Flavour physics 2

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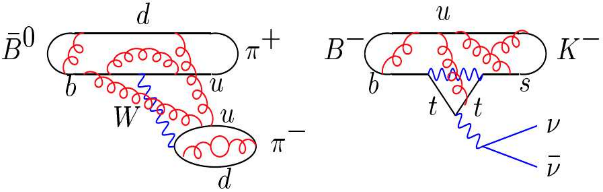
Outline

The QCD challenge



physics of interest: weak quark-transition process
problem: hidden by QCD effects

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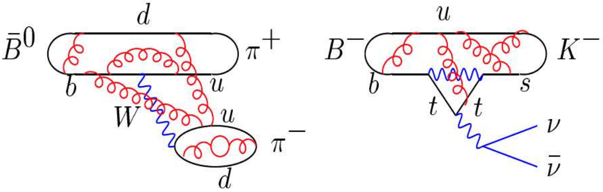

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- large perturbative corrections with strong coupling $\alpha_s(\mu)$ for $\mu \gtrsim m_b$ potentially enhanced by large logs
- non-perturbative hadronic effects
  quark-confinement in hadrons (baryons and mesons)

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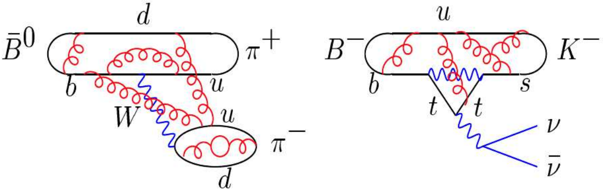

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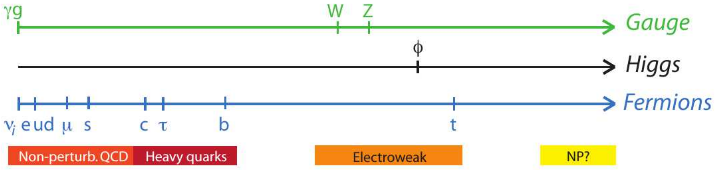
basic strategy:
facorise non-perturbative effects into process-independent decay constants and form factors
$\rightarrow$ to be determined in reference measurements or calculated with non-perturbative methods (lattice QCD, light-cone sum rules, ...)

Separated scales



- QCD corrections involve separated mass scales m_1^2, m_2^2 $\rightarrow$ logarithmic enhancement $\log(m_1^2/m_2^2)$

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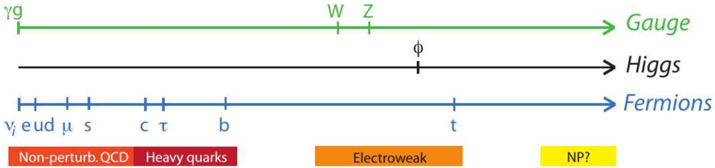
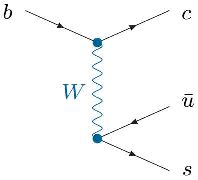


- QCD corrections involve separated mass scales m_1^2, m_2^2
  $\rightarrow$ logarithmic enhancement $\log(m_1^2/m_2^2)$
- construct sequence of effective theories:
  decouple heavier particles by encoding their effects into higher dimensional operators

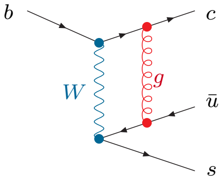
$$\begin{aligned}
\mathcal{L}_{\text{eff}}(q^2 \sim v_{EW}^2) &= \mathcal{L}_{SM} + \sum_{d \geq 5} \frac{1}{\Lambda_{NP}^{d-4}} C_n \mathcal{O}_n(\{\psi_{SM}\}) \\
\mathcal{L}_{\text{eff}}(q^2 \sim m_b^2) &= \mathcal{L}_{QCD}^{5f} + \sum_{d \geq 5} \frac{1}{v_{EW}^{d-4}} C_n \mathcal{O}_n(\{\psi_{QCD}^{5f}\}) \\
\mathcal{L}_{\text{eff}}(q^2 \sim \Lambda_{QCD}^2) &= \mathcal{L}_{HQET} + \mathcal{O}(\Lambda_{QCD}/m_b)
\end{aligned}$$

