Bulk & Shear Viscosities in Nuclear Matter

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1. Hydrodynamics of viscous fluid
2. Scheme of calculation of damping times of stellar oscillations
3. Mechanism of effective bulk viscosity generation in nuclear matter
4. Shear viscosity coefficient
5. Dissipation in Ekman layer
6. Application to r-mode physics

Hydrodynamics of non-dissipative fluid composed of identical particles.

$$ds^2 = g_{\mu\nu} dx^\nu dx^\mu \qquad \mu, \nu = t, x, y, z$$

four-velocity $\quad u^\mu \equiv \frac{dx^\mu}{ds} \qquad u^\nu u_\nu = -1$

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2. **energy-momentum conservation law** $\quad T^{\mu\nu}_{;\nu} = 0 \qquad T^{\mu\nu} = (P + \varepsilon) u^\mu u^\nu + P g^{\mu\nu}$

$\varepsilon = T^{00}$ in the comoving locally flat frame

P – pressure

Since $n,\ \varepsilon$ are defined in the same reference frame, the standard thermodynamic equalities hold:

the expression for pressure $$P = -\varepsilon + \mu\, n + TS$$

the second law of thermodynamics $$d\varepsilon = \mu\, dn + TdS$$

In non-dissipative hydrodynamics
u^μ is both particle transport velocity and energy transport velocity

$$j^\nu = nu^\nu$$

$$T^{\mu\nu} = (P+\varepsilon)u^\mu u^\nu + Pg^{\mu\nu}$$

Account for dissipation.

We will assume that dissipation is *small* and it just *slightly* modifies j^μ and $T^{\mu\nu}$

$$j^\nu = nu^\nu + \Delta J^\nu$$

$$T^{\mu\nu} = (P+\varepsilon)u^\mu u^\nu + Pg^{\mu\nu} + \Delta T^{\mu\nu}$$

Conservation laws hold:

$$j^\nu_{;\nu} = 0 \qquad T^{\mu\nu}_{;\nu} = 0$$

What are n, ε now? Not obvious.

We define that again $n = j^0$ $\varepsilon = T^{00}$ in the comoving locally flat frame,

this definition imposes constraints on dissipative corrections:

$$\Delta T^{\mu\nu} u_\mu u_\nu = 0$$

$$\Delta J^\nu u_\nu = 0$$

pressure is given by the same equation as in non-dissipative fluid: $P = -\varepsilon + \mu n + TS$

$$d\varepsilon = \mu\, dn + T dS$$

Dissipation makes also notion of velocity indefinite.

What is u^μ? Not obvious.

$$j^\nu = nu^\nu + \Delta J^\nu$$ neither particle transport velocity

$$T^{\mu\nu} = (P+\varepsilon)u^\mu u^\nu + Pg^{\mu\nu} + \Delta T^{\mu\nu}$$ nor energy transport velocity

We will demand u^μ to be the velocity of particle transport

Eckart formulation (Eckart, Physical Review, 58, 1940; Weinberg, ApJ, 168, 175, 1971)

$$j^\nu = nu^\nu \qquad \Delta J^\mu = 0$$

Alternative: u^μ is velocity of energy transport

Landau formulation (Landau, Lifshitz, Hydrodynamics 1966)

$$\Delta T^{0i} = 0$$

Eckart formulation: $$\Delta T^{\mu\nu} u_\mu u_\nu = 0$$

$$\Delta J^\mu = 0$$

Determine $\Delta T^{\mu\nu}$

What we know:

1. Uniform flow in uniform fluid does not lead to dissipation

$$\Delta T^{\mu\nu} \propto \text{ gradients } \mathrm{u}^{\mu}_{;\nu}, \ \mathrm{f}_{;\nu}$$

2. Dissipation is small => gradients are small => no $(u^{\mu}_{;\nu})^2$, no $u^{\mu}_{;\nu;\gamma}$ ....
3. Solid body rotation should not lead to dissipation
4. $\Delta T^{\mu\nu}$ should satisfy $\Delta T^{\mu\nu} u_{\mu} u_{\nu} = 0$ ($\varepsilon = T^{00}$)
5. $\Delta T^{\mu\nu}$ should be symmetric (to satisfy angular momentum conservation)
6. Entropy should ***increase*** due to dissipation

the last requirement will be guiding for us

$$(\text{entropy current}^{\nu})_{;\nu} = f(\Delta T^{\mu\nu}, \ldots)$$

source of entropy
(entropy generation rate)

must be always positive

construct vanishing combination $u_\mu T^{\mu\nu}_{;\nu} = 0$

$$u_\mu T^{\mu\nu}_{;\nu} = u_\mu \left[(P+\varepsilon) u^\mu u^\nu + P g^{\mu\nu} + \Delta T^{\mu\nu} \right]_{;\nu} = 0$$

Using: $u^\mu u_\mu = -1$

$$j^\nu_{;\nu} = 0$$

$$d\varepsilon = \mu dn + T dS$$

$$P = -\varepsilon + \mu n + TS$$

we transform it to $\left(S u^\nu - \frac{u_\mu}{T} \Delta T^{\mu\nu} \right)_{;\nu} = -\Delta T^{\mu\nu} \left(\frac{u_\mu}{T} \right)_{;\nu}$

$$(\text{entropy current}^\nu)_{;\nu} = f(\Delta T^{\mu\nu}, ...)$$

we interpret $S u^\nu - \frac{u_\mu}{T} \Delta T^{\mu\nu}$ as entropy four-current

$\Delta T^{\mu\nu} \propto \text{gradients}$, thus rhs is quadratic form,
we demand it to be positively defined

The most general form of such $\Delta T^{\mu\nu}$ is

$$\Delta T^{\mu\nu} = -\eta H^{\mu\gamma} H^{\nu\delta} \left(u_{\gamma;\delta} + u_{\delta;\gamma} - \frac{2}{3} g_{\gamma\delta} u^{\alpha}_{;\alpha} \right)$$
$$-\zeta H^{\mu\nu} u^{\gamma}_{;\gamma} - \chi \left(H^{\mu\gamma} u^{\nu} + H^{\nu\gamma} u^{\mu} \right) \left(T_{;\gamma} + T u_{\gamma;\delta} u^{\delta} \right)$$

$$H^{\mu\nu} = g^{\mu\nu} + u^{\mu} u^{\nu}$$

The most general form of such $\Delta T^{\mu\nu}$ is

shear viscosity

$$\Delta T^{\mu\nu} = -\eta H^{\mu\gamma} H^{\nu\delta}\left(u_{\gamma;\delta} + u_{\delta;\gamma} - \frac{2}{3} g_{\gamma\delta} u^{\alpha}_{;\alpha}\right)$$

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bulk viscosity

heat conductivity

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heat conductivity

$$H^{\mu\nu} = g^{\mu\nu} + u^{\mu} u^{\nu}$$

entropy current: $Su^{\nu} - \frac{u_{\mu}}{T} \Delta T^{\mu\nu} = Su^{\mu} - \frac{\chi}{T} H^{\mu\nu} \left(T_{;\nu} + T u_{\nu;\gamma} u^{\gamma} \right)$

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$$H^{\mu\nu} = g^{\mu\nu} + u^{\mu} u^{\nu}$$

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---

only $\eta, \ \zeta$:

entropy generation equation: $T \left(S u^{\nu} \right)_{;\nu} =$

$$\frac{\eta}{2} H^{\delta\nu} H^{\gamma\mu} \left(u_{\gamma;\delta} + u_{\delta;\gamma} - \frac{2}{3} g_{\gamma\delta} u^{\alpha}_{;\alpha} \right) \left(u_{\mu;\nu} + u_{\nu;\mu} - \frac{2}{3} g_{\mu\nu} u^{\alpha}_{;\alpha} \right) +$$

$$\zeta \left(u^{\nu}_{;\nu} \right)^{2}$$

only $\eta,\ \zeta$:

$$T\left(Su^{\nu}\right)_{;\nu} =$$

$$\frac{\eta}{2} H^{\delta\nu} H^{\gamma\mu} \left(u_{\gamma;\delta} + u_{\delta;\gamma} - \frac{2}{3} g_{\gamma\delta} u^{\alpha}_{;\alpha} \right) \left(u_{\mu;\nu} + u_{\nu;\mu} - \frac{2}{3} g_{\mu\nu} u^{\alpha}_{;\alpha} \right) +$$

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Newtonian limit

$$T\Delta\Gamma_S = \frac{\eta}{2} \left(\frac{\partial u_i}{\partial x^k} + \frac{\partial u_k}{\partial x^i} - \frac{2}{3} \delta_{ik} \mathrm{div}\, \boldsymbol{u} \right)^2 + \zeta \left(\mathrm{div}\, \boldsymbol{u} \right)^2$$

compression/decompression

shear motions and compression/decompression

Sound wave in x direction. $$T\Delta\Gamma_S = \left(\frac{4\eta}{3} + \zeta \right) \left(\frac{\partial u}{\partial x} \right)^2$$

both bulk and shear viscosity lead to dissipation in sound wave

The fact that entropy does not decrease allowed us to find dissipative corrections to the dynamic equations

$$T^{\mu\nu}_{;\nu} = 0$$

$$T^{\mu\nu} = (P+\varepsilon)u^\mu u^\nu + Pg^{\mu\nu} + \Delta T^{\mu\nu}$$

$$\Delta T^{\mu\nu} = -\eta H^{\mu\gamma} H^{\nu\delta}\left(u_{\gamma;\delta} + u_{\delta;\gamma} - \frac{2}{3} g_{\gamma\delta} u^\alpha_{;\alpha}\right)$$
$$-\zeta H^{\mu\nu} u^\gamma_{;\gamma} - \chi\left(H^{\mu\gamma}u^\nu + H^{\nu\gamma}u^\mu\right)\left(T_{;\gamma} + T u_{\gamma;\delta} u^\delta\right)$$

Kinetic coefficients η, ζ, χ should be determined from microphysics.

One of the most important applications of the viscosity is damping of oscillations

Assume that neutron star experiences global oscillations.
What will be the damping time of the oscillations $\tau_{\rm Damp}$?
It can be calculated in two ways.

1. Look for a solution of dynamic equations with dissipative terms
   In the form of $\propto \exp(\imath \omega t)$ $\quad \omega$ – complex $\qquad \frac{1}{\tau_{\rm Damp}} = {\rm Im}\, \omega$

2. Oscillation energy notion $\qquad \frac{1}{\tau_{\rm Damp}} = -\frac{\dot{E}}{2E}$

If dissipation is small ($\frac{1}{\tau_{\rm Damp}} \ll \omega$) and does not affect eigenfunctions significantly:

solve non-dissipative dynamic equations $\Rightarrow$ non-dissipative eigenfunctions $\Rightarrow$ $E,\ \dot{E}$

$$\frac{1}{\tau_{\mathrm{Damp}}} = -\frac{\dot{E}}{2E}$$

$$E = \int_V \epsilon_{\mathrm{osc}} \mathrm{d}V$$

sound waves or p-mode $\quad E = \int_V (\epsilon_{\mathrm{pot}} + \epsilon_{\mathrm{kin}}) \mathrm{d}V = 2\int_V \left\langle \frac{\rho u^2}{2} \right\rangle \mathrm{d}V$

To calculate $\dot{E}$ notice that dissipated energy is deposited into the thermal energy of the star

Thus $\dot{E}$ can be calculated from the entropy generation equation:

$$\dot{E} = -\int_V T\,(Su^\nu)_{;\nu}\,\mathrm{d}V = -\int_V \left[\frac{\eta}{2}\left(\frac{\partial u_i}{\partial x^k} + \frac{\partial u_k}{\partial x^i} - \frac{2}{3}\delta_{ik}\mathrm{div}\,\boldsymbol{u}\right)^2 + \zeta\,(\mathrm{div}\,\boldsymbol{u})^2\right]\mathrm{d}V$$

eigenfunctions of non-dissipative dynamic equations

Be careful! For example, in the case of viscous secular instability total energy decreases, but oscillation amplitude increases.

to get numerical result we need to know η, ζ ,
that is we need to understand what mechanisms drive viscosity

Bulk viscosity

Neutron star matter is composed of fermions (neutrons, protons, electrons, muons, hyperons, quarks) and is strongly degenerate.

