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COSMOLOGY: THEORY
Vacuum Energy
and the
Accelerated Universe (I)

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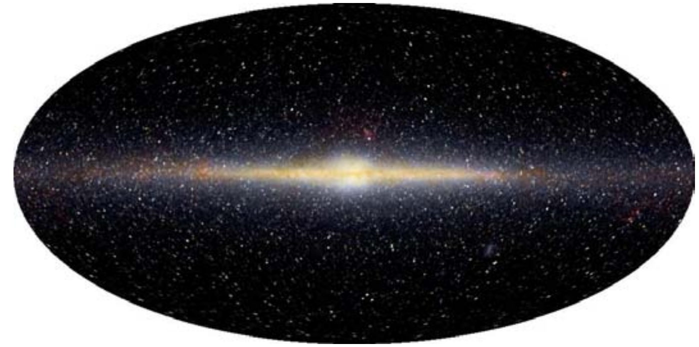
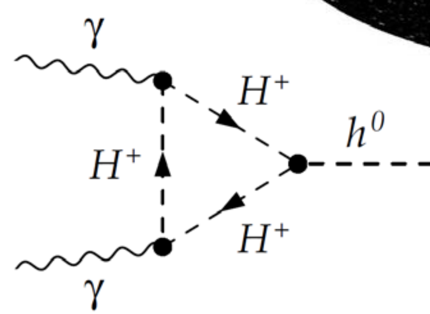
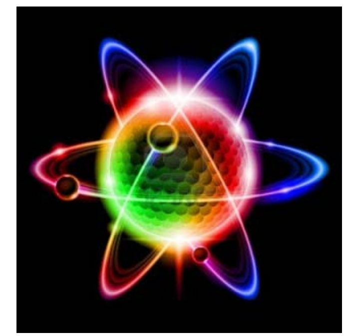
Guidelines of the Talk

- The CC term in Einstein's equations
- Vacuum energy and the CC Problem
- Some aspects of dark matter and SUSY
- Running vacuum in the early and current universe
- Fundamental "constants" and their possible cosmic time evolution
- Conclusions

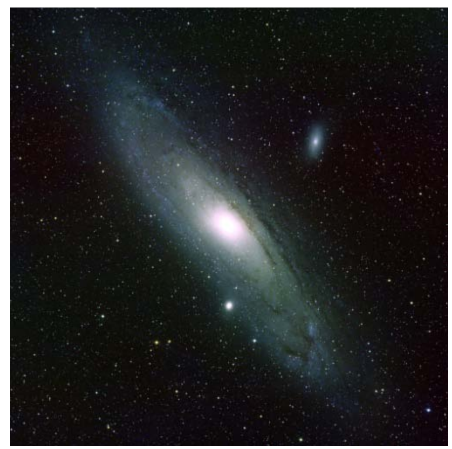
➢ DE and DM: **Finding the Rings…**

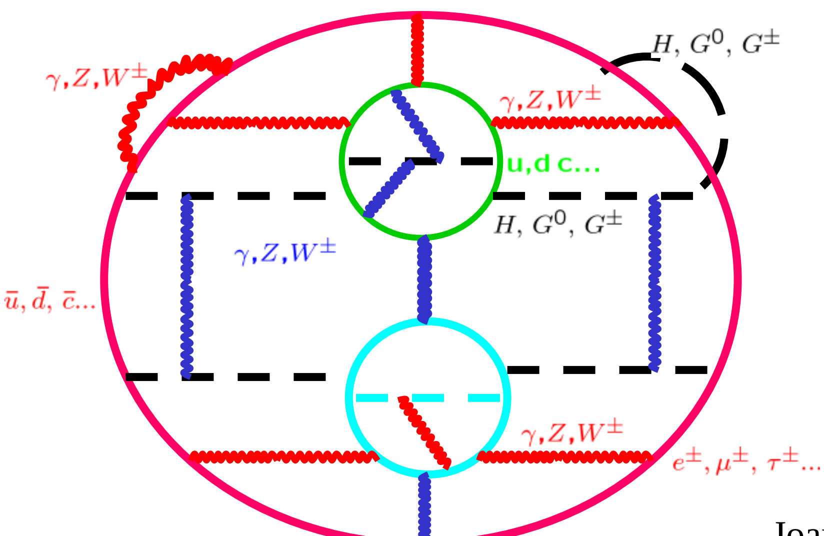


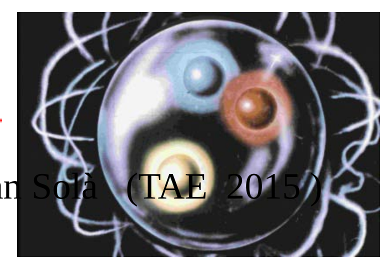




Λ ring







The Big Universe around us

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Andròmeda M 31
M32
Island Universe
M 110
0.7 Mpc = 2.2 millions l.yr.
Joan Solà (TAE 2015)


other island universes...
M 81
M 82
3.3 Mpc = 11 millions l.yr.
Joan Solà (TAE 2015)

