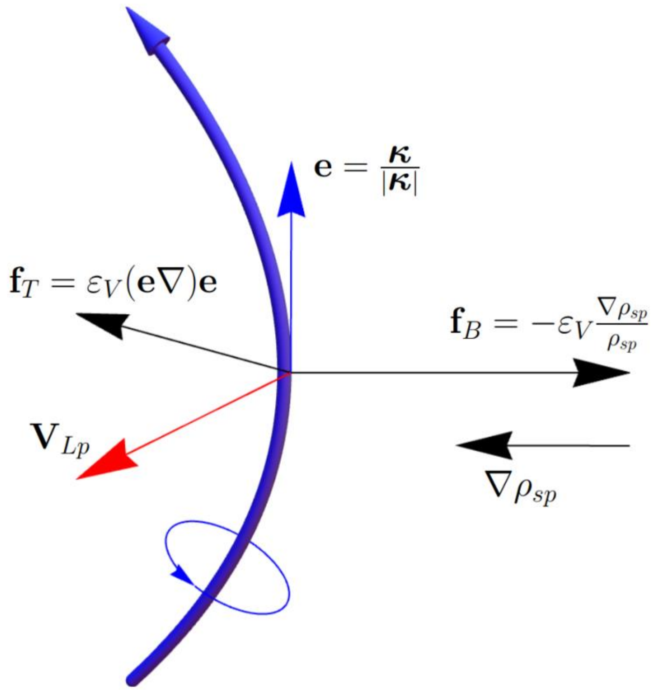
Vortex buoyancy in superfluid and superconducting neutron stars



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Introduction

- At $T < 10^8 - 10^{10} K$ protons in NS core condense into superconducting state.
- In the case of type-II superconductivity magnetic field penetrates the core in the form of quantized fluxtubes (Abrikosov vortices).
- Each fluxtube (proton vortex) carries single flux quantum, $\phi_0 \equiv \frac{\pi \hbar c}{e}$.
- Since all the magnetic field is locked to proton vortices, its evolution is determined by vortex motion.

Proton vortex structure

coherence length (≈size of kernel): $\xi_p \sim 10^{-12}$ cm

London penetration depth: $\delta_p \sim 10^{-11}$ cm

velocity field:

$$\mathbf{v}(r) = \frac{\kappa}{2\pi\delta_p} K_1\left(\frac{r}{\delta_p}\right)\mathbf{e}_\phi, \quad \kappa = \frac{\pi\hbar}{m_p}$$

$$v(r) \approx \frac{\kappa}{2\pi r}, \quad r \ll \delta_p$$

$$v(r) \sim \mathrm{e}^{-r/\delta_p}, \quad r \gg \delta_p$$

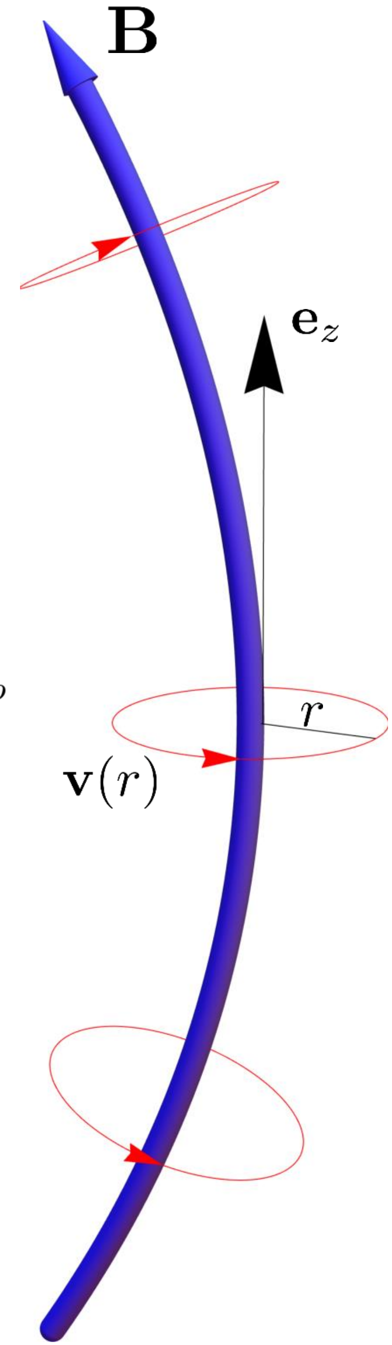
magnetic field:

