Why Does the Standard Model Need the Higgs Boson ?

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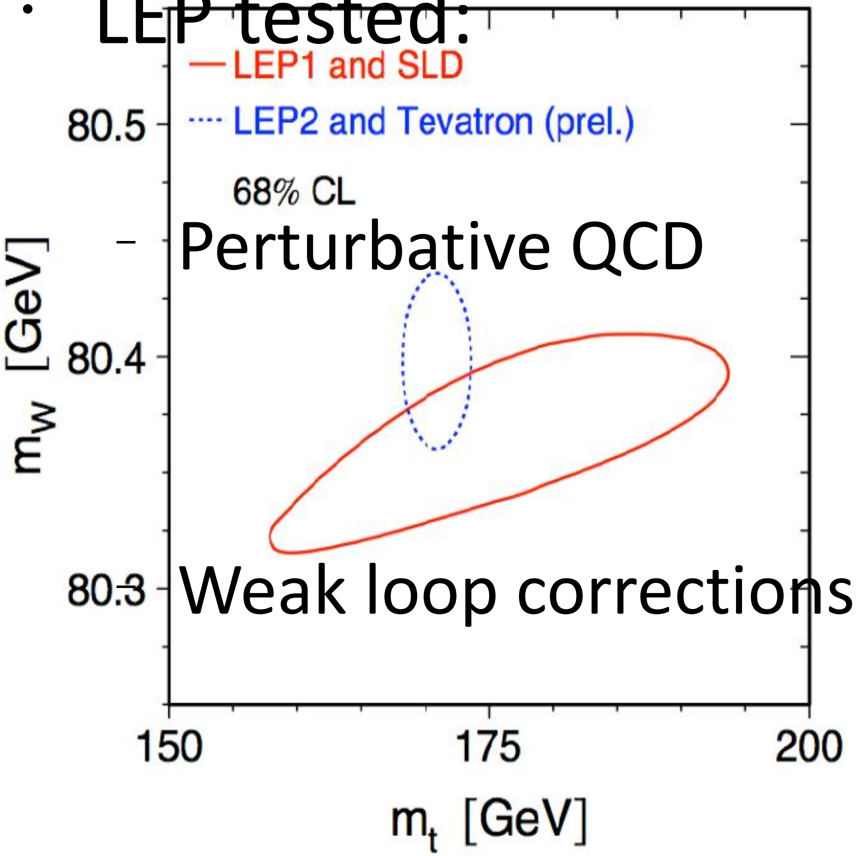
Starting Point

- The SM is described by a Quantum Field Theory.
- Perturbation theory gives reliable results.

Praise the SM I (LEP working group)

- LEP tested:
  - Perturbative QCD
  - Weak loop corrections



$$\alpha_s(M_Z) = 0.1216 \pm 0.0017$$

$$m = 174.2 \pm 3.3 \, GeV/c^2$$ Direct

$$m = 172.3 \pm^{10.2}_{7.6} \, GeV/c^2$$

The Standard Model

- SU(3)xSU(2)xU(1) QFT
  - This means:

1 - We know the symmetries of the building blocks;

2 - We have a general principle to obtain the equations of motion;

$$S = \int d^4 x L(fields, \partial^{\mu} fields)$$

3 - We don't know how to solve these equations. But we know how to use perturbation theory.

Example: QED

- - Building Blocks

$$e^- \longrightarrow \psi$$

$$\gamma \longrightarrow A^\mu$$

  - Symmetries

$$\psi \longrightarrow e^{ie\alpha(x)}\psi \qquad \partial^\mu \longrightarrow \partial^\mu + ieA^\mu$$

$$A^\mu \longrightarrow A^\mu - \partial^\mu \alpha$$

- Lagrangian

$$L = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\psi}A\!\!\!/\psi$$

- Equations of Motion

$$(i\partial\!\!\!/ - m)\psi = eA\!\!\!/\psi$$

$$\partial_\mu F^{\mu\nu} = e\bar{\psi}\gamma^\nu\psi$$

Laws of Nature are invariant under:

Lorentz Transformations $J^{\mu\nu}$

Space x Time Translations P^{μ}

Poincaré Group

↓

The building blocks of any QFT must be Irreducible Representations of the Poincaré Group

Representations of the Poincaré group are classified

according to the values of P^2 and W^2

$$P^2 = P^\mu P_\mu$$

$$W^\mu = -\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} J^{\nu\alpha} P^\beta$$

Eigenvalues:

$$P^2 \longrightarrow m^2$$

$$W^2 \longrightarrow -m^2 s(s+1) \qquad s = 0, 1/2, 1, \ldots$$

Lorentz Transformations

- Rotations

$$t' = t$$
$$x' = \cos\theta\, x + \sin\theta\, y$$
$$y' = -\sin\theta\, x + \cos\theta\, y$$
$$z' = z$$