Collisional bulk viscosity of neutron-star degenerate matter is small.

$$\zeta \sim \left(\frac{T}{\mu}\right)^4 \eta \qquad \frac{T}{\mu} \sim 10^{-4} - 10^{-6}$$

Sykes, Brooker, Ann. Phys., 56, 1 (1970)

However, non-equilibrium processes of particle transformations can effectively “generate” bulk viscosity.

First who noticed: *Mandelshtam, Leontovich (1937)*
(in application to laboratory fluids)

Consider a minimal composition of NS core: neutrons + protons + electrons

The most powerful process of particle transformations in npe-matter is

1. Neutron decay or **direct Urca** process:

$$n \to p + e + \bar{\nu}_e, \qquad p + e \to n + \nu_e$$

threshold process, operates at sufficiently high densities,
in not too heavy NSs it is forbidden,
for some equations of state of nuclear matter it is forbidden at any density.

If direct Urca process is closed then modified Urca process is the most efficient:

2. **Modified Urca** process:

$$n + N \to p + e + N + \bar{\nu}_e, \qquad p + e + N \to n + N + \nu_e$$

not threshold, operates at any density

In equilibrium the rates of direct and inverse reactions are equal, thus the net number of electrons (neutrons, protons), appearing because of beta-reactions in unit volume per second is zero: $\Delta\Gamma_i = 0$ $\quad \delta\mu \equiv \mu_n - \mu_p - \mu_e = 0$

Perturbed npe-matter

$$\delta\mu = \mu_n - \mu_p - \mu_e \neq 0$$

The rates of direct and inverse reactions are NOT equal, $\Delta\Gamma_i \neq 0$

$$\Delta\Gamma_n = -\Delta\Gamma_p = -\Delta\Gamma_e$$

Non-equilibrium reactions try to restore the equilibrium.
This results in energy dissipation.

Deviation from the equilibrium is small $\delta\mu \ll k_B T => \Delta\Gamma_e = \lambda\delta\mu$

$$\lambda_{\mathrm{D}} \sim (10^{39} - 10^{41}) \times T_9^4 \ \mathrm{cm}^{-3}\mathrm{s}^{-1}\mathrm{erg}^{-1}$$
$$\lambda_{\mathrm{M}} \sim (10^{33} - 10^{35}) \times T_9^6 \ \mathrm{cm}^{-3}\mathrm{s}^{-1}\mathrm{erg}^{-1}$$

To see how non-equilibrium reactions lead to dissipation and generate effective bulk viscosity we will consider

non-dissipative hydrodynamics ($\eta = 0, \ \zeta = 0$)
with non-equilibrium reactions.

1. continuity equations obtain non-zero source in rhs:

$$j^{\nu}_{i\,;\nu} = \Delta\Gamma_i \qquad i = n, p, e$$

baryon number is conserved: $$j^{\nu}_{b\,;\nu} = 0 \qquad n_b = n_n + n_p$$

2. energy-momentum conservation law and energy-momentum tensor are not affected by particle source:

$$T^{\mu\nu}_{;\nu} = 0 \qquad T^{\mu\nu} = (P + \varepsilon) u^{\mu} u^{\nu} + P g^{\mu\nu}$$

construct vanishing combination $u_\mu T^{\mu\nu}_{;\nu} = 0$

$$u_\mu T^{\mu\nu}_{;\nu} = u_\mu \left[(P+\varepsilon)u^\mu u^\nu + P g^{\mu\nu} + \Delta T^{\mu\nu}\right]_{;\nu} = 0$$

Using: $u^\mu u_\mu = -1$

$$j^{\nu}_{b\,;\nu} = 0 \qquad j^{\nu}_{e\,;\nu} = \Delta\Gamma_e$$

$$d\varepsilon = \textstyle\sum_i \mu_i dn_i + T dS = \mu_n dn_b - \delta\mu\, dn_e + T dS$$

$$P = -\varepsilon + \textstyle\sum_i \mu_i n_i + TS = -\varepsilon + \mu_n n_b - \delta\mu\, n_e + TS$$

we obtain $T\,(S u^\nu)_{;\nu} = \delta\mu\,\Delta\Gamma_e$

$\delta\mu \ll k_B T \Rightarrow \Delta\Gamma_e = \lambda\delta\mu$

$$\boxed{T\,(S u^\nu)_{;\nu} = \lambda\delta\mu^2}$$

no reactions ($\lambda = 0$) => no dissipation

more complicated composition =>
entropy generation should be calculated by summation over all reactions

$$T\left(Su^{\nu}\right)_{;\nu} = \sum_i \lambda_i \delta {\mu_i}^2$$

Non-equilibrium reactions lead to entropy generation

How dissipation due to non-equilibrium reactions can be expressed in terms of bulk viscosity

entropy generation by non-equilibrium reactions $T\left(Su^{\nu}\right)_{;\nu} = \lambda \delta\mu^2$

entropy generation by bulk viscosity $T\left(Su^{\nu}\right)_{;\nu} = \zeta \left(u^{\nu}_{;\nu}\right)^2$

express $\delta\mu$ through $u^{\nu}_{;\nu}$

Taylor series $\delta\mu(n_n, n_p, n_e, T) \underset{T \ll \mu}{=} \delta\mu(n_n, n_p, n_e) \underset{n_p = n_e}{=} \delta\mu(n_b, n_e) =$

$$\frac{\partial \delta\mu(n_b, n_e)}{\partial n_b} \delta n_b + \frac{\partial \delta\mu(n_b, n_e)}{\partial n_e} \delta n_e$$

now we will express δn_b and δn_e through $u^{\nu}_{;\nu}$ from continuity equations

$$(n_b u^\nu)_{;\nu} = 0 \qquad\qquad (n_e u^\nu)_{;\nu} = \Delta\Gamma_e = \lambda\delta\mu$$

$$\delta n_b \qquad\qquad \delta n_e$$

Urca processes are slow in comparison with typical oscillation frequency, the composition is almost frozen and we can neglect the source deriving δn_e :

$$(n_e u^\nu)_{;\nu} \approx 0$$

$$\delta n_e$$

substituting the result into entropy generation $T\left(Su^{\nu}\right)_{;\nu} = \lambda\delta\mu^{2}$

we obtain:

$$T\left(Su^{\nu}\right)_{;\nu} = \lambda\left(\frac{n_b}{\omega u^0}\right)^2\left(\frac{\partial\delta\mu(n_b,x_e)}{\partial n_b}\right)^2\left(u^{\nu}_{;\nu}\right)^2$$

frequency of perturbation

comparing with the standard expression for entropy generation by bulk viscosity:

$$T\left(Su^{\nu}\right)_{;\nu} = \zeta\left(u^{\nu}_{;\nu}\right)^2$$

We derive the effective bulk viscosity coefficient due to non-equilibrium reactions:

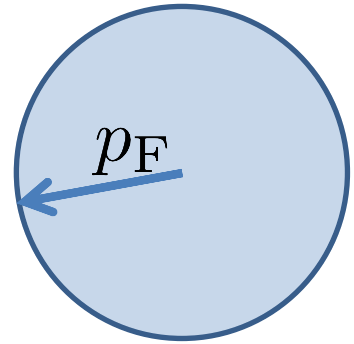
$$\zeta_{\text{eff}} = \lambda\left(\frac{n_b}{\omega u^0}\right)^2\left(\frac{\partial\delta\mu(n_b,x_e)}{\partial n_b}\right)^2$$

depends on the reaction rate and oscillation frequency

We expressed the dissipation due to non-equilibrium reactions in terms of effective bulk viscosity.

Temperature dependence of ζ_{eff} is determined by the reaction rate λ.

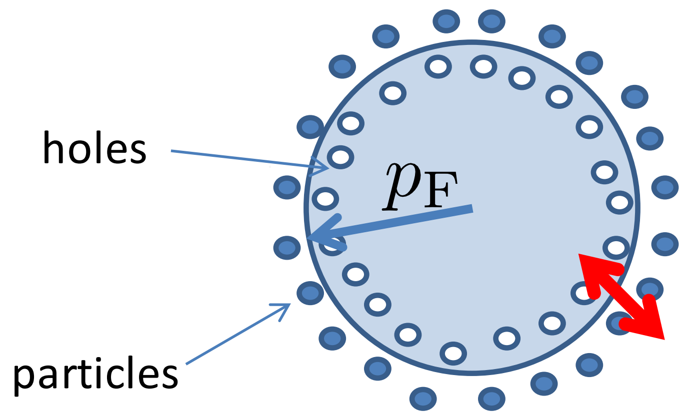
Particles are fermions. At *T=0* they fill in Fermi sphere in the momentum space completely and there are no particles above Fermi surface.



Fermi sphere at *T=0*

Temperature allows for the excitations near the Fermi surface in the layer of the order of

$$T/\mu \sim 10^{-4} - 10^{-6}$$



Fermi sphere at finite *T*

$$\delta p/p_F \sim T/\mu \sim 10^{-4} - 10^{-6}$$

Only particles from this thin layer near Fermi surface can participate in reactions (to satisfy energy conservation).

$$\zeta_{\mathrm{eff}} \propto \lambda \quad \begin{array}{ll} \propto T^4 & \text{direct Urca} \\ \propto T^6 & \text{modified Urca} \end{array}$$

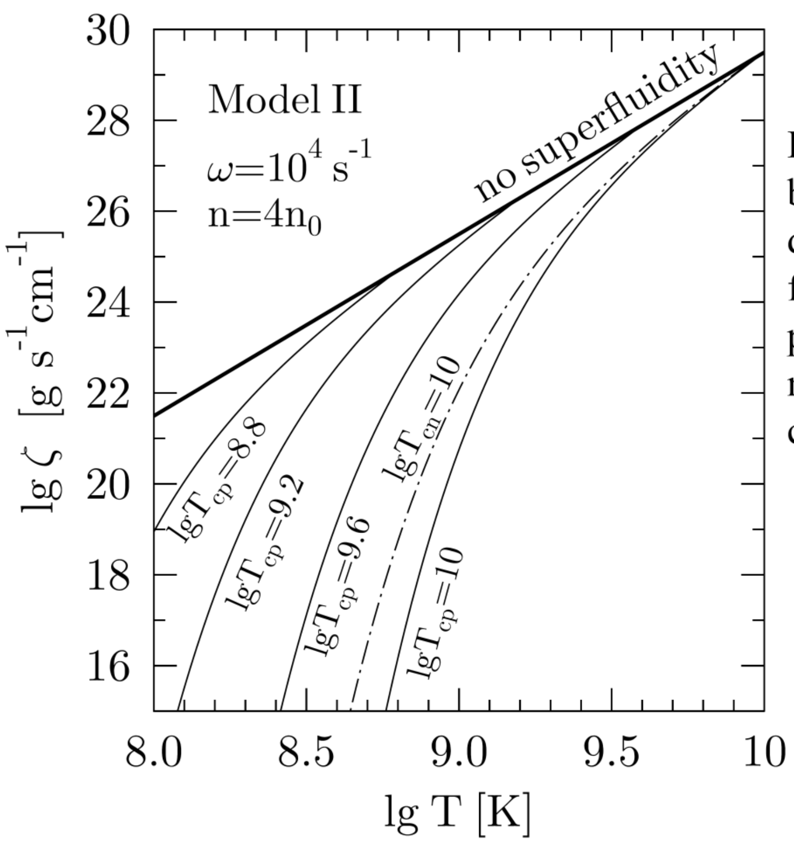
Bulk viscosity in superfluid matter

Baryons can be in superfluid state $T_c \sim 10^8 - 10^{10}\,\mathrm{K}$

Superfluidity affects bulk viscosity in two ways:

(i) Superfluidity suppresses the number of excitations near Fermi surface, and hence the reaction rates

P. Haensel, K. Levenfish, D. Yakovlev, A&A (2000,2001,2002)



Haensel, Levenfish, Yakovlev, 2000

Fig. 7. Bulk viscosity ζ of superfluid $npe\mu$ matter (model II) produced by the electron and muon direct Urca processes at the baryon number density $n_b = 4\,n_0$ and $\omega = 10^4\,\mathrm{s}^{-1}$ as a function of temperature for non-superfluid matter (thick solid line), for matter with superfluid protons (solid curves, $T_{cp} = 10^{10}$, $10^{9.6}$, $10^{9.2}$ and $10^{8.8}$) and normal neutrons, and for matter with superfluid neutrons (dash-and-dotted curve, $T_{cn} = 10^{10}$ K) and normal protons.

