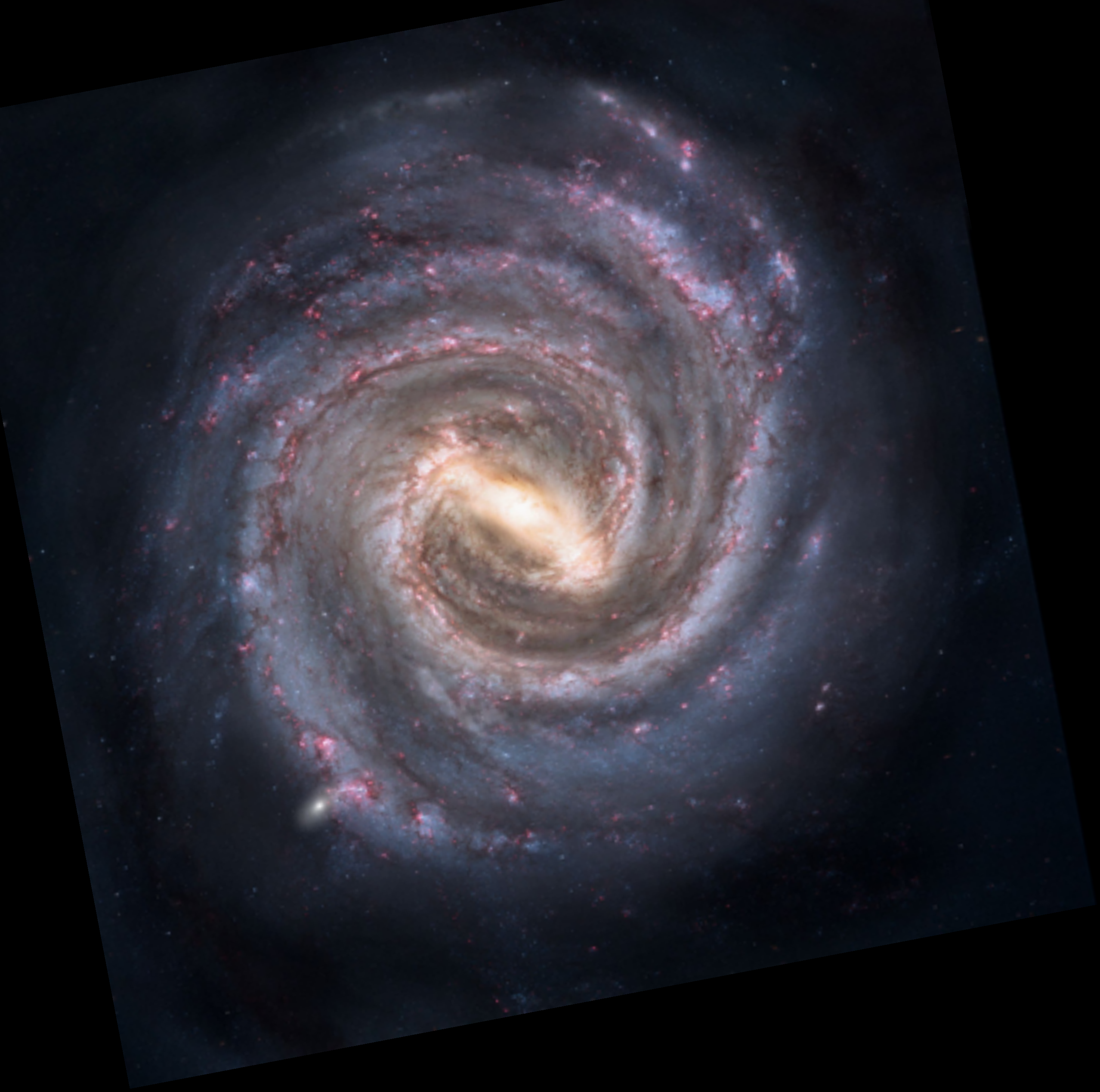
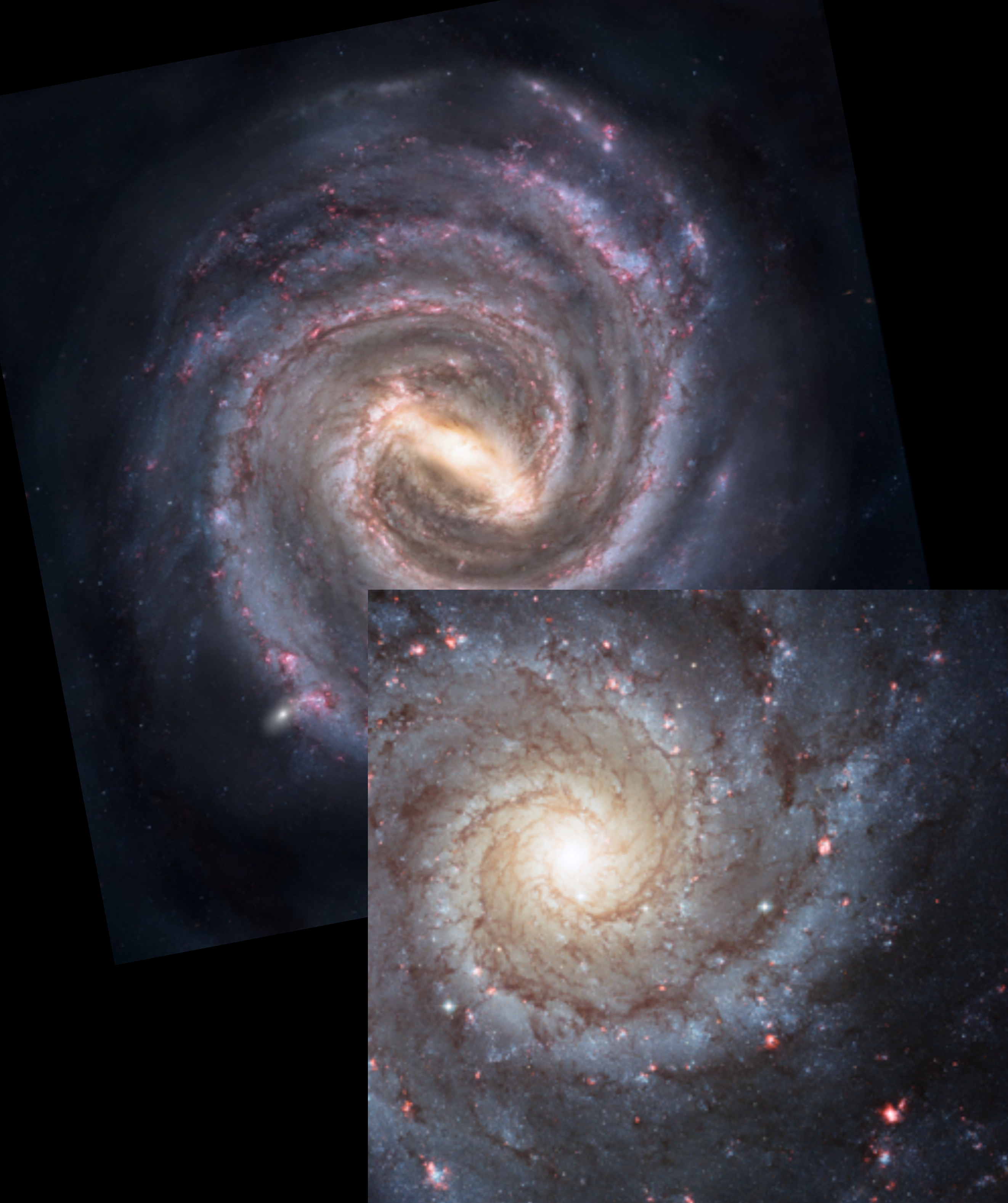
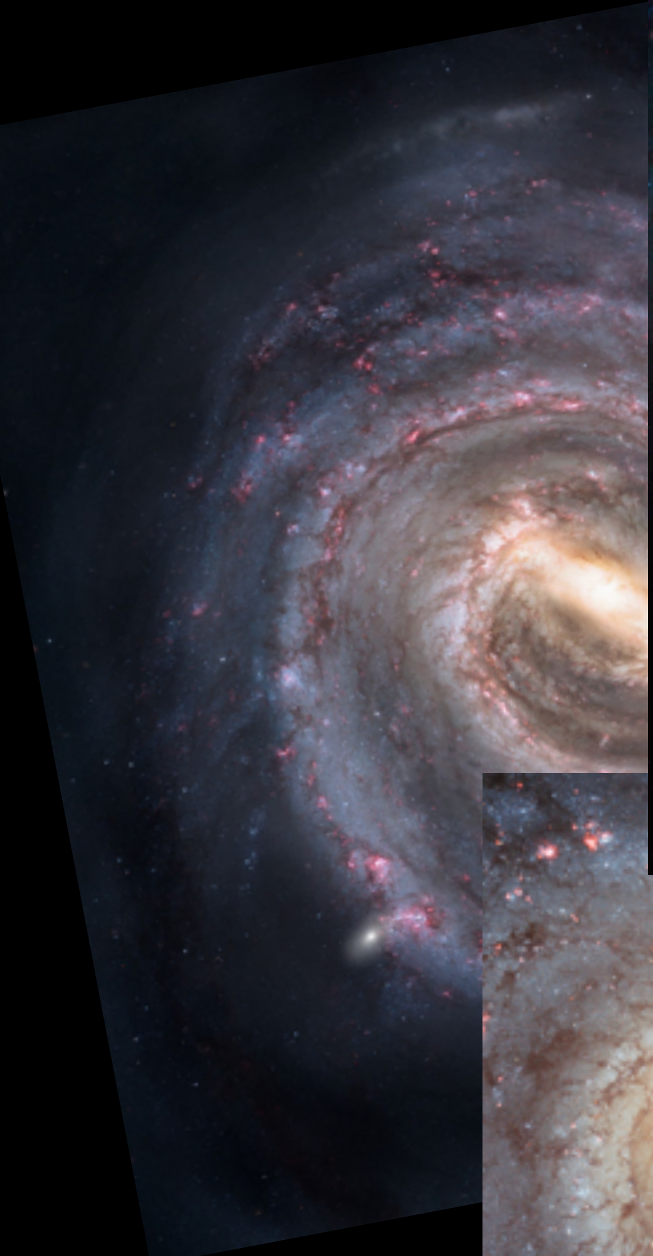
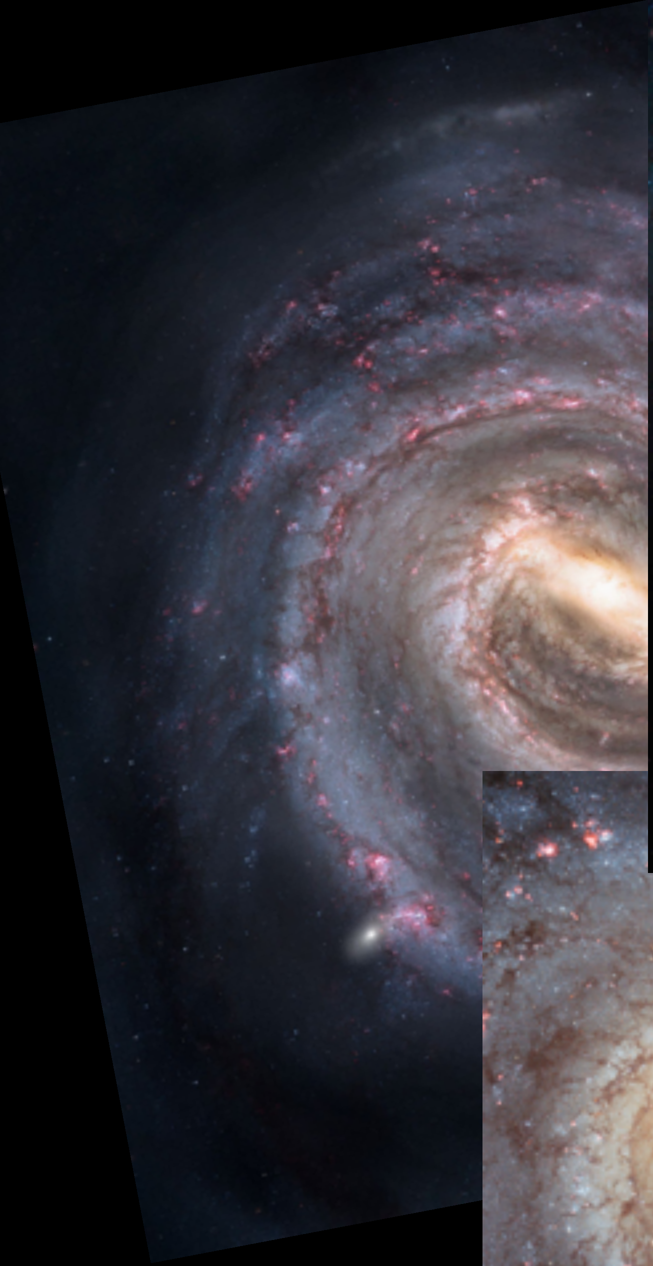
Galaxies

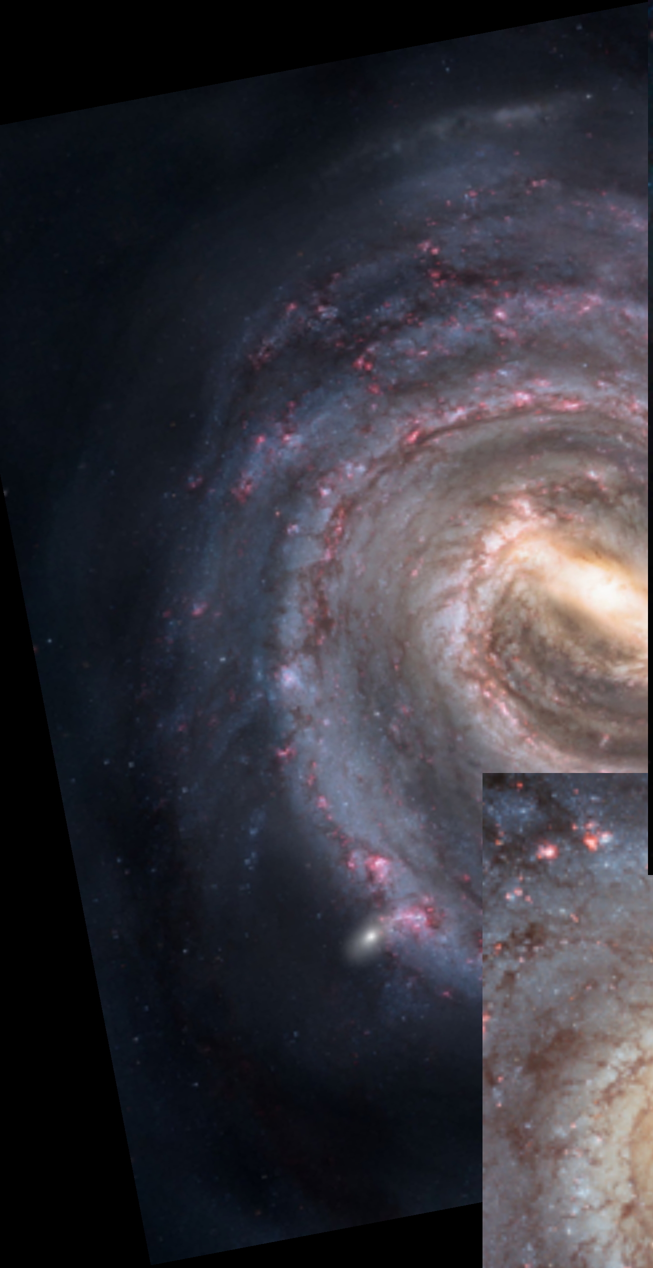
Jo Bovy (University of Toronto)



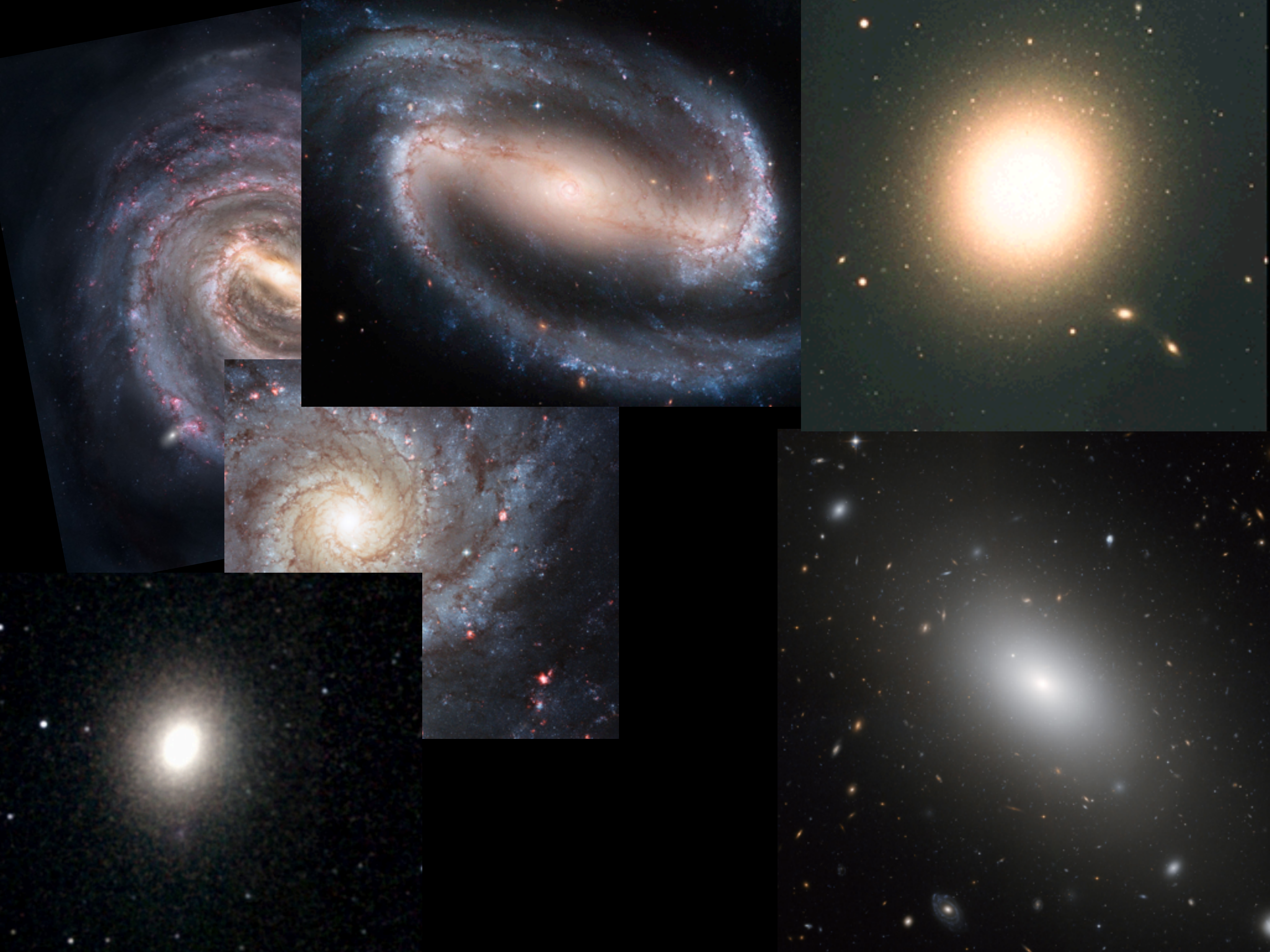


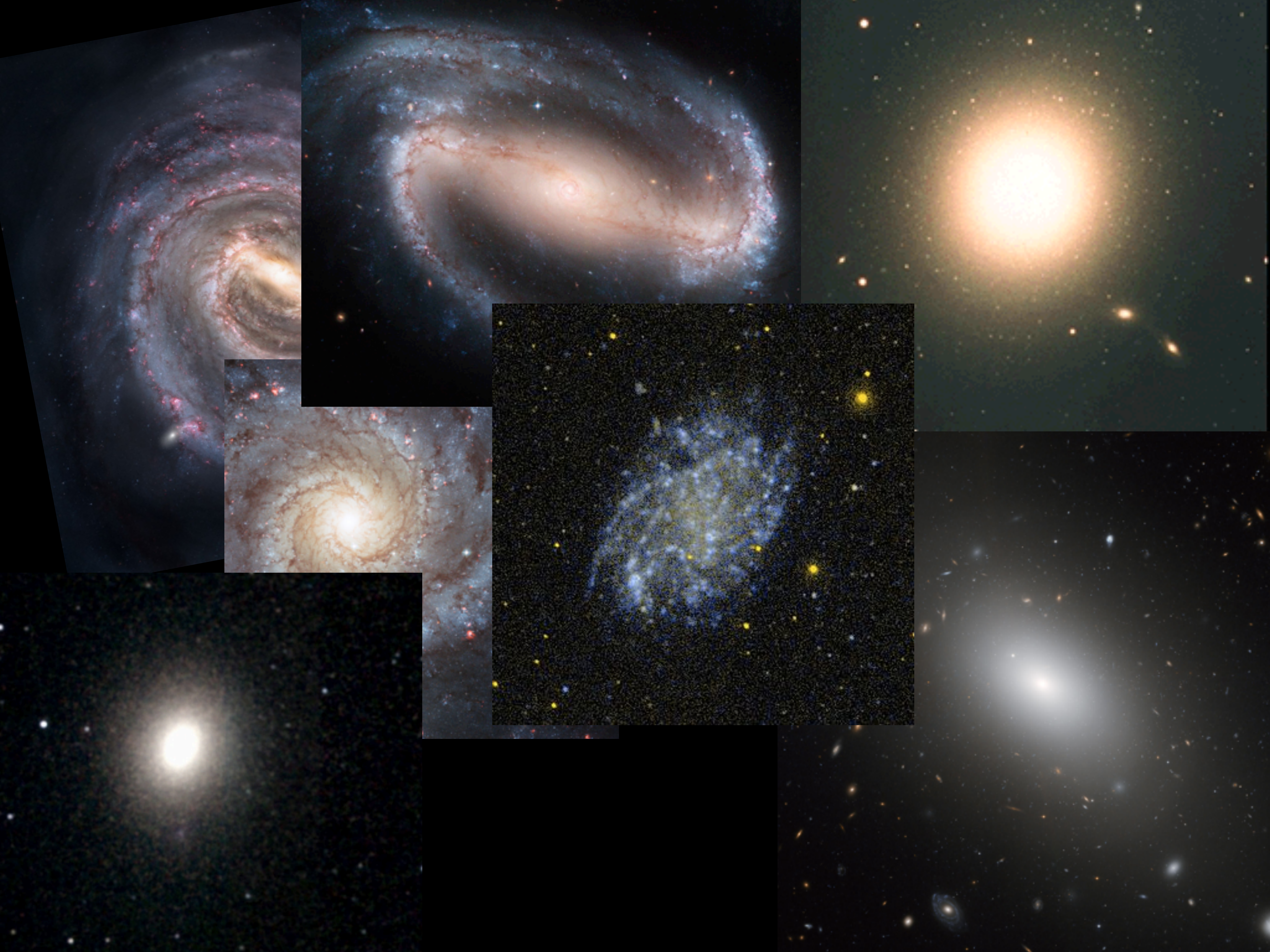


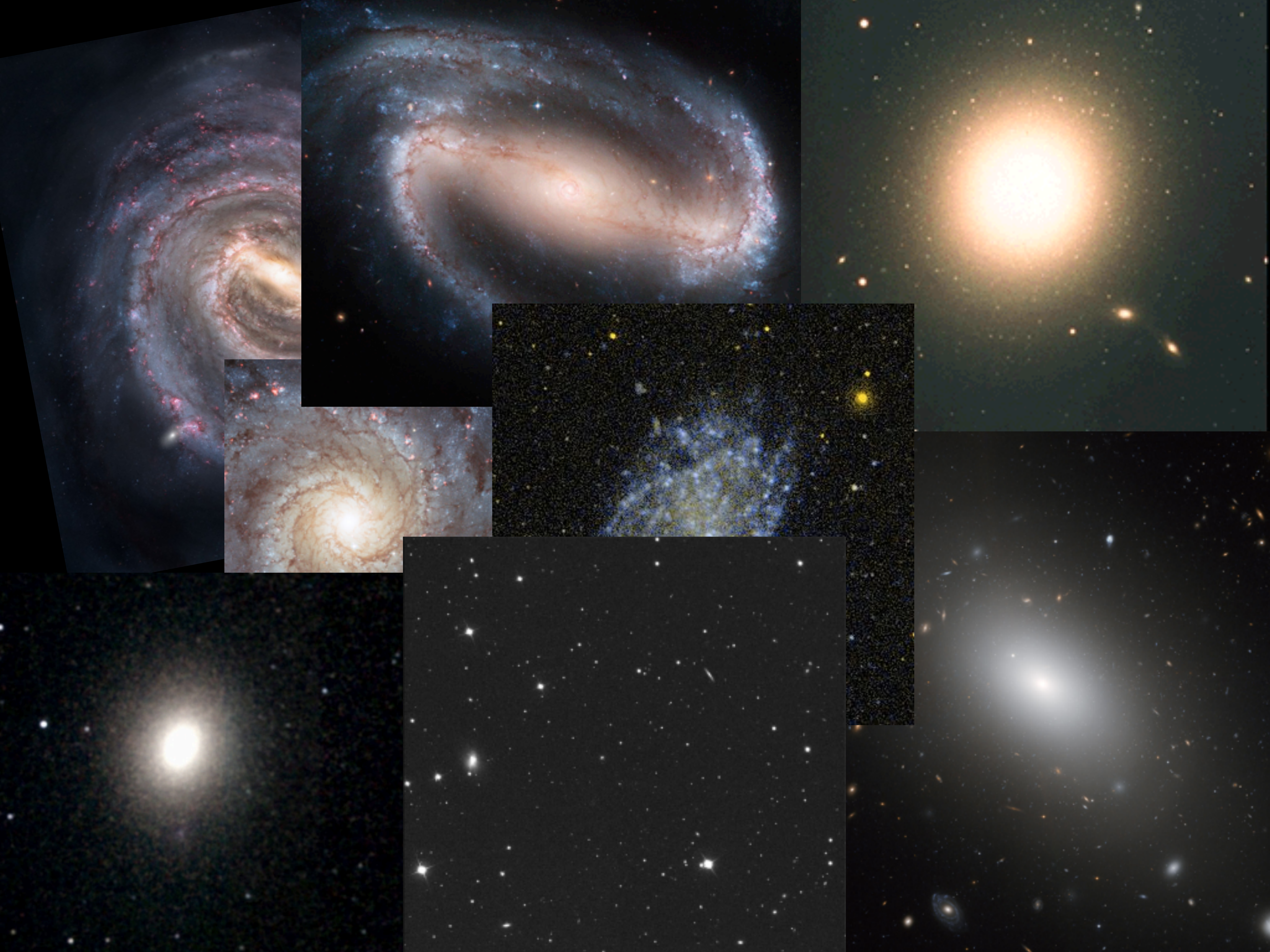




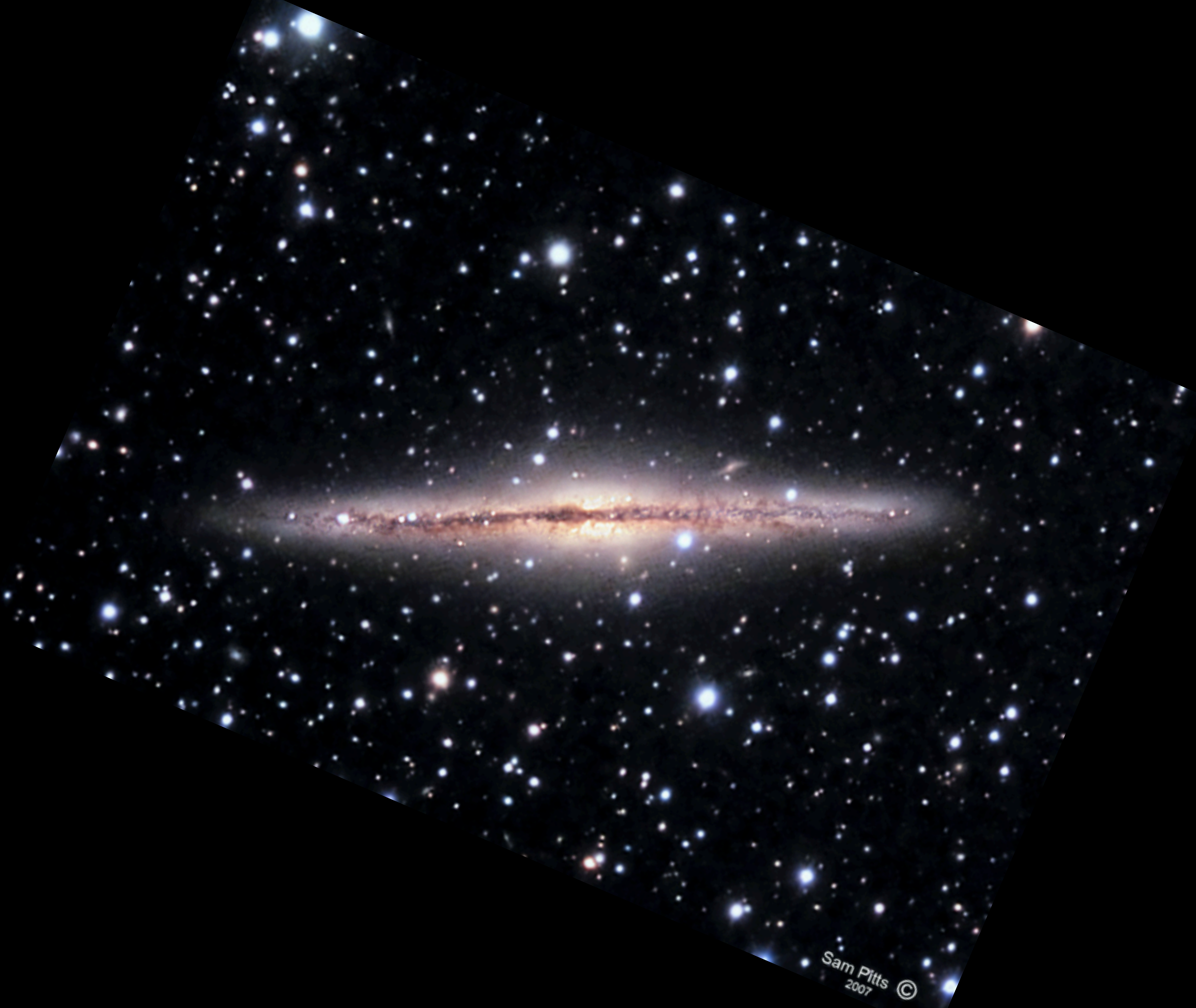






Overview

- 1st lecture: General introduction to galaxies and their properties
- 2nd lecture: Galactic dynamics
- 3rd lecture: Galaxy evolution and Dark matter in the Milky Way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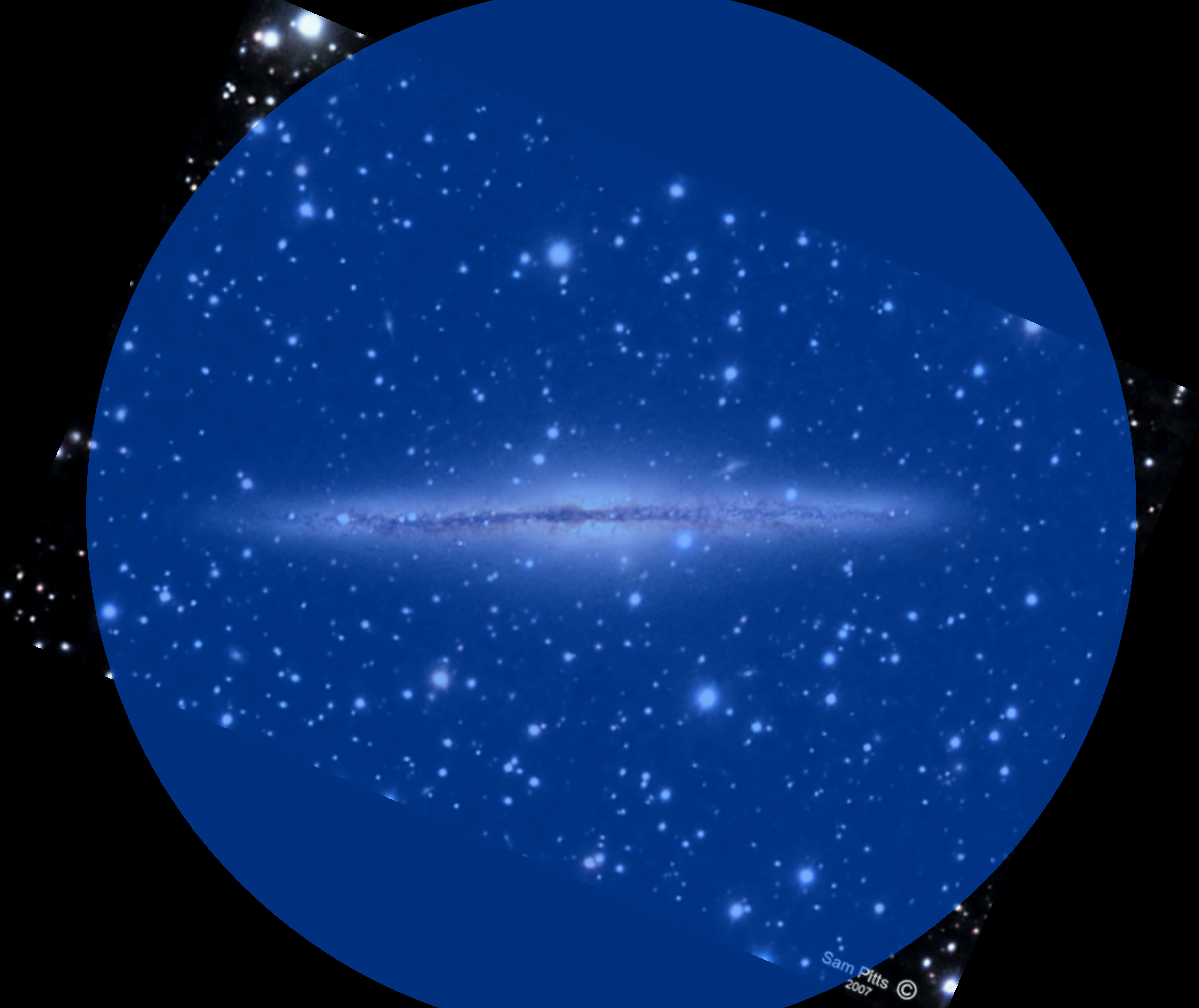
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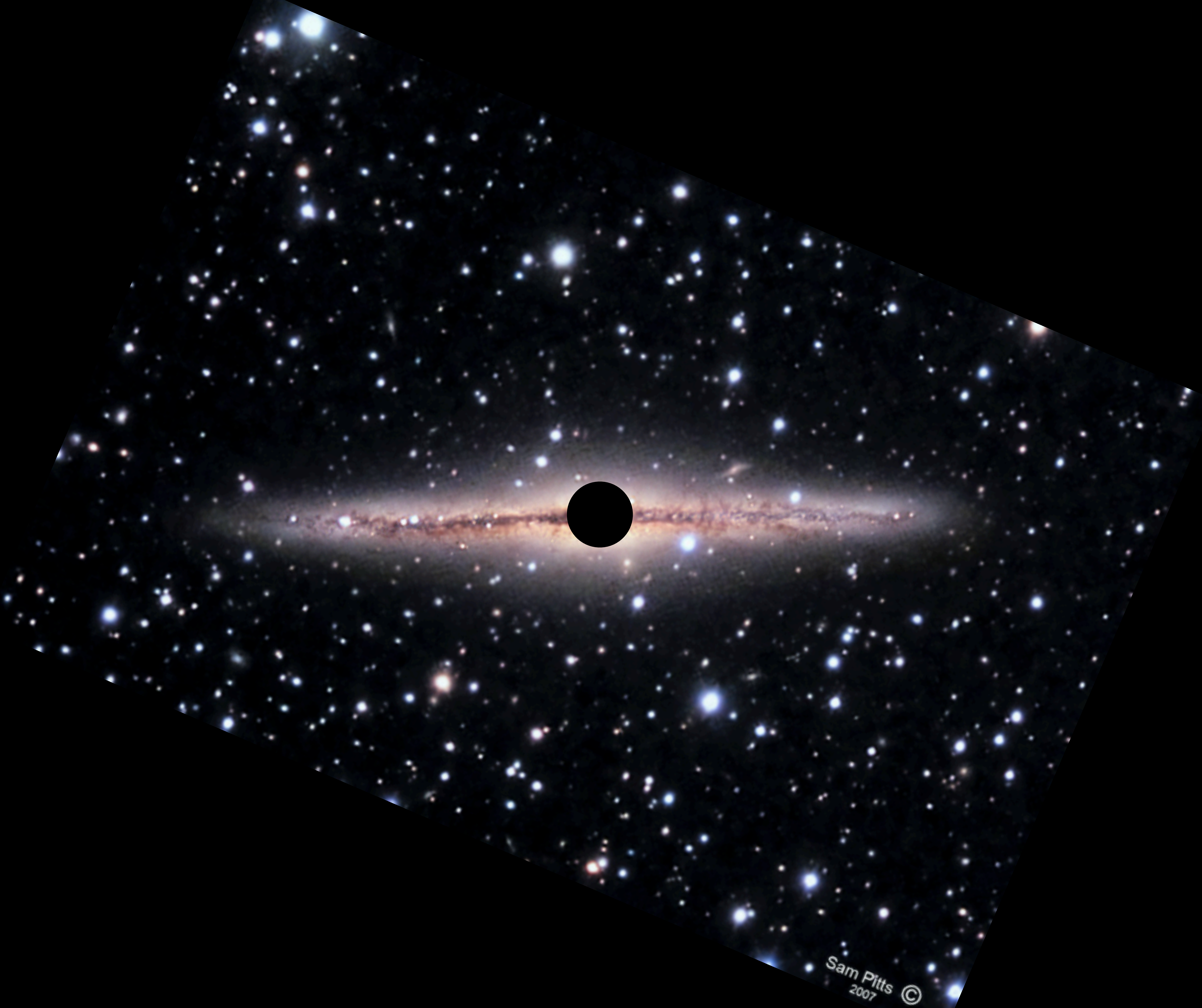
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What are galaxies made up of?