$b \to c\bar{u}s$: effective theory

LO: $\mathcal{O}(\alpha_s^0)$



NLO: $\mathcal{O}(\alpha_s^1)$



+ further diagrams

$$\mathcal{M}_{\text{LO}} \propto \frac{1}{\underbrace{q^2}_{\mathcal{O}(m_b^2)} - m_W^2}$$

$$\mathcal{M}_{\text{NLO}} \supset \mathcal{M}_{\text{NLO}}^{\text{LL}} \propto \alpha_s \log \frac{M_W^2}{\underbrace{q_i^2}_{\mathcal{O}(m_b^2)}}$$

- hierarchy between scales $q_i^2 \ll M_W^2$:
  large logs $\log(M_W^2/p_i^2)$ spoil perturbative expansion
- solution: effective theory
  decouple heavy scale $M_W^2 \to \infty$

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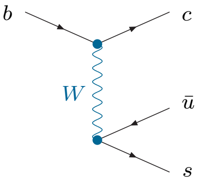

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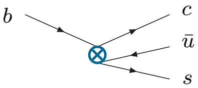

$$\mathcal{M}_{\text{LO}} \propto \frac{1}{m_W^2}$$

- expansion of amplitude in $p_i^2/M_W^2 \ll 1$:
  heavy particle propagator $\to$ point-like interaction
  $\Rightarrow$ heavy particle disapears as dynamical particle (decoupling)

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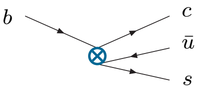

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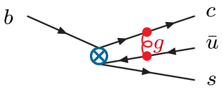

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- expansion of amplitude in $p_i^2/M_W^2 \ll 1$:
  heavy particle propagator $\to$ point-like interaction
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- effective Hamiltonian:

$$\mathcal{H}_{\mathrm{eff}} \propto C_1\,[\bar{c}_L^\alpha \gamma^\mu b_L^\beta][\bar{u}_L^\beta \gamma_\mu s_L^\alpha] + C_2\,[\bar{c}_L^\alpha \gamma^\mu b_L^\alpha][\bar{u}_L^\beta \gamma_\mu s_L^\beta]$$

  C_1, C_2 : Wilson coefficients
  first colour structure induced by QCD corrections

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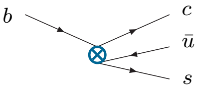

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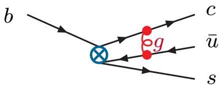

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$$\mathcal{M}_{\text{LO}} \propto \frac{1}{m_W^2}$$

$$\mathcal{M}_{\text{NLO}} \supset \mathcal{M}_{UV} \propto \alpha_s \int \frac{d^4q}{2\pi^4} \frac{q^\mu q^\nu}{(\cancel{q^2} - M_W^2)(q^2)^3}$$

log-divergence for $q \to \infty$

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- additional UV divergences in effective theory compared to full theory

mass m in homogenous gravitational field

- absolute potential:

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- divergence is consequence of unhandy normalisation

Dimensional regularisation

- perform calculation in $D = 4 - 2\epsilon$ space-time dimensions
- integral converges for suitable choice of ϵ
  $\rightarrow$ analytic continuation of the result for arbitrary complex ϵ
- UV divergence appears as $1/\epsilon$ pole
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- dimensional regularisation respects gauge invariance
- $S = \int d^D x \mathcal{L} \quad \Rightarrow \quad \mathcal{L}$ has mass dimension D
  gauge coupling: replace $g \rightarrow \mu^\epsilon g \quad \Rightarrow \quad g$ is dimensionless
  $\Rightarrow$ dimensional regularisation introduces energy scale μ !
- $1:1$ correspondence between $1/\epsilon$ pole and μ dependence
  $\Rightarrow$ amplitude contains piece proportional to

$$\Delta_{UV}(\mu) = \underbrace{\frac{1}{\epsilon} - \gamma_E + \log(4\pi)}_{\equiv \Delta_{UV}} + \log \mu^2$$