(ii) Hydrodynamics becomes multifluid, there are several velocity fields => several bulk viscosity coefficients at divergences of different velocities.

$$\begin{aligned}
\Delta T^{\mu\nu} = & -\kappa \left(H^{\mu\gamma} u^{\nu} + H^{\nu\gamma} u^{\mu}\right) \left(\partial_\gamma T + T u^{\delta} \partial_\delta u_\gamma\right) \\
& - \eta H^{\mu\gamma} H^{\nu\delta} \left(\partial_\delta u_\gamma + \partial_\gamma u_\delta - \frac{2}{3} \eta_{\gamma\delta} \partial_\varepsilon u^{\varepsilon}\right) \\
& - \xi_{1i} H^{\mu\nu} \partial_\gamma \left[Y_{ik} w^{\gamma}_{(k)}\right] - \xi_2 H^{\mu\nu} \partial_\gamma u^{\gamma}, \\
\varkappa_{\mathrm{n}} = & -\xi_{3i} \partial_\mu \left[Y_{ik} w^{\mu}_{(k)}\right] - \xi_{4\mathrm{n}} \partial_\mu u^{\mu}, \\
\varkappa_{\mathrm{p}} = & -\xi_{5i} \partial_\mu \left[Y_{ik} w^{\mu}_{(k)}\right] - \xi_{4\mathrm{p}} \partial_\mu u^{\mu}.
\end{aligned}$$

dissipative corrections to the equations on superfluid velocities

Calculations of bulk viscosity in superfluid neutron star matter in the frame of multifluid hydrodynamics:

M. Gusakov, Phys. Rev. D, 76, 083001 (2007)
M. Gusakov, E. Kantor, Phys. Rev. D, 78, 083006 (2008)
B. Haskell, N. Andersson, MNRAS, 408, 1897 (2010)

Superfluidity significantly complicates dissipative dynamic equations.

However, if we are interested in dissipation rate of oscillations we can use

$$\dot{E} = -\int_V T\left(Su^\nu\right)_{;\nu} \mathrm{d}V = -\int_V \sum_i \lambda_i \delta{\mu_i}^2 \mathrm{d}V$$

If dissipation is small ($\frac{1}{\tau_{\mathrm{Damp}}} \ll \omega$) $\delta\mu_i$ can be calculated

from non-dissipative dynamic equations.

Bulk viscosity in hyperon matter

Internal layers $n\, p\, e\, \mu \quad + \quad \Lambda\ \Xi^-\ \Xi^0\ \Sigma^-\ \Sigma^0$

Hyperons participate in non-leptonic reactions which proceed much faster than leptonic Urca processes. Reactions with Λ hyperon:

$$\begin{aligned} n + p &\leftrightarrow p + \Lambda \\ n + n &\leftrightarrow n + \Lambda \\ n + \Lambda &\leftrightarrow \Lambda + \Lambda \end{aligned}$$

In contrast to Urca processes, reaction rates can be comparable or even higher than typical oscillation frequency. The sources in continuity equations are not negligible. As a result the dependence of $\zeta_{\rm eff}$ on the reaction rate is more complicated:

$$\zeta_{\rm eff} \propto \frac{1}{\lambda\left(1+C\,\frac{\omega^2}{\lambda^2}\right)}$$

$$\lambda \to 0 \quad \zeta_{\rm eff} \propto \frac{\lambda}{\omega^2} \to 0 \qquad\qquad \lambda \to \infty \quad \zeta_{\rm eff} \propto \frac{1}{\lambda} \to 0$$

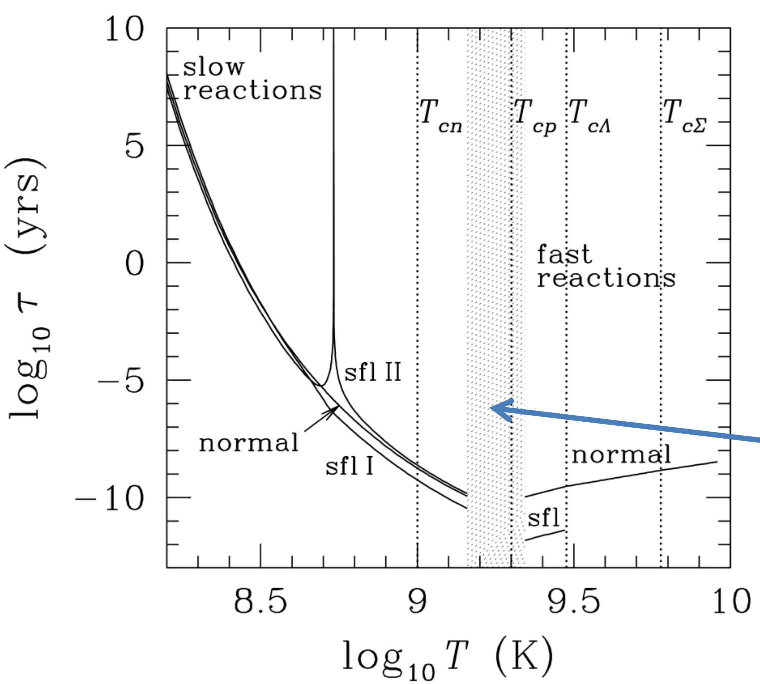
Urca reactions

$$\lambda \to 0 \quad \zeta_{\text{eff}} \propto \frac{\lambda}{\omega^2} \to 0 \qquad\qquad \lambda \to \infty \quad \zeta_{\text{eff}} \propto \frac{1}{\lambda} \to 0$$

Dissipation vanish in two limiting cases:

(i) low reaction rate ($\lambda \to 0$), $T\,(Su^{\nu})_{;\nu} = \sum_i \lambda_i \delta\mu_i{}^2 \to 0$ (low temperature)
(ii) high reaction rate ($\lambda \to \infty$), reactions are so fast that composition is maintained almost in equilibrium, $\delta\mu \to 0$, $T\,(Su^{\nu})_{;\nu} = \sum_i \lambda_i \delta\mu_i{}^2 \to 0$ (high temperatures)

Maximum of dissipation is somewhere in between.



Kantor, Gusakov 2009

Damping time of sound modes: dissipation is most efficient at intermediate reaction rate.

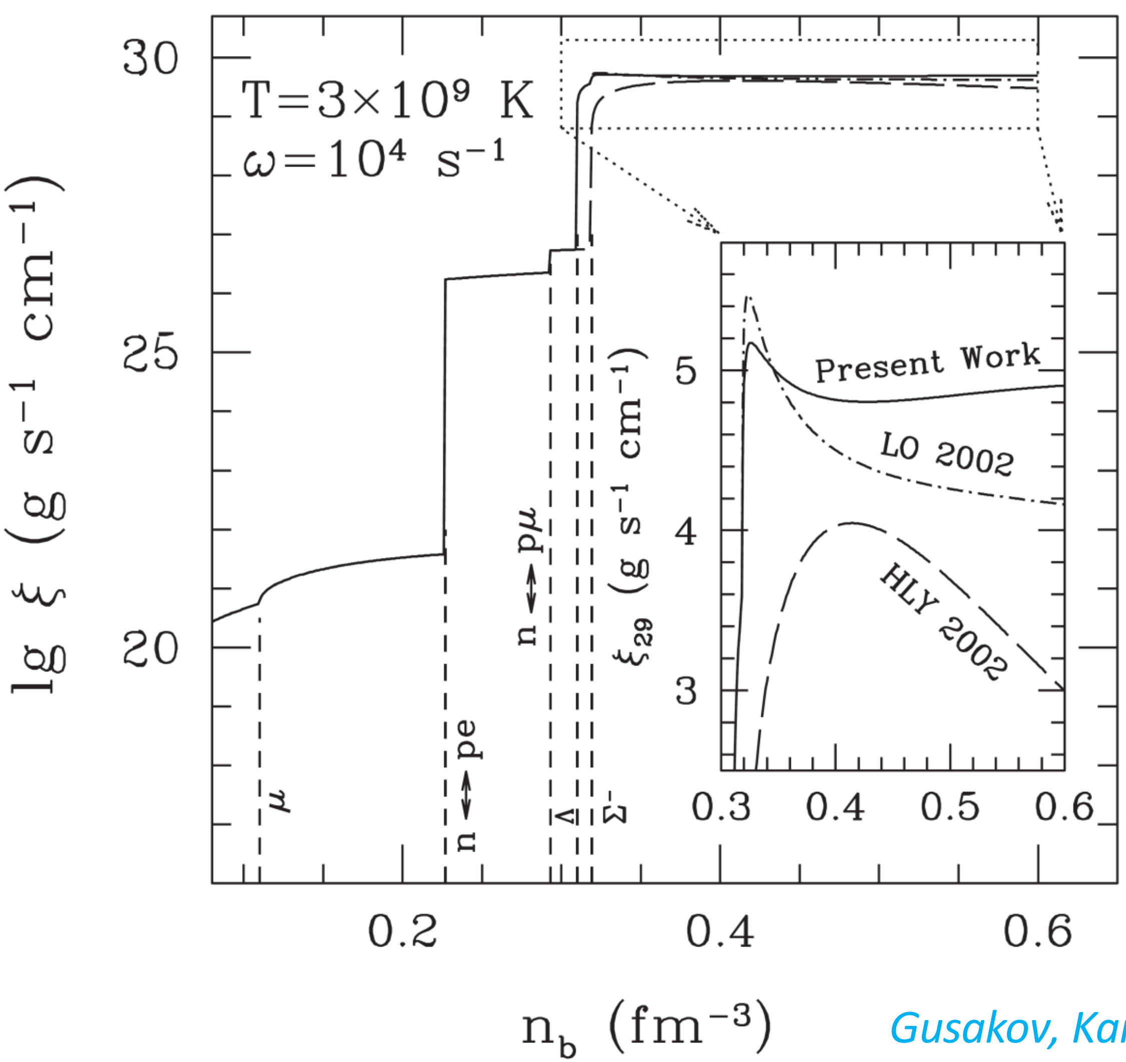
damping time is of the order of oscillation period

Calculations of the bulk viscosity in hyperon matter:

P.B. Jones, Phys. Rev. D, 64, 084003 (2001)

L. Lindblom, B. Owen, Phys. Rev. D, 65, 063006 (2002)

M. Gusakov, E. Kantor, Phys. Rev. D, 78, 083006 (2008) ... and many others ...



Direct Urca process increases bulk viscosity

Hyperon non-leptonic reactions increase it even more.