- Milky-Way like:
  - Stars! 100 billion, typical mass about half that of the Sun
  - Gas:
    - Thin layer that forms stars
    - Hot gas in a spherical halo (not very well constrained!)
  - Dark matter: 20x the mass in stars, spanning 1000x as much volume
  - Black hole: supermassive at the center
- Other disk galaxies: roughly scale by total mass
- Elliptical galaxies: Stars and dark matter, but little gas

Galactic time scales

- Hubble time t_H: ~age of the Universe (13.8 Gyr)
- Rotation of the Sun: about 200 km/s at 8 kpc (~24 ly) (1 km/s ~ 1 pc/Myr) $\rightarrow 2\pi x 8000 pc/(200 km/s)$ =~ 250 Myr ~ $40 t_H$
- Closest stars near the black hole: ~10 yr
- Orbits in the outer halo: ~100 km/s at 200 kpc $\rightarrow 2\pi x 200{,}000 pc/(100 km/s)$ ~ 10 Gyr ~ $\rightarrow t_H$
- Stellar evolution: few Myr (massive stars) to many t_H (low-mass stars)
- Gas depletion: gas-mass (~few$x10^9$ M_{sun}) / star-formation-rate (~few M_{sun} / yr) ~ 1 Gyr << t_H

Cosmological context

Millenium II fly through http://www.mpa-garching.mpg.de/galform/millennium-II/Movies/msII_lowres_slow.mp4

Credit: Volker Springel

Cosmological context

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Cosmological context

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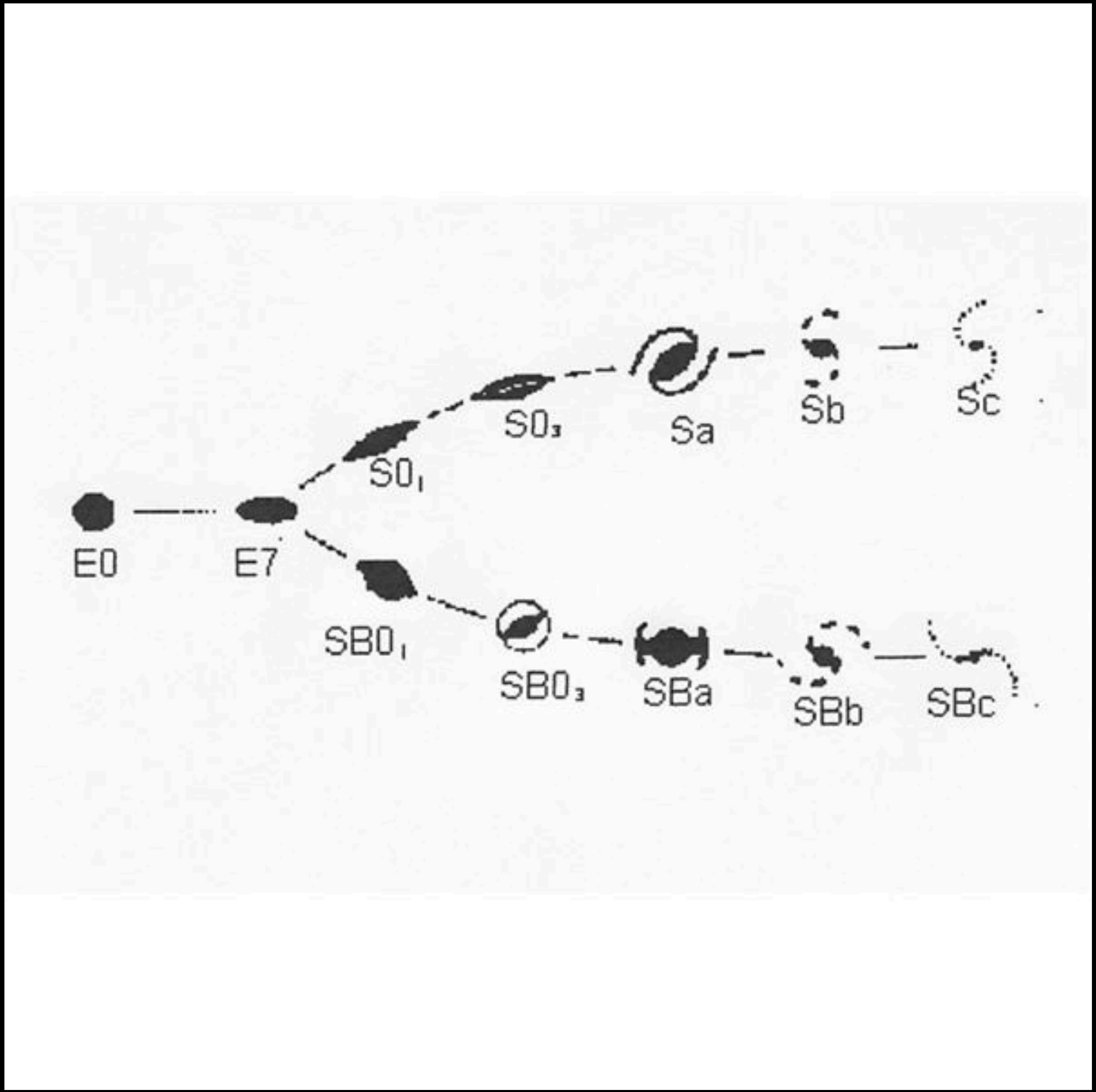
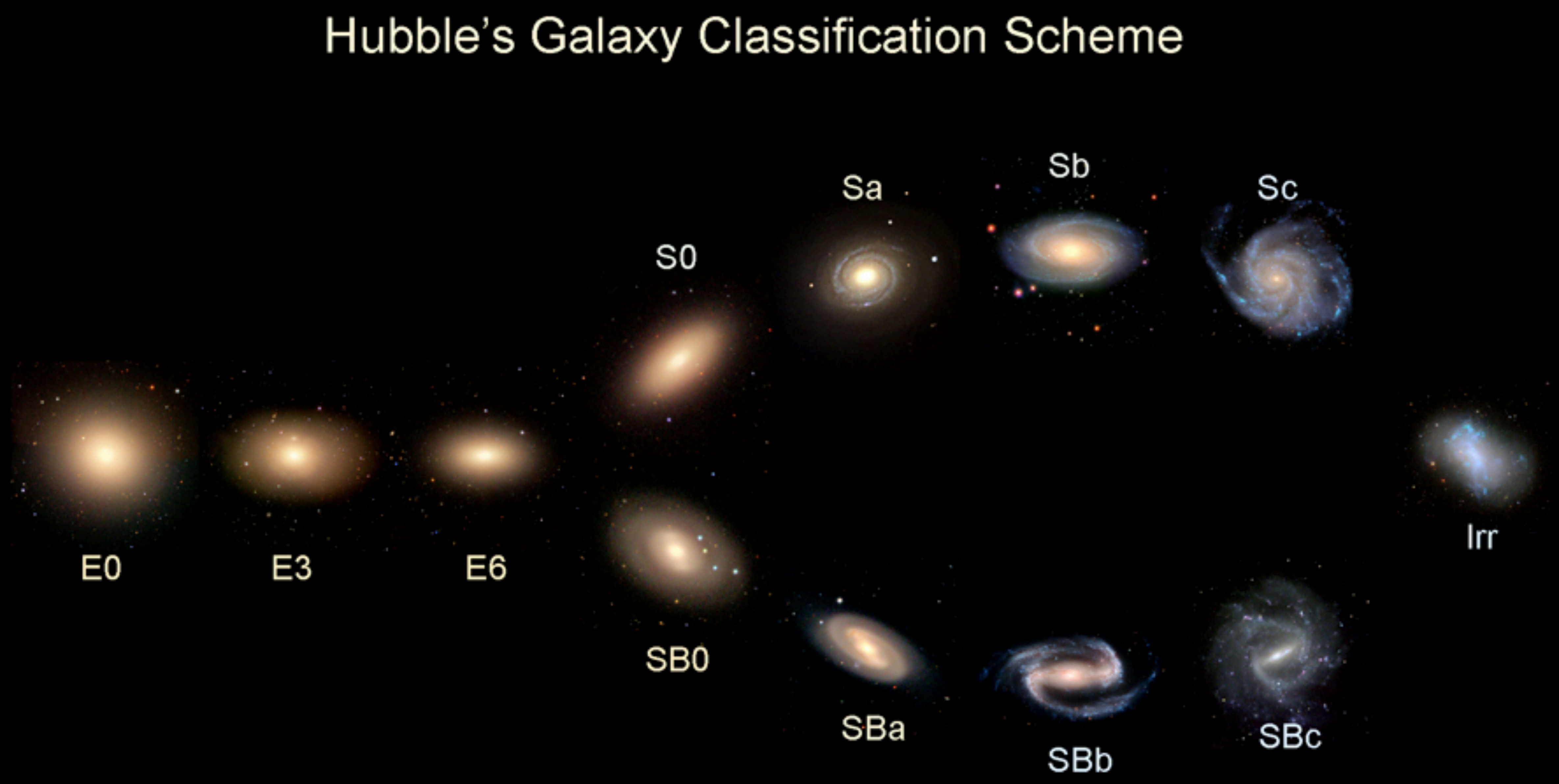
Hubble *tuning-fork* diagram



Hubble *tuning-fork* diagram

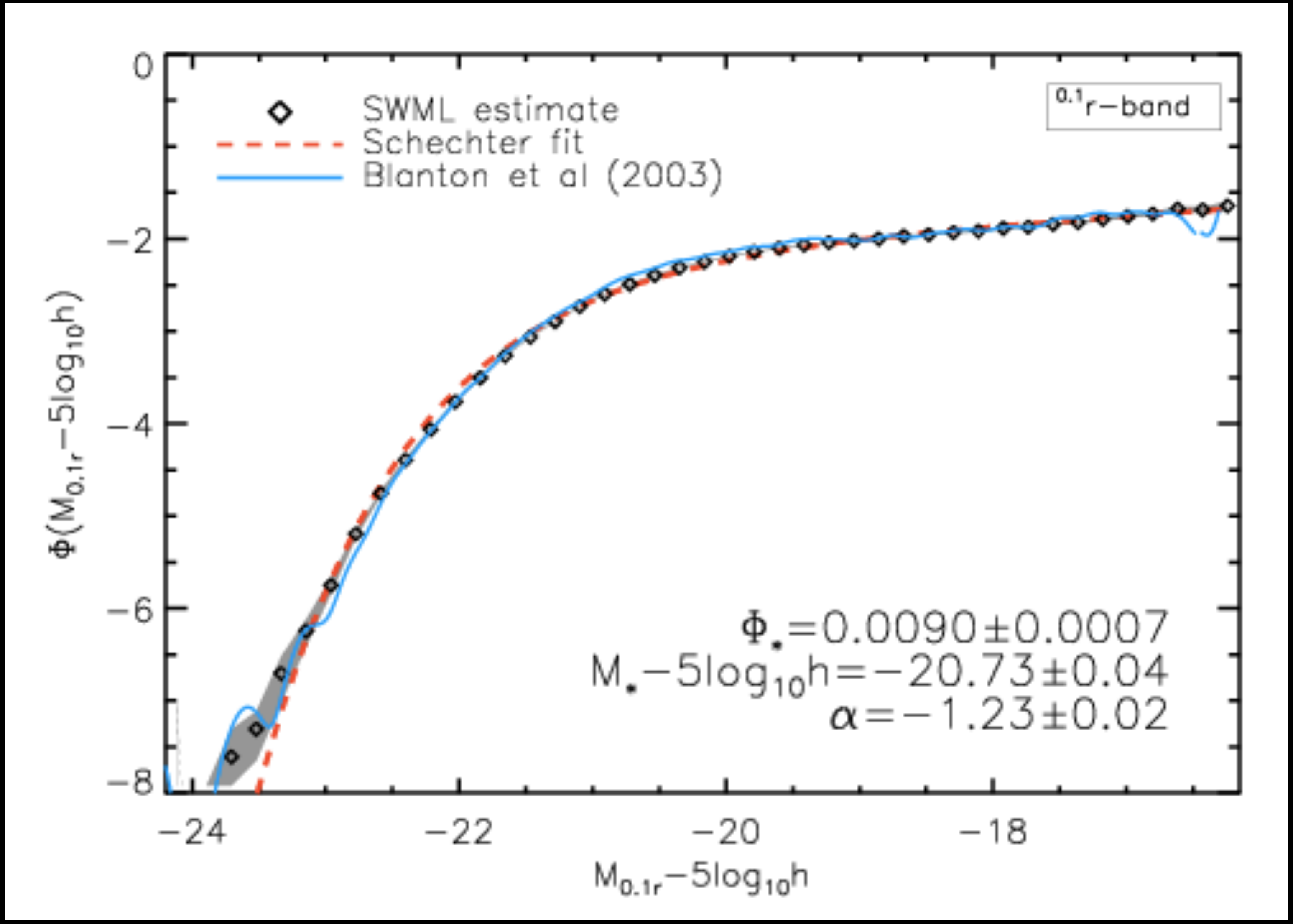


Early type

Late type

Galaxy luminosity function

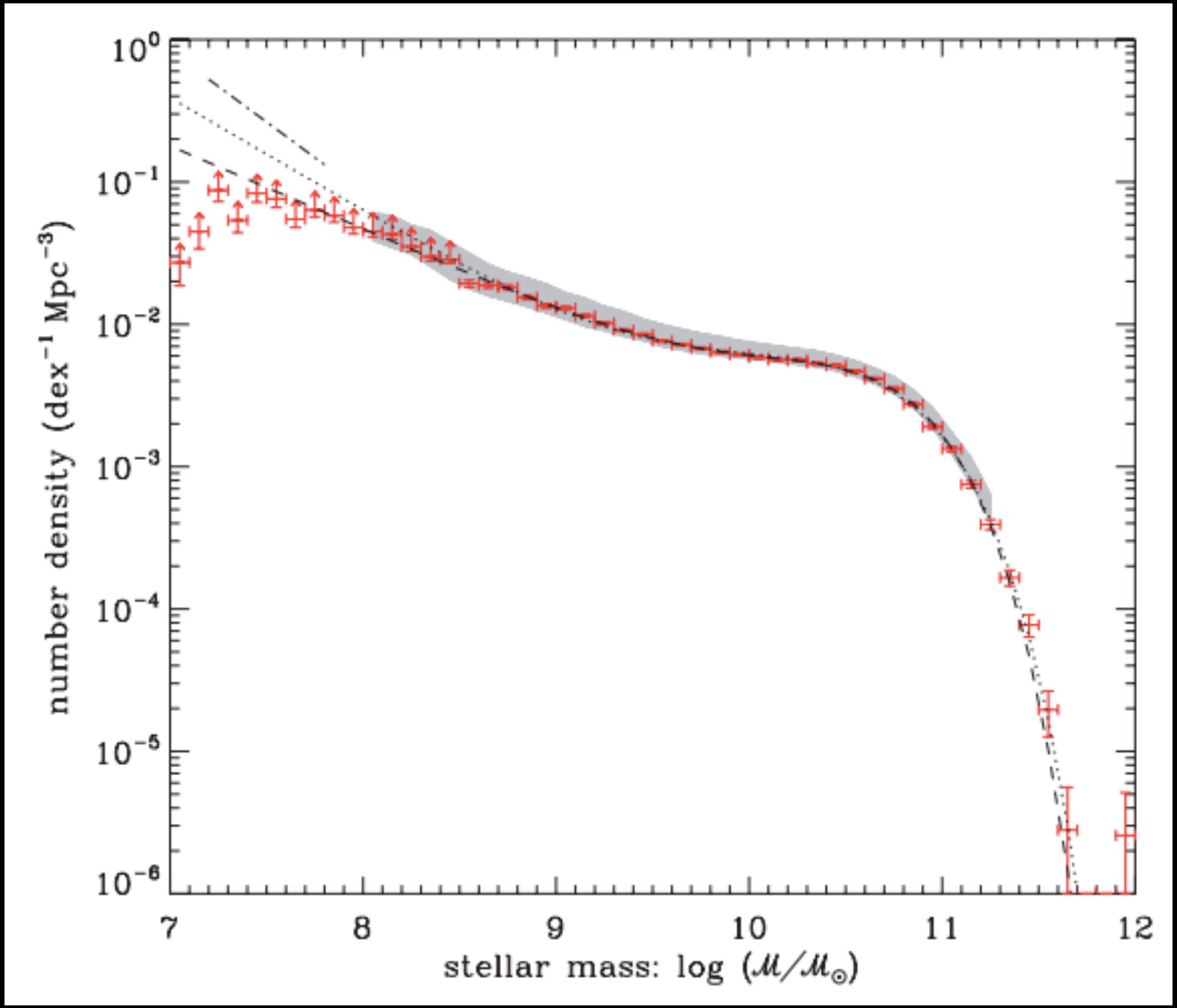
- Number of galaxies as a function of luminosity $\phi(\log L)$
- Can be fit with Schechter function:
  $\phi(\log L) \sim (L/L_*)^\alpha \exp(-[L/L_*])$
- $L_* \sim$ Milky Way



Montero-Dorta & Prada (2009)

Galaxy stellar mass function

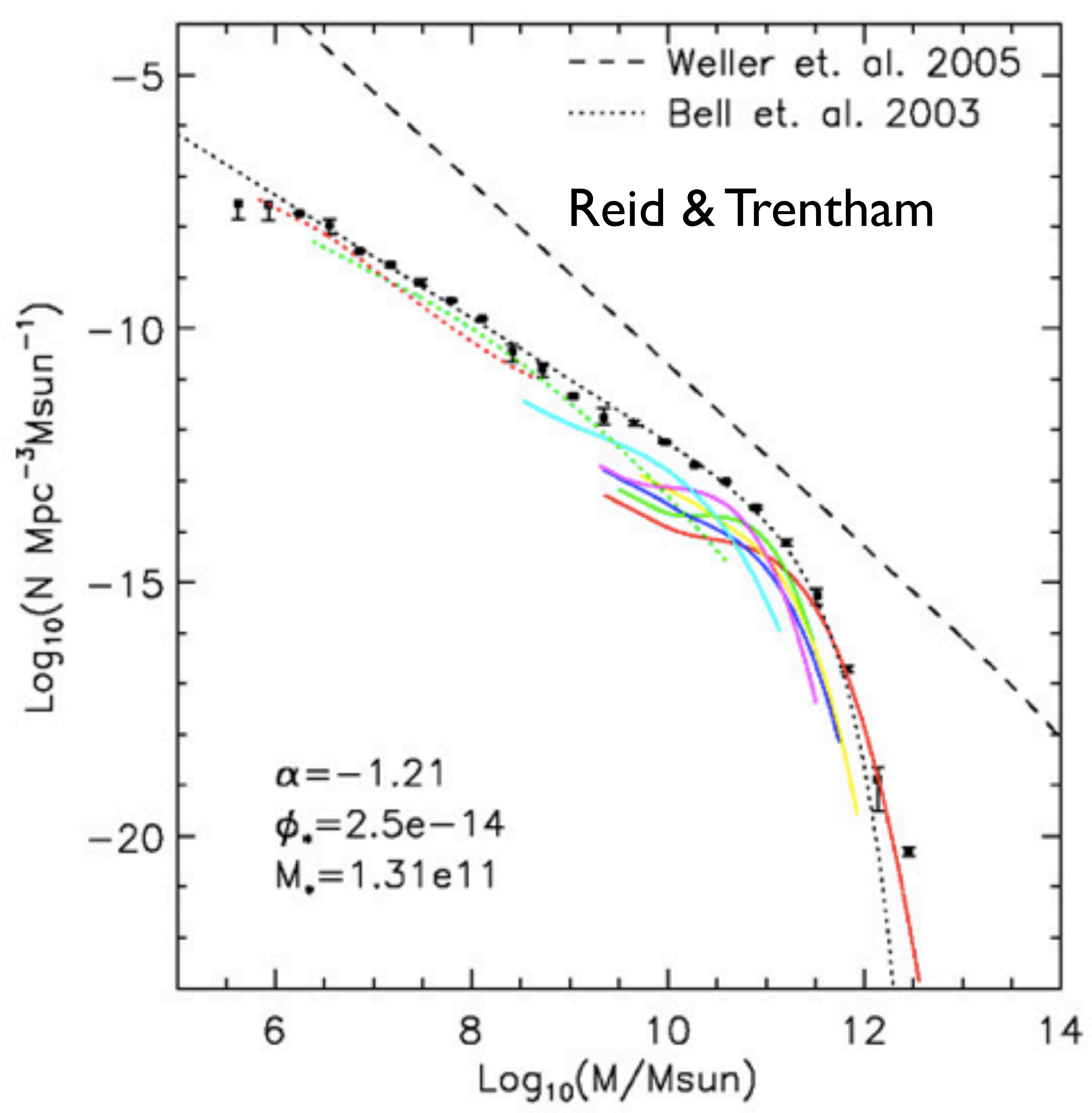
- Can estimate stellar mass from observed colors of galaxies



Baldry et al. (2008)

Galaxy stellar mass to dark-matter mass ratio

- Very difficult to measure! Currently estimated based on simulated Universes and assumptions of relation between galaxies and dark-matter halos



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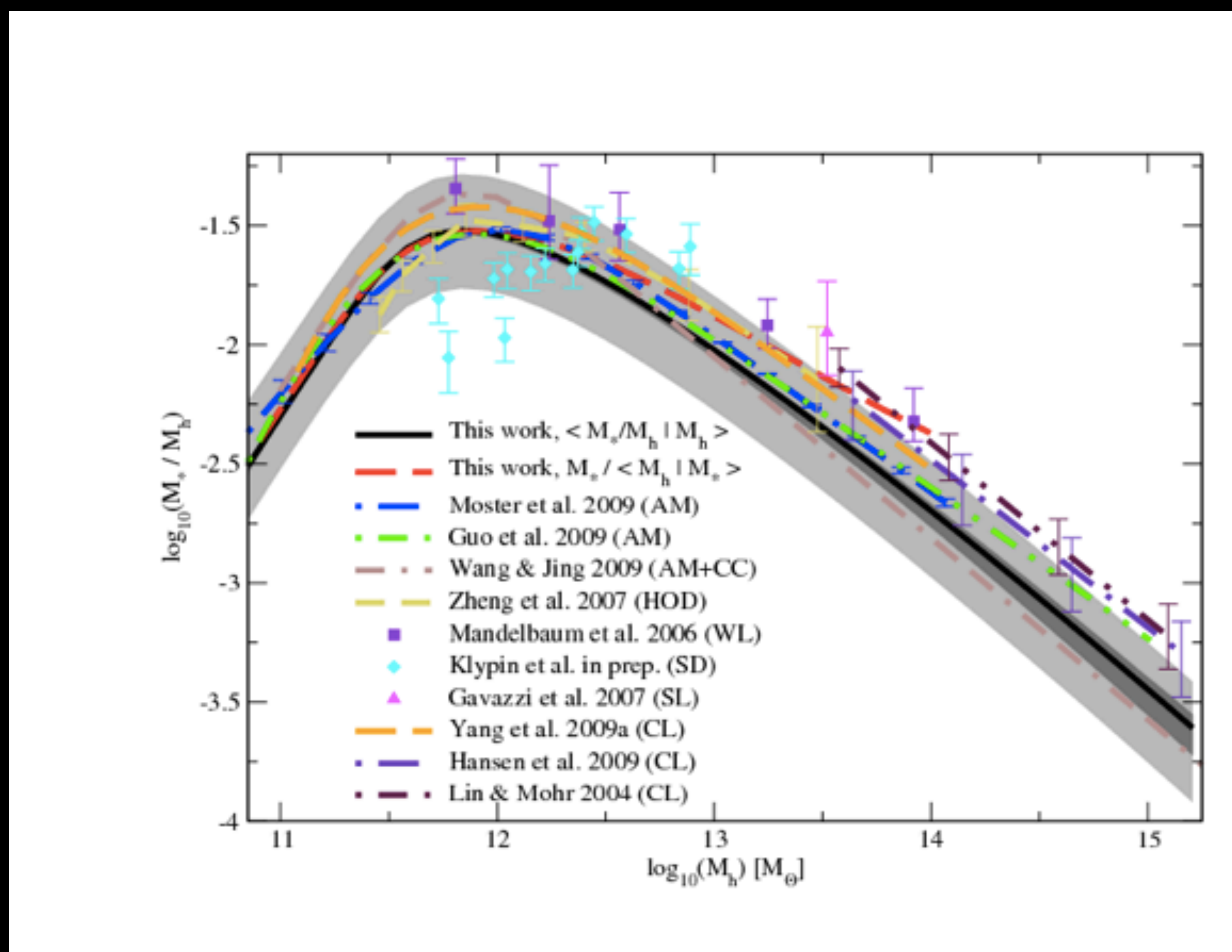
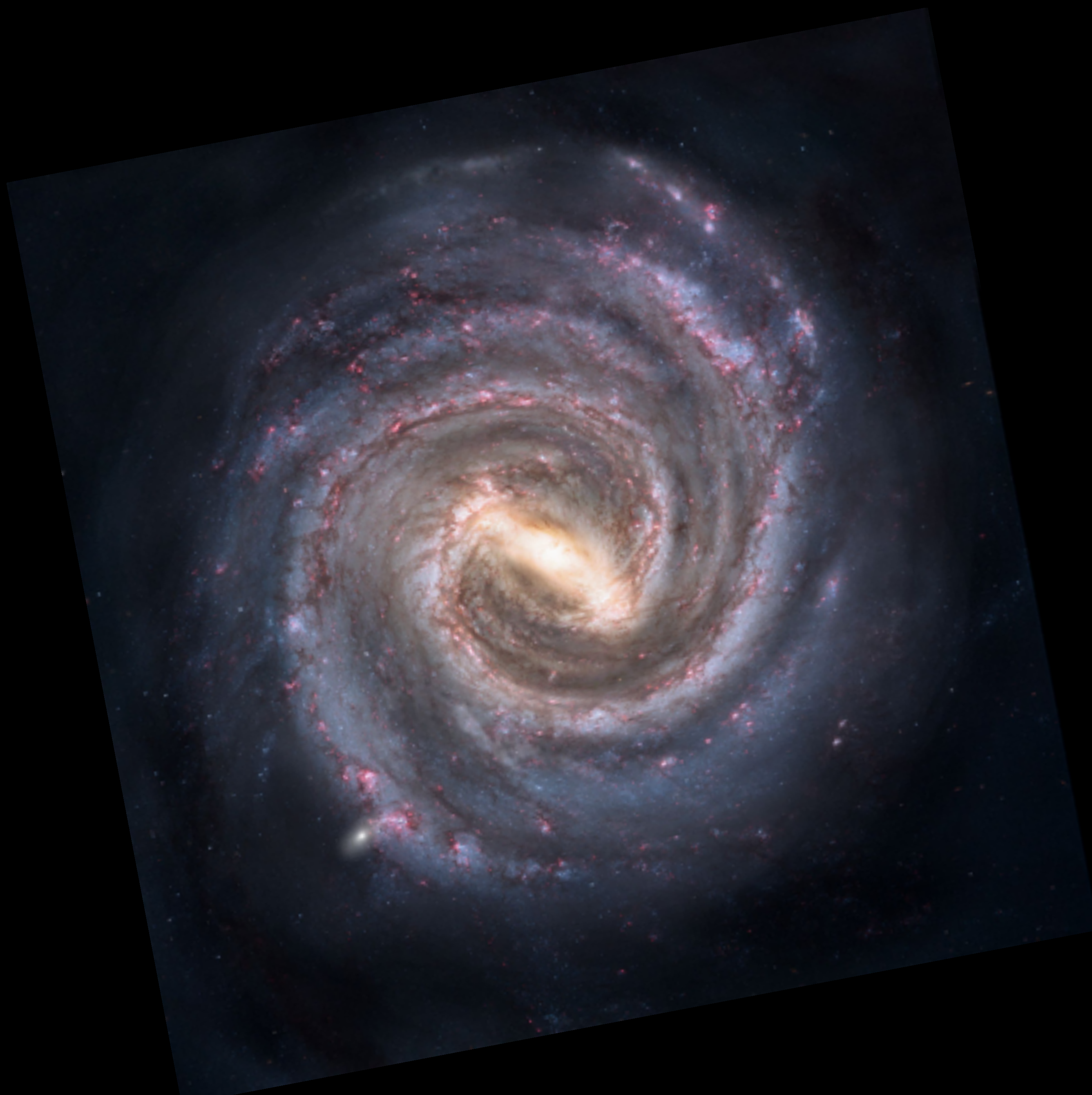
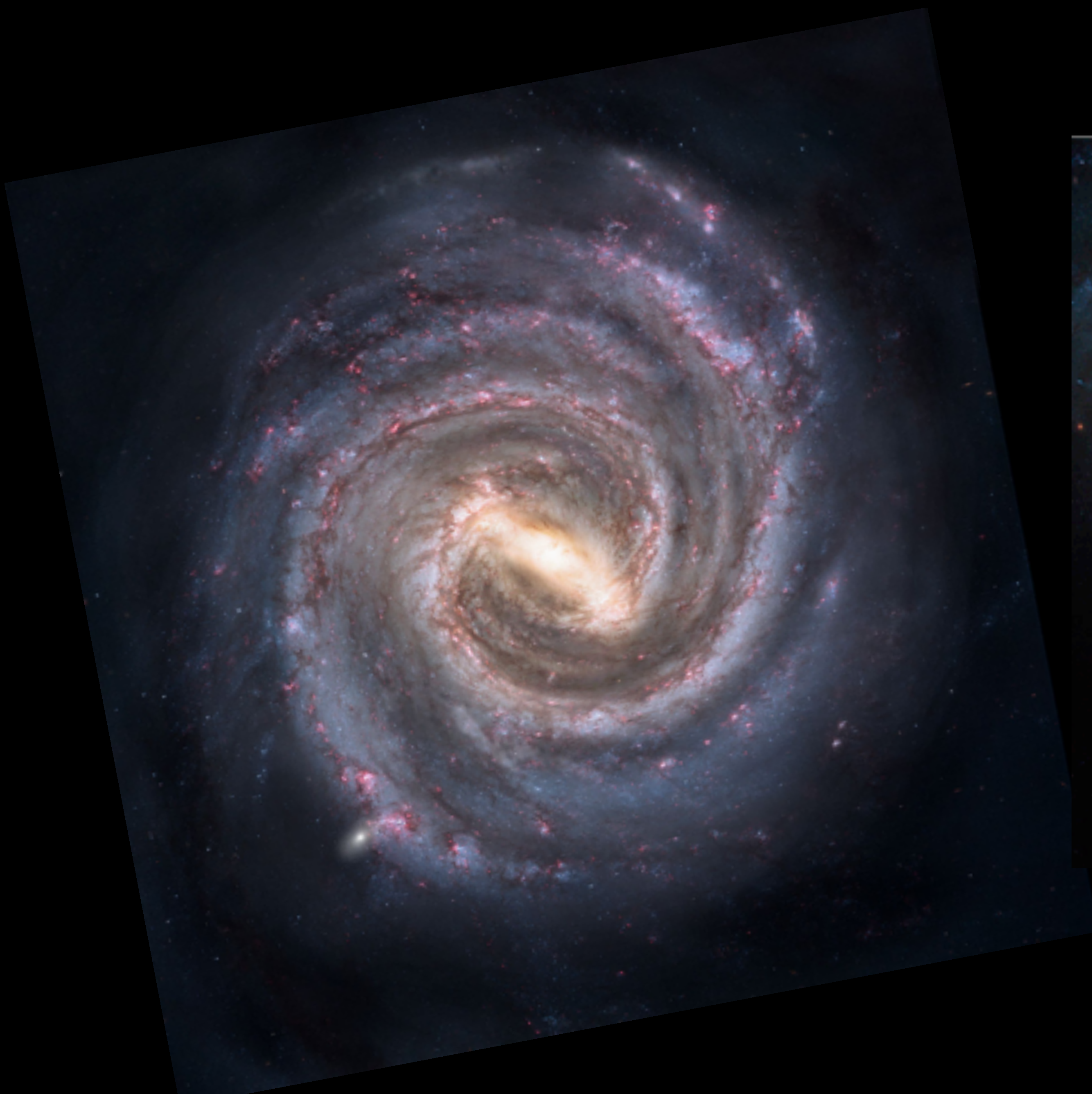


Behroozi et al.