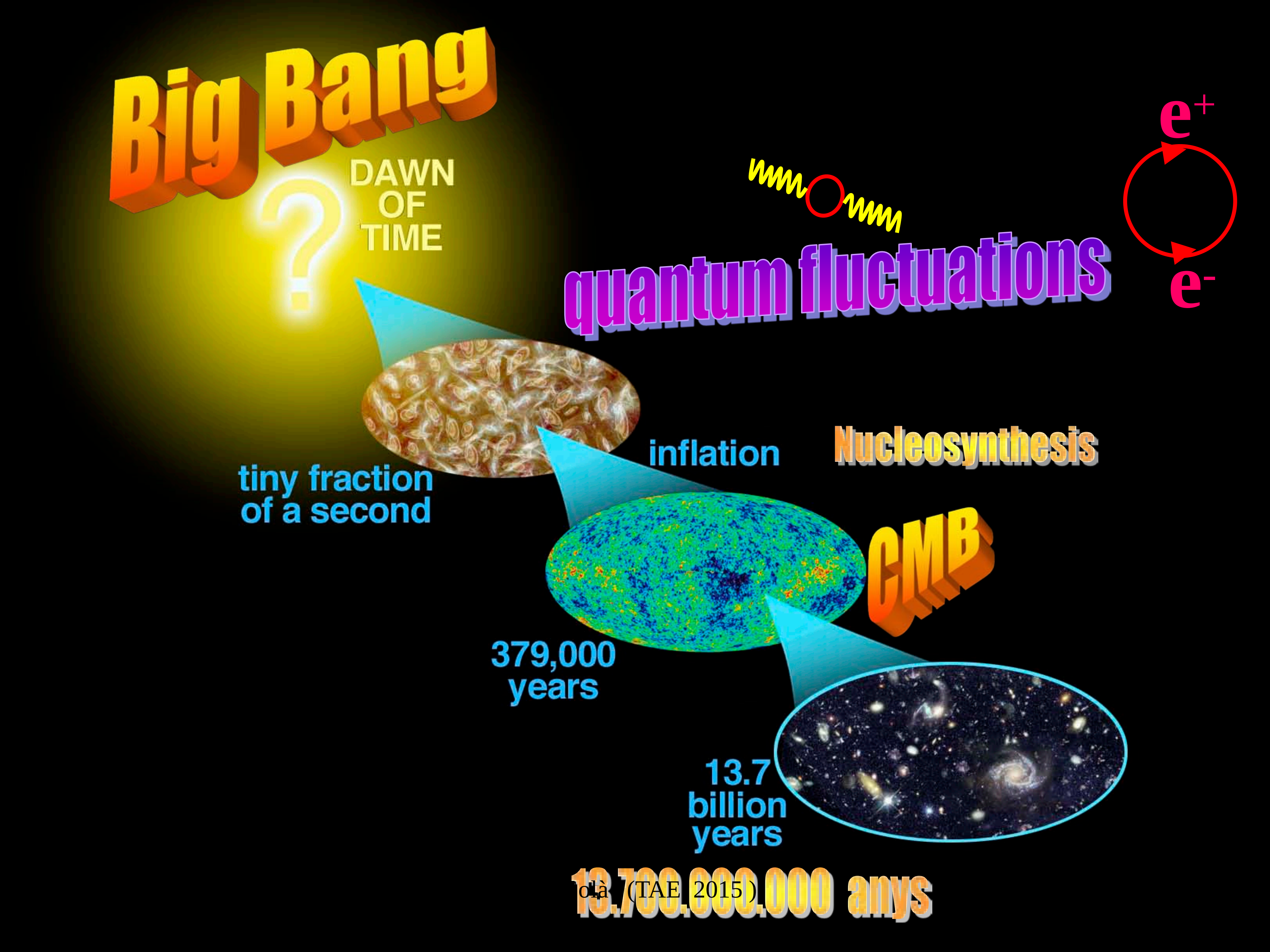

9 Mpc = 30 millions l.yr.
M 104
The sombrero galaxy
The Sombrero Galaxy — M104
HUBBLESITE.org

A wider and deeper cosmic sightseeing

10^{11} Galaxies

10^{22} Stars

Hubble Ultra Deep Field


Big Bang
DAWN
OF
TIME
quantum fluctuations
e^+
e^-
tiny fraction
of a second
inflation
Nucleosynthesis
CMB
379,000
years
13.7
billion
years
13.700.000.000 anys

The Universe is expansing (1929). Edwin Hubble:

$$v = H\, d$$

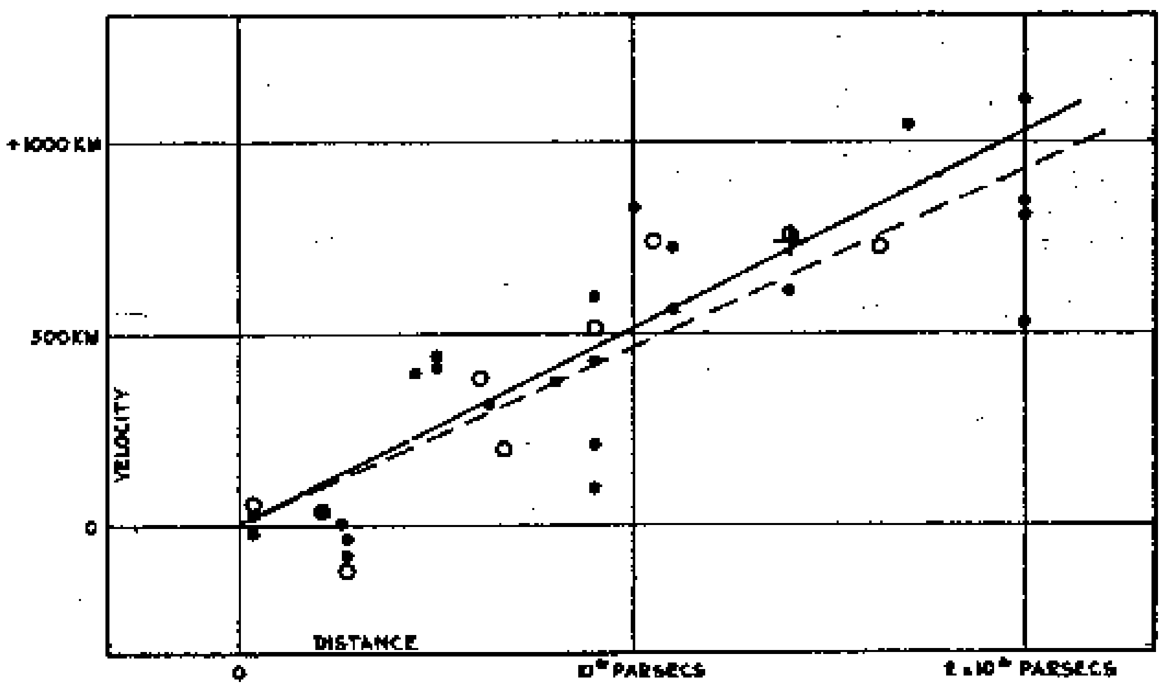
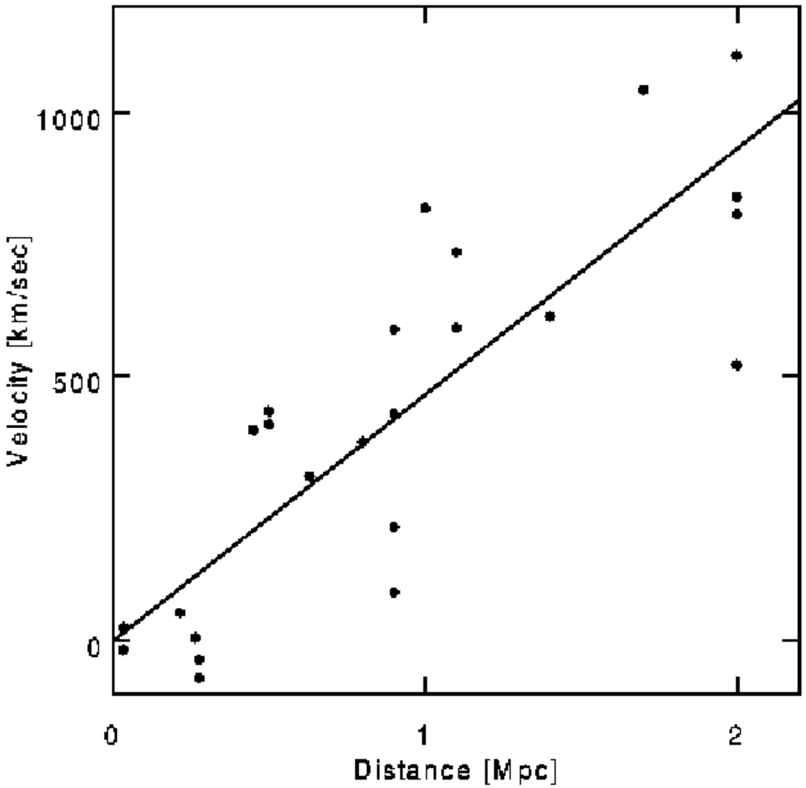