Gusakov, Kantor (2008)

Non-equilibrium reactions lead to entropy generation.

The dissipation due to non-equilibrium reactions can be expressed in terms of the effective bulk viscosity.

$$\zeta_{\mathrm{eff}} \propto \frac{1}{\lambda\left(1+C\frac{\omega^2}{\lambda^2}\right)}$$

$$\lambda \to 0 \quad \zeta_{\mathrm{eff}} \propto \frac{\lambda}{\omega^2} \to 0 \qquad\qquad \lambda \to \infty \quad \zeta_{\mathrm{eff}} \propto \frac{1}{\lambda} \to 0$$

λ	$\propto T^4$	direct Urca
	$\propto T^6$	modified Urca

Shear viscosity coefficient

Shear viscosity coefficient enters energy momentum conservation law:

$$T^{\mu\nu}_{;\nu} = 0 \qquad T^{\mu\nu} = (P+\varepsilon)u^\mu u^\nu + Pg^{\mu\nu} + \Delta T^{\mu\nu}$$

$$\Delta T^{\mu\nu} = -\eta H^{\mu\gamma} H^{\nu\delta} \left(u_{\gamma;\delta} + u_{\delta;\gamma} - \frac{2}{3} g_{\gamma\delta} u^\alpha_{;\alpha} \right)$$

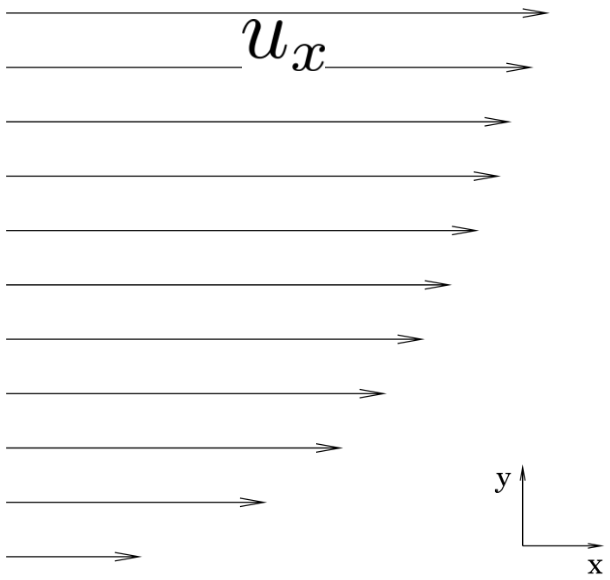
$$-\zeta H^{\mu\nu} u^\gamma_{;\gamma} - \chi \left(H^{\mu\gamma} u^\nu + H^{\nu\gamma} u^\mu \right) \left(T_{;\gamma} + T u_{\gamma;\delta} u^\delta \right)$$

In the Newtonian limit momentum conservation reduces to the Navier-Stokes equation

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \Pi_{ik}}{\partial x_k} = 0 \qquad \Pi_{ik} = P\delta_{ik} + \rho u_i u_k + \pi_{ik}$$

stress tensor or momentum flux density

dissipative correction $$\pi_{ik} = -\eta \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3} \delta_{ik} \frac{\partial u_l}{\partial x_l} \right) - \zeta \delta_{ik} \frac{\partial u_l}{\partial x_l}$$



flat-parallel flow of uniform fluid $u = u_x$

The only non-zero components of

$$\pi_{ik} = -\eta \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial u_l}{\partial x_l} \right) - \zeta \delta_{ik}\frac{\partial u_l}{\partial x_l}$$

are

$$\pi_{xy} = \pi_{yx} = -\eta \frac{\partial u}{\partial y}$$

On the other hand,
momentum flux density can be presented as the integral over all particles:

$$\Pi_{xy} = P\delta_{xy} + \rho u_x u_y + \pi_{xy} = \pi_{xy} = -\frac{2}{(2\pi\hbar)^3} \int p_x v_y f(\boldsymbol{p}) \mathrm{d}\boldsymbol{p}$$

particle momentum

particle velocity
(velocity of momentum transfer)

distribution function
of particles

To find $f(\boldsymbol{p})$ we will use Boltzmann kinetic equation:

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\nabla f + \boldsymbol{F}\frac{\partial f}{\partial \boldsymbol{p}} = \mathrm{St}(f)$$

external force

collision integral

The continuity equation for distribution function $f(\boldsymbol{x},\boldsymbol{p})$ in 6-dimentional space: $\boldsymbol{x},\boldsymbol{p}$. $\mathrm{St}(f)$ is the source due to collisions.

For stationary flow without external forces it reduces to

$$\boldsymbol{v}\nabla f = \mathrm{St}(f)$$

Assume that we have uniform particle flow, $u = \mathrm{const}$. The corresponding distribution function is shifted Fermi sphere

$$f_0(\boldsymbol{p}) = \frac{1}{\exp\left(\frac{\varepsilon(\boldsymbol{p}) - \mu - \boldsymbol{p}\boldsymbol{u}}{k_\mathrm{B}T}\right) + 1}$$

$$\mathrm{St}(f_0) = 0$$

Account for the velocity gradient. We consider small dissipation, thus gradient is small and perturbs distribution function slightly:

$$f(\boldsymbol{p}) = f_0(\boldsymbol{p}) + \delta f(\boldsymbol{p}) \qquad \delta f(\boldsymbol{p}) \ll f_0(\boldsymbol{p})$$

$$\boldsymbol{v}\nabla(f_0 + \delta f) = \mathrm{St}(f_0 + \delta f)$$

small

Expand the rhs, $\mathrm{St}(f_0) = 0$ => $\mathrm{St}(f) \propto \delta f$

We will assume that the proportionality coefficient is constant (relaxation time approximation):

$$\mathrm{St}(f) \approx -\frac{\delta f}{\tau}$$

τ has a meaning of a typical time of distribution function relaxation, or the typical time between collisions

$$\boldsymbol{v}\nabla(f_0) = -\frac{\delta f}{\tau} \qquad \left(\delta f \sim \frac{\lambda}{l} f_0\right)$$

$$\Downarrow$$

$$\delta f = -\tau \boldsymbol{v}\nabla f_0 = \tau \boldsymbol{v}\nabla(\boldsymbol{p}\boldsymbol{u})\frac{\partial f_0}{\partial \varepsilon}$$

Coming back to the dissipative momentum flux density:

$$\pi_{xy} = -\frac{2}{(2\pi\hbar)^3}\int p_x v_y f(\boldsymbol{p})\mathrm{d}\boldsymbol{p} = -\frac{2}{(2\pi\hbar)^3}\left[\overbrace{\cancel{\int p_x v_y f_0(\boldsymbol{p})\mathrm{d}\boldsymbol{p}}}^{0} + \int p_x v_y \delta f(\boldsymbol{p})\mathrm{d}\boldsymbol{p}\right]$$

$$= -\frac{2}{(2\pi\hbar)^3}\int p_x v_y \tau \boldsymbol{v}\nabla(\boldsymbol{p}\boldsymbol{u})\frac{\partial f_0}{\partial \varepsilon} p^2 \mathrm{d}p\mathrm{d}\Omega$$

$$\frac{\partial f_0}{\partial \varepsilon} \approx -\delta(\varepsilon - \varepsilon_{\mathrm{F}})$$

degenerate gas

integrating => $\pi_{xy} = -\frac{p_{\mathrm{F}} v_{\mathrm{F}} n\tau}{5}\frac{\partial u}{\partial y}$

comparing with $\pi_{xy} = -\eta\frac{\partial u}{\partial y}$

$$\boxed{\eta = \frac{p_{\mathrm{F}} v_{\mathrm{F}} n\tau}{5}}$$

Several particle species:

$$\pi_{xy} = -\frac{2}{(2\pi\hbar)^3} \sum_i \int p_{x\,i} v_{y\,i} f_i(\boldsymbol{p}) \mathrm{d}\boldsymbol{p}_i$$

$$\eta = \sum_i \eta_i = \sum_i \frac{p_{\mathrm{F}i} v_{\mathrm{F}i} n_i \tau_i}{5}$$

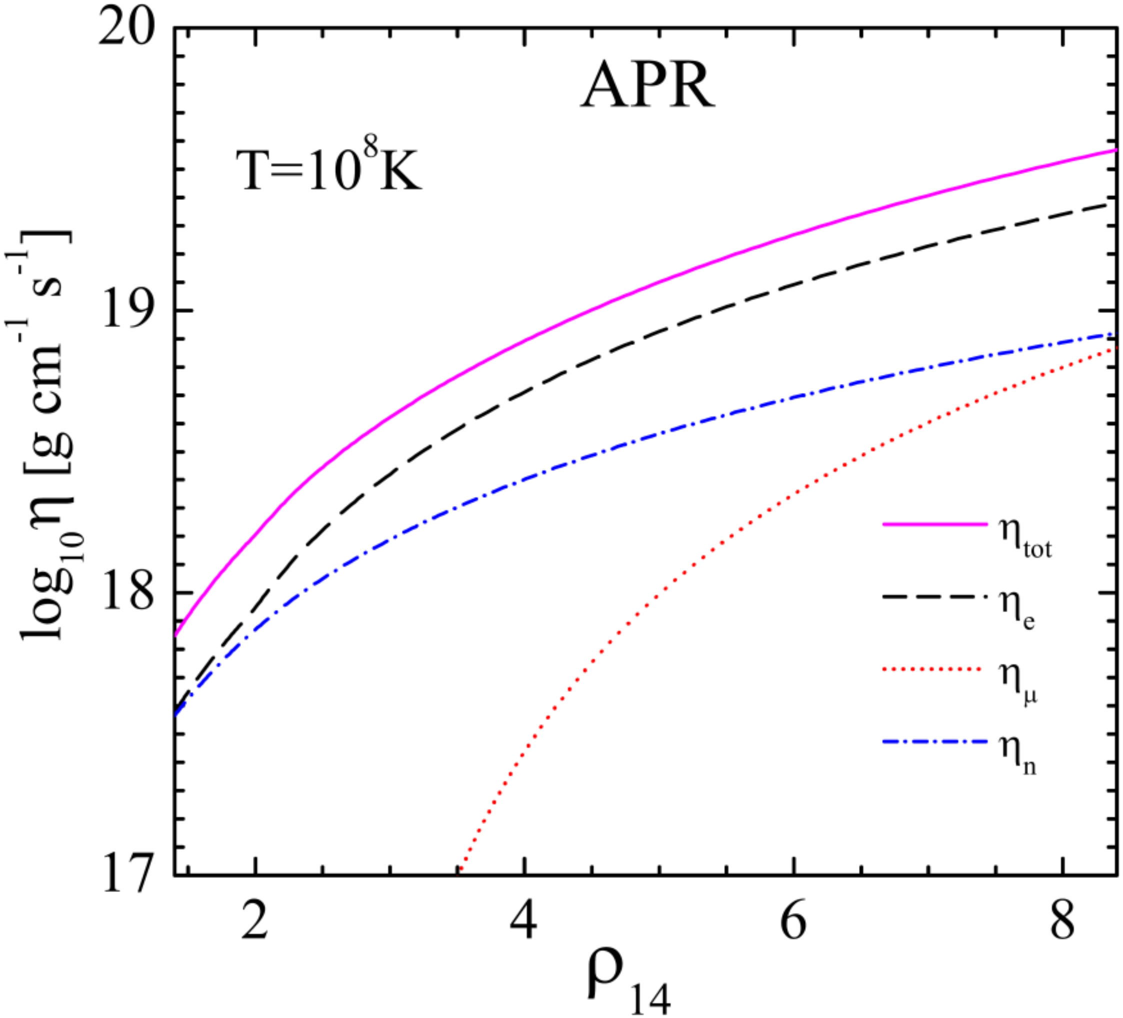
What is the contribution of different particle species to η in NS matter?