Predictions for observables

Effective Lagrangian: $\mathcal{L} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{eff}}(C_1^0, C_2^0)$

- consider n observables $\mathcal{O}_1, ..., \mathcal{O}_n$
- calculate these observables in effective theory up to order α_s^k:

$$\mathcal{O}_1^{\text{th}} = \mathcal{O}_1^{(k)}(C_1^0, C_2^0), \quad ... \quad , \mathcal{O}_n^{\text{th}} = \mathcal{O}_n^{(k)}(C_1^0, C_2^0)$$

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$\rightarrow \widetilde{\mathcal{O}}_i^{(k)}$ UV finite functions of $\mathcal{O}_1^{\text{exp}}, \mathcal{O}_2^{\text{exp}}$?

Renormalisability

Renormalisable theory:

Predictions $\widetilde{\mathcal{O}}_i^{(k)}(\mathcal{O}_1^{\mathsf{exp}}, \mathcal{O}_2^{\mathsf{exp}})$ in terms of observables $\mathcal{O}_1^{\mathsf{exp}}, \mathcal{O}_2^{\mathsf{exp}}$ are UV-finite.

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arbitrary order $k = 1, ..., \infty$ in effective couplings C_i:

- new effective couplings $C_i^{(k)}$ have to be introduced at each order k to absorb UV-divergences
- infinite number of $C_i^{(k)}$ to be fixed from measurements

$\Rightarrow$ not renormalisable and not predictive

Phenomenology: fixed order sufficient because higher coefficients are suppressed by higher powers of p_i^2/M_{heavy}

Renormalisation

Renormalisation:
split of bare parameters C_i^0 into a finite part C_i and a counterterm δC_i

$$C_i^0 = C_i + \delta C_i, \qquad \delta C_i = \frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon}\zeta_i^{(1)} + \zeta_i^{(2)}\right)$$

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but: perturbative evaluation
treat C_i as $C_i = \mathcal{O}(1)$ and δC_i as $\delta C_i = \mathcal{O}(\alpha_s)$

$\rightarrow$ dependence on renormalisation scheme:

calculation of $\mathcal{O}(\alpha_s^n)$ $\rightarrow$ scheme dependence of $\mathcal{O}(\alpha_s^{n+1})$

Renormalisation

to first order in effective couplings C_i:

$$\delta C_i = \sum_j \delta Z_{ij} C_j \quad \Rightarrow \quad \vec{C}^0 = Z\vec{C}, \quad \text{with } Z_{ij} = \delta_{ij} + \delta Z_{ij}$$

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UV-divergent amplitudes contain piece

$$\propto \Delta_{UV}(\mu) = \underbrace{\frac{1}{\epsilon} - \gamma_E + \log(4\pi)}_{\equiv \Delta_{UV}} + \log \mu^2$$

$\overline{\text{MS}}$-scheme: subtract only this piece $\quad \rightarrow \quad \delta Z_{ij} = \frac{\alpha_s}{4\pi} z_{ij} \Delta_{UV}$

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predictions for observables cannot depend on artificial scale μ:

- explicit μ-dependence of $\Delta_{UV}(\mu)$ inside renormalised Wilson-coefficients: $\vec{C} = \vec{C}(\mu)$
- in addition: implicit μ-dependence via $\alpha_s = \alpha_s(\mu)$ in $\vec{C}$ and $\delta\vec{C}$
- <u>but:</u> $\vec{C}^0 = \vec{C} + \delta\vec{C}$ is μ-independent