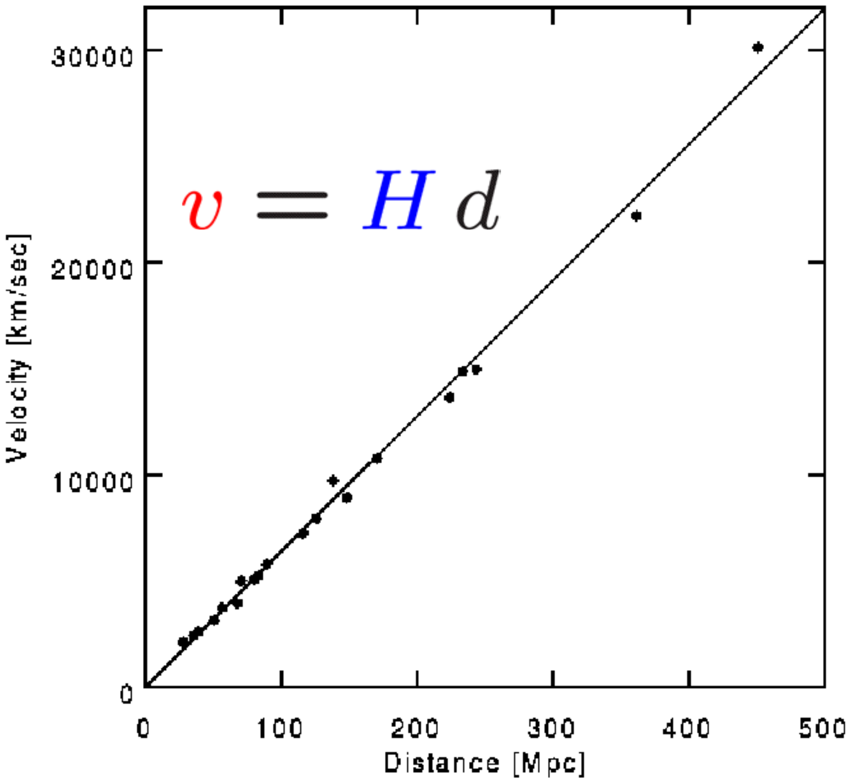
FIGURE 1

Experimental Hubble's law

Original measurements



New measurements



➢ Cosmological and astrophysical scales

1) Cosmological Scales (>150 Mpc)
Large scale structure formation process (CMB, BAOs)

2) Clusters Scales (1 Mpc)
Observations from Gravitational lensing, X-ray & SZ effect
(Total Mass - Bullet Clusters)

3) Galactic Scales (30 Kpc)
The simplest explanation for Galaxy Rotation Curves.

Einstein's Equations

1915

1917

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda\, g_{\mu\nu} = 8\pi G_N\, T_{\mu\nu}$$

Geometry ⟷ **Energy**

$$\nabla^\mu G_{\mu\nu} = 0, \text{ where } G_{\mu\nu} = R_{\mu\nu} - (1/2) g_{\mu\nu} R$$

$$\nabla_\mu \Lambda = \partial_\mu \Lambda = 0 \quad \Rightarrow \quad \Lambda = \text{const}\,. \quad !!$$

if $\nabla^\mu (G_N T_{\mu\nu}) = 0 \ldots$!!!

$$\rho_\Lambda = \frac{\Lambda}{8\,\pi\, G_N}$$

Cosmological Constant

Dark Energy

➢ Einstein's equations and FLRW metric

FLRW metric $ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{1-K\,r^2} + r^2\,d\Omega^2\right]$

$$d\,\Omega^2 = r^2\,d\theta^2 + r^2\sin^2\theta\,d\phi^2$$

$$G_{\mu\nu} - g_{\mu\nu}\,\Lambda = 8\pi G\,T_{\mu\nu}$$

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}\,R$$

$$\begin{cases} H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_m + \frac{\Lambda}{3} - \frac{K}{a^2} \\ \frac{\ddot{a}}{a} = -\frac{4\pi\,G}{3}\left(\rho_m + 3p_m\right) + \frac{\Lambda}{3} \end{cases}$$

Local covariant conservation of matter

$$\dot{\rho}_m + 3\,H\,(\rho_m + p_m) = 0 \quad \Rightarrow \quad \rho_m(a) = \rho_m^0\,a^{3(1+\omega_m)}$$

where $p_m = \omega_m \rho_m$

EoS

- But in general we expect

$$\nabla^\mu \left(G \tilde{T}_{\mu\nu} \right) = \nabla^\mu \left[G \left(T_{\mu\nu} + g_{\mu\nu}\, \rho_\Lambda \right) \right] = 0$$

$$\boxed{p_\Lambda = -\rho_\Lambda, \text{ i.e. } \omega_\Lambda = -1}$$ for **vacuum !**

$$T_{\mu\nu} = (\rho_m + p_m) U_\mu U_\nu - p_m\, g_{\mu\nu}$$

⬇ FLRW metric

$$\frac{d}{dt}\left[G(\rho_m + \rho_\Lambda) \right] + 3\, G\, H\, (\rho_m + p_m) = 0$$

- Particular case with $\dot{\rho}_\Lambda \neq 0$ at $G =$const.

⬇

$$\boxed{\dot{\rho}_\Lambda + \dot{\rho}_m + 3\, H\, (\rho_m + p_m) = 0}$$

- Notice that the previous laws are covariant and **local**

  We do **not** have a **global** conservation energy law in gravity !!

- Why? Think of QED, where

$$\nabla_\mu J^\mu = 0 \quad \Longleftrightarrow \quad \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,J^\mu\right) = 0$$

Integrating in a 4-dim. region $\mathcal{V}$ using Gauss theorem:

$$\int_{\mathcal{V}} d^4x\sqrt{-g}\nabla_\mu J^\mu = \int_{\partial\mathcal{V}} dS_\mu\sqrt{-g}\,J^\mu = 0$$

so, indeed, the current does not flow through $\partial\mathcal{V}$ out of $\mathcal{V}$.

