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Summer School in Particle and Astroparticle Physics

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NEUTRINO PHYSICS
(II LECTURE)

ALESSANDRO MIRIZZI

(University of Bari & Sez. INFN Bari)

NEUTRINO MIXING

If neutrino have masse, then in general the mass eigenstates will not be the same as the weak-interaction eigenstates. The leptonic charged current interactions will possess a unitary matrix U, analogously to the CKM quark mixing matrix.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

The neutral-current interactions are unaffected, since they are flavor conserving. For antineutrinos $\nu \rightarrow \overline{\nu}$ and U $\rightarrow$ U*

ν masses are much smaller than charged-fermion masses. ν propagation very different from typical charged-fermion propagation. When a ν is produced via EW interactions, it is a flavor eigenstate. After production, it propagates in its mass eigenstates. Because these bases are not the same, the ν flavor will not be conserved by propagation. Since mass is small, the change in flavor can occur on macroscopic distances and thus be observable.

The neutrino wave equation

For propagation of ultrarelativistic neutrinos the full spin structure is not probed. Weak-interactions couple only to l.h. component of ν. For ultrarelativistic particles, chirality conservation is good to order (m/E). For E>>m, only the propagation of ν_L is relevant.
Eliminating the spin structure one gets the Klein-Gordon equation, whether the ν is a Dirac or Majorana particle.

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2\right)|\nu\rangle = 0 \qquad \text{(in mass basis)}$$

If we imagine neutrinos to be produced with a fixed energy E at some source, their wave functions vary as e^{-iEt} so that their spatial propagation is governed by the equation

$$\left(-E^2 - \partial_x^2 + m^2\right)|\nu\rangle = 0$$

where we have reduced the spatial variation to one dimension, i.e. we consider plane waves.

In the relativistic limit $E \gg m^2_j$ we may linearize this wave equation by virtue of the decomposition

$$(-E^2 - \partial_x^2) = -(E + i\partial_x)(E - i\partial_x)$$

Since $-i\partial_x \nu_j = p_j \nu_j$ with $p_j = (E^2 - m_j^2)^{1/2} \simeq E$

it is enough to keep the differential in the sum term, while replacing it with E in the difference, leading to

$$(-E^2 - \partial_x^2) \to -2E(E + i\partial_x)$$

Therefore, in the relativistic limit the evolution along the beam is governed by a Schrödinger-type equation

$$i\partial_x|\nu\rangle = (-E + \Omega)|\nu\rangle, \quad \Omega = \frac{m^2}{2E}$$

In the mass basis H is diagonal

$$H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \simeq -E + \begin{pmatrix} m_1^2/2E & 0 \\ 0 & m_2^2/2E \end{pmatrix}$$

Solution to the wave equation (2ν)

The solution for the ν wave function can be written for two neutrino species in mass basis as

$$\begin{pmatrix} |\nu_1(t)\rangle \\ |\nu_2(t)\rangle \end{pmatrix} = \begin{pmatrix} e^{-iE_1t} & 0 \\ 0 & e^{-iE_2t} \end{pmatrix} \begin{pmatrix} |\nu_1(0)\rangle \\ |\nu_2(0)\rangle \end{pmatrix}$$

$$E_i = (p^2 + m_i^2)^{1/2}$$

Neutrinos are produced and detected via weak interactions. The physical quantity that one observes is the flavor at the production and detection position. Using the mixing matrix

$$U = \begin{pmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{pmatrix}$$

we can write the solution of the ν wave-function that describes ν production and detection

If we produce an electron neutrino, the probability of detecting this neutrino as an electron neutrino after a time t,

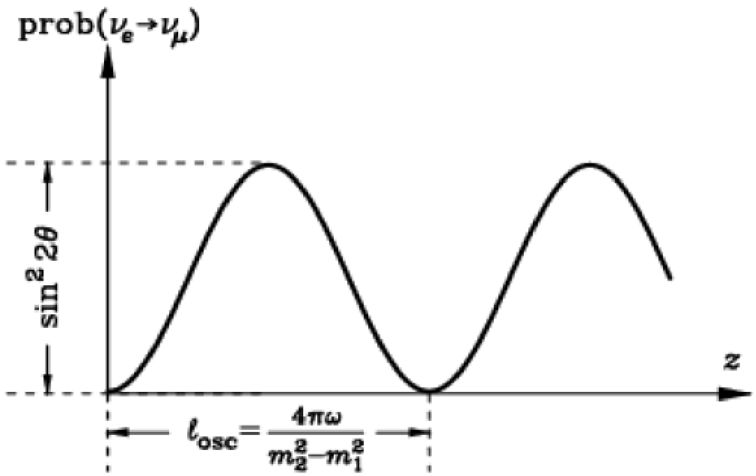
$P(\nu_e \to \nu_e) = |\langle \nu_e(t)|\nu_e(0)\rangle|^2$ can be written as

$$
\begin{aligned}
P(\nu_e \to \nu_e) &= |(1\ \ 0)\begin{pmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{pmatrix}\begin{pmatrix} e^{-iE_1 t} & 0 \\ 0 & e^{-iE_2 t} \end{pmatrix} \\
&\times \begin{pmatrix} C_\theta & -S_\theta \\ S_\theta & C_\theta \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}|^2 \\
&= 1 - \frac{1}{2}\sin^2 2\theta[1 - \cos(E_2 - E_1)t]
\end{aligned}
$$

Flavor oscillations !!

