Theory of Higgs Bosons: The Standard Model and Beyond

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7–8 September 2011
Idpasc Higgs School
Foz do Arelho, Portugal

Lectures

Lecture I: Electroweak symmetry breaking and the Standard Model Higgs boson

Lecture II: The Higgs Sector of the minimal supersymmetric extension of the Standard Model (MSSM)

A brief bibliography

1. J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, *The Higgs Hunter's Guide* (Perseus Publishing, Cambridge, MA, 1990).

2. H.E. Haber, "Lectures on Electroweak Symmetry Breaking," in *Testing the Standard Model*, Proceedings of the Theoretical Advanced Study Institute (TASI-90), Boulder, Colorado, 3–27 June, 1990, edited by M. Cvetic and P. Langacker (World Scientific, Singapore, 1991), pp. 340–475.

3. M. Carena and H.E. Haber, "Higgs boson theory and phenomenology," Prog. Part. Nucl. Phys. **50**, 63 (2003) [arXiv:hep-ph/0208209].

4. A. Djouadi, "The anatomy of electro-weak symmetry breaking. I: The Higgs boson in the Standard Model," Phys. Rept. **457**, 1 (2008) [arXiv:hep-ph/0503172].

5. A. Djouadi, "The anatomy of electro-weak symmetry breaking. II: The Higgs bosons in the minimal supersymmetric model," Phys. Rept. **459**, 1 (2008) [arXiv:hep-ph/0503173].

6. J.D. Wells, "Lectures on Higgs Boson Physics in the Standard Model and Beyond," arXiv:0909.4541 [hep-ph] (2009).

Lectures

I Electroweak symmetry breaking and the Standard Model (SM) Higgs boson

- Mass generation and the Goldstone boson
- Theory of the Standard Model Higgs boson

II Expanding the Higgs sector of the Standard Model

- The Two Higgs Doublet Extension of the Standard Model (2HDM)
- Theory of the MSSM Higgs sector
- Higgs physics beyond the MSSM

Lecture I: Electroweak symmetry breaking and the Standard Model Higgs boson

Outline

- The Standard Model—what's missing?
- mass generation and the Goldstone boson
- The significance of the TeV scale—Part 1
- theory of the Standard Model (SM) Higgs boson

What's missing?

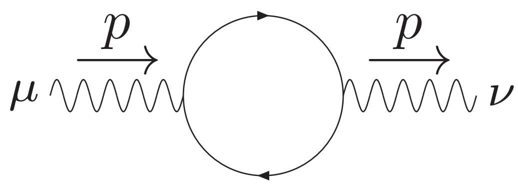
The theory of $W^\pm$ and Z gauge bosons must be *gauge invariant*; otherwise the theory is mathematically inconsistent. You may have heard that "gauge invariance implies that the gauge boson mass must be zero," since a mass term of the form $m^2 A^a_\mu A^{\mu a}$ is not gauge invariant.

So, what is the origin of the $W^\pm$ and Z boson masses? Gauge bosons are massless at tree-level, but perhaps a mass may be generated when quantum corrections are included. The tree-level gauge boson propagator $G^0_{\mu\nu}$ (in the Landau gauge) is:

$$G^0_{\mu\nu}(p) = \frac{-i}{p^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) .$$

The pole at $p^2 = 0$ indicates that the tree-level gauge boson mass is zero. Let's now include the radiative corrections.

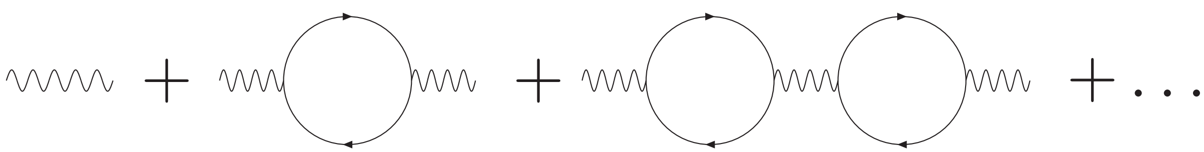
The polarization tensor $\Pi_{\mu\nu}(p)$ is defined as:



$$i\,\Pi_{\mu\nu}(p) \equiv i(p_\mu p_\nu - p^2 g_{\mu\nu})\Pi(p^2)$$

where the form of $\Pi_{\mu\nu}(p)$ is governed by covariance with respect to Lorentz transformations, and is constrained by gauge invariance, i.e. it satisfies $p^\mu \Pi_{\mu\nu}(p) = p^\nu \Pi_{\mu\nu}(p) = 0$.

The renormalized propagator is the sum of a geometric series



$$= \frac{-i(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})}{p^2[1+\Pi(p^2)]}$$

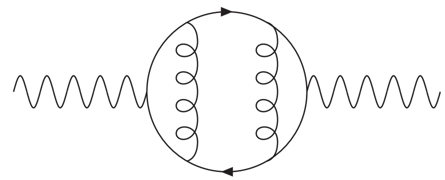
The pole at $p^2 = 0$ is shifted to a non-zero value if:

$$\Pi(p^2) \underset{p^2 \to 0}{\simeq} \frac{-g^2 v^2}{p^2}\,.$$

Then $p^2[1 + \Pi(p^2)] = p^2 - g^2 v^2$, yielding a gauge boson mass of gv.

Interpretation of the $p^2 = 0$ pole of $\Pi(p^2)$

The pole at $p^2 = 0$ corresponds to a propagating massless scalar. For example, the sum over intermediate states includes a quark-antiquark pair with many gluon exchanges, e.g.,

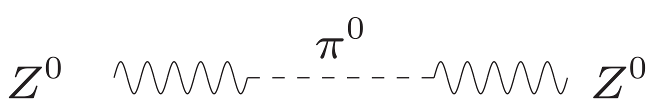


This is a strongly-interacting system—it is possible that one of the contributing intermediate states is a massless spin-0 state (due to the strong binding of the quark/antiquark pair).

We know that the Z and $W^\pm$ couple to neutral and charged weak currents

$$\mathcal{L}_{\rm int} = g_Z j_\mu^Z Z^\mu + g_W (j_\mu^W W^{+\mu} + {\rm h.c.}) \,,$$

which are known to create neutral and charged pions from the vacuum. In the absence of quark masses, the pions are massless bound states of $q\bar{q}$ [they are Goldstone bosons of chiral symmetry which is spontaneously broken by the strong interactions]. Thus, the diagram:



yields $\Pi(p^2) = -g_Z^2 f_\pi^2/p^2$, where $f_\pi = 93$ MeV is the amplitude for creating a pion from the vacuum. Thus, $m_Z = g_Z f_\pi$. Similarly $m_W = g_W f_\pi$.

Gauge boson mass generation and the Goldstone boson

We have demonstrated a mass generation mechanism for gauge bosons that is both Lorentz-invariant and gauge-invariant! The $p^2 = 0$ pole of $\Pi(p^2)$ corresponds to a propagating massless scalar state called the Goldstone boson. We showed that the W and Z are massive in the Standard Model (without Higgs bosons!!). Moreover, the ratio

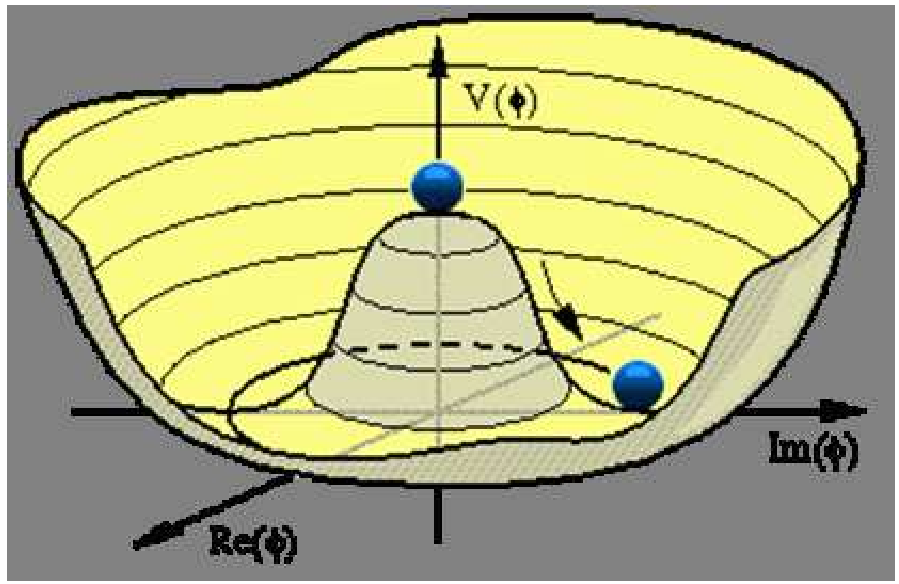
$$\frac{m_W}{m_Z} = \frac{g_W}{g_Z} \equiv \cos\theta_W \simeq 0.88$$

is remarkably close to the measured ratio. Unfortunately, since $g_Z \simeq 0.37$ we find $m_Z = g_Z f_\pi = 35$ MeV, which is too small by a factor of 2600.

There must be another source for the gauge boson masses, i.e. new fundamental dynamics that generates the Goldstone bosons that are the main sources of mass for the $W^\pm$ and Z.

Possible choices for electroweak-symmetry-breaking (EWSB) dynamics

- weakly-interacting self-coupled elementary (Higgs) scalar dynamics



- strong-interaction dynamics involving new fermions and gauge fields [technicolor, dynamical EWSB, little Higgs models, composite Higgs bosons, Higgsless models, extra-dimensional EWSB, . . .]

Both mechanisms generate new phenomena with significant experimental consequences.

Significance of the TeV Scale—Part 1

Let $\Lambda_{\rm EW}$ be energy scale of EWSB dynamics. For example:

- Elementary Higgs scalar ($\Lambda_{\rm EW} = m_h$).

- Strong EWSB dynamics ($e.g.$, $\Lambda_{\rm EW}^{-1}$ is the characteristic scale of bound states arising from new strong dynamics).

Consider $W_L^+W_L^- \to W_L^+W_L^-$ (L = longitudinal or equivalently, zero helicity) for $m_W^2 \ll s \ll \Lambda_{\rm EW}^2$. The corresponding amplitude, to leading order in g^2, but to all orders in the couplings that control the EWSB dynamics, is equal to the amplitude for $G^+G^- \to G^+G^-$ (where $G^\pm$ are the charged Goldstone bosons). The latter is universal, independent of the EWSB dynamics. This is a rigorous low-energy theorem.

Applying unitarity constraints to this amplitude yields a critical energy $\sqrt{s_c}$, above which unitarity is violated. This unitarity violation must be repaired by EWSB dynamics and implies that $\Lambda_{\rm EW} \lesssim \mathcal{O}\left(\sqrt{s_c}\,\right)$.

Unitarity of scattering amplitudes

Unitarity is equivalent to the conservation of probability in quantum mechanics. A violation of unitarity is tantamount to a violation of the principles of quantum mechanics—this is too sacred a principle to give up!

Consider the helicity amplitude $\mathcal{M}(\lambda_3\lambda_4\,;\,\lambda_1\lambda_2)$ for a $2 \to 2$ scattering process with initial [final] helicities λ_1, λ_2 [λ_3, λ_4]. The Jacob-Wick partial wave expansion is:

$$\mathcal{M}(\lambda_3\lambda_4\,;\,\lambda_1\lambda_2) = \frac{8\pi\sqrt{s}}{(p_i p_f)^{1/2}} e^{i(\lambda_i-\lambda_f)\phi} \sum_{J=J_0}^{\infty} (2J+1)\mathcal{M}^J_\lambda(s) d^J_{\lambda_i\lambda_f}(\theta)\,,$$

where p_i [p_f] is the incoming [outgoing] center-of-mass momentum, $\sqrt{s}$ is the center-of-mass energy, $\lambda \equiv \{\lambda_3\lambda_4\,;\,\lambda_1\lambda_2\}$ and

$$J_0 \equiv \max\{\lambda_i\,,\,\lambda_f\}\,, \quad \text{where} \quad \lambda_i \equiv \lambda_1 - \lambda_2\,, \quad \text{and} \quad \lambda_f \equiv \lambda_3 - \lambda_4\,.$$

Orthogonality of the d-functions allows one to project out a given partial wave amplitude. For example, for $W_L^+ W_L^- \to W_L^+ W_L^-$ (L stands for *longitudinal* and corresponds to $\lambda = 0$),

$$\mathcal{M}^{J=0} = \frac{1}{16\pi s}\int_{-s}^{0} dt\, \mathcal{M}(L, L\,;\, L, L)\,,$$

where $t = -\frac{1}{2}s(1-\cos\theta)$ in the limit where $m_W^2 \ll s$.

The $J = 0$ partial wave for $W_L^+ W_L^- \to W_L^+ W_L^-$ in the limit of $m_W^2 \ll s \ll \Lambda_{\rm EW}^2$ is equal to the corresponding amplitude for $G^+G^- \to G^+G^-$:

$$\mathcal{M}^{J=0} = \frac{G_F s}{16\pi\sqrt{2}} \,.$$

Partial wave unitarity implies that:

$$|\mathcal{M}^J|^2 \leq |{\rm Im}\ \mathcal{M}^J| \leq 1 \,,$$

which gives

$$({\rm Re}\ \mathcal{M}^J)^2 \leq |{\rm Im}\ \mathcal{M}^J| \left(1 - |{\rm Im}\ \mathcal{M}^J|\right) \leq \tfrac{1}{4} \,.$$

Setting $|{\rm Re}\ \mathcal{M}^{J=0}| \leq \frac{1}{2}$ yields $\sqrt{s_c}$. The most restrictive bound arises from the isospin zero channel $\sqrt{\frac{1}{6}}(2W_L^+W_L^- + Z_L Z_L)$:

$$s_c = \frac{4\pi\sqrt{2}}{G_F} = (1.2\ {\rm TeV})^2 \,.$$

Since unitarity cannot be violated, we conclude that $\Lambda_{\rm EW} \lesssim \sqrt{s_c}$. That is,

The dynamics of electroweak symmetry breaking must be exposed at or below the 1 TeV energy scale.

EWSB Dynamics of the Standard Model

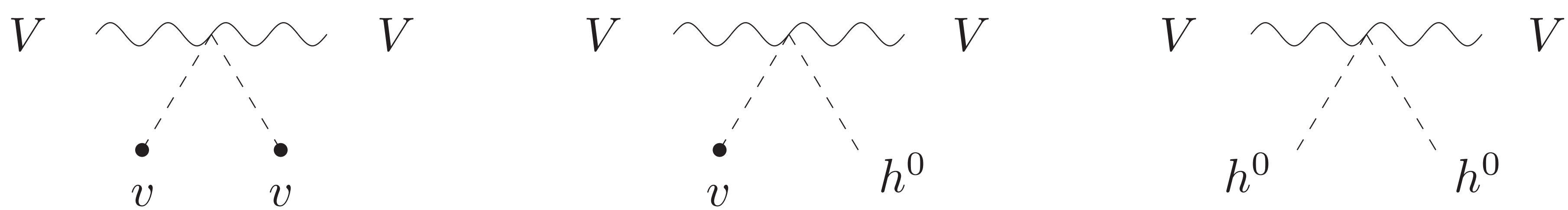
- Add a new sector of "matter" consisting of a complex SU(2) doublet, hypercharge-one self-interacting scalar fields, $\Phi \equiv (\Phi^+ \;\; \Phi^0)$ with four real degrees of freedom. The scalar potential is:
$$V(\Phi) = \frac{\lambda}{4}(\Phi^\dagger\Phi - \tfrac{1}{2}v^2)^2\,,$$
so that in the ground state, the neutral scalar field takes on a constant non-zero value $\langle\Phi^0\rangle = v/\sqrt{2}$, where $v = 246$ GeV.

- The non-zero scalar vacuum expectation value breaks the electroweak symmetry, thereby generating three Goldstone bosons (exactly massless), which become the longitudinal components of the $W^\pm$ and Z. Here, v plays the role of f_π, so we get $m_Z = g_Z v \simeq 91$ GeV.

- One scalar degree of freedom is left over—the Higgs boson, $h^0 \equiv \sqrt{2}\,\mathrm{Re}(\Phi^0 - \frac{v}{\sqrt{2}})$. It is a neutral CP-even scalar boson, whose interactions are precisely predicted, but whose mass $m_h = \frac{1}{2}\lambda v^2$ depends on the unknown strength of the scalar self-coupling—the only unknown parameter of the model.

Mass generation and Higgs couplings in the SM

Gauge bosons ($V = W^\pm$ or Z) acquire mass via interaction with the Higgs vacuum condensate.



Thus,

$$g_{hVV} = 2m_V^2/v\,, \qquad \text{and} \qquad g_{hhVV} = 2m_V^2/v^2\,,$$

$i.e.$, the Higgs couplings to vector bosons are proportional to the corresponding boson squared-mass.

Likewise, by replacing V with the Higgs field h^0 in the above diagrams, the Higgs self-couplings are also proportional to the square of the Higgs mass:

$$g_{hhh} = \tfrac{3}{2}\lambda v = \frac{3m_h^2}{v}\,, \qquad \text{and} \qquad g_{hhhh} = \tfrac{3}{2}\lambda = \frac{3m_h^2}{v^2}\,.$$

Fermions in the Standard Model

Given a four-component fermion f, we can project out the right and left-handed parts:

$$f_R \equiv P_R f\,, \qquad f_L \equiv P_L f\,, \qquad \text{where} \quad P_{R,L} = \tfrac{1}{2}(1 \pm \gamma_5)\,.$$

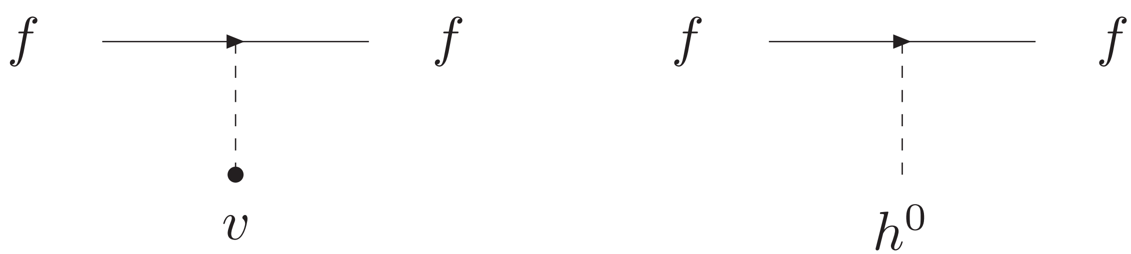
Under the electroweak gauge group, the right and left-handed components of each fermion has different SU(2)×U(1)$_{\rm Y}$ quantum numbers:

fermions	SU(2)	U(1)$_{\rm Y}$
$(\nu\,,\, e^-)_L$	2	-1
e^-_R	1	-2
$(u\,,\, d)_L$	2	$1/3$
u_R	1	$4/3$
d_R	1	$-2/3$

where the electric charge is related to the U(1)$_{\rm Y}$ hypercharge by $Q = T_3 + \frac{1}{2}Y$.

Before electroweak symmetry breaking, Standard Model fermions are massless, since the fermion mass term $\mathcal{L}_m = -m(\bar{f}_R f_L + \bar{f}_L f_R)$ is not gauge invariant.

The generation of masses for quarks and leptons is especially elegant in the SM. The fermions couple to the Higgs field through the gauge invariant Yukawa couplings (see below). The quarks and charged leptons acquire mass when Φ^0 acquires a vacuum expectation value:



Thus, $g_{hf\bar{f}} = m_f/v$, *i.e.*, Higgs couplings to fermions are proportional to the corresponding fermion mass.

It is remarkable that the neutral Higgs boson coupling to fermions is flavor-diagonal. This is a consequence of the Higgs-fermion Yukawa couplings:

$$\mathcal{L}_{\rm Yukawa} = -h_u^{ij}(\bar{u}_R^i u_L^j \Phi^0 - \bar{u}_R^i d_L^j \Phi^+) - h_d^{ij}(\bar{d}_R^i d_L^j \Phi^{0\,*} + \bar{d}_R^i u_L^j \Phi^-) + {\rm h.c.}\,,$$

where i, j are generation labels and h_u and h_d are arbitrary complex 3×3 matrices. Writing $\Phi^0 = (v + h^0)/\sqrt{2}$, we identify the quark mass matrices,

$$M_u^{ij} \equiv h_u^{ij} \frac{v}{\sqrt{2}}\,, \qquad\qquad M_d^{ij} \equiv h_d^{ij} \frac{v}{\sqrt{2}}\,.$$

One is free to redefine the quark fields:

$$u_L \to V_L^U u_L\,, \qquad u_R \to V_R^U u_R\,, \qquad d_L \to V_L^D d_L\,, \qquad d_R \to V_R^D d_R\,,$$

where V_L^U, V_R^U, V_L^D, and V_R^D are unitary matrices chosen such that

$$V_R^{U\,\dagger} M_u V_L^U = \mathrm{diag}(m_u\,,\, m_c\,,\, m_t)\,, \qquad V_R^{D\,\dagger} M_d V_L^D = \mathrm{diag}(m_d\,,\, m_s\,,\, m_b)\,,$$

such that the m_i are the positive quark masses (this is the *singular value decomposition* of linear algebra).