- Electrons and muons do not interacting via strong forces (only electro-magnetic interaction), τ is high.

- Neutrons have lower τ, but they are the most abundant particles.

- Protons have low τ and they are not abundant.

Generally, in $npe\mu$ matter the main contribution to η comes from **electrons**, and also from muons and neutrons.



Shternin, Yakovlev 2008

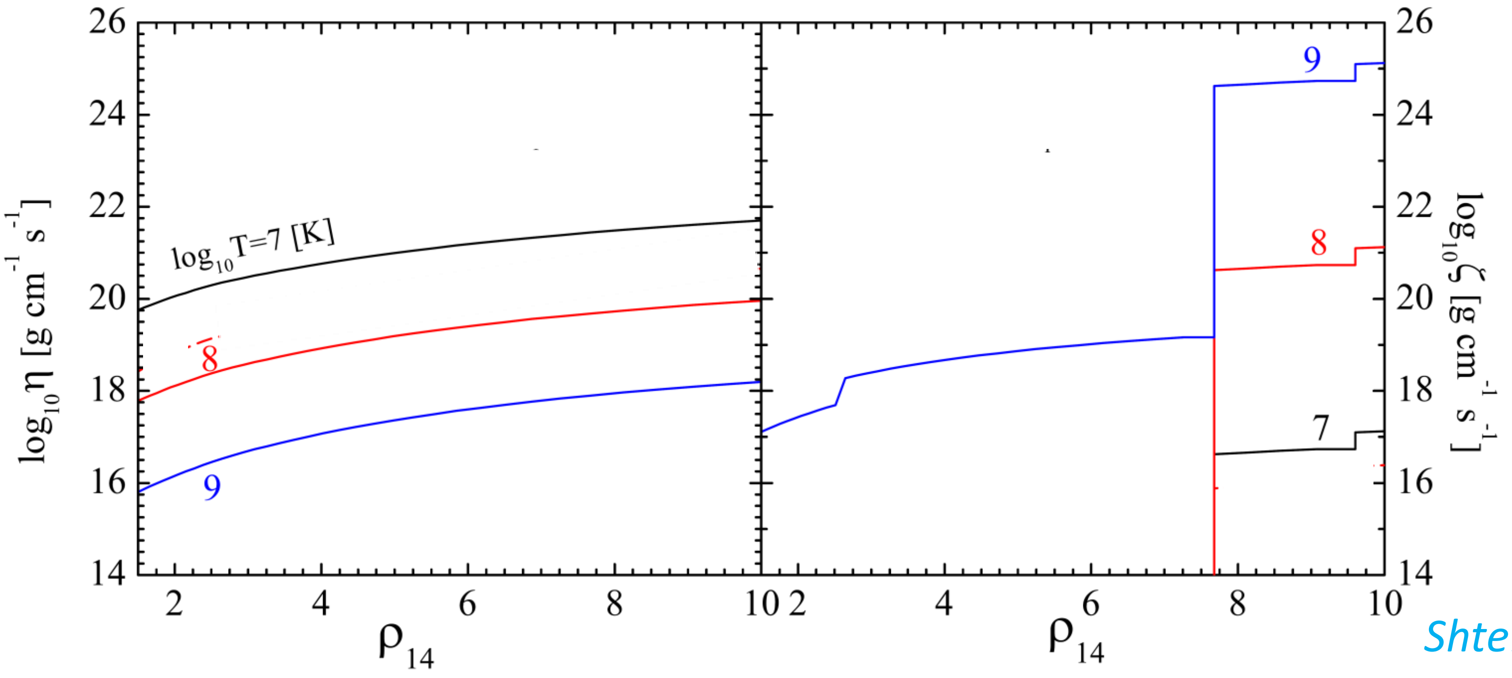
Partial shear viscosities in non-superfluid core.

Temperature dependence of shear viscosity coefficient

$$\eta = \frac{p_{\rm F} v_{\rm F} n \tau}{5}$$

Only particles from thin layer ($\delta p/p_F \sim T/\mu \sim 10^{-4} - 10^{-6}$) near Fermi surface can participate in scattering (to satisfy energy conservation).

$\tau \propto \left(\frac{\mu}{T}\right)^2$ $\eta \propto \left(\frac{\mu}{T}\right)^2$ (however, in-medium effects can alter the exponent)



Shternin, Yakovlev 2008

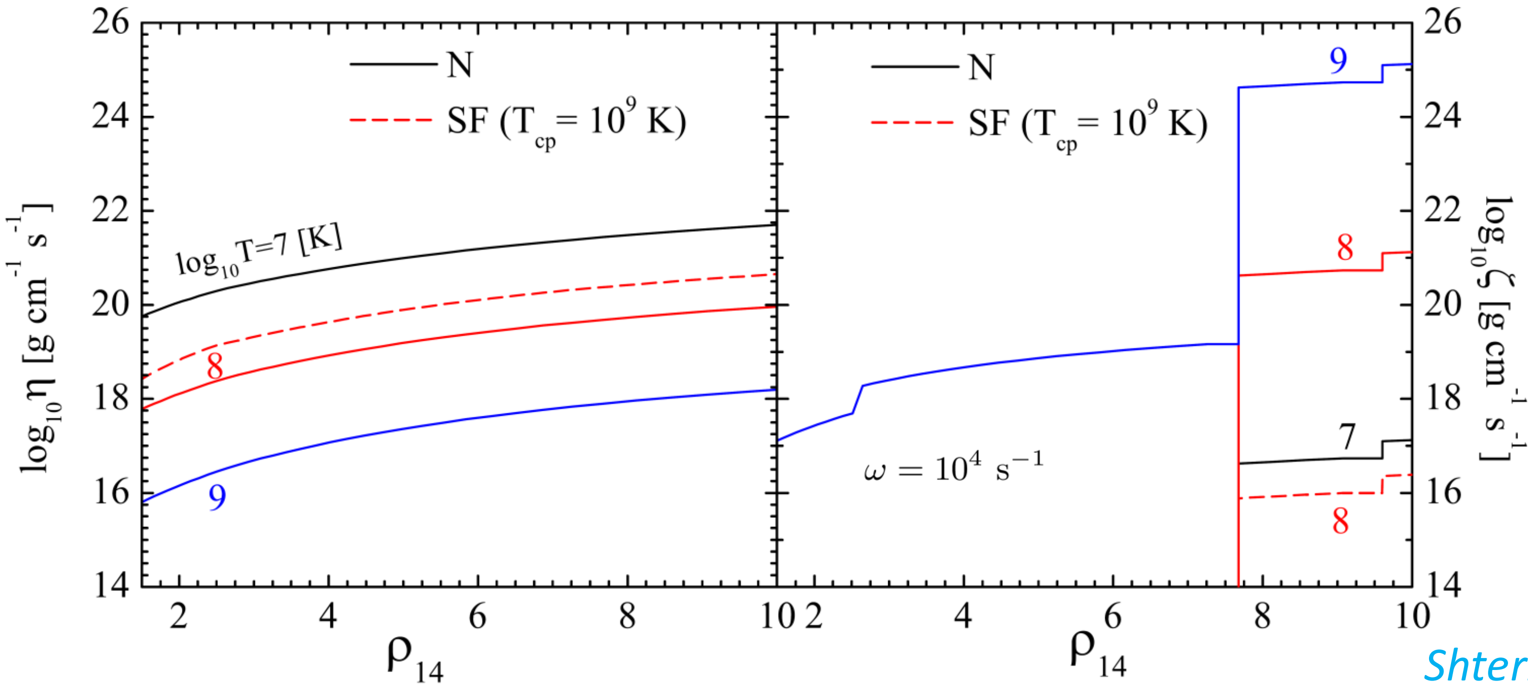
Shear viscosity in non-superfluid matter of neutron star core for different temperatures.

What is the effect of proton superconductivity?

$$\eta = \frac{p_F v_F n \tau}{5}$$

Proton superconductivity increases η (number of scatterers decreases, the screening of electromagnetic interactions is affected -> τ increases).

Superconducting matter is more viscous.



Shternin, Yakovlev 2008

Shear and bulk viscosity at different temperatures for non-superconducting and superconducting matter.

$$\eta = \frac{p_{\rm F} v_{\rm F} n \tau}{5}$$

To determine τ one has to calculate collision integral $\mathrm{St}(f)$:

transition probability
rather uncertain

$$\mathrm{St}(f) = \frac{4}{(2\pi\hbar)^9} \int \mathrm{d}\boldsymbol{p}_2 \mathrm{d}\boldsymbol{p}_{1'} \mathrm{d}\boldsymbol{p}_{2'} w(12|1'2')$$
$$\times \left[f_{1'} f_{2'} (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_{1'})(1 - f_{2'}) \right]$$

Poorly known bare (in-vacuum)
nucleon-nucleon potentials
at high energies

in-medium effects,
polarization effects
(screening of
electromagnetic and strong
interaction by medium)

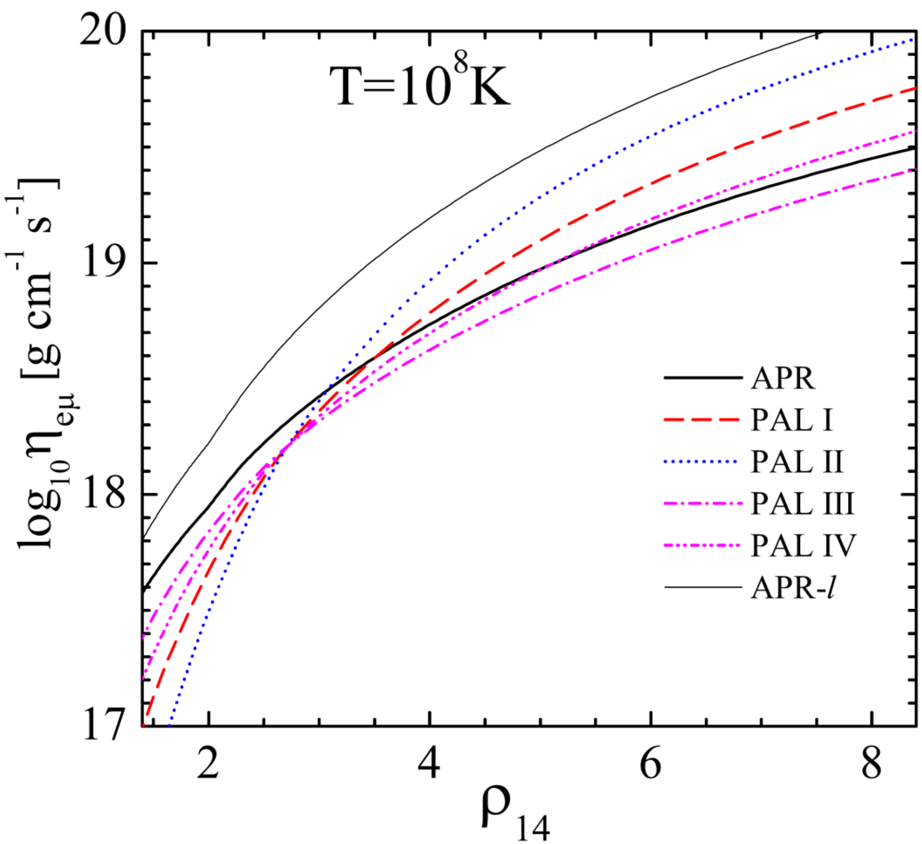
transition probability

"Self-energy" effects
(the energy of particles in the
medium differs from that in vacuum).

effects of superfluidity and
superconductivity

effect of impurities in the crust

Sensitivity to the equation of state



Shear viscosity of electrons and muons for different EOSs.