Physical meaning of scale μ

a priori: scale μ is not physical:
cancels order by order in perturbation theory

schematically:

$$\mathcal{M} \supset \sum_i \underbrace{a_i C_i(\mu)}_{\boxed{1}} + \underbrace{\frac{\alpha_s}{4\pi} b_i C_i \log \frac{m^2}{\mu^2}}_{\boxed{2}} + \mathcal{O}(\alpha_s^2)$$

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  $\Rightarrow$ by varying μ contributions can be reshuffled between $\boxed{1}$ and $\boxed{2}$
- for $\mu \sim m$: log in $\boxed{2}$ becomes small
  $\Rightarrow$ dominant NLO effects absorbed into LO result
  $\Rightarrow$ better convergence of perturbative series

Resummation of large logs

amplitude dependending on two separated scales $m_1 \ll m_2$:

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$$= \underbrace{\left[1 + \alpha_s \log \frac{m_1^2}{\mu^2} + \mathcal{O}(\alpha_s^2)\right]}_{\mathcal{M}_1(m_1^2,\mu^2)} \underbrace{\left[1 + \alpha_s \log \frac{\mu^2}{m_2^2} + \mathcal{O}(\alpha_s^2)\right]}_{\mathcal{M}_2(m_2^2,\mu^2)}$$

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strategy:

1. calculate $\mathcal{M}_1$ up to order α_s^n at the scale $\mu_1^2 \sim m_1^2$
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3. calculate $\mathcal{M}_2$ up to order α_s^n at the scale $\mu^2 \sim m_2^2$
   $\Rightarrow$ good convergence of perturbative expansion

$\Rightarrow$ RGE-improved result for $\mathcal{M}$ at order $\alpha_s^n \sum_k \alpha_s^k \log^k(m_1^2/m_2^2)$

Renormalisation group equation

bare couplings do not depend on scale μ:

$$0 = \mu \frac{d}{d\mu} \vec{C}^0 = \mu \frac{d}{d\mu} (Z\vec{C}) = \left(\mu \frac{d}{d\mu} Z\right) \vec{C} + Z \left(\mu \frac{d}{d\mu} \vec{C}\right)$$

$\Rightarrow$ renormalisation group equation (RGE):

$$\left[\mu \frac{d}{d\mu} - \gamma\right] \vec{C} = 0 \qquad \text{with } \gamma \equiv - \left(\mu \frac{d}{d\mu} Z\right) Z^{-1}$$

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anomalous dimension marix γ:

$$\gamma = -\left(\mu \frac{d}{d\mu} Z\right) Z^{-1} = -\underbrace{\left(\mu \frac{da_s}{d\mu}\right)}_{\substack{= \mu \frac{d(\mu^{-2\epsilon} Z_\alpha^{-1} a_s^0)}{d\mu} \\ = -2\epsilon a_s + \mathcal{O}(a_s^2)}} \underbrace{\left(\frac{dZ}{da_s}\right) Z^{-1}}_{= z\Delta_{UV} + \mathcal{O}(a_s)}, \qquad a_s = \frac{\alpha_s}{4\pi}$$

$$= a_s(2z) + \mathcal{O}(a_s^2)$$

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express RGE for $\vec{C}$ in terms of a_s:

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final RGEs for a_s and $\vec{C}$ at leading order (LO):

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perturbative in α_s but exact in $\alpha_s(\mu)/\alpha_s(\mu_0)$!

geometric series:

$$\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} = 1 - \alpha_s(\mu_0)2\beta_0\log\frac{\mu}{\mu_0} + \left(\alpha_s(\mu_0)2\beta_0\log\frac{\mu}{\mu_0}\right)^2 - \ldots$$

$\Rightarrow$ LO RGE resums logs $[\alpha_s\log(\mu/\mu_0)]^k$ to all orders $k = 1, 2, \ldots$

(NLO RGE resums logs $\alpha_s\,[\alpha_s\log(\mu/\mu_0)]^k$ etc.)

