Delta \simeq (20-100)$ MeV at densities relevant for compact stars (from perturbative QCD and phenomenological models)

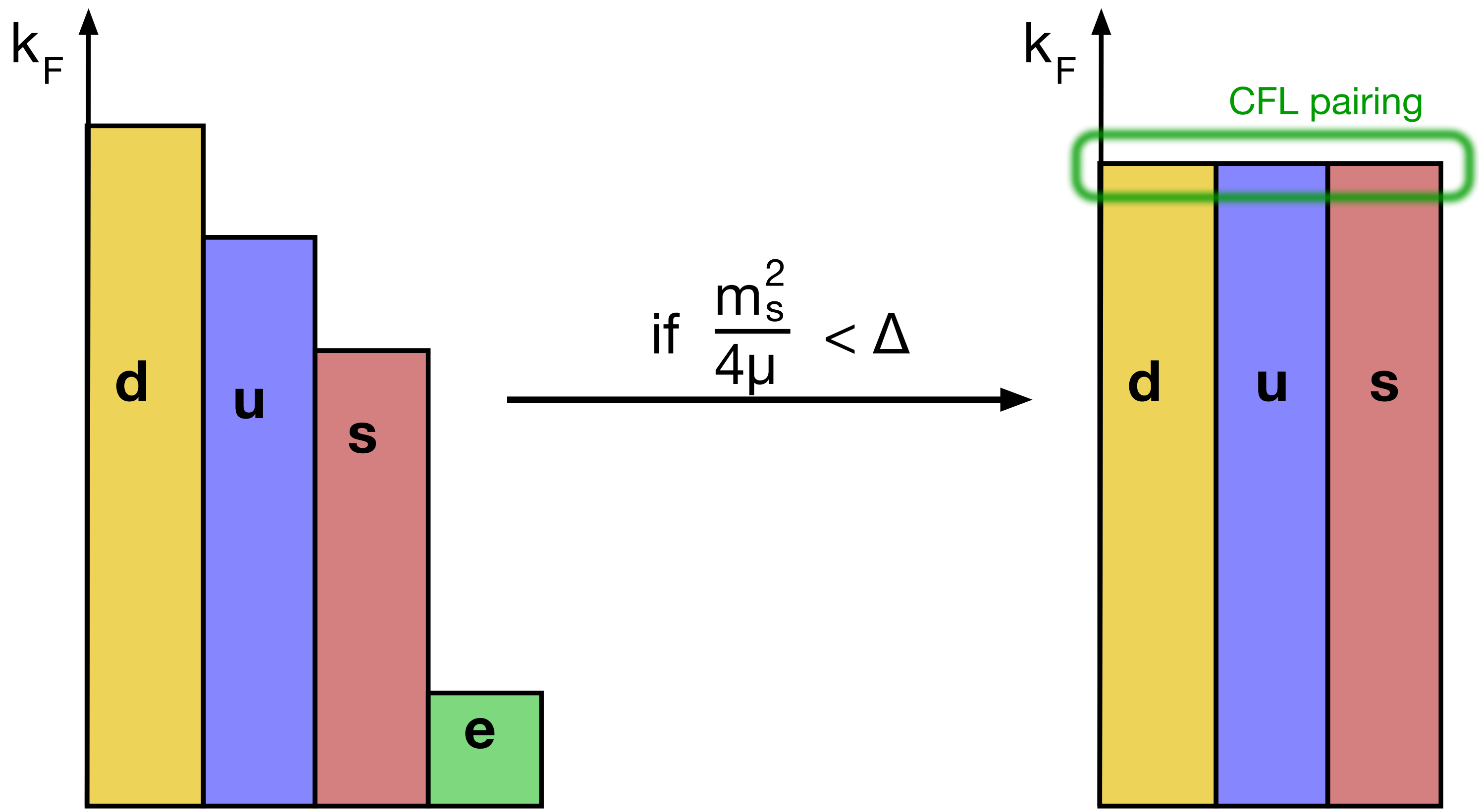
  - for $T \ll \Delta$: transport dominated by Goldstone modes ϕ, K^0, ...

• Moderate densities: CFL variants and non-CFL phases



- **Pairing with mismatch in Fermi momenta**

  - CFL favored if mismatch sufficiently small



- **Stressed pairing in quark matter: variants of CFL and non-CFL color superconductors (page 1/3)**