Spiral galaxies



Spiral galaxies



Spiral galaxies

- Highly flattened systems that rotate around their centers
  - Form because gas cools efficiently, but maintains angular momentum (rotation)
- Contain: disk of stars and gas, sometimes a spheroidal bulge, dark matter
- Prominent feature: spirals! Formation through dynamical instabilities in the disk or induced by interactions with other galaxies
- The Milky Way is an excellent example of a spiral galaxy, Hubble type SBb or SBc
- Relations: Tully-Fisher (Vrot vs. luminosity), Kennicutt-Schmidt (star-formation vs. gas density)

Elliptical galaxies

Elliptical galaxies

Elliptical galaxies

Elliptical galaxies

- Ellipsoidal systems with no coherent rotation, but pressure-supported (velocity dispersion)
- Characterization in terms of Sersic profile, n~4 (de Vaucouleurs profile
- Scaling relations: Faber-Jackson (velocity dispersion vs. luminosity), Fundamental Plane (velocity dispersion, effective radius, surface brightness)
- Common in clusters and high-density environments in general
- Span wide range of masses, mostly old, some have rotation, exhibiting a variety of different dynamical equilibria
- Formation: Mergers

Lenticular galaxies

Lenticular galaxies

- Transition objects between spirals and ellipticals
- No gas or signs of recent star formation
- High-density regions
- Spirals stripped of their gas by interactions with hot gas in clusters

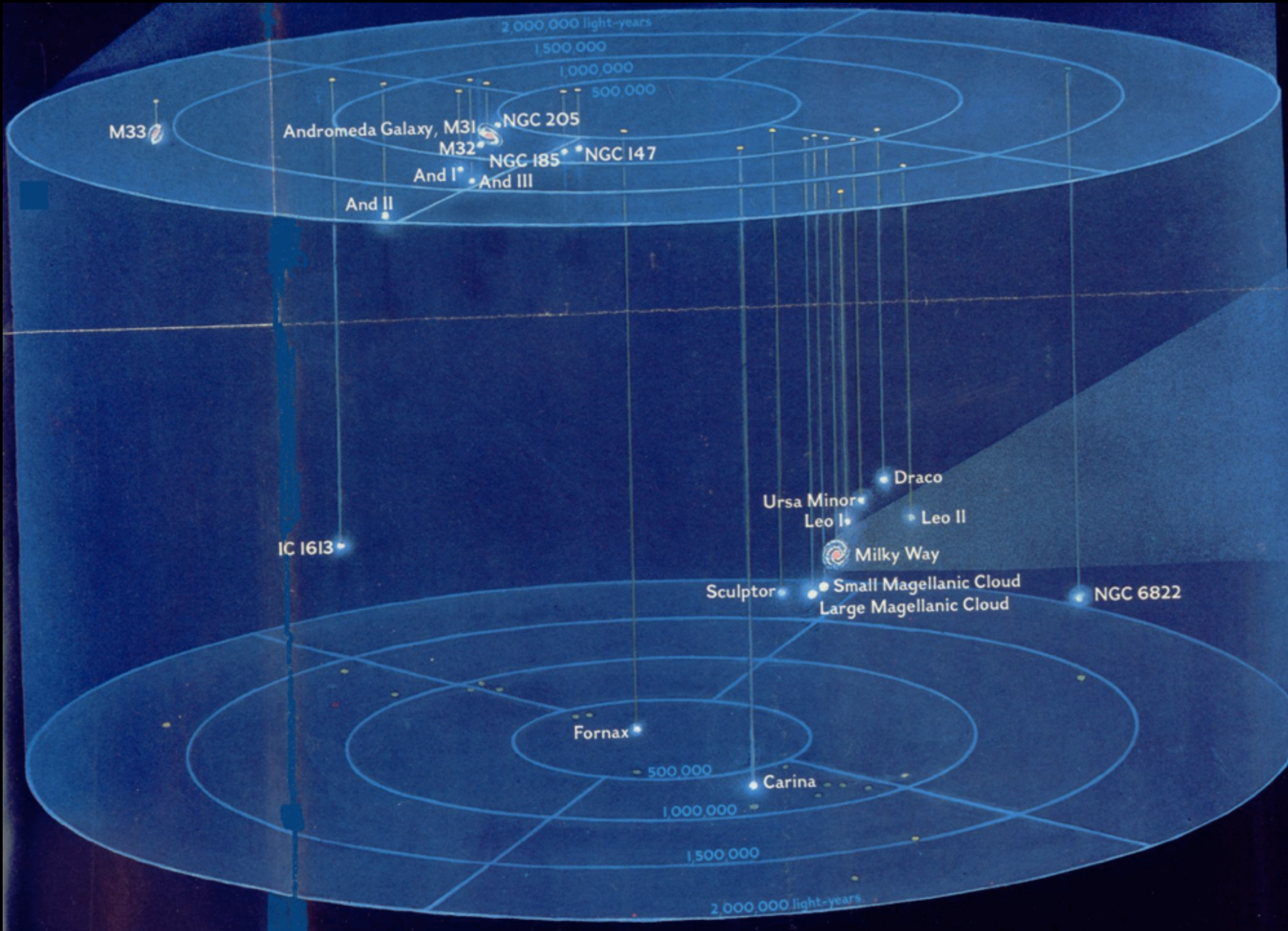
Irregular galaxies

Irregular galaxies

- Many galaxies are low-luminosity galaxies in which the young stars are arranged haphazardly
- Very gas rich
- LMC/SMC
- Also irregular galaxies that are the result of mergers or intense star formation

Groups and clusters of galaxies

- On large scales, galaxies are arranged in groups (few galaxies) and clusters (many galaxies)
- Example: Galaxies within ~1 Mpc are in the *Local Group*: MW, Andromeda (M31), LMC, SMC, ...
- While galaxies are typically ~40 rotation periods old, clusters are only a few orbits old → not as relaxed as galaxies
- Collisions between galaxies in clusters are common, while collisions between stars in galaxies are not



2,000,000 light-years
1,500,000
1,000,000
500,000
M33
Andromeda Galaxy, M31
NGC 205
M32
NGC 185
NGC 147
And I
And III
And II
IC 1613
Draco
Ursa Minor
Leo I
Leo II
Milky Way
Sculptor
Small Magellanic Cloud
Large Magellanic Cloud
NGC 6822
Fornax
500,000
Carina
1,000,000
1,500,000
2,000,000 light-years

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Star clusters

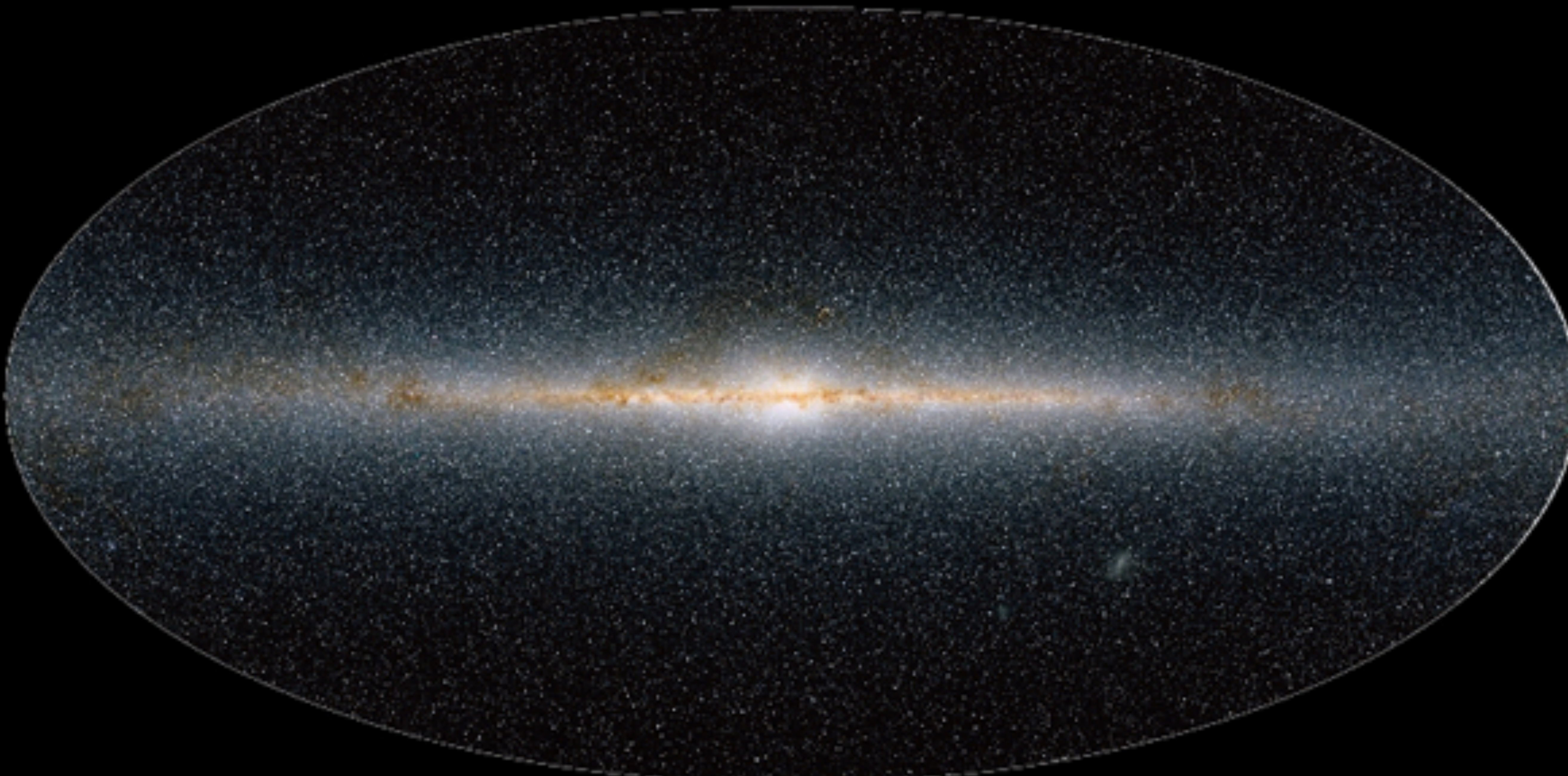
Star clusters

- Prominent component of galaxies are different types of star clusters
  - Open clusters: loosely-bound or unbound associations of stars in the galactic disk: 100s to 10,000s of stars; believed to be the formation sites of *all* stars
  - Globular clusters: tightly-bound groups of 10,000s to millions of stars. Old, metal-poor, without gas, DM, or young stars; 1000s of dynamical times old
- Both of these are typically 10 pc in size

Black holes and AGN

- Many galaxies contain a very luminous point-source at their center (AGN) that often outshines the rest of the galaxy (quasars)
- Galaxies without an AGN are found to contain a large-mass concentration at their centers. For example, the MW has a black hole with a mass of 4 million M_{sun}
- AGN are thought to be powered by an accretion disk of hot gas surrounding the black hole that gives off powerful radiation
- The mass of a galaxy is strongly correlated with the mass of the black hole at its center (M-σ relation), demonstrating that the black holes form an integral part of a galaxy's evolution

Milky Way and solar neighborhood



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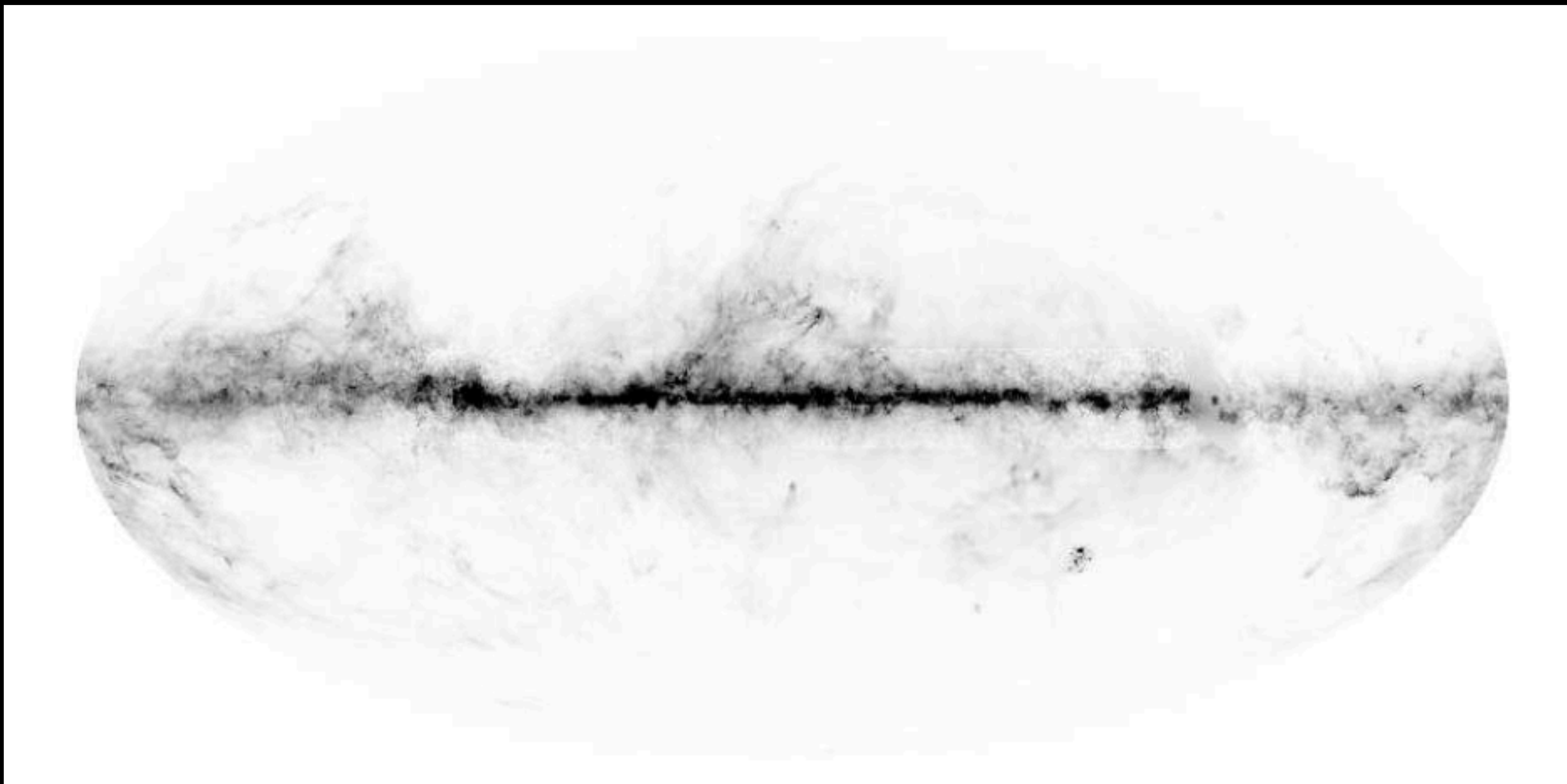
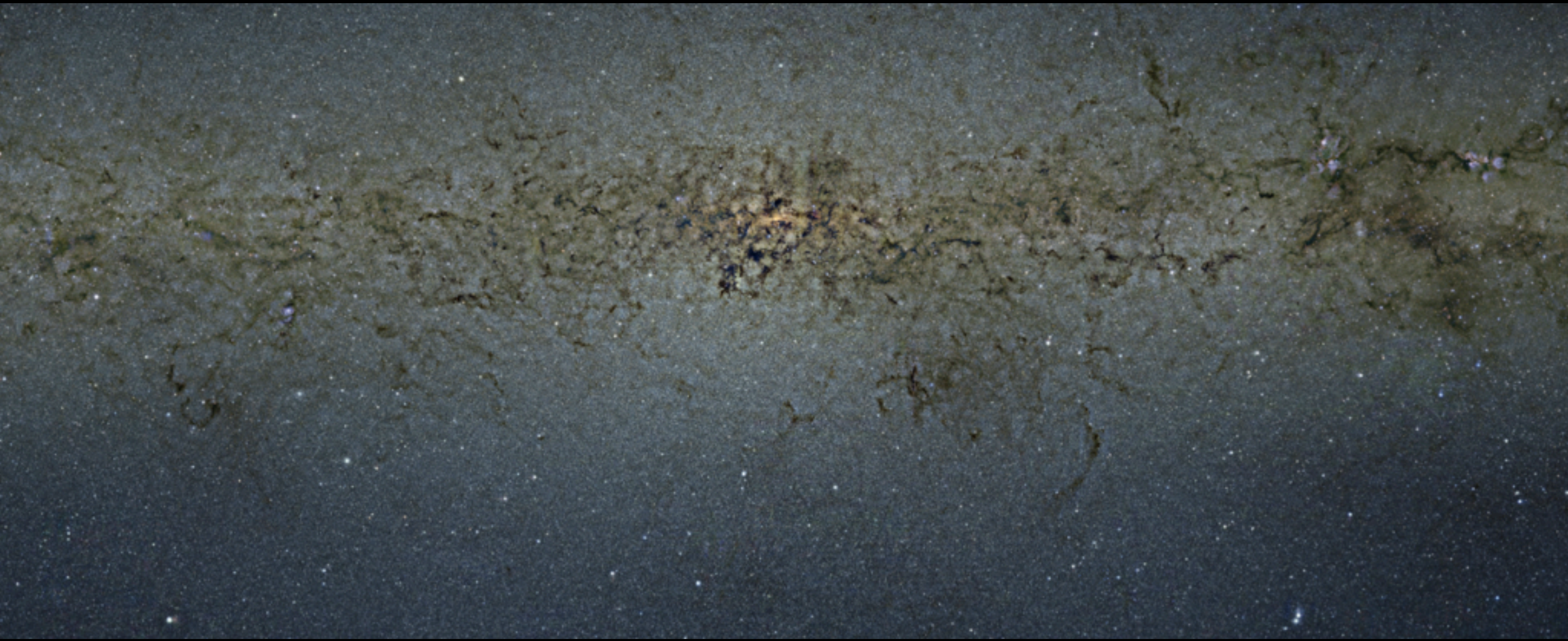
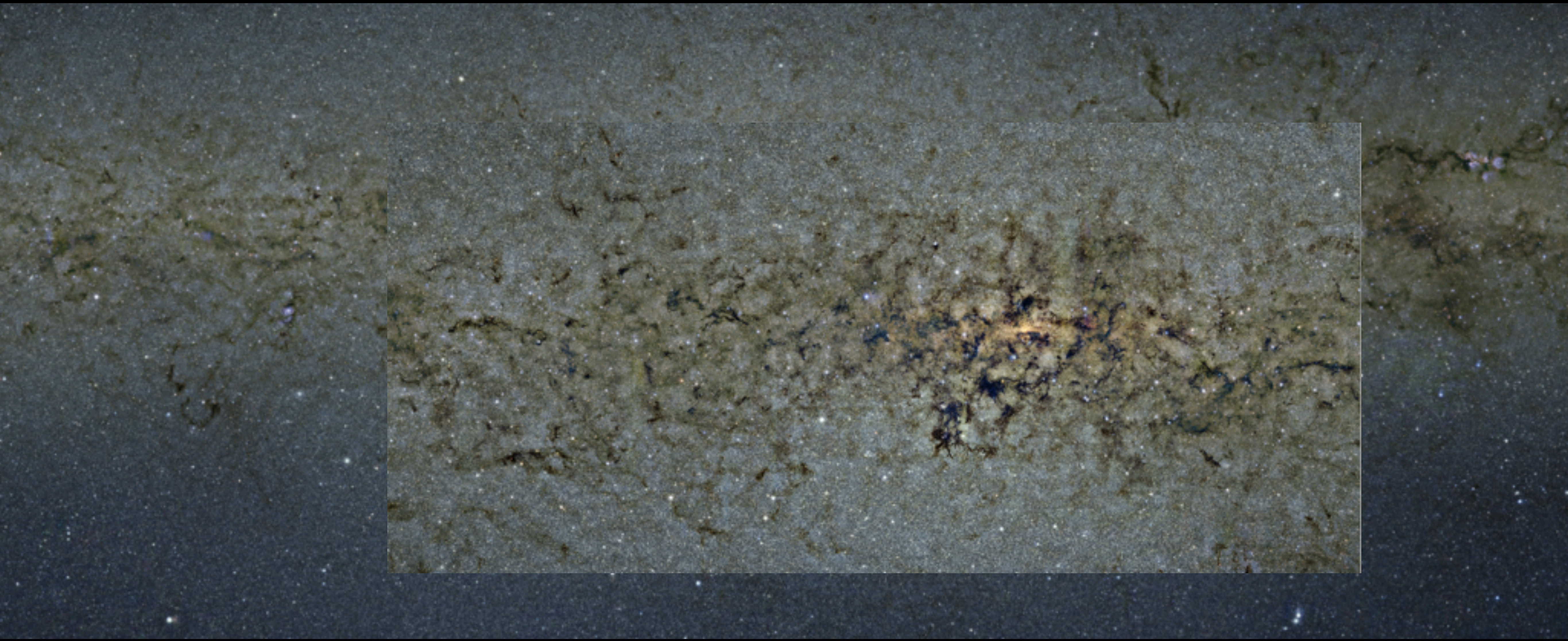
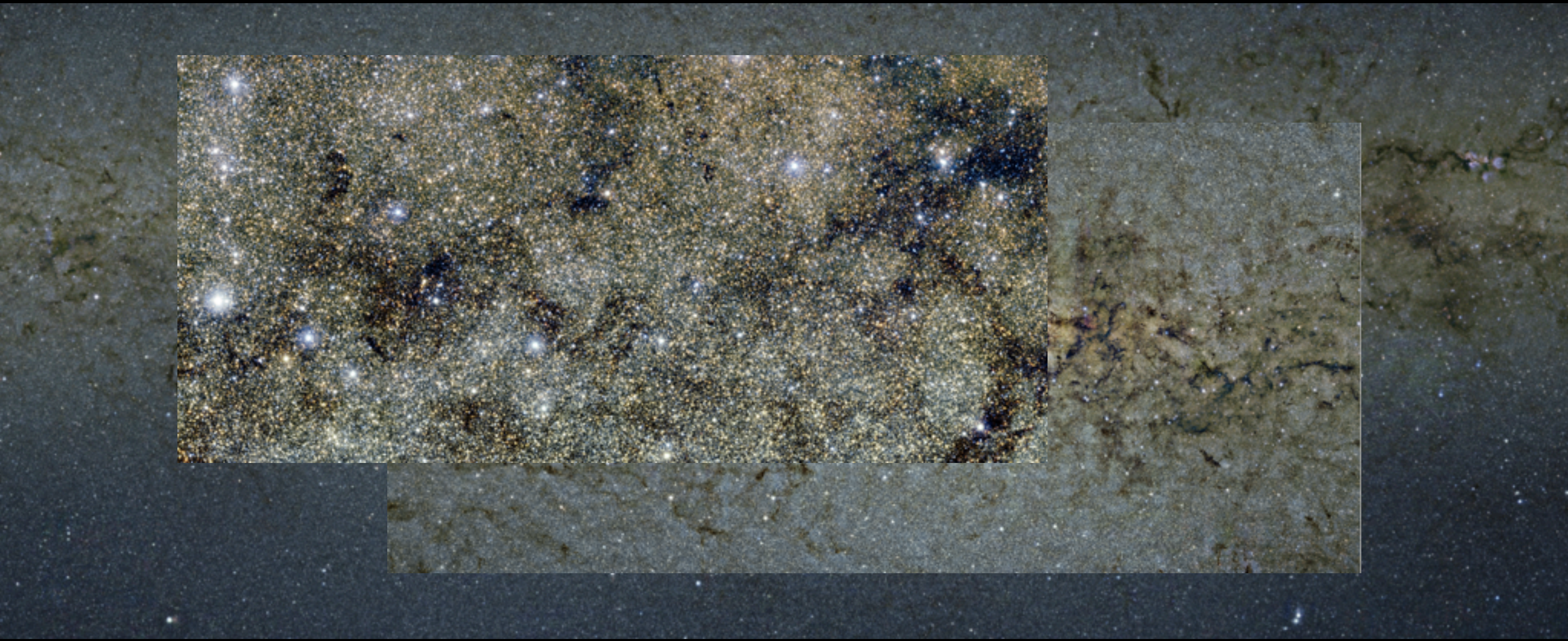
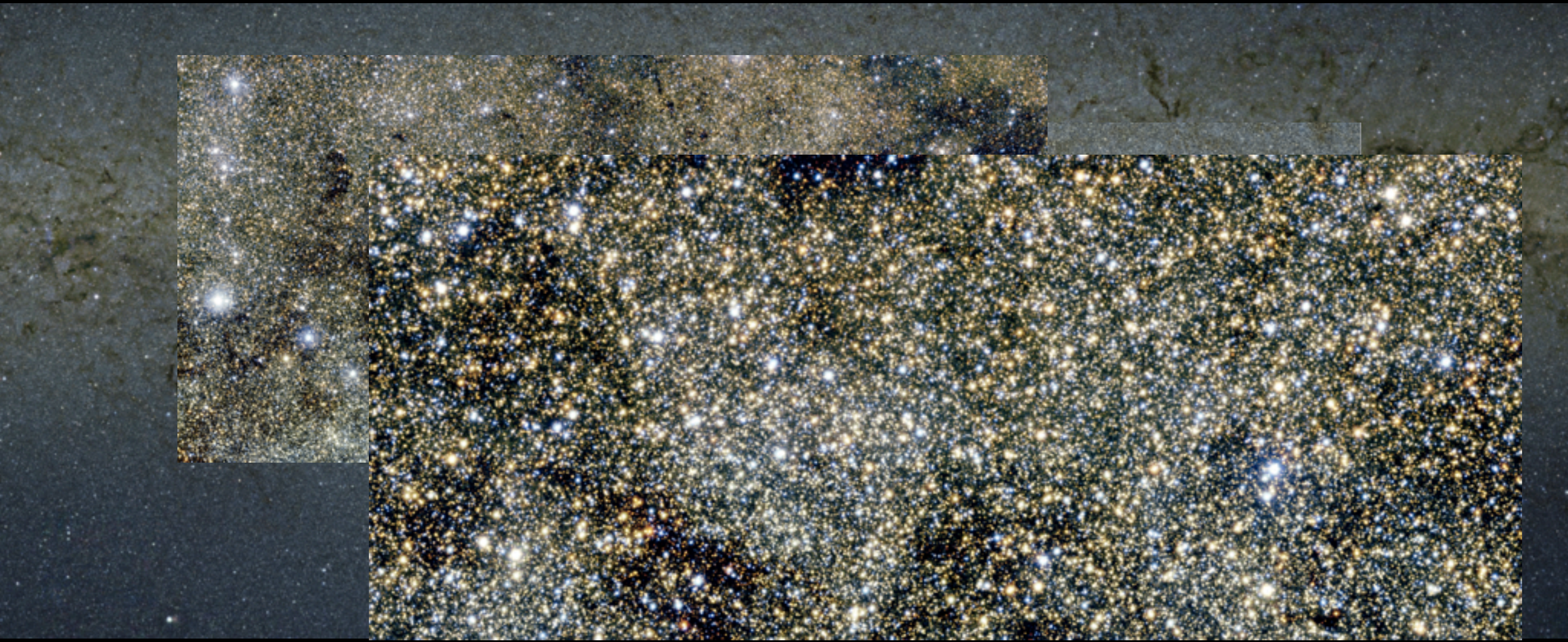