Matching

effective theory based on a more fundamental theory:
$\rightarrow$ determine Wilson coefficients from matching to the full theory

effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} \propto C_1\,[\bar{c}_L^{\alpha}\gamma^{\mu}b_L^{\beta}][\bar{u}_L^{\beta}\gamma_{\mu}s_L^{\alpha}] + C_2\,[\bar{c}_L^{\alpha}\gamma^{\mu}b_L^{\alpha}][\bar{u}_L^{\beta}\gamma_{\mu}s_L^{\beta}]$$

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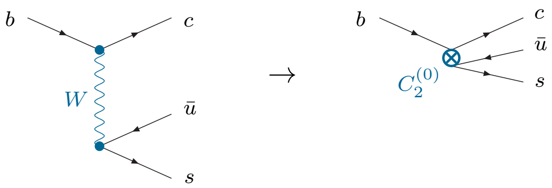
LO: $\mathcal{O}(\alpha_s^0)$



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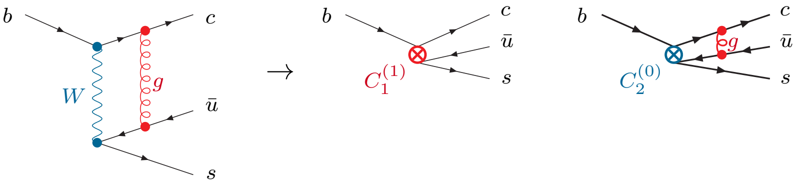

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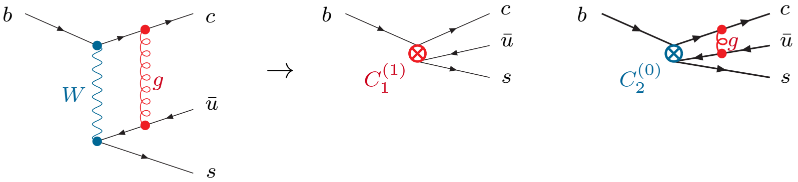
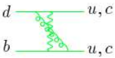


$\mathcal{M}^{\text{LL}}_{\text{full}} \propto \alpha_s \log(M_W^2/q_i^2)$, $\quad\quad \mathcal{M}^{\text{LL}}_{\text{eff}} \propto C_2^{(0)} \alpha_s \log(\mu^2/q_i^2)$

$\Rightarrow (C_1^{(1)})^{\text{LL}} \propto \alpha_s \log(M_W^2/\mu^2)$

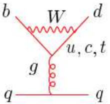
μ should be chosen of order $\mathcal{O}(m_W)$ for matching

Effective $\Delta F = 1$ hamiltonian



$$Q_1^p = (\bar{d}p)_{V-A}(\bar{p}b)_{V-A} \qquad p = u, c$$

$$Q_2^p = (\bar{d}_i p_j)_{V-A}(\bar{p}_j b_i)_{V-A}$$

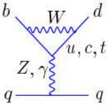


$$Q_3 = (\bar{d}b)_{V-A} \sum_q (\bar{q}q)_{V-A}$$

$$Q_4 = (\bar{d}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}$$

$$Q_5 = (\bar{d}b)_{V-A} \sum_q (\bar{q}q)_{V+A}$$

$$Q_6 = (\bar{d}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}, \qquad Q_{7\gamma} = \frac{e m_b}{4\pi^2} \bar{d}_L \sigma_{\mu\nu} F^{\mu\nu} b_R$$



$$Q_7 = (\bar{d}b)_{V-A} \sum_q \tfrac{3}{2} e_q (\bar{q}q)_{V+A}$$

$$Q_8 = (\bar{d}_i b_j)_{V-A} \sum_q \tfrac{3}{2} e_q (\bar{q}_j q_i)_{V+A}$$

$$Q_9 = (\bar{d}b)_{V-A} \sum_q \tfrac{3}{2} e_q (\bar{q}q)_{V-A}$$

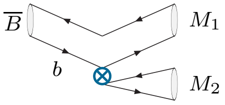
$$Q_{10} = (\bar{d}_i b_j)_{V-A} \sum_q \tfrac{3}{2} e_q (\bar{q}_j q_i)_{V-A}, \qquad Q_{8g} = \frac{g m_b}{4\pi^2} \bar{d}_L \sigma_{\mu\nu} G^{\mu\nu} b_R$$