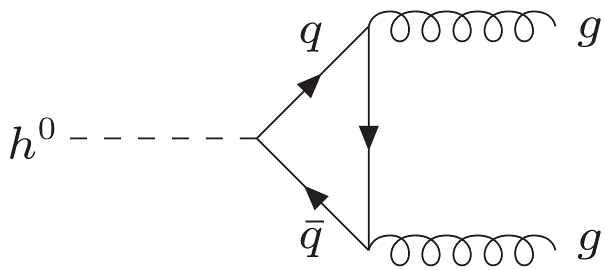
Having diagonalized the quark mass matrices, the neutral Higgs Yukawa couplings are automatically flavor-diagonal.* Hence the SM possesses no flavor-changing neutral currents (FCNCs) mediated by neutral Higgs boson (or gauge boson) exchange at tree-level.

*Independently of the Higgs sector, the quark couplings to Z and γ are automatically flavor diagonal. Flavor dependence only enters the quark couplings to the $W^\pm$ via the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $K \equiv V_L^{U\,\dagger} V_L^D$.

Loop induced Higgs boson couplings

Higgs boson coupling to gluons

At one-loop, the Higgs boson couples to gluons via a loop of quarks:



This diagram leads to an effective Lagrangian

$$\mathcal{L}^{\rm eff}_{hgg} = \frac{g\alpha_s N_g}{24\pi m_W} h^0 G^a_{\mu\nu} G^{\mu\nu a} \,,$$

where N_g is roughly the number of quarks heavier than h^0. More precisely,

$$N_g = \sum_i F_{1/2}(x_i) \,, \qquad\qquad x_i \equiv \frac{m^2_{q_i}}{m^2_h} \,,$$

where the loop function $F_{1/2}(x) \to 1$ for $x \gg 1$.

Note that heavy quark loops do *not* decouple. Light quark loops are negligible, as $F_{1/2}(x) \to \frac{3}{2}x^2 \ln x$ for $x \ll 1$.

The dominant mechanism for Higgs production at the LHC is gluon-gluon fusion. At leading order,

$$\frac{d\sigma}{dy}(pp \to h^0 + X) = \frac{\pi^2 \Gamma(h^0 \to gg)}{8m_h^3} g(x_+, m_h^2) g(x_-, m_h^2)\,,$$

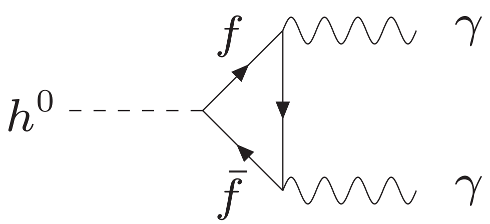
where $g(x, Q^2)$ is the gluon distribution function at the scale Q^2 and

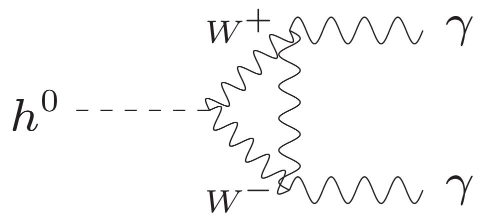
$$x_\pm \equiv \frac{m_h e^{\pm y}}{\sqrt{s}}\,, \qquad\qquad y = \tfrac{1}{2} \ln \left(\frac{E + p_{||}}{E - p_{||}} \right)\,.$$

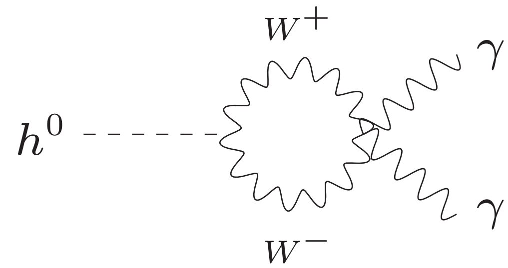
The rapidity y is defined in terms of the Higgs boson energy and longitudinal momentum in the pp center-of-mass frame.

Higgs boson coupling to photons

At one-loop, the Higgs boson couples to photons via a loop of charged particles:







If charged scalars exist, they would contribute as well. These diagrams lead to an effective Lagrangian

$$\mathcal{L}^{\rm eff}_{h\gamma\gamma} = \frac{g\alpha N_\gamma}{12\pi m_W} h^0 F_{\mu\nu}F^{\mu\nu} ,$$

where

$$N_\gamma = \sum_i N_{ci} e_i^2 F_j(x_i) , \qquad\qquad x_i \equiv \frac{m_i^2}{m_h^2} .$$

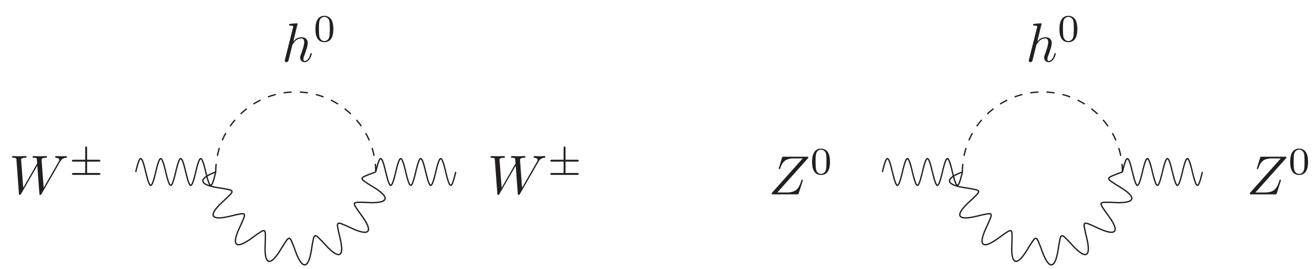
In the sum over loop particles i of mass m_i, $N_{ci} = 3$ for quarks and 1 for color singlets, e_i is the electric charge in units of e and $F_j(x_i)$ is the loop function corresponding to ith particle (with spin j). In the limit of $x \gg 1$,

$$F_j(x) \longrightarrow \begin{cases} 1/4 , & j = 0 , \\ 1 , & j = 1/2 , \\ -21/4 , & j = 1 . \end{cases}$$

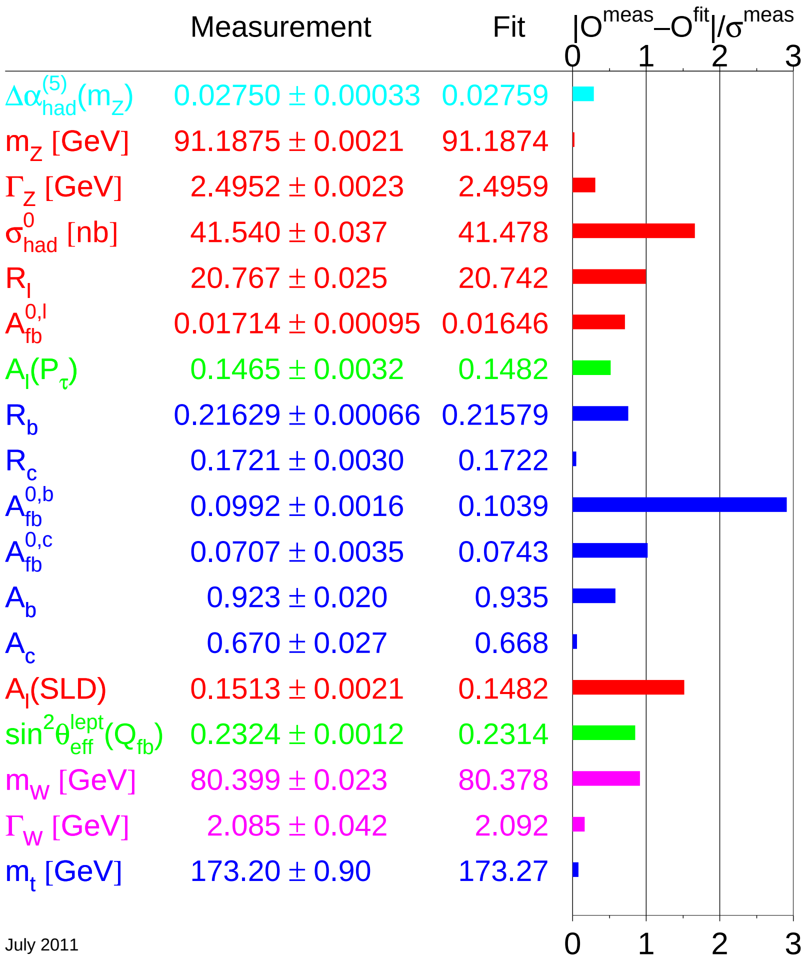
Expectations for the SM Higgs mass

1. Consequences of precision electroweak data.

Very precise tests of the Standard Model are possible given the large sample of electroweak data from LEP, SLC and the Tevatron. Although the Higgs boson mass (m_h) is unknown, electroweak observables are sensitive to m_h through quantum corrections. For example, the W and Z masses are shifted slightly due to:



The m_h dependence of the above radiative corrections is logarithmic. Nevertheless, a global fit of many electroweak observables can determine the preferred value of m_h (assuming that the Standard Model is the correct description of the data).



Measurement
Fit
$|O^{meas}-O^{fit}|/\sigma^{meas}$
0 1 2 3
$\Delta\alpha_{had}^{(5)}(m_Z)$ 0.02750 ± 0.00033 0.02759
m_Z [GeV] 91.1875 ± 0.0021 91.1874
Γ_Z [GeV] 2.4952 ± 0.0023 2.4959
σ_{had}^0 [nb] 41.540 ± 0.037 41.478
R_l 20.767 ± 0.025 20.742
$A_{fb}^{0,l}$ 0.01714 ± 0.00095 0.01646
$A_l(P_\tau)$ 0.1465 ± 0.0032 0.1482
R_b 0.21629 ± 0.00066 0.21579
R_c 0.1721 ± 0.0030 0.1722
$A_{fb}^{0,b}$ 0.0992 ± 0.0016 0.1039
$A_{fb}^{0,c}$ 0.0707 ± 0.0035 0.0743
A_b 0.923 ± 0.020 0.935
A_c 0.670 ± 0.027 0.668
A_l(SLD) 0.1513 ± 0.0021 0.1482
$\sin^2\theta_{eff}^{lept}(Q_{fb})$ 0.2324 ± 0.0012 0.2314
m_W [GeV] 80.399 ± 0.023 80.378
Γ_W [GeV] 2.085 ± 0.042 2.092
m_t [GeV] 173.20 ± 0.90 173.27
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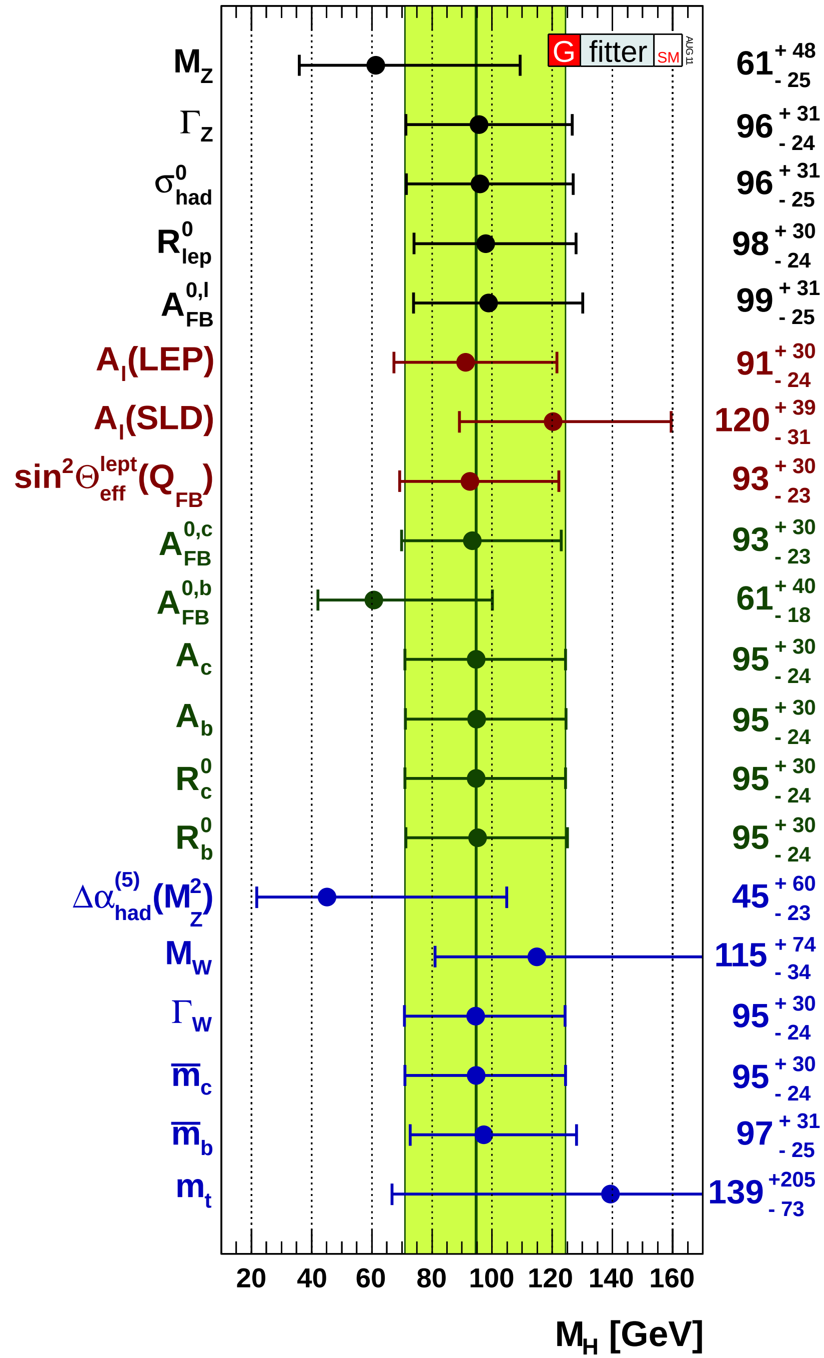
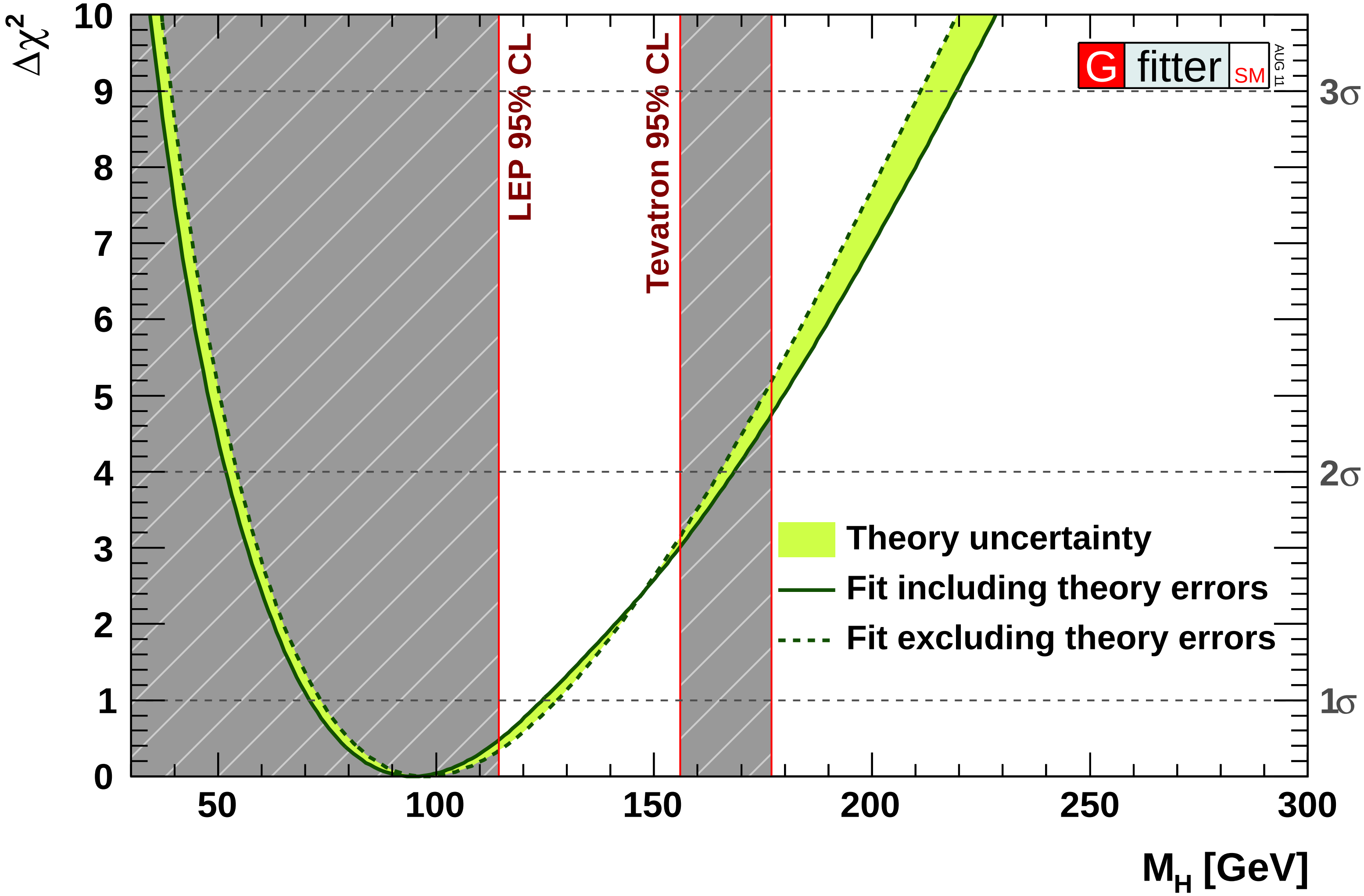
0 1 2 3

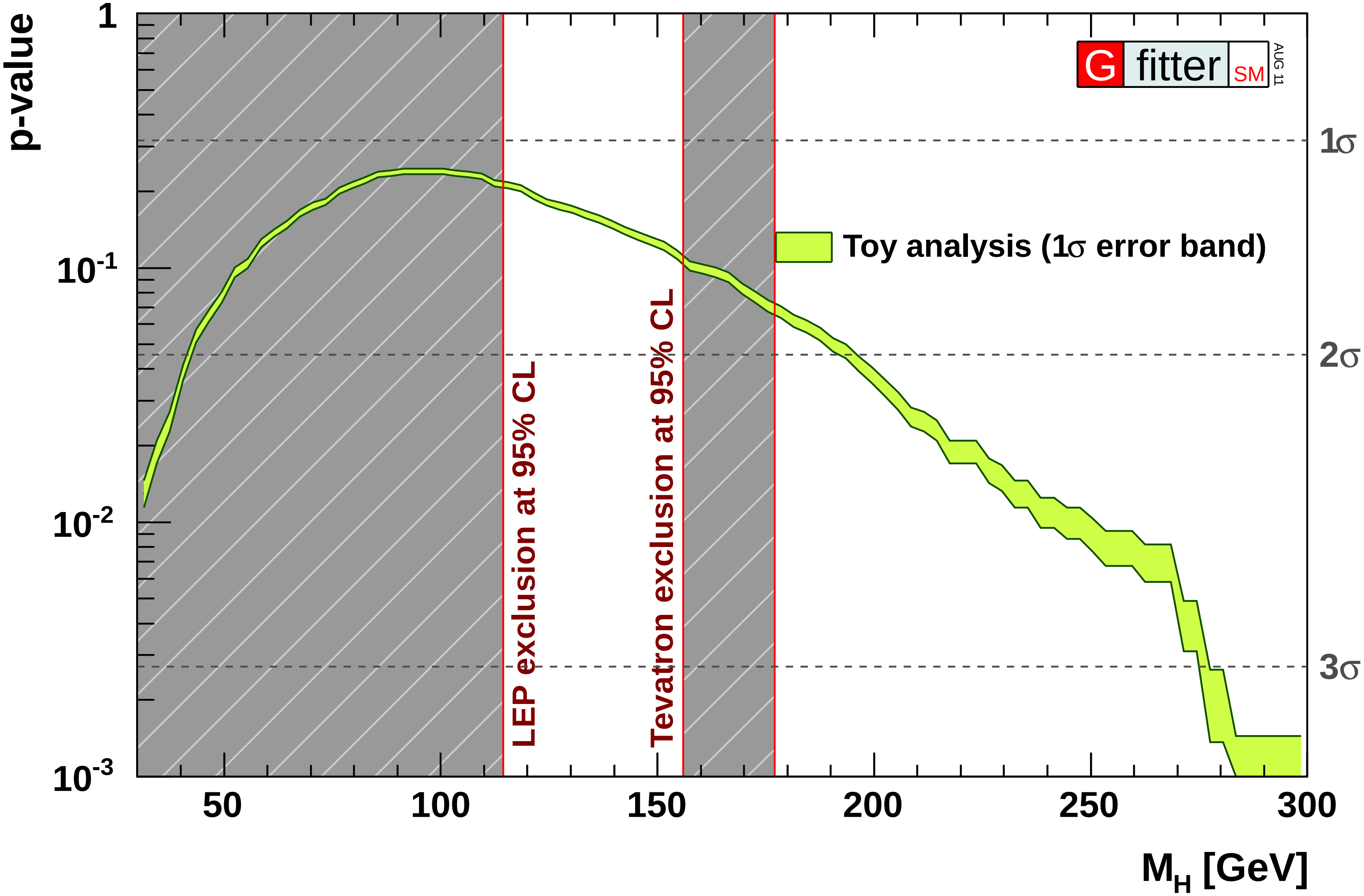


G fitter SM
AUG 11
M_Z 61^{+48}_{-25}
Γ_Z 96^{+31}_{-24}
σ_{had}^0 96^{+31}_{-25}
R_{lep}^0 98^{+30}_{-24}
$A_{FB}^{0,l}$ 99^{+31}_{-25}
A_l(LEP) 91^{+30}_{-24}
A_l(SLD) 120^{+39}_{-31}
$\sin^2\Theta_{eff}^{lept}(Q_{FB})$ 93^{+30}_{-23}
$A_{FB}^{0,c}$ 93^{+30}_{-23}
$A_{FB}^{0,b}$ 61^{+40}_{-18}
A_c 95^{+30}_{-24}
A_b 95^{+30}_{-24}
R_c^0 95^{+30}_{-24}
R_b^0 95^{+30}_{-24}
$\Delta\alpha_{had}^{(5)}(M_Z^2)$ 45^{+60}_{-23}
M_W 115^{+74}_{-34}
Γ_W 95^{+30}_{-24}
$\overline{m}_c$ 95^{+30}_{-24}
$\overline{m}_b$ 97^{+31}_{-25}
m_t 139^{+205}_{-73}
20 40 60 80 100 120 140 160
M_H [GeV]