↓

Total energy-momentum is conserved inside $\mathcal{V}$!!

- In gravity we cannot repeat the trick because $T_{\mu\nu}$ has two indices !!

Now

$$\nabla_\mu T^{\mu\nu} = 0 \iff \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}T^{\mu\nu}\right) + \Gamma^\nu_{\rho\sigma}T^{\rho\sigma} = 0$$

obstruction

As a result, Gauss theorem

$$0 = \int_{\mathcal{V}} d^4x\sqrt{-g}\nabla_\mu T^{\mu\nu}$$
$$= \int_{\partial\mathcal{V}} dS_\mu\sqrt{-g}\,T^{\mu\nu} + \int_{\mathcal{V}} d^4x\sqrt{-g}\Gamma^\nu_{\rho\sigma}T^{\rho\sigma}$$

Now we can no longer conclude

$$\int_{\partial\mathcal{V}} dS_\mu\sqrt{-g}\,T^{\mu\nu} = 0 \;!!$$

- Not too surprising, though. Think of the **Equivalence Principle**.

  We can always transform away the gravitational fields in any local region $\mathcal{V}$.

➢ Einstein's Universe

- The field equations were proposed by A. Einstein in 1915: "Die Feldgleichungen der Gravitation". Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin: 844–847.

- Until about 1930 almost everybody ``knew'' that the universe was static, in spite of the two important papers by Friedmann in 1922 and 1924 and Lemaitre's no less important work in 1927 (recall that Hubble's law is from 1929)

condition for static universe:

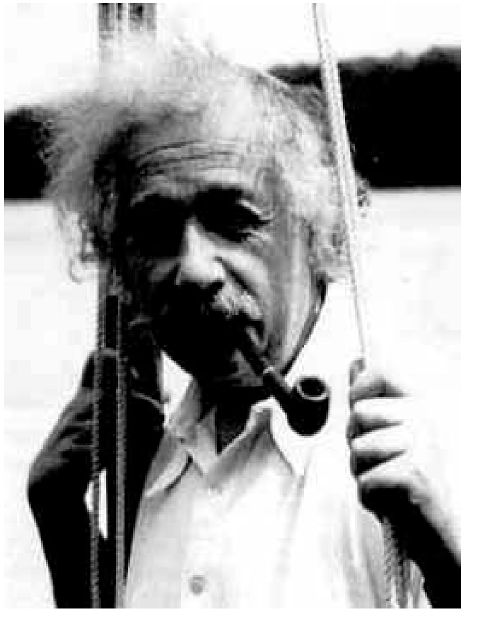
$$4\pi G \rho_m = \frac{1}{a^2} = \Lambda$$

- Einstein himself accepted the idea of an expanding universe only much later

  In fact, only after Lemaitre's successful explanation of Hubble's discovery changed the viewpoint of cosmologists. At this point Einstein rejected the cosmological term as superfluous and no longer justified. He published his new view on it in: Einstein A. Sitzungsber. Preuss. Akad. Wiss. (1931) 235-37.

- Funny enough, after the CC was rejected by Einstein, it was later re-introduced by Eddington in order to explain the age of the Universe when H_0 was thought too large (500 Km/s/Mpc) implying t_0 of only 2 billion years (younger than the Earth!).

In a letter to P. Ehrenfest on 4 February 1917 Einstein wrote about his introduction of the CC term:



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

A. Einstein, Sitzungsber. Konigl. Preuss. Akad. Wiss., phys.-math. Klasse VI, 142 (1917)

"Ich habe wieder etwas verbrochen in der Gravitationstheorie, was mich ein wenig in Gefahr bringt, in ein Tollhaus interniert zu werden"

(I have again perpetrated something relating to the theory of gravitation that might endanger me of being interned to a madhouse.)

1915 – 1916

A. Einstein, *Grundgedanken der allgemeinen Relativitätstheorie und Anwendung diese Theorie in der Astronomie*, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin (1915) 315; *Zur allgemeinen Relativitätstheorie*, Sitzungsber. Preuss. Akad. Wiss. Berlin (1915) 778; *Zur allgemeinen Relativitätstheorie (Nachtrag)*, Sitzungsber. Preuss. Akad. Wiss. Berlin (1915) 799; *Die Feldgleichungen der Gravitation*, Sitzungsber. Preuss. Akad. Wiss. Berlin (1915) 844; *Die Grundlage der allgemeinen Relativitätstheorie*, Annalen der Physik **49** (1916) 770.

1917 Λ appears

A. Einstein, *Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie*, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin, phys.-math. Klasse VI (1917) 142-152.

1931 Λ disappears $\rightarrow$ (Einstein's "blunder"?)

A. Einstein, *Zum kosmologischen Problem der allgemeinen Relativitätstheorie*, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin, phys.-math. Klasse, XII, (1931) 235.

G. Gamow, *My World Line, an Informal Autobiography* (The Viking Press, New York 1970)

The relevant paragraph says (I quote it from my own copy of this book, p. 44): "Much later, when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder he ever made in his life. ".

If we continue reading G. Gamow's autobiography, just after the sentence mentioned we find (I continue quoting from my own copy of this book, p. 44): "But this "blunder", rejected by Einstein, is still sometimes used by cosmologists even today, and the cosmological constant denoted by the Greek letter Λ rear its ugly head again and again and again. "

Y. B. Zeldovich, *Cosmological constant and elementary particles*, Zh. Eksp. Teor. Fiz. **6** (1967) 883 (JETP Lett **6** (1967) 316).

1932 Λ is **completely neglected** $\rightarrow$ (**CDM** model)

A. Einstein, W. de Sitter, *On the relation between the expansion and the mean density of the universe*, Proceedings of the National Academy of Sciences **18** (1932) 213214.

One can read there the following sentence : “Historically the term containing the ”cosmological constant” λ [written lowercase in the text of the article] was introduced into the field equations in order to enable us to account theoretically for the existence of a finite mean density in a static universe. It now appears that in the dynamical case this end can be reached without the introduction of λ”.