  - Kaon-condensed phases: CFL-K^0, curCFL-K^0

    P. Bedaque, T. Schäfer, NPA 697, 802 (2002)

    T. Schäfer, PRL 96, 012305 (2006)

    A. Schmitt, NPA 820, 49C (2009)

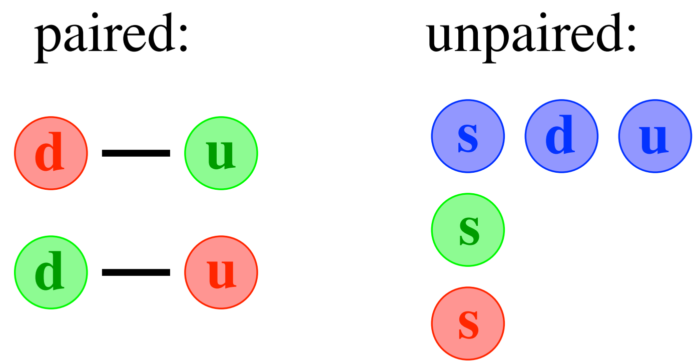
curCFL-K^0

counterpropagating currents: K^0-condensate + gapless fermions

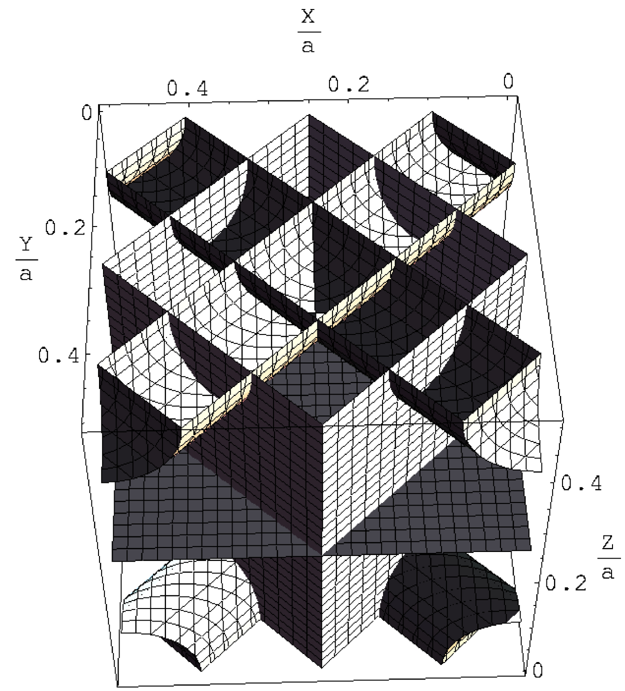
  - 2SC phase

    R. Rapp, T. Schäfer, E.V. Shuryak, M. Velkovsky, PRL 81, 53 (1998)

    M.G. Alford, K. Rajagopal, F. Wilczek, PLB 422, 247 (1998)

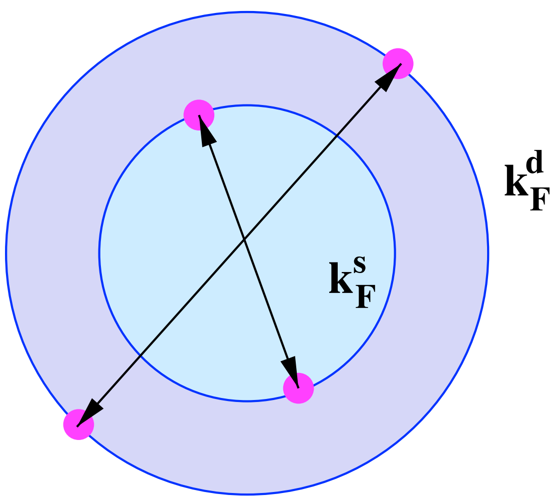


- **Stressed pairing in quark matter: variants of CFL and non-CFL color superconductors (page 2/3)**



- Crystalline phases: LOFF

  M. Alford, J. Bowers, K. Rajagopal, PRD 63, 074016 (2001)
  M. Mannarelli, K. Rajagopal and R. Sharma, PRD 73, 114012 (2006)



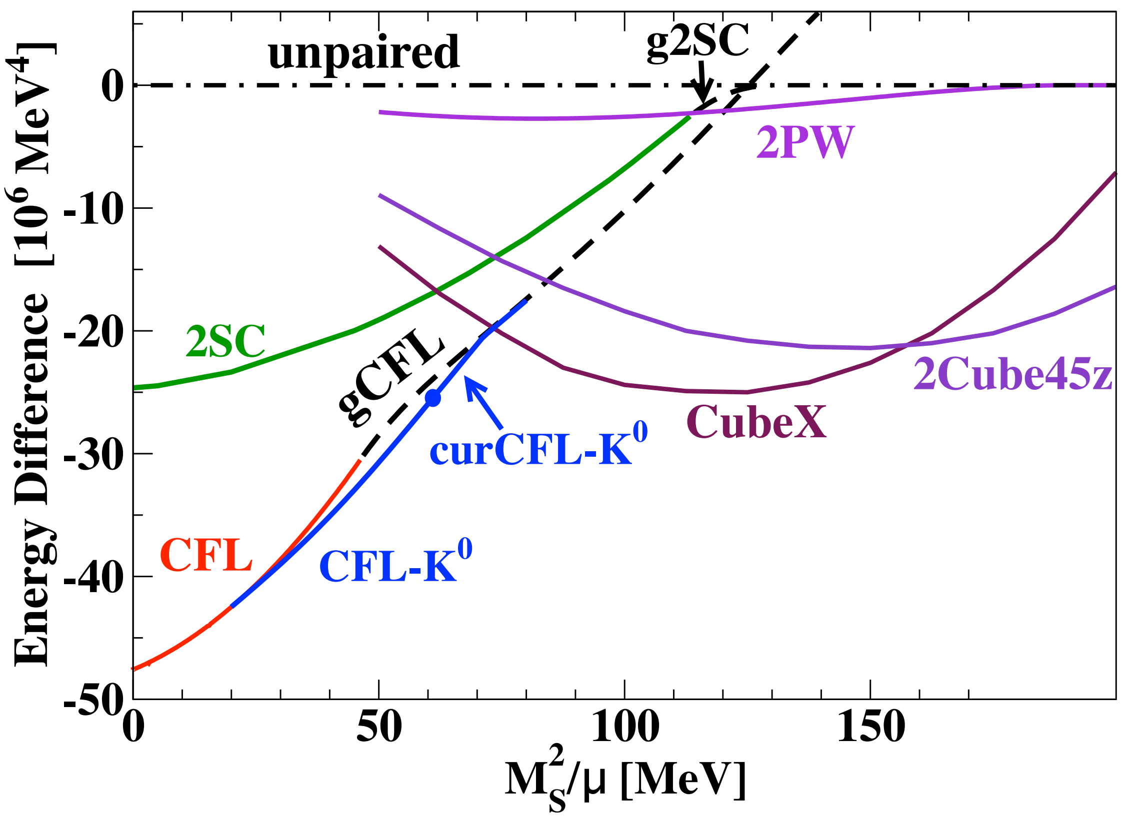
- Single-flavor pairing:
  CSL, *A*-phase, polar phase …