- Boosts

$$t' = \cosh\theta\, t - \sinh\theta\, x$$
$$x' = -\sinh\theta\, t + \cosh\theta\, x$$
$$y' = y$$
$$z' = z$$

$$\tanh\theta = \beta$$

Generators

$$[J_i, J_j] = i\varepsilon_{ijk} J_k \qquad [J_i, K_j] = i\varepsilon_{ijk} K_k \qquad [K_i, K_j] = -i\varepsilon_{ijk} J_k$$

$$J_{\pm i} = \frac{1}{2}(J_i \pm iK_i)$$

$$[J_{+i}, J_{+j}] = i\varepsilon_{ijk} J_{+k}$$
$$[J_{-i}, J_{-j}] = i\varepsilon_{ijk} J_{-k}$$
$$[J_{+i}, J_{-j}] = 0$$

Representations

Two SU(2) Algebras

(0,0) Scalar

(1/2,0) two dimensional $\Longleftrightarrow$ (0,1/2) two dimensional

Parity

Because QED conserves parity we have to stack up both representations.
Electron belongs to (1/2,0)+(0,1/2).
But we obtain 4 degrees of freedom! Discover the Positron!

Difference between $m \neq 0$ and $m = 0$

- Spin ½

$m \neq 0$ Two degrees of freedom

$m = 0$ Two degrees of freedom

- Spin 1

$m \neq 0$ Three degrees of freedom

$m = 0$ Two degrees of freedom

Lorentz boost

$$t = \gamma(t' + \beta z')$$
$$z = \gamma(z' + \beta t')$$

- Frame S'(Rest frame)

$$P^{\mu} = (E,0,0,p) = E(1,0,0,\beta)$$

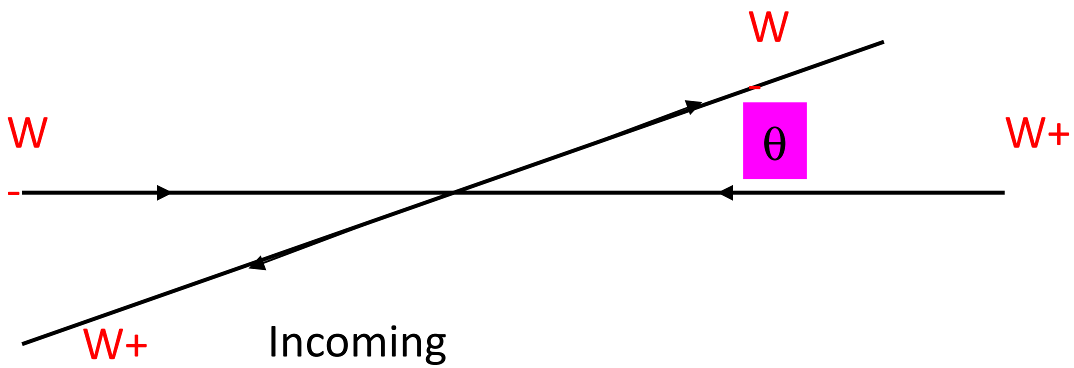
$$P'^{\mu} = (m,0,0,0)$$

$$\varepsilon^{\mu}_{L} = (\gamma\beta,0,0,\gamma) = \frac{E}{m}(\beta,0,0,1)$$

$$\varepsilon'^{\mu} = \begin{cases} (0,1,0,0) & \\ (0,0,1,0) & \text{T} \\ (0,0,0,1) & \text{L} \end{cases}$$

$$E = \gamma m$$

$W_L^- W_L^+ \longrightarrow W_L^- W_L^+$ Scattering



$$x = \frac{s}{4M^2}$$

Incoming

W-

$$p_-^\mu = \frac{\sqrt{s}}{2}(1,0,0,\beta)$$

$$\eta_-^\mu = \frac{\sqrt{s}}{2M}(\beta,0,0,1)$$

W+

$$p_+^\mu = \frac{\sqrt{s}}{2}(1,0,0,-\beta)$$

$$\eta_+^\mu = \frac{\sqrt{s}}{2M}(\beta,0,0,-1)$$

Outgoing

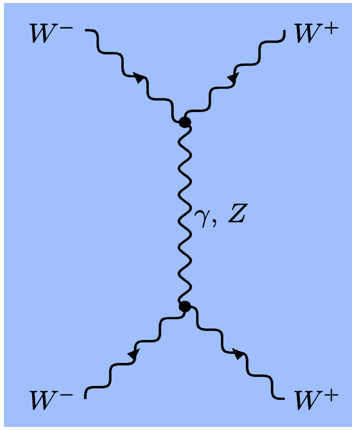
$$k_-^\mu = \frac{\sqrt{s}}{2}(1,\beta\hat{k})$$