Hadronic matrix elements

- hadronic B-decay into two mesons:

$$\overline{B} \to M_1 M_2$$

M_1: picks up the spectator quark



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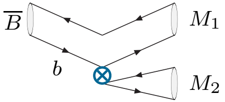

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  calculated non-perturbatively (lattice, light-cone sum rules)

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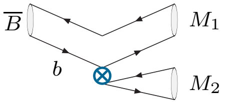

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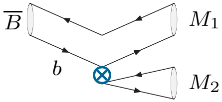
- does factorisation work?
  what about gluon exchange between the factorised matrix elements?

QCD factorisation

- consider situation that quarks $q, \bar{q}'$ composing M_2 are light (u, d, s)



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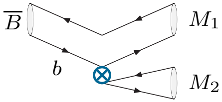


- consider situation that quarks $q, \bar{q}'$ composing M_2 are light (u, d, s)
- q and $\bar{q}'$ are very energetic and originate from a common space-time point (they are created by a point-like interaction)
  $\Rightarrow$ highly collinear with small transverse extension
- low-energetic gluons see $q\bar{q}'$ as colourless object because they cannot resolve the inner structure (colour-transperancy)
  $\Rightarrow$ non-perturbative QCD interactions confined to B-M_1 and M_2 systems separately
- QCD interactions between $B - M_1$ and M_2 can be treated perturbatively

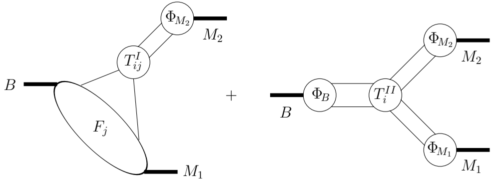
QCD factorisation

factorisation formula:

$$\langle M_1 M_2|Q_i|\bar{B}\rangle = \sum_j F_j^{B\to M_1}(m_2^2)\int_0^1 du_2\, T_{ij}^I(u_2)\,\Phi_{M_2}(u_2) + (M_1 \leftrightarrow M_2)$$
$$+ \int_0^1 du_B\, du_1\, du_2\, T_i^{II}(u_B,u_1,u_2)\,\Phi_B(u_B)\,\Phi_{M_1}(u_1)\,\Phi_{M_2}(u_2),$$

T^I, T^{II} : hard scattering kernels
(perturbative QCD corrections of $\mathcal{O}(\alpha_s(\mu_b))$)

$\Phi_M(u)$: light-cone distribution amplitude
$\rightarrow$ probability for the quark to carry momentum fraction up of the meson momentum p



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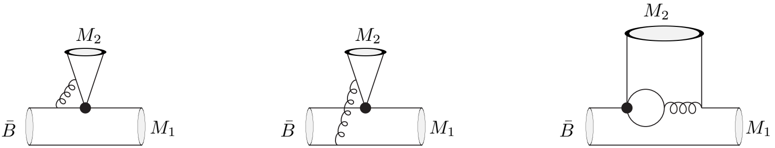

Properties and limitations of QCDF

- concept of QCDF valid in the limit $m_b \to \infty$ (heavy-quark limit)
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  $\Rightarrow$ QCDF predicts small strong phases with large uncertainties
- In $B \to VV$ decays (V : vector mesons) three helicity configurations are possible:
  both longitudinally, both positively or both negatively polarised.
  In the SM the generation of transversely polarised vector mesons requires helicity flips of the energetic light quarks

hierarchy: $$\mathcal{A}_0 : \mathcal{A}_- : \mathcal{A}_+ = 1 : \frac{\Lambda_{\text{QCD}}}{m_b} : \left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^2$$