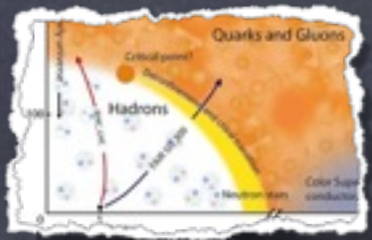

Microscopic description of the dense matter EoS

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NewCompStar School 2016
"Neutron Stars : gravitational physics theory and observations"
Coimbra, 5-9 September 2016

Preliminary notes

You can find material in the following books :

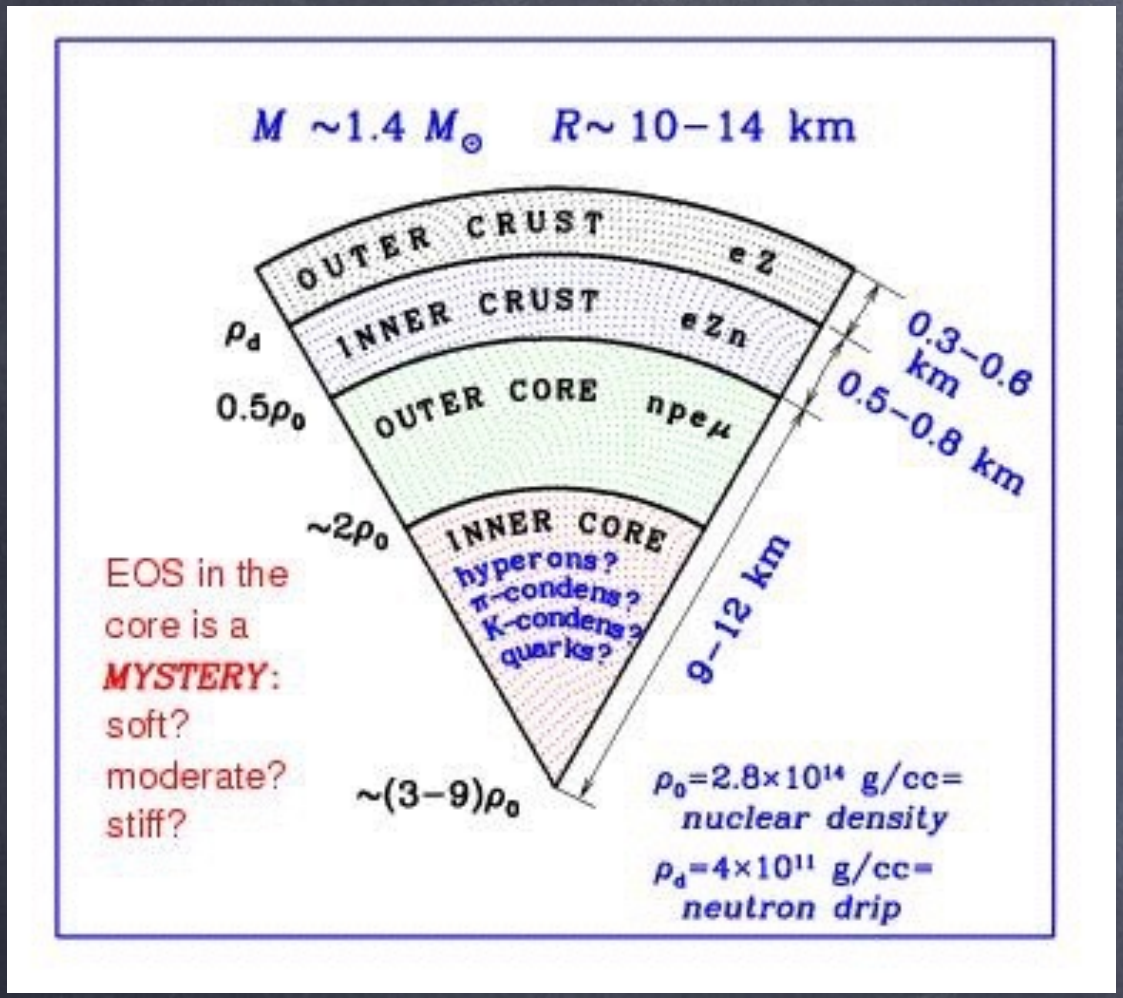
1. "Nuclear Methods and the Nuclear Equation of State", edited by M. Baldo, International Review of Nuclear Physics 8, (1999), World Scientific,.

2. "Neutron Stars I", Equation of State and Structure, P. Haensel, A. Potekhin, D. Yakovlev, 2007, Springer

3. "Properties of the nuclear medium", M. Baldo & G.F. Burgio, Reports on Progress in Physics, 75 (2012) 026301.

For current research I'll give references during the lectures.

Schematic view of a neutron star



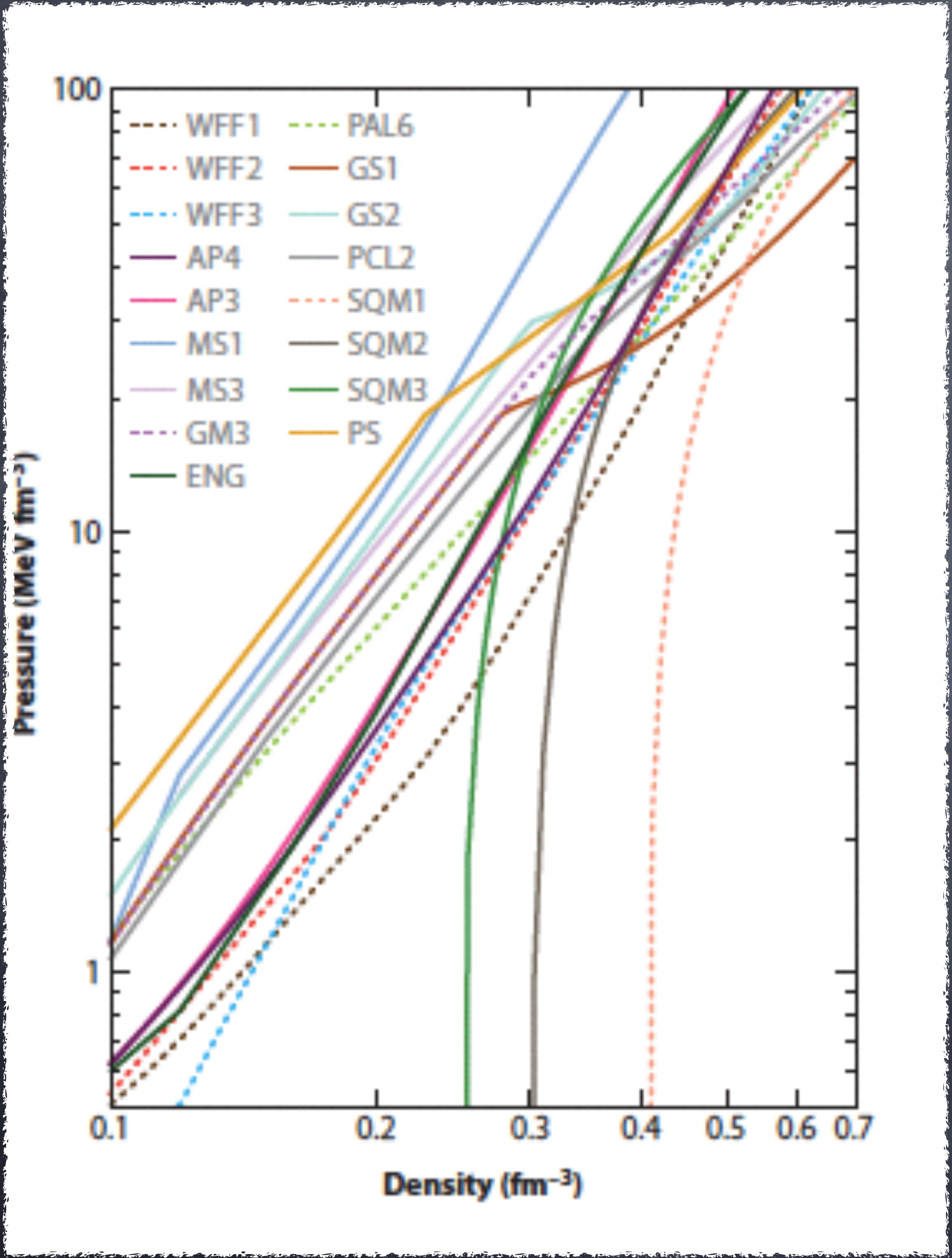
Outer crust. Nuclei immersed in an electron gas, as in normal metal. In the outer part, matter is expected to contain ^{56}Fe.

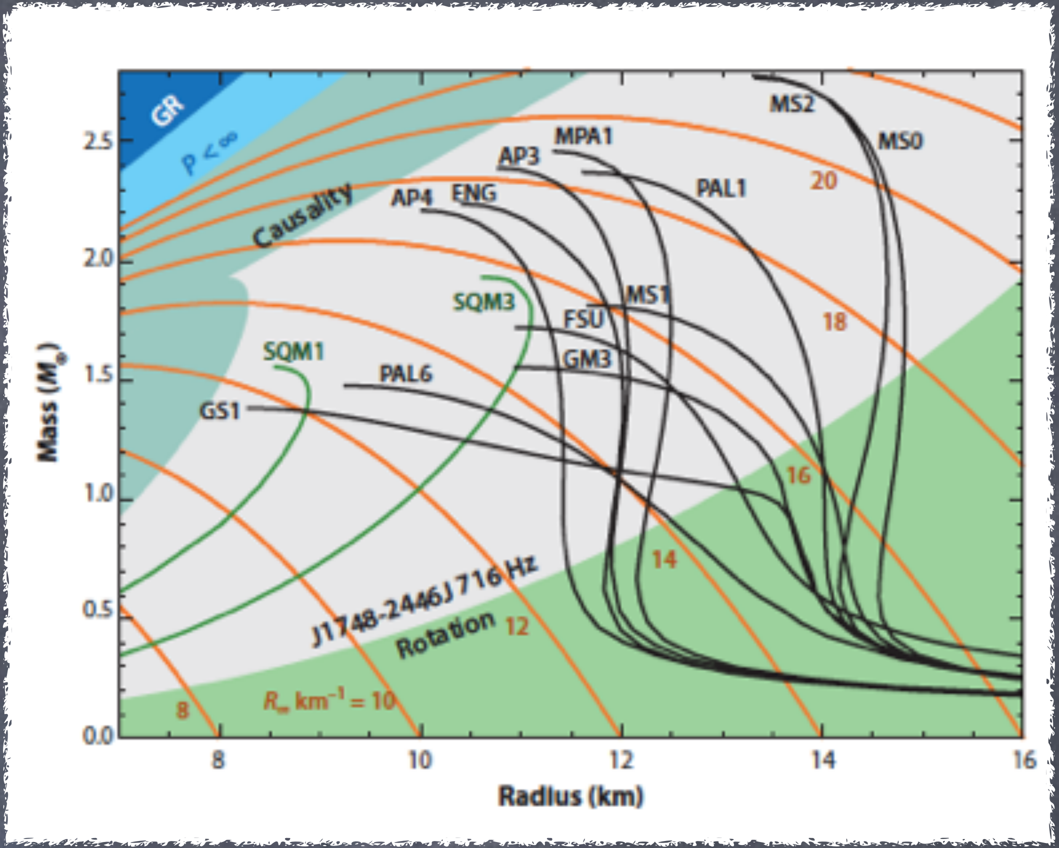
Inner crust. By increasing density, electrons are beta-captured by nuclei, which become more and more neutron-rich. Due to large asymmetry, the neutron chemical potential becomes positive and the neutrons drip out of the nuclei. At drip point, besides electrons and nuclei, a gas of free neutrons in chemical equilibrium is present. Nuclei melt down and nuclear matter sets in starting from drip point up to about half the saturation density.

Outer core. Asymmetric nuclear matter above saturation. Mainly composed by neutrons, protons, electrons and muons. Its exact composition depends on the nuclear matter Equation of State (EoS).

Inner core. The most unknown region. "Exotic matter" . Hyperons ? Kaons ? Quarks ?

1. EOS in the crust is known reasonably well
2. EOS in the outer core is not very certain
3. EOS in the inner core is a mystery





J. Lattimer, Ann. Rev. Nucl. Part. Sci. (2012)

Need of a microscopic description of the EoS !

Strong interactions and Neutron Stars

In the first model of a neutron star strong interactions were neglected => a pure Fermi gas. Oppenheimer-Volkoff (1939) obtained a maximum mass value equal to $0.71 M_\odot$. Subsequent observations of the Hulse-Taylor binary pulsar, with $1.44 M_\odot$, tell us that strong interactions are crucial for understanding neutron stars.

Hadronic interactions in dense matter

- Hadronic Interaction is given by the quantum chromodynamics (QCD). The weak interaction enters the problem only indirectly by opening some channels for reaching the ground state of the matter, whereas the electromagnetic interaction plays almost no role for the EOS.

- Hadronic Hamiltonian cannot be presently derived from the QCD, → we have to use phenomenological models of hadronic interaction, based on mesonic theories, where strong interaction is modeled by the exchange of mesons.

★Most refined and complete phenomenological models constructed for the NN interactions. Tested using thousands of experimental data on NN scattering cross sections supplemented with experimental properties on deuteron.

★Experimental information on the nucleon-hyperon interactions available only for the lowest-mass hyperons Λ and Σ. Mainly obtained from studies of hypernuclei. A few data (35 !) : the interaction models are incomplete. No experimental data on hyperon-hyperon interaction.

★Three body interactions. Two-body hadronic interactions yield only a part of the hadronic Hamiltonian of dense matter. At densities typical of NS core, interactions involving three and more hadrons might be important. Our experimental knowledge of three-body interaction is restricted to nucleons. The three-nucleon (NNN) force is necessary to reproduce properties of ^{3}H and ^{3}He and to obtain correct parameters of symmetric nuclear matter at saturation.