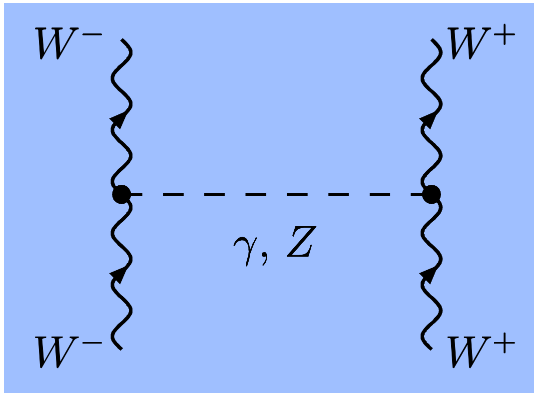
$$\eta_-^\mu = \frac{\sqrt{s}}{2M}(\beta,\hat{k})$$

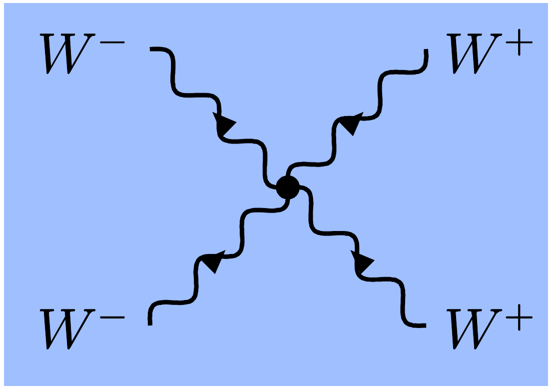
$$k_+^\mu = \frac{\sqrt{s}}{2}(1,-\beta\hat{k})$$

$$\eta_+^\mu = \frac{\sqrt{s}}{2M}(\beta,-\hat{k})$$

Scattering $W_L^- W_L^+ \to W_L^- W_L^+$







Examine the amplitudes and take the $x \to \infty$ Limit

S-channel

Photon + Z

$$T_s = -g^2\left[\sin^2\theta_W + \frac{\cos^2\theta_W}{1 - M_Z^2/s}\right]\cos\theta\,(4x^2 - 3 - \frac{1}{x})$$

$$\sin^2\theta_W + \frac{\cos^2\theta_W}{1 - M_Z^2/s} \; ; \; 1 + \cos^2\theta_W \frac{M_Z^2}{s} = 1 + \frac{1}{4x}$$

$$T_s \; ; -g^2\cos\theta\left[\,4x^2 + x - 3 + \ldots\right]$$

T-channel

$$T_t = g^2\left[\frac{\sin^2\theta_W}{1-\beta^2\cos\theta - 1/x} + \frac{\cos^2\theta_W}{1-\beta^2\cos\theta - 1/x + 2M_Z^2/s}\right]$$

Photon+Z

$$\left[(1-\cos\theta)^2(3+\cos\theta)x^2 + (1-\cos\theta)(-3+10\cos\theta+\cos^2\theta)x \right.$$
$$\left. + 1-7\cos\theta+10\cos^2\theta-(1+\cos\theta)/x\right]$$

$$\frac{\sin^2\theta_W}{1-\beta^2\cos\theta - 1/x} ; \sin^2\theta_W\frac{1}{1-\cos\theta}(1+\frac{1}{x})$$

$$\frac{\cos^2\theta_W}{1-\beta^2\cos\theta - 1/x + 2M_Z^2/s} ; \cos^2\theta_W\frac{1}{1-\cos\theta}(1+\frac{1}{x}-\frac{1}{2x(1-\cos\theta)}\frac{M_Z^2}{M^2})$$

$$T_t ; g^2\left[(3-2\cos\theta-\cos^2\theta)x^2 + (-\frac{3}{2}+\frac{15}{2}\cos\theta)x + \ldots\right]$$

Examine the amplitudes in the $x \to \infty$ Limit

Summary

$$T_s \; ; -g^2\left[4\cos\theta\, x^2 + x - 3 + \ldots \right]$$

$$T_t \; ; \; g^2\left[(3 - 2\cos\theta - \cos^2\theta\,)x^2 + (-\frac{3}{2} + \frac{15}{2}\cos\theta\,)x + \ldots \right]$$

4 W vertex

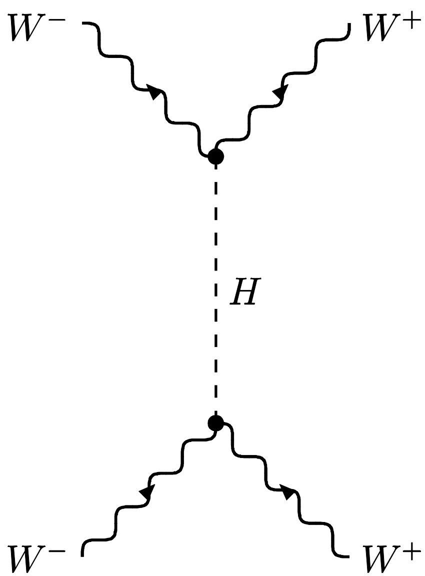
$$T_v = g^2\left[(\cos^2\theta - 3 + 6\cos\theta\,)x^2 + 2(1 - 3\cos\theta\,)x \right]$$

The gauge structure cancels the quadratic term. But the sum grows as s!