Shternin, Yakovlev 2008

Papers on shear viscosity calculations:

E. Flowers, N. Itoh, Astrophys. J., 206, 218 (1976)
E. Flowers, N. Itoh, Astrophys. J., 230, 847 (1979)
A. Chugunov. D. Yakovlev, Astron. Rep., 49, 724 (2005)
O. Benhar and M. Valli, Phys. Rev. Lett., 232501 (2007)
P. Shternin, D. Yakovlev, Phys. Rev. D, 78, 063006 (2008)
C. Manuel, L. Tolos, Phys. Rev. D, 84, 123007 (2011)
P. Shternin, M. Baldo, P. Haensel, Phys. Rev. C, 88, 065803 (2013)
B. Bertoni, S. Reddy, E. Rrapaj, Phys. Rev. C, 91, 025806 (2015)
... and many others …

Shear viscosity coefficient

$$\eta = \frac{p_\mathrm{F} v_\mathrm{F} n \tau}{5}$$

in the mixtures is the sum of partial shear viscosities

$$\eta = \sum_i \eta_i$$

in degenerate matter decreases with temperature

$$\eta \propto \left(\frac{\mu}{T}\right)^2$$

is higher in superconducting matter;

Ekman layer

One of the most efficient dissipative mechanisms related to shear viscosity

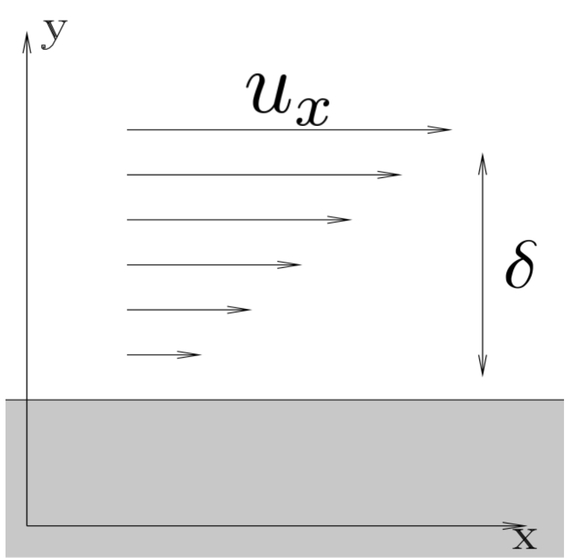
Consider axially oscillating liquid core and (perfect) rigid crust that does not couple to the core oscillations.
If viscosity is zero then fluid can slip at the core-crust interface, fluid motions are discontinuous.
However, non-zero viscosity forbids discontinuities in the fluid displacements and leads to "no-slip" condition at the core-crust interface, resulting in a thin transition layer where fluid displacements fall to zero.

Flat-parallel flow

$$u = u_x$$

fluid oscillates with frequency ω

velocity profile is determined by shear viscosity



Navier-Stokes equation:

$$\frac{\partial \rho u_x}{\partial t} = \eta \frac{\partial^2 u_x}{\partial y^2}$$

$$\omega \rho u_x \approx \eta u_x / \delta^2$$

$$\delta \approx \left(\frac{\eta}{\rho \omega} \right)^{1/2} \sim 1\,\mathrm{cm} \ll \mathrm{R}$$

Dissipation in the viscous boundary layer:

$$\frac{1}{\tau_{\rm Damp}} = -\frac{\dot{E}}{2E} \qquad E = \int_V \left\langle \frac{\rho u^2}{2} \right\rangle {\rm d}V$$

$$\dot{E} = -\int_V \left\langle T\,(S u^\nu)_{;\nu} \right\rangle {\rm d}V = -\int_V {\rm d}V \left\langle \frac{\eta}{2}\left(\frac{\partial u_i}{\partial x^k} + \frac{\partial u_k}{\partial x^i} - \frac{2}{3}\delta_{ik}{\rm div}\,\boldsymbol{u}\right)^2 \right\rangle$$

VBL:

$$E \sim \frac{\rho u^2}{2}\frac{4\pi}{3}R^3 \qquad \dot{E} \sim \eta\frac{u^2}{\delta^2}\,4\pi R^2\delta$$

$$\frac{1}{\tau_{\rm Ek}} \sim -\frac{\dot{E}}{2E} \sim \frac{\eta}{\rho\delta R} \propto \left(\frac{\eta\omega}{\rho}\right)^{1/2}\frac{1}{R} \propto \frac{1}{T}$$

$$\delta \approx \left(\frac{\eta}{\rho\omega}\right)^{1/2}$$

shear viscosity:

$$E \sim \frac{\rho u^2}{2}\frac{4\pi}{3}R^3 \qquad \dot{E} \sim \eta\frac{u^2}{R^2}\,\frac{4\pi}{3}R^3$$

$$\frac{1}{\tau_{\rm shear}} \sim \frac{\eta}{\rho R^2} \sim \frac{1}{T^2}$$

Model of perfectly rigid crust oversimplifies the problem.

Crust is not absolutely rigid, but it is elastic (it couples to the oscillating core)

Calculations show that the spectrum of torsional oscillations of the crust is spread above ~50 Hz. This means that core oscillations with frequencies >50 Hz can penetrate the crust (lattice stresses are small in comparison to driving force of the oscillations).

Yu. Levin, G. Ushomirsky, MNRAS, 324, 917 (2001):

“at spin frequencies in excess of 50 Hz, the r-modes strongly penetrate the crust. This reduces the relative motion (slippage) between the crust and the core compared with the rigid-crust limit.“

The degree of crust coupling to the core is characterized by "slippage" factor S

$$S \sim \delta u/u = (\underset{\text{co-motion}}{0} - \underset{\text{rigid crust}}{1})$$

Depends on the crustal physics (shear modulus) and on the oscillation frequency.

velocity gradients in the boundary viscous layer $\propto S$

Dissipation rate $\frac{1}{\tau} \sim \left(\frac{\eta\omega}{\rho}\right)^{1/2} \frac{1}{R} \; S^2$

Other uncertainties:

Magnetic field effect (superconductor effect as well)
Neutron superfluidity effect on hydrodynamic flows
Compressibility and stratification

L. Bildsten, G. Ushomirsky, ApJ, 529, L33 (2000)
Yu. Levin, G. Ushomirsky, MNRAS, 324, 917 (2001)
G. Mendell, Phys. Rev. D, 64, 044009 (2001)
K. Glampedakis, N. Andersson, MNRAS, 371, 1311; Phys. Rev. D, 74, 044040 (2006)
… and many others …

Application of the dissipative processes to r-mode physics

see Haskell, IJMPE, 24, 1541007 (2015) for the recent review

NSs rotate. Rotation frequencies of NSs lie in a wide range. The most rapidly rotating neutron star observed so far is the millisecond pulsar PSR J1748-2446ad with $\nu = 716\ \mathrm{Hz}$. Kepler (or mass-shedding) frequency:

$\nu_{\rm K} \approx 1233\ (M/1.4\,M_\odot)^{1/2}\ (R/10\,{\rm km})^{-3/2}\ {\rm Hz.}$ (*Lattimer & Prakash 2007*).

Rotating NSs support inertial oscillation modes, in particular r-modes, which are subject to gravitational-driven instability at *any* rotation frequency (gravitational waves emitted by r-mode excite this r-mode and its amplitude increases with time; excitation of r-mode decreases total energy of NS). Emitted gravitational waves carry off the angular momentum and the star spins down.

Dissipation suppresses *r*-mode instability in a wide range of parameters. Only rapidly rotating and warm NSs are unstable with respect to r-mode excitation ("instability window").

The most effective dissipative mechanisms

Shear viscosity

$$\propto \frac{1}{T^2}$$

$$\tau_{sv} = 2.2 \times 10^5 \left(\frac{M}{1.4 M_\odot}\right)^{-1} \left(\frac{R}{10\,\mathrm{km}}\right)^5 \left(\frac{T}{10^8\,\mathrm{K}}\right)^2 \mathrm{s}$$

Ekman layer

$$\propto \frac{1}{T}$$

$$\tau_{ek} = 3 \times 10^4 \left(\frac{P}{10^{-3}\,\mathrm{s}}\right)^{1/2} \left(\frac{T}{10^8\,\mathrm{K}}\right) \mathrm{s} \qquad \mathcal{S} = 0.05$$

Bulk viscosity

$\propto T^6$ (modified Urca)

$\propto T^4$ (direct Urca)

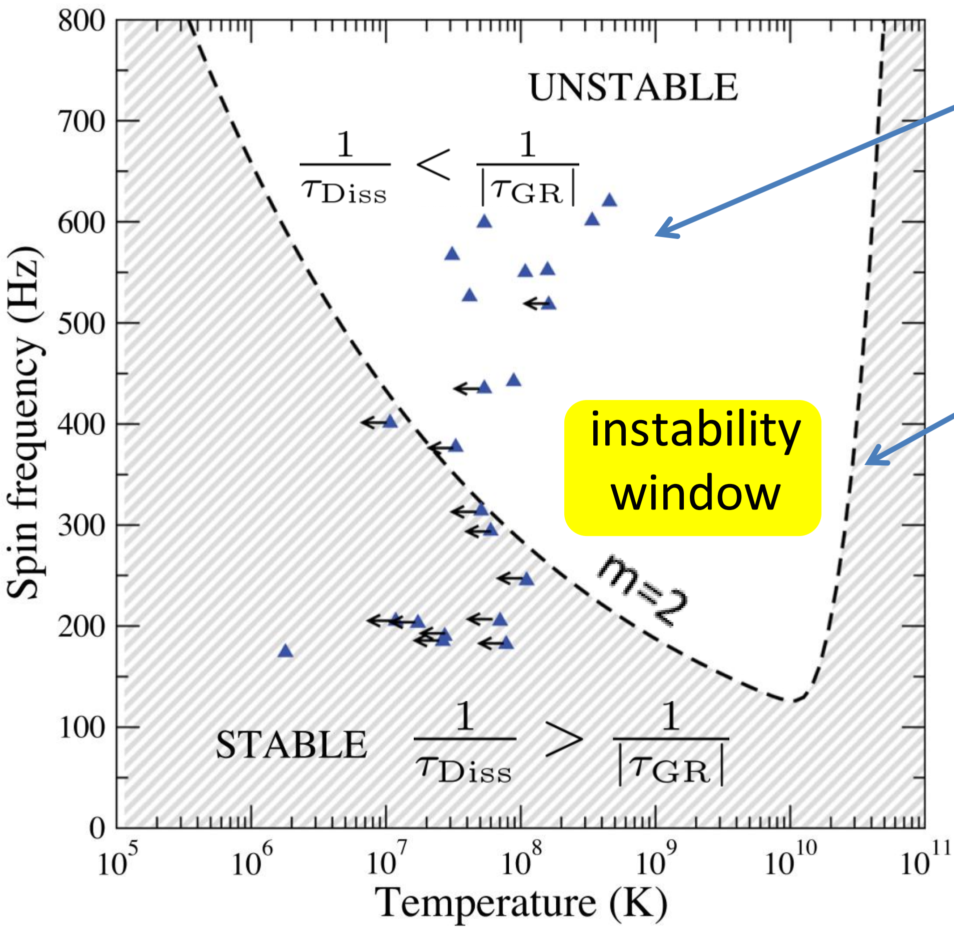
$$\tau_{bv} = 2.7 \times 10^{17} \left(\frac{M}{1.4 M_\odot}\right) \left(\frac{R}{10\,\mathrm{km}}\right)^{-1} \left(\frac{P}{10^{-3}\,\mathrm{s}}\right)^2 \left(\frac{T}{10^8\,\mathrm{K}}\right)^{-6} \mathrm{s}$$