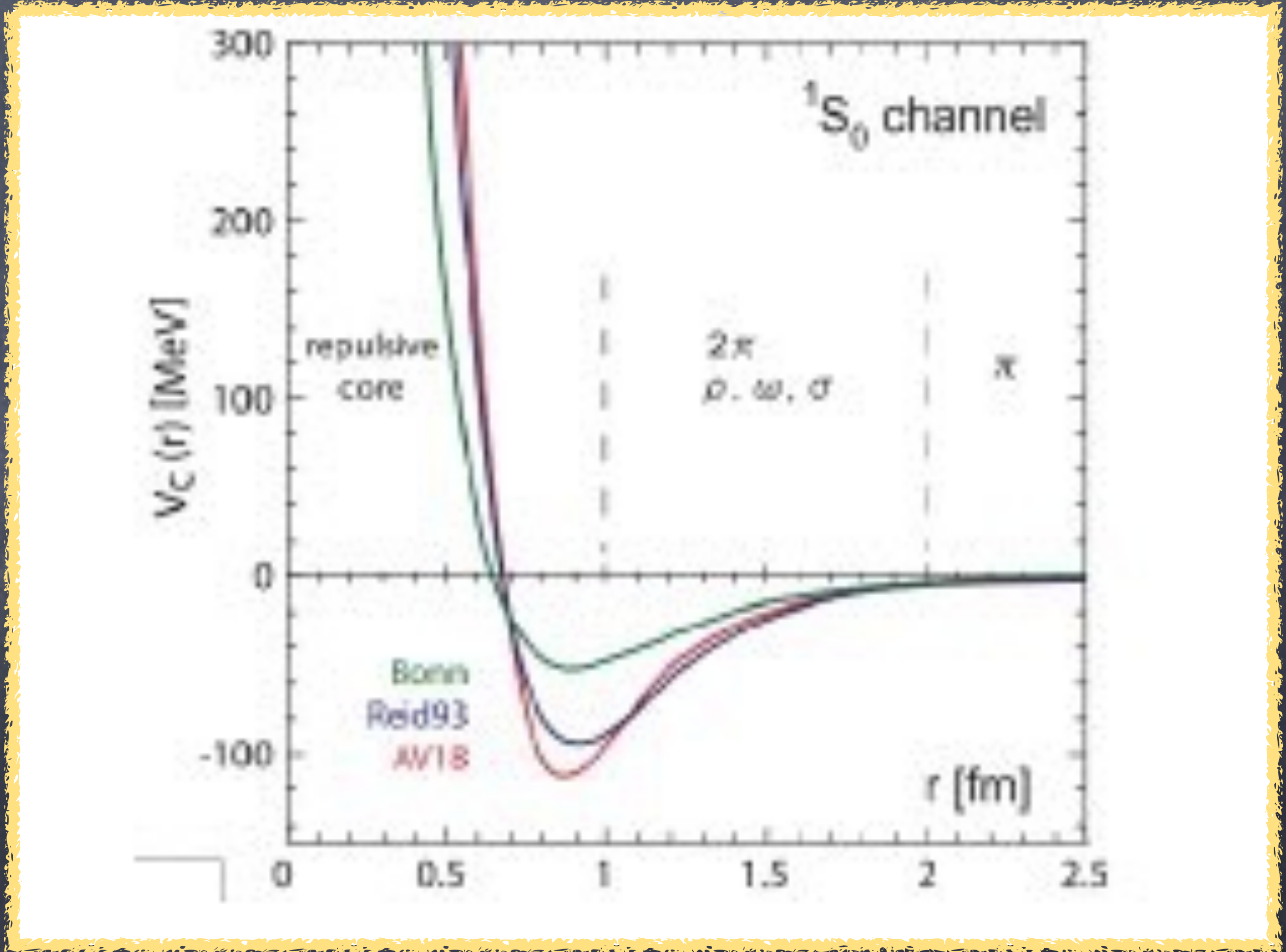
Sketch of the NN interaction

At large distance, r > 1 fm, the interaction is attractive with an exponential tail.

At intermediate distance, 0.5 < r < 1 fm, a stronger attraction is present, at least once an average is made over the different channels.

At short distance, r < 0.5 fm, a strong repulsive core is in any case present (an infinite impenetrable barrier was assumed in early calculations).

- CAVEAT ! Divergency problem in many-body calculations. Standard perturbation theory not applicable !



Several NN potentials available in literature

Fit to pp data   Reid('68), Njimegen ('78), Paris ('80)

Fit to np data   Urbana v14 ('81), Argonne v14 ('84), Bonn ('87)

Potential models which have been fit only to the np data often give a poor description of the pp data, and viceversa →

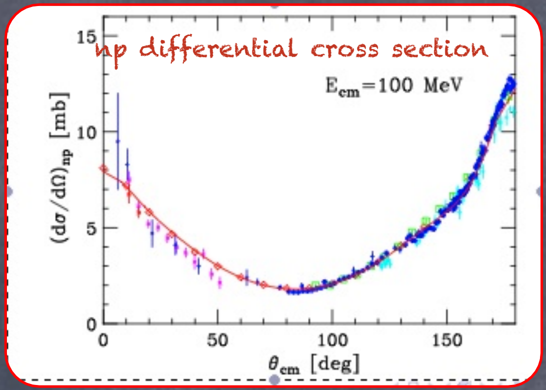
Fit to both np and pp data : only a limited set of forces remain

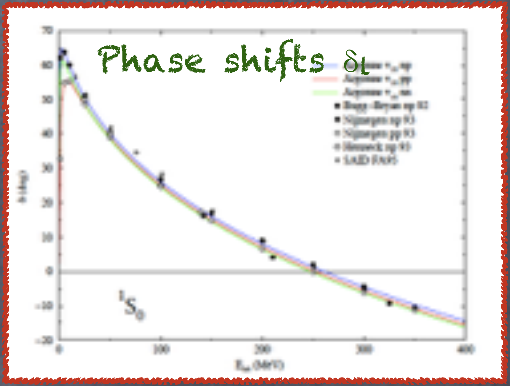
1. Argonne v_{18} (strictly local in each channel, Wiringa 1995)
2. CD Bonn potential (OBE, Machleidt 2001)
3. IS potential (non-local modifications of v_{18}, Doleschall 2004)

A modern NN potential : Argonne v18

The Argonne v_{18} potential has been fit to the Nijmegen pp and np scattering cross section, NN phase shifts and deuteron binding energy.

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} \left| \sum_l (2l+1)(e^{2i\delta_l} - 1) P_l(\vartheta) \right|^2$$





A non-relativistic NN potential can be expressed in terms of a set of operators acting on the spin (σ) and isospin (τ) variables of the two nucleons, as well as on the relative angular momentum (L), the total spin operators S, and r the relative coordinate.

The form of the operators is dictated by symmetry requirements : translational and rotational invariance, charge independence of the nuclear forces, parity and time-reversal symmetry.

1	*central*
$\sigma_1 \cdot \sigma_2$	*spin − spin*
$(\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2)$	*spin − isospin*
$\frac{3(\sigma_1 \cdot \mathbf{r})(\sigma_2 \cdot \mathbf{r})}{r^2} - (\sigma_1 \cdot \sigma_2)$	*tensor*
$(\frac{3(\sigma_1 \cdot \mathbf{r})(\sigma_2 \cdot \mathbf{r})}{r^2} - (\sigma_1 \cdot \sigma_2))(\tau_1 \cdot \tau_2)$	*tensor − isospin*
$\mathbf{l} \cdot \mathbf{S}$	*spin − orbit*
$\mathbf{l}^2$	*ang. mom. square*

In operatorial form the Argonne v18 NN potential is expressed by :

$$v_{18} = \sum_{p=1,18} v_p(r_{ij}) O_{ij}^p$$

tensor force

$$O_{ij}^p = \left[1, \sigma_i \cdot \sigma_j, S_{ij}, L \cdot S, L^2, L^2(\sigma_i \cdot \sigma_j), (L \cdot S)^2\right] \otimes \left[1, \tau_i \cdot \tau_j\right],$$

isotensor force

$$\left[1, \sigma_i \cdot \sigma_j, S_{ij}\right] \otimes T_{ij}, \text{ and } \left(\tau_{zi} + \tau_{zj}\right)$$

Wiringa et al., PRC51, 38 (1995)

The first fourteen terms express charge independence (corresponding to $v_{nn}=v_{np}=v_{pp}$). The four additional operators are small and break the charge independence.
In coordinate representation each term is multiplied by a form factor v_p which is in general a non-local potential and describes the possible velocity dependence of the NN potential.

Solving the nuclear many-body problem

Phenomenological vs. ab initio approaches

Phenomenological approaches

Based on effective density-dependent NN force with parameters fitted on nuclei properties.

- Liquid Drop models
  - ✧ BPS Baym et al, *ApJ* ***170****, 299 (1971)*
  - ✧ BBP Baym et al., *NPA* ***175****, 225 (1971)*
  - ✧ LS Lattimer&Swesty, *NPA* ***535****, 331 (1991)*
  - ✧ DH Douchin&Haensel, *A&A* ***380****, 151 (2001)*
- TF + RMF
  - ✧ Shen et al., *NPA* ***637****, 435 (1998)*
- ETFSI + Eff. Skyrme force
  - ✧ BSk Goriely et al.,*PRC* ***82****, 035804 (2010)*
- Hartree-Fock
  - ✧ NV Negele&Vautherin, *NPA* ***207****, 298 (1973)*
  - ✧ RMF Serot&Walecka, *Adv. NP 16, 1 (1986)*
  - ✧ RHF Boussy et al., *PRL* ***55****, 1731 (1985)*
  - ✧ QMC Guichon et al., *NPA* ***814****, 66 (2008)*
- Statistical models
  - ✧ NSE Raduta&Gulminelli. *PRC* ***82****, 065801 (2010)*
  - ✧ HS Hempel&Schaffner-Bielich, *NPA* ***837****, 210 (2010)*

Chiral SU(3) model

Dexheimer&Schramm, ApJ 683, 943 (2008)

Ab initio approaches

The nuclear problem is solved starting from the two- and three-body realistic nucleon interaction.

- Diagrammatic
  - ✧ BBG *Day*, *RMP****39****, 719 (1967)*
  - ✧ SCGF Kadanoff&Baym, Quantum Statistical Mechanics (1962)
  - ✧ DBHF Ter Haar&Malfiet, *Phys, Rep.* ***149****, 207 (1987);*
- Variational
  - ✧ APR *Akmal et al.*, *PRC* ***58****, 1804 (1998)*
  - ✧ FHNC Fantoni&Rosati, *Nuovo Cimento A****20****, 179 (1974)*
  - ✧ CBF Fabrocini&Fantoni, *PLB* ***298****, 263(1993)*
  - ✧ LOCV Owen et al., *NPA* ***277****, 45 (1978)*
- Monte Carlo
  - ✧ VMC Wiringa, *PRC****43****, 1585 (1991)*
  - ✧ GFMC Carlson, *PRC****68****, 025802 (2003)*
  - ✧ AFDMC Schmidt&Fantoni, *PLB****446****, 99 (1999)*

Diagrammatic technique I:
The Bruckner theory of nuclear matter

The Brueckner-Hartree-Fock theory is based on the Goldstone expansion, which is a perturbation series for the ground-state energy of a many- body system.
The theory amounts to ordinary perturbation theory expressed in a tractable form.

We will first consider a system of a certain number A of identical nucleons whose Hamiltonian is the sum of the kinetic energies of all the particles plus the sum of the two-body interactions between them, i.e.,

$$H = \sum_{i=1}^{A} \frac{\hbar^2}{2m} k_i^2 + \sum_{i<j=1}^{A} v_{ij} = H_0 + H_1$$

The above equation splits H into two parts. The unperturbed Hamiltonian

$$H_0 = \sum_{i=1}^{A} \left(\frac{\hbar^2}{2m} k_i^2 + U_i \right)$$

is the sum of the kinetic energy T and a one-body potential operator U. The perturbation

$$H_1 = \sum_{i<j=1}^{A} v_{ij} - \sum_{i=1}^{A} U_i$$

is what is left over.

CAVEATS

- The introduction of the single-particle potential U (auxiliary potential) is intended to make numerical calculation easier. Since the total Hamiltonian does not involve U, the final result should in principle be independent of U. However, the energy is to be calculated as an expansion in powers of H_1, and the expansion will converge more rapidly for some choices of U than for others. Thus we must try to choose U in such a way that the energy expansion converges rapidly enough to be useful for practical calculations.

- Ordinary perturbation theory cannot be used in its commonly used form for nuclear calculations because the strong short-range repulsion in the NN potential makes all the matrix elements very large, and the series cannot converge.

The strong short-range repulsion causes a similar difficulty in the problem of NN scattering. If one calculates the scattering matrix T to first order in V (Born approximation), then one obtains a large and inaccurate result. But if one calculates to all orders in V (two-particle Schroedinger eq.), then one obtains the correct result.

$$T = V + V\frac{1}{H_0 - E + i\varepsilon}T \;; \quad H = H_0 + V$$

$$G_0 = (H_0 - E + i\varepsilon)^{-1}$$

$$T = V + VG_0V + VG_0VG_0V + \ldots\ldots\ldots\ldots$$

The Bethe-Goldstone equation

The procedure followed for nuclear matter is analogous to the treatment of NN scattering. All terms in the expansion of the Hamiltonian are rearranged in such a way that each matrix element of V is replaced by an infinite series which takes account the two-body interaction to all orders of the potential.
The quantity that replaces the two-body potential V is called the reaction matrix G; and calculating the reaction matrix is equivalent to solving a Schrodinger equation which describes the scattering of two particles in the presence of all the others. The G-matrix is well-behaved even for a singular two-body force, all terms in this new perturbation series are finite and of reasonable size.

$$G(\rho;\omega) = V + \sum_{k_a k_b} V \frac{|k_a k_b\rangle Q \langle k_a k_b|}{\omega - e(k_a) - e(k_b)} G(\rho;\omega)$$

ω=starting energy

single-particle energy

$$e(k;\rho) = \frac{k^2}{2m} + U(k;\rho)$$

$$U(k;\rho) = \mathrm{Re} \sum_{k' \leq k_F} \langle kk' | G(\rho;\omega) | kk' \rangle_a$$

Binding energy

$$\boxed{\frac{E}{A}(\rho) = T_0 + D_2 + D_3 + \ldots. D_n}$$

$$T_0 = \frac{3}{5}\frac{k_F^2}{2m}$$

D_n = contribution of all diagrams with n-body correlations

Keeping the two-body correlations, one gets the Brueckner-Hartree-Fock approximation for the binding energy

$$\frac{E}{A}(\rho) = \frac{3}{5}\frac{k_F^2}{2m} + \frac{1}{2\rho}\sum_{k,k'\le k_F} \langle kk' | G[\rho;\omega] | kk' \rangle_a$$

Is the perturbative expansion convergent ? YES !