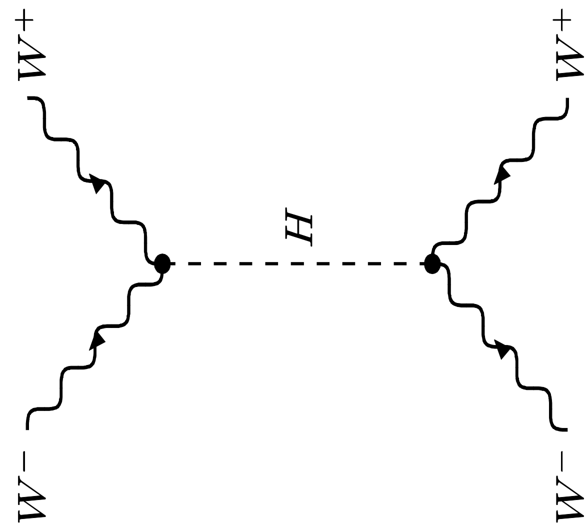
$$T_{s+t+v} \; ; \; g^2 \frac{1}{2}\left[1 + \cos\theta \,\right] x + \ldots$$

Amplitudes can grow at most as $\ln^2(s/m)$ Froissart bound

Let us save the theory. Invent a Scalar H.



H s-channel



H t-channel

The scalar Boson H couples to W with λH

Then, there are two additional amplitudes:

$$T_{Hs} = -\left(\frac{\lambda_H}{M}\right)^2 \frac{1}{1 - M_H^2/s}\left[x - 1 + \frac{1}{4x}\right]$$

$$T_{Ht} = \left(\frac{\lambda_H}{M}\right)^2 \frac{1}{1 - \beta^2 \cos\theta - 1/x + 2M_H^2/s}\left[\frac{1}{2}(1 - \cos\theta)^2 x - 1 + \cos\theta + \frac{1}{2x}\right]$$

In the high energy limit the sum is:

$$T_H \simeq -\left(\frac{\lambda_H}{M}\right)^2\left[\frac{1}{2}(1 + \cos\theta)x\right]$$

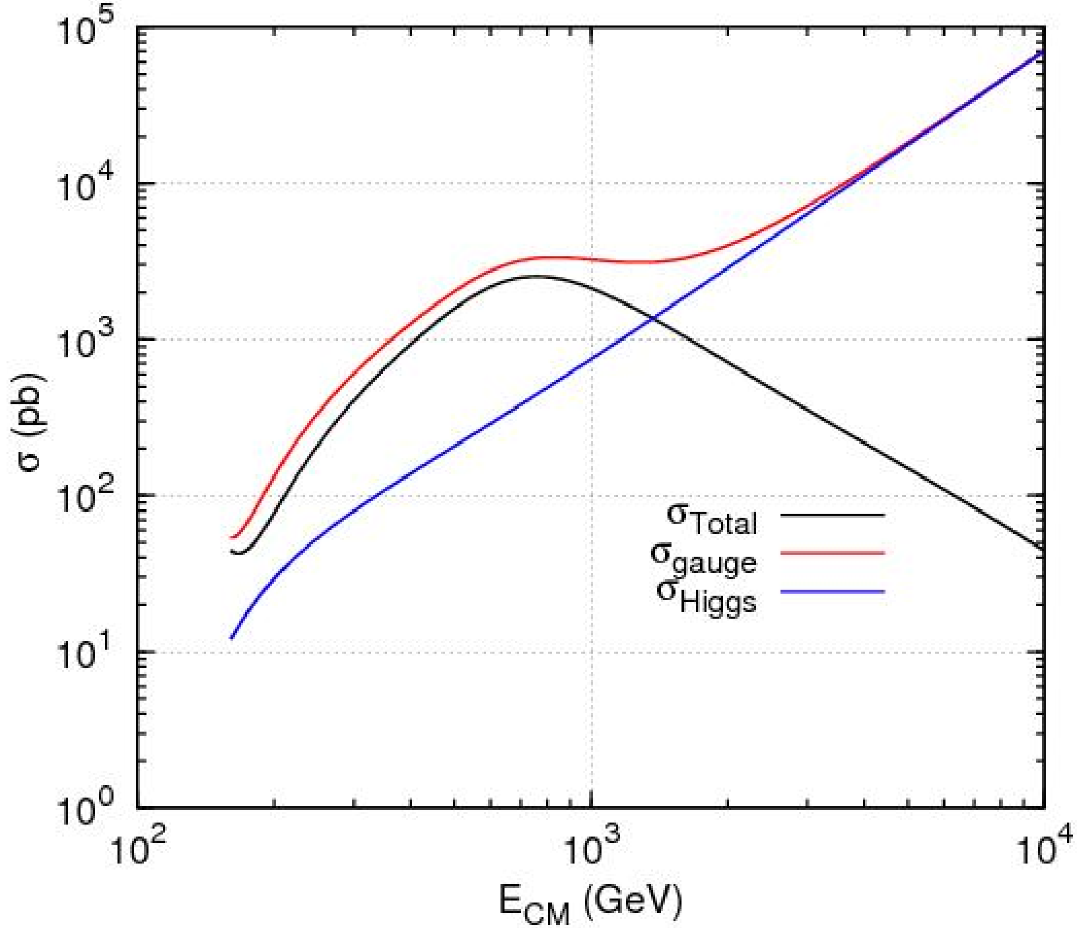
The "normal" contributions give: $$T_{s+t+v} \; ; \; g^2 \frac{1}{2}\left[1+\cos\theta \right] x + \ldots$$