$$\mathbf{B}(r) = \frac{\phi_0}{2\pi\delta_p^2} K_0\left(\frac{r}{\delta_p}\right)\mathbf{e}_z$$

$$\int_0^\infty B(r) 2\pi r \mathrm{d}r = \phi_0 \equiv \frac{\pi\hbar c}{e}$$

energy per unit length:

$$\varepsilon_V = \int_{\xi_p}^\infty \left[\rho_{sp}\frac{v^2(r)}{2} + \frac{B^2(r)}{8\pi}\right] 2\pi r dr \approx \rho_{sp}\frac{\kappa^2}{4\pi}\ln\frac{\delta_p}{\xi_p}$$



Buoyancy force: standard derivation

[Muslimov & Tsygan (1985)]

see also Sharvey, Ruderman & Shaham (1986)

Bernoulli's theorem:

$$P_{\text{fluid}} + \rho_{sp}\frac{v^2(r)}{2} + \frac{B^2(r)}{8\pi} = const$$

pressure drop:

$$\Delta P(r) = \rho_{sp}\frac{v^2(r)}{2} + \frac{B^2(r)}{8\pi} \quad \Rightarrow$$

density drop:

$$\Delta\rho = \frac{1}{c_s^2}\Delta P, \quad c_s^2 = \frac{\mathrm{d}P}{\mathrm{d}\rho}$$

buoyancy force per unit length:

$$\mathbf{f}_B = -\mathbf{g}\int_{\xi_p}^{\infty}\Delta\rho(r)2\pi r dr = -\varepsilon_V\frac{\mathbf{g}}{c_s^2} = -\varepsilon_V\frac{\nabla\rho}{\rho}$$

Buoyancy force: standard derivation

$$\mathbf{f}_B = -\varepsilon_V \frac{\mathbf{g}}{c_s^2} = -\varepsilon_V \frac{\nabla \rho}{\rho}$$

Muslimov & Tsygan (1985)

Also used by:

- Jones (1987)
- Ruderman et al. (1998)
- Jahan-Miri (2000)
- Elfritz et al. (2016)
- …

Hydrodynamical approach at
$\boldsymbol{r \sim \delta_p \sim 10^{-11}\mathrm{cm}}$ **???**

Buoyancy force: alternative derivation

energy per unit length:

$$\varepsilon_V = \rho_{sp} \frac{\kappa^2}{4\pi} \ln \frac{\delta_p}{\xi_p}$$

Let's shift the vortex from x to x':

$$\varepsilon_V(x') - \varepsilon_V(x) = [\rho_{sp}(x') - \rho_{sp}(x)] \frac{\kappa^2}{4\pi} \ln \frac{\delta_p}{\xi_p}$$

Generally, the force is given by the (minus) energy gradient:

$$\mathbf{f}_B = -\nabla \varepsilon_V(\mathbf{r}) = -\varepsilon_V \frac{\nabla \rho_{sp}}{\rho_{sp}}$$

$$\mathbf{f}_B = -\varepsilon_V \frac{\nabla \rho}{\rho}$$

Muslimov & Tsygan (1985)

Our derivation is also supported by microscopic Ginzburg–Landau theory [see Rubinstein & Pismen (1991)].

Is this difference significant?

$$\mathbf{f}_B = -\varepsilon_V \frac{\nabla \rho_{sp}}{\rho_{sp}}$$

VS

$$\mathbf{f}_B = -\varepsilon_V \frac{\nabla \rho}{\rho}$$

Muslimov & Tsygan (1985)

near the boundary
of superconducting region:

$$\left| \frac{\rho_{sp}}{\nabla \rho_{sp}} \right| \sim \text{meters}$$

$$\left| \frac{\rho}{\nabla \rho} \right| \sim R \sim 10 \text{ km}$$

⇒ several orders of magnitude!
⇒ faster magnetic field expulsion

NB: our buoyancy force may act in opposite direction!