1934 Λ **resuscitates !!**

G. Lemaître discusses for the first time the idea of Λ as vacuum energy.

$$\rho_\Lambda = \frac{\Lambda}{8\,\pi\,G}$$

G. Lemaître, Proc. of the Nat. Acad. of Sci. **20** (1934) 12

He had actually been discussing EE’s with Λ since 1927...
independent of Friedmann first discussions
of dynamical solutions of EE’s (1922)

Some remarks on the Einstein Universe...

$$\begin{cases} H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_m + \frac{\Lambda}{3} - \frac{K}{a^2} \\ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_m + 3p_m) + \frac{\Lambda}{3} \end{cases}$$

$\Downarrow$

When $a = \text{const.} \Rightarrow$

$$\left.\begin{array}{r} \rho_m = \frac{1}{4\pi G\, a^2} = 2\,\rho_\Lambda \\ \rho_\Lambda = \frac{\Lambda}{8\pi G} \end{array}\right\} 4\pi\, G\, \rho_m = \frac{1}{a^2} = \Lambda$$

However, Einstein's universe is unstable!

$$a \to a_{\rm eq} + \delta a \text{ with } \rho_m = \rho_m^0 a_0^3 / a^3$$

where $a_{\rm eq}$ is the presumed position of equilibrium

$$\Rightarrow \qquad \delta\ddot{a} = 4\pi G \rho_m^0 \, (a_0/a)^3 \, \delta a.$$

Since the acceleration has the same sign as the perturbation

⬇

point $a_{\rm eq}$ is unstable. **!!!**

(as remarked by Eddington in 1931)

Mon. Not. Roy. Astron. Soc. **91** (1931) 490

It is instructive to find this result in an alternative way:

Newtonian limit of Einstein's equations.

$|\Phi(x)/c^2| \ll 1$ ⇓ $g_{00} = 1 + 2\Phi(x)/c^2$

$$\nabla^2\Phi = 4\pi\, G \sum_{N=m,\Lambda} (\rho_N + 3\, p_N/c^2)$$

Take a point mass, $\rho_m = m\,\delta^3(\mathbf{r})$
Assuming negligible pressure

$$= 4\pi\, G\, (\rho_m + 3\, p_m/c^2 - 2\rho_\Lambda) \simeq 4\pi\, G\, \rho_m - \Lambda\, c^2$$

Using $\nabla^2(1/r) = -4\pi\,\delta^3(\mathbf{r})$ $\Rightarrow$ for $\Lambda = 0$ the solution is $\Phi = -G\, m/r$

The Λ part is solved by $f(r) = -\Lambda\, r^2/6$

$$\nabla^2 f(r) = (1/r^2)\, d/dr\,(r^2 df/dr)$$

Corrected Newtonian law ⇓

$$\Phi = -G\,\frac{m}{r} - \frac{\Lambda\, c^2}{6}\, r^2$$

New repulsive term for $\Lambda > 0$

$$\mathbf{g}(r) = -\nabla\Phi = -G\,\frac{m}{r^2}\mathbf{u}_r + \frac{1}{3}\,\Lambda\, c^2\, r\,\mathbf{u}_r$$

We may now rephrase Eddington's objection to Einstein's Universe:

If for some reason Einstein's U. expands slightly

$$r \to r + \delta r$$

$$\Phi = -G\,\frac{m}{r} - \frac{\Lambda\, c^2}{6}\, r^2$$

$\left\{ \begin{array}{l} \text{Gravitation attraction diminishes} \\ \text{Repulsive } \Lambda\text{-force increases} \end{array} \right.$

If, on the contrary, Einstein's U. contracts slightly

$$r \to r - \delta r$$

$\left\{ \begin{array}{l} \text{Gravitation attraction increases} \\ \text{Repulsive } \Lambda\text{-force decreases} \end{array} \right.$

Runaway solution

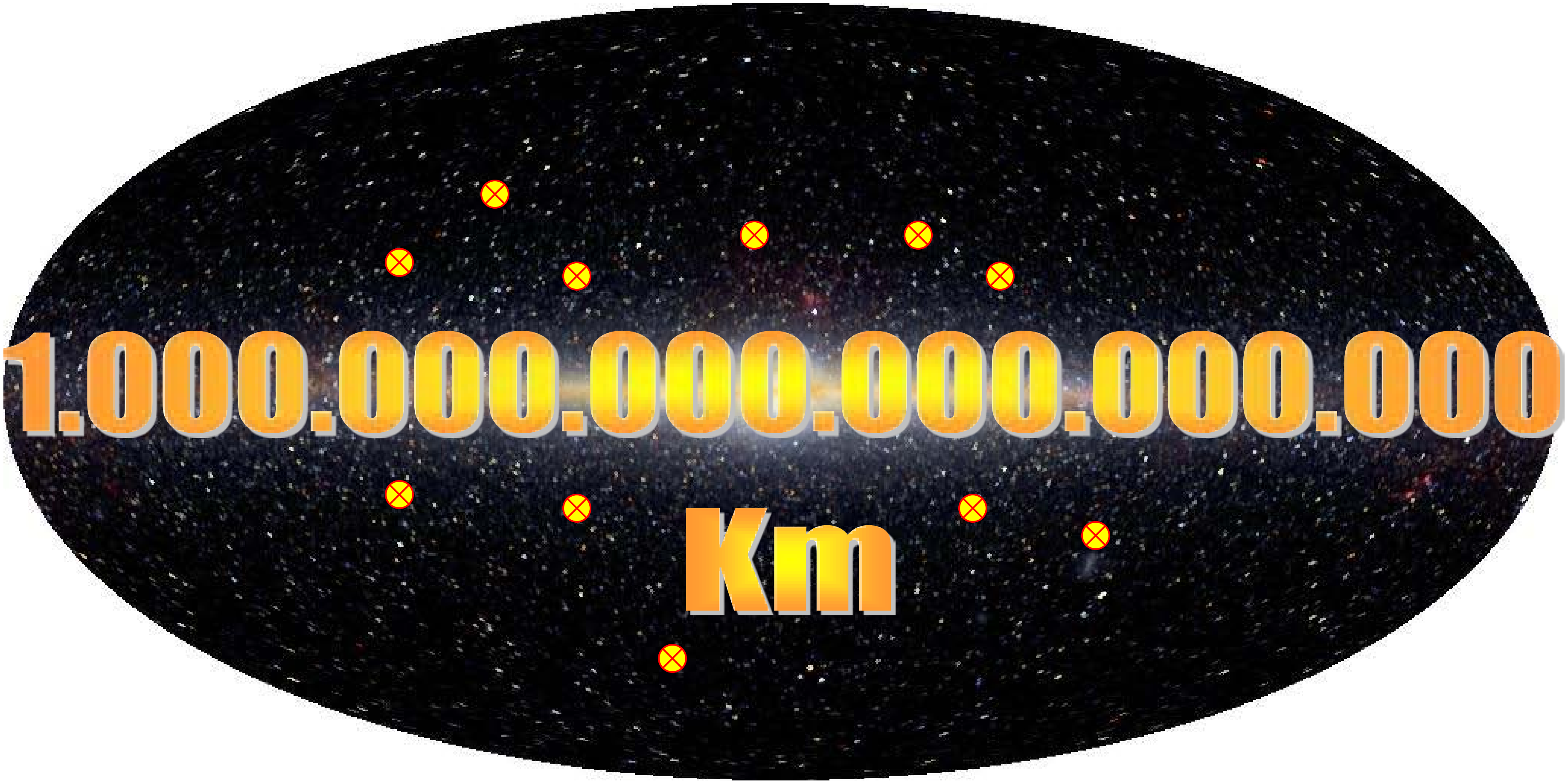
Cosmological Parameters

$$\Omega_M = \frac{\rho_M}{\rho_c} \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \qquad \Omega_K = -\frac{k}{H^2 a^2}$$