Phys. Rev. C65, 017303 (2001).

- The contribution of the three body correlations is small.

Diagrammatic technique II : Self-consistent Green's functions (SCGF)

- ✓Elegant method based on the Martin-Schwinger hierarchy of Green's Functions
- ✓More complete treatment of the NN correlations.

EoS of nuclear matter :

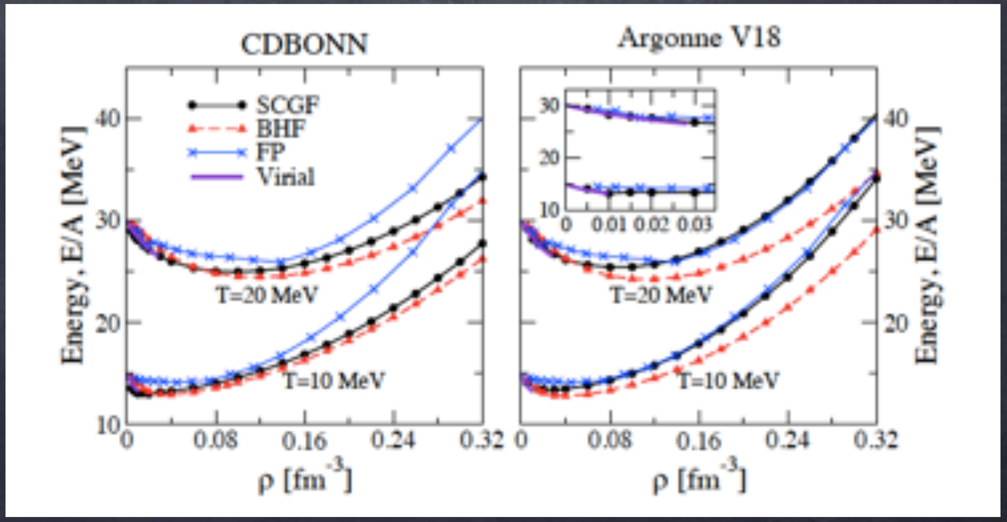
$$\frac{E}{N}(\rho,T)=\frac{\nu}{\rho}\int\frac{d^3k}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{2}\left(\frac{\hbar^2k^2}{2m}+\omega\right)A(k,\omega)f(\omega).$$

Spectral function

$$A(k,w)=\frac{-2Im\Sigma(k,\omega)}{[\omega-\frac{\hbar^2k^2}{2m}-Re\Sigma(k,\omega)]^2+[Im\Sigma(k,\omega)]^2}.$$

Self-energy

Results for hot neutron matter :



Rios, Polls, & Vidana, PRC79 (2009)

More in :

Ramos, Polls & Dickhoff, Nucl. Phys. A 503, 1 (1989)

Muether & Dickhoff, Phys. Rev. C 72, 054313 (2005)

Somà & Boz˙ek, Phys. Rev. C 78, 054003 (2008)

The variational method in its practical form

Pandharipande & Wiringa, 1979; Lagaris & Pandharipande, 1981

The variational method is based on the Ritz's principle, according to which the expectation value of the Hamiltonian is stationary with respect to variations about the eigenvectors

$$\delta \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0 \qquad \text{for an arbitrary variation } \delta\Psi \text{ of } \Psi$$

In the variational method one assumes that the ground state wave function Ψ can be written in the following form

$$\Psi(r_1, r_2, \ldots) = \prod_{i<j} f(r_{ij}) \Phi(r_1, r_2, \ldots)$$

where Φ is the unperturbed ground state wave function, properly antisymmetrized, and the product runs over all possible distinct pairs of particles.

The correlation factor $f(r_{ij})$ is here determined by the variational principle, i.e. by assuming that the mean value of the Hamiltonian gets a minimum (or in general stationary point)

$$\frac{\delta}{\delta f}\frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} = 0$$

This is a functional equation for the correlation function f, which can be expanded in the same spin-isospin, spin-orbit and tensor operators appearing in the NN interaction.

The best known and most used variational nuclear matter EoS is the one by Akmal, Pandharipande, Ravenhall (APR EOS, PRC 58, 1804 (1998))

Quantum Monte Carlo methods

- VMC, GFMC, AFDMC : MC sampling of a probability density

- Variational MC : variational method for the approximation of the g.s. A specific class of trial wave functions is considered, and using Monte Carlo quadrature to evaluate the multidimensional integrals, the energy with respect to changes in a set of variational parameters is minimized.

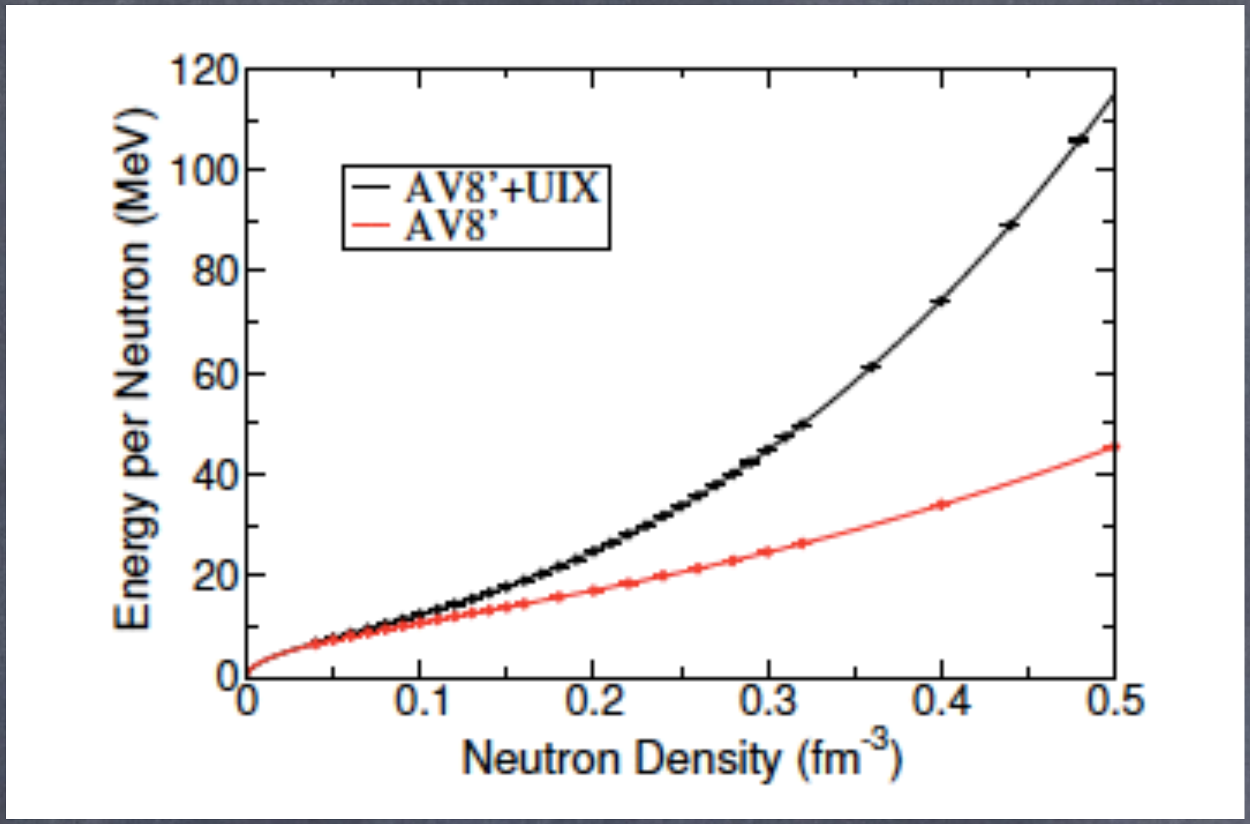
$$\min\left\{\frac{<\Psi|\hat{H}|\Psi>}{<\Psi|\Psi>}\right\} \geq E$$

- GFMC : best when an accurate trial wave function (VM) is available, Very accurate for light nuclei, but increasingly more difficult for larger systems (Exponential growth of the computing time). The largest nuclear GFMC calculations are for the 12C nucleus, and for systems of 16 neutrons.

- AFDMC : extended GFMC to include a diffusion in the spin and isospin states of the individual nucleons. More efficient in treating homogeneous neutron matter It does require the use of simpler trial wave functions -> not yet quite flexible in the treating complex nuclear Hamiltonians.

Advantages : finite nuclei - virtually exact, BUT only local NN potentials

AFDMC EoS for neutron matter



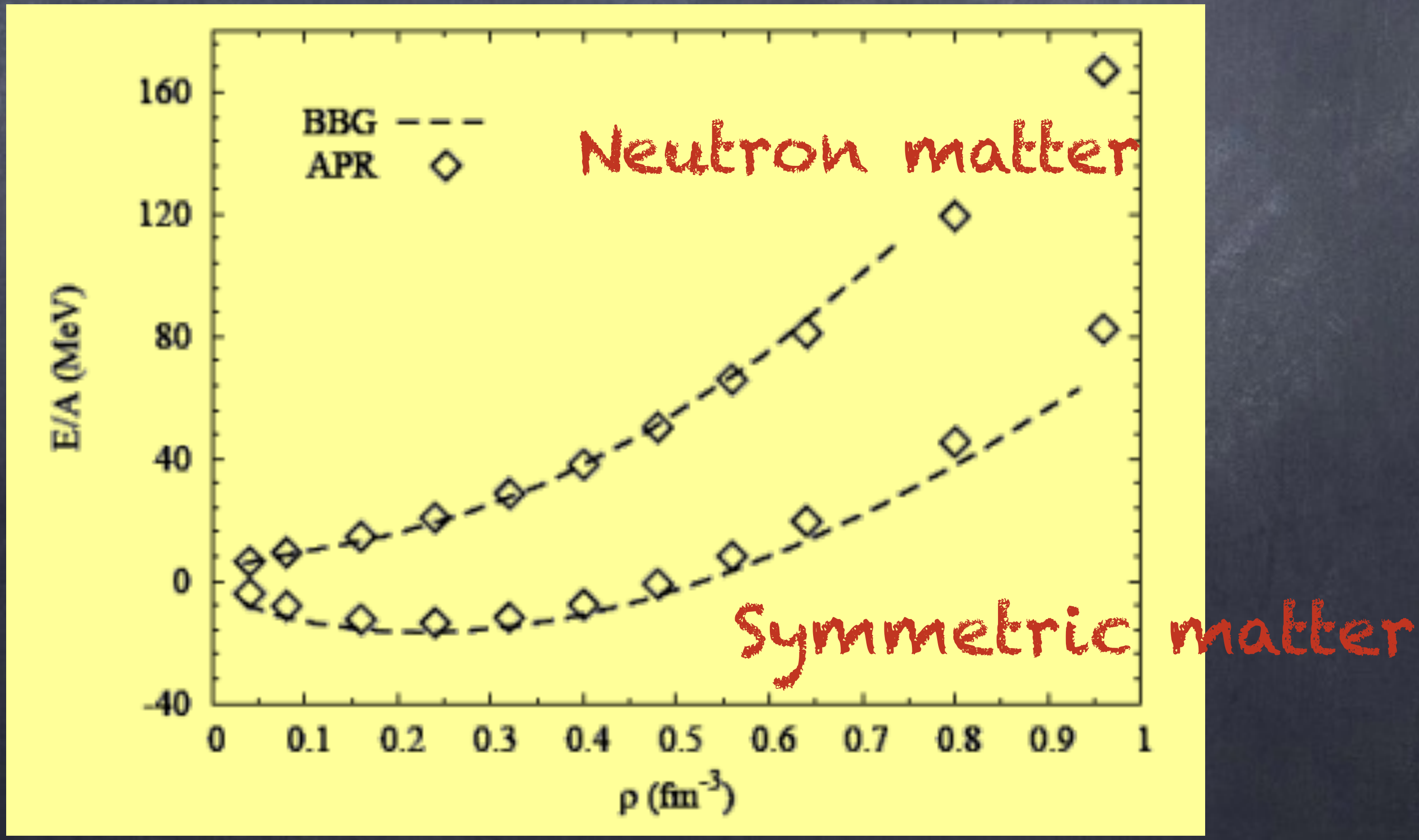
Gandolfi et al, (2014)

The EoS of neutron matter with only 2BF (red line) and including 3BF (black line). Results obtained with Av8' potential.

Dependence on the many-body scheme: BHF vs. APR

- For the full interaction (Av18) good agreement between var. and BBG up to 0.6 fm-3 (symmetric and neutron matter).

- The many-body treatment of nuclear matter EOS can be considered well understood up to density below 0.6 fm^{-3}



The main differences between BBG and variational method:

a) In BBG the kinetic energy contribution is kept at its unperturbed value at all orders of the expansion, while all correlations are embodied in the interaction energy part. In the variational, both kinetic and interaction parts are directly modified by the correlation factors.

b) No single particle potential is introduced in variational. In BBG the s.p. potential is introduced in the expansion and improves the rate of convergence.

At two-body level, both methods give quite similar results.

All non-relativistic many-body methods fail to reproduce the correct saturation point.

Three-body forces need to be included.