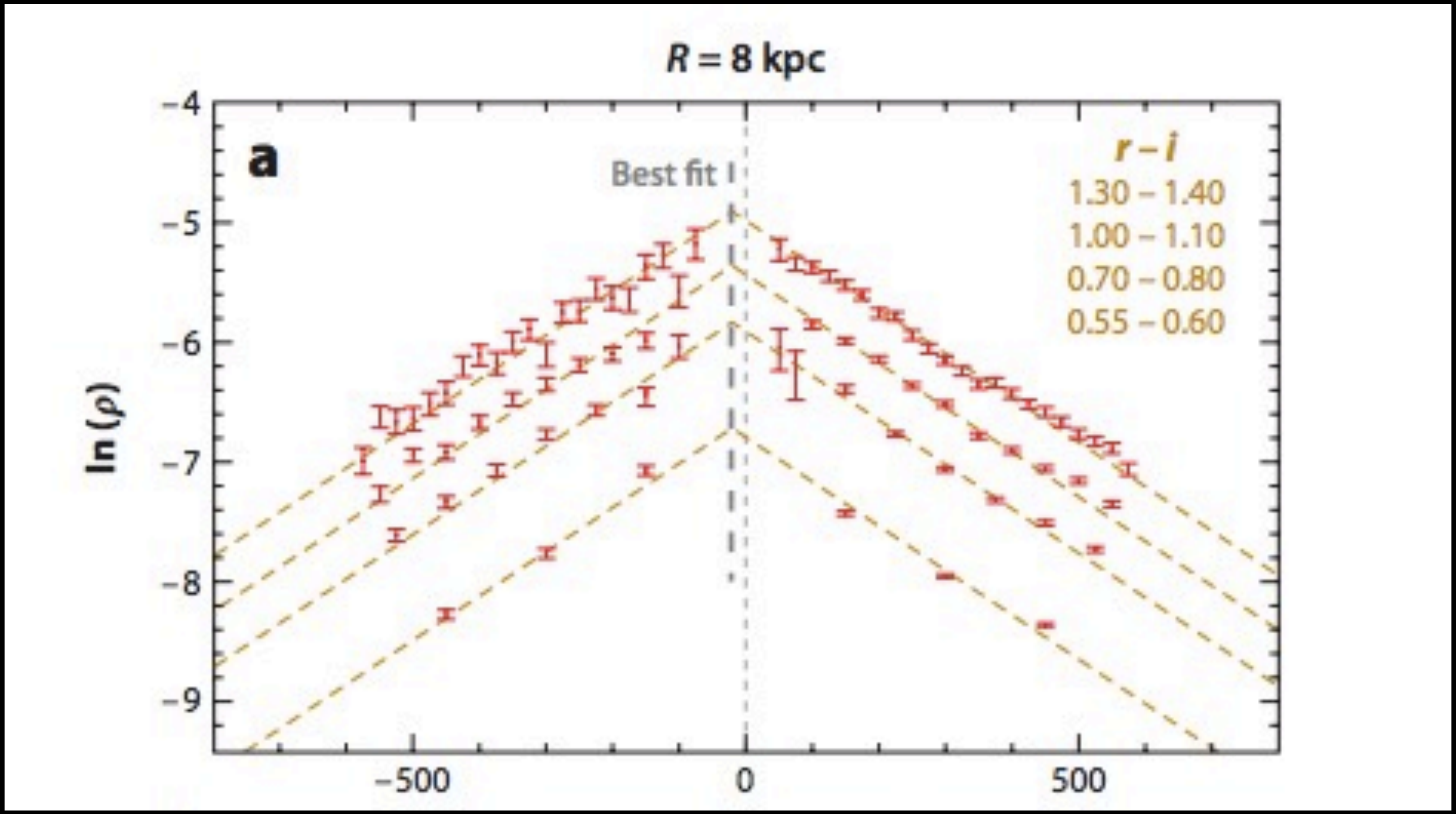






Milky Way and solar neighborhood

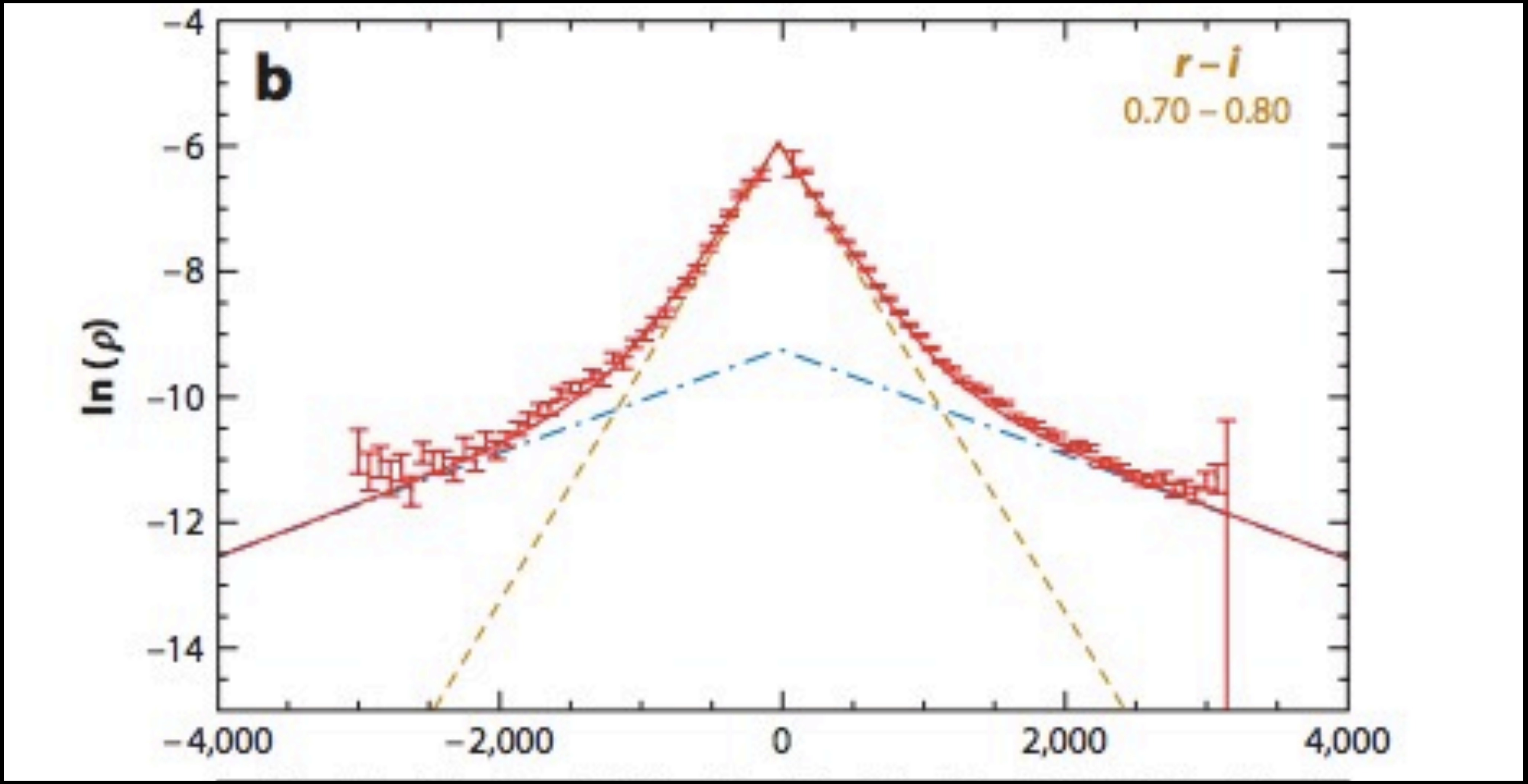
Ivezic et al. (2012), ARAA



Vertical distance from mid-plane

Milky Way and solar neighborhood

Ivezic et al. (2012), ARAA



Vertical distance from mid-plane

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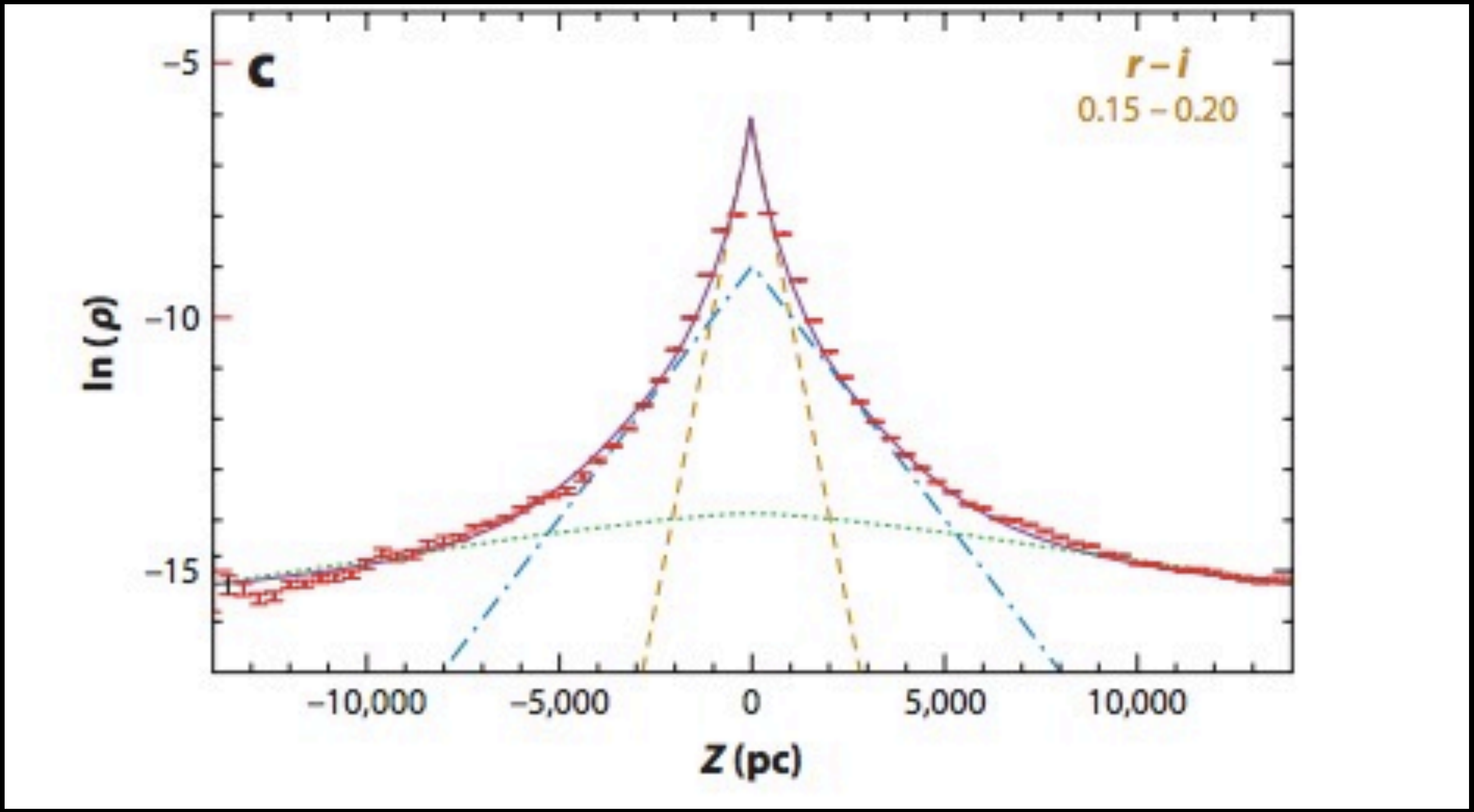


Vertical distance from mid-plane

Milky Way and solar neighborhood

- Disk: radial exponential, scale length ~2.5 kpc, vertical exponential, scale height ~400 pc, mass ~5×10^{10} M_{sun}
- Bulge: extent ~4 kpc, mass ~10^{10} M_{sun}, cylindrical rotation, mostly a bar
- Gas: mass ~1/10 of stars, much more extended, thickness ~100 pc
- Dust: smoke-like, <~ 1 μm, very little mass but extinguishes background light by orders of magnitude when looking in the plane
- Stellar halo: extended spheroidal distribution, density ~ r^{-3}, mass ~ 10^9 M_{sun}, old stars
- Hot gas corona: halo of gas too hot to cool, mass and properties largely unknown
- Dark halo: extends to ~250 kpc, mass ~10^{12} M_{sun}
- Circular velocity: 220 km/s and constant (see later)

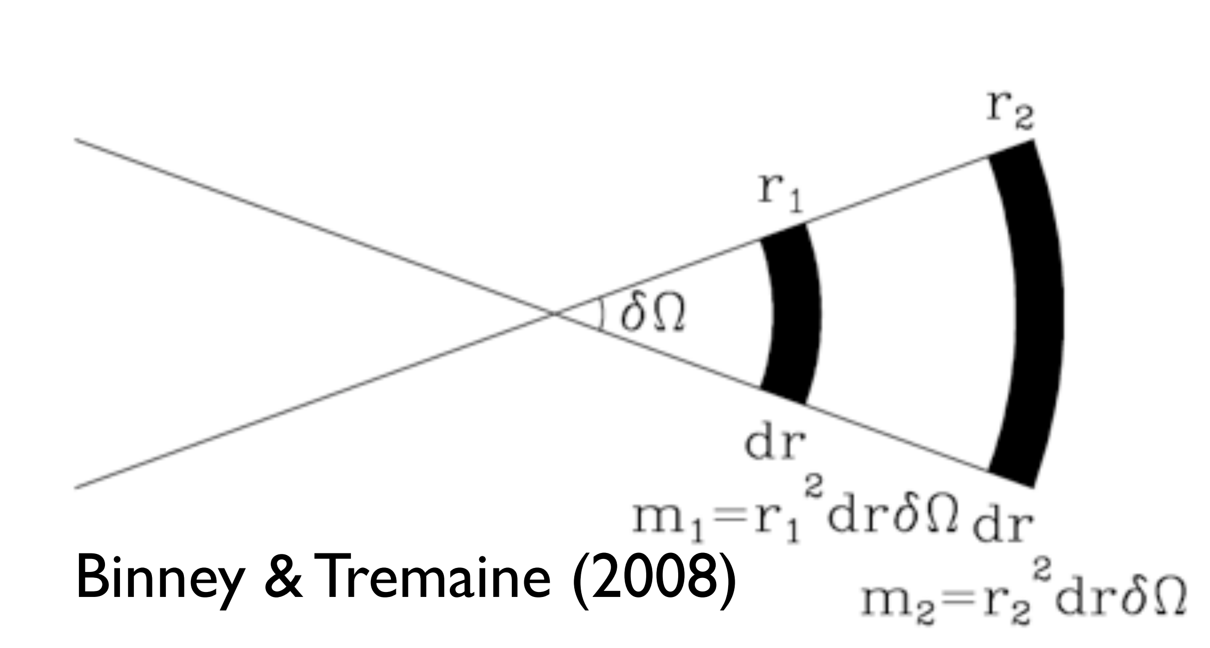
Milky Way and solar neighborhood

- Distance between the Sun and the Galactic center: ~8 kpc
- SN inventory: density (M_{sun}/pc^3) | surface density (M_{sun}/pc^2)

	density (M_{sun}/pc^3)	surface density (M_{sun}/pc^2)
visible stars:	0.033	30
BH, WD, etc.:	0.006	5
BD:	0.002	2
Gas:	0.050	13
Total:	**0.091**	**49**

- SN dynamics:
  - Rotational period: 220 Myr
  - Radial oscillation period: 170 Myr
  - Vertical period: 90 Myr
  - Sun's motion wrt local stars: ~20 km/s
  - Local velocity dispersion of old stars: ~35 km/s
  - Escape speed: ~500 km/s

Galactic dynamics



- Galaxies consist of many stars (~100 billion, with ~0.5 M_{sun} each) that interact gravitationally
- But a given star is mainly influenced by interactions by distant stars (large-scale structure of the galaxy)
- Similarly, close encounters between stars are exceedingly rare:
  - Typical densities N: ~ 1 star / pc^3
  - Typical velocities v: 10 km/s =~ 10 pc/Myr =~ 10^{17} m / Myr
  - Stellar cross section σ: ~10^{18} m^2 to AU^2
  - Interaction rate = N x v x σ ~ 1/ (10^8 to 10^{11} Gyr)

Galactic potentials

- Force field from smooth density ρ most easily described in terms of a potential ϕ

$$\nabla^2\Phi = 4\pi G\rho.$$

- Forces are derivatives of the potential
- Gravitational acceleration can then be computed from Newton's law
- Newton's theorems for spherical potentials:
  - Inside spherical shell do not experience force from shell
  - Outside spherical shell, force same as if shell were concentrated in point
  - Thus, mass distribution as a function of r is all that matters

Galactic potentials

- Rotation galaxies have a *circular velocity*: velocity $V_c(R)$ of star on a circular orbit at R

$$V_c^2 = RF_R$$

- For a spherical mass distribution, only the mass within R matters

$$V_c^2 = \frac{G(M < R)}{R}$$

- Escape velocity: required velocity to escape potential

Galactic potentials: spherical

- Kepler, point-mass potential: $$\Phi(R) = -\frac{GM}{R}$$
- Isochrone potential: $$\Phi(R) = -\frac{GM}{b+\sqrt{b^2+R^2}}$$
- Logarithmic potential: $$\Phi(R) = V_c^2 \, \ln R$$
- Power-law: $$\rho(R) = \rho_0 \left(\frac{R}{R_0}\right)^{\alpha}$$
- Double power-law: $$\rho(R) = \frac{\rho_0}{(r/r_s)^{\alpha}\,(1+r/r_s)^{\beta-\alpha}}$$
- Latter: α=1 and β=3: Navarro-Frenk-White (NFW)

Galactic potentials: disk

- Many components of galaxies are strongly flattened, so cannot just use spherical potentials
- Potential-density pairs for flattened densities are typically very complicated
- Simple models:
  - Kuzmin potential:

$$\Phi_{\mathrm{K}}(R,z) = -\frac{GM}{\sqrt{R^2+(a+|z|)^2}} \quad (a \geq 0).$$

  - Miyamoto-Nagai:

$$\Phi_{\mathrm{M}}(R,z) = -\frac{GM}{\sqrt{R^2+\left(a+\sqrt{z^2+b^2}\,\right)^2}}.$$

  - Flattened logarithmic potential:

$$\Phi_{\mathrm{L}} = \tfrac{1}{2}v_0^2 \ln\left(R_c^2 + R^2 + \frac{z^2}{q_\Phi^2}\right)$$

  - Double exponential disk

FLATTENING OF POTENTIALS

- The density of the flattened logarithmic potential:

$$\rho(R, z) = \frac{v_0}{4\pi G} \frac{(2q^2+1)R_c^2 + R^2 + (2 - q^{-2})z^2}{(R_c^2 + R^2 + z^2 q^{-2})^2}$$

- We can define a flattening of the potential as the ratio of the forces:

$$q_\Phi^2 = \frac{z}{R} \frac{F_R}{F_z}$$

- For the flattened logarithmic potential we have that:

$$q_\Phi = q$$

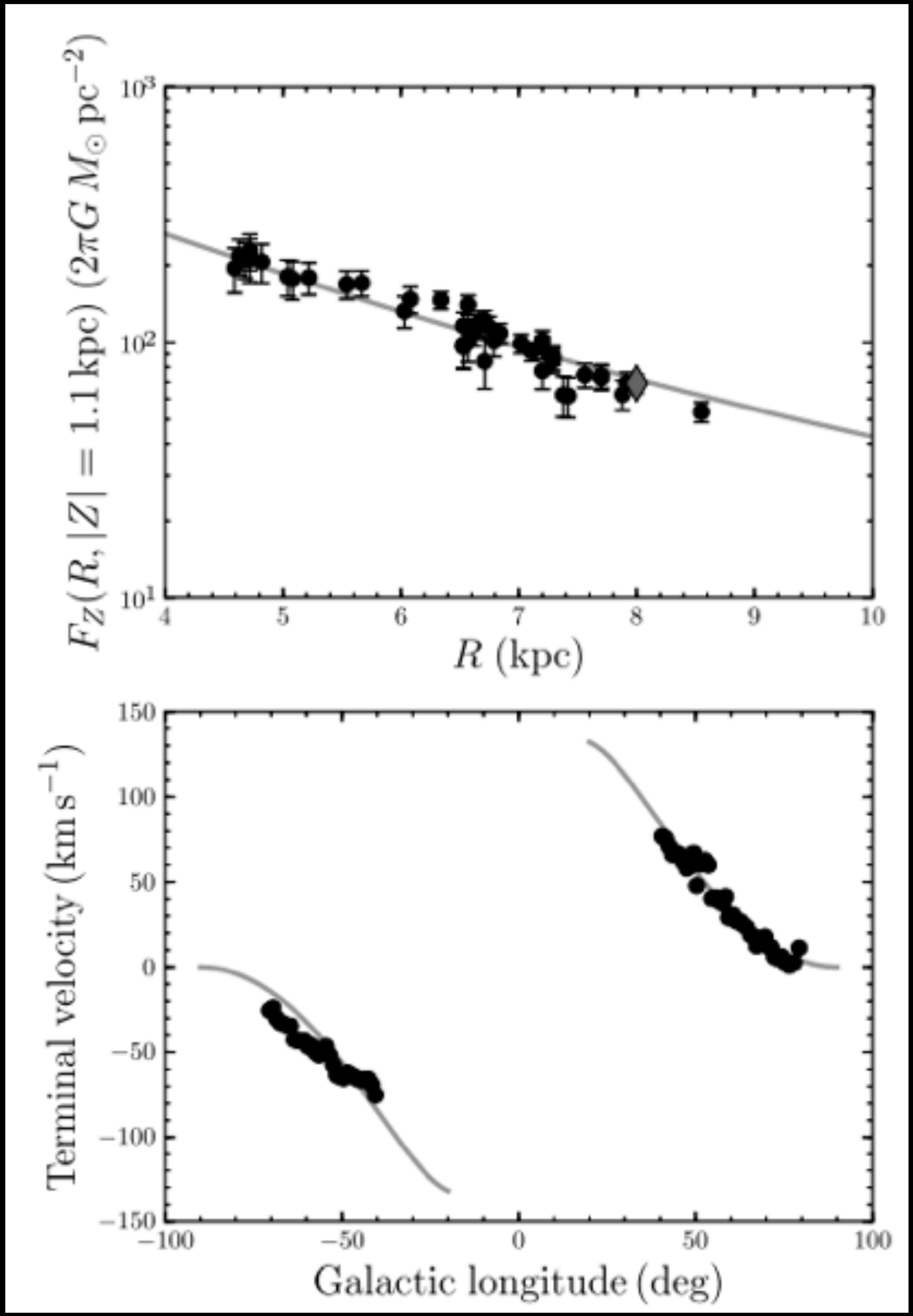
FLATTENING OF POTENTIALS

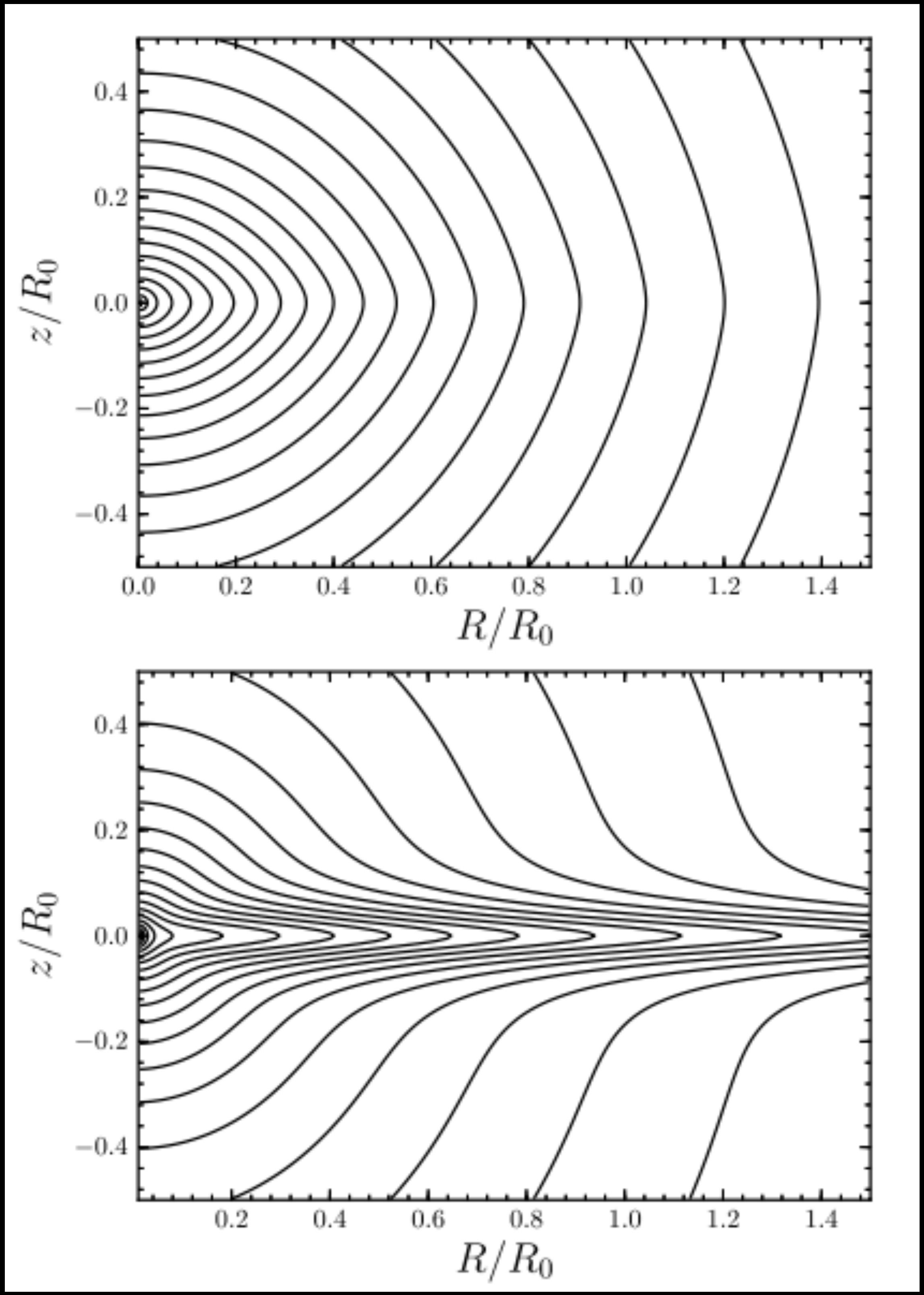
- For the flattening in the density, we need to look at the density contours: flattening = ratio of z and R at which isodensity contour cuts through R=0, z=0, respectively

- Outside R_c approximately

$$q_\rho^2 = q^4\left(2 - \frac{1}{q^2}\right) \,, \qquad 1 - q \approx \frac{1}{3}\left(1 - q_\rho\right)$$

Galactic potentials: the Milky Way





ORBITS IN SPHERICAL POTENTIALS

$$\ddot{r} = g(r)\,\hat{\mathbf{e}}_r$$

$$\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\mathbf{r} \times \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}\right) = 0$$

- Full angular momentum vector is conserved → orbit is confined to plane

- Equations of motion (EOM):

$$\ddot{\mathbf{r}} - r\dot{\psi}^2 = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\dot{\psi}\right) = 0$$

Constants and integrals of the motion

- Any property of an orbit that does not change along the orbit is a *constant of motion*
- *Integrals of the motion* are constants of the motion that do not depend on time
- Isolating integrals confine orbits to a subspace of the full 6D phase space: for example
  - **L** in a spherical potential: confines the orbit to a plane
  - E confines the orbit to a range of radii between peri and apocenter

ORBITS IN AXISYMMETRIC POTENTIALS: MOTION IN THE MERIDIONAL PLANE

- In axisymmetric potentials, we use cylindrical coordinates to investigate orbits; symmetry plane z=0

- Equations of motion:

$$\ddot{R} - R\dot{\theta}^2 = -\frac{\partial \Phi}{\partial R}$$

$$\ddot{z} = -\frac{\partial \Phi}{\partial z}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(R^2\dot{\theta}\right) = -\frac{\partial \Phi}{\partial \theta} = 0$$

- The last equation shows that the *z*-component of the angular momentum is conserved

ORBITS IN AXISYMMETRIC POTENTIALS: MOTION IN THE MERIDIONAL PLANE

- We can rewrite the equations of motion in terms of the angular momentum

$$\begin{aligned}\ddot{R} &= -\frac{\partial \Phi}{\partial R} + R\dot{\theta}^2 \\ &= \frac{\partial}{\partial R}\left(\Phi + \frac{L_z}{2R^2}\right) \\ &= -\frac{\partial \Phi_{\rm eff}}{\partial R}\end{aligned}$$

- and

$$\ddot{z} = -\frac{\partial \Phi_{\rm eff}}{\partial z}$$

- Motion is essentially in four dimensions (R,z,v_R,v_z) and the (R,z) plane is known as the *meridional plane*

ORBITS IN AXISYMMETRIC POTENTIALS: MOTION IN THE MERIDIONAL PLANE

- Examples of effective potentials w/ galpy

- Specify w/ angular momentum L_z and energy E

$$E = \Phi(R,z) + \frac{L_z}{2R^2} + \left(\frac{v_R^2}{2} + \frac{v_z^2}{R}\right)$$

- Naively expect orbits to fully fill space allowed by

$$\Phi_{\rm eff} \leq E$$

POINCARE SECTIONS

- To investigate the dimensionality of orbits further, we make use of *Poincare sections*

- These are defined by considering the motion in the meridional plane (R,z,v_R,v_z):

    - Four dimensions hard to visualize, but can remove 1 because of energy constraint → (R,z,v_R)

    - Only look at cuts through $z=0$

    - If (E,L_z) fully specify the orbit, then orbit should be 3-dimensional in (R,z,v_R) and 2-dimensional in (R,$z=0$,v_R)

- Example in galpy

POINCARE SECTIONS

- In most galactic potentials, these Poincare sections are 1 dimensional → potential has a third isolating integral in addition to (E,L)
- Hard to calculate!

CLOSE-TO-CIRCULAR ORBITS AND THE EPICYCLE APPROXIMATION

- Circular orbits: R does not change, so from the EOM we find that these are minima of Φ_{eff}

- These circular orbits satisfy

$$\frac{L_z^2}{R^3} = \frac{\partial \Phi}{\partial R} \qquad \frac{V_c^2}{R} = \frac{\partial \Phi}{\partial R} \qquad R\,\Omega^2 = \frac{\partial \Phi}{\partial R}$$

- at some radius R_c

- We can expand the effective potential around R_c

$$\Phi_{\mathrm{eff}}(R, z) \approx \Phi_{\mathrm{eff}}(R_c, 0) + \frac{\partial \Phi_{\mathrm{eff}}}{\partial R}(R_c, 0)\,(R - R_c)$$
$$+ \frac{\partial^2 \Phi_{\mathrm{eff}}}{\partial R^2}(R_c, 0)^2\,(R - R_c) + \frac{\partial^2 \Phi_{\mathrm{eff}}}{\partial z^2}(R_c, 0)^2\,z$$

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- at some radius R_c

- We can expand the effective potential around R_c

$$\Phi_{\text{eff}}(R,z) \approx \Phi_{\text{eff}}(R_c,0) + \frac{\partial \Phi_{\text{eff}}}{\partial R}(R_c,0)\,(R-R_c)$$
$$+ \frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2}(R_c,0)^2\,(R-R_c) + \frac{\partial^2 \Phi_{\text{eff}}}{\partial z^2}(R_c,0)^2\,z$$

CLOSE-TO-CIRCULAR ORBITS AND THE EPICYCLE APPROXIMATION

- which leads to the equations of motion:

$$\frac{\mathrm{d}^2(R-R_c)}{\mathrm{d}t^2} = -\frac{\partial^2\Phi_{\mathrm{eff}}}{\partial R^2}(R_c, 0)\,(R-R_c)^2$$
$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} = -\frac{\partial^2\Phi_{\mathrm{eff}}}{\partial z^2}(R_c, 0)\,z^2$$

- These are just two decoupled harmonic oscillators with frequencies

$$\kappa^2 \equiv \frac{\partial^2\Phi_{\mathrm{eff}}}{\partial R^2}$$
$$\nu^2 \equiv \frac{\partial^2\Phi_{\mathrm{eff}}}{\partial z^2}$$

Action-angle coordinates

- In position-velocity space, dynamics follows from Hamilton's equations: $\dot{\mathbf{x}} = \mathbf{v};\ \dot{\mathbf{v}} = -\mathrm{d}\Phi/\mathrm{d}\mathbf{x}$
- However, we can express dynamics in any other set of canonical coordinates, using a generating function S(**x**,**J**):

$$\theta = \frac{\partial S}{\partial \mathbf{J}},\ \mathbf{v} = \frac{\partial S}{\partial \mathbf{x}}$$

- Then $H \equiv H\left(\mathbf{x}, \frac{\partial S}{\partial \mathbf{x}}(\mathbf{x},\mathbf{J})\right)$ and we can solve the Hamilton-Jacobi equation for S

$$H\left(\mathbf{x}, \frac{\partial S}{\partial \mathbf{x}}(\mathbf{x},\mathbf{J})\right) = E$$

- As a PDE this is hard to solve and explicit solutions are rare

What are action-angle coordinates?