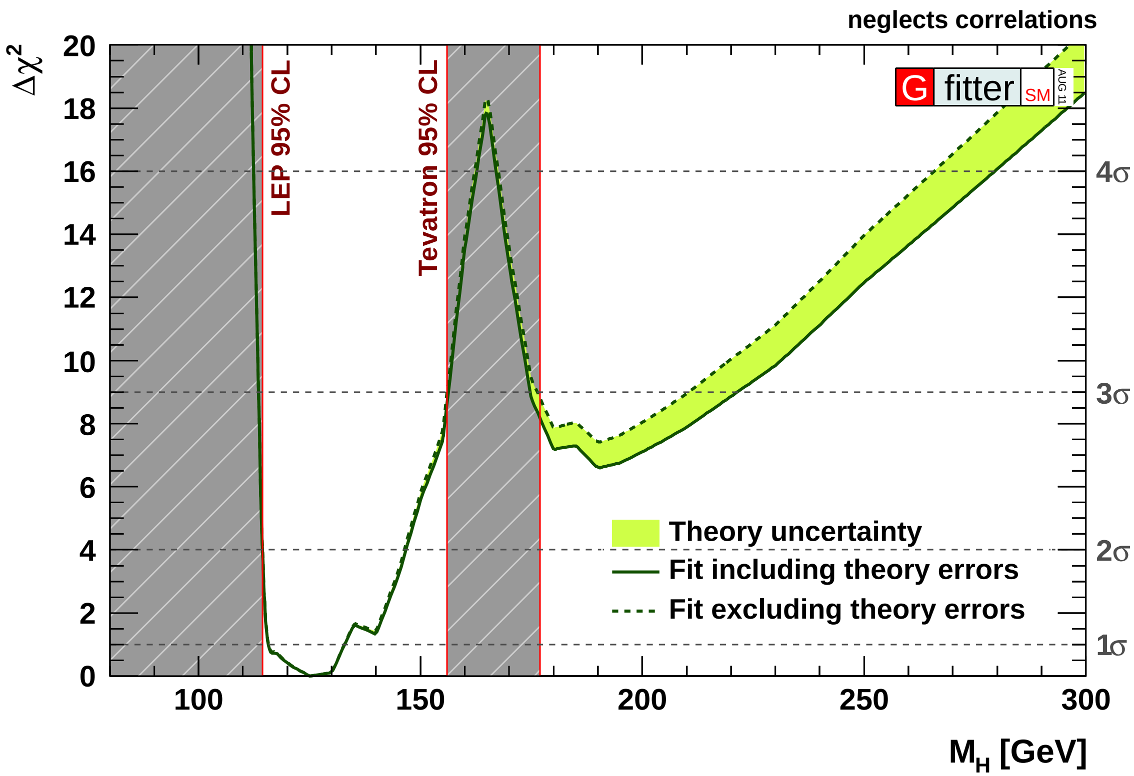


This result, which does *not* employ the direct Higgs search limits, corresponds to a upper bound of $m_h < 169$ GeV at 95% CL and $m_h < 200$ GeV at 99% CL. A similar result of the LEP Electroweak Working group quotes $m_h < 161$ GeV at 95% CL.

Moreover, the global fit to the SM is not too bad if 114 GeV $\lesssim m_h \lesssim 200$ GeV.



Including the direct searches from LEP, Tevatron and the initial LHC Higgs search data prior to the summer of 2011, the GFITTER collaboration obtains a stronger constraint:



These results imply the existence of either:

- a SM-like Higgs boson with $114\ \mathrm{GeV} < m_h < 143\ \mathrm{GeV}$ at 95% CL; or
- new physics beyond the Standard Model, which provides additional corrections to precision electroweak observables that can compensate the effects of a heavier Higgs boson (or no Higgs boson at all!).

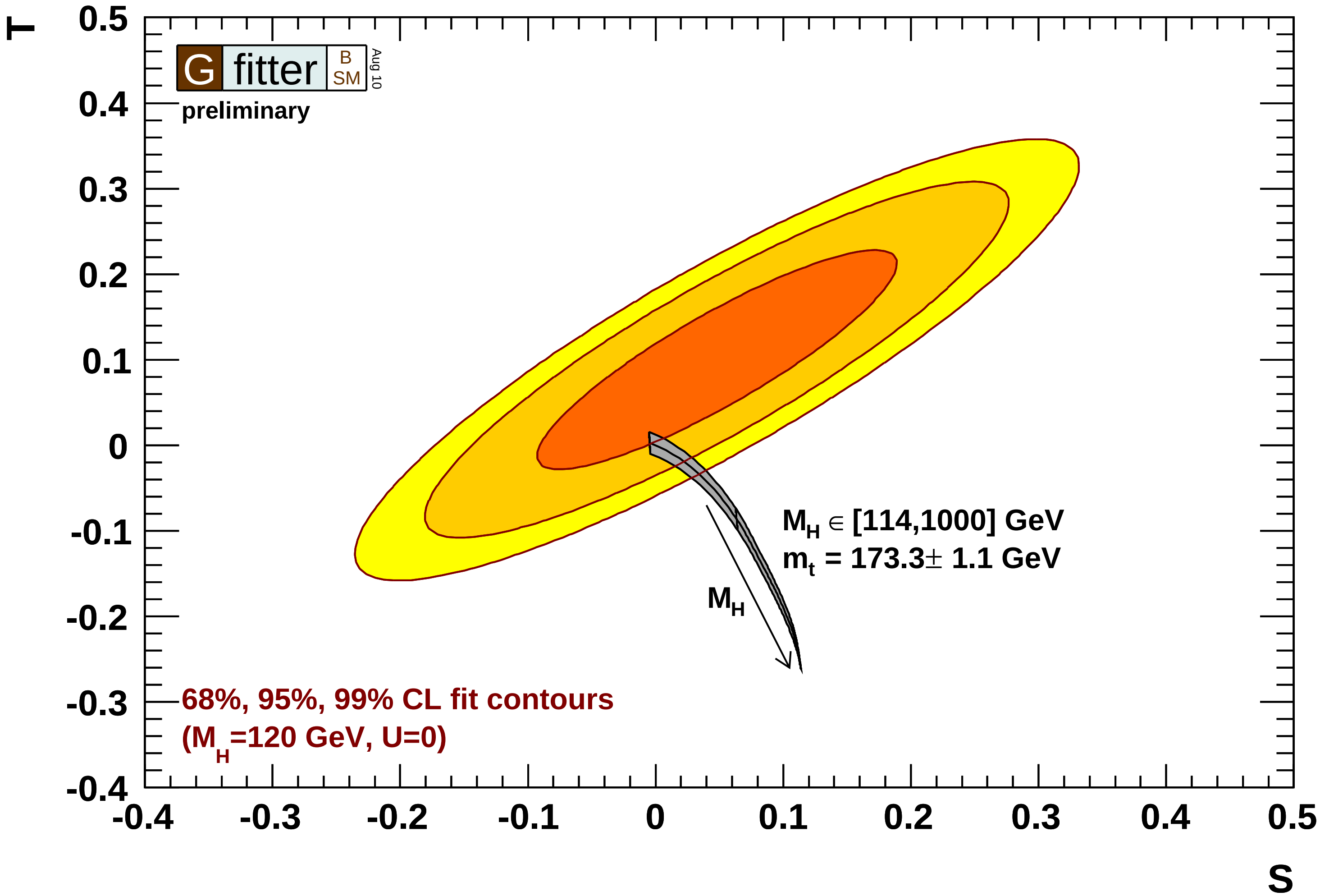
Can a Light Higgs Boson be avoided?

If new physics beyond the Standard Model (SM) exists, it almost certainly couples to W and Z bosons. Then, there will be additional shifts in the W and Z mass due to the appearance of new particles in loops. In many cases, these effects can be parameterized in terms of two quantities, S and T [Peskin and Takeuchi]:

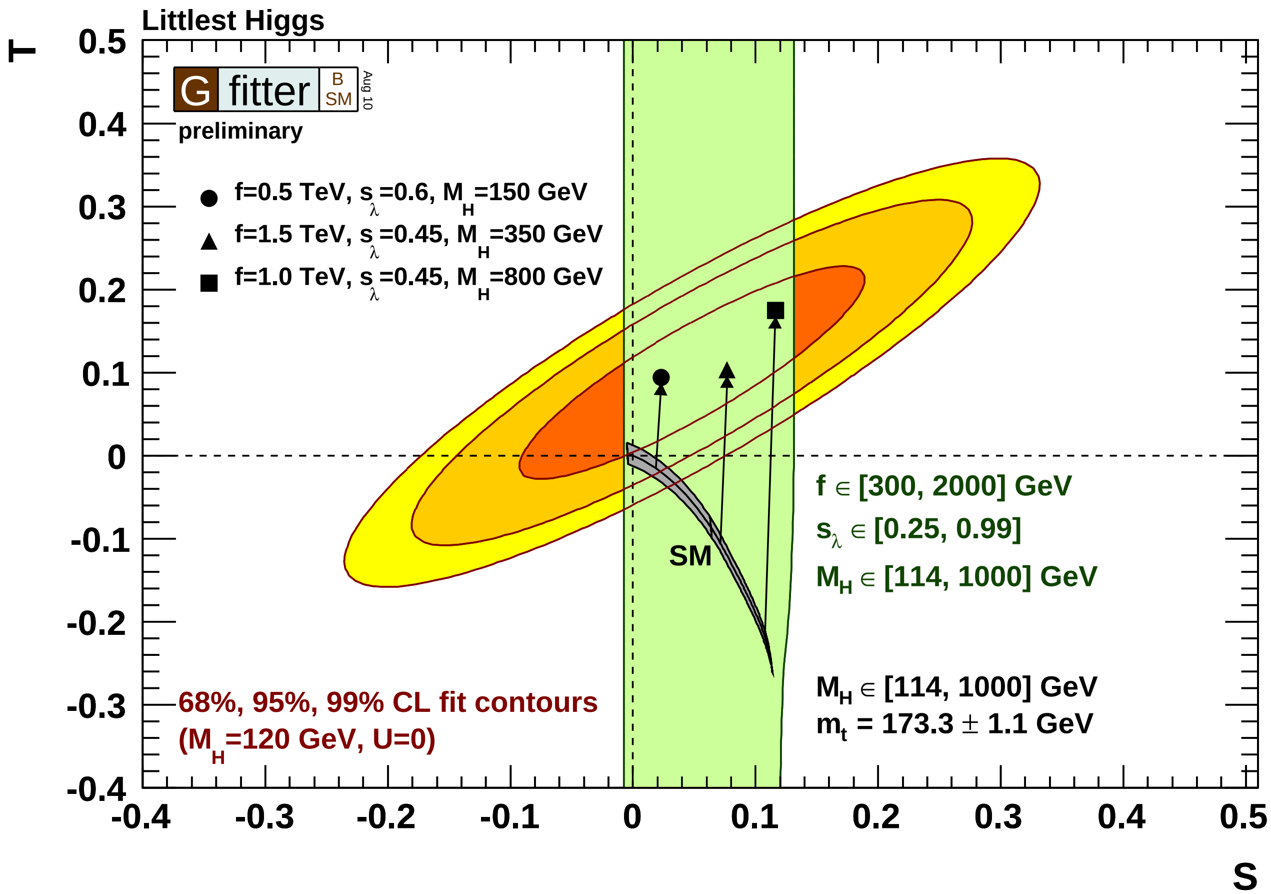
$$\overline{\alpha}\, T \equiv \frac{\Pi^{\rm new}_{WW}(0)}{m_W^2} - \frac{\Pi^{\rm new}_{ZZ}(0)}{m_Z^2}\,,$$

$$\frac{\overline{\alpha}}{4\overline{s}^2_Z\overline{c}^2_Z}\, S \equiv \frac{\Pi^{\rm new}_{ZZ}(m_Z^2) - \Pi^{\rm new}_{ZZ}(0)}{m_Z^2} - \left(\frac{\overline{c}^2_Z - \overline{s}^2_Z}{\overline{c}_Z\overline{s}_Z}\right)\frac{\Pi^{\rm new}_{Z\gamma}(m_Z^2)}{m_Z^2} - \frac{\Pi^{\rm new}_{\gamma\gamma}(m_Z^2)}{m_Z^2}\,,$$

where $s \equiv \sin\theta_W$, $c \equiv \cos\theta_W$, and barred quantities are defined in the $\overline{\rm MS}$ scheme evaluated at m_Z. The $\Pi^{\rm new}_{V_aV_b}$ are the new physics contributions to the one-loop V_a—V_b vacuum polarization functions.



In order to avoid the conclusion of a light Higgs boson, new physics beyond the SM must be accompanied by a variety of new phenomena at an energy scale between 100 GeV and 1 TeV.



This new physics will be detected at future colliders

- either through direct observation of new physics beyond the SM
- or by improved precision measurements that can detect small deviations from SM predictions.

Although the precision electroweak data is suggestive of a weakly-coupled Higgs sector, one cannot definitively rule out another source of EWSB dynamics (although the measured S and T impose strong constraints on alternative approaches).

In alternative models of EWSB, there may be a scalar state with the properties of the Higgs boson that is significantly heavier. Unitarity of $W_L^+ W_L^-$ scattering (which is violated in the SM in the absence of a Higgs boson) can be restored either by new physics beyond the Standard Model (e.g., the techni-rho of technicolor or Kaluza-Klein states of extra-dimensional models) or by the heavier Higgs boson itself. Suppose we assume the latter. How heavy can this Higgs boson be?

Can the Higgs Boson mass be large?

A Higgs boson with a mass greater than 200 GeV requires additional new physics beyond the Standard Model. But, how heavy can this Higgs boson be?

Let us return to the unitarity argument. Consider the scattering process $W_L^+(p_1)W_L^-(p_2) \to W_L^+(p_3)W_L^-(p_4)$ at center-of-mass energies $\sqrt{s} \gg m_W$. Each contribution to the tree-level amplitude is proportional to

$$[\varepsilon_L(p_1) \cdot \varepsilon_L(p_2)]\,[\varepsilon_L(p_3) \cdot \varepsilon_L(p_4)] \sim \frac{s^2}{m_W^4}\,,$$

after using the fact that the helicity-zero polarization vector at high energies behaves as $\varepsilon_L^\mu(p) \sim p^\mu/m_W$. Due to the magic of gauge invariance and the presence of Higgs-exchange contributions, the bad high-energy behavior is removed, and one finds for $s,\, m_h^2 \gg m_W^2$:

$$\mathcal{M} = -\sqrt{2}G_F m_H^2 \left(\frac{s}{s-m_h^2} + \frac{t}{t-m_h^2}\right)\,.$$

Projecting out the $J = 0$ partial wave and taking $s \gg m_h^2$,

$$\mathcal{M}^{J=0} = -\frac{G_F m_h^2}{4\pi\sqrt{2}}\,.$$

Imposing $|\mathrm{Re}\ \mathcal{M}^J| \leq \frac{1}{2}$ yields an upper bound on m_h. The most stringent bound is obtained by all considering other possible final states such as $Z_L Z_L$, $Z_L h^0$ and $h^0 h^0$. The end result is:

$$\boxed{m_h^2 \leq \frac{4\pi\sqrt{2}}{3G_F} \simeq (700\ \mathrm{GeV})^2\,.}$$

However, in contrast to our previous analysis of the unitarity bound, the above computation relies on the validity of a tree-level computation. That is, we are implicitly assuming that perturbation theory is valid. If $m_h \gtrsim 700$ GeV, then the Higgs-self coupling parameter, $\lambda = 2m_h^2/v^2$ is becoming large and our perturbative analysis is becoming suspect.

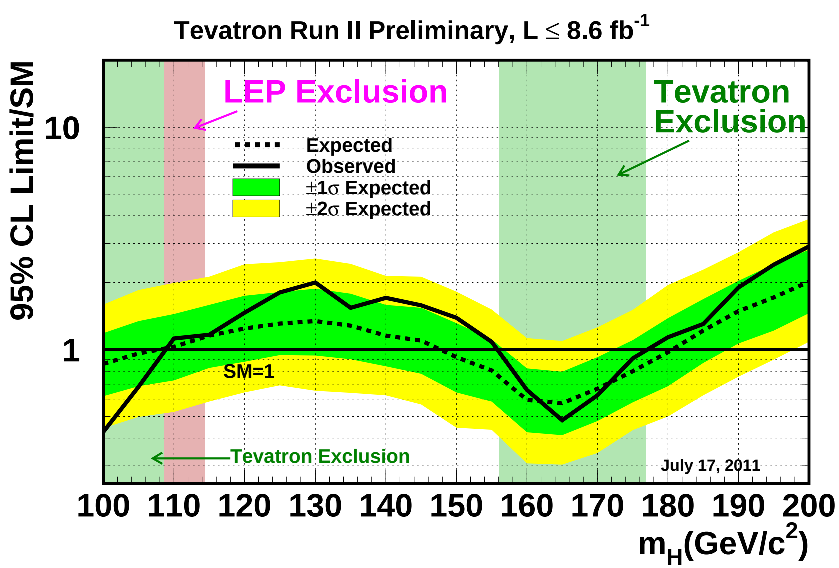
Nevertheless, lattice studies suggest that an upper Higgs mass bound below 1 TeV remains valid even in the strong Higgs self-coupling regime.

2. Higgs mass bounds from collider searches.

From 1989–2000, experiments at LEP searched for $e^+e^- \to Z \to h^0 Z$ (where one of the Z-bosons is on-shell and one is off-shell). No significant evidence was found leading to a lower bound on the SM Higgs mass

$$\boxed{m_h > 114.4 \text{ GeV} \quad \text{at} \quad 95\% \text{ CL.}}$$

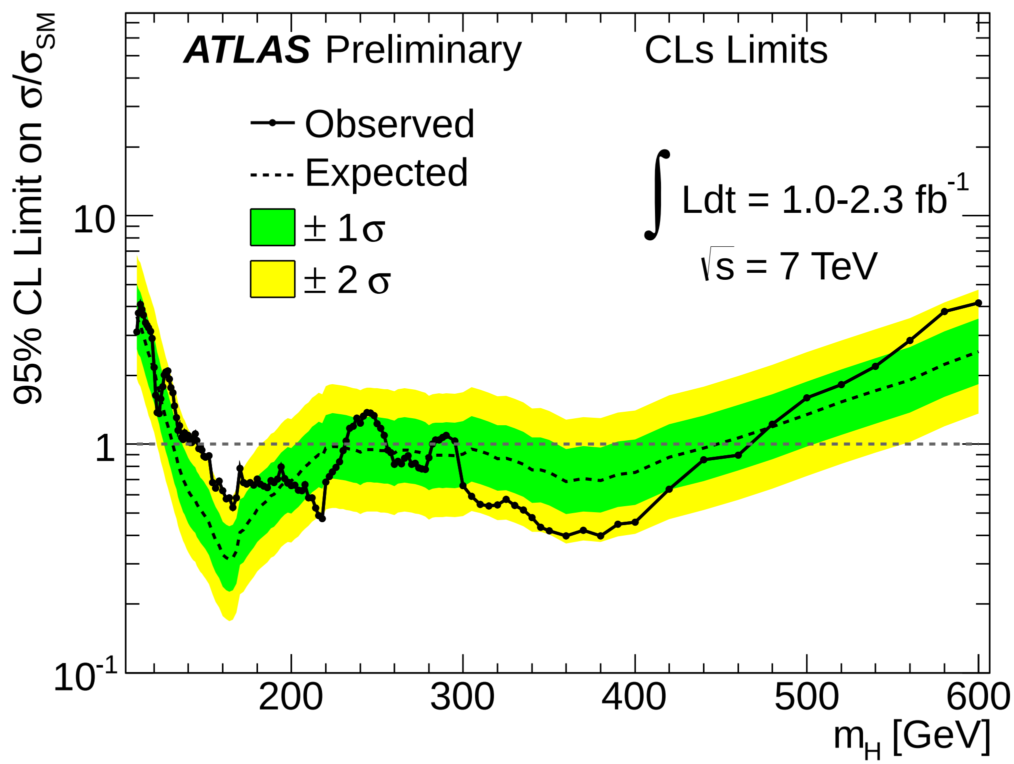
Searches at the Tevatron and LHC extend the 95% excluded region of Higgs masses. Tevatron data excludes $156 \text{ GeV} < m_h < 177 \text{ GeV}$ at 95% CL.