Vortex motion

Force balance:

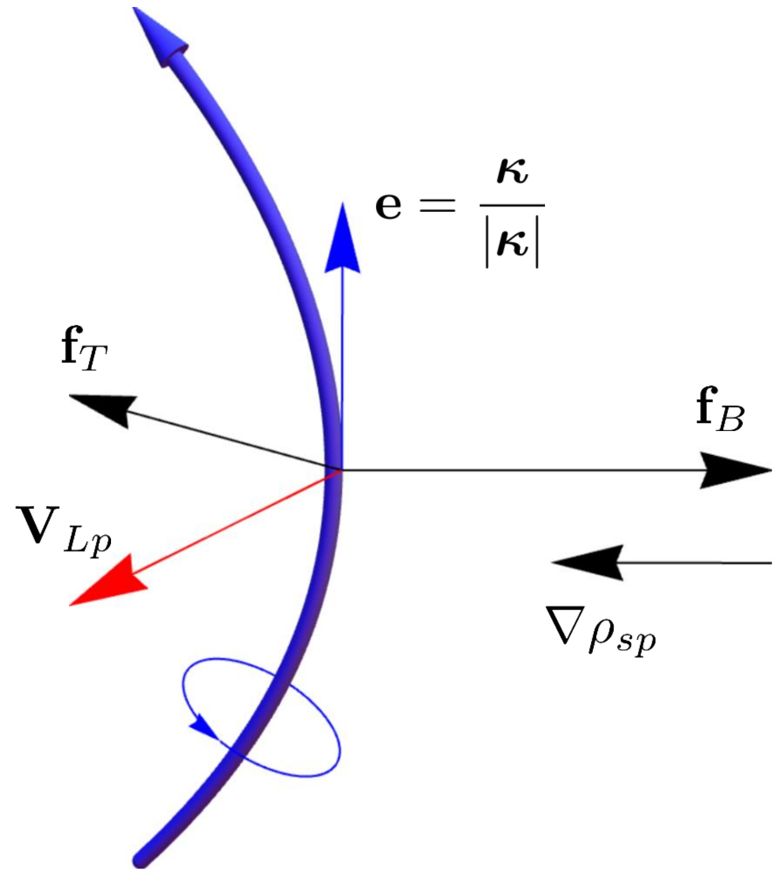
$$\mathbf{f}_B + \mathbf{f}_M + \mathbf{f}_T + \mathbf{f}_D = 0$$

$\mathbf{f}_B = -\varepsilon_V \dfrac{\nabla \rho_{sp}}{\rho_{sp}}$ buoyancy force

$\mathbf{f}_M = \rho_{sp} \boldsymbol{\kappa} \times (\mathbf{V}_{Lp} - \mathbf{V}_{sp})$ Magnus force

$\mathbf{f}_T = \varepsilon_V (\mathbf{e}\nabla)\mathbf{e}$ tension force

$\mathbf{f}_D = -\kappa \rho_{sp} \mathcal{R}(\mathbf{V}_{Lp} - \mathbf{V}_e)$ drag force (=0)



$$\mathbf{V}_{Lp} = \mathbf{V}_{sp} + \underbrace{\frac{\varepsilon_V}{\rho_{sp}\kappa} \operatorname{rot} \mathbf{e}}_{\text{(tension)}} - \frac{\varepsilon_V}{\rho_{sp}\kappa} \mathbf{e} \times \underbrace{\frac{\nabla \rho_{sp}}{\rho_{sp}}}_{\text{(buoyancy)}}$$

Is there a buoyancy force in superfluid hydrodynamics?

Hydrodynamics of the one-component uncharged superfluid liquid (e.g. superfluid helium):
Hall & Vinen (1956), Hall (1960), Bekarevich & Khalatnikov (1961)

All quantities are averaged over a volume containing large number of vortices

for protons in NS core:

$$d_p \sim \mathcal{N}_p^{1/2} \sim 5 \times 10^{-10} \left(\frac{B}{10^{12}\mathrm{G}} \right)^{-1/2} \mathrm{cm}$$

Macroscopic superfluid velocity:

$$\mathbf{V}_s = \left\langle \mathbf{v}_s^{\mathrm{micro}} \right\rangle$$

Its vorticity is produced by vortices:

$$\mathrm{rot}\, \mathbf{V}_s \equiv \boldsymbol{\omega} = \boldsymbol{\kappa} \mathcal{N}$$

$$\kappa = \frac{\pi \hbar}{m_p}$$

Is there a buoyancy force in superfluid hydrodynamics?