- Hamilton's equations for action-angle coordinates:

$$\dot{\mathbf{J}} = -\frac{\partial H}{\partial \theta} = 0;\ \dot{\theta} = \frac{\partial H}{\partial \mathbf{J}} = \boldsymbol{\Omega}(\mathbf{J}) = \mathrm{constant}$$

- Dynamics is extremely simple:
  - Actions are conserved along orbit
  - Angles increase linearly in time

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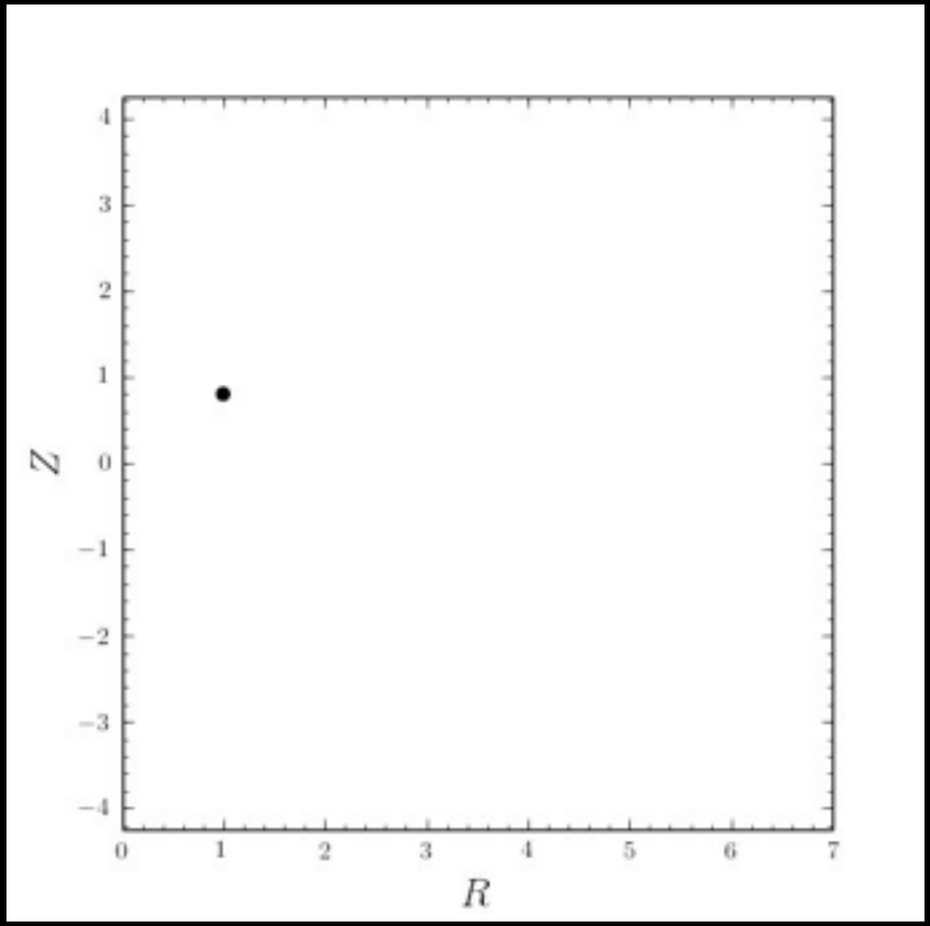
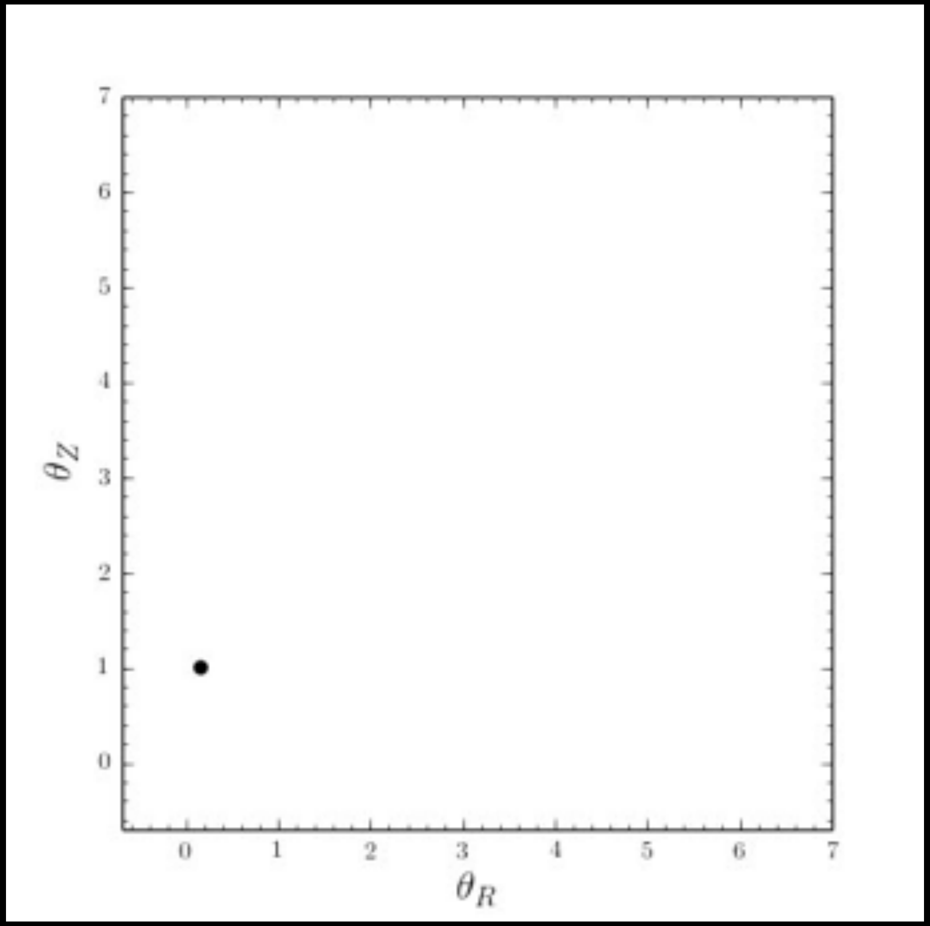




Action-angle coordinates: some solutions

- Only analytic case: isochrone potential (incl. Kepler and harmonic oscillator)
- Spherical: $J_\phi=L_z$, $J_z=L-|L_z|$, J_r = integral, frequencies and angles can be calculated as integrals (one frequency is zero)
- Axisymmetric: L_z, no general expressions for J_r and J_z (~ third-integral problem)
- Staeckel potentials: integral expressions for class of potential, incl. triaxial, but realistic galactic potentials are not of this form
- For orbits in and near the galactic plane, can approximate vertical and planar motion as decoupled, allows action-angle coordinates to be calculated (e.g., Binney 2010), or approximate potential as Staeckel potential (Binney 2012)
- General solutions for time-independent potentials now available (Bovy 2014, Sanders & Binney 2014)

galpy: A Python Library for Galactic Dynamics

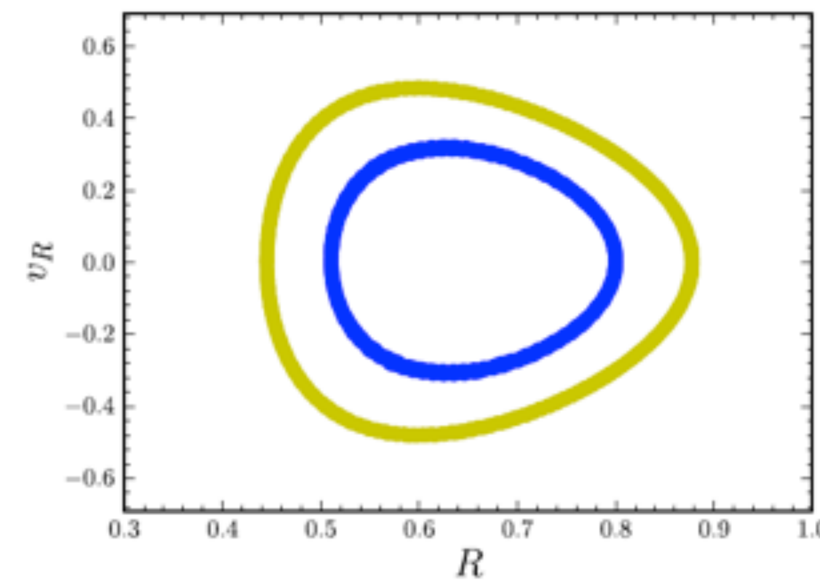
Jo Bovy (IAS) https://github.com/jobovy/galpy

Galactic Dynamics in python

build passing | coverage 100% | C coverage 99% | docs latest | pypi v1.0 | license New BSD

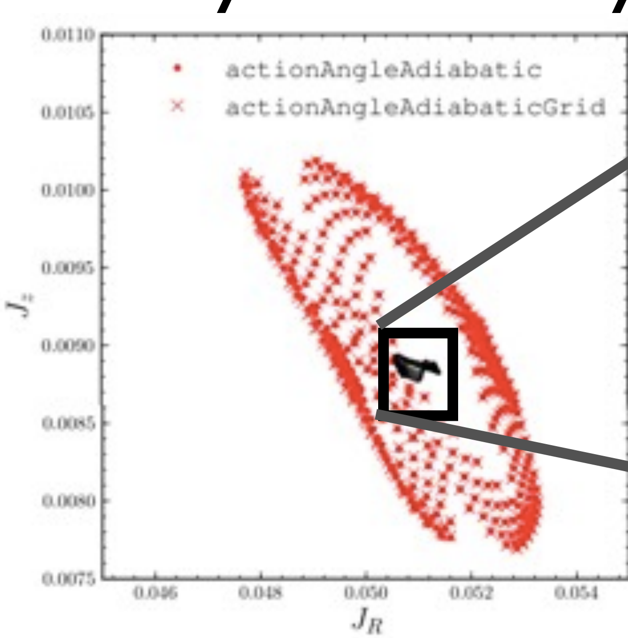
See Bovy (2015, ApJS) and online documentation

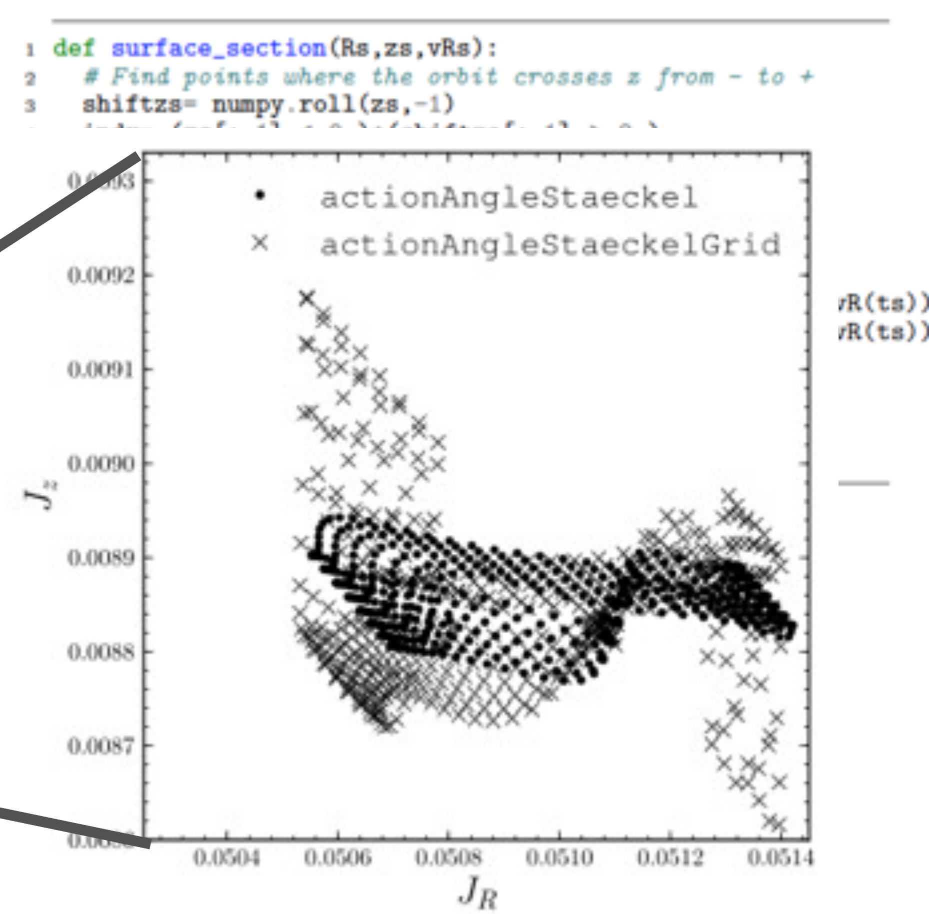
- *galpy*: general-purpose Galactic dynamics package; 23,000 lines + 11,000 lines of test code + 20,000 lines of documentation; test coverage of 99.6%



- Large variety of potentials, incl. a MW potential (`galpy.potential.MWPotential2014`)

- Fast orbit integration in variety of potentials, steady-state kinematics of disk galaxies (e.g., asymmetric drift), non-axisymmetric dynamics, all sorts of action-angle coordinates, this talk's stream model, and much more





Equilibrium of collisionless systems

- Because galaxies are smooth, we can work with the distribution function of orbits f(x,v) rather than individual orbits directly
- In equilibrium D f(x,v) / D t == 0, leading to the collisionless Boltzmann equation
- Only works for low-mass stars, such that few stars are born or die in a dynamical time
- CBE in cylindrical coordinates:

$$\frac{\partial f}{\partial t} + p_R \frac{\partial f}{\partial R} + \frac{p_\phi}{R^2}\frac{\partial f}{\partial \phi} + p_z \frac{\partial f}{\partial z} - \left(\frac{\partial \Phi}{\partial R} - \frac{p_\phi^2}{R^3}\right)\frac{\partial f}{\partial p_R} - \frac{\partial \Phi}{\partial \phi}\frac{\partial f}{\partial p_\phi} - \frac{\partial \Phi}{\partial z}\frac{\partial f}{\partial p_z} = 0.$$

Jeans equations

- Jeans equations are obtained as *moments* of the CBE: multiply by v_i^a and integrate over **v**
- Spherical:

$$\frac{\mathrm{d}(\nu\overline{v_r^2})}{\mathrm{d}r}+\nu\left(\frac{\mathrm{d}\Phi}{\mathrm{d}r}+\frac{2\overline{v_r^2}-\overline{v_\theta^2}-\overline{v_\phi^2}}{r}\right)=0.$$

- Cylindrical:

$$p_R\frac{\partial f}{\partial R}+p_z\frac{\partial f}{\partial z}-\left(\frac{\partial\Phi}{\partial R}-\frac{p_\phi^2}{R^3}\right)\frac{\partial f}{\partial p_R}-\frac{\partial\Phi}{\partial z}\frac{\partial f}{\partial p_z}=0.$$

$$F_R(R,Z)=-\frac{\partial\Phi(R,Z)}{\partial R}=\frac{1}{\nu}\frac{\partial\left(\nu\sigma_U^2\right)}{\partial R}+\frac{1}{\nu}\frac{\partial\left(\nu\sigma_{UW}^2\right)}{\partial Z}+\frac{\sigma_U^2-\sigma_V^2-\bar{V}^2}{R},$$

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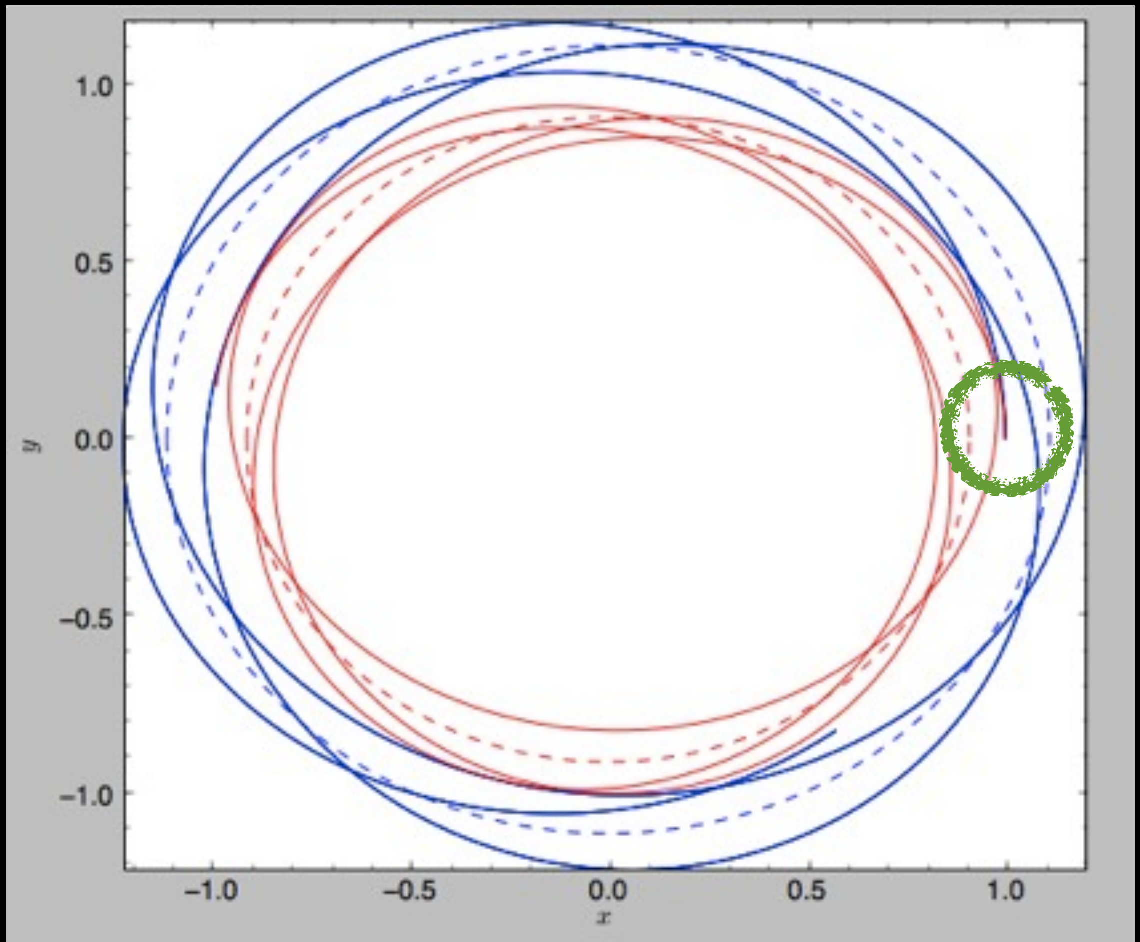
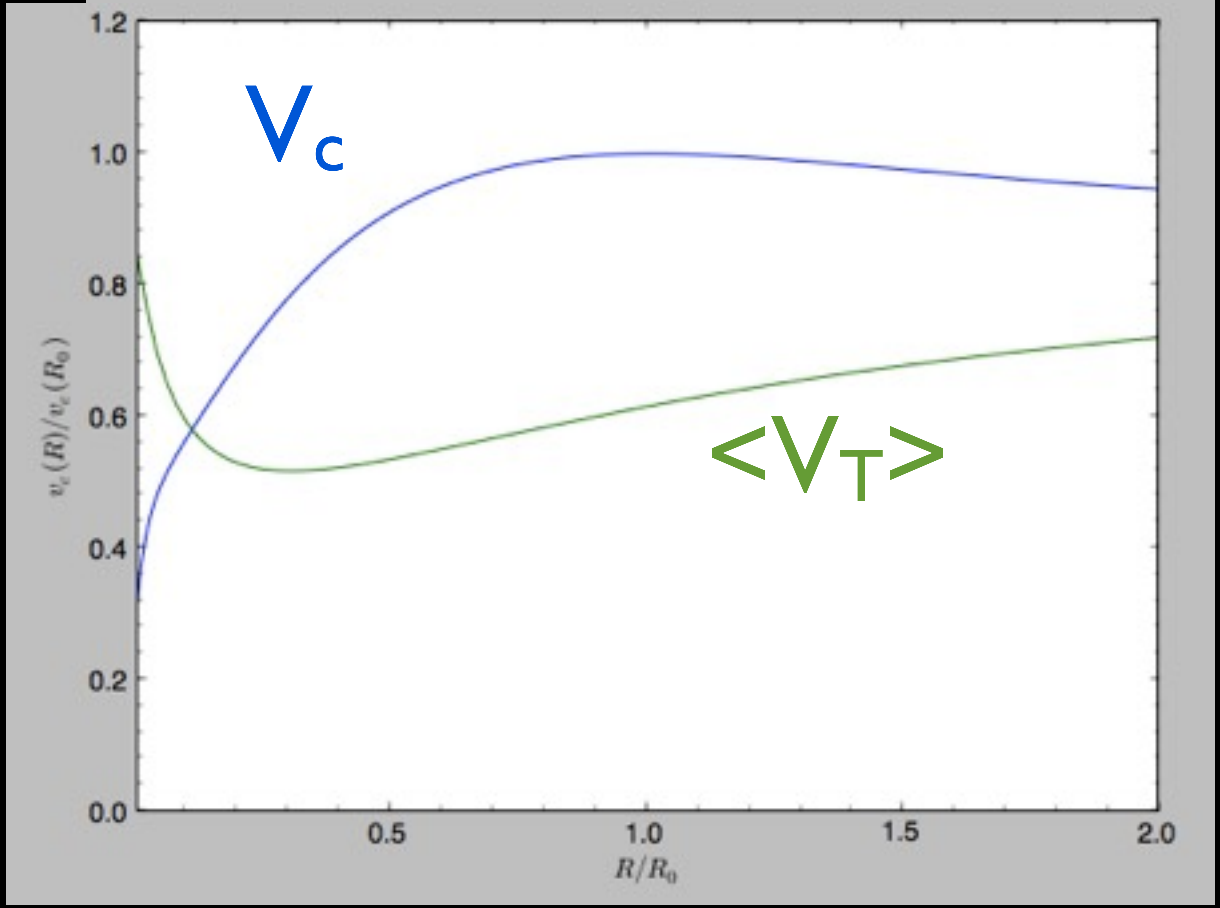
Jeans equations and asymmetric drift

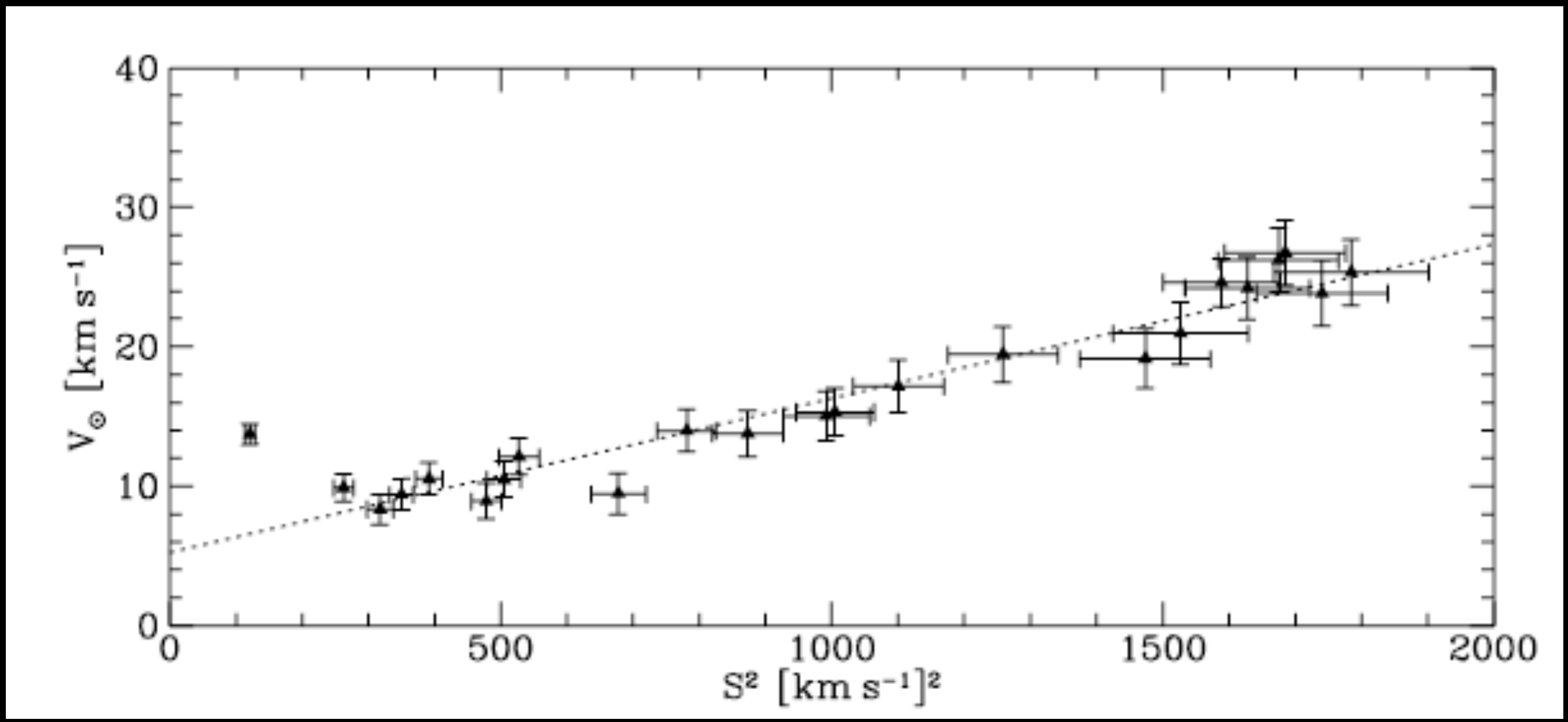
ASYMMETRIC DRIFT

- Galaxy disks have decreasing density and dispersion profiles with radius
- Therefore, there are more stars coming from the inner Galaxy than from the outer Galaxy
- Because of conservation of L, inner-Galaxy stars move slower than V_c in the solar neighborhood
- The mean V_T is therefore $< V_c$
- This effect is bigger for larger dispersions





ASYMMETRIC DRIFT

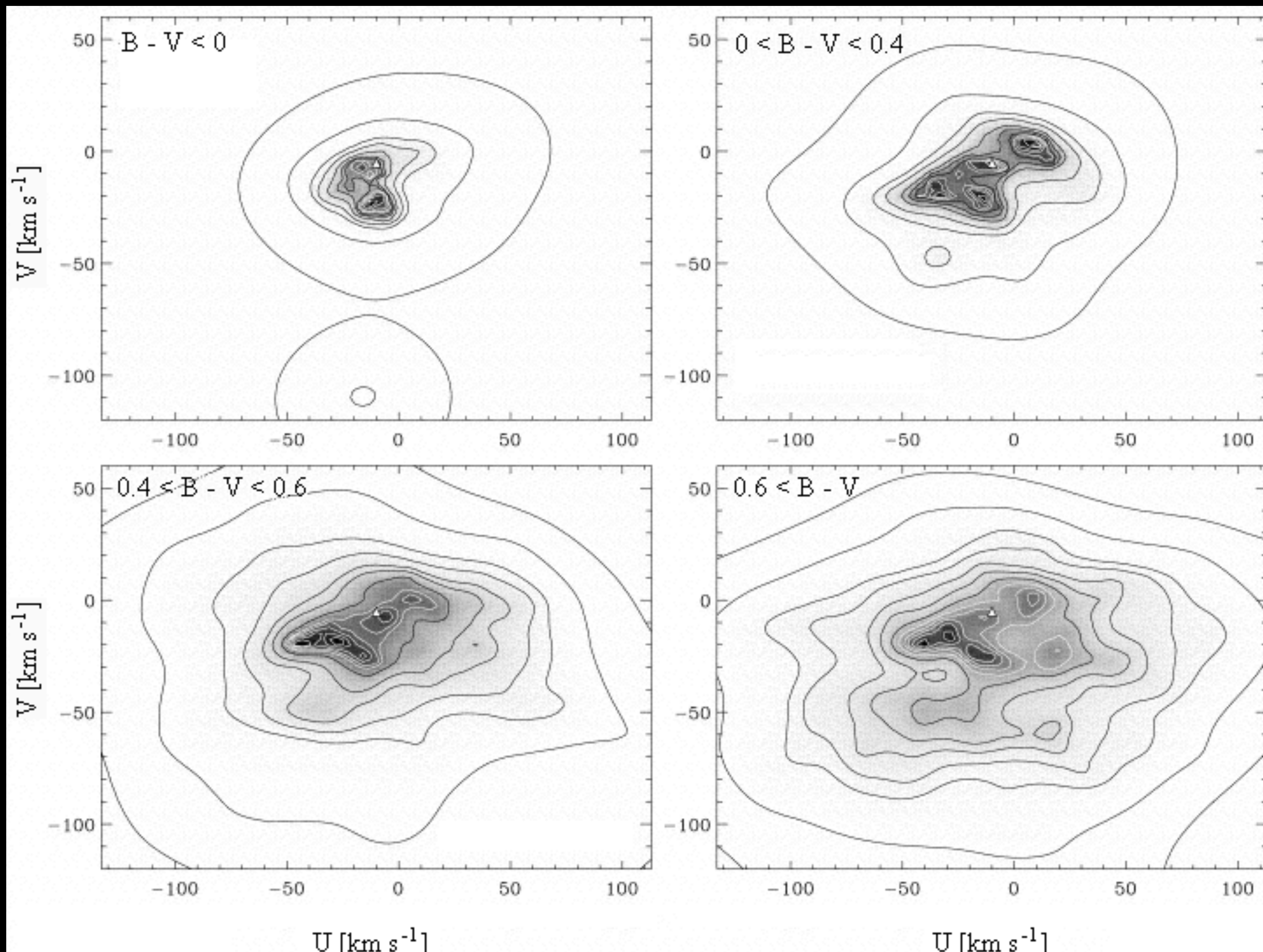


Dehnen & Binney (1998)

Jeans theorem

- Rather than working with the Jeans equations, we can work with the distribution itself
- Jeans theorem: any steady-state solution of the CBE depends on the phase-space coordinates (x,v) only through integrals of the motion
- and any function of the integrals of the motion is a solution to the steady-state CBE
- Can use *actions* as these integrals

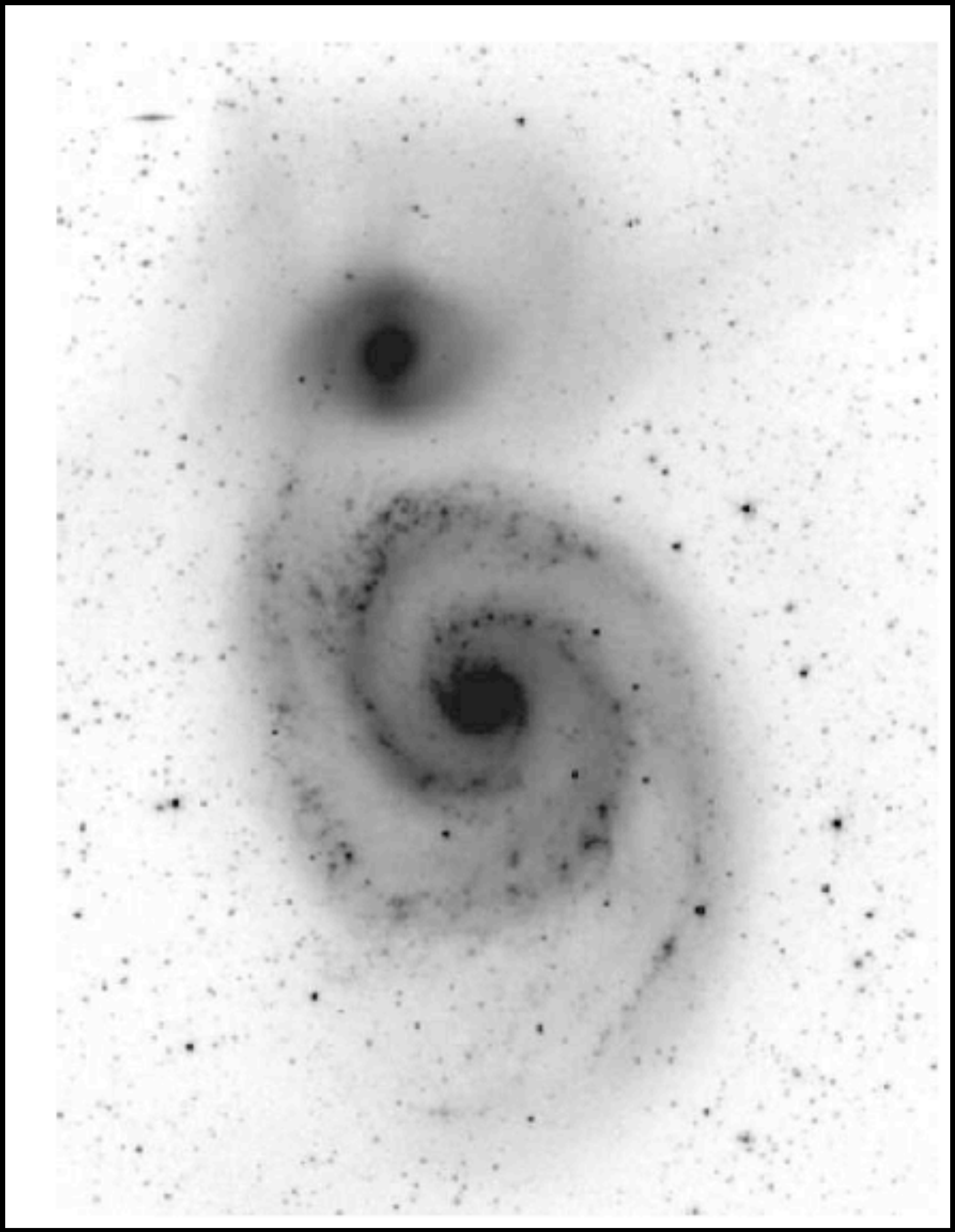
BREAKDOWN OF SIMPLE AXISYMMETRIC, TIME-INDEPENDENT PICTURE IN THE SN



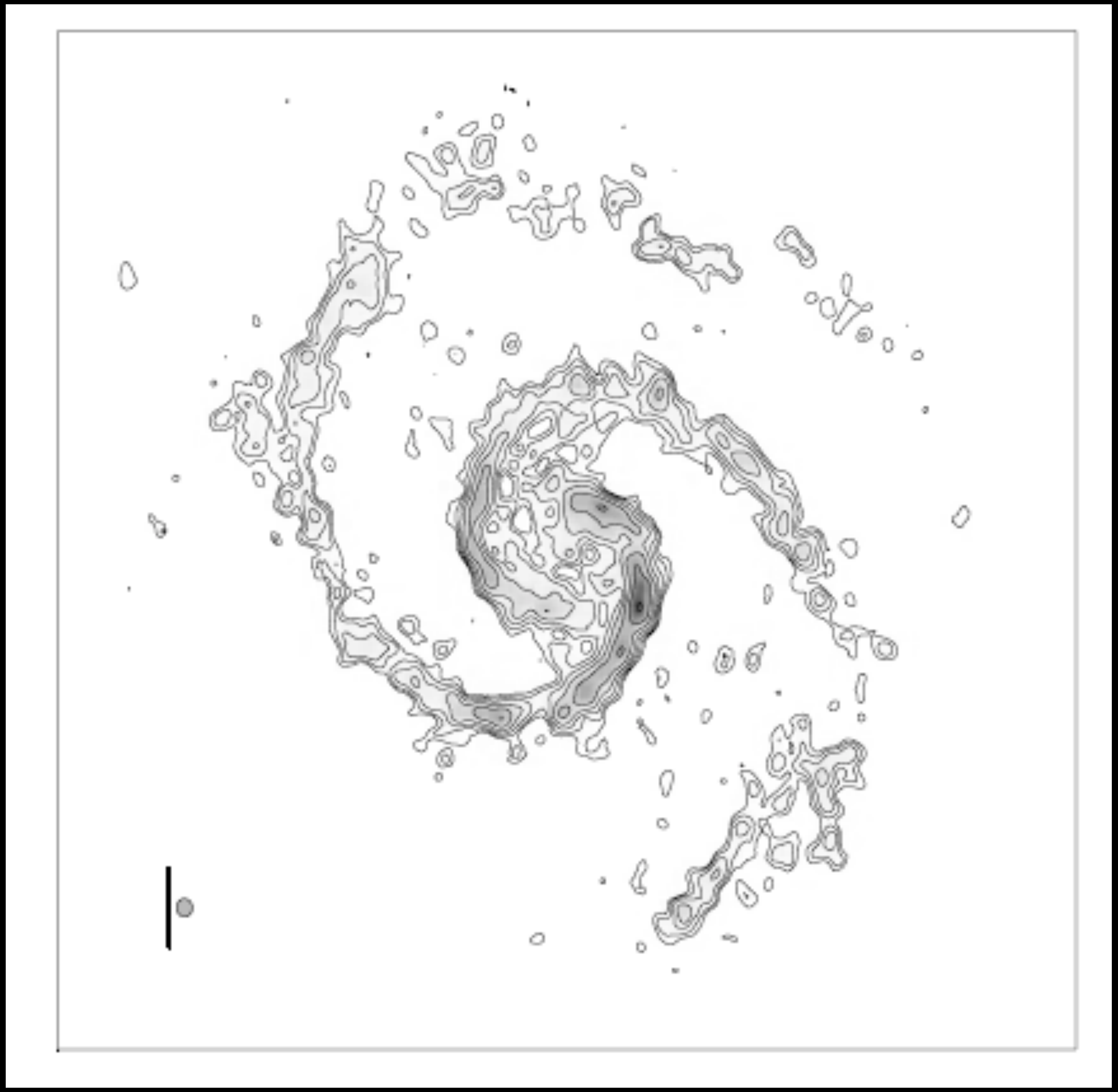
NON-AXISYMMETRY

- Disks develop non-axisymmetric perturbations to evolve to a lower energy state
- Collisionless systems can only transfer angular momentum through gravitational torques from non-axisymmetry