The scalar contributions are: $$T_H \; ; \; -\left(\frac{\lambda_H}{M} \right)^2 \frac{1}{2}\left[1+\cos\theta \right] x + \ldots$$

The divergent terms cancel if $$\lambda_H = gM$$

A massive spin 1 particle "cries out" for a spin zero companion coupled with a strength proportional to g.

$$W_L^- W_L^+ \rightarrow W_L^- W_L^+$$



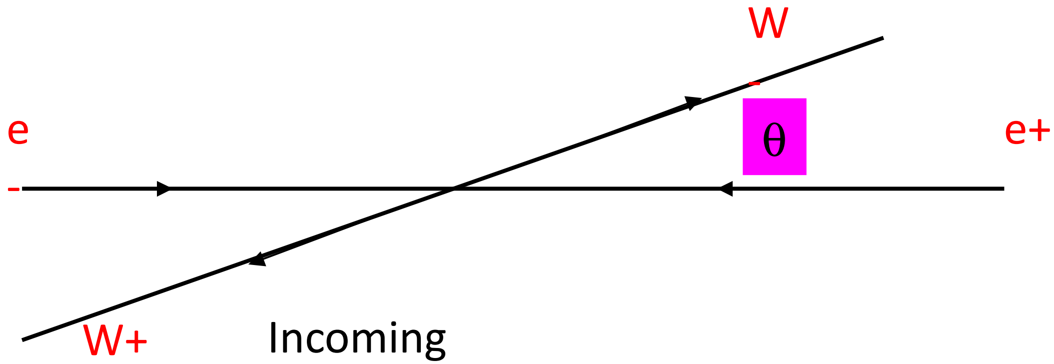
Summary

- Massive Spin 1 particles have a bad High Energy Behaviour;
- The introduction of a Spin 0 companion can cure the disease;
- There is no known alternative medicine.

$e^- e^+ \longrightarrow W_L^- W_L^+$ Scattering

$$\beta_i = \sqrt{1 - \frac{4m^2}{s}}$$

$$\beta_f = \sqrt{1 - \frac{4M^2}{s}}$$



Outgoing

e- $\quad p^\mu = \frac{\sqrt{s}}{2}(1,0,0,\beta_i)$

e+ $\quad p'^\mu = \frac{\sqrt{s}}{2}(1,0,0,-\beta_i)$

W
-

$$k_-^\mu = \frac{\sqrt{s}}{2}(1,\beta_f \hat{k})$$

$$\eta_-^\mu = \frac{\sqrt{s}}{2M}(\beta_f, \hat{k})$$

W+

$$k_+^\mu = \frac{\sqrt{s}}{2}(1,-\beta_f \hat{k})$$

$$\eta_+^\mu = \frac{\sqrt{s}}{2M}(\beta_f, -\hat{k})$$

$e^+ e^- \longrightarrow W_L^- W_L^+$ Scattering

γ and *Z* s-channel

$$T_s = e^2 \frac{s\sqrt{s}}{4M^2}\beta_f(\beta_f - 3)\left[\frac{1}{s} - \frac{1}{s - M_Z^2}\right]\bar{v}(p')\bar{\gamma}.\hat{k}u(p)$$

$$+\, g^2 \frac{s\sqrt{s}}{8M^2}\beta_f(\beta_f - 3)\frac{1}{s - M_Z^2}\bar{v}(p')\bar{\gamma}.\hat{k}\gamma_L u(p)$$

$$= \frac{\sqrt{s}}{2m}\sin\theta\,\chi_\alpha^\dagger\sigma_2\chi_\beta + \ldots \qquad\qquad = -i\frac{\sqrt{s}}{2m}\sin\theta\,\chi_\alpha^\dagger\sigma_+\chi_\beta$$

Both terms grow as s. However, notice the cancelation between gamma and Z.