  T. Schäfer, PRD 62, 094007 (2000)
  A. Schmitt, PRD 71, 054016 (2005)

- **Stressed pairing in quark matter: variants of CFL and non-CFL color superconductors (page 3/3)**

Free energy comparison of 3-flavor quark phases for Δ_{CFL} = 25 MeV:

M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, RMP 80, 1455 (2008)



• Quark matter transport in a nutshell

	All fermions paired (CFL, CFL-K^0)	Ungapped fermions (2SC, LOFF, single-flavor pairing, ...)
transport dominated by	Goldstone modes (phonon ϕ and meson K^0)	quarks ("blue" quarks in 2SC, ...)
typical size of transport for $\mu \gg T$	small (no Fermi surface)	large (Fermi surface)

(cf. specific heat of bosons $c_V \propto T^3$ vs. fermions $c_V \propto \mu^2 T$)

• Bulk viscosity: definition

- bulk viscosity $\zeta(\omega)$ = dissipative response to compression and expansion $V(t) = V_0 + \delta V \cos\omega t$,

$$\zeta(\omega) = 2\langle\dot{E}\rangle\left(\frac{V_0}{\delta V}\right)^2\frac{1}{\omega^2}$$

with the dissipated power in an oscillation period $\tau = 2\pi/\omega$

$$\langle\dot{E}\rangle = -\frac{1}{\tau V_0}\int_0^\tau dt\, P(t)\frac{dV}{dt} = B\frac{\gamma}{A}\langle\delta\mu(t)\delta V(t)\rangle$$

- A, B susceptibilities in equilibrium, and $\delta\mu \equiv \mu_d - \mu_s$
- Γ number of d quarks produced per volume and time in $u + d \leftrightarrow u + s$

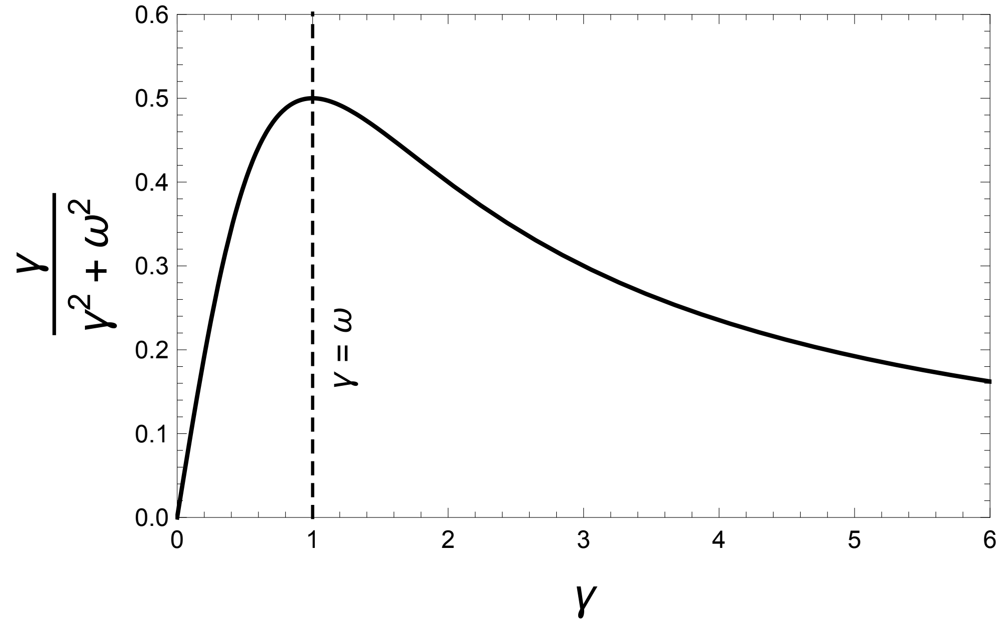
$$\Gamma \simeq \frac{\gamma}{A}\delta\mu\,, \qquad \frac{\partial\delta\mu}{\partial t} = B\frac{\partial\delta V}{\partial t} - \Gamma$$

$$\Rightarrow \qquad \zeta(\omega) = \frac{B^2}{A}\frac{\gamma}{\gamma^2+\omega^2}$$

[for more details of the derivation see e.g. M. G. Alford and A. Schmitt, JPG 34, 67 (2007)]