ρ_c **critical density**

10^{-23} **grams/m**^{3} → 5 **protons / m**^{3}

Milky Way



100.000 lyr = 30 kpc = 1 billion billion Km

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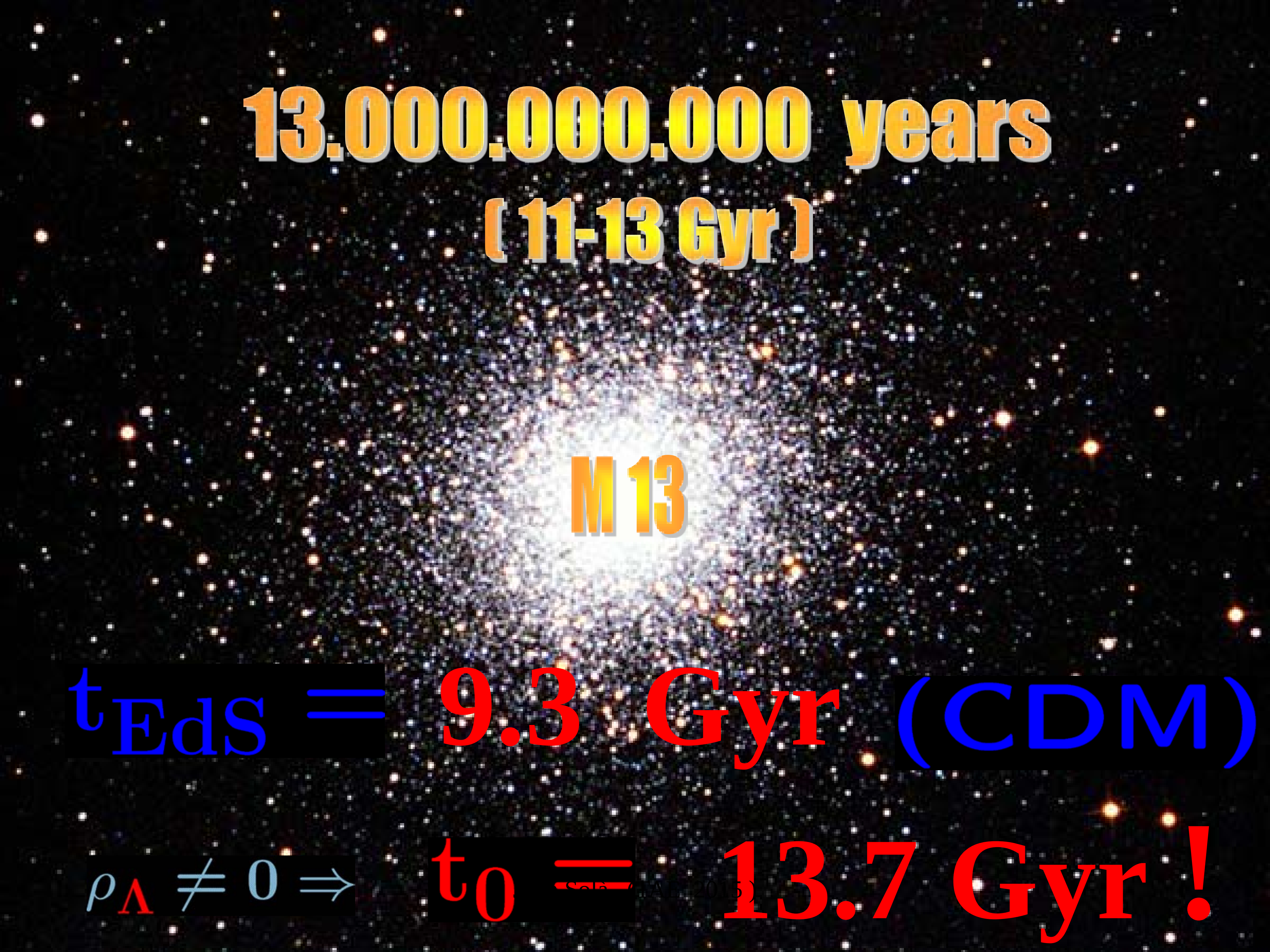
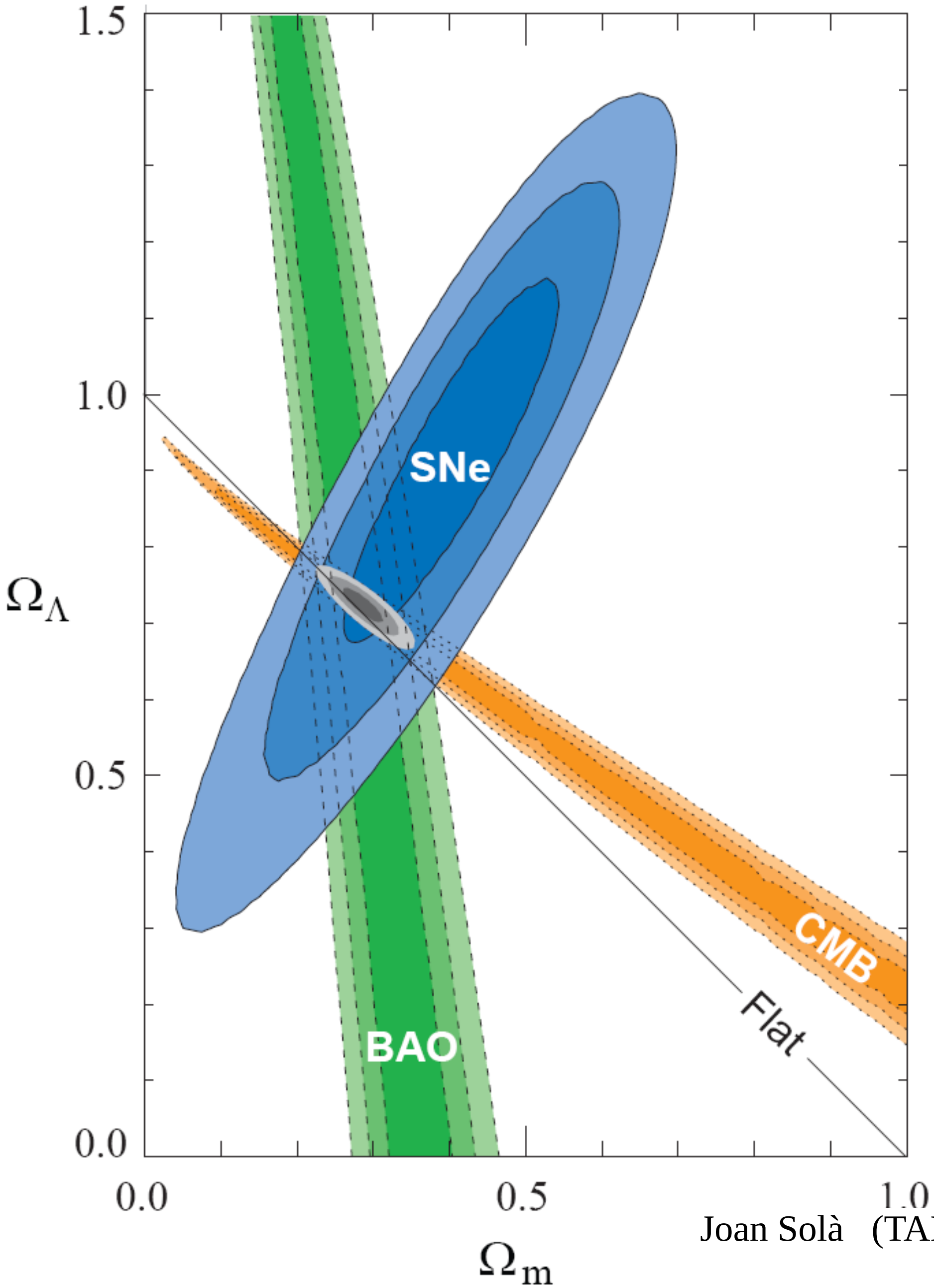