t-channel diagram

$$T_t = -g^2 \frac{s\sqrt{s}}{8M^2}\left[\beta_f(\beta_f^2 - 3) + 2\beta_i \cos\theta\right] \frac{1}{s(1-\beta_i\beta_f \cos\theta) - 2(M^2 + m^2)}$$

$$\bar{v}(p')\bar{\gamma}.\hat{k}\gamma_L u(p)$$

It cancels with the similar contribution from the s-channel, i. e.

$$T_s = \ldots + g^2 \frac{s\sqrt{s}}{8M^2}\beta_f(\beta_f - 3)\frac{1}{s - M_Z^2}\bar{v}(p')\bar{\gamma}.\hat{k}\gamma_L u(p)$$

Conclusion: All contributions that grow with s or $\sqrt{s}$ cancel

Notice that:

$$\beta_i = \sqrt{1 - \frac{4m^2}{s}}$$

$$\beta_f = \sqrt{1 - \frac{4M^2}{s}}$$

The previous results were obtained with *m=0.* Otherwise there would be an additional contribution to the t-channel diagram, namely:

$$T_t = \ldots - g^2 \frac{sm}{8M^2} \frac{1}{s(1 - \beta_i \beta_f \cos\theta) - 2(M^2 + m^2)}$$

$$\bar{v}(p')\left[1 + \beta_f - 2\beta_f \gamma^0 \vec{\gamma}.\hat{k}\right] u(p)$$

$$= -\frac{\sqrt{s}}{2m}\left[\beta_i(1 + \beta_f^2) - 2\beta_f \cos\theta\right] \chi_\alpha^\dagger \sigma_3 \chi_\beta + \ldots$$

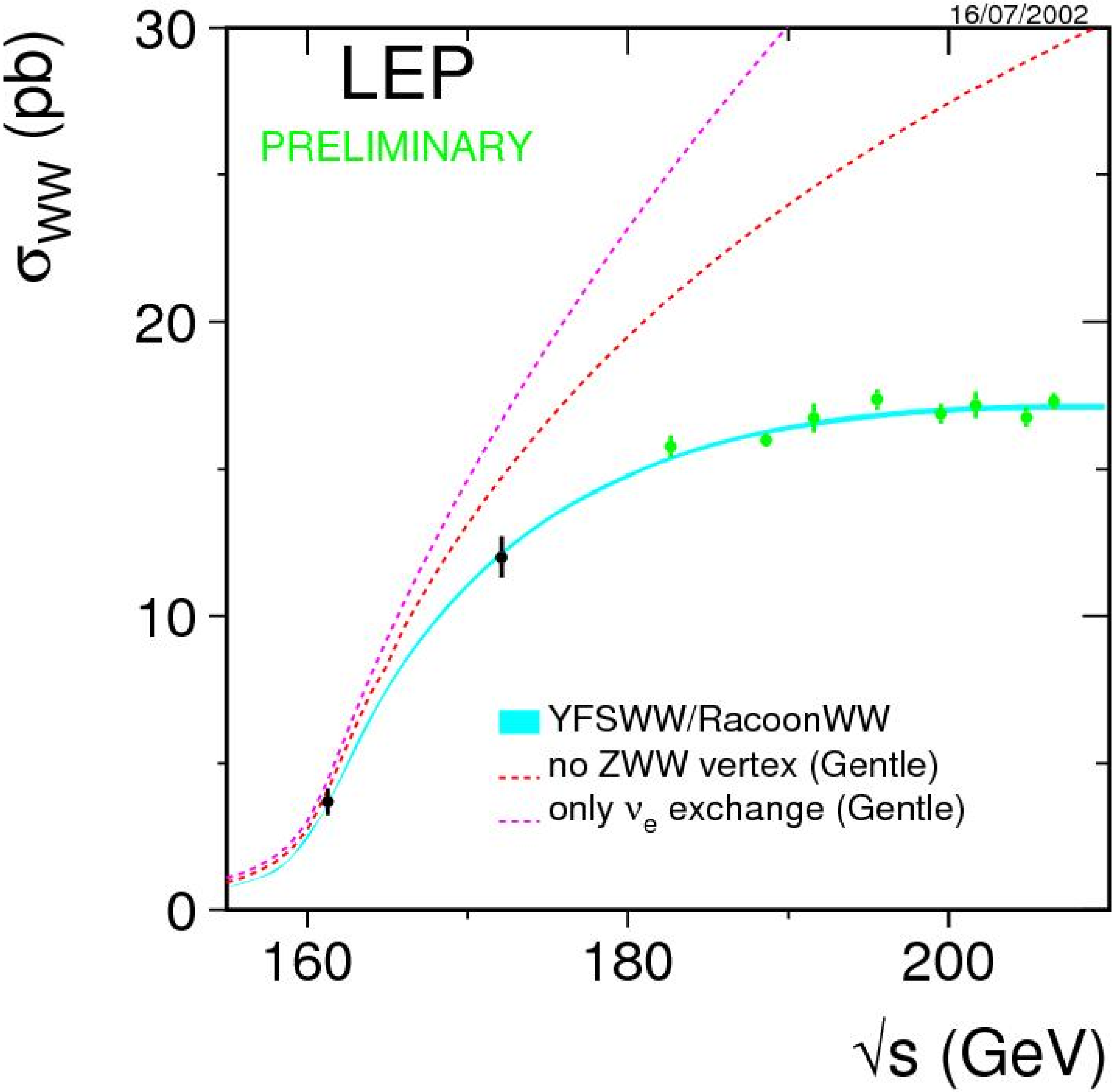
Again the scalar exchange rescues the theory!