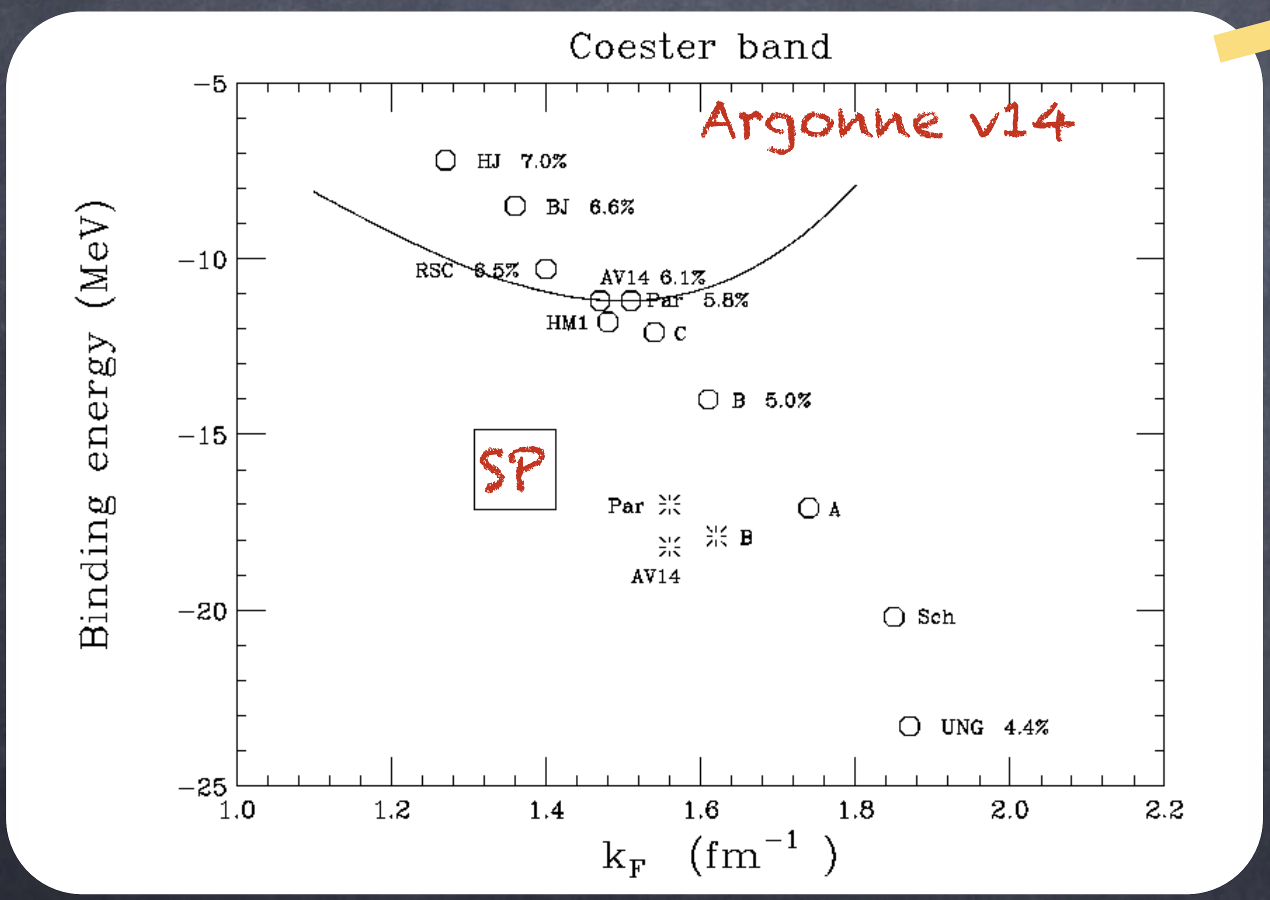
They must allow to reproduce "reasonably well" also the data on three and four nucleon systems.

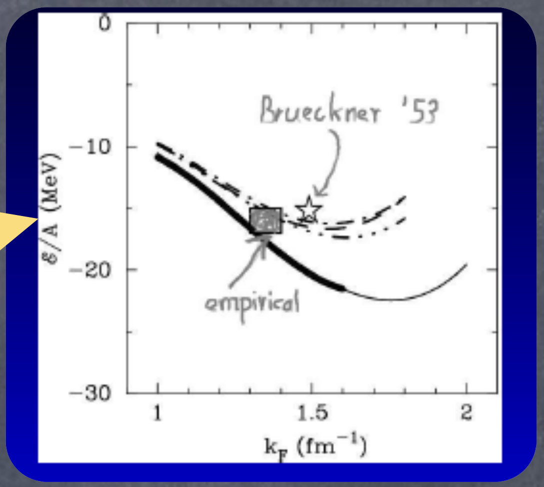
They must be consistent with the two-body force adopted . Only partially explored !

Missing the saturation point ...

Systematics by R. Machleidt,
Adv. Nucl. Phys. 1989

Coester et al.,
Phys. Rev. C1, 769 (1970)





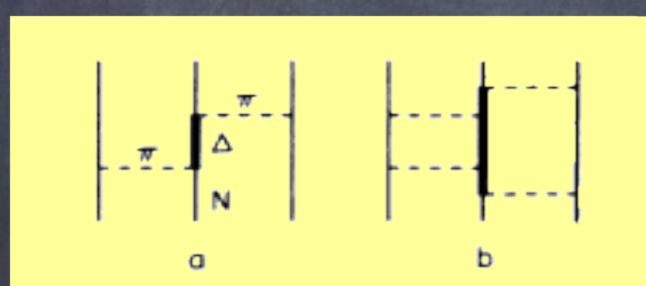
- Results depend on the adopted NN potential.
- The saturation point is missed even including the 3hl.

Role of TBF's on the saturation point

- No complete theory available yet .
- Compare phenomenological and microscopic approaches.

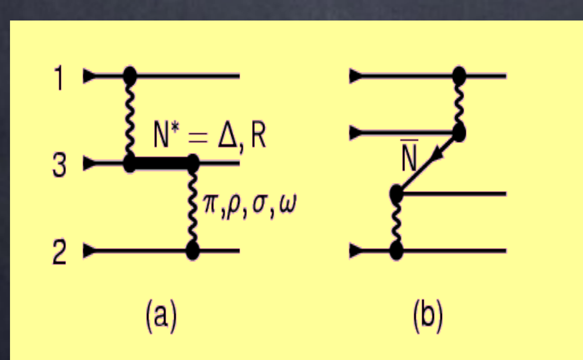
Urbana IX model

Carlson et al., NP A401,(1983) 59

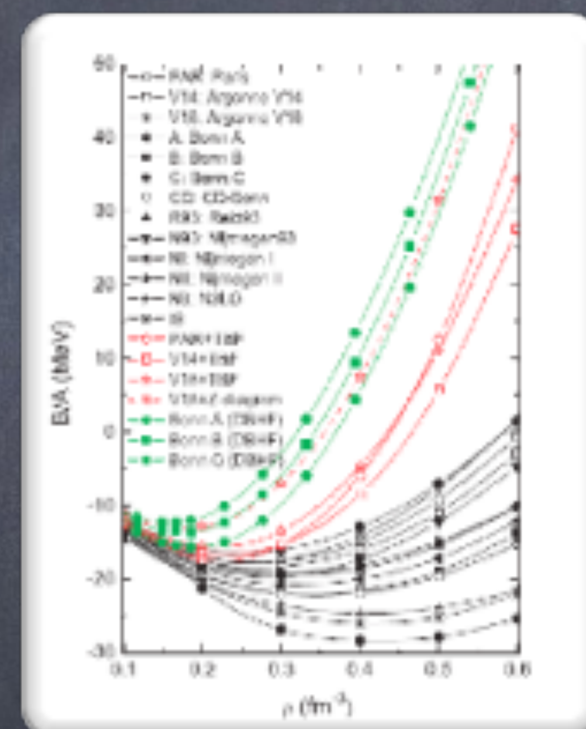


Microscopic model

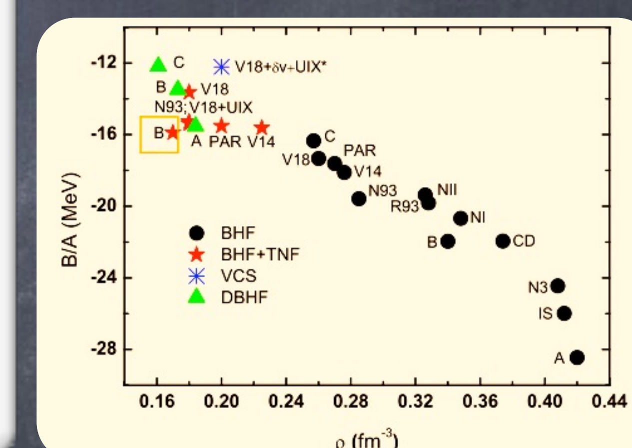
P. Grange' et al, PR C40, (1989) 1040



Z.H. Li, U. Lombardo, H.-J. Schulze, W. Zuo,
PRC 74, 047304 (2006)



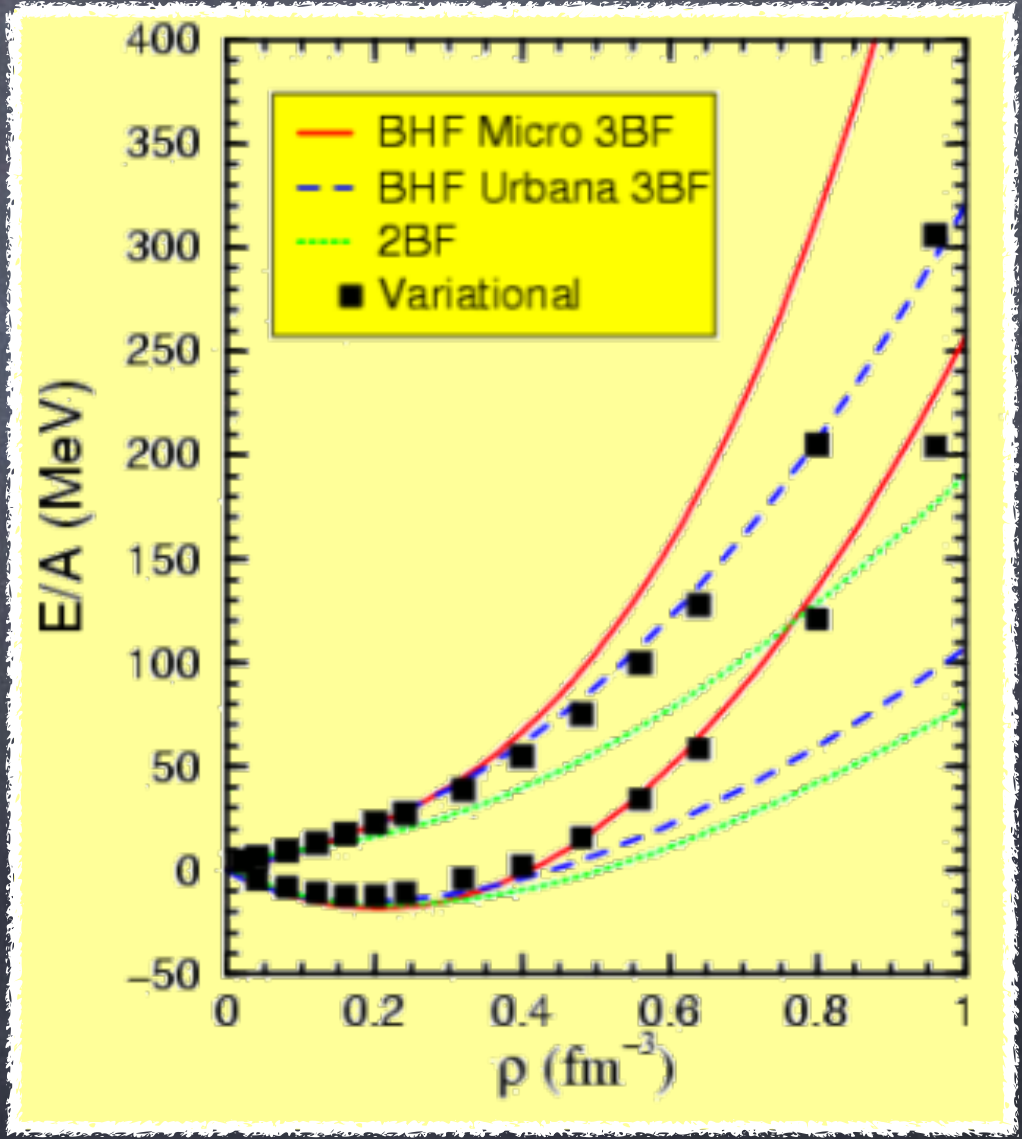
New Coester band



- ✳ TBF needed to improve saturation point.
- ✳ Dependence on NN potential.
- ✳ Uncertain high-density behaviour due to unknown TBF.

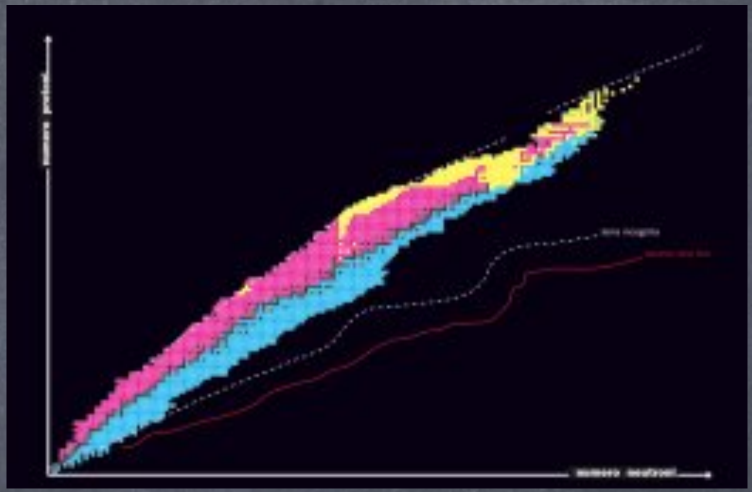
Including TBF's and comparing up to high density

- TBF's parameters fitted either to NM saturation point or to finite nuclei g.s.
- TBF's are different in either methods.
- Good agreement in SNM up to 0.4 fm-3
- Large discrepancy at the high density typical of a NS core.



Compare to experimental and observational data

The EoS : where do we stand ?