• Bulk viscosity is a resonance phenomenon

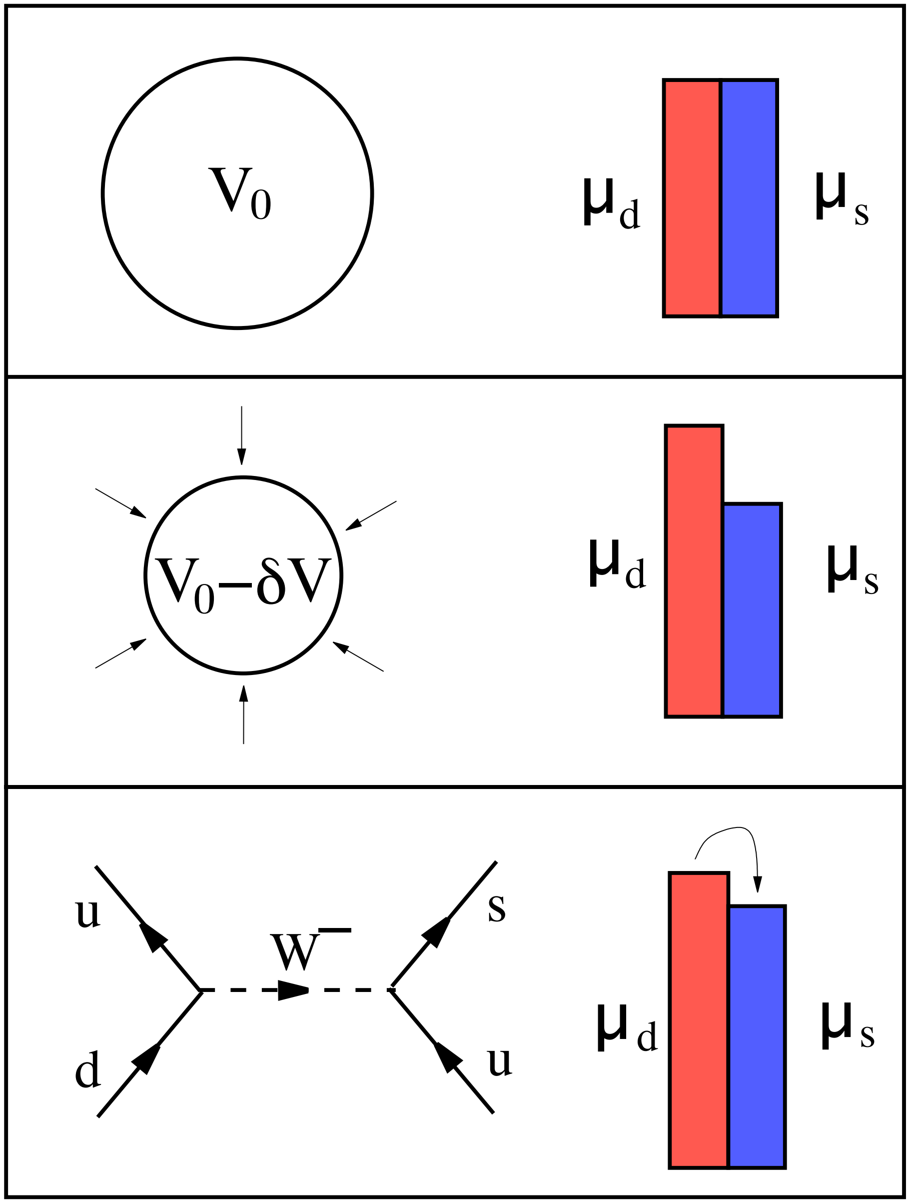
• ζ maximal for $\gamma = \omega$



• need microscopic rate γ to be of the order of star oscillations ω
$\rightarrow$ ζ dominated by electroweak interactions

• γ is typically monotonically increasing with T
$\rightarrow$ maximum of ζ at a certain T

- **Bulk viscosity schematically**



- volume oscillation
  → chemical
  non-equilibrium

$$\mu_d - \mu_s \neq 0$$

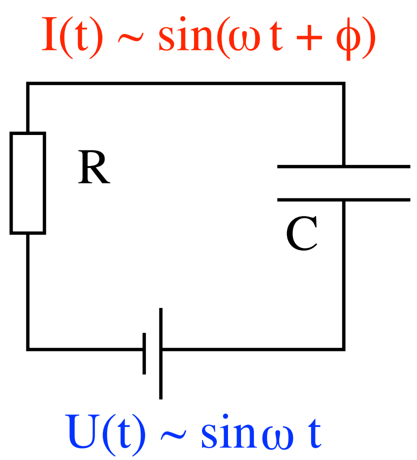
- re-equilibration via

$$\boxed{u + d \leftrightarrow u + s}$$

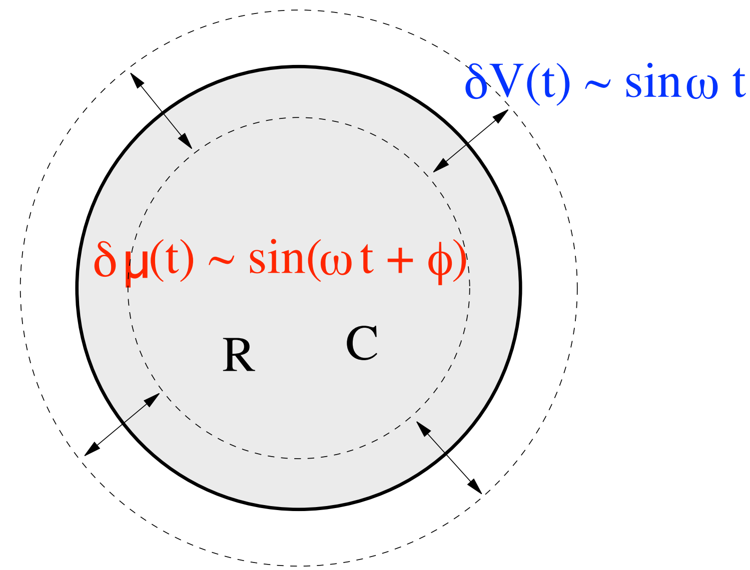
- resonance phenomenon:
  external oscillation
  vs. microscopic rate