13.000.000.000 years
(11-13 Gyr)
M 13
$t_{EdS} = 9.3$ Gyr (CDM)
$\rho_\Lambda \neq 0 \Rightarrow t_0 = 13.7$ Gyr !

Cosmic sum rule



$\Omega_\Lambda \simeq 0.73$ $\Omega_M \simeq 0.27$

↓

$\Omega_M + \Omega_\Lambda + \Omega_K = 1$

↓

$\Omega_K \simeq 0$

Universe spatially "flat"
→ Euclidian

Geometries of the Universe

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{1 - K\,r^2} + r^2\,d\Omega^2\right]$$

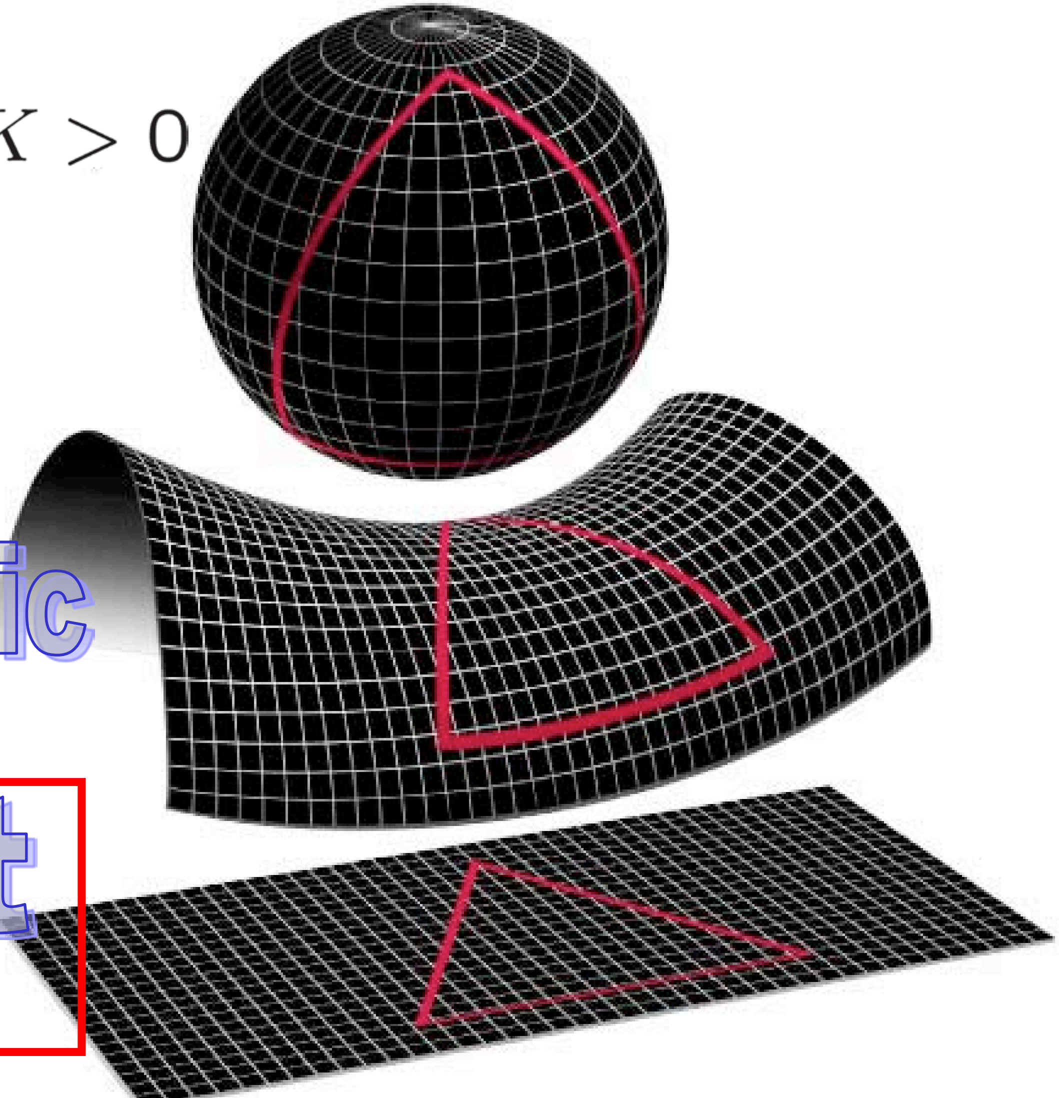
Spheric $K > 0$

$K < 0$

Hyperbolic

Flat

$K = 0$



What objects to measure?