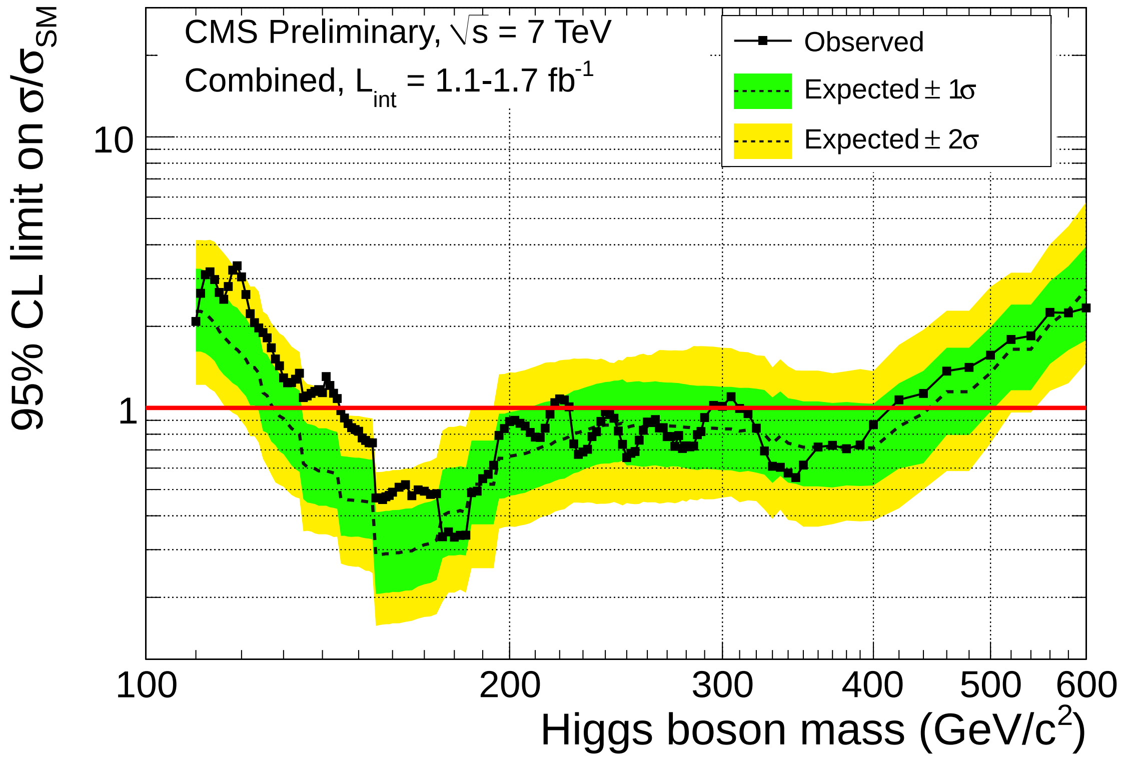


The excluded mass region above the LEP Higgs mass bound obtained by the CMS collaboration is:

$$\boxed{144 \text{ GeV} < m_h < 440 \text{ GeV} \quad \text{at} \quad 90\% \text{ CL.}}$$

CMS and ATLAS also obtain 95% CL exclusion regions that are roughly similar with a few small intervals in the above mass range that cannot quite be excluded with present data.





Bill Murray will tell us more about the Higgs searches at LHC. Abdelhak Djouadi will discuss in detail the phenomenological profile of the SM Higgs boson.

Lecture II: Weakly coupled Higgs bosons beyond the SM

Outline

- Expanding the Higgs sector
- The Two-Higgs Doublet Model
- The decoupling limit
- The significance of the TeV-scale—Part 2
- The MSSM Higgs sector at tree-level
- Saving the MSSM Higgs sector—the impact of radiative corrections
- Constraints from present data
- Beyond the MSSM Higgs sector

Constraints on the non-minimal Higgs sector

Three generations of fermions appear in nature, with each generation possessing the same quantum numbers under the SU(3)×SU(2)×U(1)$_{\rm Y}$ gauge group. So, why should the scalar sector be of minimal form?

For an arbitrary Higgs sector, the tree-level ρ-parameter is given by

$$\rho_0 \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{\sum_{T,Y}[4T(T+1)-Y^2]|V_{T,Y}|^2 c_{T,Y}}{\sum_{T,Y} 2Y^2|V_{T,Y}|^2}\,,$$

where $V_{T,Y} \equiv \langle\phi(T,Y)\rangle$ defines the vacuum expectation values (vevs) of each neutral Higgs field, and T and Y specify the total SU(2) isospin and the hypercharge of the Higgs representation to which it belongs. Y is normalized such that the electric charge of the scalar field is $Q = T_3 + Y/2$, and

$$c_{T,Y} = \begin{cases} 1, & (T,Y) \in \text{complex representation,} \\ \frac{1}{2}, & (T, Y=0) \in \text{real representation.} \end{cases}$$

For the complex ($c = 1$) Higgs doublet of the Standard Model with $T = 1/2$ and $Y = 1$, it follows that $\rho_0 = 1$ as strongly suggested by the electroweak data. The same result follows from a Higgs sector consisting of multiple complex Higgs doublets (independent of the neutral Higgs vevs). One can also add Higgs singlets ($T = Y = 0$) without changing the value of ρ_0.

But, one cannot add arbitrary Higgs multiplets in general[†] unless their corresponding vevs are very small (typically $|V_{T,Y}| \lesssim 0.05v \sim 10~\mathrm{GeV}$).

Thus, we shall consider non-minimal Higgs sectors consisting of multiple Higgs doublets (and perhaps Higgs singlets), but no higher Higgs representations, in order to avoid the fine-tuning of Higgs vevs.

[†]To automatically have $\rho_0 = 1$ independently of the Higgs vevs, one must satisfy

$$(2T + 1)^2 - 3Y^2 = 1$$

for integer values of $(2T, Y)$. The smallest nontrivial solution beyond the complex $Y = 1$ Higgs doublet is a Higgs multiplet with $T = 3$ and $Y = 4$.

The Two-Higgs doublet model (2HDM)

Consider the two-Higgs-doublet model, consisting of two-complex hypercharge-one scalar doublets Φ_1 and Φ_2. Of the eight initial degrees of freedom, five are physical (after three Goldstone bosons provide masses for the $W^\pm$ and Z). The five physical scalars are: a charged Higgs pair, $H^\pm$, and three neutral scalars. In contrast to the SM, where the Higgs-sector is CP-conserving, the 2HDM allows for Higgs-mediated CP-violation. If CP is conserved, the three scalars can be classified as two CP-even scalars, h^0 and H^0 (where $m_h < m_H$ as the notation suggests) and a CP-odd scalar A^0.

Thus, new features of the extended Higgs sector include:

- Charged Higgs bosons
- A CP-odd Higgs boson (if CP is conserved in the Higgs sector)
- Higgs-mediated CP-violation (and neutral Higgs states of indefinite CP)

More exotic Higgs sectors allow for doubly-charged Higgs bosons, etc.

Consider the most general renormalizable scalar Higgs potential,

$$\begin{aligned}\mathcal{V} = {} & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}] + \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 \\ & + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ & + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right\} ,\end{aligned}$$

where m_{12}^2, λ_5, λ_6 and λ_7 are potentially complex parameters. There is a significant region of the 2HDM parameter space in which the vacuum expectation values (vevs) of the two Higgs fields are:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} , \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix} ,$$

where $v^2 \equiv |v_1|^2 + |v_2|^2 = (246~\mathrm{GeV})^2$. The vevs are aligned along the neutral direction, in which case the SU(2)$\times$U(1) electroweak symmetry is spontaneously broken to U(1)$_{\mathrm{EM}}$ as it is in the Standard Model.

It is convenient to define new Higgs doublet fields:

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \equiv \frac{v_1^* \Phi_1 + v_2^* \Phi_2}{v}, \qquad\qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \equiv \frac{-v_2 \Phi_1 + v_1 \Phi_2}{v}.$$

It follows that

$$\langle H_1^0 \rangle = \frac{v}{\sqrt{2}}, \qquad\qquad \langle H_2^0 \rangle = 0\,.$$

This is the so-called *Higgs basis*, which is uniquely defined up to a possible rephasing of H_2.

In the Higgs basis, the scalar potential is given by:

$$\begin{aligned}
\mathcal{V} = {} & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2 \\
& + \tfrac{1}{2} Z_2 (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\
& + \left\{ \tfrac{1}{2} Z_5 (H_1^\dagger H_2)^2 + \big[Z_6 (H_1^\dagger H_1) + Z_7 (H_2^\dagger H_2) \big] H_1^\dagger H_2 + \text{h.c.} \right\},
\end{aligned}$$

The scalar potential minimum conditions: $Y_1 = -\frac{1}{2} Z_1 v^2$ and $Y_3 = -\frac{1}{2} Z_6 v^2$.

The Higgs mass-eigenstate basis

In the Higgs basis, we immediately identify $H_1^+ = G^+$ (the charged Goldstone boson that provides the longitudinal component of the W^+) and $H_2^+ = H^+$ (the physical charged Higgs boson, with $m^2_{H^\pm} = Y_2 + \frac{1}{2}Z_3 v^2$). The three physical neutral Higgs boson mass-eigenstates are determined by diagonalizing a 3×3 real symmetric squared-mass matrix

$$\mathcal{M}^2 = v^2 \begin{pmatrix} Z_1 & \mathrm{Re}(Z_6) & -\mathrm{Im}(Z_6) \\ \mathrm{Re}(Z_6) & \frac{1}{2}Z_{345} + Y_2/v^2 & -\frac{1}{2}\mathrm{Im}(Z_5) \\ -\mathrm{Im}(Z_6) & -\frac{1}{2}\mathrm{Im}(Z_5) & \frac{1}{2}Z_{345} - \mathrm{Re}(Z_5) + Y_2/v^2 \end{pmatrix},$$

where $Z_{345} \equiv Z_3 + Z_4 + \mathrm{Re}(Z_5)$. The real symmetric squared-mass matrix $\mathcal{M}^2$ can be diagonalized by an orthogonal transformation

$$R\mathcal{M}^2 R^T = \mathcal{M}_D^2 \equiv \mathrm{diag}\,(m_1^2\,,\, m_2^2\,,\, m_3^2)\,,$$

where $RR^T = I$ and the m_k^2 are the eigenvalues of $\mathcal{M}^2$.

An explicit form for R is:

$$R = R_{12}R_{13}R_{23} = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & -s_{13} \\ 0 & 1 & 0 \\ s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix}$$

$$= \begin{pmatrix} c_{13}c_{12} & -c_{23}s_{12} - c_{12}s_{13}s_{23} & -c_{12}c_{23}s_{13} + s_{12}s_{23} \\ c_{13}s_{12} & c_{12}c_{23} - s_{12}s_{13}s_{23} & -c_{23}s_{12}s_{13} - c_{12}s_{23} \\ s_{13} & c_{13}s_{23} & c_{13}c_{23} \end{pmatrix},$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.

It is then convenient to define the quantities $q_{k\ell}$,

k	q_{k1}	q_{k2}
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - is_{12}s_{13}$
3	s_{13}	ic_{13}
4	i	0

One can express the Higgs fields of the Higgs basis in terms of the mass eigenstate neutral Higgs fields h_1, h_2 and h_3, the neutral Goldstone boson $h_4 \equiv G^0$, the charged Higgs field H^+ and the charged Goldstone field G^+,

$$H_1 = \begin{pmatrix} G^+ \\ \dfrac{v}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} \displaystyle\sum_{k=1}^{4} q_{k1} h_k \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H^+ \\ \dfrac{1}{\sqrt{2}} \displaystyle\sum_{k=1}^{4} q_{k2} e^{-i\theta_{23}} h_k \end{pmatrix},$$

Since H_2 is only defined up to an overall phase, one can always choose $\theta_{23} = 0$ without loss of generality, and absorb the remaining θ_{23} dependence by a rephasing of the definition of the charged Higgs field.

Plugging the above into the Higgs Lagrangian (in the Higgs basis), one derives a compact form for all the Higgs interactions with gauge bosons and Higgs bosons.

Reference: H.E. Haber and D. O'Neil, "Basis-independent methods for the two-Higgs doublet model. II: The significance of $\tan\beta$," *Phys. Rev.* **D74**, 015018 (2006).

The gauge boson–Higgs boson interactions

$$\mathscr{L}_{VVH} = \left(g m_W W_\mu^+ W^{\mu\,-} + \frac{g}{2c_W} m_Z Z_\mu Z^\mu\right) \mathrm{Re}(q_{k1}) h_k + e m_W A^\mu (W_\mu^+ G^- + W_\mu^- G^+)$$

$$- g m_Z s_W^2 Z^\mu (W_\mu^+ G^- + W_\mu^- G^+)\,,$$

$$\mathscr{L}_{VVHH} = \left[\tfrac{1}{4} g^2 W_\mu^+ W^{\mu\,-} + \frac{g^2}{8c_W^2} Z_\mu Z^\mu\right] \mathrm{Re}(q_{j1}^* q_{k1} + q_{j2}^* q_{k2})\, h_j h_k$$

$$+ \left[\tfrac{1}{2} g^2 W_\mu^+ W^{\mu\,-} + e^2 A_\mu A^\mu + \frac{g^2}{c_W^2}\left(\tfrac{1}{2} - s_W^2\right)^2 Z_\mu Z^\mu + \frac{2ge}{c_W}\left(\tfrac{1}{2} - s_W^2\right) A_\mu Z^\mu\right] (G^+ G^- + H^+ H^-)$$

$$+ \left\{\left(\tfrac{1}{2} e g A^\mu W_\mu^+ - \frac{g^2 s_W^2}{2c_W} Z^\mu W_\mu^+\right) (q_{k1} G^- + q_{k2}\, e^{-i\theta_{23}} H^-) h_k + \text{h.c.}\right\},$$

$$\mathscr{L}_{VHH} = \frac{g}{4c_W} \mathrm{Im}(q_{j1} q_{k1}^* + q_{j2} q_{k2}^*) Z^\mu h_j \overleftrightarrow{\partial}_\mu\, h_k - \tfrac{1}{2} g \left\{ i W_\mu^+ \left[q_{k1} G^- \overleftrightarrow{\partial}^\mu\, h_k + q_{k2} e^{-i\theta_{23}} H^- \overleftrightarrow{\partial}^\mu\, h_k\right] + \text{h.c.}\right\}$$

$$+ \left[i e A^\mu + \frac{ig}{c_W}\left(\tfrac{1}{2} - s_W^2\right) Z^\mu\right] (G^+ \overleftrightarrow{\partial}_\mu\, G^- + H^+ \overleftrightarrow{\partial}_\mu\, H^-)\,,$$

where $s_W \equiv \sin\theta_W$ and $c_W \equiv \cos\theta_W$.

The cubic and quartic Higgs couplings

$$\mathscr{L}_{3h} = -\tfrac{1}{2} v\, h_j h_k h_\ell \Big[q_{j1} q^*_{k1} \mathrm{Re}(q_{\ell 1}) Z_1 + q_{j2} q^*_{k2}\, \mathrm{Re}(q_{\ell 1})(Z_3+Z_4) + \mathrm{Re}(q^*_{j1} q_{k2} q_{\ell 2} Z_5\, e^{-2i\theta_{23}})$$

$$+\mathrm{Re}\Big([2q_{j1} + q^*_{j1}] q^*_{k1} q_{\ell 2} Z_6\, e^{-i\theta_{23}}\Big) + \mathrm{Re}(q^*_{j2} q_{k2} q_{\ell 2} Z_7\, e^{-i\theta_{23}})\Big]$$

$$-v\, h_k G^+ G^- \Big[\mathrm{Re}(q_{k1}) Z_1 + \mathrm{Re}(q_{k2}\, e^{-i\theta_{23}} Z_6)\Big] + v\, h_k H^+ H^- \Big[\mathrm{Re}(q_{k1}) Z_3 + \mathrm{Re}(q_{k2}\, e^{-i\theta_{23}} Z_7)\Big]$$

$$-\tfrac{1}{2} v\, h_k \Big\{ G^- H^+\, e^{i\theta_{23}} \Big[q^*_{k2} Z_4 + q_{k2}\, e^{-2i\theta_{23}} Z_5 + 2\mathrm{Re}(q_{k1}) Z_6\, e^{-i\theta_{23}}\Big] + \mathrm{h.c.}\Big\}\,,$$

$$\mathscr{L}_{4h} = -\tfrac{1}{8} h_j h_k h_l h m \Big[q_{j1} q_{k1} q^*_{\ell 1} q^*_{m1} Z_1 + q_{j2} q_{k2} q^*_{\ell 2} q^*_{m2} Z_2 + 2 q_{j1} q^*_{k1} q_{\ell 2} q^*_{m2} (Z_3 + Z_4)$$

$$+2\mathrm{Re}(q^*_{j1} q^*_{k1} q_{\ell 2} q_{m2} Z_5\, e^{-2i\theta_{23}}) + 4\mathrm{Re}(q_{j1} q^*_{k1} q^*_{\ell 1} q_{m2} Z_6\, e^{-i\theta_{23}}) + 4\mathrm{Re}(q^*_{j1} q_{k2} q_{\ell 2} q^*_{m2} Z_7\, e^{-i\theta_{23}})\Big]$$

$$-\tfrac{1}{2} h_j h_k G^+ G^- \Big[q_{j1} q^*_{k1} Z_1 + q_{j2} q^*_{k2} Z_3 + 2\mathrm{Re}(q_{j1} q_{k2} Z_6\, e^{-i\theta_{23}})\Big]$$

$$-\tfrac{1}{2} h_j h_k H^+ H^- \Big[q_{j2} q^*_{k2} Z_2 + q_{j1} q^*_{k1} Z_3 + 2\mathrm{Re}(q_{j1} q_{k2} Z_7\, e^{-i\theta_{23}})\Big]$$

$$-\tfrac{1}{2} h_j h_k \Big\{ G^- H^+\, e^{i\theta_{23}} \Big[q_{j1} q^*_{k2} Z_4 + q^*_{j1} q_{k2} Z_5\, e^{-2i\theta_{23}} + q_{j1} q^*_{k1} Z_6\, e^{-i\theta_{23}} + q_{j2} q^*_{k2} Z_7\, e^{-i\theta_{23}}\Big] + \mathrm{h.c.}\Big\}$$

$$-\tfrac{1}{2} Z_1 G^+ G^- G^+ G^- - \tfrac{1}{2} Z_2 H^+ H^- H^+ H^- - (Z_3 + Z_4) G^+ G^- H^+ H^-$$

$$-\tfrac{1}{2}(Z_5 H^+ H^+ G^- G^- + Z_5^* H^- H^- G^+ G^+) - G^+ G^- (Z_6 H^+ G^- + Z_6^* H^- G^+) - H^+ H^- (Z_7 H^+ G^- + Z_7^* H^- G^+)\,.$$

Higgs-fermion Yukawa couplings in the 2HDM

The 2HDM Higgs-fermion Yukawa Lagrangian is:

$$-\mathscr{L}_{\rm Y} = \overline{Q}_L\widetilde{\Phi}_a h_a^U U_R + \overline{Q}_L\,\Phi_a h_a^{D\,\dagger} D_R + {\rm h.c.}\,,$$

where $\widetilde{\Phi}_a \equiv i\sigma_2\Phi_a^*$, $Q_L \equiv (U_L, D_L)$ is the weak isospin quark doublet, and U_R, U_R are weak isospin quark singlets. There is an implicit sum over $a = 1, 2$ and flavor indices are suppressed. As before, we redefine the quark fields

$$U_L \to V_L^U U_L\,, \qquad U_R \to V_R^U U_R\,, \qquad D_L \to V_L^D D_L\,, \qquad D_R \to V_R^D D_R\,,$$

and the CKM matrix is defined by $K \equiv V_L^U V_L^{D\,\dagger}$. Likewise we redefine the 3×3 Yukawa coupling matrices, $h_a^U \to V_L^U h_a^U V_R^{U\,\dagger}$ and $h_a^D \to V_R^D h_a^D V_L^{D\,\dagger}$. These redefinitions yield:

$$-\mathscr{L}_{\rm Y} = \overline{U}_L\Phi_a^{0\,*}h_a^U U_R - \overline{D}_L K^\dagger\Phi_a^- h_a^U U_R + \overline{U}_L K\Phi_a^+ h_a^{D\,\dagger} D_R + \overline{D}_L\Phi_a^0 h_a^{D\,\dagger} D_R + {\rm h.c.}$$

In the Higgs basis, the Yukawa coupling matrices will be denoted $\kappa^{U,D}$ and $\rho^{U,D}$, and

$$\begin{aligned}-\mathscr{L}_{\rm Y} = &\overline{U}_L(\kappa^U H_1^{0\,\dagger} + \rho^U H_2^{0\,\dagger})U_R - \overline{D}_L K^\dagger(\kappa^U H_1^- + \rho^U H_2^-)U_R \\ &+\overline{U}_L K(\kappa^{D\,\dagger} H_1^+ + \rho^{D\,\dagger} H_2^+)D_R + \overline{D}_L(\kappa^{D\,\dagger} H_1^0 + \rho^{D\,\dagger} H_2^0)D_R + {\rm h.c.}\end{aligned}$$

By setting $H_1^0 = v/\sqrt{2}$ and $H_2^0 = 0$, we see that κ^U and κ^D are proportional to the quark mass matrices M_U and M_D, respectively. The matrices V_L^U, V_R^U, V_L^D and V_R^D introduced above are chosen such that κ^U and κ^D are diagonal with non-negative elements (via the singular value decomposition):

$$M_U = \frac{v}{\sqrt{2}}\kappa^U = \mathrm{diag}(m_u\,,\, m_c\,,\, m_t)\,, \qquad M_D = \frac{v}{\sqrt{2}}\kappa^{D\,\dagger} = \mathrm{diag}(m_d\,,\, m_s\,,\, m_b)\,.$$

The matrices ρ^U and ρ^D are independent complex 3×3 matrices. The final form for the Yukawa couplings of the mass-eigenstate Higgs bosons and the Goldstone bosons to the quarks is

$$\begin{aligned}
-\mathscr{L}_Y = {} & \frac{1}{v}\overline{D}\Big\{M_D(q_{k1}P_R + q^*_{k1}P_L) + \frac{v}{\sqrt{2}}\left[q_{k2}\,[e^{i\theta_{23}}\rho^D]^\dagger P_R + q^*_{k2}\,e^{i\theta_{23}}\rho^D P_L\right]\Big\}Dh_k \\
& + \frac{1}{v}\overline{U}\Big\{M_U(q_{k1}P_L + q^*_{k1}P_R) + \frac{v}{\sqrt{2}}\left[q^*_{k2}\,e^{i\theta_{23}}\rho^U P_R + q_{k2}\,[e^{i\theta_{23}}\rho^U]^\dagger P_L\right]\Big\}Uh_k \\
& + \Big\{\overline{U}\left[K[\rho^D]^\dagger P_R - [\rho^U]^\dagger K P_L\right]DH^+ + \frac{\sqrt{2}}{v}\,\overline{U}\,[KM_DP_R - M_UKP_L]\,DG^+ + \mathrm{h.c.}\Big\}\,.
\end{aligned}$$

where $P_{L,R} = \frac{1}{2}(1\mp\gamma_5)$. The couplings of the neutral Higgs bosons to quark pairs are generically CP-violating due to the fact that the q_{k2} and the matrices $e^{i\theta_{23}}\rho^Q$ are not generally either pure real or pure imaginary.