• Just like an electric circuit!

electric circuit with alternating voltage



volume element with oscillating volume

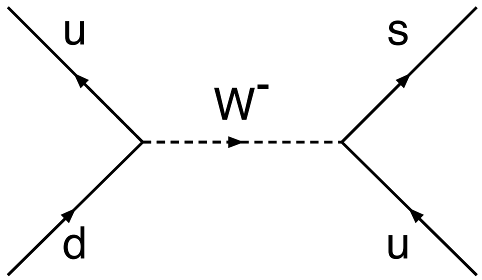


$\langle \dot{E} \rangle \propto \langle U(t) I(t) \rangle$	$\langle \dot{E} \rangle \propto \langle V(t) \delta\mu(t) \rangle$
capacitance C	inverse microscopic rate γ^{-1} (slow process → store large chemical energy)
resistance R	$\left(n_d \frac{\partial \mu_d}{\partial n_d} - n_s \frac{\partial \mu_s}{\partial n_s} \right)^{-1}$ (same dispersion for d and s → infinite "resistance" → no dissipation)
inductance L	0

• Flavor changing rates

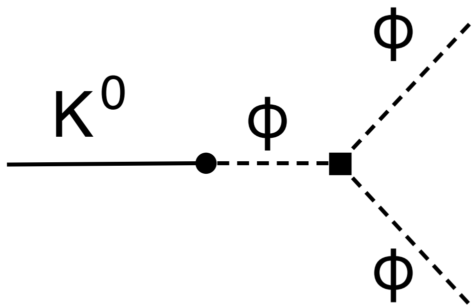
- unpaired quark matter
  J. Madsen, PRD 46, 3290 (1992)

$$\frac{\Gamma_d}{\delta\mu} \simeq \frac{64 G_F^2 V_{us}^2 V_{ud}^2}{5}\,\mu^5 T^2$$



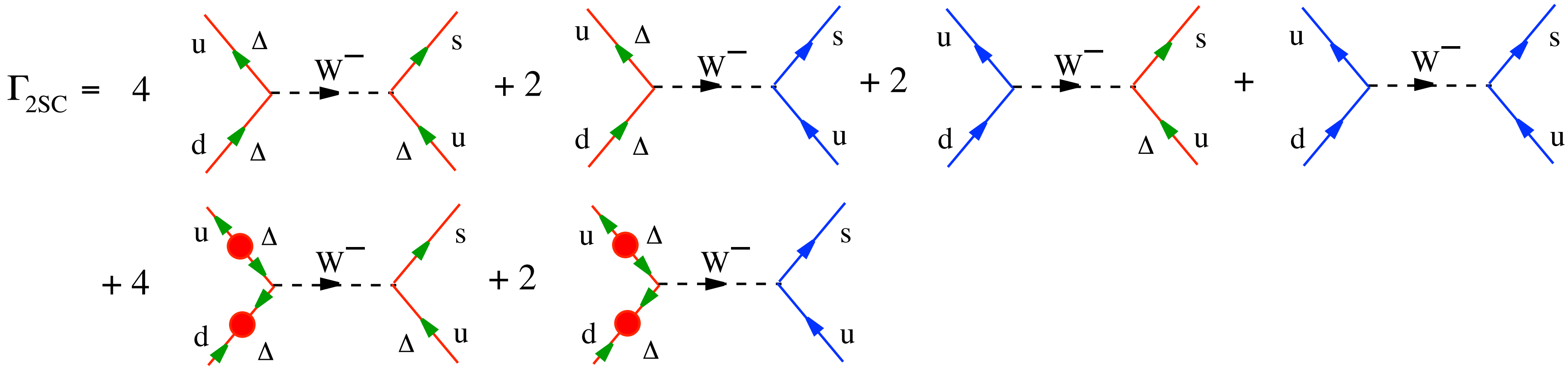
- CFL-K^0
  M.G. Alford, M. Braby, A. Schmitt, JPG 35, 115007 (2008)

$$\frac{\Gamma_{K^0}}{\delta\mu} \simeq \frac{G_F^2 V_{us}^2 V_{ud}^2}{\pi}\,\frac{m_{K^0}^4}{\mu_{K^0}^4}\,Q(c)\,f_\pi^2 f_\phi^2\,\frac{T^7}{\mu^4}$$