\+ Mutual friction (superfluid NSs)

complicated temperature behavior,
resonances at certain temperatures

Typical instability window of an NS

B. Haskell, N. Degenaar, W. Ho, MNRAS, 424, 93 (2012)



Observed spin frequencies and internal temperatures of NSs in LMXBs.

$$\frac{1}{\tau_{\rm GR}} + \frac{1}{\tau_{\rm Diss}} = 0$$

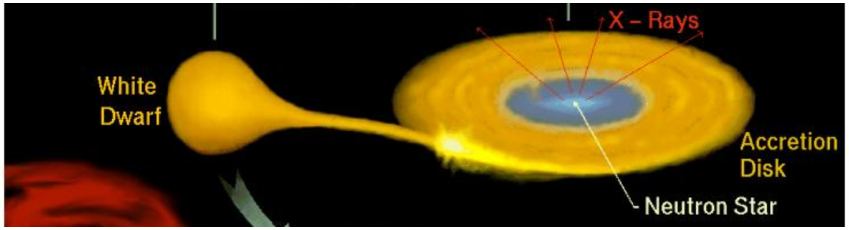
excitation time

viscous damping time

$$\frac{1}{\tau_{\rm Diss}} = \frac{1}{\tau_{\rm Ek}} + \frac{1}{\tau_{\rm bv}}$$

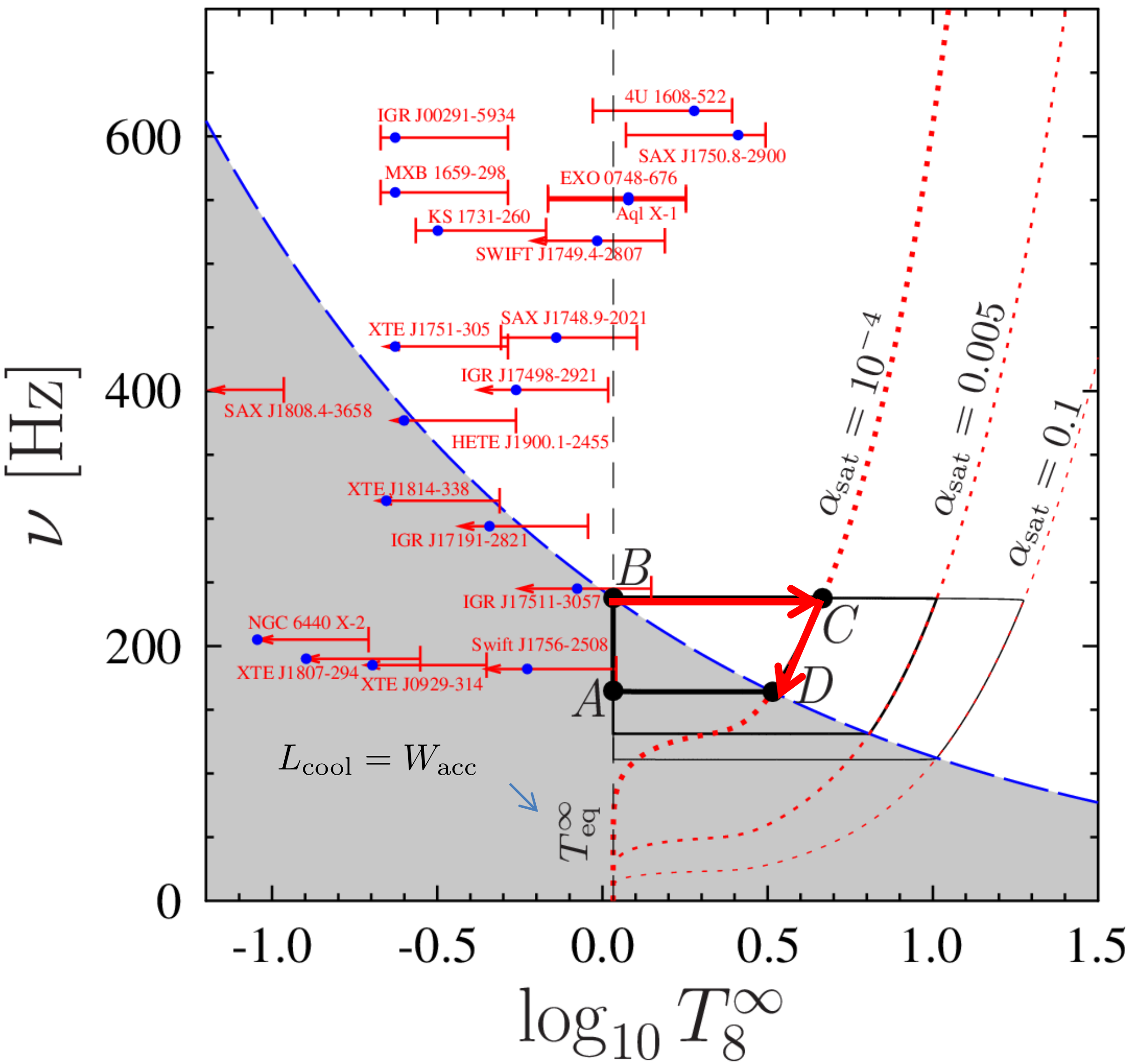
$S = 0.05$ modified Urca

Such rapidly rotating and warm NSs are observed in LMXBs (NS + low-mass companion), where they are spun up and heated by the accretion from the companion.



Artist's impression of an LMXB

What happens with an NS in instability region?



Standard scenario of NS evolution in LMXB.

Levin 1999

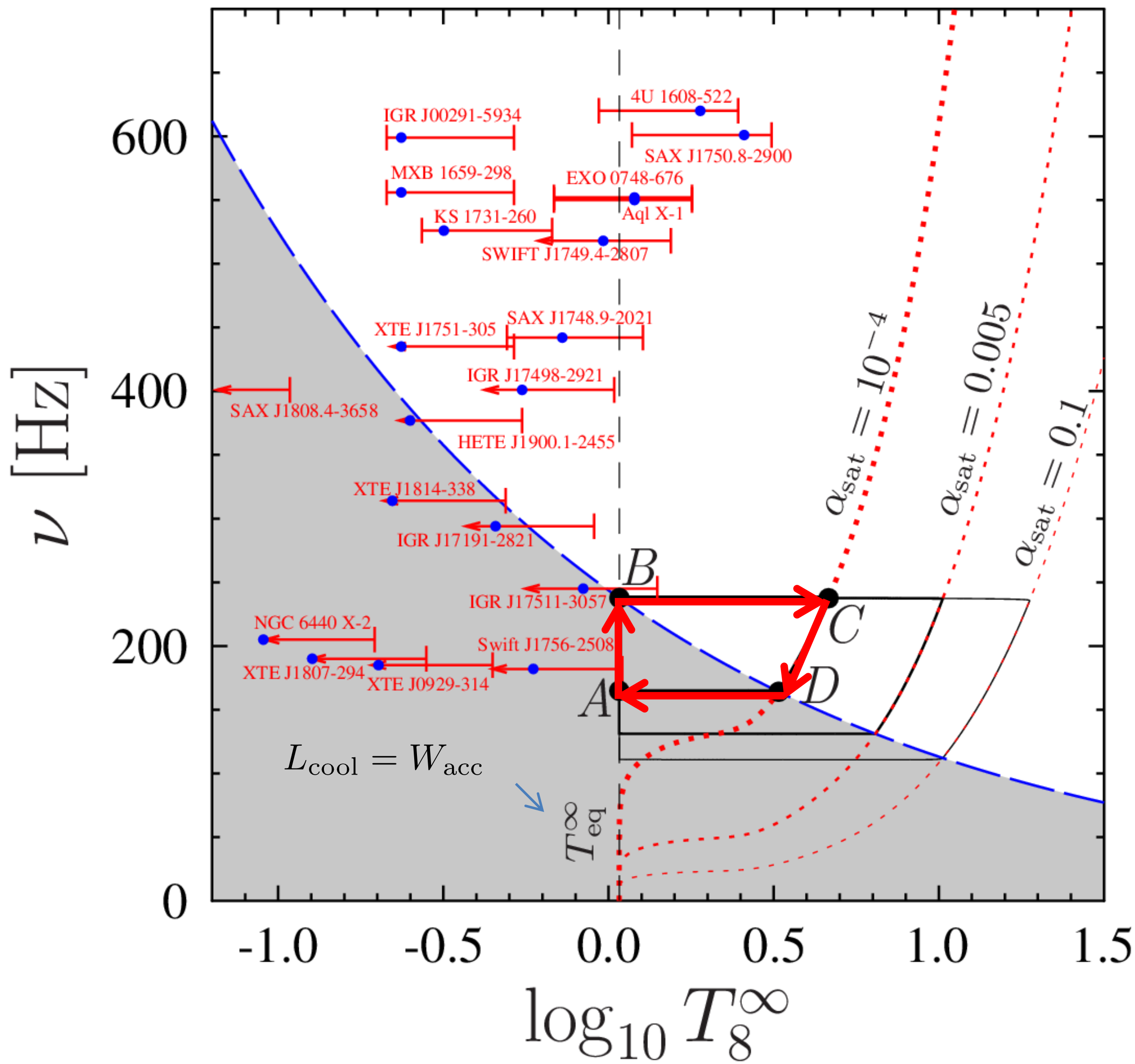
Once an NS crosses the instability curve (in point B), r-mode amplitude increases to the saturation value (typically $\sim 10^{-4}R$) and rapidly heats the star and spins it down, so that the NS leaves the instability window.

$$t_{\rm BCD} \ll t_{\rm DAB}$$

The probability to observe an NS in instability region is very small.

But we see a lot of NSs there!

What happens with an NS in instability region?



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$$t_{\rm BCD} \ll t_{\rm DAB}$$

The probability to observe an NS in instability region is very small.

But we see a lot of NSs there!

Two standard ways to solve this contradiction:

1. **r-mode saturates at very small amplitude** $10^{-6}R - 10^{-9}R$

Theoretical justifications:

R. Bondarescu, I. Wasserman, ApJ, 778, 13 (2013)
mechanism – coupling to other oscillation modes

B. Haskell, K. Glampedakis, N. Andersson, MNRAS, 441,1662 (2014)
mechanism – vortex unpinning and cutting through flux tubes

M. Alford, S. Han, K. Schwenzer, Phys. Rev. C, 91, 055804 (2015)
mechanism – the periodic conversion between different phases (movement of the interface) in hybrid stars

L. Rezzolla, F. Lamb, S. Shapiro, ApJ, 531, L139 (2000)
mechanism – saturation of r-mode by the magnetic field

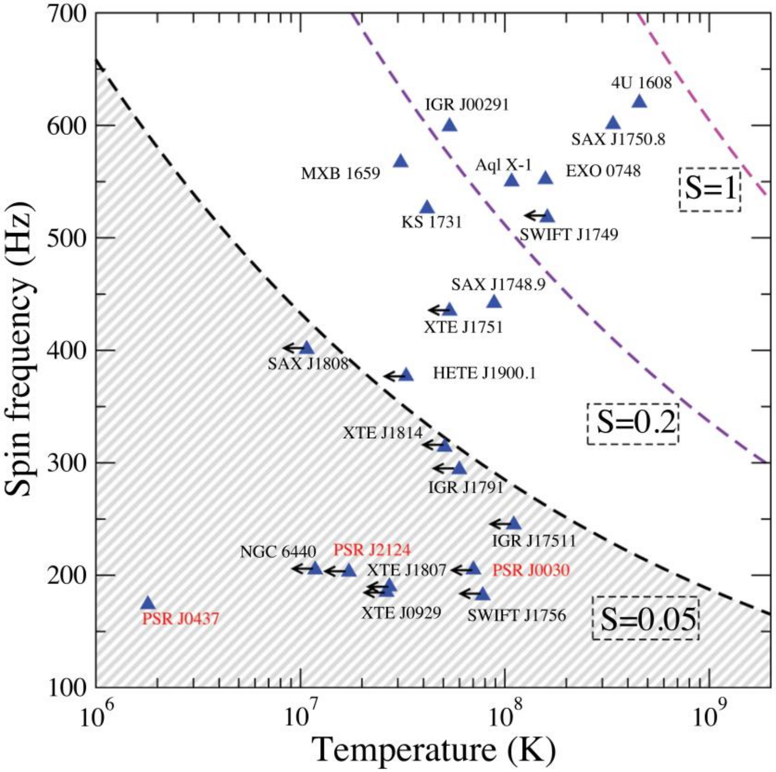
2. stability region spreads over all sources

Requires enhanced dissipation.