Higgs-mediated FCNCs exist if $\rho^{U,D}$ are non-diagonal. This is a generic feature of multi-Higgs doublet models, as more than one Yukawa coupling matrix (one for each Higgs doublet) contributes to each of the up and down-type fermion mass matrices. Diagonalizing the quark mass matrix diagonalizes only one linear combination of the Yukawa coupling matrices.

However, one can recover flavor-diagonal Yukawa couplings by restricting the form of the Higgs-fermion Lagrangian. Glashow and Weinberg showed that a sufficient condition is to require that at most one neutral Higgs field couple to fermions of a given electric charge.

To avoid FCNCs in the 2HDM, one can impose a discrete symmetry to restrict the structure of the Higgs-fermion Yukawa Lagrangian (consistent with the Glashow-Weinberg theorem) in the original basis of scalar fields. In this basis, we define $\tan\beta = \langle\Phi_2^0\rangle/\langle\Phi_1^0\rangle$. Possible choices for the discrete symmetry are:

- Type-I Yukawa couplings: $h_2^U = h_2^D = 0$,

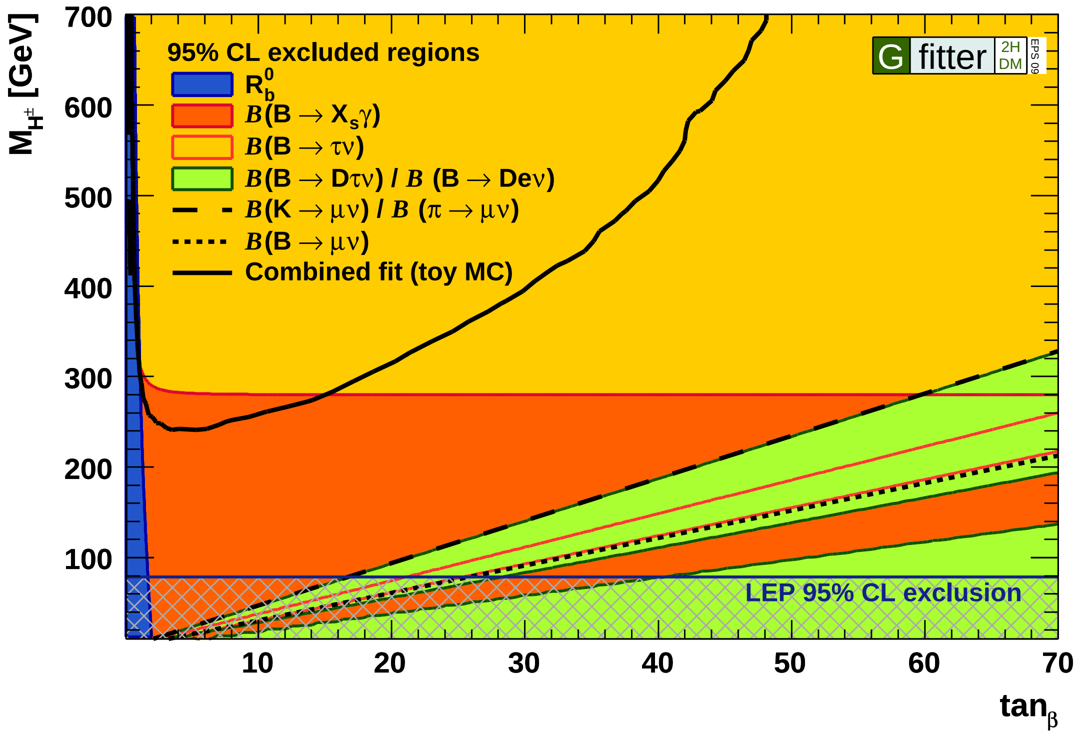
$$\rho^D = -\frac{\sqrt{2}M_d\tan\beta}{v}\,, \qquad \rho^U = -\frac{\sqrt{2}M_u\tan\beta}{v}\,.$$

- Type-II Yukawa couplings: $h_1^U = h_2^D = 0$,

$$\rho^D = -\frac{\sqrt{2}M_d \tan\beta}{v}\,, \qquad\qquad \rho^U = \frac{\sqrt{2}M_u \cot\beta}{v}\,.$$

In both cases, the $\rho^{U,D}$ are diagonal and real, in which case there are no tree-level FCNCs and no CP-violating neutral Higgs–fermion couplings. (Type-II Higgs-fermion Yukawa couplings can also be imposed by supersymmetry. More on that shortly.)

There are interesting experimental constraints on the Type-II 2HDM.



The decoupling limit of the 2HDM

The decoupling limit corresponds to the limiting case in which one of the two Higgs doublets of the 2HDM receives a very large mass and is therefore decoupled from the theory. This can be achieved by assuming that $Y_2 \gg v^2$ and $|Z_i| \lesssim \mathcal{O}(1)$ [for all i]. It is critical that all Higgs self-coupling parameters remain small in this limit. The effective low energy theory is then a one-Higgs-doublet model that corresponds to the Higgs sector of the Standard Model.

We shall order the neutral scalar masses according to $m_1 < m_{2,3}$ and define the Higgs mixing angles accordingly. Thus, we expect one light CP-even Higgs boson, h_1, with couplings identical (up to small corrections) to those of the Standard Model (SM) Higgs boson. One can show that the conditions for the decoupling limit are:

$$|s_{12}| \lesssim \mathcal{O}\left(\frac{v^2}{m_2^2}\right) \ll 1\,, \qquad |s_{13}| \lesssim \mathcal{O}\left(\frac{v^2}{m_3^2}\right) \ll 1\,,$$

$$\mathrm{Im}(Z_5\, e^{-2i\theta_{23}}) \lesssim \mathcal{O}\left(\frac{v^2}{m_3^2}\right) \ll 1\,.$$

One can explicitly verify that in the approach to the decoupling limit, we have $m_1 \ll m_2, m_3, m_{H^\pm}$. In particular, $m_1^2 = Z_1 v^2$, with corrections of $\mathcal{O}(v^4/m_{2,3}^2)$, and $m_2 \simeq m_3 \simeq m_{H^\pm}$ with squared mass splittings of $\mathcal{O}(v^2)$.

In the exact decoupling limit, where where $s_{12} = s_{13} = \mathrm{Im}(Z_5\, e^{-2i\theta_{23}}) = 0$, it is a simple exercise to show that the interactions of h_1 are precisely those of the SM Higgs boson. In particular, the interactions of the h_1 in the decoupling limit are CP-conserving and diagonal in quark flavor space. In the most general 2HDM, CP-violating and neutral Higgs-mediated FCNCs are suppressed by factors of $\mathcal{O}(v^2/m_{2,3}^2)$ in the decoupling limit. In contrast, the interactions of the heavy neutral Higgs bosons (h_2 and h_3) and the charged Higgs bosons ($H^\pm$) in the decoupling limit can exhibit both CP-violating and quark flavor non-diagonal couplings (proportional to the ρ^Q).

The decoupling limit is a generic feature of extended Higgs sectors.[‡] Hence,

- The observation of a SM-like Higgs boson does not rule out the possibility of an extended Higgs sector in the decoupling regime.
- The SM Higgs search at colliders is applicable to a much larger class of extended Higgs models (including the MSSM Higgs sector).

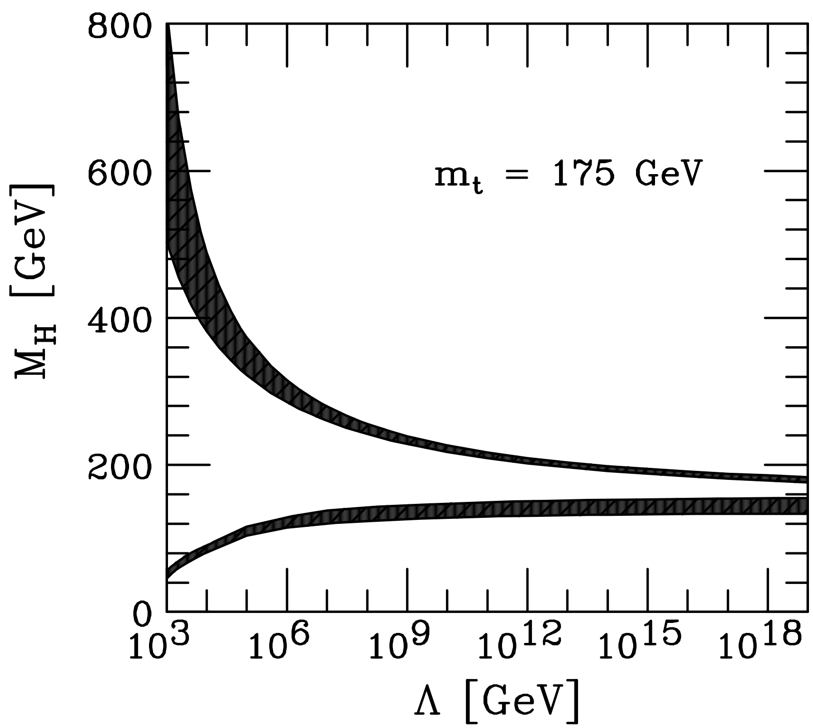
[‡]However, if some terms of the Higgs potential are absent, it is possible that no decoupling limit may exist. In this case, the only way to have very large Higgs masses is to have large Higgs self-couplings.

The significance of the TeV scale—Part 2

If a SM-like Higgs boson is discovered, should we expect any additional new physics phenomena at the TeV scale?

The Standard Model describes quite accurately physics near the EWSB scale. But, the SM is only a "low-energy" approximation to a more fundamental theory, whose degrees of freedom are revealed at some high energy scale Λ.

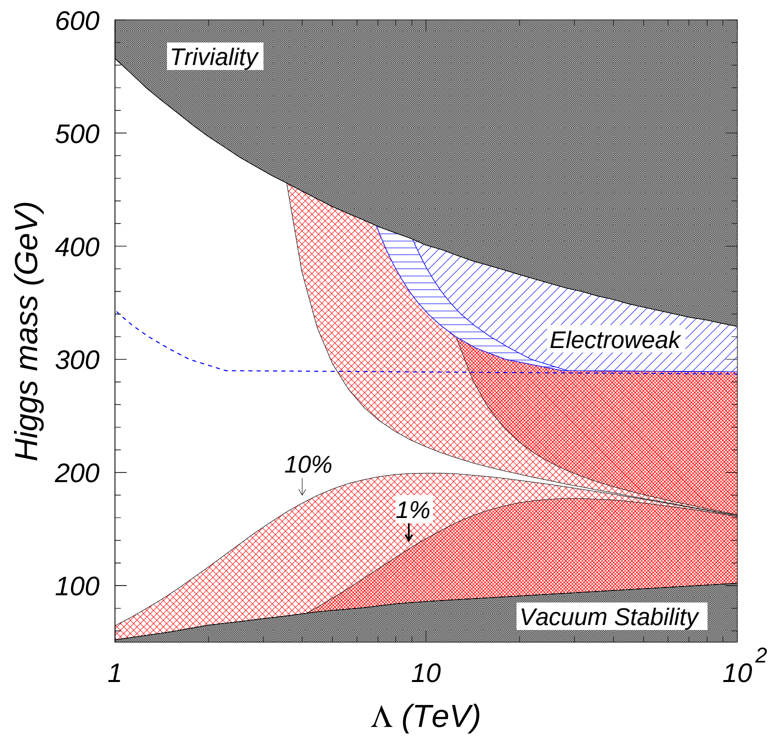
- The SM cannot be valid at energies above the Planck scale, $M_{\rm PL} \equiv (c\hbar/G_N)^{1/2} \simeq 10^{19}$ GeV, where gravity can no longer be ignored.
- Neutrinos are exactly massless in the Standard Model. But, the neutrino mixing data imply that neutrinos have very small masses ($m_\nu/m_e \lesssim 10^{-7}$). Neutrino masses can be incorporated in a theory whose fundamental scale is $M \gg v$. Neutrino masses of order v^2/M are generated, which suggest that $M \sim 10^{15}$ GeV.
- The radiatively-corrected Higgs potential is unstable at large values of the Higgs field ($|\Phi| > \Lambda$) if the Higgs mass is too small.
- The value of the Higgs self-coupling runs off to infinity at an energy scale above Λ if the Higgs mass is too large.



The present-day theoretical uncertainties on the lower [Altarelli and Isidori; Casas, Espinosa and Quirós] and upper [Hambye and Riesselmann] Higgs mass bounds as a function of energy scale Λ at which the Standard Model breaks down, assuming $m_t = 175$ GeV and $\alpha_s(m_Z) = 0.118$. The shaded areas above reflect the theoretical uncertainties in the calculations of the Higgs mass bounds.

Depending on the observed Higgs mass, we may be able to conclude that the SM breaks down at an energy Λ that is considerably below 10^{19} GeV.

The significance of the TeV-scale as the energy scale where new physics beyond the SM must emerge follows from the field-theoretic observation that m_h^2 (more precisely, the square of the Higgs vev) is sensitive to Λ^2. Demanding that the value of m_h is natural, $i.e.$, without substantial fine-tuning, then Λ cannot be significantly larger than 1 TeV.



Following Kolda and Murayama [*JHEP* **0007** (2000) 035], a reconsideration of the Λ $vs.$ Higgs mass plot with a focus on $\Lambda < 100$ TeV. Precision electroweak measurements restrict the parameter space to lie below the dashed line, based on a 95% CL fit that allows for nonzero values of S and T and the existence of higher dimensional operators suppressed by v^2/Λ^2. The unshaded area has less than one part in ten fine-tuning.

The Principle of Naturalness

JULY 1, 1939 PHYSICAL REVIEW VOLUME 56

On the Self-Energy and the Electromagnetic Field of the Electron

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(Received April 12, 1939)

In 1939, Weisskopf announces in the abstract to this paper that "the self-energy of charged particles obeying Bose statistics is found to be quadratically divergent"....

.... and concludes that in theories of elementary bosons, new phenomena must enter at an energy scale of order m/e (e is the relevant coupling)—the first application of naturalness.

which is about 10^{-58} times smaller than the classical electron radius. The "critical length" of the positron theory is thus infinitely smaller than usually assumed.

The situation is, however, entirely different for a particle with Bose statistics. Even the Coulombian part of the self-energy diverges to a first approximation as $W_{st} \sim e^2h/(mca^2)$ and requires a much larger critical length that is $a = (hc/e^2)^{-\frac{1}{2}} \cdot h/(mc)$, to keep it of the order of magnitude of mc^2. This may indicate that a theory of particles obeying Bose statistics must involve new features at this critical length, or at energies corresponding to this length; whereas a theory of particles obeying the exclusion principle is probably consistent down to much smaller lengths or up to much higher energies.

Principle of naturalness in modern times

How can we understand the magnitude of the EWSB scale? In the absence of new physics beyond the Standard Model, its natural value would be the Planck scale (or perhaps the GUT scale or seesaw scale that controls neutrino masses). The alternatives are:

- Naturalness is restored by a symmetry principle—supersymmetry—which ties the bosons to the more well-behaved fermions.

- The Higgs boson is an approximate Goldstone boson—the only other known mechanism for keeping an elementary scalar light.

- The Higgs boson is a composite scalar, with an inverse length of order the TeV-scale.

- The naturalness principle does not hold in this case. Unnatural choices for the EWSB parameters arise from other considerations (landscape?).

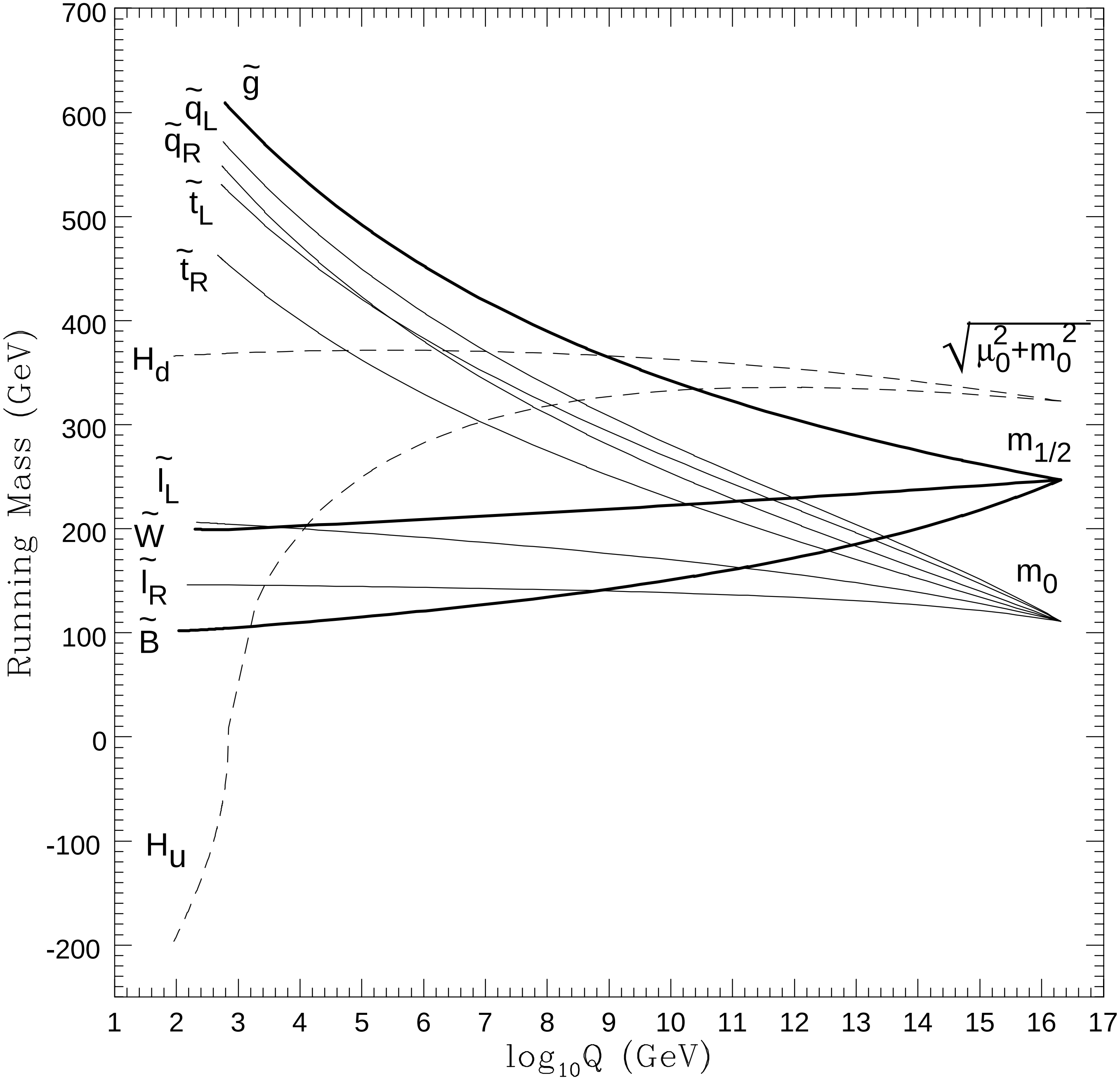
Low-Energy Supersymmetry

Supersymmetry (SUSY) provides a mechanism in which the quadratic sensitivity of scalar squared-masses to very high-energy scales is exactly canceled. Since SUSY is not an exact symmetry of nature, the supersymmetry must be broken. To maintain the naturalness of the theory, the SUSY-breaking scale cannot be significantly larger than 1 TeV.

> The scale of supersymmetry-breaking must be of order 1 TeV or less, if supersymmetry is associated with the scale of electroweak symmetry breaking.

We shall initially focus on the minimal supersymmetric extension of the Standard Model (MSSM), which is constructed by starting with the 2HDM and adding the associated superpartners.[§] One bonus of this construction is the elegant way in which EWSB is radiatively generated (providing a nice connection between SUSY-breaking and the mechanism of EWSB).