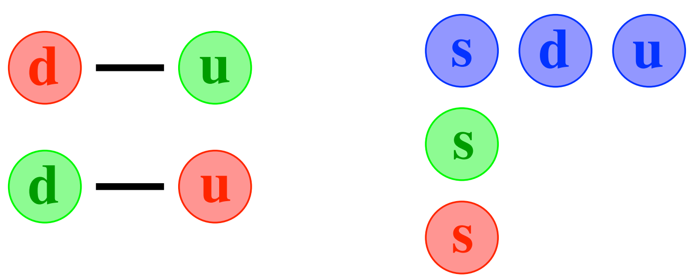


with $Q(c)$ function of the slope of the kaon dispersion c

• 2SC phase



paired: unpaired:



small temperatures,

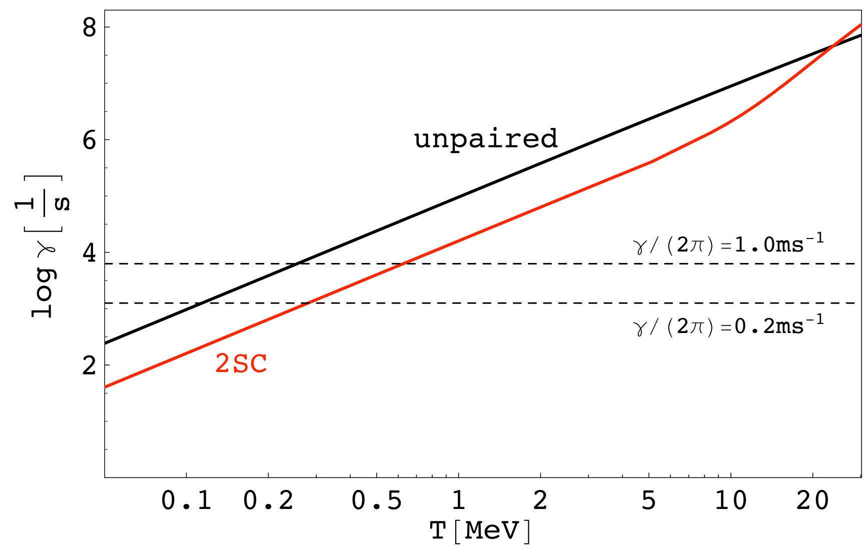
$$T \ll T_c \simeq 20\text{MeV}$$

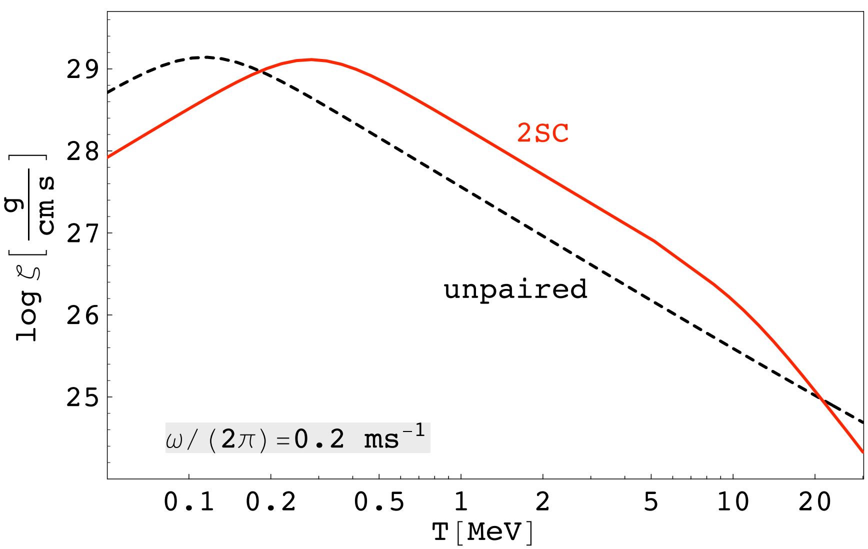
$$\Gamma_{2SC} = \frac{1}{9}\Gamma_{\text{unpaired}}$$

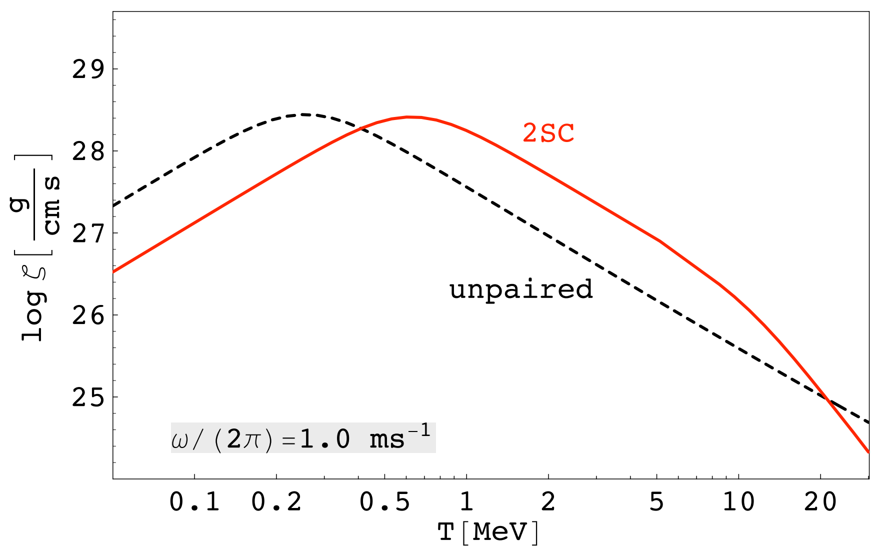
due to exponential suppression $\exp(-\Delta/T)$ of gapped modes