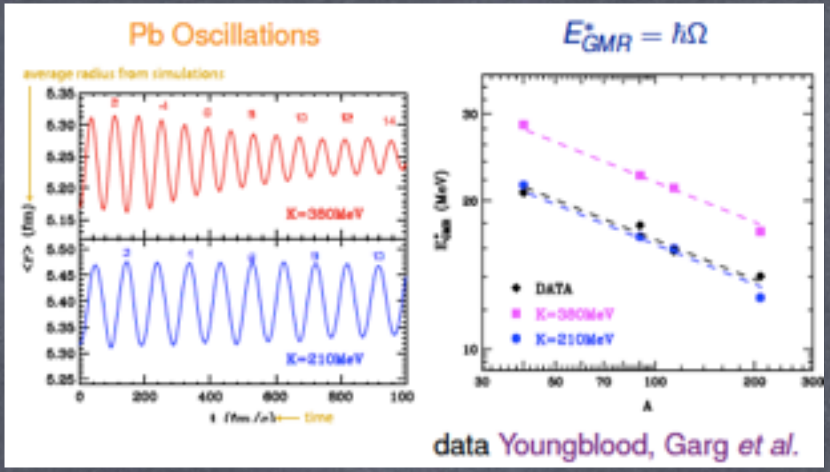
- Structure properties known for about 3339 nuclides
- Binding energy in the Liquid Drop Model
- Extrapolating the mass formula for A -> ∞ in the symmetric case, the binding energy close to saturation is usually expanded as

$$
\begin{aligned}
\frac{E}{A}(\rho,\beta) &= E_0 + \frac{1}{18}K_0\epsilon^2 + \left[S_0 - \frac{1}{3}L\epsilon + \frac{1}{8}K_{sym}\epsilon^2\right]\beta^2 \\
\beta &= \frac{\rho_n - \rho_p}{\rho}, \qquad \epsilon = \frac{\rho - \rho_0}{\rho_0}
\end{aligned}
$$

Close to saturation ...

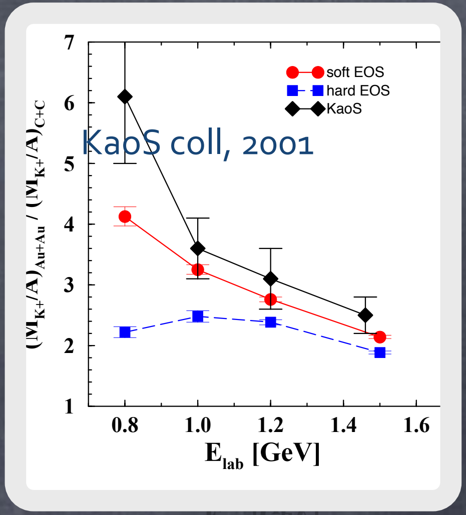
★ $E_0 \approx -16$ MeV

★ $K_0 = k_F^2 \left(\dfrac{d^2 E/A}{dk_F^2} \right)_{\rho_0} \approx 230 \pm 20$ MeV



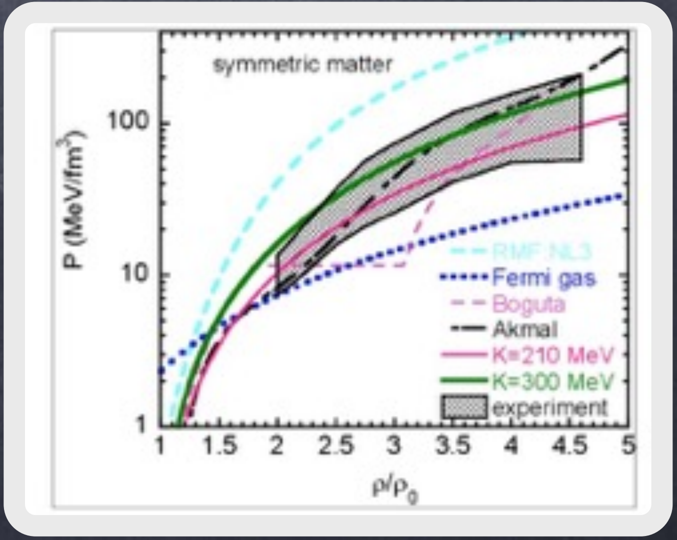
Results confirmed by expt. on K^+ production in HIC

- Largest density explored : $\rho \approx$ 2-3 ρ_0
- Only calculations with a compression $180 \leq K_0 \leq 250$ MeV can describe the data (Fuchs, 2001)



... and by collective flow data

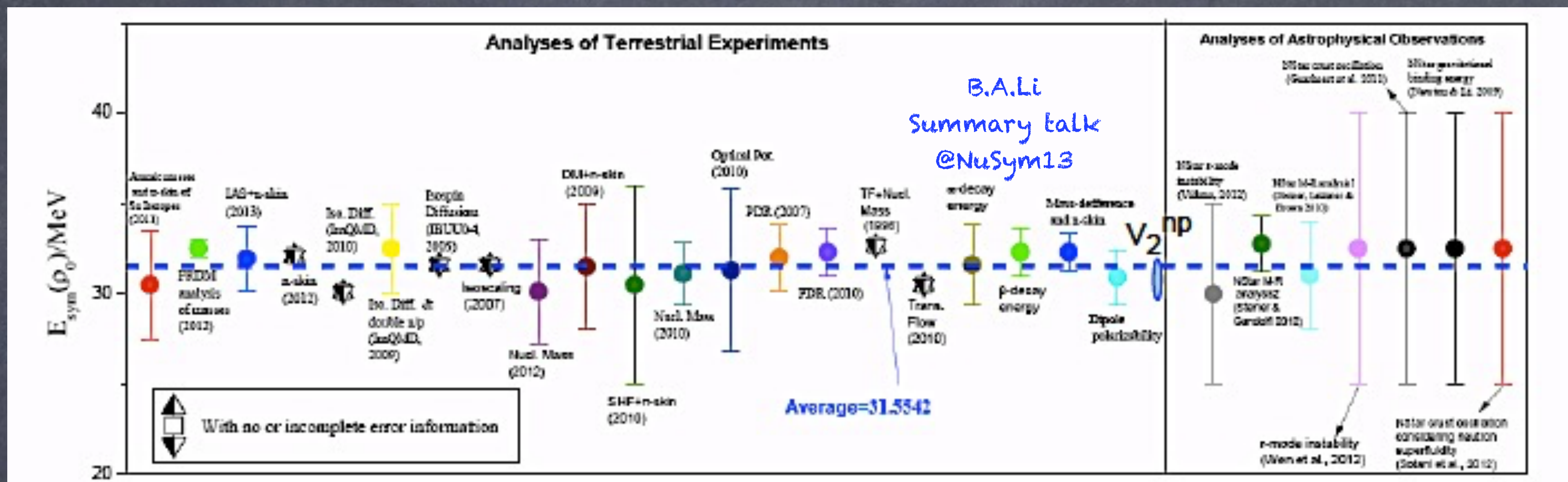
- Transverse flow measurements in Au + Au collisions at E/A=0.5 to 10 GeV
- Flow data exclude very repulsive equations of state



P. Danielewicz, Science 298, 1592 (2002)

The Symmetry Energy S₀

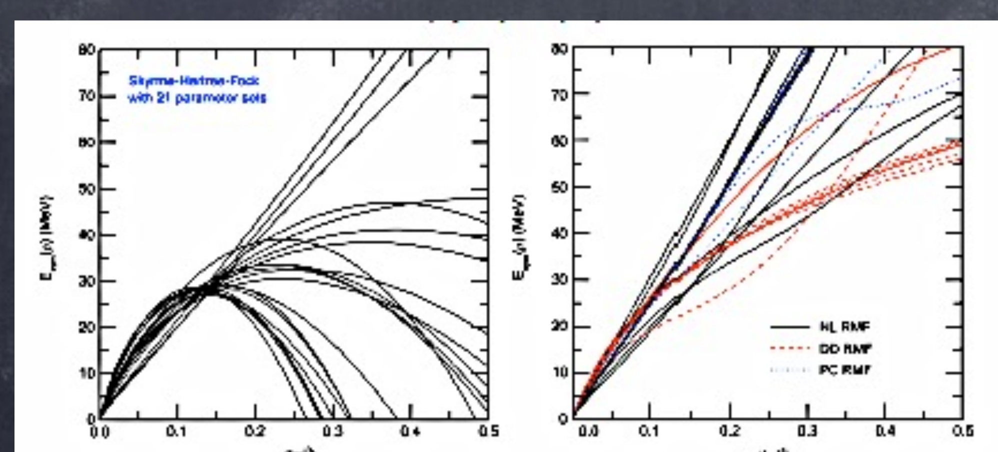
$$S_0 = \frac{1}{2}\frac{\partial^2 E/A}{\partial \beta^2}\Big|_{\beta=0}$$



The key problem is its density dependence

Ex: 21 Skyrme forces vs RMF

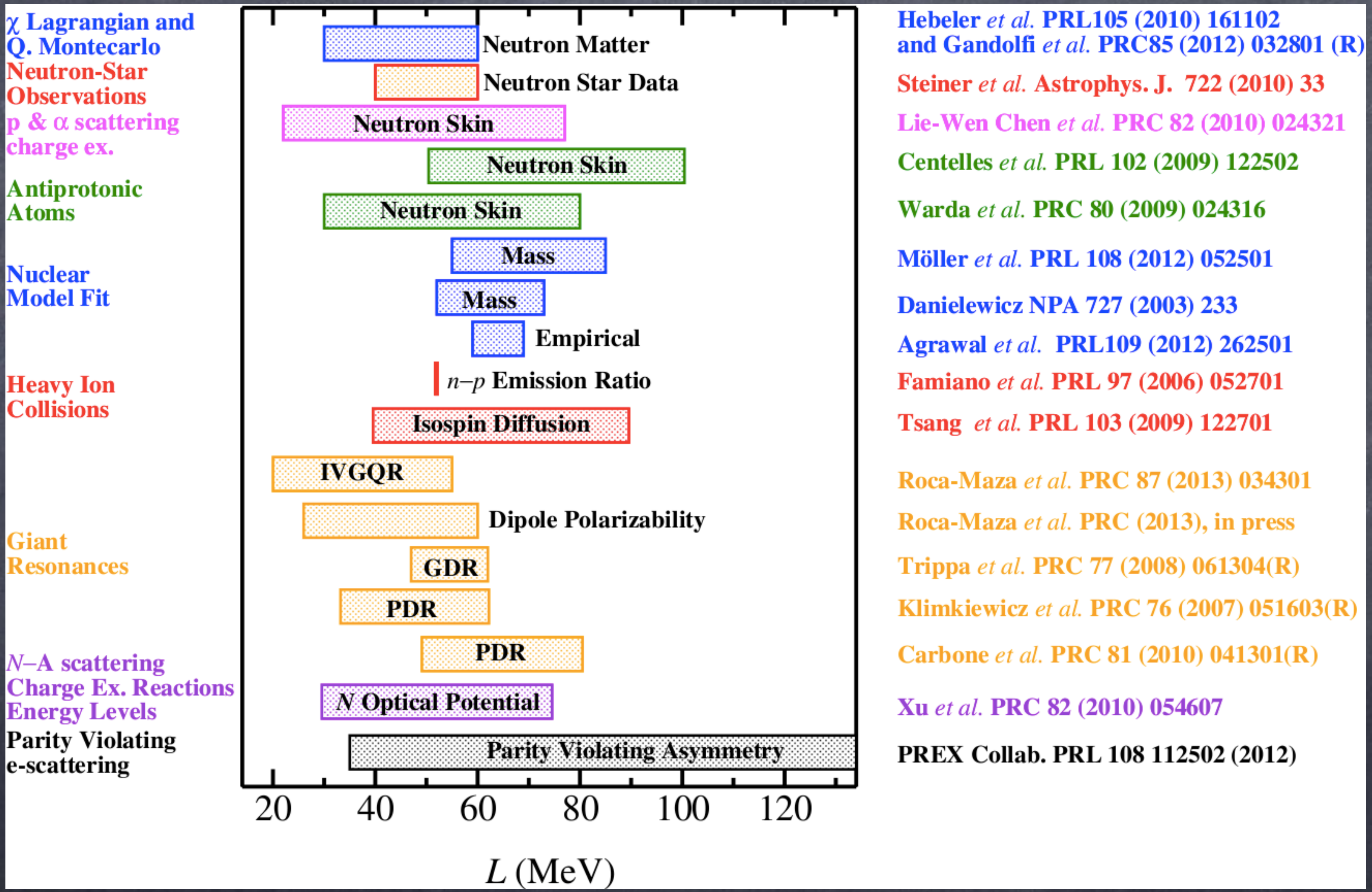
Low density : Sensitivity to observables dependent on the N/Z asymmetry



High density : a few experimental probes.