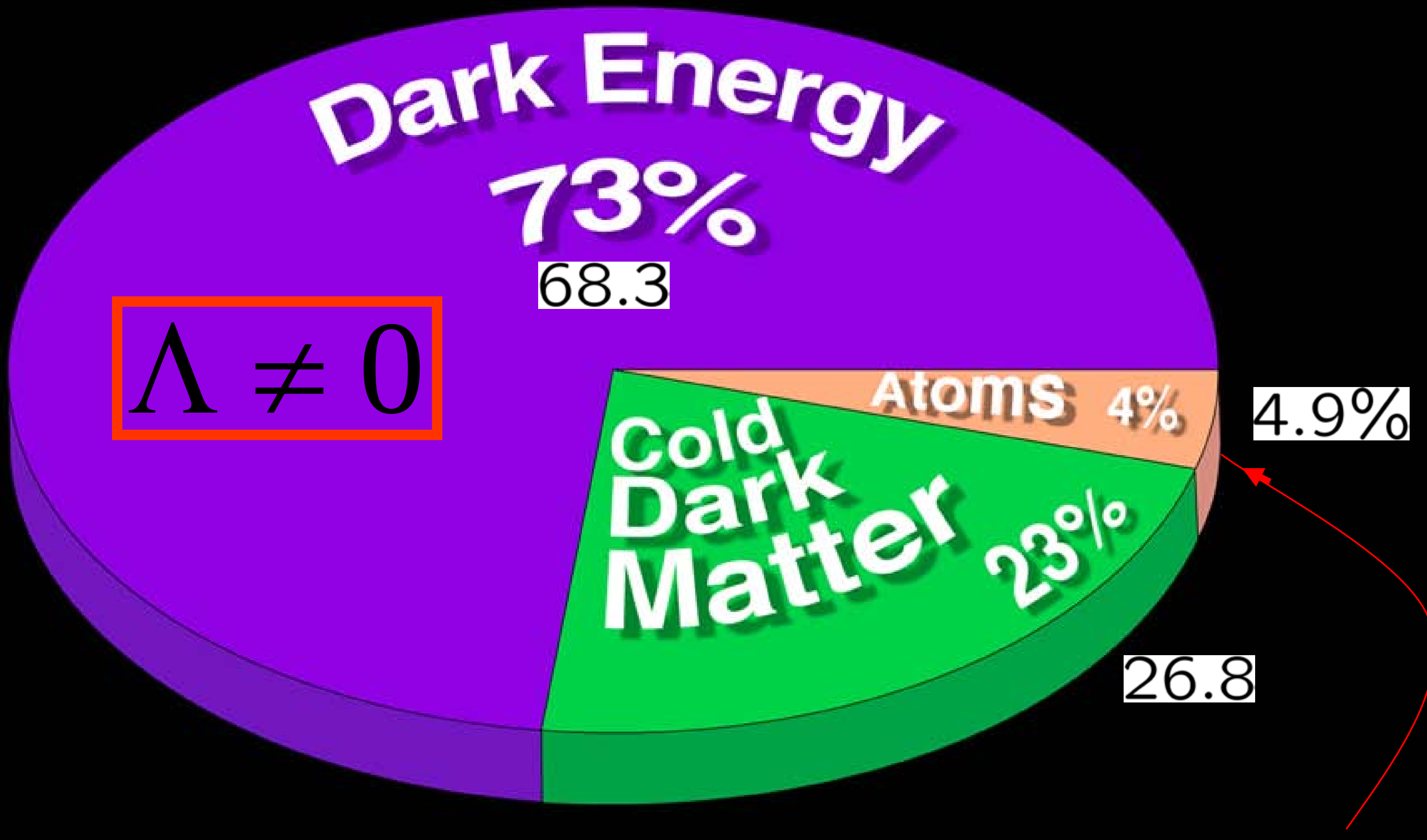
↓

Fundamental observers !!

The most distant supernovae!!

Measuring the expansion of the Universe....

Cosmological Cake



We are nobody...

$h_0 \simeq 0.67$ PLANCK J 2015)

Vacuum energy: zero-point energy and some cosmic numerology

- Zeldovich (1967) first raised the connection $\mathbf{QFT} \Longleftrightarrow \Lambda$, $\mathbf{QFT} \Longleftrightarrow \rho_\Lambda$, $\rho_\Lambda \Longleftrightarrow \Lambda$

$$\rho_\Lambda = \frac{\Lambda}{8\,\pi\,G_N}$$ (vacuum energy density)

First thought: $\rho_\Lambda \propto m_p^4 \sim 1\mathrm{GeV}^4$

Impossible since $\rho_\Lambda \simeq \rho_c^0$ and $\rho_c^0 = \dfrac{3\,H_0^2}{8\pi\,G} \sim 10^{-47}\,\mathrm{GeV}^4$

Second thought:

$$\rho_\Lambda \simeq G\,m_p^6 = \frac{m_p^6}{M_P^2} \sim 10^{-38}\,\mathrm{GeV}^4$$

much better, but still unacceptable...

- Weinberg in 1972 "cosmic prediction" of the pion mass

$$m_\pi^3 \sim \frac{H_0}{G} = H_0\, M_P^2 \sim 10^{-4}\,\mathrm{GeV}^3 \quad \Rightarrow \quad m_\pi = \mathcal{O}(0.1)\ \mathrm{GeV}$$

Using now Zeldovich's second thought with $m_p \to m_\pi$:

$$\rho_\Lambda \sim G\, m_\pi^6 = \frac{m_\pi^6}{M_P^2} \sim H_0^2\, M_P^2 = \frac{H_0^2}{G} \sim \rho_c^0$$

correct order of magnitude **!!**

From the previous formulae, one findsthe disguised form:

$$\rho_\Lambda \sim m_\pi^3\, H_0 \sim H_0\, \Lambda_{QCD}^3$$

The last form became popular more recently by several authors

But it is **unacceptable** $\Rightarrow$ ~~covariance~~ covariance of the effective action

- Of all SM* particles there is one that may realize the "CC dream":

$$\text{neutrino of a few meV} = 10^{-3}\ \text{eV}$$

$$\rho_\Lambda \sim m_\nu^4 \sim 10^{-11}\,\text{meV}^4 \sim 10^{-47}\,\text{GeV}^4 \quad !!$$

Fine, but why only this d.o.f.?

- Perhaps the most "cabalistic" attempt at finding the CC "number" is the following one using G, m_e and α:

$$\Lambda = \frac{G^2}{\hbar^4}\left(\frac{m_e}{\alpha}\right)^6$$

Or in nat. unit.

$$\rho_\Lambda = \frac{G}{8\pi}\left(\frac{m_e}{\alpha}\right)^6 \simeq 3\times 10^{-47}\ \text{GeV}^4 \ !!$$

Kind of "improved" form of Zeldovich's second thought