We can define a wavelength for the oscillations

$$
\begin{aligned}
E_2 - E_1 &= (m_2^2 + p^2)^{1/2} - (m_1^2 + p^2)^{1/2} \\
&\approx \Delta m^2/2p \equiv 2\pi/\lambda
\end{aligned}
$$



We can often set $\beta \approx 1$, $t \approx z$, $d/dt \approx d/dz$

$$\boxed{P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}}$$

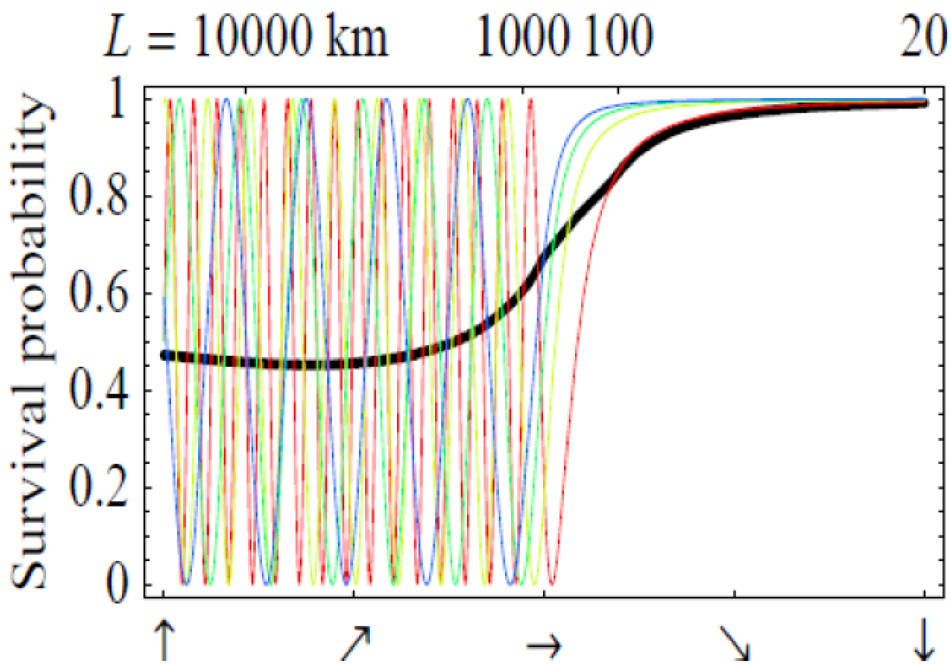
A convenient numerical relation is found restoring h/2pi and c:

$$S_{ij} \equiv \sin^2 \frac{c^3}{\hbar} \frac{\Delta m^2 L}{4E} = \sin^2 1.27 \frac{\Delta m^2}{\mathrm{eV}^2} \frac{L}{\mathrm{km}} \frac{\mathrm{GeV}}{E}$$

The oscillation wave-length is

$$\lambda = \frac{4\pi E}{\Delta m^2} = 2.48\ \mathrm{km} \frac{E}{\mathrm{GeV}} \frac{\mathrm{eV}^2}{\Delta m^2} \Rightarrow$$ Macroscopic !!

- The classical probability



In a realistic setup, the neutrino beam is not monochromatic, and the energy resolution of the detector is not perfect: one need to average the oscillation probability around some energy range ΔE. Furthermore, the productions and detection regions are not points: one need to average around some path-length range ΔL

Phase information is lost

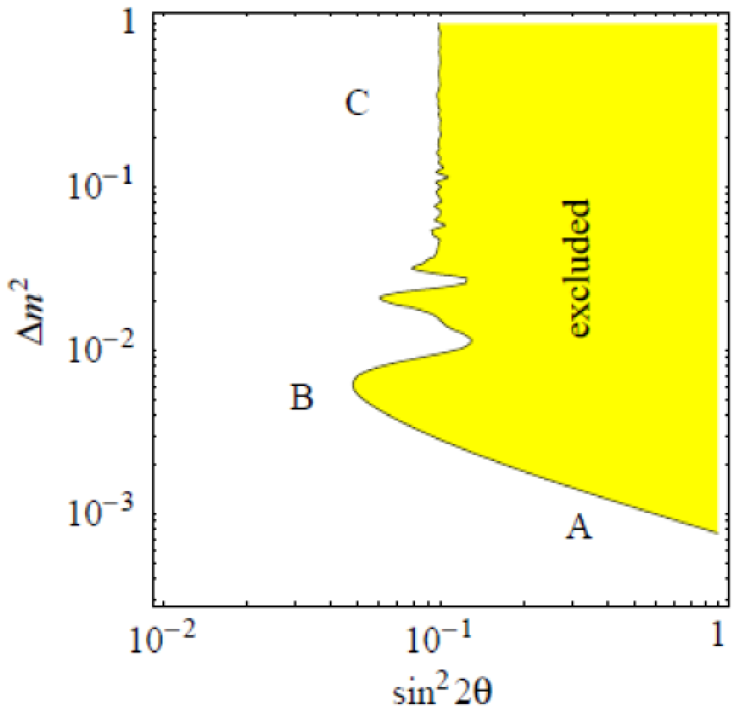
If the phase information is lost, then the probability is just the classical probability. The classical probability can be calculated without using the equation for neutrino propagation. The phase acquired during neutrino propagation is averaged out; so we can sum incoherently over the propagation eigenstates, the mass eigenstates