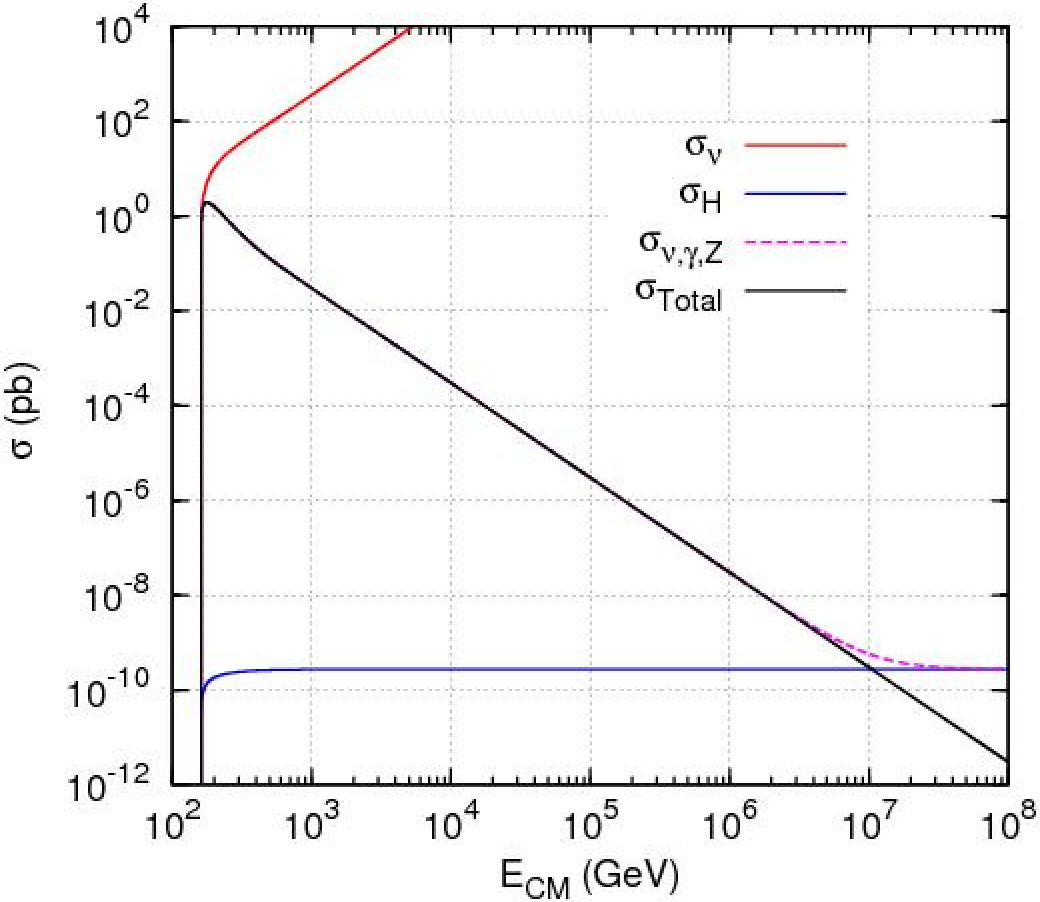
$$T_H = g^2 \frac{sm}{8M^2}(1+\beta_f^2)\frac{1}{s-M_H^2}\bar{v}(p')u(p)$$