[§]Two Higgs doublets are required for anomaly cancelation by higgsino pairs of opposite hypercharge.



Running Mass (GeV)
$\log_{10}Q$ (GeV)
700
600
500
400
300
200
100
0
-100
-200
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
$\tilde{g}$
$\tilde{q}_L$
$\tilde{q}_R$
$\tilde{t}_L$
$\tilde{t}_R$
H_d
$\sqrt{\mu_0^2+m_0^2}$
$m_{1/2}$
$\tilde{l}_L$
$\tilde{W}$
$\tilde{l}_R$
m_0
$\tilde{B}$
H_u

The Higgs sector of the MSSM

The Higgs sector of the MSSM is a 2HDM, whose Yukawa couplings and Higgs potential are constrained by SUSY. Instead of employing to hypercharge-one scalar doublets $\Phi_{1,2}$, it is more convenient to introduce a $Y=-1$ doublet $H_d \equiv i\sigma_2 \Phi_1^*$ and a $Y=+1$ doublet $H_u \equiv \Phi_2$:

$$H_d = \begin{pmatrix} H_d^1 \\ H_d^2 \end{pmatrix} = \begin{pmatrix} \Phi_1^{0\,*} \\ -\Phi_1^- \end{pmatrix}, \qquad H_u = \begin{pmatrix} H_u^1 \\ H_u^2 \end{pmatrix} = \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix}.$$

The origin of the notation originates from the Higgs Yukawa Lagrangian:

$$\mathcal{L}_{\text{Yukawa}} = -h_u^{ij}(\bar{u}_R^i u_L^j H_u^2 - \bar{u}_R^i d_L^j H_u^1) - h_d^{ij}(\bar{d}_R^i d_L^j H_d^1 - \bar{d}_R^i u_L^j H_d^2) + \text{h.c.}\,.$$

Note that the neutral Higgs field H_u^2 couples exclusively to up-type quarks and the neutral Higgs field H_d^1 couples exclusively to down-type quarks.¶

¶This is an example of the so-called Type-II 2HDM, which satisfies the Glashow-Weinberg condition and has no tree-level Higgs-mediated FCNCs.

The Higgs potential of the MSSM is:

$$V = \left(m_d^2 + |\mu|^2\right) H_d^{i*} H_d^i + \left(m_u^2 + |\mu|^2\right) H_u^{i*} H_u^i - m_{ud}^2 \left(\epsilon^{ij} H_d^i H_u^j + \text{h.c.}\right)$$
$$+\tfrac{1}{8}\left(g^2 + g'^2\right)\left[H_d^{i*} H_d^i - H_u^{j*} H_u^j\right]^2 + \tfrac{1}{2} g^2 |H_d^{i*} H_u^i|^2 \,,$$

where $\epsilon^{12} = -\epsilon^{21} = 1$ and $\epsilon^{11} = \epsilon^{22} = 0$, and the sum over repeated indices is implicit. Above, μ is a supersymmetric Higgsino mass parameter and m_d^2, m_u^2, m_{ud}^2 are soft-supersymmetry-breaking masses. The quartic Higgs couplings are related to the SU(2) and $\text{U}(1)_{\text{Y}}$ gauge couplings as a consequence of SUSY.

Minimizing the Higgs potential, the neutral components of the Higgs fields acquire vevs:[‖]

$$\langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \qquad \langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix},$$

where $v^2 \equiv v_d^2 + v_u^2 = 4m_W^2/g^2 = (246~\text{GeV})^2$. The ratio of the two vevs is an important parameter of the model:

$$\tan\beta \equiv \frac{v_u}{v_d}\,, \qquad 0 \leq \beta \leq \tfrac{1}{2}\pi\,.$$

[‖]The phases of the Higgs fields can be chosen such that the vacuum expectation values are real and positive. That is, the tree-level MSSM Higgs sector conserves CP, which implies that the neutral Higgs mass eigenstates possess definite CP quantum numbers.

In the Higgs basis, the phase of H_2 can be chosen such that Z_5, Z_6 and Z_7 are real:

$$Z_1 = Z_2 = \tfrac{1}{4}(g^2 + g'^2)\cos^2 2\beta\,, \qquad Z_3 = Z_5 + \tfrac{1}{4}(g^2 - g'^2)\,, \qquad Z_4 = Z_5 - \tfrac{1}{2}g^2\,,$$
$$Z_5 = \tfrac{1}{4}(g^2 + g'^2)\sin^2 2\beta\,, \qquad Z_7 = -Z_6 = \tfrac{1}{4}(g^2 + g'^2)\sin 2\beta\cos 2\beta\,.$$

The 3×3 squared-mass matrix of the neutral Higgs bosons reduces in block form to a 2×2 block, which is easily diagonalized and yields the CP-even mass eigenstates h^0 and H^0 (with corresponding diagonalizing angle α), and a 1×1 block corresponding to the CP-odd mass eigenstate A^0. To make contact with our previous analysis of the 2HDM in the Higgs basis, we identify the neutral Higgs fields as $h_1 = h^0$, $h_2 = H^0$, $h_3 = A^0$ and $h_4 = G^0$. The diagonalization of the squared-mass matrix of the neutral Higgs bosons yields $\theta_{13} = \theta_{23} = 0$ and $\theta_{12} = \frac{1}{2}\pi - \beta + \alpha$. In particular,

k	q_{k1}	q_{k2}
1	c_{12}	$-s_{12}$
2	s_{12}	c_{12}
3	0	i
4	i	0

where $c_{12} = \sin(\beta - \alpha)$ and $s_{12} = \cos(\beta - \alpha)$.

The five physical Higgs particles consist of a charged Higgs pair

$$H^{\pm} = H_d^{\pm} \sin\beta + H_u^{\pm} \cos\beta \,,$$

one CP-odd scalar

$$A^0 = \sqrt{2} \left(\mathrm{Im}\, H_d^0 \sin\beta + \mathrm{Im}\, H_u^0 \cos\beta \right) \,,$$

and two CP-even scalars

$$h^0 = -(\sqrt{2}\,\mathrm{Re}\, H_d^0 - v_d) \sin\alpha + (\sqrt{2}\,\mathrm{Re}\, H_u^0 - v_u) \cos\alpha \,,$$
$$H^0 = (\sqrt{2}\,\mathrm{Re}\, H_d^0 - v_d) \cos\alpha + (\sqrt{2}\,\mathrm{Re}\, H_u^0 - v_u) \sin\alpha \,,$$

where we have now labeled the Higgs fields according to their electric charge. The angle α arises when the CP-even Higgs squared-mass matrix (in the H_d^0—H_u^0 basis) is diagonalized to obtain the physical CP-even Higgs states.

All Higgs masses and couplings can be expressed in terms of two parameters usually chosen to be m_A and $\tan\beta$.

Tree-level MSSM Higgs masses

The charged Higgs mass is given by

$$m^2_{H^\pm} = m^2_A + m^2_W \,,$$

and the CP-even Higgs bosons h^0 and H^0 are eigenstates of the squared-mass matrix

$$\mathcal{M}_0^2 = \begin{pmatrix} m_A^2 \sin^2\beta + m_Z^2 \cos^2\beta & -(m_A^2 + m_Z^2)\sin\beta\cos\beta \\ -(m_A^2 + m_Z^2)\sin\beta\cos\beta & m_A^2\cos^2\beta + m_Z^2 \sin^2\beta \end{pmatrix} .$$

The eigenvalues of $\mathcal{M}_0^2$ are the squared-masses of the two CP-even Higgs scalars

$$m^2_{H,h} = \tfrac{1}{2}\left(m_A^2 + m_Z^2 \pm \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta} \right) ,$$

and α is the angle that diagonalizes the CP-even Higgs squared-mass matrix. It follows that

$$\boxed{m_h \leq m_Z |\cos 2\beta| \leq m_Z \,.}$$

Note the contrast with the SM where the Higgs mass is a free parameter, $m_h^2 = \frac{1}{2}\lambda v^2$. In the MSSM, all Higgs self-coupling parameters of the MSSM are related to the squares of the electroweak gauge couplings.

Aside: the decoupling limit of the MSSM

In the limit of $m_A \gg m_Z$, the expressions for the Higgs masses and mixing angle simplify and one finds

$$
\begin{aligned}
m_h^2 &\simeq m_Z^2 \cos^2 2\beta \,, \\
m_H^2 &\simeq m_A^2 + m_Z^2 \sin^2 2\beta \,, \\
m_{H^\pm}^2 &= m_A^2 + m_W^2 \,, \\
\cos^2(\beta - \alpha) &\simeq \frac{m_Z^4 \sin^2 4\beta}{4 m_A^4} \,.
\end{aligned}
$$

Two consequences are immediately apparent. First, $m_A \simeq m_H \simeq m_{H^\pm}$, up to corrections of $\mathcal{O}(m_Z^2/m_A)$. Second, $\cos(\beta - \alpha) = 0$ up to corrections of $\mathcal{O}(m_Z^2/m_A^2)$. This is the decoupling limit, since at energy scales below approximately common mass of the heavy Higgs bosons $H^\pm$ H^0, A^0, the effective Higgs theory is precisely that of the SM.

In particular, we will see that in the limit of $\cos(\beta - \alpha) \to 0$, all the h^0 couplings to SM particles approach their SM limits.

Tree-level MSSM Higgs couplings

1. Higgs couplings to gauge boson pairs ($V = W$ or Z)

$$g_{h^0VV} = g_V m_V \sin(\beta - \alpha)\,, \qquad\qquad g_{H^0VV} = g_V m_V \cos(\beta - \alpha)\,,$$

where $g_V \equiv 2m_V/v$. There are no tree-level couplings of A^0 or $H^\pm$ to VV.

2. Higgs couplings to a single gauge boson

The couplings of V to two neutral Higgs bosons (which must have opposite CP-quantum numbers) is denoted by $g_{\phi A^0 Z}(p_\phi - p_A^0)$, where $\phi = h^0$ or H^0 and the momenta p_ϕ and p_A^0 point into the vertex, and

$$g_{h^0A^0Z} = \frac{g\cos(\beta-\alpha)}{2\cos\theta_W}\,, \qquad\qquad g_{H^0A^0Z} = \frac{-g\sin(\beta-\alpha)}{2\cos\theta_W}\,.$$

3. Summary of Higgs boson–vector boson couplings

The properties of the three-point and four-point Higgs boson-vector boson couplings are conveniently summarized by listing the couplings that are proportional to either $\sin(\beta-\alpha)$ or $\cos(\beta-\alpha)$ or are angle-independent. As a reminder, $\cos(\beta-\alpha)\to 0$ in the decoupling limit.

$\cos(\beta-\alpha)$	$\sin(\beta-\alpha)$	angle-independent
$H^0W^+W^-$	$h^0W^+W^-$	—
H^0ZZ	h^0ZZ	—
ZA^0h^0	ZA^0H^0	$ZH^+H^-\,,\ \gamma H^+H^-$
$W^\pm H^\mp h^0$	$W^\pm H^\mp H^0$	$W^\pm H^\mp A^0$
$ZW^\pm H^\mp h^0$	$ZW^\pm H^\mp H^0$	$ZW^\pm H^\mp A^0$
$\gamma W^\pm H^\mp h^0$	$\gamma W^\pm H^\mp H^0$	$\gamma W^\pm H^\mp A^0$
—	—	$VV\phi\phi\,,\ VVA^0A^0\,,\ VVH^+H^-$

where $\phi=h^0$ or H^0 and $VV=W^+W^-$, ZZ, $Z\gamma$ or $\gamma\gamma$.

4. Higgs-fermion couplings

Supersymmetry imposes a Type-II structure for the Higgs-fermion Yukawa couplings. Since the neutral Higgs couplings to fermions are flavor-diagonal, we list only the Higgs coupling to 3rd generation fermions. The couplings of the neutral Higgs bosons to $f\bar{f}$ relative to the Standard Model value, $gm_f/2m_W$, are given by

$$
\begin{aligned}
h^0 b\bar{b} \quad (\text{or } h^0\tau^+\tau^-): &\qquad -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)\,, \\
h^0 t\bar{t}: &\qquad \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)\,, \\
H^0 b\bar{b} \quad (\text{or } H^0\tau^+\tau^-): &\qquad \frac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha)\,, \\
H^0 t\bar{t}: &\qquad \frac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \cot\beta\sin(\beta-\alpha)\,, \\
A^0 b\bar{b} \quad (\text{or } A^0\tau^+\tau^-): &\qquad \gamma_5\tan\beta\,, \\
A^0 t\bar{t}: &\qquad \gamma_5\cot\beta\,,
\end{aligned}
$$

where the γ_5 indicates a pseudoscalar coupling. Note that the $h^0 f\bar{f}$ couplings approach their SM values in the decoupling limit, where $\cos(\beta-\alpha)\to 0$.

Similarly, the charged Higgs boson couplings to fermion pairs, with all particles pointing into the vertex, are given by**

$$
\begin{aligned}
g_{H^-t\bar{b}} = &\quad \frac{g}{\sqrt{2}m_W}\left[m_t \cot\beta\, P_R + m_b \tan\beta\, P_L\right], \\
g_{H^-\tau^+\nu} = &\quad \frac{g}{\sqrt{2}m_W}\left[m_\tau \tan\beta\, P_L\right].
\end{aligned}
$$

Especially noteworthy is the possible $\tan\beta$-enhancement of certain Higgs-fermion couplings. The general expectation in MSSM models is that $\tan\beta$ lies in a range:

$$1 \lesssim \tan\beta \lesssim \frac{m_t}{m_b}.$$

Near the upper limit of $\tan\beta$, we have roughly identical values for the top and bottom Yukawa couplings, $h_t \sim h_b$, since

$$h_b = \frac{\sqrt{2}\, m_b}{v_d} = \frac{\sqrt{2}\, m_b}{v\cos\beta}, \qquad\qquad h_t = \frac{\sqrt{2}\, m_t}{v_u} = \frac{\sqrt{2}\, m_t}{v\sin\beta}.$$

**Including the full flavor structure, the CKM matrix appears in the charged Higgs couplings in the standard way for a charged-current interaction.

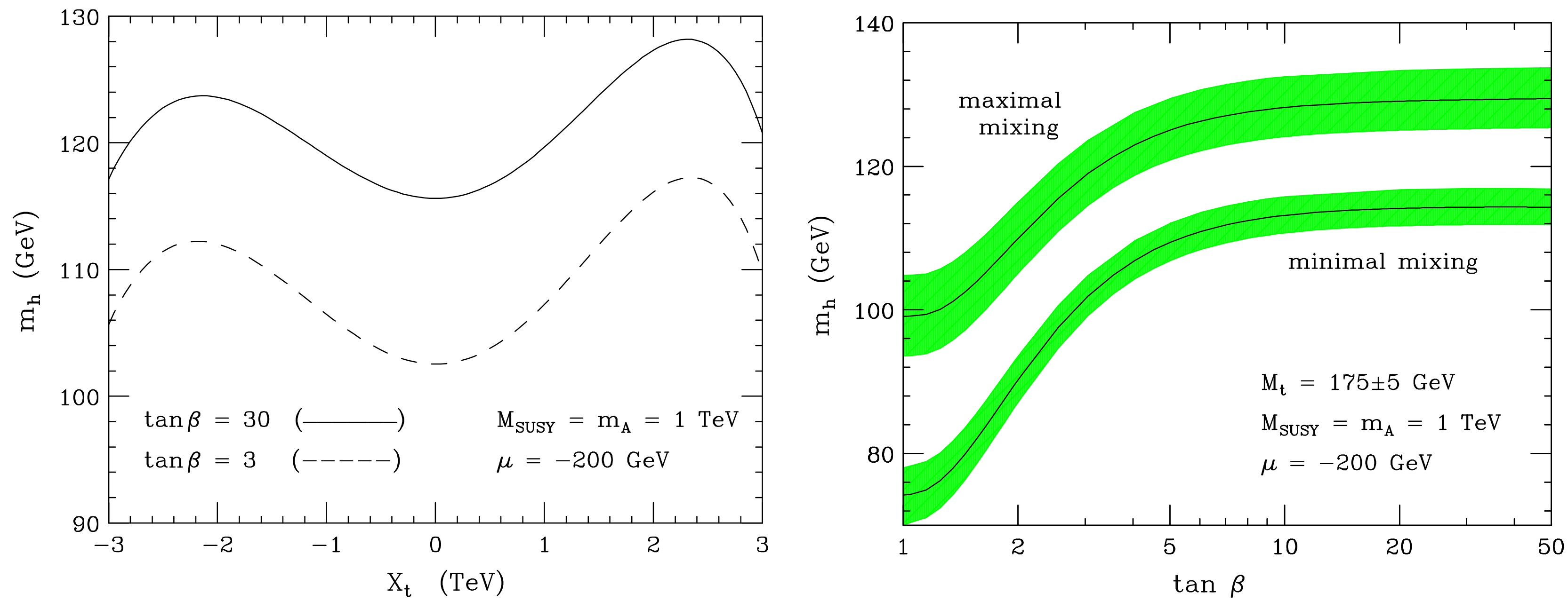
Saving the MSSM Higgs sector—the impact of radiative corrections

We have already noted the tree-level relation $m_h \leq m_Z$, which is already ruled out by LEP data. But, this inequality receives quantum corrections. The Higgs mass can be shifted due to loops of particles and their superpartners (an incomplete cancelation, which would have been exact if supersymmetry were unbroken):

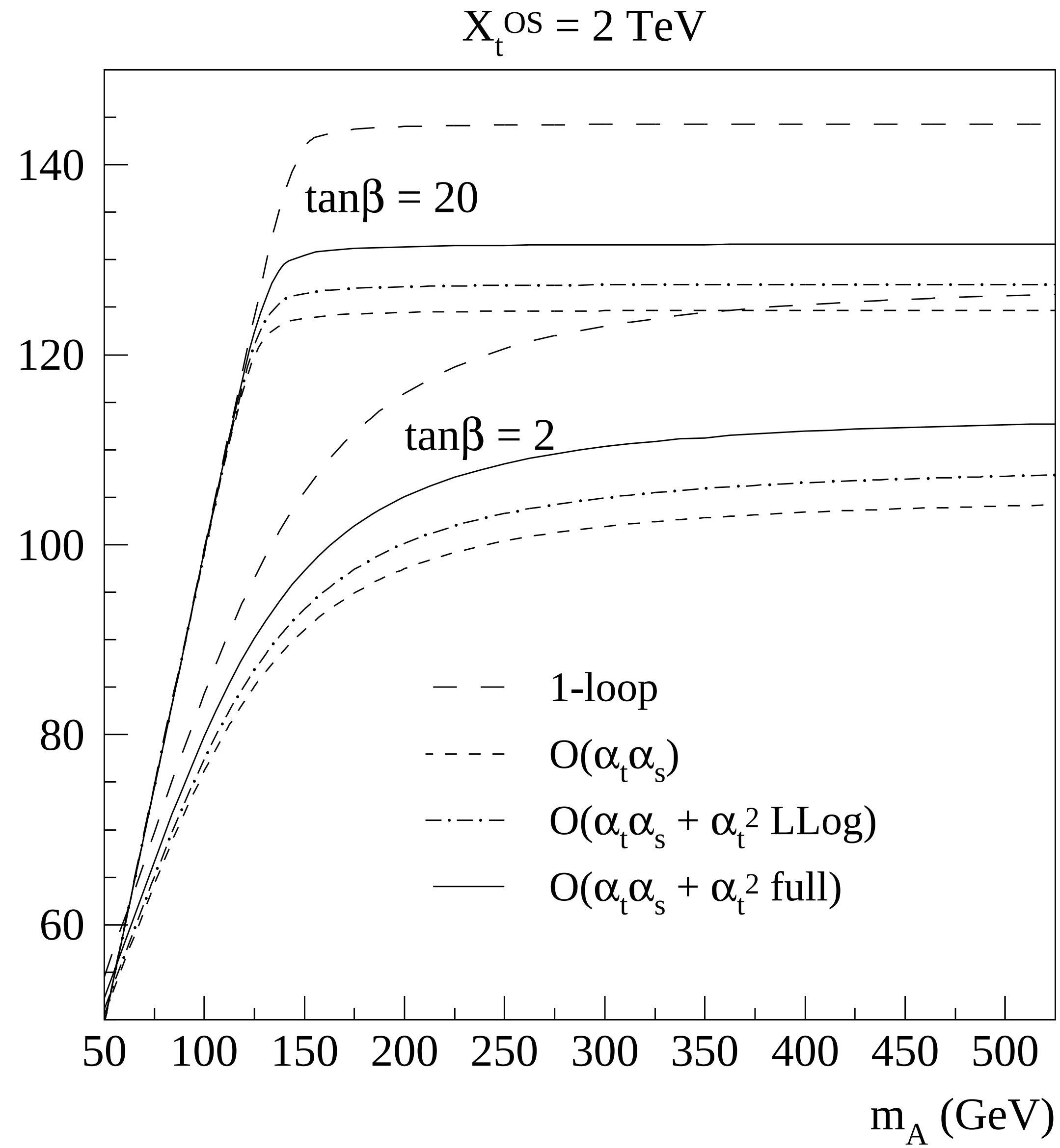
$$m_h^2 \lesssim m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right],$$

where $X_t \equiv A_t - \mu\cot\beta$ governs stop mixing and M_S^2 is the average squared-mass of the top-squarks $\widetilde{t}_1$ and $\widetilde{t}_2$ (which are the mass-eigenstate combinations of the interaction eigenstates, $\widetilde{t}_L$ and $\widetilde{t}_R$).