• From the microscopic rate to the bulk viscosity

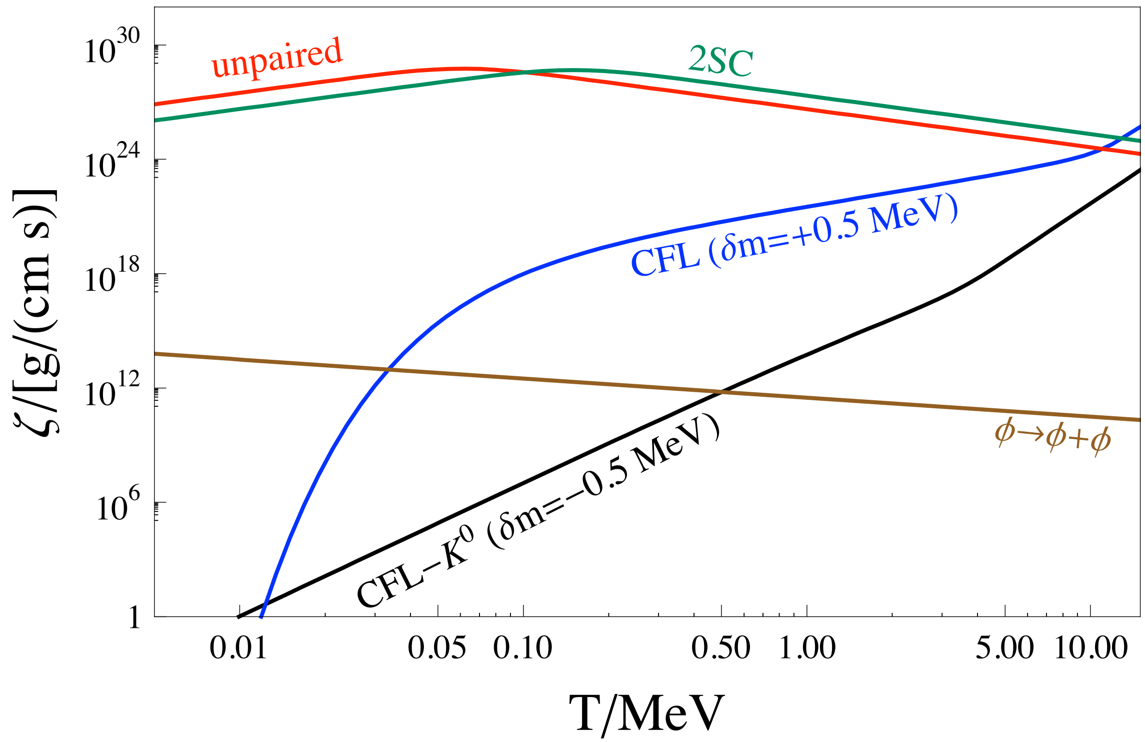
$$\zeta = \frac{B^2}{A} \frac{\gamma}{\gamma^2 + \omega^2}$$







• Quark matter bulk viscosity: different phases



$\omega/(2\pi) = 1\,\mathrm{ms}^{-1}$

$\mu = 400\,\mathrm{MeV}$

$\delta m \equiv m_{K^0} - \mu_{K^0}$

unpaired from $u + d \leftrightarrow u + s$ J. Madsen, PRD 46, 3290 (1992)

unpaired from $u + e \leftrightarrow d + \nu_e$ B. A. Sa'd, I. A. Shovkovy and D. H. Rischke, PRD 75, 125004 (2007)

2SC from $u + d \leftrightarrow u + s$ M.G. Alford, A. Schmitt, JPG 34, 67-101 (2007)

CFL from $K^0 \leftrightarrow \phi + \phi$ M.G. Alford, M. Braby, S. Reddy, T. Schäfer, PRC 75, 055209 (2007)

CFL-K^0 from $K^0 \leftrightarrow \phi + \phi$ M.G. Alford, M. Braby, A. Schmitt, JPG 35, 115007 (2008)

CFL from $\phi \leftrightarrow \phi + \phi$ C. Manuel, F. Llanes-Estrada, JCAP 0708, 001 (2007)

Spin-one from $u + d \leftrightarrow u + s$, $u + e \leftrightarrow d + \nu_e$ X. Wang and I. A. Shovkovy, PRD 82, 085007 (2010)

• Beyond the "standard" calculation

- a superfluid has three bulk viscosity coefficients
  (due to the presence of two fluids: "superfluid" and "normal fluid")
  I.M. Khalatnikov, *An introduction to the theory of superfluidity* (New York, 1989)

  – apply to CFL

    CFL from $\phi \leftrightarrow \phi + \phi$ M. Mannarelli and C. Manuel, PRD 81, 043002 (2010)
    CFL from $K^0 \leftrightarrow \phi + \phi$ R. Bierkandt and C. Manuel, PRD 84, 023004 (2011)
    r-modes with CFL N. Andersson, B. Haskell and G. L. Comer, PRD 82, 023007 (2010)

- electroweak rate beyond linear order $\Gamma \propto \delta\mu$
  J. Madsen, PRD 46, 3290 (1992)
  M. G. Alford, S. Mahmoodifar and K. Schwenzer, JPG 37, 125202 (2010)
  $\rightarrow$ relevant for large r-mode amplitudes