$$= -\frac{\sqrt{s}}{2m}\beta_i \chi_\alpha^\dagger \sigma_3 \chi_\beta$$

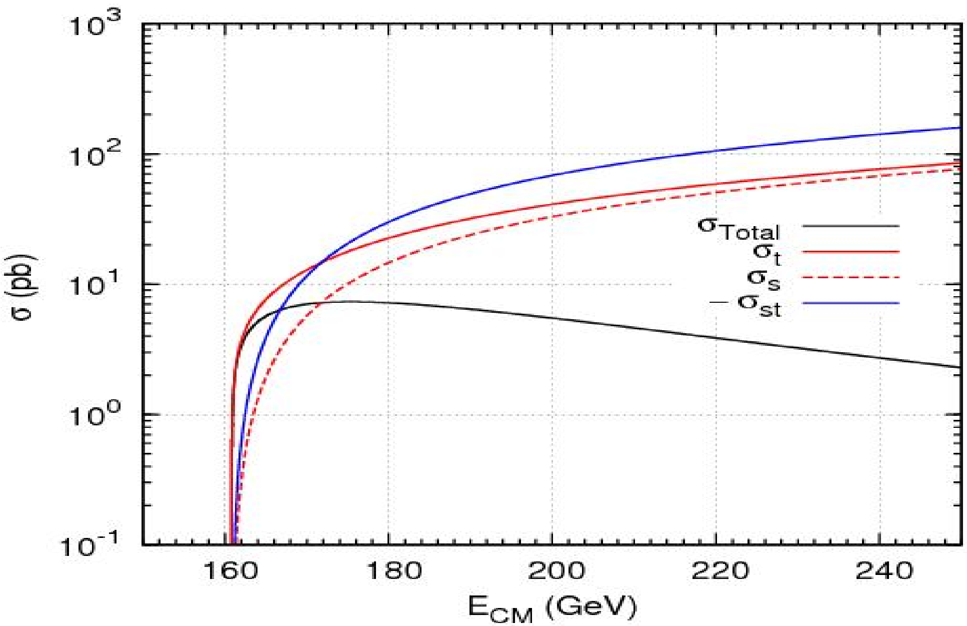
$$T_H = -g^2 \frac{\sqrt{s}}{16M^2}(1+\beta_f^2)\beta_i \frac{1}{1-M_H^2/s}\chi_\alpha^\dagger \sigma_3 \chi_\beta$$

$$T_t = \ldots + g^2 \frac{\sqrt{s}}{16M^2}\frac{\beta_i(1+\beta_f^2)-2\beta_f\cos\theta}{(1-\beta_i\beta_f\cos\theta)-2(M^2+m^2)/s}\chi_\alpha^\dagger \sigma_3 \chi_\beta$$


16/07/2002
LEP
PRELIMINARY
σ_WW (pb)
30
20
10
0
160
180
200
√s (GeV)
YFSWW/RacoonWW
no ZWW vertex (Gentle)
only ν_e exchange (Gentle)



10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
σ (pb)
σ_ν
σ_H
$\sigma_{\nu,\gamma,Z}$
σ_{Total}
10^2
10^3
10^4
10^5
10^6
10^7
10^8
E_{CM} (GeV)



10^3
10^2
10^1
10^0
10^{-1}
σ (pb)
160
180
200
220
240
E_{CM} (GeV)
σ_{Total}
σ_t
σ_s
$-\sigma_{st}$

References

- The previous Figures are from:
- J. C. Romão
- Departamento de Física and CFTP, Instituto Superior Técnico
- Avenida Rovisco Pais 1, 1049-001 Lisboa, P
- Portugal

Summary:

Parity conservation in QED → Electron in Dirac Rep.

↓

Mass terms are Left-right

Parity violation in Weak Int. → The weak currents are left

$$\begin{pmatrix}\nu \\ e\end{pmatrix}_L \quad \nu_R \quad e_R$$

Bad behaviour of WW scattering → Scalar boson couples to W

↓

Scalar is a doublet of SU(2)

$$\Phi = \begin{pmatrix}\varphi^{+} \\ \varphi^{0}\end{pmatrix}$$

Bad behaviour of e+e- → WW → The scalar couples to fermions

$$\mathcal{L}_Y = g_Y (\bar{\nu} \quad \bar{e})_L \begin{Bmatrix}\varphi^{+} \\ \varphi^{0}\end{Bmatrix} e_R + \ldots$$

Solution

$$\mathcal{L}_s = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi^\dagger \Phi)$$

with

$$V = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

Spontaneous symmetry breaking.

$$\langle 0 | \Phi | 0 \rangle = \begin{bmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{bmatrix}$$

$$v = \sqrt{\mu^2 / \lambda}$$

Three degrees of freedom become the longitudinal polarizations of W and Z. The remaining is the Higgs Boson.

Higgs Boson

From the value of GF one obtains:

The parameter λ can be traded off with the Higgs mass.

Inserting the LEP upper bound: • *MH < 144 GeV* (95%CL)

gives

One can safely use perturbation theory.

FAQ

- The Higgs mechanism is a “clever trick”. Can we do it without ending up with an

  Higgs Boson?

NO. Cif. the example on WW scattering.

Unless …

FAQ

- If we are not careful can we also break the electromagnetic gauge group and give the photon a mass?

- No, in the minimal model with a single doublet.

$$V = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$\Phi^\dagger \Phi \equiv (\varphi_1 - i\varphi_2 \quad \varphi_3 - i\varphi_4) \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix} = \sum_{i=1,4} \varphi_i^2$$

Conclusion

- LEP results tested the SM in such a way that it is hard to believe that one is not checking the loop expansion of a QFT.
- Then, the standard model is a QFT valid at the same level as, for instance, QED.
- Hence, the Higgs boson ought to exist.