The state-of-the-art computation includes the full one-loop result, all the significant two-loop contributions, some of the leading three-loop terms, and renormalization-group improvements. The final conclusion is that $m_h \lesssim 130$ GeV [assuming that the top-squark mass is no heavier than about 2 TeV].



Maximal mixing corresponds to choosing the MSSM Higgs parameters in such a way that m_h is maximized (for a fixed $\tan\beta$). This occurs for $X_t/M_S \sim 2$. As $\tan\beta$ varies, m_h reaches is maximal value, $(m_h)_{\max} \simeq 130$ GeV, for $\tan\beta \gg 1$ and $m_A \gg m_Z$.



taken from Brignole et al., Nucl. Phys. **B631**, 195 (2002).

Radiatively-corrected Higgs couplings

Although radiatively-corrections to couplings tend to be at the few-percent level, there is some potential for significant effects:

- large radiative corrections due to a $\tan\beta$-enhancement (assuming $\tan\beta \gg 1$)
- CP-violating effects induced by complex SUSY-breaking parameters that enter in loops

In the MSSM, the tree-level Higgs–quark Yukawa Lagrangian is supersymmetry-conserving and is given by Type-II structure,

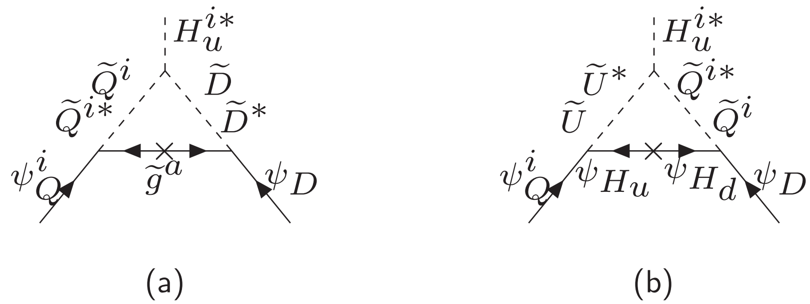
$$\mathcal{L}_{\rm yuk}^{\rm tree} = -\epsilon_{ij} h_b H_d^i \psi_Q^j \psi_D + \epsilon_{ij} h_t H_u^i \psi_Q^j \psi_U + {\rm h.c.}$$

Two other possible dimension-four gauge-invariant non-holomorphic Higgs-quark interactions terms, the so-called wrong-Higgs interactions,

$$H_u^{k*} \psi_D \psi_Q^k \qquad \text{and} \qquad H_d^{k*} \psi_U \psi_Q^k \,,$$

are not supersymmetric (since the dimension-four supersymmetric Yukawa interactions must be holomorphic), and hence are absent from the tree-level Yukawa Lagrangian.

Nevertheless, the wrong-Higgs interactions can be generated in the effective low-energy theory below the scale of SUSY-breaking. In particular, one-loop radiative corrections, in which supersymmetric particles (squarks, higgsinos and gauginos) propagate inside the loop can generate the wrong-Higgs interactions.



One-loop diagrams contributing to the wrong-Higgs Yukawa effective operators. In (a), the cross ($\times$) corresponds to a factor of the gluino mass M_3. In (b), the cross corresponds to a factor of the higgsino Majorana mass parameter μ. Field labels correspond to annihilation of the corresponding particle at each vertex of the triangle.

If the superpartners are heavy, then one can derive an effective field theory description of the Higgs-quark Yukawa couplings below the scale of SUSY-breaking ($M_{\rm SUSY}$), where one has integrated out the heavy SUSY particles propagating in the loops.

The resulting effective Lagrangian is:

$$\mathcal{L}_{\rm yuk}^{\rm eff} = -\epsilon_{ij}(h_b + \delta h_b)\psi_b H_d^i \psi_Q^j + \Delta h_b \psi_b H_u^{k*} \psi_Q^k$$
$$+\epsilon_{ij}(h_t + \delta h_t)\psi_t H_u^i \psi_Q^j + \Delta h_t \psi_t H_d^{k*} \psi_Q^k + {\rm h.c.}$$

In the limit of $M_{\rm SUSY} \gg m_Z$,

$$\Delta h_b = h_b \left[\frac{2\alpha_s}{3\pi} \mu M_3 \mathcal{I}(M_{\tilde{b}_1}, M_{\tilde{b}_2}, M_g) + \frac{h_t^2}{16\pi^2} \mu A_t \mathcal{I}(M_{\tilde{t}_1}, M_{\tilde{t}_2}, \mu) \right] ,$$

where, M_3 is the Majorana gluino mass, μ is the supersymmetric Higgs-mass parameter, and $\widetilde{b}_{1,2}$ and $\widetilde{t}_{1,2}$ are the mass-eigenstate bottom squarks and top squarks, respectively. The loop integral is given by

$$\mathcal{I}(a,b,c) = \frac{a^2b^2 \ln(a^2/b^2) + b^2c^2 \ln(b^2/c^2) + c^2a^2 \ln(c^2/a^2)}{(a^2-b^2)(b^2-c^2)(a^2-c^2)} .$$

In the limit where at least one of the arguments of $\mathcal{I}(a,b,c)$ is large,

$$\mathcal{I}(a,b,c) \sim 1/\max(a^2, b^2, c^2) .$$

Thus, in the limit where $M_3 \sim \mu \sim A_t \sim M_{\tilde{b}} \sim M_{\tilde{t}} \sim M_{\rm SUSY} \gg m_Z$, the one-loop contributions to Δh_b do *not* decouple.

Phenomenological consequences of the wrong-Higgs Yukawas

The effects of the wrong-Higgs couplings are $\tan\beta$-enhanced modifications of some physical observables. To see this, rewrite the Higgs fields in terms of the physical mass-eigenstates (and the Goldstone bosons):

$$
\begin{aligned}
H_d^1 &= \frac{1}{\sqrt{2}}(v\cos\beta + H^0\cos\alpha - h^0\sin\alpha + iA^0\sin\beta - iG^0\cos\beta)\,,\\
H_u^2 &= \frac{1}{\sqrt{2}}(v\sin\beta + H^0\sin\alpha + h^0\cos\alpha + iA^0\cos\beta + iG^0\sin\beta)\,,\\
H_d^2 &= H^-\sin\beta - G^-\cos\beta\,,\\
H_u^1 &= H^+\cos\beta + G^+\sin\beta\,,
\end{aligned}
$$

with $v^2 \equiv v_u^2 + v_d^2 = (246~\mathrm{GeV})^2$ and $\tan\beta \equiv v_u/v_d$. For simplicity, we neglect below possible CP-violating effects due to complex couplings. Then, the b-quark mass is:

$$
m_b = \frac{h_b v}{\sqrt{2}}\cos\beta\left(1 + \frac{\delta h_b}{h_b} + \frac{\Delta h_b \tan\beta}{h_b}\right) \equiv \frac{h_b v}{\sqrt{2}}\cos\beta(1+\Delta_b)\,,
$$

which defines the quantity Δ_b.

In the limit of large $\tan\beta$ the term proportional to δh_b can be neglected, in which case,

$$\Delta_b \simeq (\Delta h_b/h_b)\tan\beta \,.$$

Thus, Δ_b is $\tan\beta$–enhanced if $\tan\beta \gg 1$. As previously noted, Δ_b survives in the limit of large $M_{\rm SUSY}$; this effect does not decouple.

From the effective Yukawa Lagrangian, we can obtain the couplings of the physical Higgs bosons to third generation fermions. Neglecting possible CP-violating effects,

$$\mathcal{L}_{\rm int} = -\sum_{q=t,b,\tau}\left[g_{h^0 q\bar{q}} h^0 q\bar{q} + g_{H^0 q\bar{q}} H^0 q\bar{q} - i g_{A^0 q\bar{q}} A^0 \bar{q}\gamma_5 q\right] + \left[\bar{b} g_{H^- t\bar{b}} t H^- + {\rm h.c.}\right] \,.$$

The one-loop corrections can generate measurable shifts in the decay rate for $h^0 \to b\bar{b}$:

$$g_{h^\circ b\bar{b}} = -\frac{m_b}{v}\frac{\sin\alpha}{\cos\beta}\left[1 + \frac{1}{1+\Delta_b}\left(\frac{\delta h_b}{h_b} - \Delta_b\right)(1 + \cot\alpha\cot\beta)\right] \,.$$

At large $\tan\beta \sim 20$—50, Δ_b can be as large as 0.5 in magnitude and of either sign, leading to a significant enhancement or suppression of the Higgs decay rate to $b\bar{b}$.

Non-decoupling effects in $h^0 \to b\bar{b}$: a closer look

The origin of the non-decoupling effects can be understood by noting that below the scale $M_{\rm SUSY}$, the effective low-energy Higgs theory is a completely general 2HDM. Thus, it is not surprising that the wrong-Higgs couplings do not decouple in the limit of $M_{\rm SUSY} \to \infty$.

However, suppose that $m_A \sim \mathcal{O}(M_{\rm SUSY})$. Then, the low-energy effective Higgs theory is a one-Higgs doublet model, and thus $g_{h^0 b\bar{b}}$ must approach its SM value. Indeed in this limit,

$$\cos(\beta-\alpha) = \frac{m_Z^2 \sin 4\beta}{2m_A^2} + \mathcal{O}\left(\frac{m_Z^4}{m_A^4}\right) \,,$$

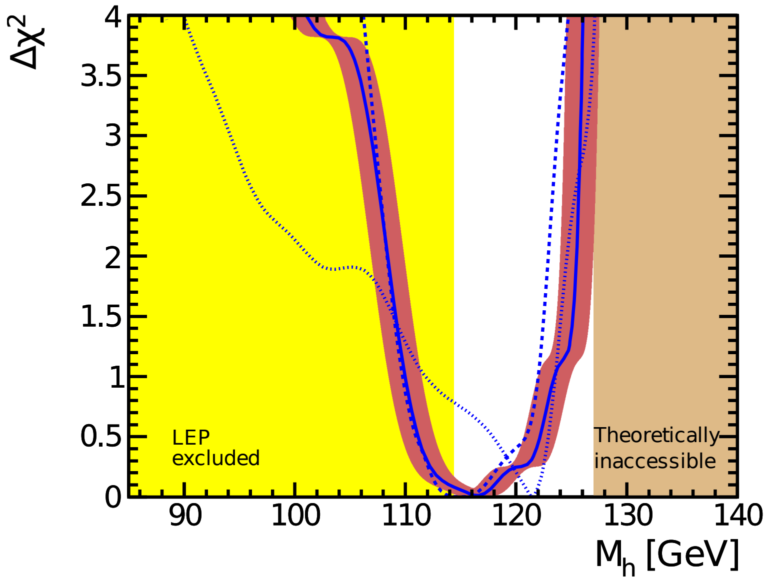
$$1 + \cot\alpha\cot\beta = -\frac{2m_Z^2}{m_A^2}\cos 2\beta + \mathcal{O}\left(\frac{m_Z^4}{m_A^4}\right) \,.$$

Thus the previously non-decoupling SUSY radiative corrections do decouple as expected.

Expectations for the MSSM Higgs masses

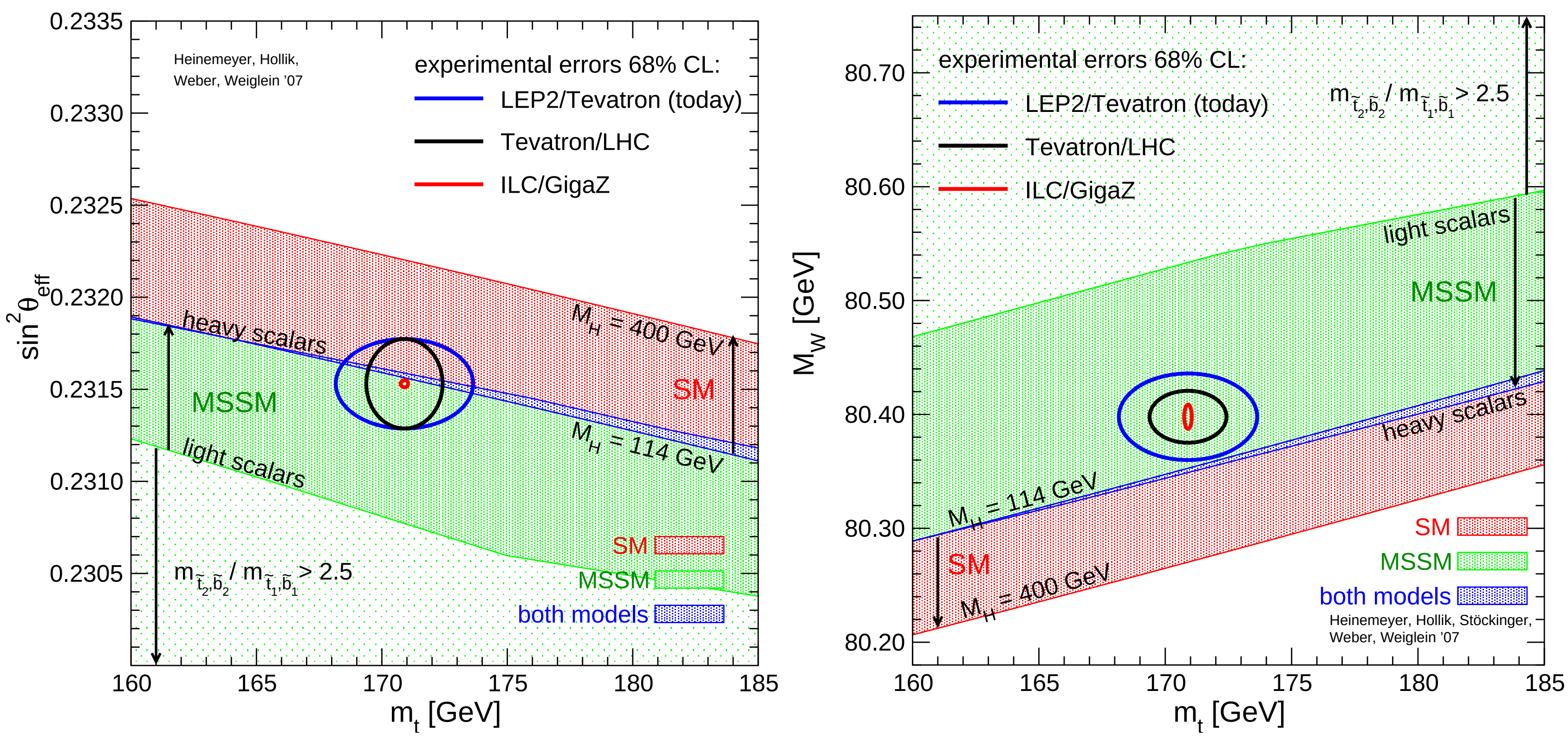
1. Consequences of precision electroweak data.

- In the decoupling limit (with SUSY particles somewhat heavy), the effects of the heavy Higgs states and the SUSY particles decouple and the global SM fit applies.
- In the latter case, h^0 is a SM-like Higgs boson whose mass lies below about 130 GeV in the *preferred* Higgs mass range!



Higgs mass constraints in the NUHM1 extension of the CMSSM, with non-universal Higgs mass parameters [taken from O. Buchmüller et al., Eur. Phys. J. **C71**, 1634 (2011)].

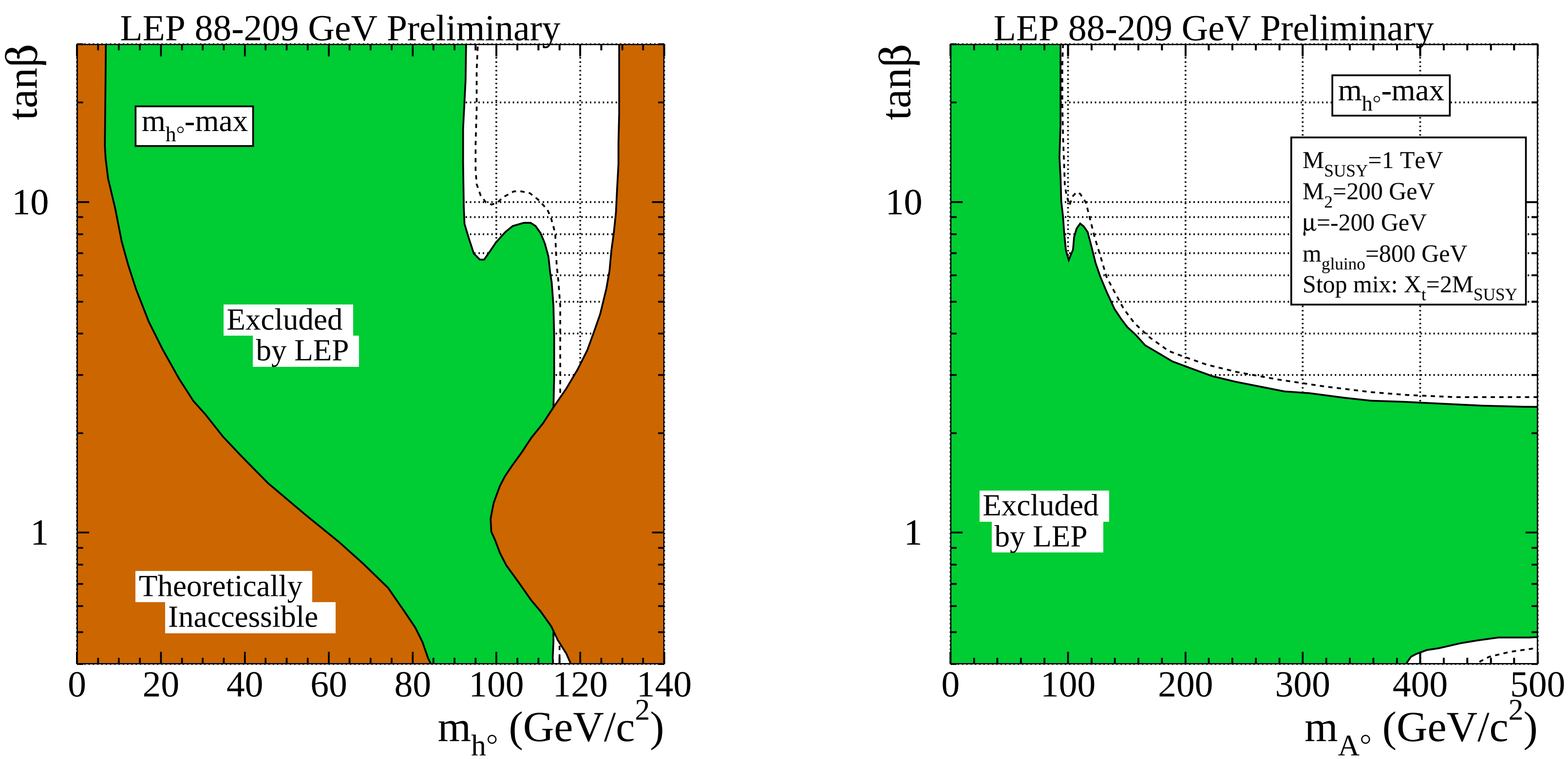
- If SUSY particle masses are not too heavy, they can have small effects on the fit to precision electroweak data. With additional degrees of freedom, the goodness of fit can be slightly improved (and possibly argue for SUSY masses close to their present experimental limits).
- The MSSM fit is further improved if one wishes to ascribe deviations of $(g-2)_\mu$ from their SM expectations to the effects of superpartners.



Taken from S. Heinemeyer, W. Hollik, A.M. Weber and G. Weiglein, JHEP **0804**, 039 (2008).

2. Constraints from collider searches

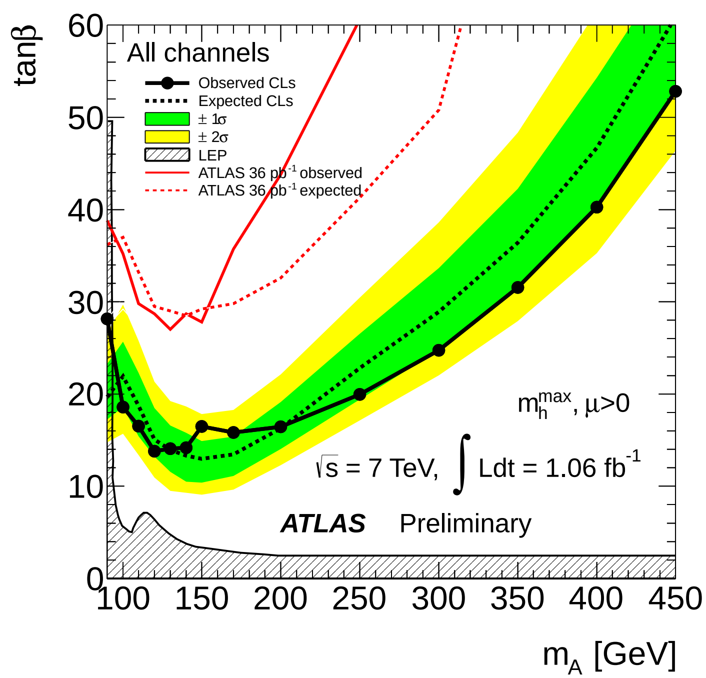
Summary of the LEP MSSM Higgs Search [95% CL limits]

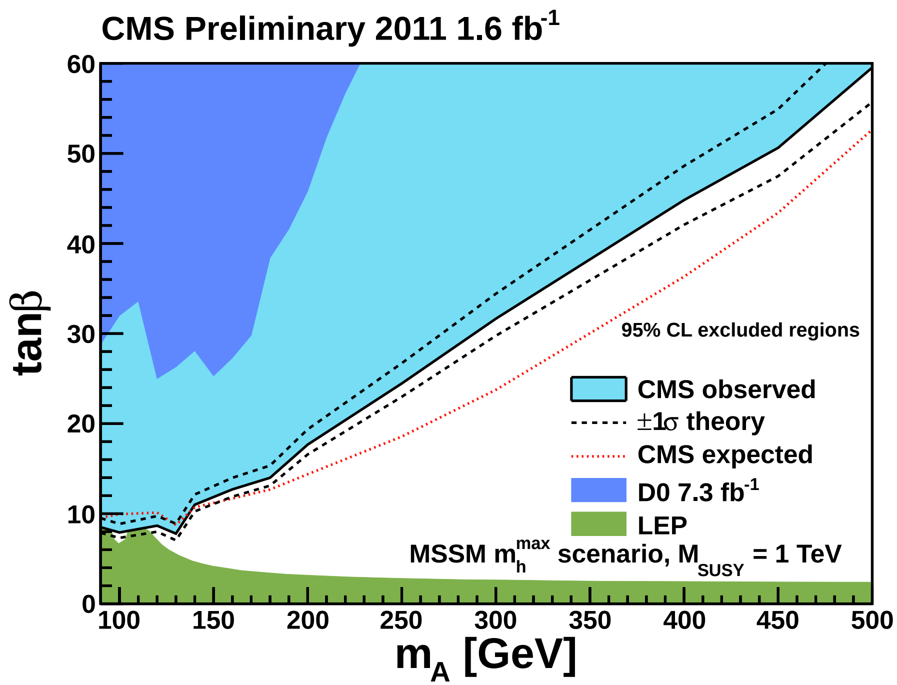


- Charged Higgs boson: $m_{H^\pm} > 79.3$ GeV
- MSSM Higgs: $m_h > 92.9$ GeV; $m_A > 93.4$ GeV [max-mix scenario]

WARNING: Allowing for possible CP-violating effects that can enter via radiative corrections, large holes open up in the Higgs mass exclusion plots.