SPIRAL STRUCTURE



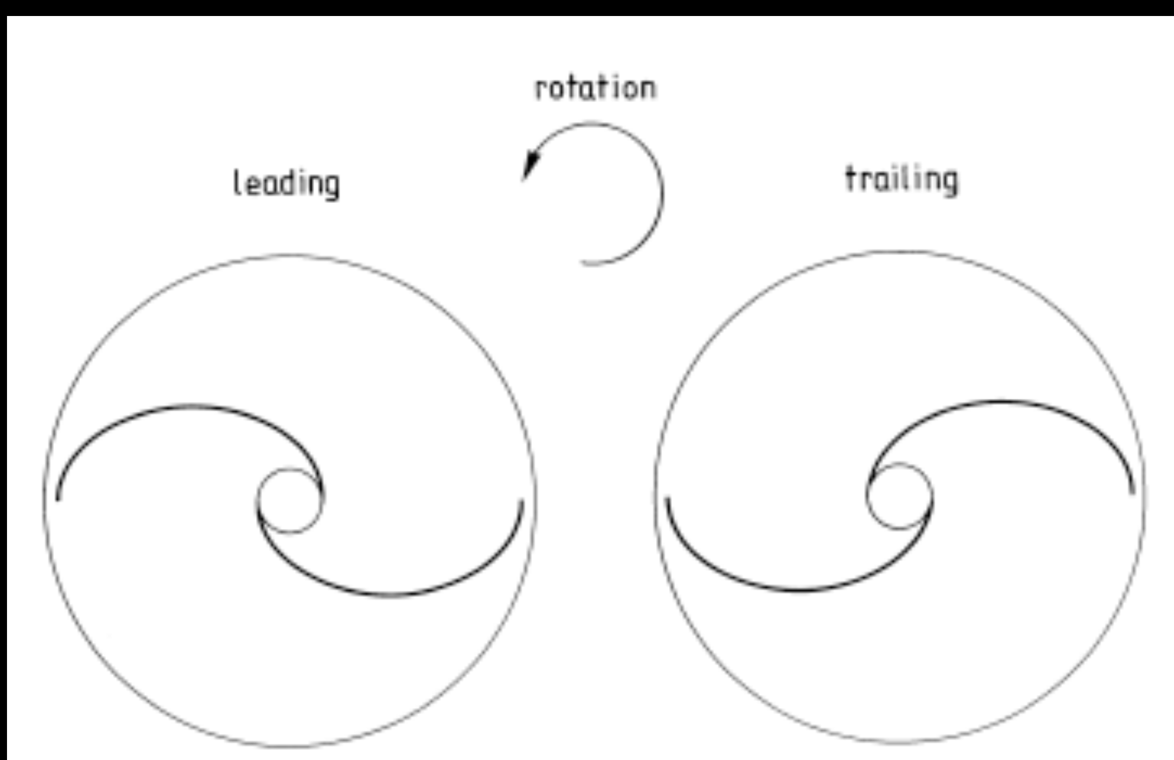
NIR

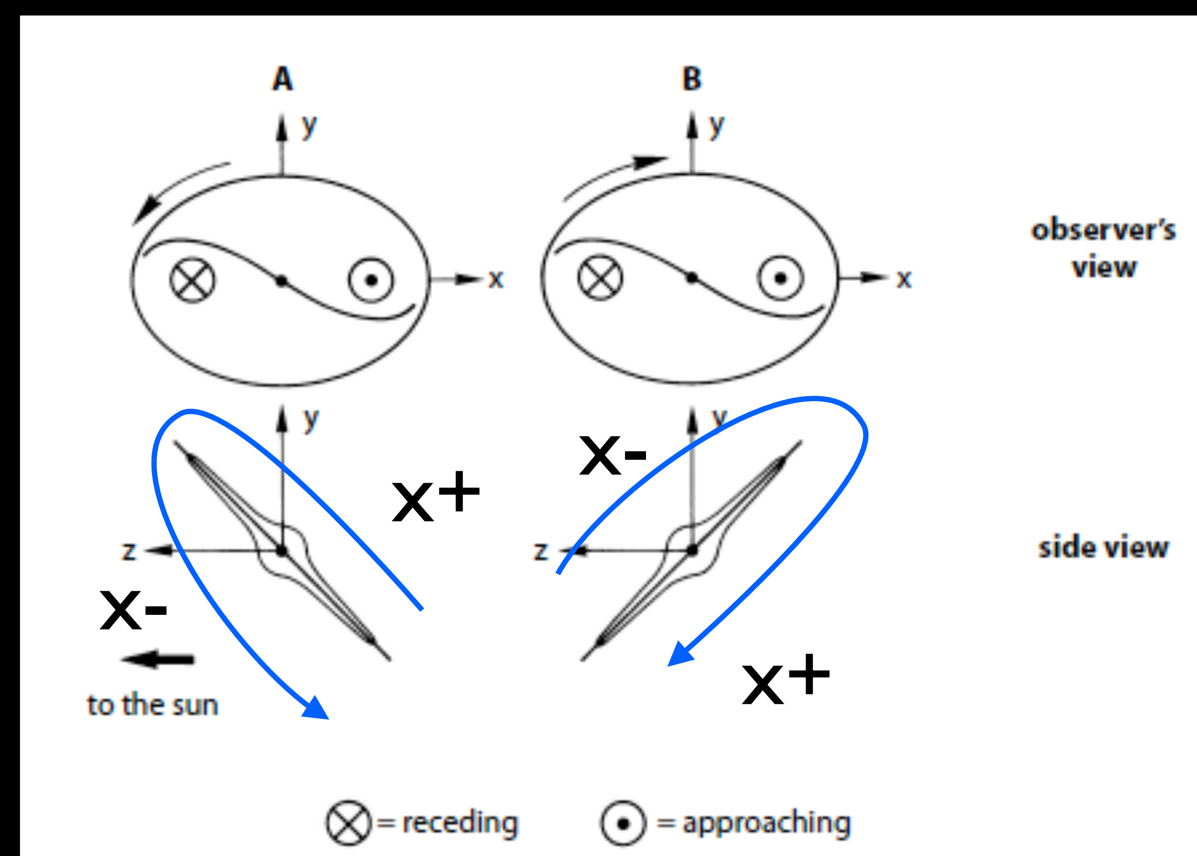


CO

SPIRAL STRUCTURE

- Strength of spiral structure: Expand the surface brightness as Fourier series → arm/inter-arm ratios of 1.5 to 4
- All spirals are trailing





diagrams from Binney & Tremaine (2008)

SPIRAL STRUCTURE

- Anti-spiral theorem: Newtonian gravity and motion is time-reversible, so in steady state, leading arms should be equivalent solution → spiral arms cannot be steady-state phenomenon
- Winding problem: spiral arms cannot be material arms, because they would wind up too much over 10 Gyr
- Long-term spirals must be a density wave
- Popular model has pattern with constant pattern speed

GRAVITATIONAL POTENTIAL OF TIGHTLY WOUND SPIRAL STRUCTURE

- Location of arms specified by a shape function:

$$m\phi + f(R,t) = \text{constant} \pmod{2\pi},$$

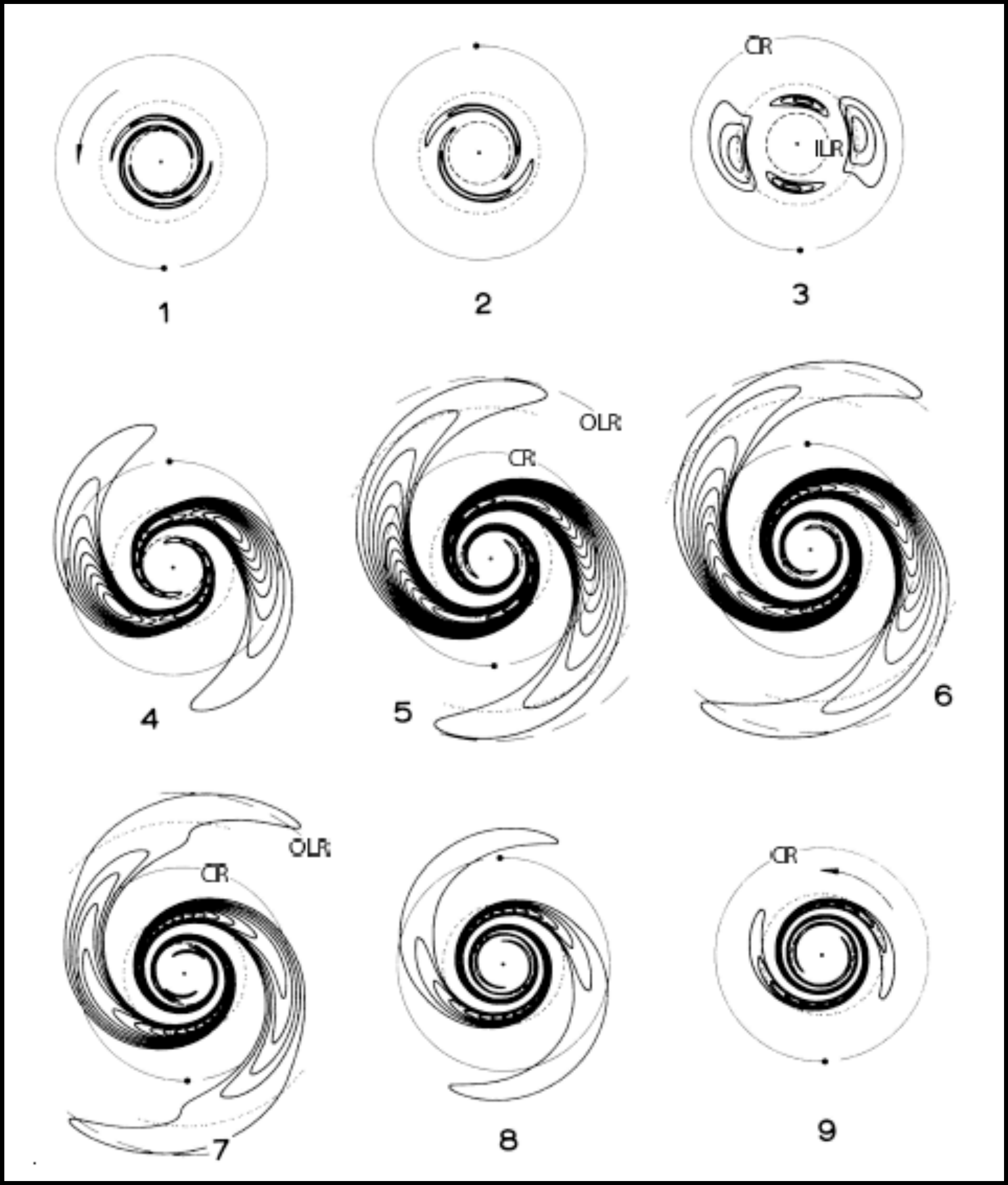
- For tightly-wound spirals, |kR| << 1, long-range coupling is negligible (WKB approximation), surface density

$$\Sigma_1(R,\phi,t) = H(R,t)\mathrm{e}^{\mathrm{i}[m\phi+f(R,t)]},$$

- Can then easily solve the Poisson equation

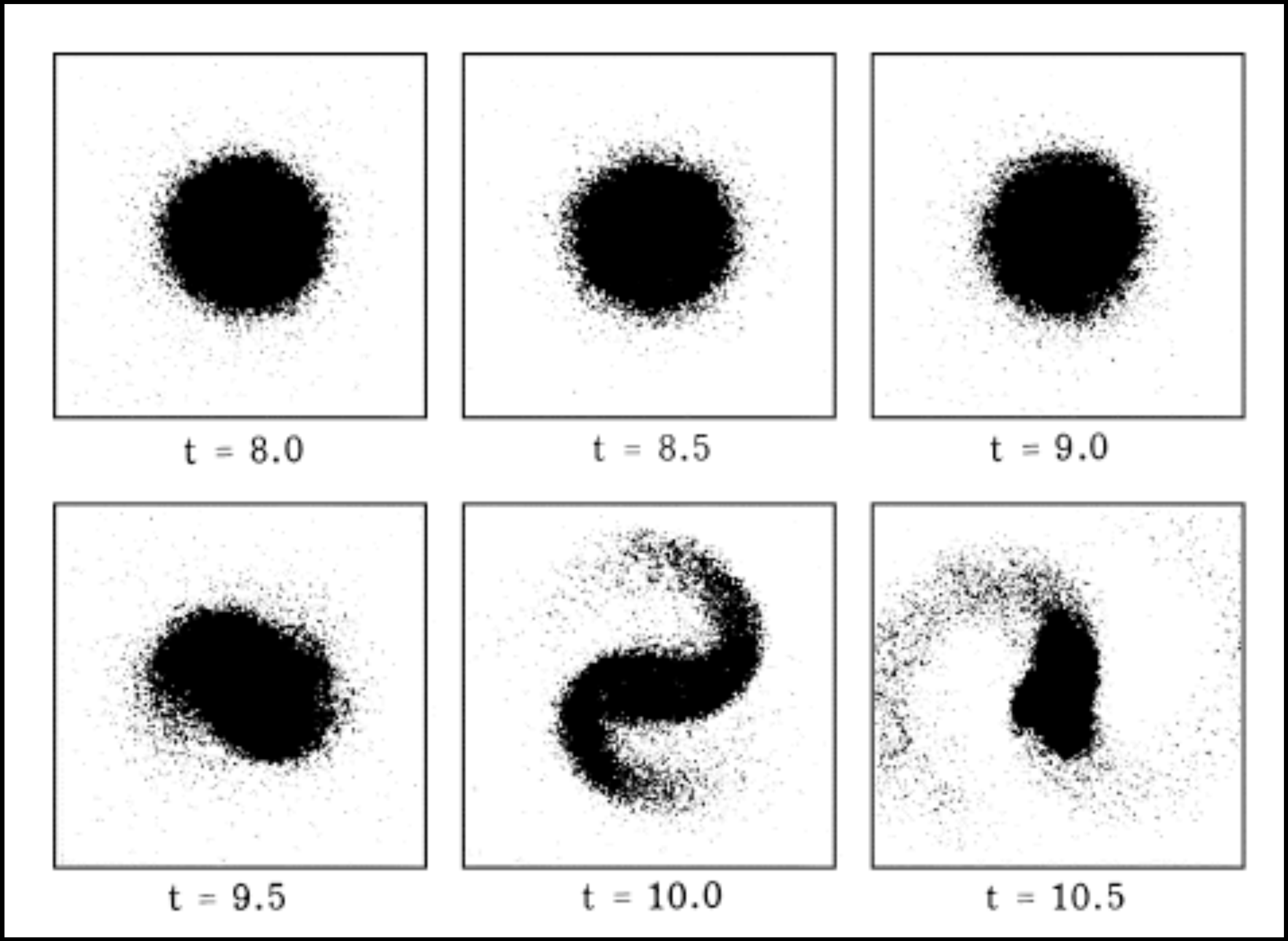
$$\Phi_1(R,\phi,t) = -\frac{2\pi G}{|k|}H(R,t)\mathrm{e}^{\mathrm{i}[m\phi+f(R,t)]}.$$

DEVELOPMENT OF SPIRAL STRUCTURE: SWING AMPLIFICATION



BARS AND THE BAR INSTABILITY

- Stellar disks have strong m=2 instability, the *bar instability*
- Bars form through the bar instability, followed by a *buckling instability*



Dark matter in the Milky Way

- Milky Way provides up-close look of distribution of dark matter (DM) in a large disk galaxy
- Local density and density profile important for dark matter detection
- Direct detection: local density and velocity distribution

- Indirect detection: e.g., Galactic center
- DM content intimately linked with formation and evolution of large disk galaxies like the Milky Way
- This lecture: overview of dynamical measurements of DM in the Milky Way

OVERVIEW

- Basics of dynamical modeling in the MW
- The Milky Way rotation curve
- Local determinations of the DM density
- The radial profile of DM near the center of the MW
- The large-scale distribution of DM in the halo
- Future developments

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BASIC PROBLEM OF DYNAMICAL MODELING

BASIC PROBLEM OF DYNAMICAL MODELING

$$\vec{F} = m\vec{a}$$

BASIC PROBLEM OF DYNAMICAL MODELING

- Gravitational potential only affects accelerations; positions and velocities are initial conditions
- We can only measure x,v for all but a few stars in the MW
- Need to make assumptions about the distribution function DF(x,v) of dynamical tracers: e.g.,
  - tracers are on circular orbits: DF(x,v) = δ(circ. orb.) $f(L_z)$
  - tracers are in dynamical equilibrium: DF(x,v) = DF(integrals) (cf., virial theorem)
  - tracers originate from common phase-space point (e.g., streams, timing argument)

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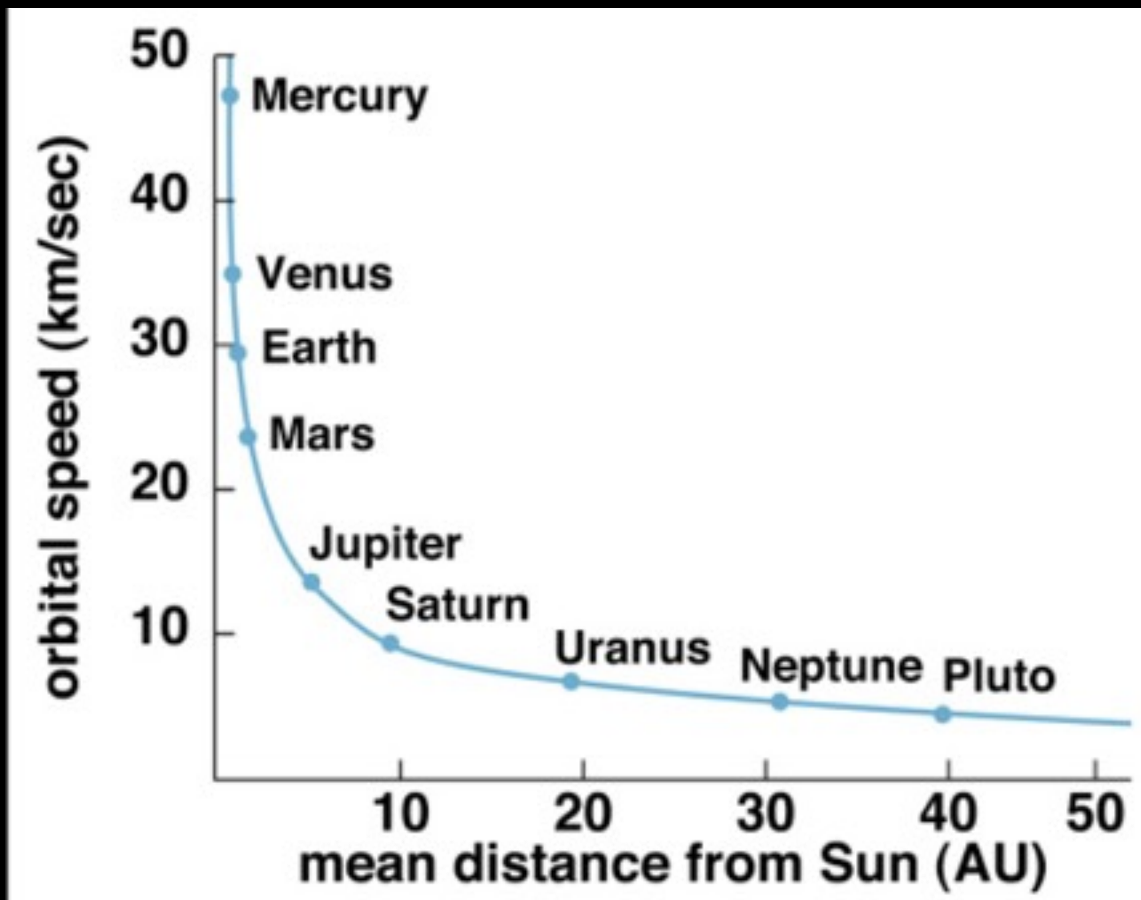
GALAXY ROTATION CURVES

- For spherical mass distribution, from Newton's laws we know that
$$V_c^2 \sim \frac{GM(< R)}{R}$$
- (does not quite hold for non-spherical distributions, but close)
- As such, the rotation curve mainly measures the *total enclosed mass*
- It does *not* distinguish clearly between roughly spherical mass distributions (~DM halo) and flattened distributions (~stellar or gas disk)

ROTATION CURVES AT LARGE DISTANCES: DISCOVERY OF DARK MATTER



$$V_c^2 \sim \frac{GM(<R)}{R}$$

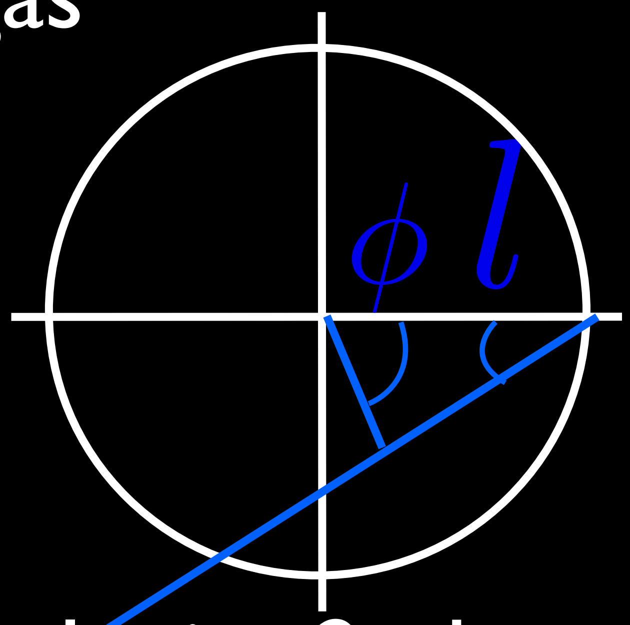
For a flat V_c, $M \sim R$, $\rho \sim R^{-2}$

Rubin+ (1970)

MEASURING THE MILKY WAY'S ROTATION CURVE

- Determining the rotation curve in external galaxies is relatively straightforward: measure motion of gas from Doppler shifts of emission lines
- In the Milky Way this is complicated by the fact that the Sun is (approximately) co-rotating with the gas
- Traditional method of measuring the MW rotation curve: terminal velocity curve:

$$V_{\rm los} = V_c(R)\,\sin(\phi + l) - V_c(R_0)\,\sin l$$



- (assuming the Sun is moving with the circular velocity, 2nd term)
- Max. $V_{\rm los}$ when $(\phi + l) = 90°$ (only in inner MW)

MEASURING THE MILKY WAY'S ROTATION CURVE

- Therefore, gas bunches up at (ϕ+l) = 90°, where V_{los} has a maximum; this can be measured from 21cm gas emission (hyperfine structure)
- From geometry: $$\frac{R_0}{\sin(\phi + l)} = \frac{R}{\sin l}$$
- So we have that

$$V_{\rm los} = \sin l \left[V_c(R)\,\frac{R_0}{R} - V_C(R_0) \right]$$

- This equation can be solved to give $V_c(R)$, but it is invariant under

$$V_c(R) \rightarrow V_c(R) + \Omega R$$

- This means that we cannot measure both solid-body rotation (ΩR) and $V_c(R_0)$

MEASURING THE MILKY WAY'S ROTATION CURVE

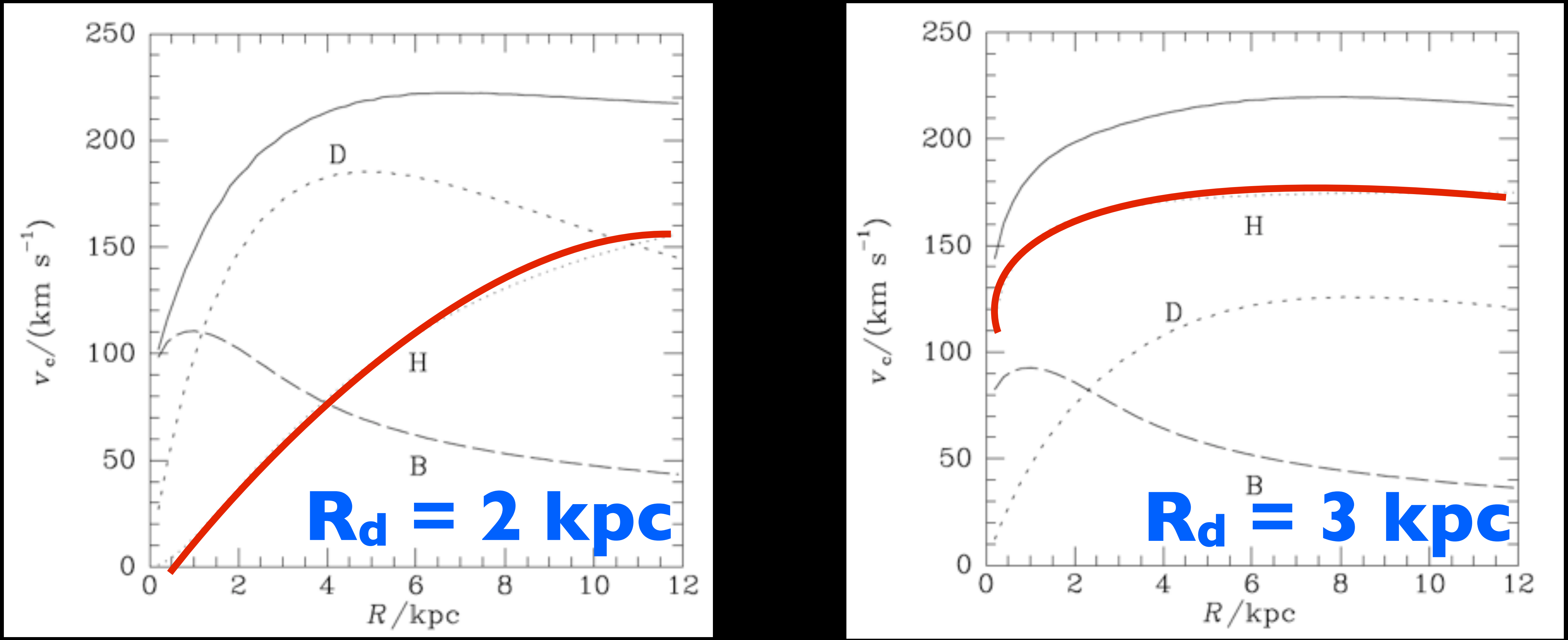
- So, measure $V_c(R_0)$ another way and use terminal-velocity curve for $V_c(R)$
- Other issues:
  - Doesn't work in outer MW (no tangent-point, $\phi + l > 90°$)
  - Only measure $V_c(R)$ at one l, or ϕ (non-axisymmetry)

MEASURING THE MILKY WAY'S ROTATION CURVE

- Solutions:
  - Many other ways to measure $V_c(R_0)$, all controversial
  - Use gas proper motions (masers; Reid et al. 2009, 2014)
  - Use stellar disk kinematics (local: Feast & Whitelock1997; global: Bovy et al. 2012)
  - Measure Sun's motion wrt population assumed to be at rest (halo globular clusters, halo stars, the black hole at the center); uncertain bc of unknown Solar motion
- Current best-knowledge: $V_c(R_0)$ = 220 to 240 km/s, rotation curve very close to flat over $4 < R/\mathrm{kpc} < 16$

MILKY WAY ROTATION CURVE IS AMBIGUOUS

- Even with perfect measurements of the rotation curve, disentangling the contributions from stars and dark matter is impossible
- "Disk-halo degeneracy"



Binney & Tremaine (2008)

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VERTICAL MASS DISTRIBUTION

- Rotation curve measurements are getting better, but they cannot tell to what extent the mass is flattened (i.e., what the relative contribution of baryonic and dark matter is)
- Measurements of the vertical mass distribution directly measure how concentrated the mass is around the Galactic mid-plane

BASIC IDEA OF VERTICAL MASS MEASUREMENT

- Throw a ball up with a known velocity v and measure its maximum height h_z

$$g = \frac{v^2}{2\,h_z}$$

- For stars we can statistically measure their velocities and the heights they reach above the plane:
  - Velocity distribution: $f(v_z|z)$ characterized by dispersion σ_z
  - Density: $\rho(z)$ ~ exponential with scale height h_z
- Assuming that the stars are in a steady state, we can relate these to the gravitational potential

$$K_z \approx \frac{\sigma_z^2}{h_z}$$

JEANS+POISSON EQUATIONS

- Jeans Eqns.: Moments of collisionless Boltzmann equation that describes the steady state

$$F_R(R,Z) = -\frac{\partial\Phi(R,Z)}{\partial R} = \frac{1}{\nu}\frac{\partial\left(\nu\sigma_U^2\right)}{\partial R} + \frac{1}{\nu}\frac{\partial\left(\nu\sigma_{UW}^2\right)}{\partial Z} + \frac{\sigma_U^2 - \sigma_V^2 - \bar{V}^2}{R},$$

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$$\Sigma(R,Z) = -\frac{1}{2\pi G}\left[\int_0^Z \mathrm{d}z\,\frac{1}{R}\frac{\partial(RF_R)}{\partial R} + F_Z(R,Z)\right]$$

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stellar density profile

$$F_Z(R,Z) = -\frac{\partial\Phi(R,Z)}{\partial Z} = \frac{1}{\nu}\frac{\partial(\nu\sigma_W^2)}{\partial Z} + \frac{1}{R\nu}\frac{\partial(R\nu\sigma_{UW}^2)}{\partial R}.$$

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radial velocity dispersion

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rotational velocity dispersion

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JEANS+POISSON EQUATIONS

- Jeans Eqns.: Moments of collisionless Boltzmann equation that describes the steady state

$$F_R(R,Z) = -\frac{\partial \Phi(R,Z)}{\partial R} = \frac{1}{\nu}\frac{\partial\left(\nu\sigma_U^2\right)}{\partial R} + \frac{1}{\nu}\frac{\partial\left(\nu\sigma_{UW}^2\right)}{\partial Z} + \frac{\sigma_U^2 - \sigma_V^2 - \bar{V}^2}{R},$$

mean rotational velocity

$$F_Z(R,Z) = -\frac{\partial \Phi(R,Z)}{\partial Z} = \frac{1}{\nu}\frac{\partial(\nu\sigma_W^2)}{\partial Z} + \frac{1}{R\nu}\frac{\partial\left(R\nu\sigma_{UW}^2\right)}{\partial R}.$$

$$\Sigma(R,Z) = -\frac{1}{2\pi G}\left[\int_0^Z \mathrm{d}z\,\frac{1}{R}\frac{\partial(RF_R)}{\partial R} + F_Z(R,Z)\right]$$

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vertical velocity dispersion

$$F_Z(R,Z) = -\frac{\partial \Phi(R,Z)}{\partial Z} = \frac{1}{\nu}\frac{\partial\left(\nu\sigma_W^2\right)}{\partial Z} + \frac{1}{R\nu}\frac{\partial\left(R\nu\sigma_{UW}^2\right)}{\partial R}.$$

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radial-vertical covariance

$$F_Z(R,Z) = -\frac{\partial\Phi(R,Z)}{\partial Z} = \frac{1}{\nu}\frac{\partial(\nu\sigma_W^2)}{\partial Z} + \frac{1}{R\nu}\frac{\partial\left(R\nu\sigma_{UW}^2\right)}{\partial R}.$$

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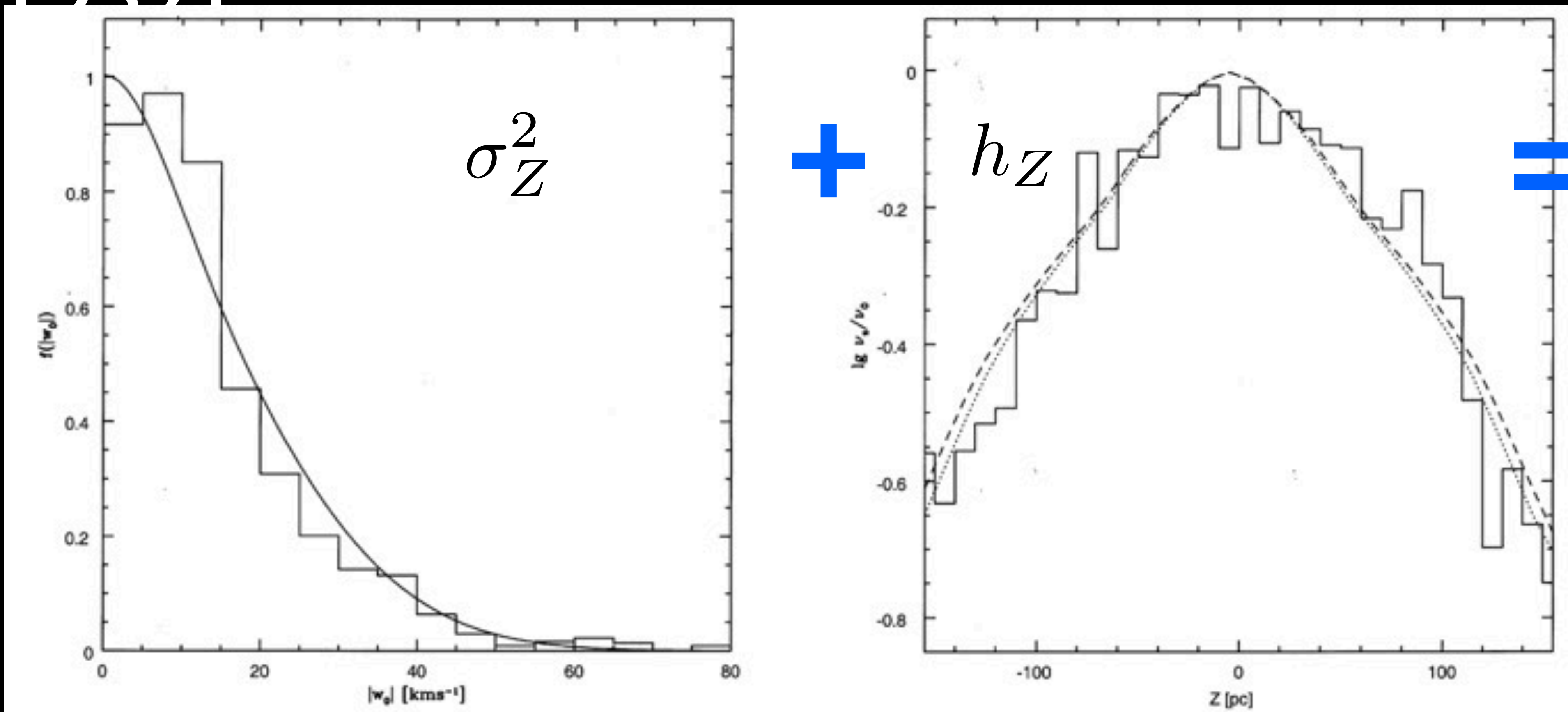
1D

Tilt =~ 0

$$\Sigma(R,Z) = -\frac{1}{2\pi G}\left[\int_0^Z \mathrm{d}z\,\frac{1}{R}\frac{\partial(RF_R)}{\partial R} + F_Z(R,Z)\right]$$

slope of rotation curve =~ 0

LOCAL DENSITY MEASUREMENTS (OORT 1932)



σ_Z^2 + h_Z = $\rho_{\rm total}(R_0, z=0)$

- Holmberg & Flynn (2000; and many earlier analyses): Model equilibrium distribution of A&F stars to measure the local mass density

- $\rho_{\rm total}(R_0, z=0)$= 0.1 +/- 0.01 $M_\odot\,{\rm pc}^{-3}$; no sign of dark matter (not expected), but strong constraint on scale height of disk dark matter

LOCAL DENSITY MEASUREMENTS ARE VERY ROBUST

Table 2. Local density estimates.