Vorticity is transferred by vortices:

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \mathrm{rot}\,[\mathbf{V}_L \times \boldsymbol{\omega}]$$

Hall (1960):

$$\mathbf{V}_L^{(Hall)} = \mathbf{V}_s + \frac{\lambda}{\rho_s}\,\mathrm{rot}\,\mathbf{e} \qquad \mathbf{e} = \frac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|}, \quad \lambda \equiv \frac{\varepsilon_V}{\kappa}$$

Bekarevich & Khalatnikov (1961):

$$\mathbf{V}_L^{(BK)} = \mathbf{V}_s + \frac{1}{\rho_s}\,\mathrm{rot}\,\lambda\mathbf{e} = \mathbf{V}_L^{(Hall)} - \frac{1}{\rho_s}\mathbf{e} \times \nabla\lambda$$

Is there a buoyancy force in superfluid hydrodynamics?

$$\mathbf{V}_L^{(BK)} = \mathbf{V}_s + \frac{\varepsilon_V}{\rho_s \kappa} \operatorname{rot} \mathbf{e} - \frac{\varepsilon_V}{\rho_s \kappa} \mathbf{e} \times \frac{\nabla \rho_s}{\rho_s}$$

(tension) (buoyancy)

Is there a buoyancy force in superfluid hydrodynamics?

Yes!

- NB: in [Bekarevich and Khalatnikov 1961], but not in [Hall 1960]
- The same as in our ‘energetic’ derivation
- The same as in Ginzburg–Landau model [Rubinstein & Pismen 1991]
- Different from the ‘standard’ one [Muslimov & Tsygan 1985]

Had anyone mentioned that fact before?

To the best of our knowledge,

no.

see, e.g.:
- Bekarevich and Khalatnikov (1961)
- "An Introduction to the Theory of Superfluidity", Khalatnikov (1965)

Extension of the superfluid hydrodynamics
to superfluid/superconducting mixtures:
- Mendell & Lindblom (1991)
- Glampedakis, Andersson & Samuelsson (2011)
- Gusakov & Dommes (2016)

Magnetic field evolution

Magnetic field in superconducting NS core
is transferred by proton vortices:

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{rot}\,[\mathbf{V}_{Lp} \times \mathbf{B}]$$

[Konenkov & Geppert (2001), Gusakov & Dommes (2016)]

$$\mathbf{V}_{Lp} = \mathbf{V}_{sp} + \frac{\varepsilon_V}{\rho_{sp}\kappa}\,\mathrm{rot}\,\mathbf{e} - \frac{\varepsilon_V}{\rho_{sp}\kappa}\mathbf{e} \times \frac{\nabla \rho_{sp}}{\rho_{sp}}$$

Generalization of superfluid hydrodynamics

- Superfluid/superconducting mixtures?
- Electromagnetic field?
- Entrainment? $\mathbf{j}_p = (\rho_p - \rho_{pp} - \rho_{np})\mathbf{V}_{norm} + \rho_{pp}\mathbf{V}_{sp} + \rho_{np}\mathbf{V}_{sn}$
- Mutual friction? $\mathbf{f}_D = -\kappa\rho_{sp}\mathcal{R}(\mathbf{V}_{Lp} - \mathbf{V}_e)$
- Finite-temperature effects?
- Relativistic effects?

see Gusakov & Dommes (2016), *ArXiv:1607.01629*
Relativistic dynamics of superfluid-superconducting mixtures in the presence of topological defects and the electromagnetic field, with application to neutron stars

Summary

1. The standard expression for the buoyancy force acting on a vortex is incorrect and should be modified:

$$\mathbf{f}_B = -\varepsilon_V \frac{\mathbf{g}}{c_s^2} = -\varepsilon_V \cancel{\frac{\nabla \rho}{\rho}} \qquad \mathbf{f}_B = -\varepsilon_V \frac{\nabla \rho_{sp}}{\rho_{sp}}$$

2. This modification can significantly increase the buoyancy force (and, thus, the magnetic field expulsion rate), and may even reverse its direction.
3. The proposed derivation is supported by microscopic theory [see Rubinstein & Pismen (1991)].
4. The same buoyancy force is already implicitly accounted for in the superfluid hydrodynamics of Bekarevich & Khalatnikov (1961) and its extensions to superfluid/superconducting mixtures [Mendell & Lindblom (1991); Gusakov & Dommes (2016)].
5. The buoyancy force should be included by hands in case of using the superfluid hydrodynamics by Hall (1960).

Thank you for your attention!