The slope parameter L

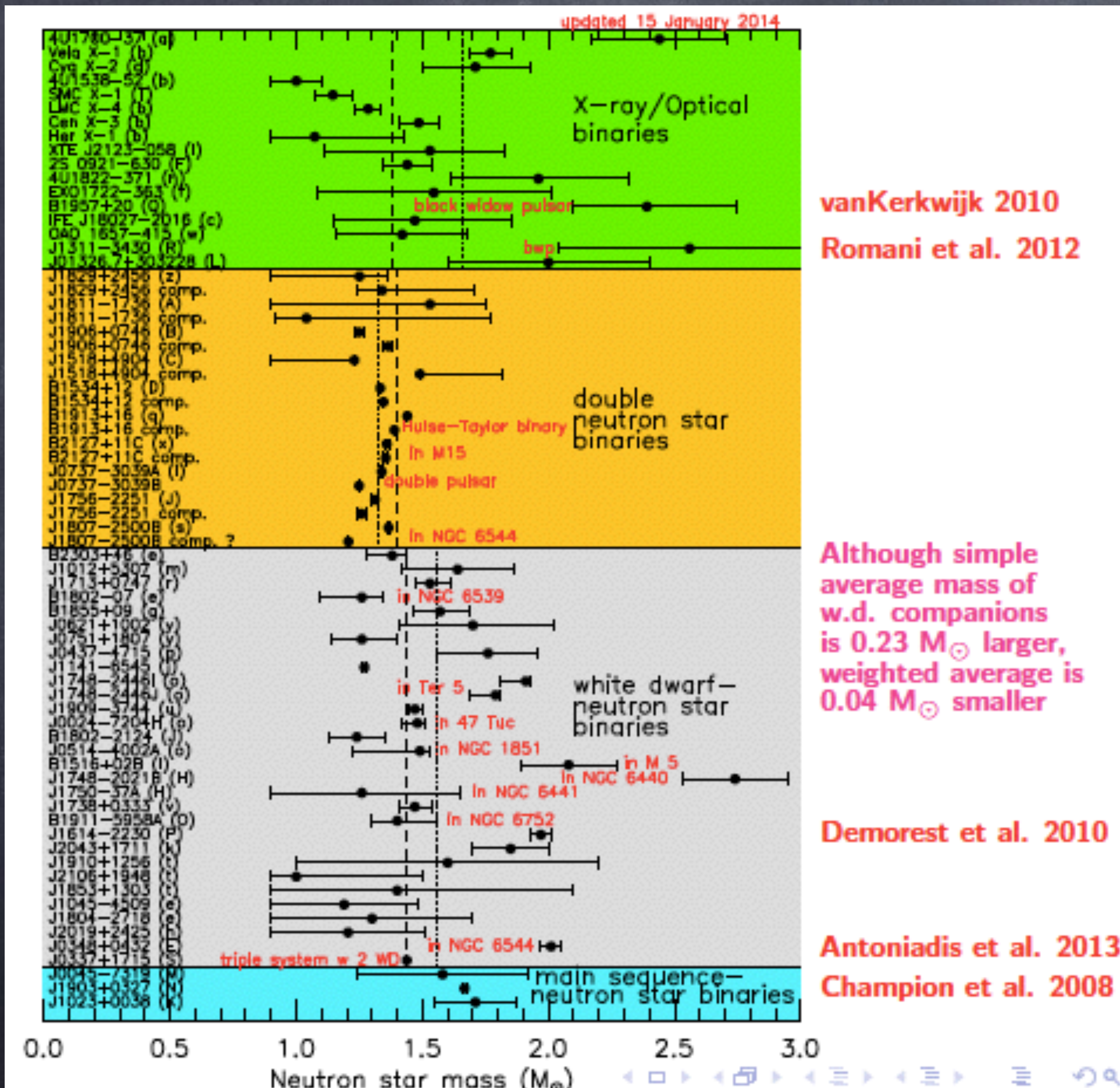
$$L = 3\rho_0 \frac{dS_0}{d\rho}\Big|_{\rho_0}$$



Taken from
X. Vinas et al
EPJA 50:27 (2014)

$44 < L < 68$ MeV

Mass measurements



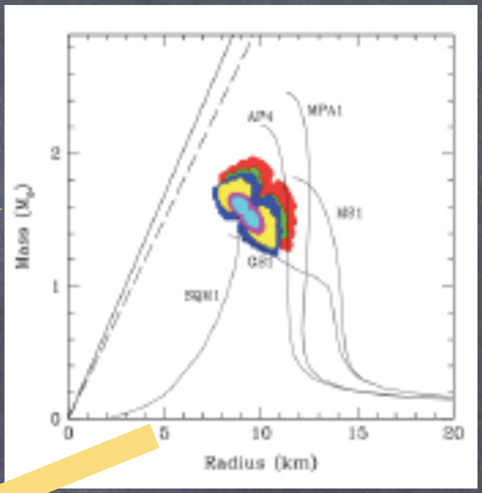
- Several soft EOS are excluded !

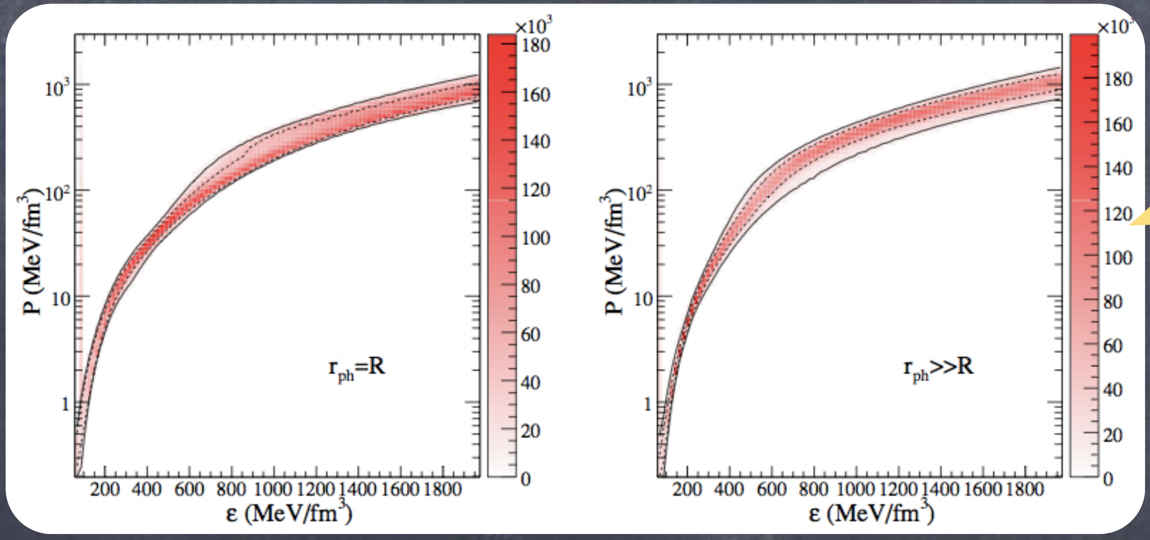
Compilation by J. Lattimer

EoS from astrophysical observations

TOV inversion to get model independent EoS

- 3 type-I X-ray bursters (F. Ozel (2009,2010))
- 3 transient low-mass X-ray binaries
- Cooling of RX J1856-3754





Steiner, Lattimer, Brown, ApJ (2010)

Parametrized EoS

$$
\begin{aligned}
\epsilon &= n_B\Big\{m_B + B + \frac{K}{18}(u-1)^2 + \frac{K'}{162}(u-1)^3 \\
&+ (1-2x)^2\Big[S_k u^{2/3} + S_p u^{\gamma}\Big] + \frac{3}{4}\hbar c x(3\pi^2 n_b x)^{1/3}\Big\}
\end{aligned}
$$

Table 3. Prior limits for the EOS parameters

Quantity	Lower limit	Upper limit
K (MeV)	180	280
K' (MeV)	-1000	-200
S_v (MeV)	28	38
γ	0.2	1.2
n_1 (fm^{-3})	0.2	1.5
n_2 (fm^{-3})	0.2	2.0
ε_1 (MeV fm^{-3})	150	600
ε_2 (MeV fm^{-3})	ε_1	1600

P. Danielewicz, Science 298, 1592 (2002)

Microscopic EoS fit well phenomenological data of heavy ion collisions.

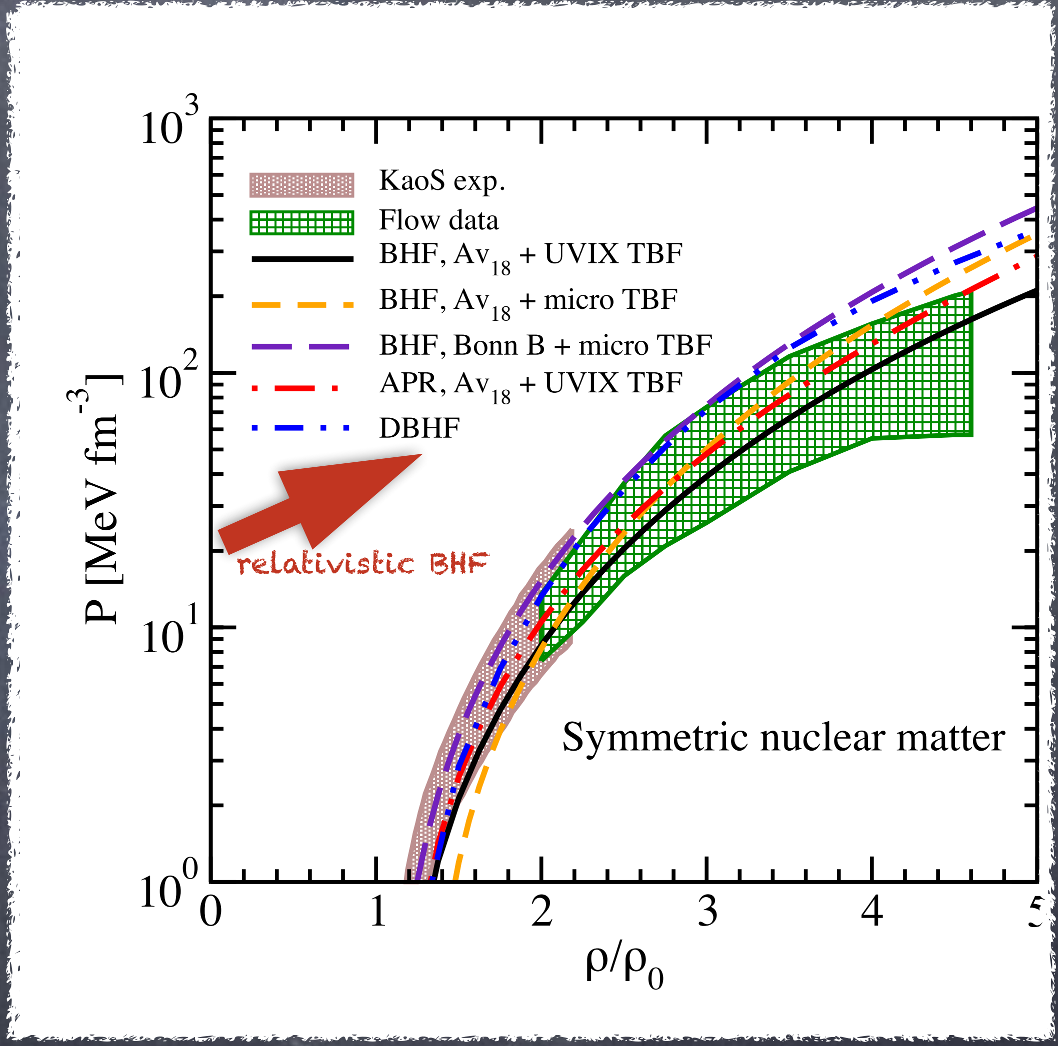
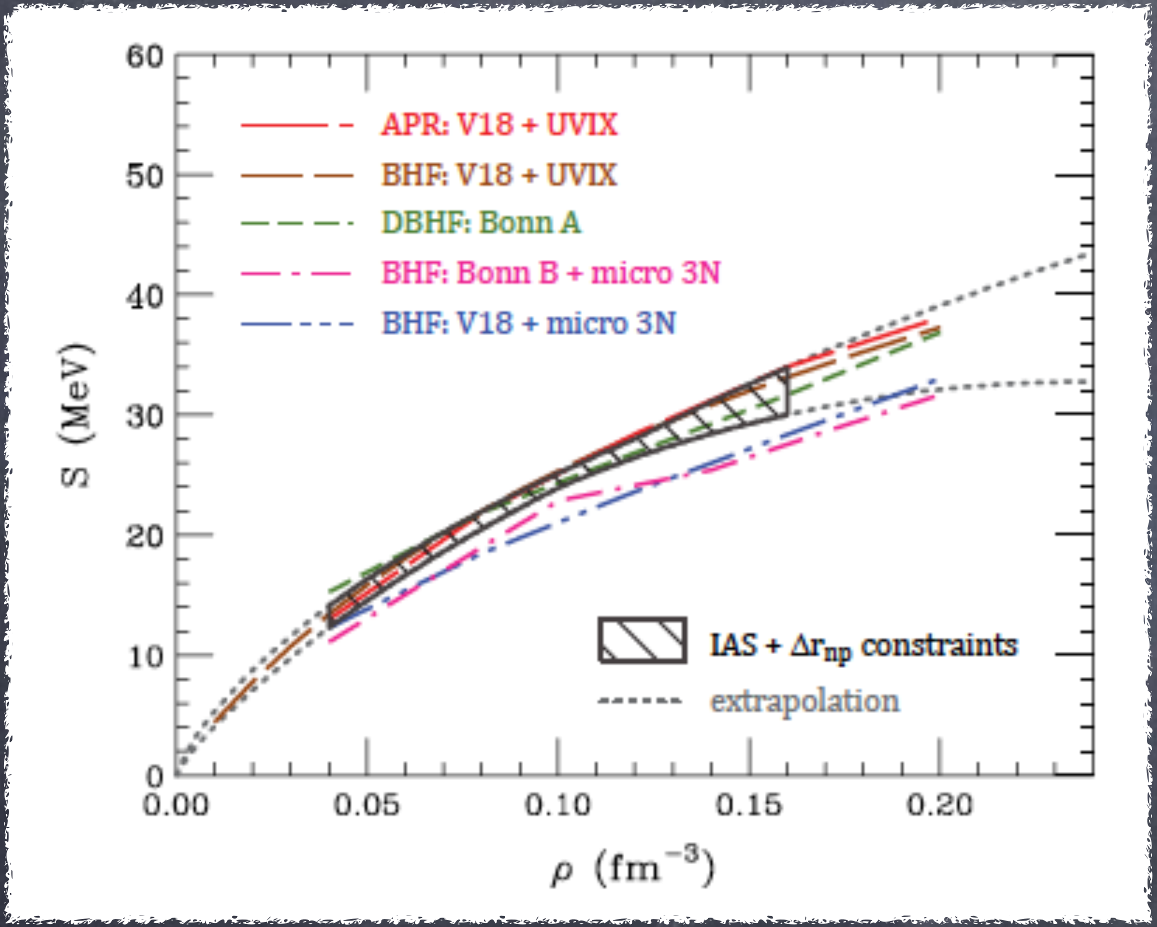
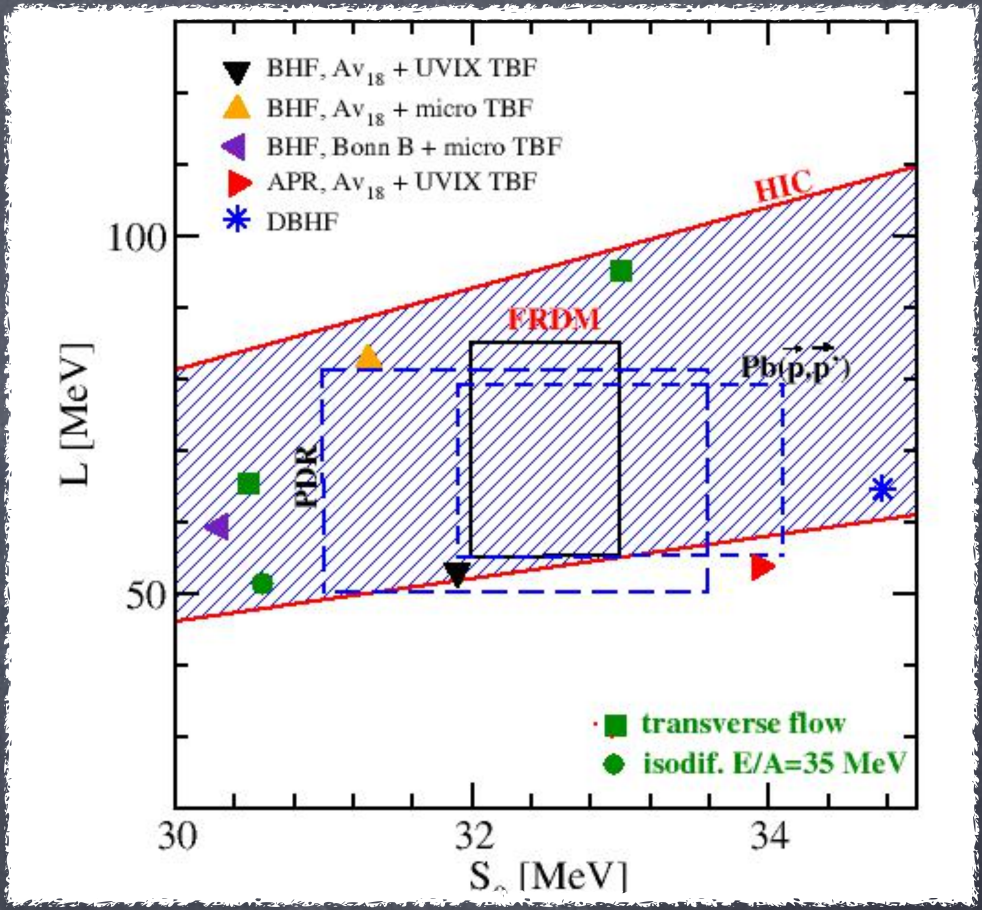