Enhanced shear viscosity, dissipation in Ekman layer, bulk viscosity…

To stabilize all the sources by shear viscosity one should increase the latter by a factor of 1000. No microscopic justification.

Dissipation in Ekman layer may be stronger



Instability window
for different values of “slip” parameter *S.*

Crust may be more rigid.

S=1 all sources are stable.
S=1 is not realistic.
However, *S* can be =1 at certain rotation frequencies (resonances with crustal torsional modes)

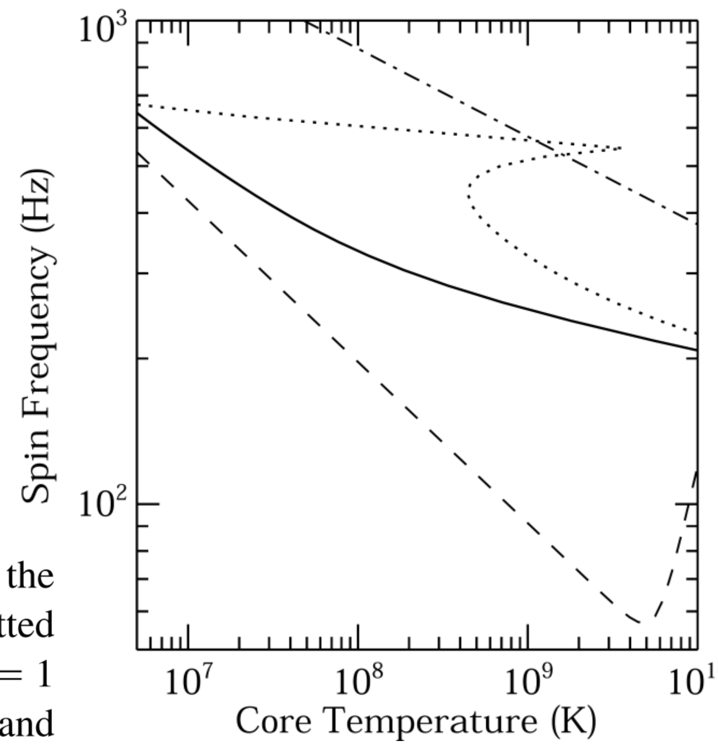


Figure 2. Critical frequencies as functions of the core temperature for the modes in the thin-crust model (solid line) and the thick-crust model (dotted line). For comparison, we also display the critical frequencies for the $n = 1$ polytrope fluid model (dashed line), and for a model with a crust and $\delta u/u = 1$ independent of the spin (dot-dashed line).

Yu. Levin, G. Ushomirsky, MNRAS, 2001

Bulk viscosity may be stronger, e.g. due to reactions with hyperons.

A. Reisenegger, A. Bonacic, PRL, 2003

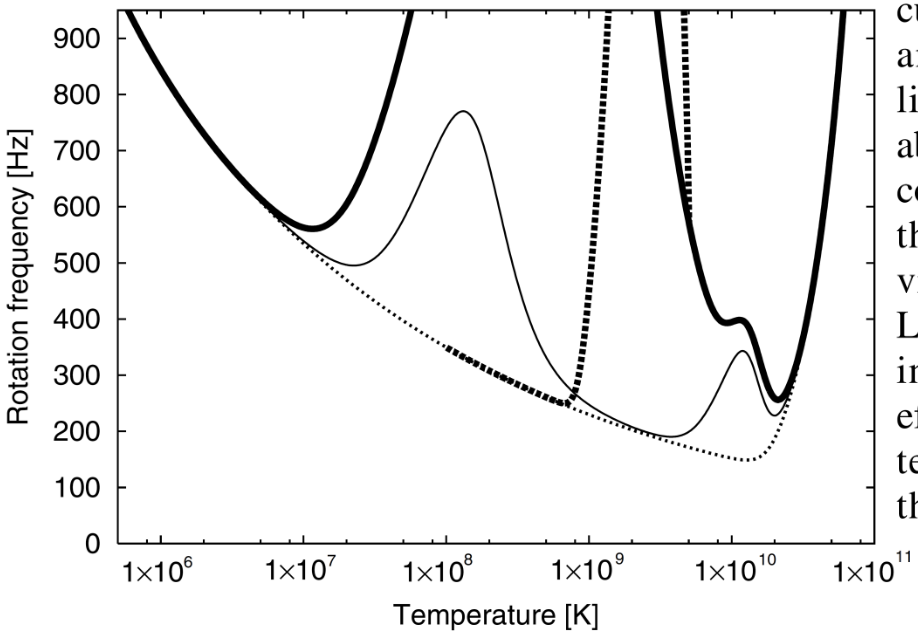


FIG. 1. Instability boundaries for the $l = m = 2$ r-mode. The curves all use the same shear viscosity, determined by Levin and Ushomirsky [9] to be active at the boundary between the liquid core and a thin, elastic crust, but different assumptions about the bulk viscosity in the core. The thin dotted curve considers only modified Urca processes, whereas the other three also include direct Urca processes and the hyperon bulk viscosities proposed by Jones [7] (thin solid line) and by Lindblom and Owen [8] (thick solid line), in both cases ignoring superfluid effects, and by Lindblom and Owen under the effects of hyperon superfluidity with a uniform, high critical temperature, $T_c \sim 5 \times 10^9$ K (thick dotted line). In each case, the unstable region lies above the curve.

Problems

1. Very uncertain physics:

potentials of hyperon interaction
in-medium effects
critical temperatures of the transition to the superfluid state.

2. How to form millisecond pulsars?

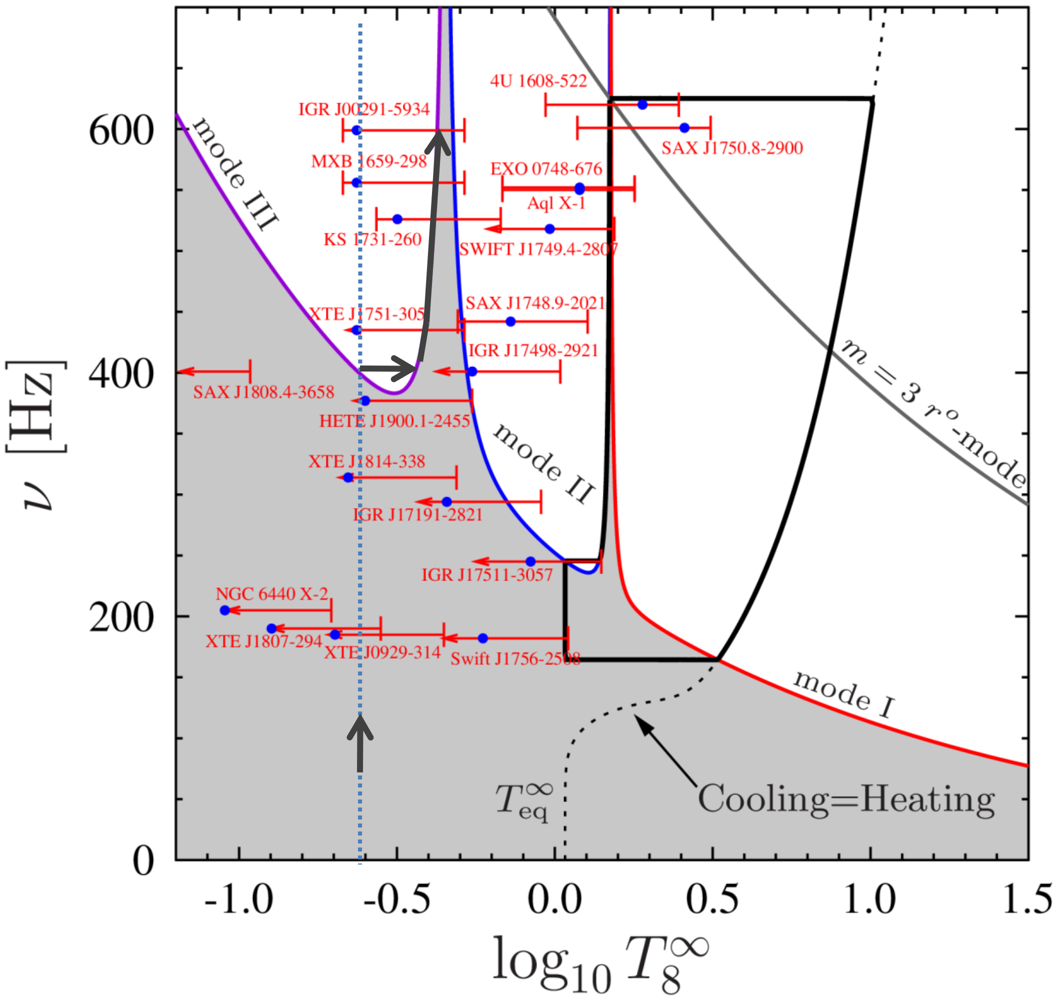
Other ways to solve the contradiction

Gusakov, Chugunov, Kantor, Phys. Rev. D, 90, 063001 (2014)
proposed the mechanism of NS stabilization by mutual friction.
Mutual friction arises in superfluid NSs
due to relative motion of normal and superfluid component.

Several velocity fields => more degrees of freedom => more oscillation modes

normal modes (r-modes)	superfluid modes
Are almost the same as in nonsuperfluid NS Eigenfrequencies are temperature independent (matter is strongly degenerate) Correspond to co-moving oscillations of superfluid component (paired neutrons) and normal component (all other particles) Almost no dissipation due to mutual friction	Are absent in nonsuperfluid NS Eigenfrequencies are strong functions of NS temperature (because superfluid density is strong function of temperature) Correspond to counter-moving oscillations of normal and superfluid components Strongly damp due to very powerful mutual friction mechanism, that tends to equalize velocities of normal and superfluid components

Generally, normal and superfluid modes decouple and almost do not interact.
But at certain temperatures, when their eigenfrequencies are close, the interaction is strong.
The eigenfunctions of the superfluid mode admix to the eigenfunctions of the normal mode, and the latter experiences enhanced (resonance) dissipation due to mutual friction.



Gusakov, Chugunov, Kantor,
Phys. Rev. D, 90, 063001 (2014)

r-mode can experience enhanced damping due to mutual friction at the temperatures corresponding to the resonances with superfluid modes.

Evolution proceeds along the stability peak.

Proper account for dissipative processes
defines the form of the instability window,
which affects the evolution of an NS in LMXB.

It also determines if r-modes are excited in the observed NSs or not,
if the observed NSs emit gravitational waves
if they can be observed by future gravitational interferometers.

The evolution of an NS after the accretion stage is finished depends on the form of the instability region.
Formation of millisecond pulsar population depends on it.

Observations of NSs in LMXBs may constrain the form of the instability window and thus properties of NS matter.

Thank you for your attention!