- non-Fermi liquid effects in unpaired quark matter
  K. Schwenzer, arXiv:1212.5242 [nucl-th]
  $\rightarrow$ seems to agree with astrophysical data in T-Ω plane
  M. G. Alford and K. Schwenzer, PRL 113, 251102 (2014)

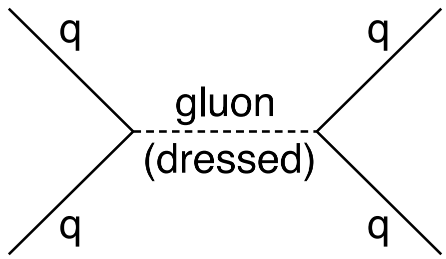
• Shear viscosity in dense quark matter (page 1/2)

- dissipation through thermal re-equilibration
  (as opposed to chemical re-equilibration for bulk viscosity)

- relevant microscopic processes from strong interaction
  (as opposed to electroweak interaction for bulk viscosity)

• Shear viscosity in dense quark matter (page 2/2)

- unpaired quark matter
  H. Heiselberg and C. J. Pethick, PRD 48, 2916 (1993)



$$\eta \propto \frac{\mu^4 m_D^{2/3}}{\alpha_s^2 T^{5/3}} \quad \text{with} \quad m_D^2 = \frac{N_f g^2 \mu^2}{2\pi^2}$$

- CFL (phonons) C. Manuel, A. Dobado, F. J. Llanes-Estrada, JHEP 0509, 076 (2005)

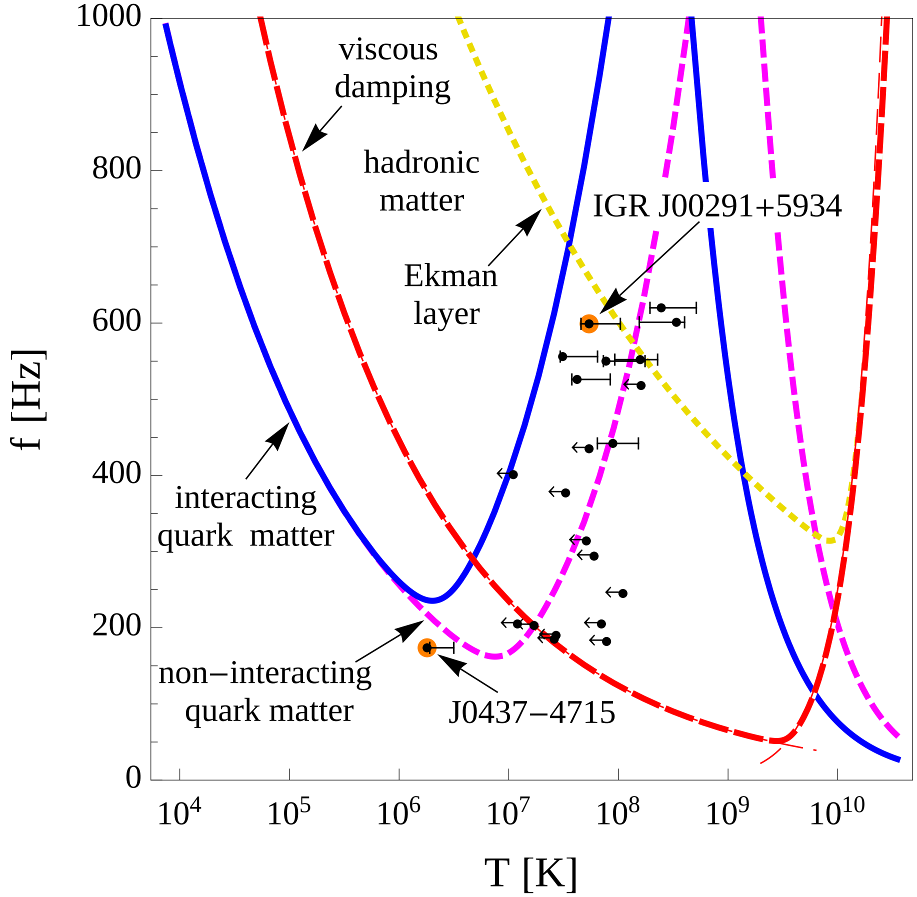
$$\eta \propto \frac{\mu^8}{T^5} \qquad \text{(not applicable at very low } T\text{)}$$

- CFL-K^0 (kaons) M. G. Alford, M. Braby, S. Mahmoodifar, PRC 81, 025202 (2010)
  → smaller η, but also smaller mean free path than phonon contribution

- 2SC (electrons and unpaired quarks)
  M. G. Alford, H. Nishimura and A. Sedrakian, PRC 90, 055205 (2014)

• Comparison to nuclear matter

- r-mode instability window for nuclear and (unpaired) quark matter



M. G. Alford, K. Schwenzer, PRL 113, 251102 (2014)

• Summary

- quark matter
  - asymptotically dense matter is a color superconductor in the color-flavor locked (CFL) state
  - at moderate densities, mismatched Fermi surfaces may lead to other phases (CFL-K^0, 2SC, ...)

- transport in quark matter
  - ... is dominated by Goldstone modes (CFL) or ungapped quarks (non-CFL)
  - bulk viscosity is a resonance phenomenon
  - bulk (shear) viscosity is dominated by electroweak (strong) interactions
  - quark matter can lead to a different instability window than nuclear matter