TABLE I. Calculated properties of symmetric nuclear matter.

EoS	ρ_0 (fm^{-3})	$\frac{E}{A}$ (MeV)	K_0 (MeV)	S_0 (MeV)	L (MeV)
BHF, Av_{18} + UVIX TBF	0.16	−15.98	212.4	31.9	52.9
BHF, Av_{18} + micro TBF	0.2	−15.5	236	31.3	82.7
BHF, Bonn B + micro TBF	0.17	−16.	254	30.3	59.2
APR, Av_{18} + UVIX TBF	0.16	−16.	247.3	33.9	53.8
DBHF, Bonn A	0.18	−16.15	230	34.4	69.4

G. Taranto et al., Phys. Rev. C87, 045803 (2013)





Microscopic models compatible with existing data !

Direct URCA processes in NS

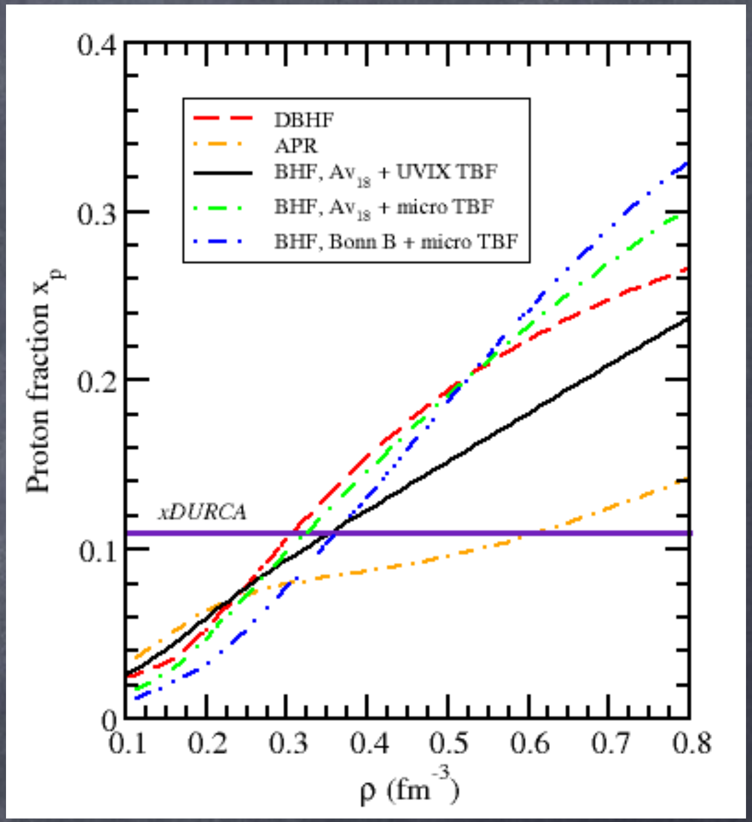
- Direct URCA processes (not considered before 1991) are

$$\boxed{\begin{array}{ll} n \longrightarrow p + e + \overline{\nu}_e \,, & p + e \longrightarrow n + \nu_e \,, \\ n \longrightarrow p + \mu + \overline{\nu}_\mu \,, & p + \mu \longrightarrow n + \nu_\mu \,. \end{array}}$$

- They are allowed only at a rather high density at which the proton fraction x_p > 0.11-0.14 (Lattimer et al. 1991).

- If Direct URCA operate, then a non-superfluid NS core cools to 10^9 K in a minute, and to 10^8 K in a year. If they are not allowed, the time scales will be one year and 10^5 years respectively.

- The symmetry energy is crucial for determining the proton fraction.

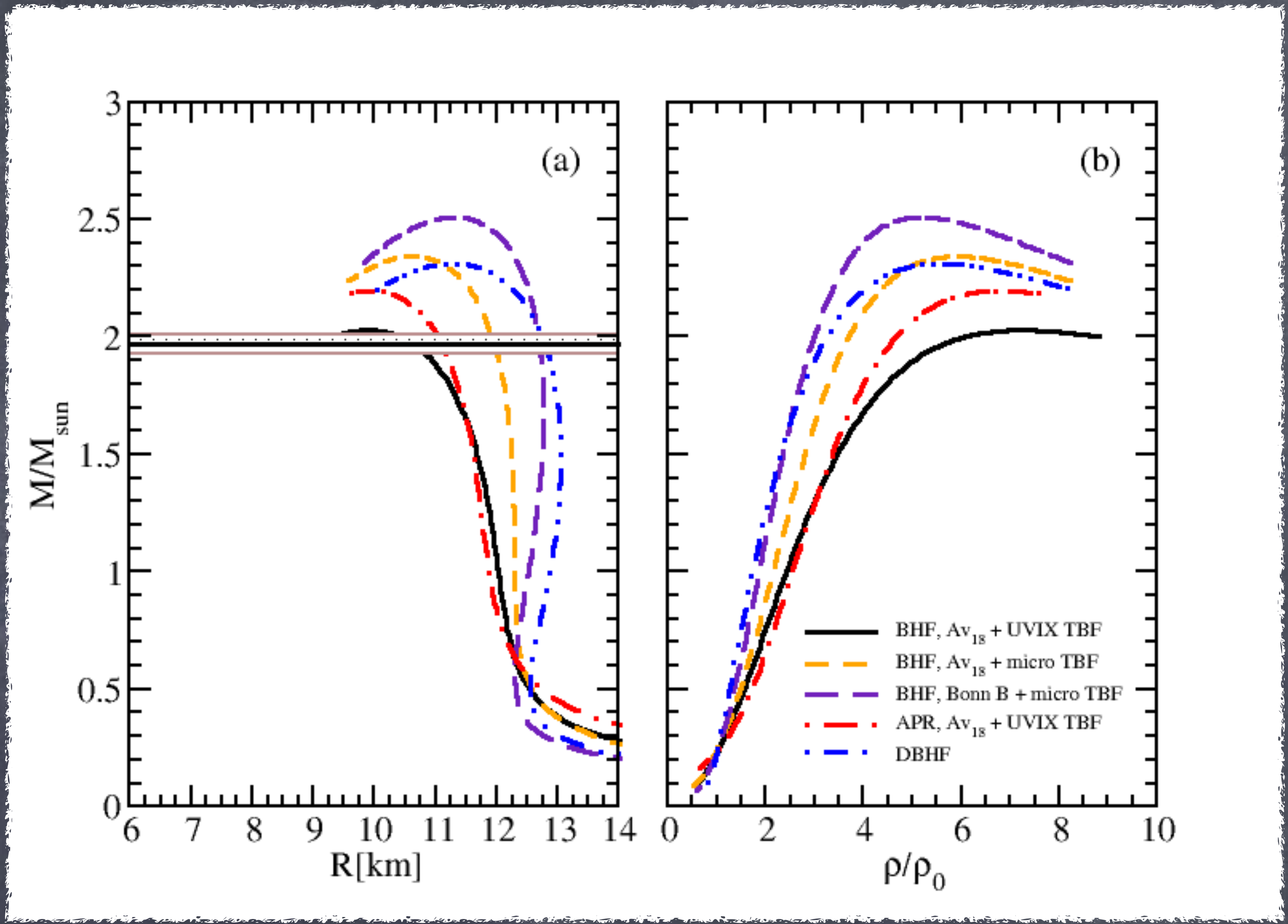


"Recipe" for neutron star structure calculations

- Energy density : $\epsilon\left(\rho_i\right); i = n, p, e, \mu, \Lambda, \Sigma, u, d, s.....$
- Chemical potentials : $\mu_i = \frac{\partial \epsilon}{\partial \rho_i}$
- Beta-equilibrium : $\mu_i = b_i \mu_n - q_i \mu_e$
- Charge neutrality : $\sum_i x_i q_i = 0$
- Composition : $x_i(\rho)$
- Equation of State : $p(\rho) = \rho^2 \frac{d(\epsilon/\rho)}{d\rho}(\rho, x_i(\rho))$
- TOV equations : $\longrightarrow$

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\frac{(\epsilon + P)(1 + 4\pi r^3 P/m)}{1 - \frac{2Gm}{r}}$$

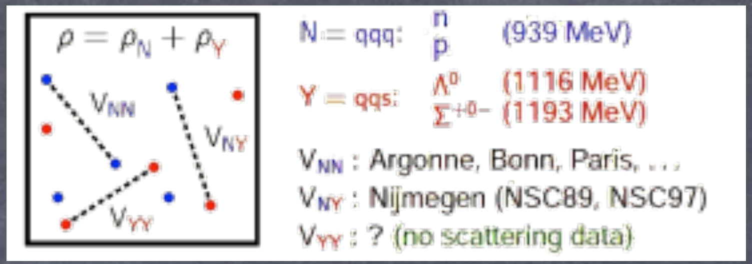
$$\frac{dm}{dr} = 4\pi r^2 \epsilon(r)$$



Neutron Star mass M and radius R

Different many-body techniques and matter compositions predict different results for the M-R relation.

Including hyperons



Extension of the BBG theory.

Several reaction channels involved, more time consuming calculations.

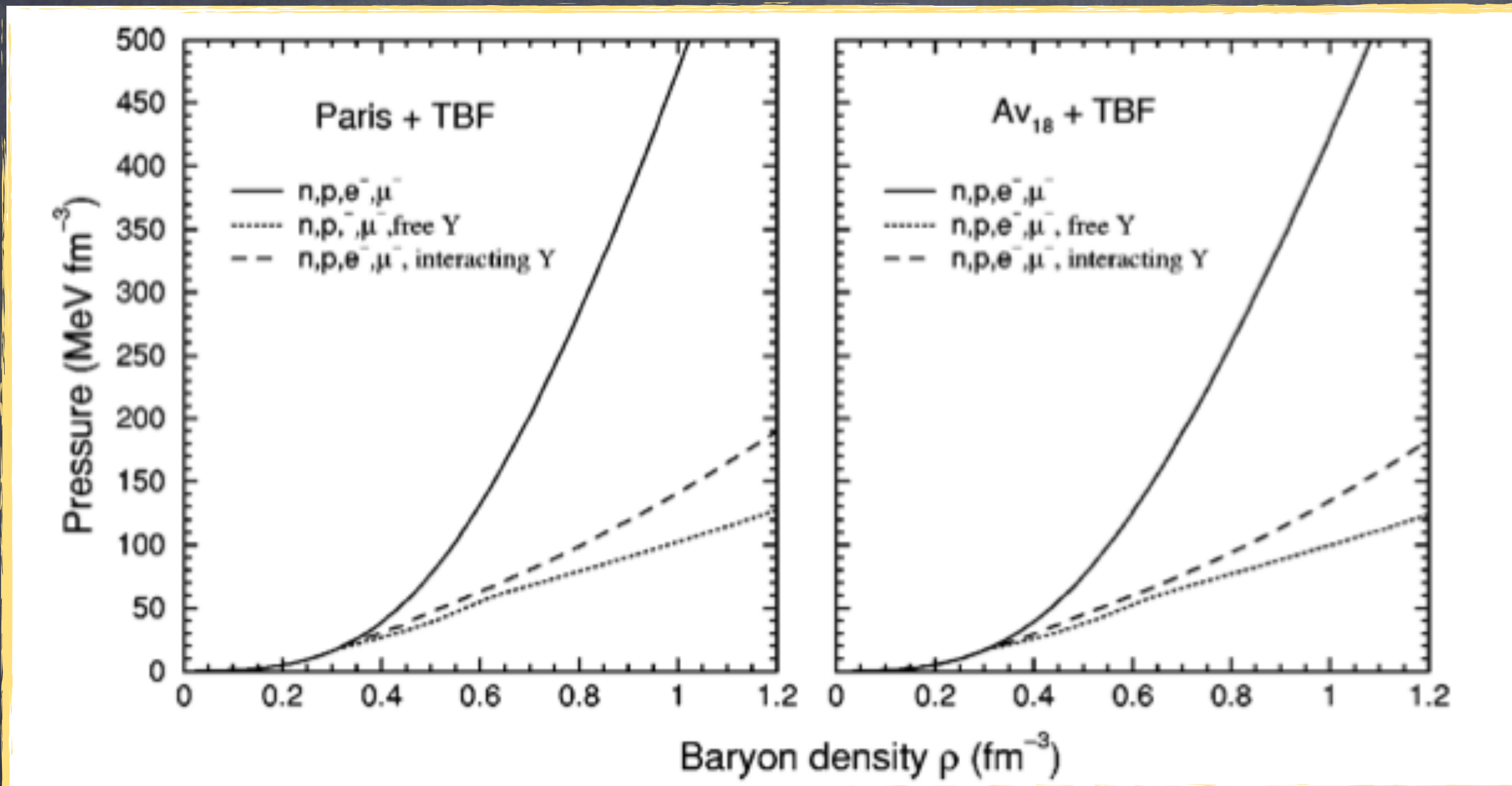
A few experimental data on nucleon-hyperon interaction.