Density ρ_0 ($M_\odot\,pc^{-3}$)	Error ($M_\odot\,pc^{-3}$)	Reference
0.185	0.020	Bahcall (1984b)
0.210	0.090	Bahcall (1984c)
0.105	0.015	Bienaymé, Robin & Crézé (1987)
0.260	0.150	Bahcall, et al. (1992)
0.110	0.010	Pham (1997)
0.076	0.015	Crézé et al. (1998)
0.102	0.010	This paper
0.150	0.026	Straight average
0.108	0.011	Variance-weighted average

Holmberg & Flynn (2000)

SURFACE DENSITY MEASUREMENTS

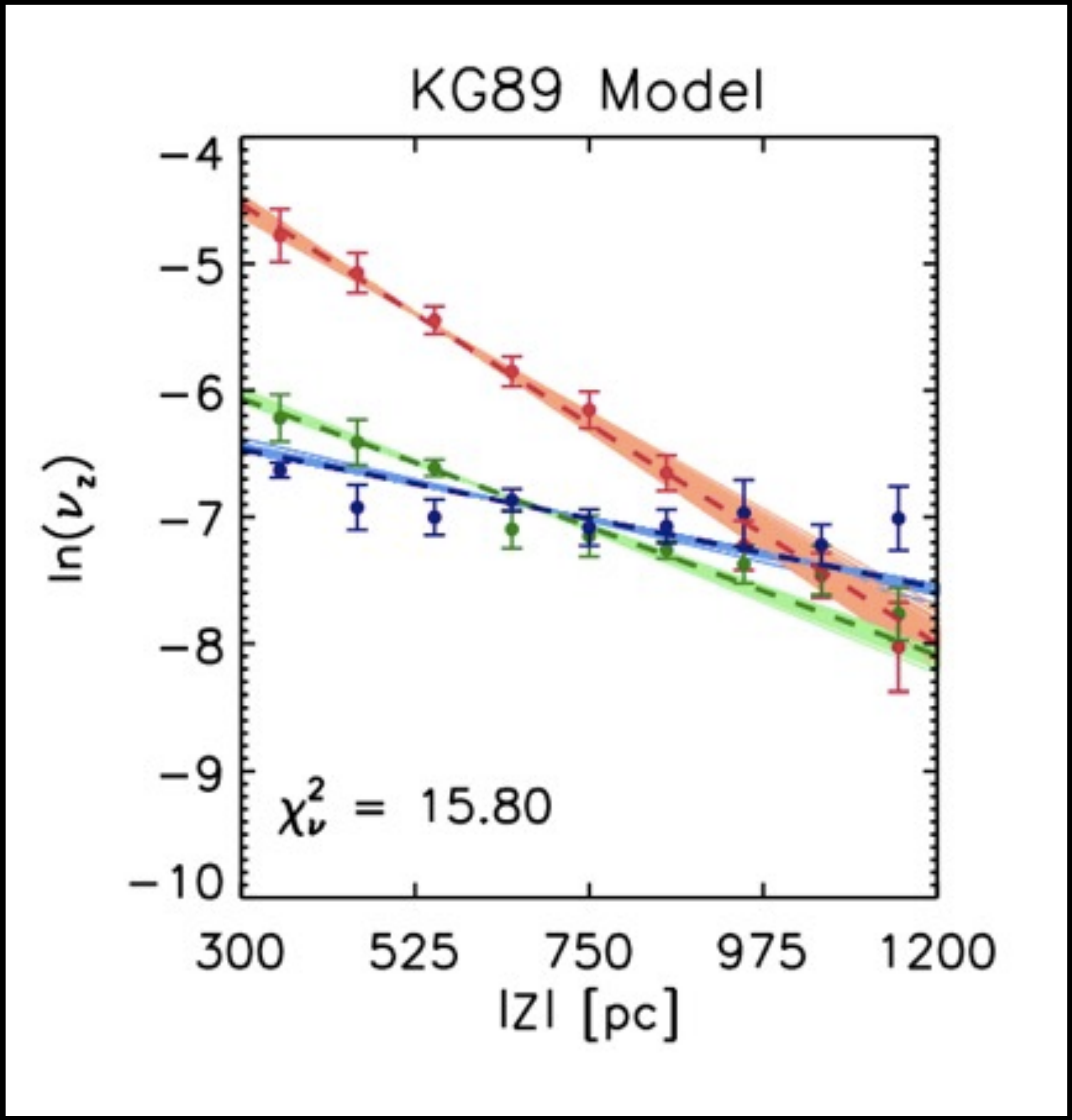
- Similar data as used in Holmberg & Flynn (2000) at larger heights measure the surface density at large heights
- Small, noisy data samples require forward modeling (e.g., Kuijken & Gilmore 1989)
- First modern measurement of Kuijken & Gilmore (1989): star counts and velocities for ~1,000 stars, measures total surface density at 1.1 kpc = 72 +/- 6 $M_\odot\,\mathrm{pc}^{-2}$
- Recent measurements of the vertical dependence of the surface density allow baryons and DM contributions to be separated
- On their own: $\rho_\mathrm{total} = \Sigma_\mathrm{total}/2h_z$ --> h_z ~ 360 pc

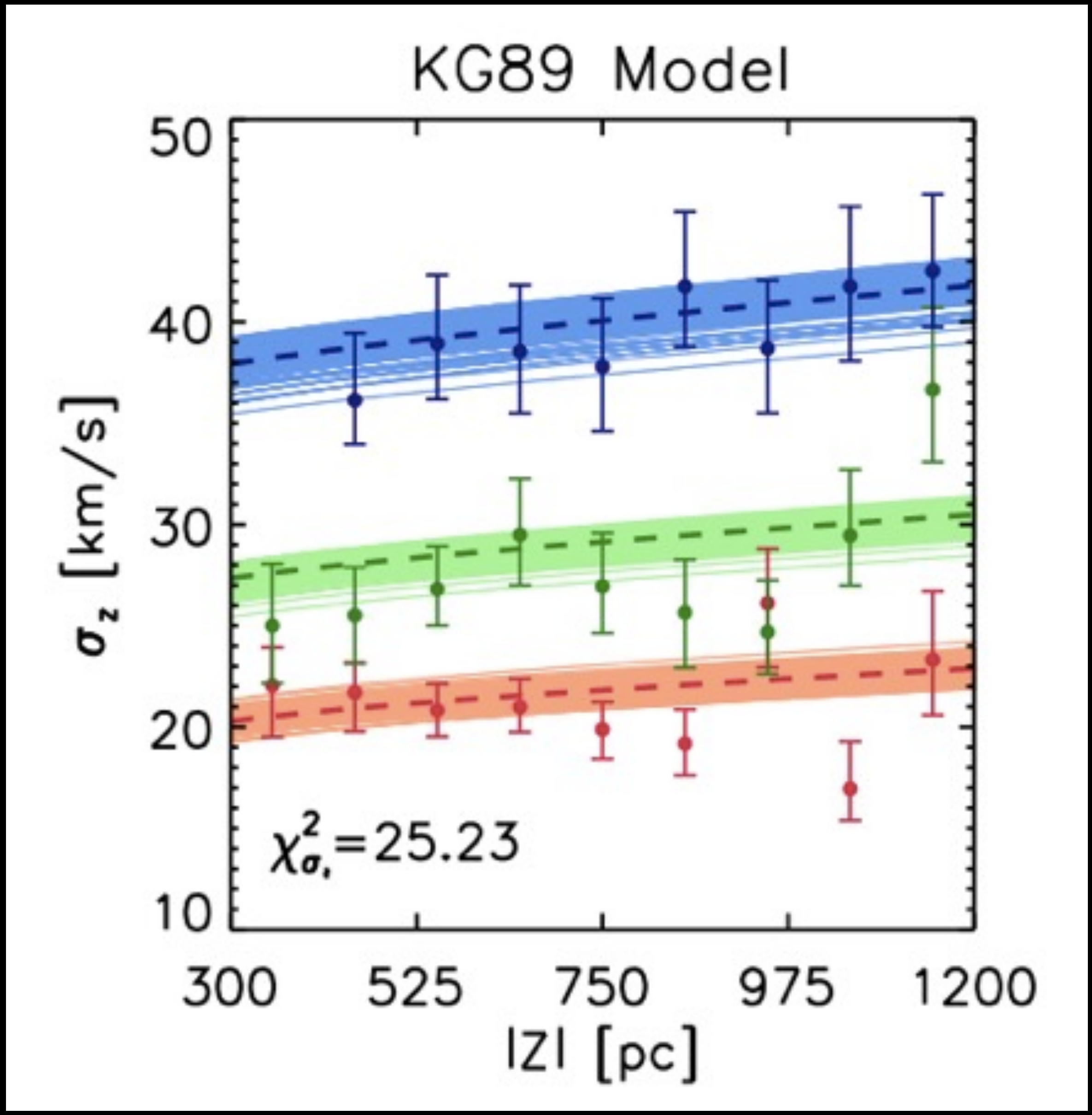
RECENT SURFACE DENSITY MEASUREMENTS

- Recently, new data have allowed the surface-density of matter around 1kpc above the mid-plane to be measured more precisely and with some Z dependence
  - Larger samples with good distances and velocity, and understood selection effects (for determining the stellar profile)
  - Improved understanding of MW disk populations
  - Improved dynamical modeling methods (beyond the Jeans equations)

ONE RECENT ANALYSIS: ZHANG ET AL. (2013)

- Jeans analysis w/ three different populations of stars: young, intermediate-age, old

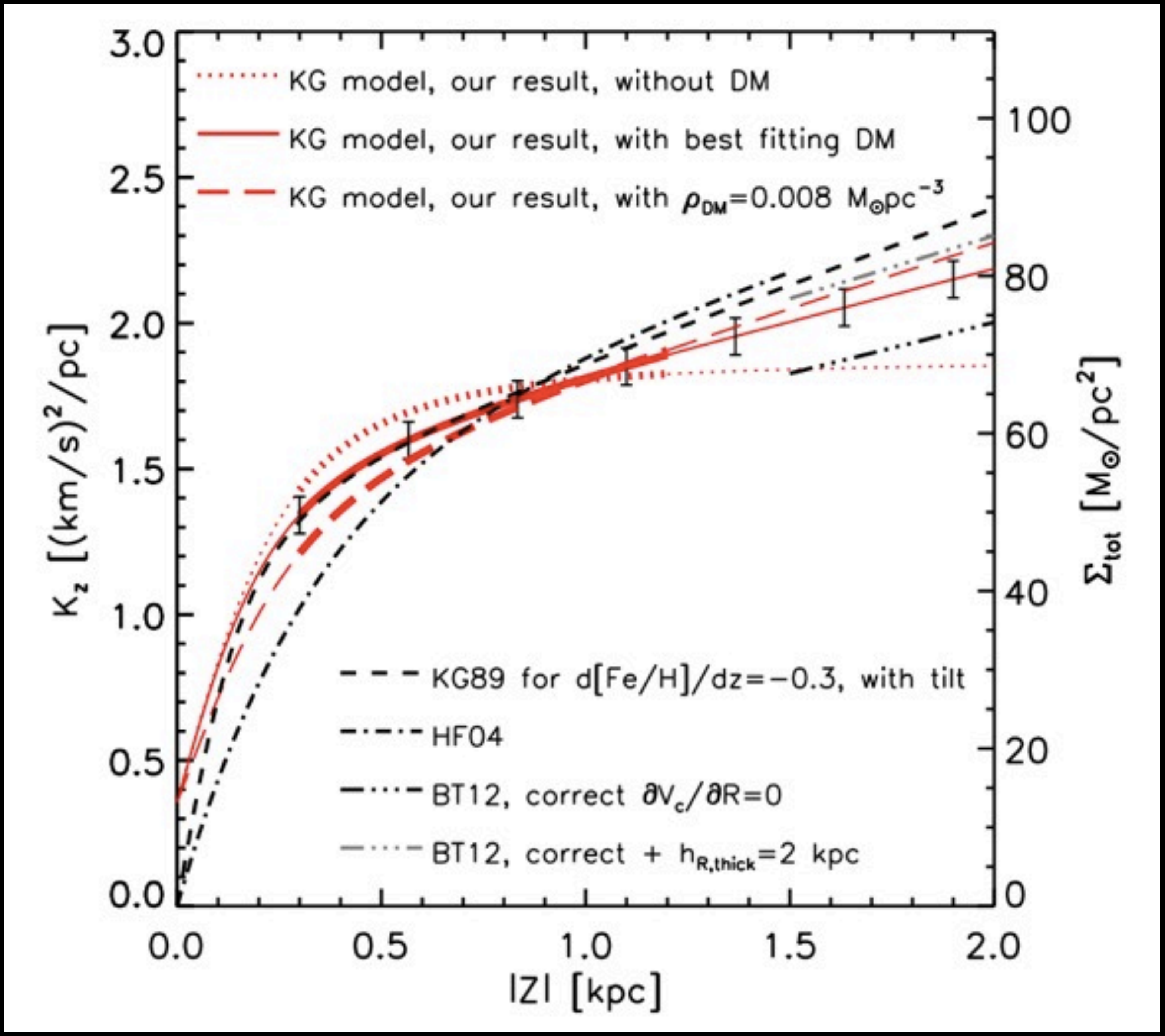




- These should all give the same gravitational potential

Zhang, Rix, van de Ven, Bovy, et al. (2013)

RESULTS FROM JOINT FIT



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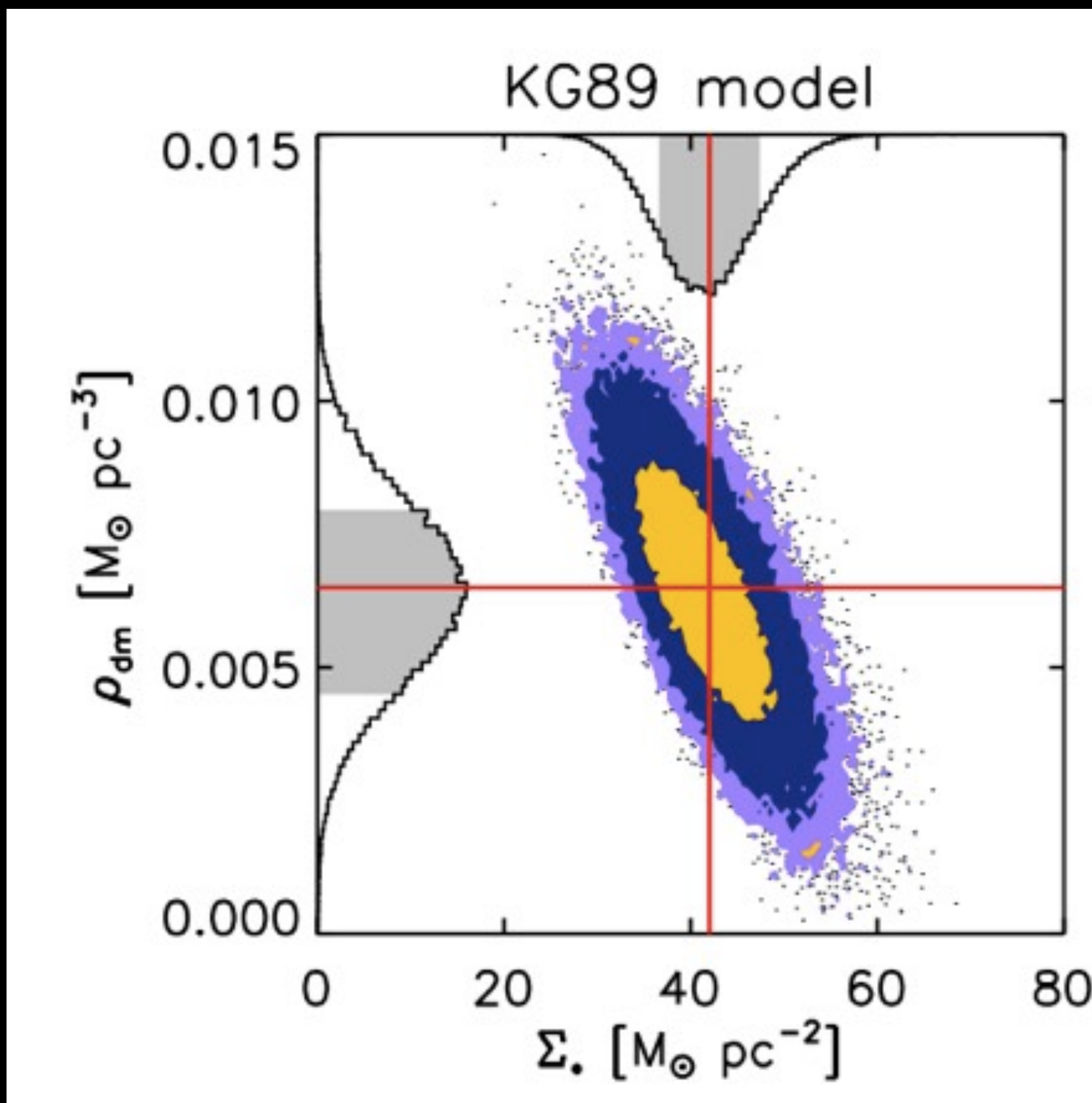

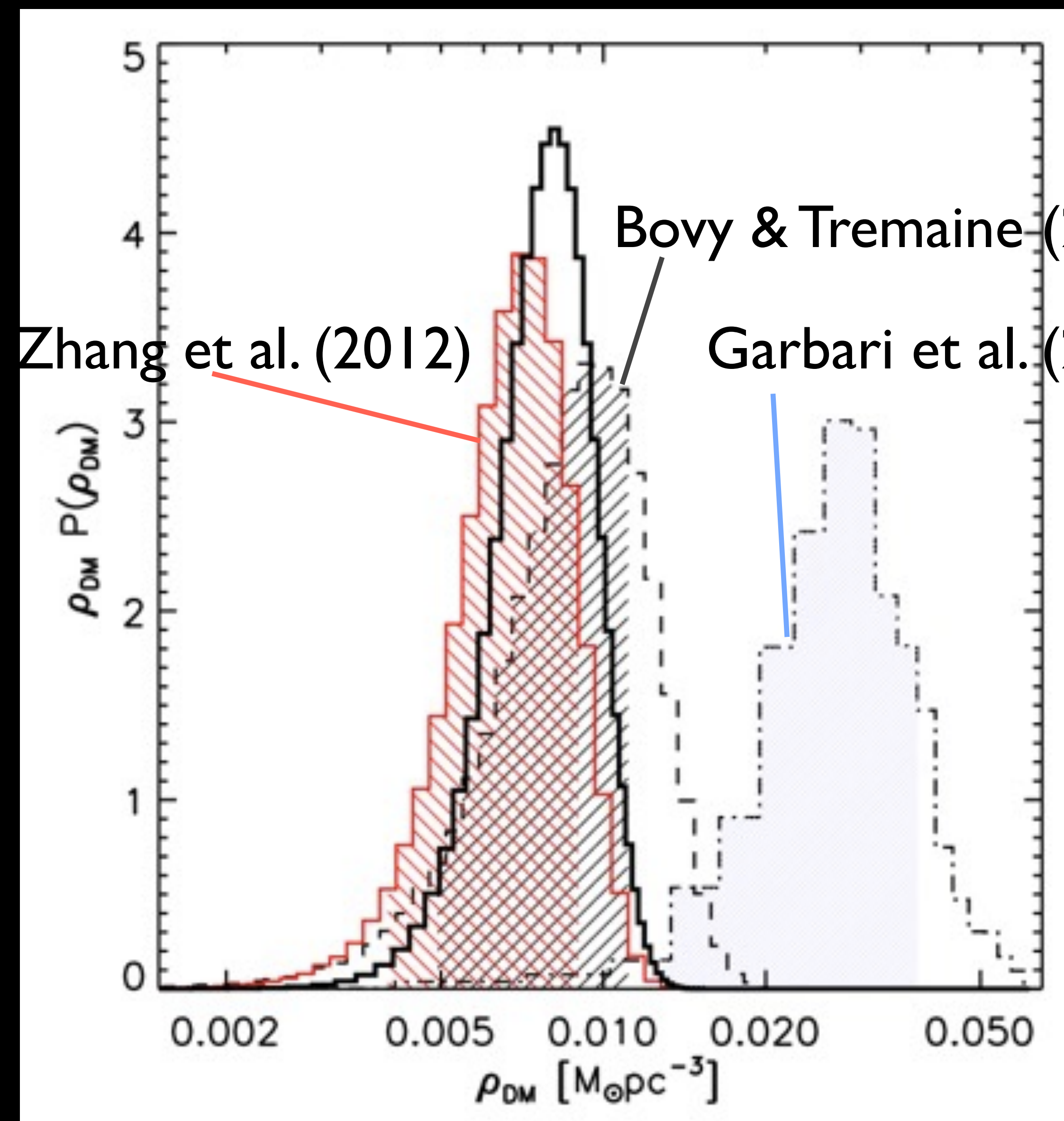

Zhang, Rix, van de Ven, Bovy, et al. (2012)

$$\Sigma(R_0, |z| \leq 1.1\,\mathrm{kpc}) = 69 \pm 6\, M_\odot\,\mathrm{pc}^{-2}$$

+ measurements of DM density and disk surface density

LOCAL MASS BUDGET e.g., Holmberg & Flynn (2000)

- ISM:
  - ~ 3 $M_\odot\,\mathrm{pc}^{-2}$ in molecular gas
  - ~ 8 $M_\odot\,\mathrm{pc}^{-2}$ in HI
  - ~ 2 $M_\odot\,\mathrm{pc}^{-2}$ in ionized gas
  - scale height ~ 100 pc
  - Total uncertainty of a few $M_\odot\,\mathrm{pc}^{-2}$
- Stars:
  - Different populations with different scale heights
  - ~38 +/- a few $M_\odot\,\mathrm{pc}^{-2}$ in stars and stellar remnants
- Dark matter: the rest (72 - 38 - 13) / 2 / 110 pc ≈ 0.01 $M_\odot\,\mathrm{pc}^{-3}$

LOCAL MASS BUDGET, CTD

- ISM + stellar disk:
  - From direct counts: ~50 $M_\odot\,\mathrm{pc}^{-2}$
  - Dynamical estimate: 51 $M_\odot\,\mathrm{pc}^{-2}$
- Dark disk: conservatively < 10 $M_\odot\,\mathrm{pc}^{-2}$
- DDDM: < 1% of dark matter in this sector, scale height must be > 300 pc (otherwise conflict with local density measurement)

OVERVIEW

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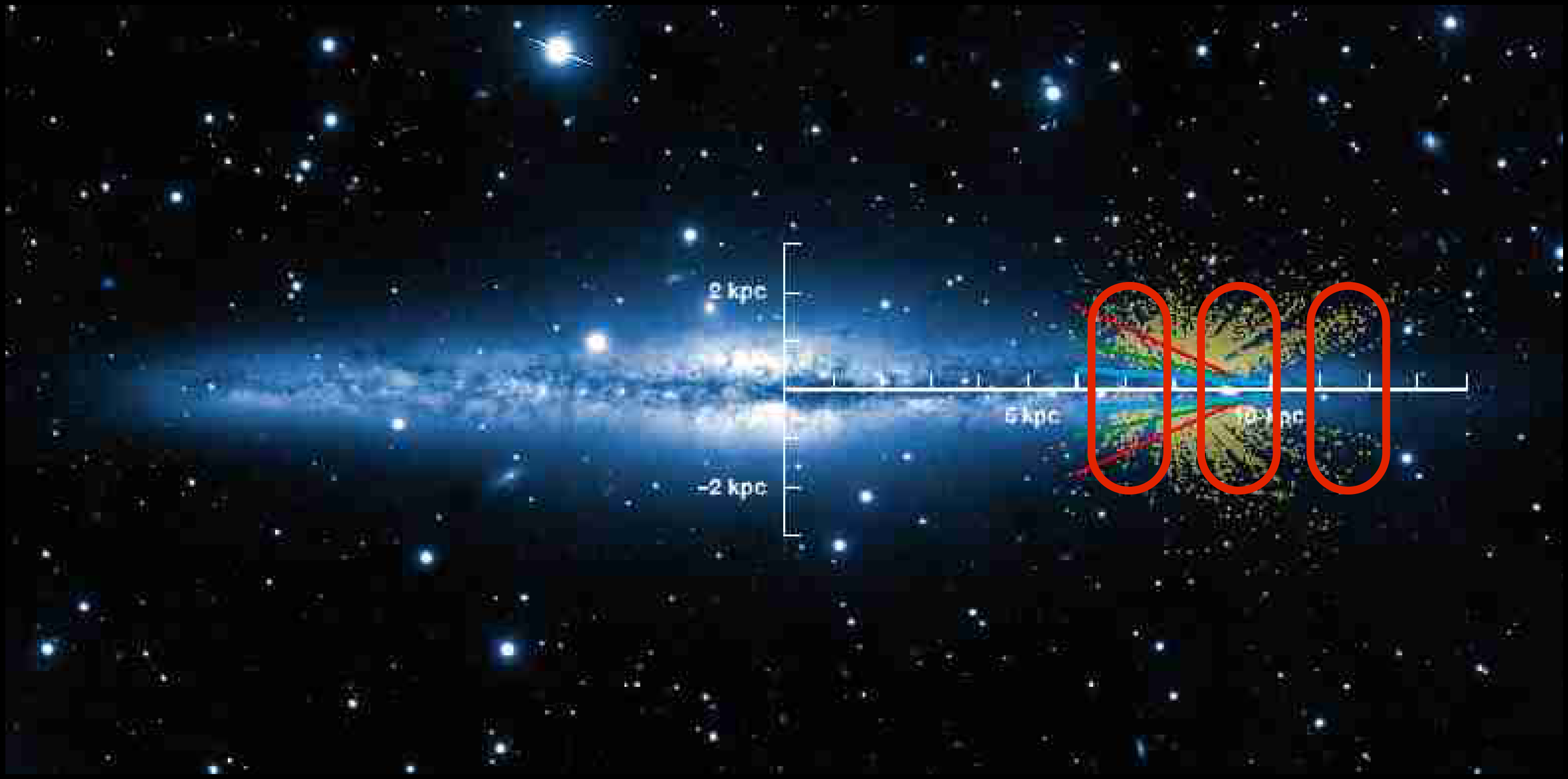
RADIAL DISK AND HALO PROFILES

- We can perform the vertical-force analysis at $R \neq R_0$ => $\Sigma(R)$ and $\rho(R)$

- This will allow us to measure the disk profile (scale length) and infer the halo profile



BEYOND THE JEANS EQUATIONS

- Jeans equations are great: no strong assumptions beyond equilibrium, can measure all ingredients in the Milky Way (in principle)
- In practice applying the Jeans equations is hard:
  - Radial gradients are difficult to measure
  - *gradients* in general are difficult to measure
  - Uncertainties and selection effects not gracefully included
- Using *Jeans theorem* instead (DF[x,v] == DF[integrals]) helps with these problems, but need to carefully choose a general enough family of DFs

DISTRIBUTION FUNCTION MODELING

$$p(\mathbf{x},\mathbf{v}|\mathrm{model}) = \frac{DF(\mathbf{x},\mathbf{v})}{\int \mathrm{d}\mathbf{x}\mathrm{d}\mathbf{v}\, DF(\mathbf{x},\mathbf{v})}$$

- Model the distribution function of stars in x,v as being in a steady state:

$$p(\mathbf{x},\mathbf{v}|\mathrm{model}) = \frac{DF(\mathbf{J}[\mathbf{x},\mathbf{v}])}{\int \mathrm{d}\mathbf{x}\mathrm{d}\mathbf{v}\, DF(\mathbf{J}[\mathbf{x},\mathbf{v}])}$$

- With selection function:

$$p(\mathbf{x},\mathbf{v}|\mathrm{model}) = \frac{DF(\mathbf{J}[\mathbf{x},\mathbf{v}])}{\int \mathrm{d}\mathbf{x}\mathrm{d}\mathbf{v}\, DF(\mathbf{J}[\mathbf{x},\mathbf{v}])\, S(\mathbf{x})}$$

STATE-OF-THE-ART: ACTIONS AS ARGUMENTS

- Jeans theorem: can use any integrals of the motion as the arguments of the DF; often use E, L_z
- Orbital actions are natural integrals to use:
  - Part of canonical (action,angle)=(J,θ) variables where dynamics is *very* simple
  - Jacobian determinant of $(x,v) \rightarrow (J,\theta)$ is unity
  - Adiabatic invariants: natural coordinates to compare orbits in different potentials
  - Simple bounds and simple interpretation: radial and vertical action range from 0 (closed orbits) to infinity (unbound orbits), give extent of radial and vertical excursions

DISK DISTRIBUTION FUNCTION MODELING

Binney (2010), Binney & McMillan (2011)

$$f(J_r, L_z, J_z) = f_{\sigma_r}(J_r, L_z) \times \frac{\nu_z}{2\pi\sigma_z^2} \mathrm{e}^{-\nu_z J_z/\sigma_z^2},$$

where

$$f_{\sigma_r}(J_r, L_z) \equiv \left.\frac{\Omega\Sigma}{\pi\sigma_r^2\kappa}\right|_{R_c} [1 + \tanh(L_z/L_0)]\mathrm{e}^{-\kappa J_r/\sigma_r^2}.$$

- Actions calculated using Staeckel fudge (Binney 2012) in four component model for Milky Way potential (2 exponential disks, bulge, halo)
- Properties of DF:

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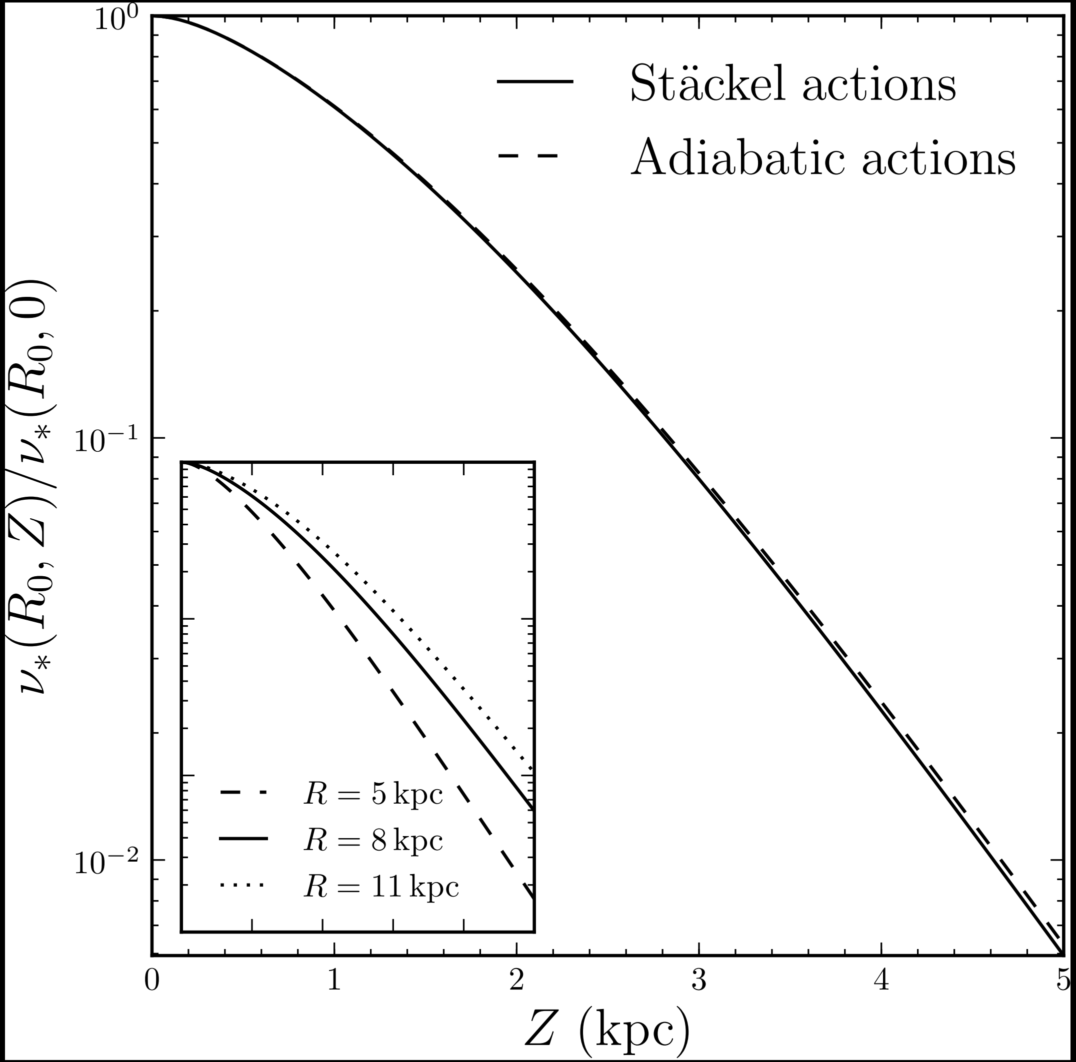

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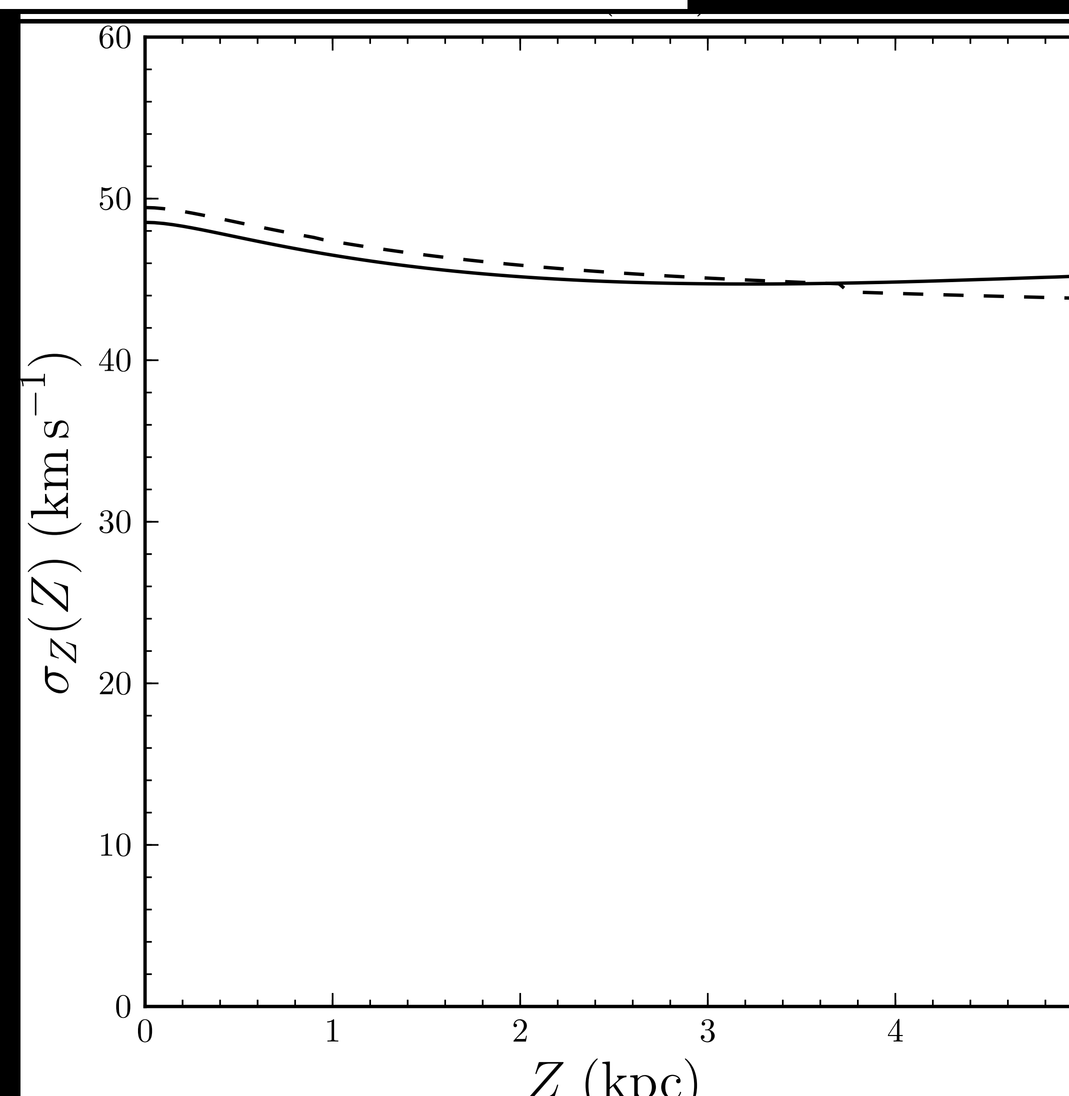

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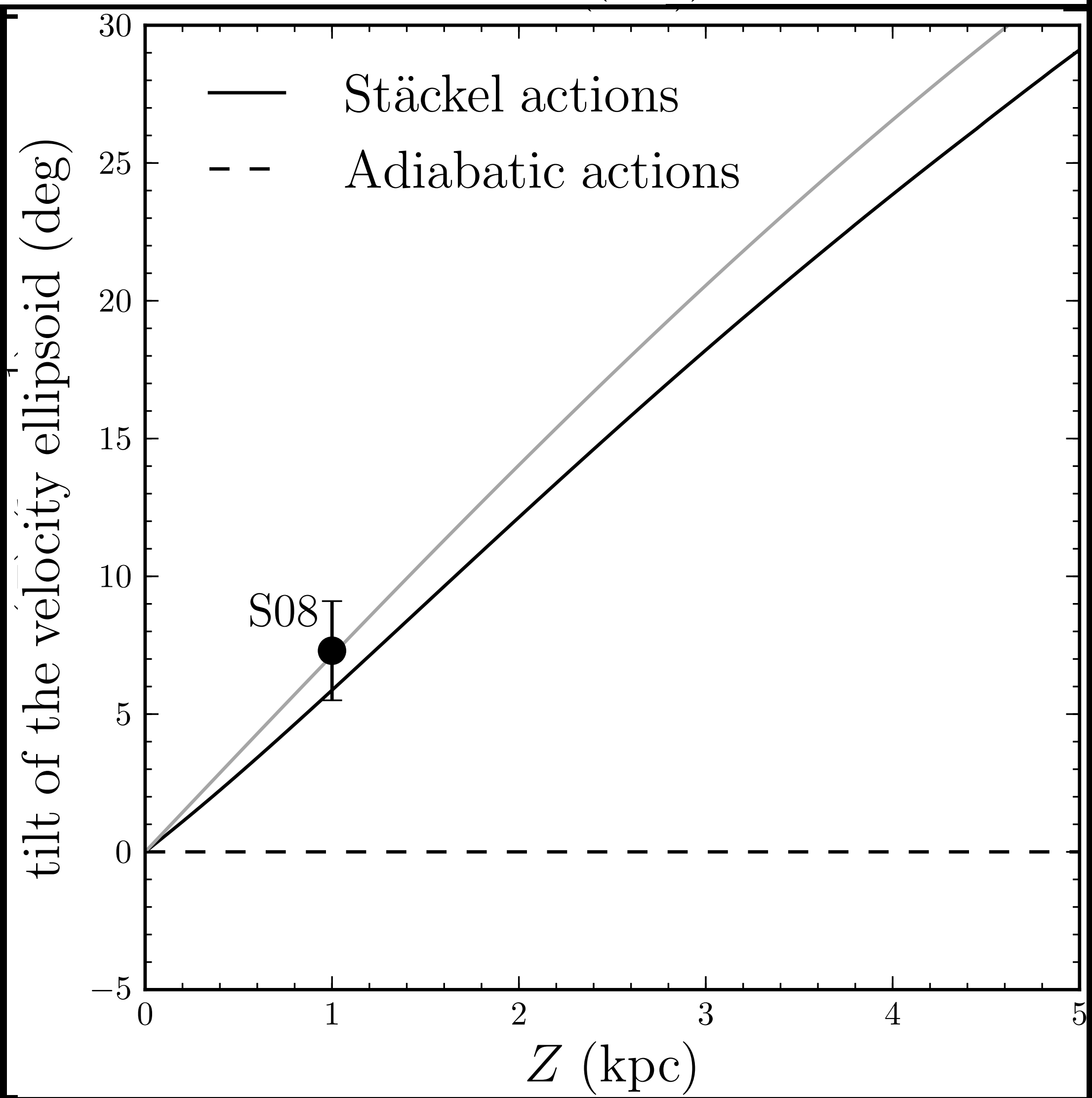


RECENT WORK MAKING USE OF THIS

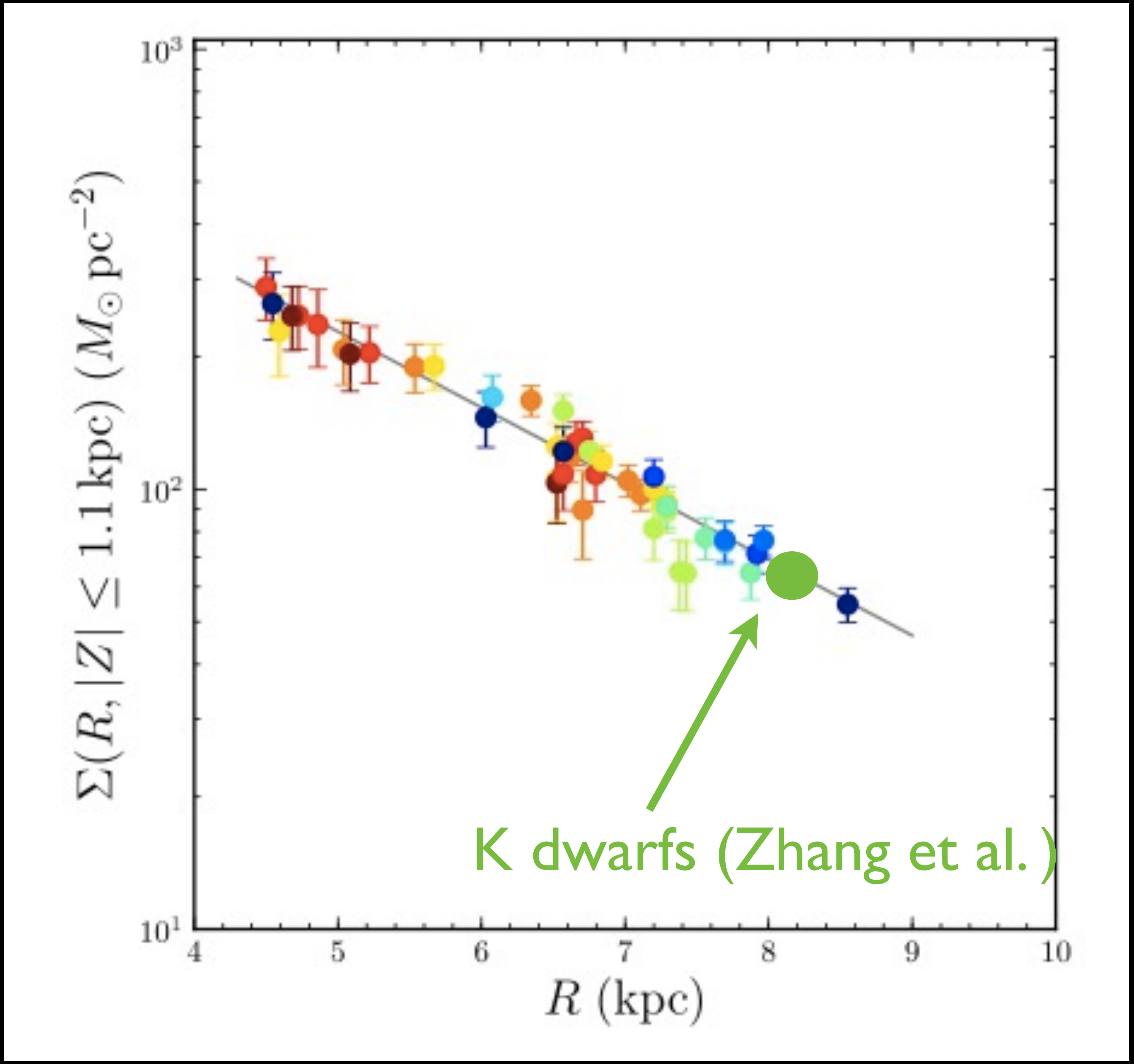
- Modeling the kinematics of the Solar neighborhood: Binney (2010, 2012): In fixed potential, fit a mixture of these basic DFs to the kinematics of stars near the Sun

- Piffl et al. (RAVE; 2014): Modeling the kinematics of ~200,000 stars within about 1.5 kpc from the Sun to constrain the local potential. Tight constraint on the dark matter density: $\rho_{DM}(R_0)$ = 0.48±0.05 GeV/cc

- Bovy & Rix (2013): Modeling the kinematics of ~16,000 SEGUE stars out to ~5 kpc to constrain the potential

SURFACE-DENSITY PROFILE

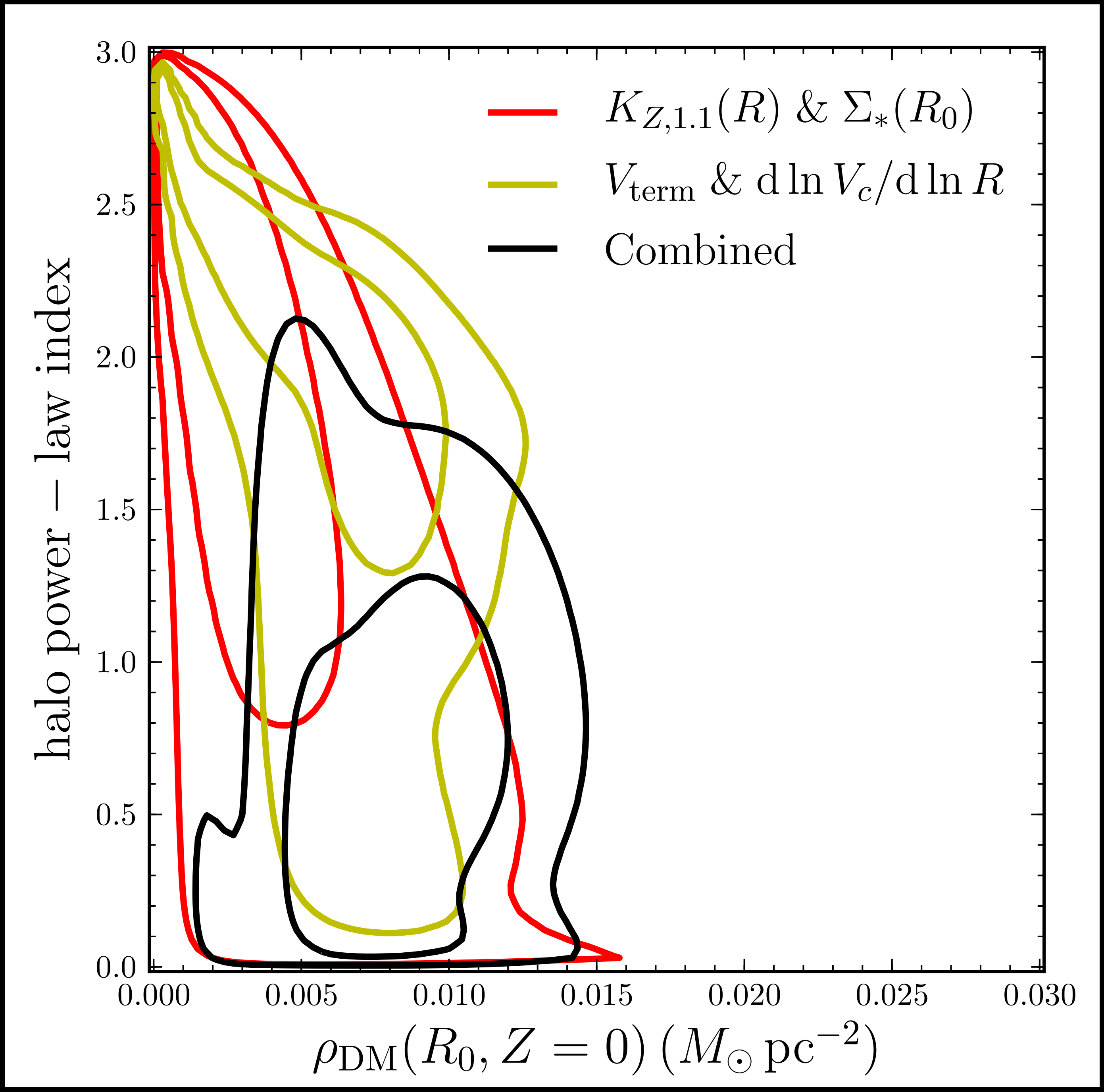
$$\Sigma(R, |z| \le 1.1\,\mathrm{kpc}) = 69\,M_\odot\,\mathrm{pc}^{-2}\,\exp\left[-\frac{R - R_0}{2.5\,\mathrm{kpc}}\right]$$



Bovy & Rix (2013), ApJ, 779, 115

CONSTRAINTS ON THE HALO

- Halo contributes little to rotation curve and surface density at R < 10 kpc
- $\alpha \leq 1.53$ (95 % confidence)



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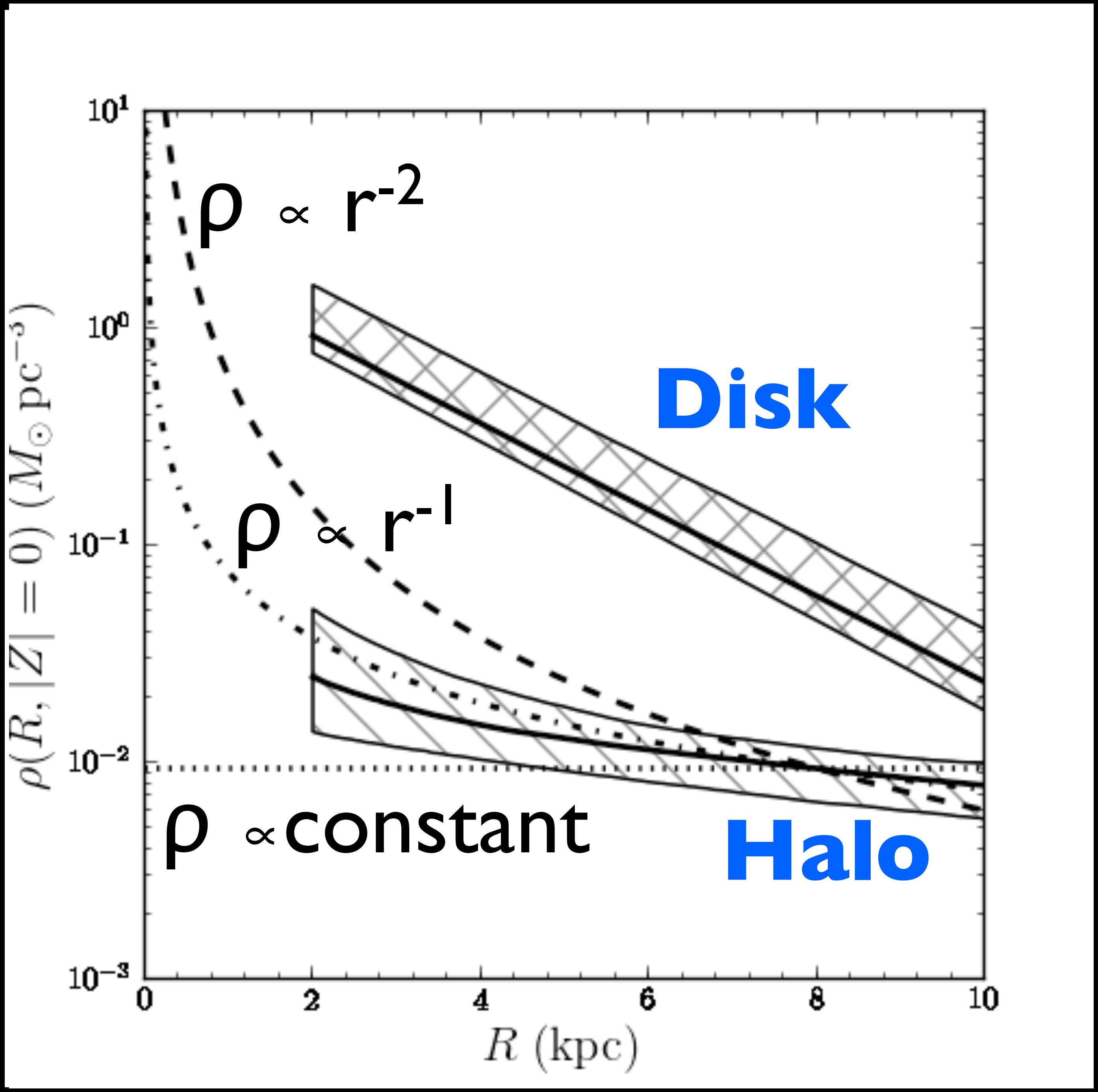

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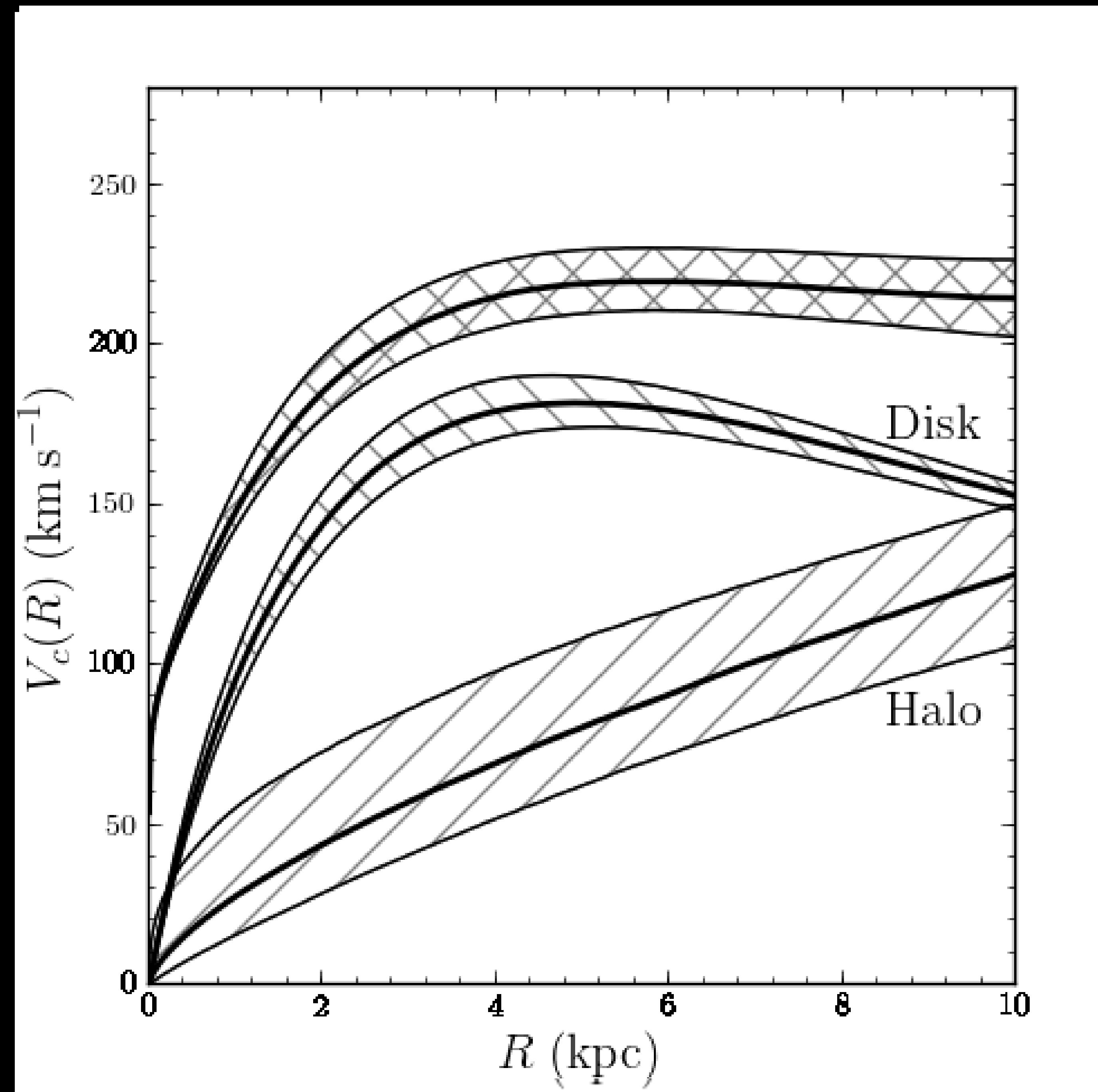
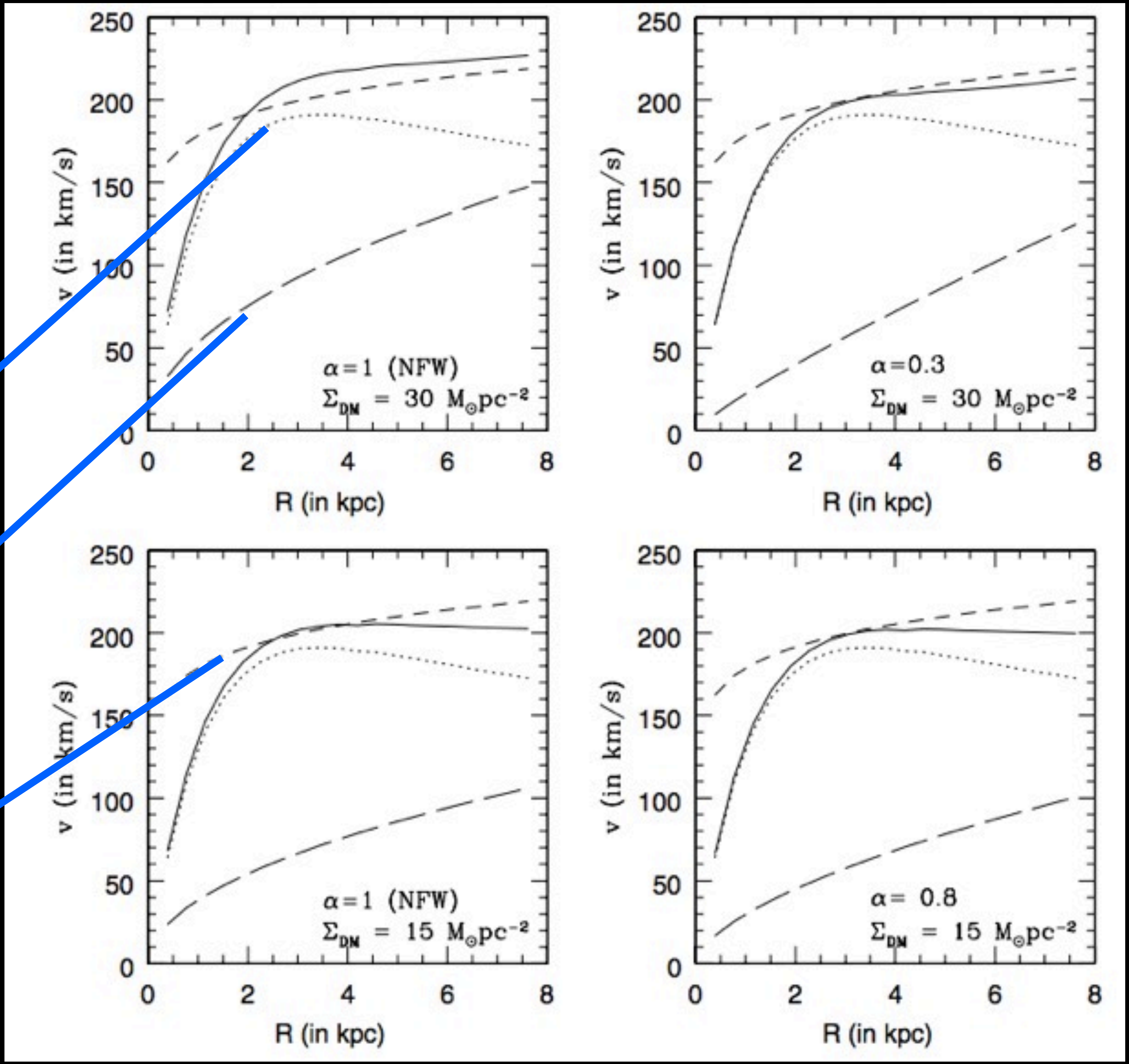


CONSTRAINTS FROM MICROLENSING

- Construct model of inner Galaxy w/ gas and stellar content constrained by optical depth to microlensing

- How much DM is allowed to remain below the observed rotation curve?



Binney & Evans (2001)

Recent updates in the optical depth to microlensing and proper modeling of baryonic components should be taken into account (see Iocco et al. 2011)

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DARK MATTER HALO

- Topic for another whole lecture...
- Techniques:
  - Jeans equations: similar to local DM Jeans equations earlier, but for spherical potential (e.g., Xue et al. 2008)
  - stellar DF modeling: DF(E,L_z) (e.g., Deason et al. 2011)
  - Streams: e.g., Sagittarius: many conflicting results
  - Satellite kinematics (Is Leo I bound?)
- Virial masses between 0.8 and 2 x 10^{12} M_{sun} are commonly reported; some measurements of the slope of the density profile (concentration)

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FUTURE IMPROVEMENTS AND CHALLENGES: GAIA

- Astrometric space mission:
  - parallaxes and proper motions for 1 billion stars out to 10 kpc and beyond, full 6D motions for up 100 million stars
  - Study of stellar populations over large volume of the disk (+spectroscopic surveys)
- Much improved stellar rotation curve with proper motions
- $\Sigma(R,Z)$ at $2 < R < 16$ kpc

SOME REFERENCES:

- *The* book: *Galactic Dynamics*, J. Binney & S. Tremaine, 2nd edition (2008) Princeton University Press

- Some recent reviews:

  - *Dynamics for Galactic Archeology*, J. Binney (2013), *New Astronomy Reviews 57, 29*: Overview of dynamics concepts useful for studying the MW disk (http://arxiv.org/abs/1309.2794)

  - *The local dark matter density*, J. Read (2014), *J. Phys. G.* 41, 063101: Overview of techniques for measuring the local DM density and overview of recent measurements (http://arxiv.org/abs/1404.1938)

  - *The Milky Way's stellar disk*, H.-W. Rix & J. Bovy (2013), *Astron. Astrophys. Rev.* 21, 61: Overview of methods for learning about MW disk stellar populations and of recent progress (http://arxiv.org/abs/1301.3168)

SOME REFERENCES: CIRCULAR VELOCITY CURVE

- van den Hulst, H.C., Muller, C.A., & Oort, J.H. 1954, BAN 12, 117: Original paper with terminal velocity measurements of rotation curve

- Gunn, J.E., Knapp, G.R., & Tremaine, S. 1979, AJ 84, 1181: Good terminal-velocity reference

- Kerr, F.J. & Lynden-Bell, D.1986, MNRAS 221, 1023: IAU standard $V_c(R_0)$ = 220 km/s

- Feast, M. & Whitelock, P. 1998, MNRAS 291, 683: Measurement of the Oort constants (combination of $V_c(R_0)/R_0$ and its derivative) from *Hipparcos* data

SOME REFERENCES: CIRCULAR VELOCITY CURVE: RECENT PROGRESS

- Reid, M.J. et al. 2009, ApJ 700, 137 and update 2014, ApJ 783, 130: Measurement of $V_c(R_0)$ and $V_c(R)$ from the kinematics of masers in the MW disk; some residual dependence on Sun's motion wrt $V_c(R_0)$ (http://arxiv.org/abs/1401.5377)

- Bovy, J. et al. 2012, ApJ 759, 131: Measurement of $V_c(R_0)$ and $V_c(R)$ from the kinematics of intermediate-age stars in the MW disk, *first measurement that is independent of the unknown value of the Sun's velocity wrt $V_c(R_0)$*: $V_c(R_0) = 218 \pm 6$ km/s (http://arxiv.org/abs/1209.0759)

SOME REFERENCES: LOCAL DARK MATTER

- Bovy, J. & Tremaine, S. 2012, ApJ 756, 89: at large heights above the plane, so largely unaffected by uncertainty in baryonic mass distribution, first 3σ detection; finds $\rho_{DM}(R_0)$ = 0.3±0.1 GeV/cc (http://arxiv.org/abs/1205.4033)

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