The LHC search for MSSM Higgs bosons also has produced interesting limits in the non-decoupling regime, where $m_A \lesssim 150$ GeV.





With more data, LHC data can be used to rule out more of the $\tan\beta$–m_A plane. However, in the region of large m_A and moderate $\tan\beta$, it will be difficult to detect H^0, A^0 and $H^\pm$ even with a significant increase of luminosity. This is the infamous *LHC wedge region*, where only the SM-like h^0 of the MSSM can be observed.

Radiatively-induced CP-violating effects: the CPX Scenario

The one-loop corrected effective Higgs-fermion Lagrangian can exhibit CP-violating effect due to possible CP-violating phases in μ, A_t and M_3. This leads to mixed-CP neutral Higgs states and CP-violating couplings. Thus instead of h^0, H^0, A^0 and mixing angle α, we have H_i^0 $(i = 1, 2, 3)$ and a real orthogonal 3×3 mixing matrix O, with

$$H_i = (\sqrt{2}\,\mathrm{Re}\,\Phi_d^0 - v_d) O_{1i} + (\sqrt{2}\,\mathrm{Re}\,\Phi_u^0 - v_u) O_{2i} + \sqrt{2}\left(\mathrm{Im}\,\Phi_d^0 \sin\beta + \mathrm{Im}\,\Phi_u^0 \cos\beta\right) O_{3i}\,.$$

The Higgs-fermion Yukawa couplings are:

$$\mathcal{L}_{H\bar{f}f} = -\sum_{i=1}^{3} H_i \left[\frac{m_b}{v}\,\bar{b}\left(g^S_{H_i bb} + i g^P_{H_i bb}\gamma_5\right) b + \frac{m_t}{v}\,\bar{t}\left(g^S_{H_i tt} + i g^P_{H_i tt}\gamma_5\right) t \right].$$

For example, the one-loop corrected $b\bar{b}$-Higgs couplings are:

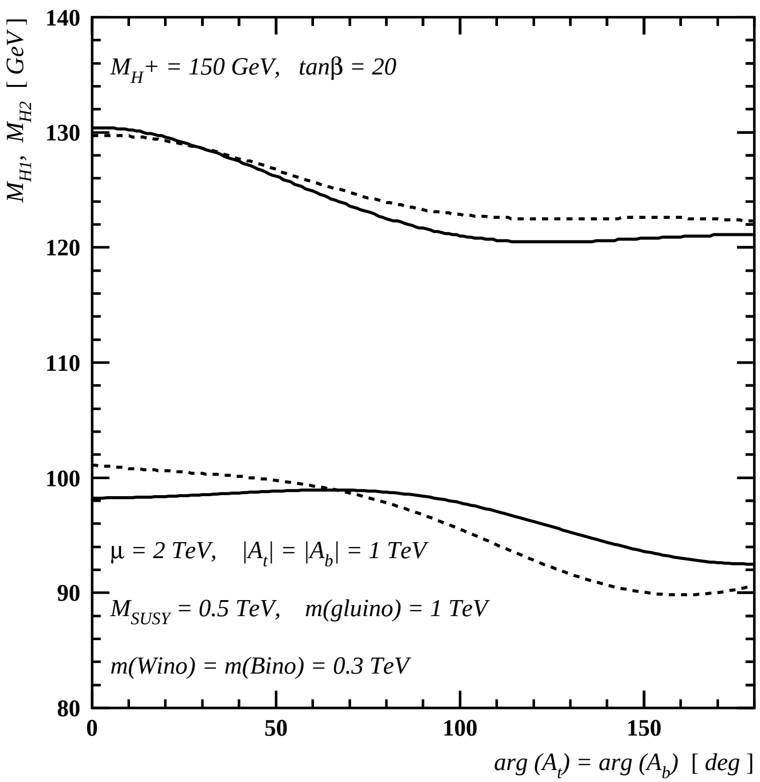
$$g^S_{H_i bb} = \frac{1}{h_b + \delta h_b + \Delta h_b \tan\beta} \left\{ \mathrm{Re}(h_b + \delta h_b)\frac{O_{1i}}{\cos\beta} + \mathrm{Re}(\Delta h_b)\frac{O_{2i}}{\cos\beta} - \left[\mathrm{Im}(h_b + \delta h_b)\tan\beta - \mathrm{Im}(\Delta h_b)\right] O_{3i} \right\},$$

$$g^P_{H_i bb} = \frac{1}{h_b + \delta h_b + \Delta h_b \tan\beta} \left\{ \left[\mathrm{Re}\,(\Delta h_b) - \mathrm{Re}(h_b + \delta h_b)\tan\beta\right] O_{3i} - \mathrm{Im}(h_b + \delta h_b)\frac{O_{1i}}{\cos\beta} - \mathrm{Im}(\Delta h_b)\frac{O_{2i}}{\cos\beta} \right\}.$$

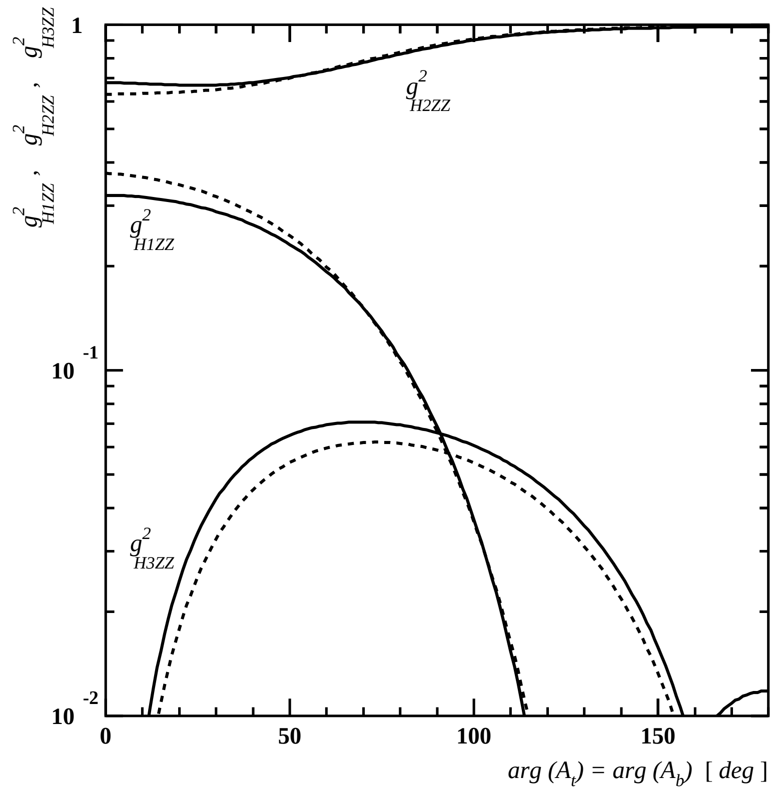
Vector bosons couple to all three neutral Higgs mass eigenstates,

$$g_{H_iVV} = O_{1i}\cos\beta + O_{2i}\sin\beta\,,$$

$$g_{H_iH_jZ} = \frac{g}{2\cos\theta_W}\left[(O_{3i}O_{1j} - O_{1i}O_{3j})\sin\beta - (O_{3i}O_{2j} - O_{2i}O_{3j})\cos\beta\right]\ .$$



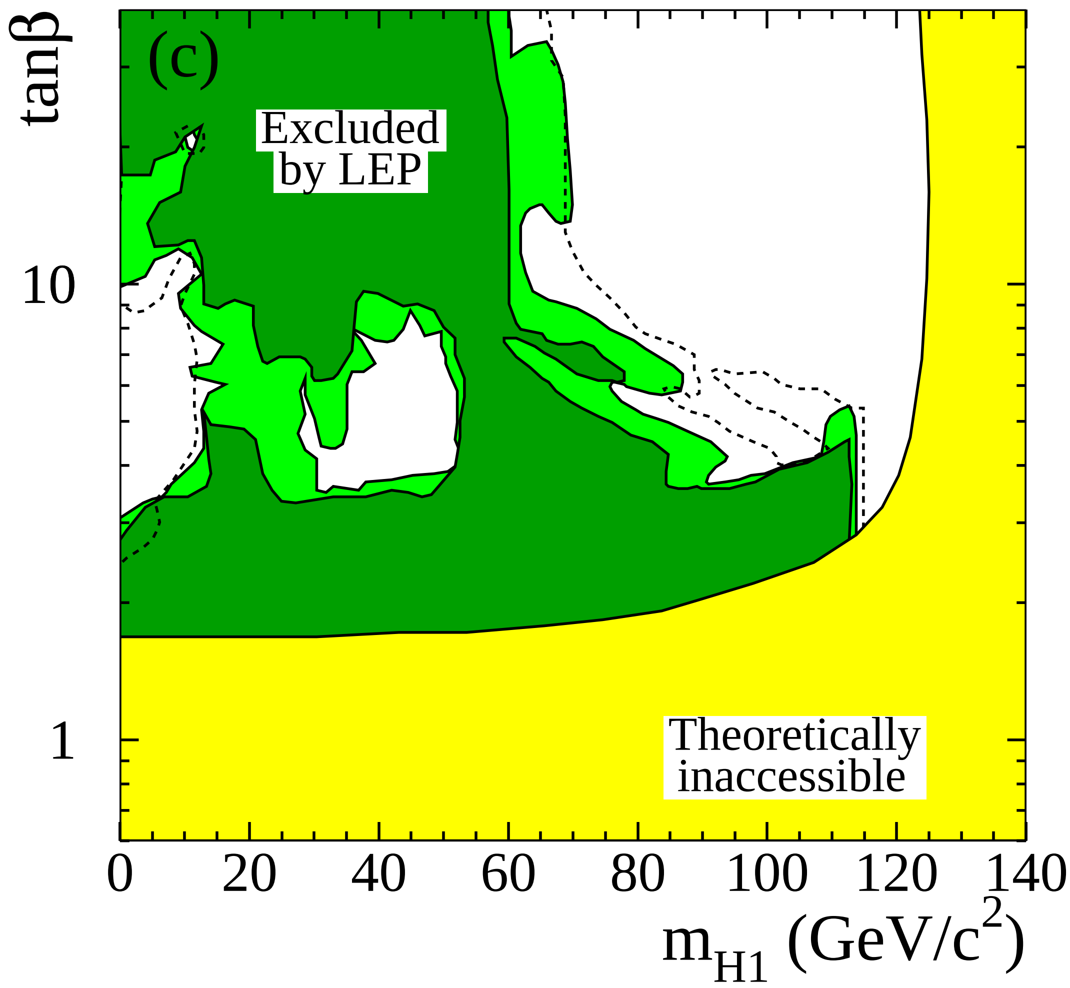
(a)



(b)

(a) Lightest and next-to-lightest neutral Higgs masses and (b) relative couplings (normalized to the SM) of the three neutral Higgs bosons to the Z (or W) as a function of the phase of A_t. Solid [dashed] lines are for $\arg(M_{\tilde{g}}) = 0^\circ$ [90°]. Taken from M. Carena, J. Ellis, A. Pilaftsis and C.E.M. Wagner, Nucl. Phys. **B586** (2000) 92.

Exclusion limits may be significantly weakened in the CPX scenario



Exclusions at 95% CL (light-green) and at 99.7% CL (dark-green) for the CP-violating CPX scenario with $m_t = 174.3$ GeV. The yellow region corresponds to the theoretically inaccessible domains. In each scan point, the more conservative of the two theoretical calculations, FeynHiggs 2.0 or CPH, was used. Taken from S. Schael *et al.* [ALEPH, DELPHI, L3 and Opal Collaborations and the LEP Working Group for Higgs Boson Searches], Eur. Phys. J. **C47** (2006) 547.

Beyond the MSSM Higgs sector

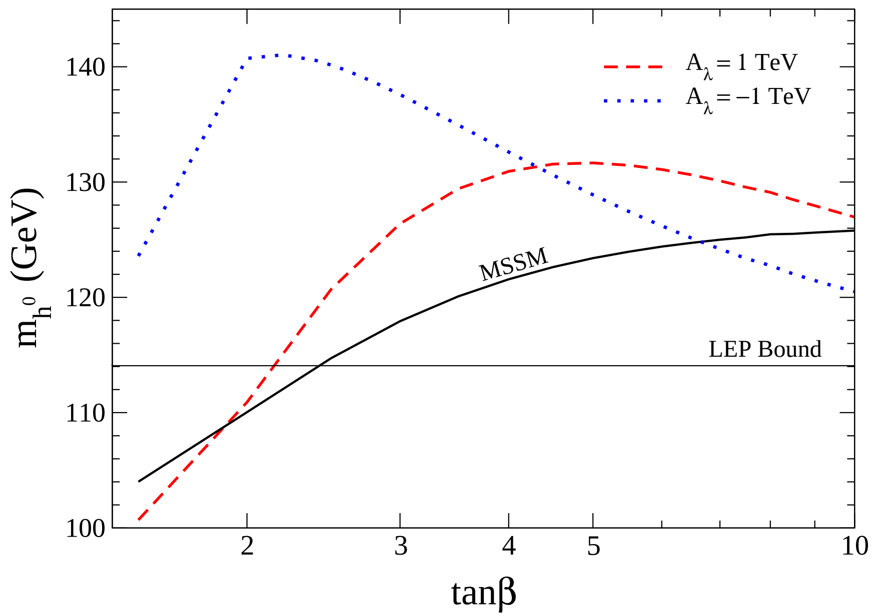
Why go beyond the MSSM? The LEP Higgs mass bounds are uncomfortable for the MSSM, as the mass of h^0 must be somewhat close to its maximally allowed value, which requires heavy stop masses and significant stop mixing. The absence of observed SUSY particles just emphasizes this apparent *little hierarchy problem* that seems to require at least 1% fine-tuning of MSSM parameters to explain the magnitude of the EWSB scale.

In the NMSSM, a Higgs singlet superfield $\hat{S}$ is added to the MSSM. The corresponding superpotential terms,

$$(\mu + \lambda \hat{S})\hat{H}_u \hat{H}_d + \tfrac{1}{2}\mu_S \hat{S}^2 + \tfrac{1}{3}\kappa \hat{S}^3 \,,$$

and soft-SUSY-breaking terms $B_s S^2 + \lambda A_\lambda S H_u H_d$ add additional parameters to the model, which can modify the bounds on the lightest Higgs mass.

For example, in a recent paper by Delgado, Kolda, Olson and de la Puente:



Other authors (e.g. Dermisek and Gunion) have advocated NMSSM models as a way to partially alleviate the little hierarchy problem. More generally, there is a large literature (beginning with Haber and Sher in 1987) suggesting the possibility of relaxing the Higgs mass upper bound in extensions of the MSSM.

In 1993, Espinosa and Quiros showed that it was relatively easy to construct extended models with the lightest Higgs boson mass as large as 155 GeV. Other authors found ways to push this bound higher (although these are perhaps less interesting in light of present experimental Higgs searches).

Where do we stand? Where are we headed?

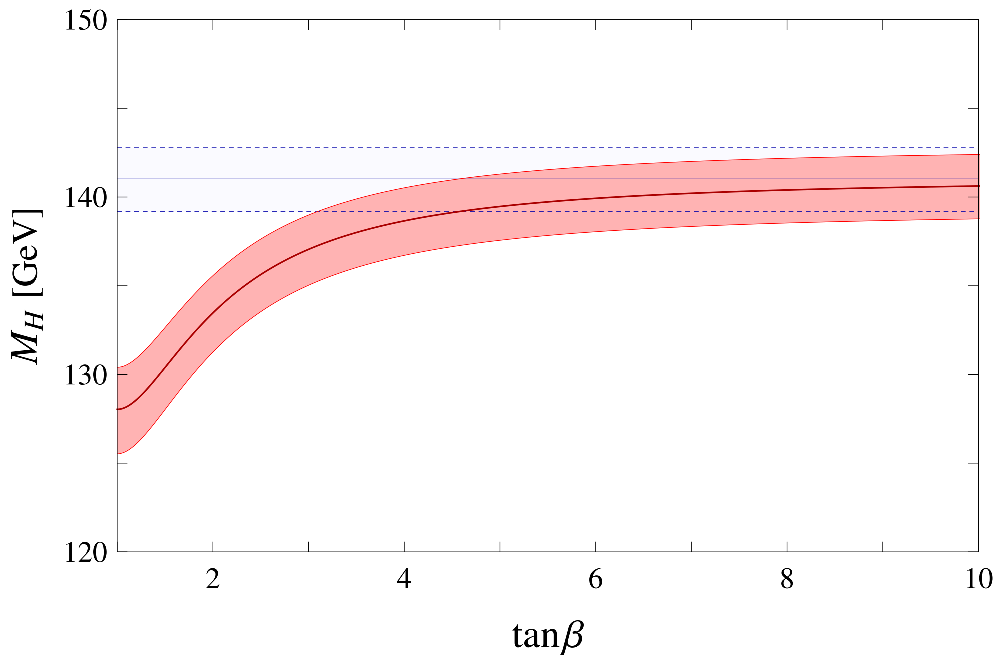
No evidence for the Higgs boson has yet been observed. But, this is precisely what is expected, given the SM global fits based on precision electroweak data. The LHC now begins to zero in on the Higgs mass range, $114~\text{GeV} < m_h < 145~\text{GeV}$, the region where the SM Higgs boson (if it exists) is likely to reside.

Beyond the potential discovery of the Higgs boson (or a clarification of the dynamics of EWSB), future progress depends on whether new physics beyond the Standard Model (BSM) is also found. Possible scenarios include:

1. A SM-like Higgs boson is discovered. No evidence for BSM physics is evident.
2. A SM-like Higgs boson is discovered. Separate evidence for BSM physics emerges.
3. A light Higgs-like scalar is discovered, with properties that deviate from the SM.
4. A very heavy scalar state is discovered.
5. No Higgs boson candidate is discovered, and the entire mass range for a SM-like Higgs boson below 1 TeV is excluded.

In the last three cases, theoretical consistency implies that BSM physics must exist at the TeV energy scale that is observable at the LHC (with sufficient luminosity). Cases 4 and 5 would likely be incompatible with low-energy supersymmetry, whereas cases 2 and 3 would strongly encourage supersymmetric enthusiasts.

Case 1 would cast doubts on the principle of naturalness. Nevertheless, is it still possible to learn about physics at higher mass scales? Consider the following Higgs mass "prediction":



The SM Higgs mass prediction for theories where the boundary condition for the quartic coupling at 10^{14} GeV is fixed by the MSSM, and $\alpha_s(m_Z) = 0.1176$ and $m_t = 173.1 \pm 1.3$ GeV. The horizontal blue lines show the asymptotes of the prediction for large $\tan\beta$. Taken from L.J. Hall, Y. Nomura, JHEP **1003**, 076 (2010).

Conclusions

• The SM is not yet complete. The nature of the dynamics responsible for EWSB (and generating the Goldstone bosons that provide the longitudinal components of the massive $W^\pm$ and Z bosons) is not yet known.

• There are strong hints that a weakly-coupled elementary Higgs boson exists in nature (although loopholes still exist).

• If low-energy supersymmetry is responsible for EWSB, then the Higgs sector will be richer than in the SM. However, in the decoupling regime, it may be difficult to to detect deviations from SM Higgs properties at the LHC or evidence for new scalar states beyond the SM-like Higgs boson.

• Ultimately, one must discover the TeV-scale dynamics associated with EWSB, e.g. low-energy supersymmetry and/or new particles and phenomena responsible for creating the Goldstone bosons. So far, no evidence for BSM physics has been forthcoming.

• If after years of LHC running, there is only a Higgs boson and no evidence for new physics beyond the SM, then . . .?