Nijmegen parametrization, Phys. Rev. C40, 2226 (1989) (NSC89)

Unknown HH interaction.

Use of NSC97 and ESC08

Strong consequences for NS structure.

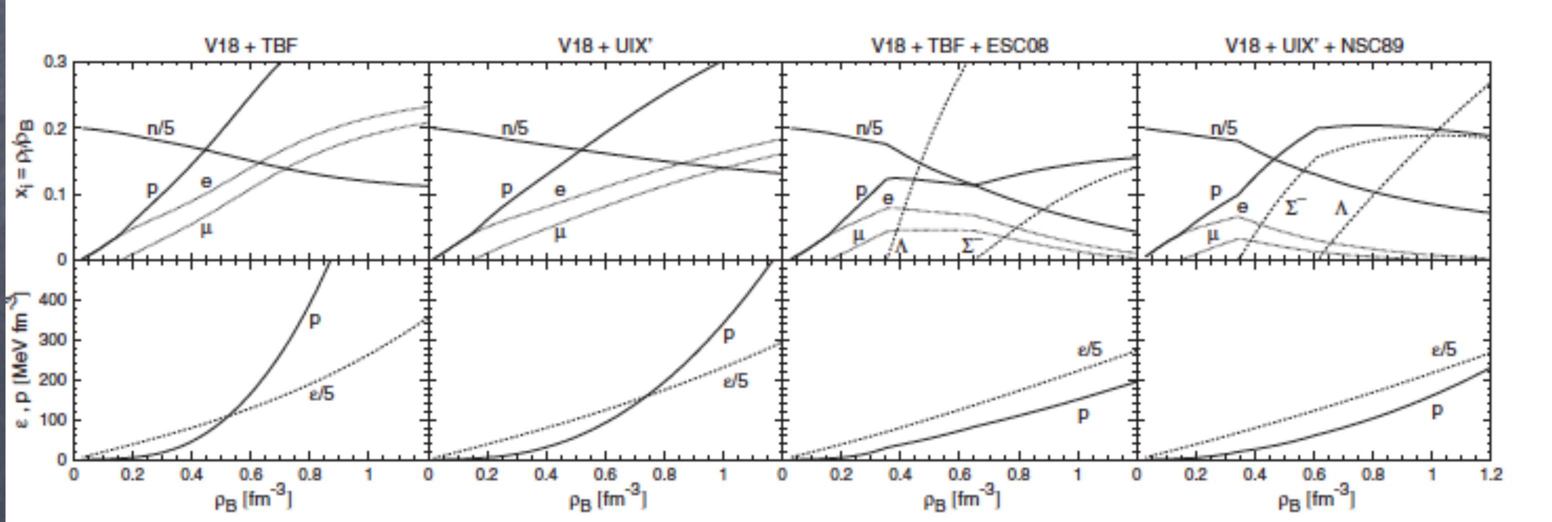


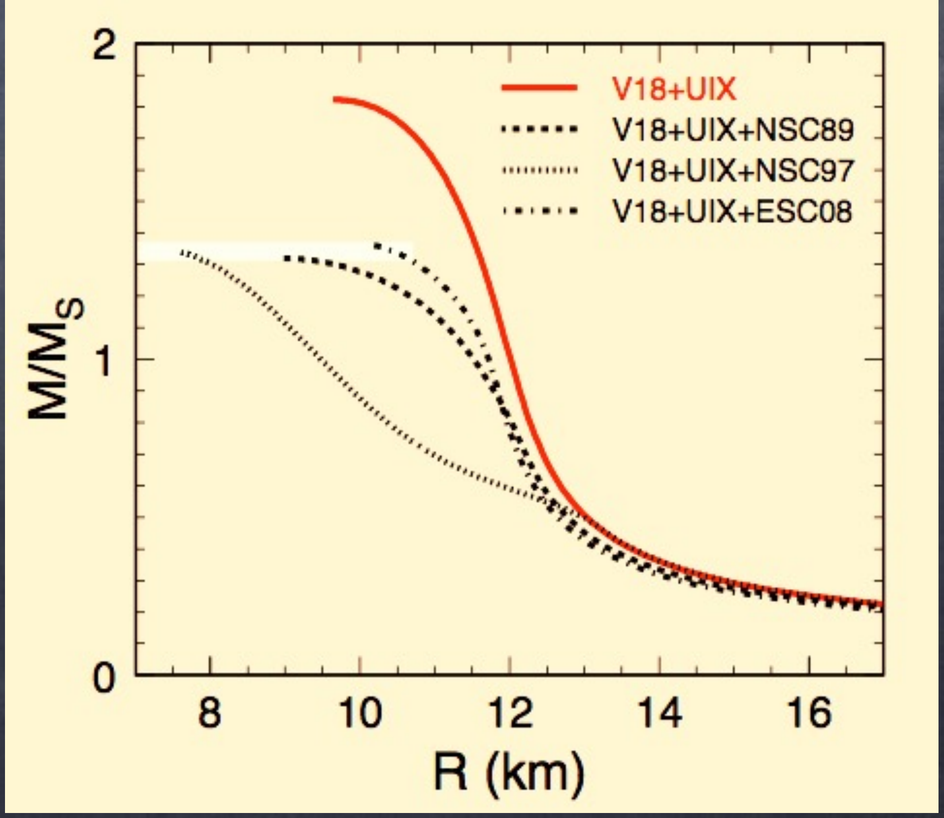
EoS of hyperonic star matter

Strong softening due to hyperons !

Using different NY potentials

- Maximum mass independent of NY potentials.
- Maximum mass too low (< 1.4 Mo !)
- Hyperonic TBF's do not help.





Hyperon puzzle ?!?

Stellar oscillations : EOS and GW emission

Why does a nuclear physicist study stellar oscillations of neutron stars ?

When a neutron star is perturbed by some external or internal event, it can be set into non-radial, damped oscillations, the quasi-normal modes (QNM) which produce GW emission. The detection of the various pulsation modes by GW detectors (AdvLIGO, AdvVirgo) will allow to measure the oscillation frequencies and damping times of the QNM, which carry information on the structure and EoS of a neutron star.

(Andersson & Kokkotas '98; Benhar et al., '04, '07)

1) To infer the value os the star mass M and the radius R

2) To discriminate among different Equations of State

3) Emitting source as NS or Quark star

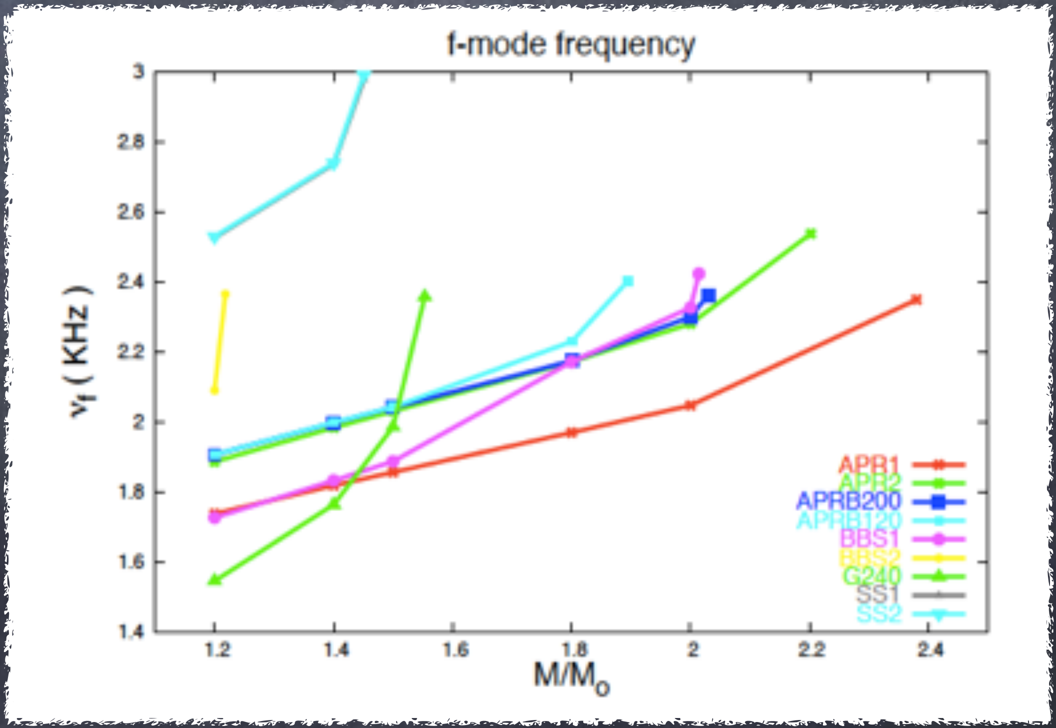
Choose a set of modern EOS :

- APR1, APR2 (n, p, l). Variational method.
- APRB200 (n, p, l) & quark matter (MIT bag)
- BBS1 (n, p, l). Brueckner approach.
- BBS2 (n,p,Σ, Λ, l)
- G240 (n,p,Σ, Λ,Ξ l). RMF model.
- SS1, SS2 (Strange quark matter)

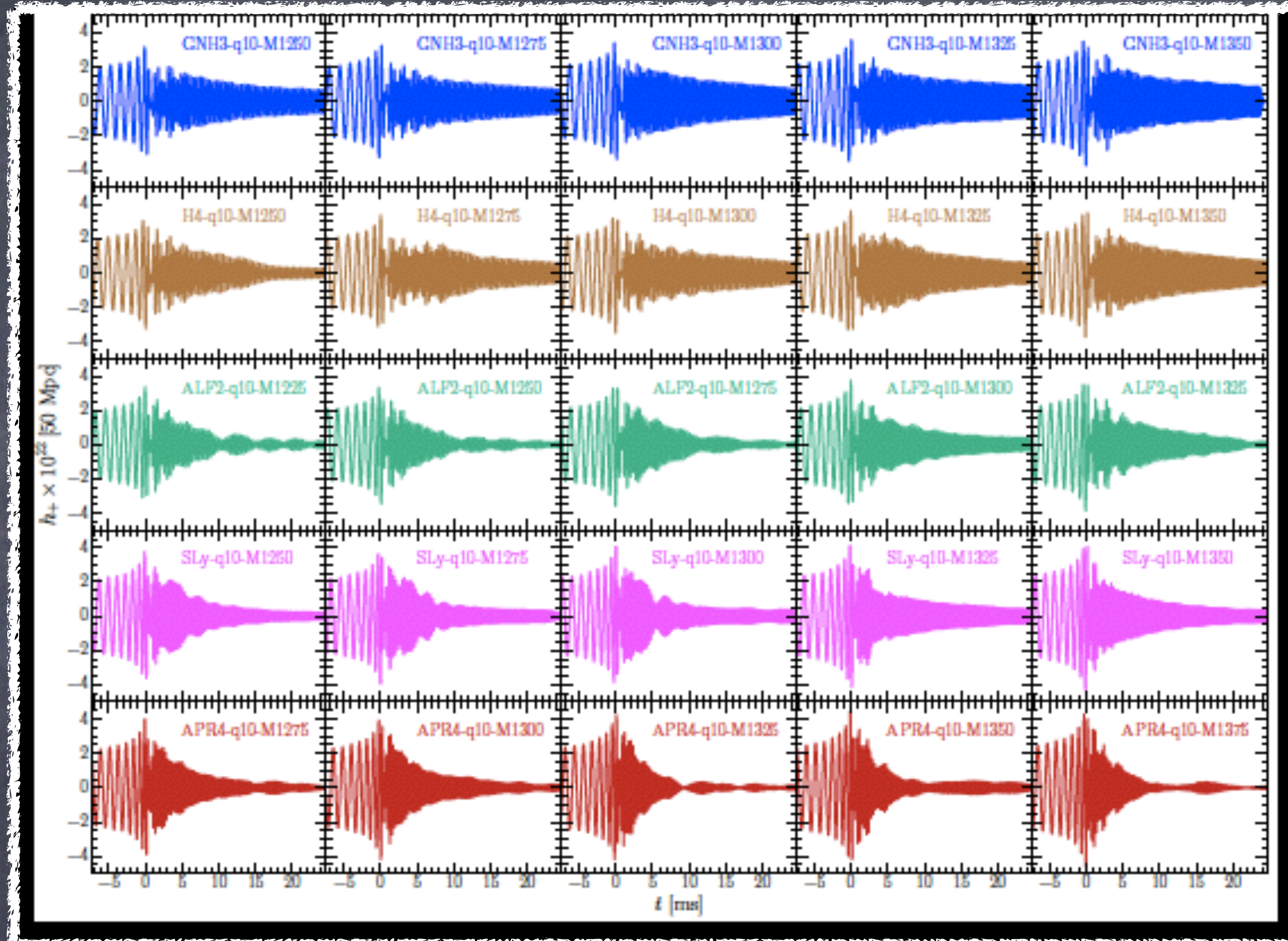
Solve the TOV eqs. for the equilibrium configurations. Find M-R.

Solve the Lindblom-Detweiler eqs. for the QNM. Find frequencies ν and damping times τ.

Empirical relations - ν vs. average density and τ vs. compactness.



from Benhar et al, PRD 70 (2004) 124015



Gravitational waveforms for all binaries with equal masses and nuclear physics EoS.

from Takami et al., PRD 91(2015)064001

Thank you !