$$
\begin{aligned}
P(\nu_e \to \nu_e) &= \sum_i P(\nu_e \to \nu_i) P(\nu_i \to \nu_e) \\
&= (1 \;\; 0) \begin{pmatrix} C_\theta{}^2 & S_\theta{}^2 \\ S_\theta{}^2 & C_\theta{}^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&\times \begin{pmatrix} C_\theta{}^2 & S_\theta{}^2 \\ S_\theta{}^2 & C_\theta{}^2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
&= 1 - \frac{1}{2} \sin^2 2\theta
\end{aligned}
$$

The classical probability is the product of the magnitudes squared instead of the the magnitude squared of the product

Limiting regimes

Iso-P_{ee} contour



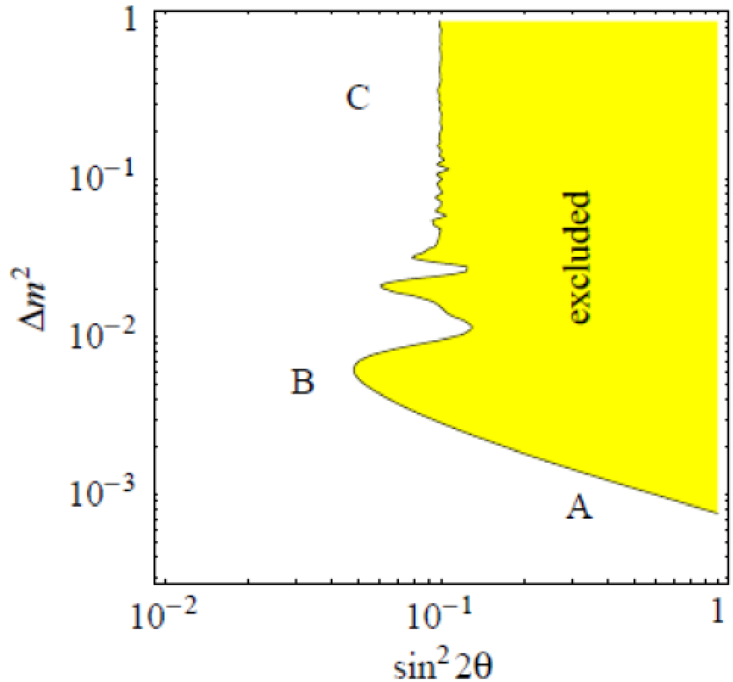
Octant symmetry

$$P_{e\mu}(\theta) = P_{e\mu}(\pi/2 - \theta)$$

A) Oscillation with short base-line. $S_{ij} \ll 1$

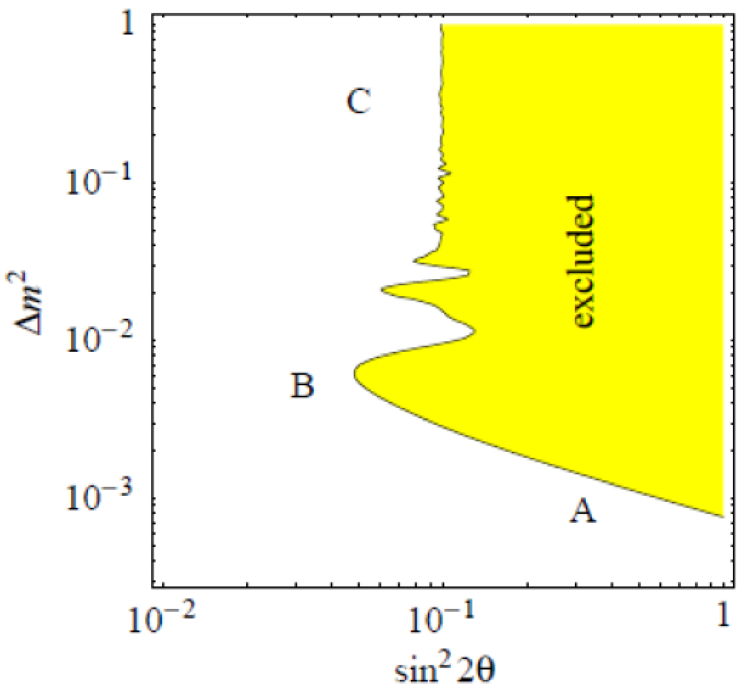
$$P(\nu_e \to \nu_\mu) \simeq (H_{e\mu} L)^2 \quad \text{with} \quad H_{e\mu} = \frac{\Delta m^2}{E} \sin 2\theta$$

$P(\nu_e \to \nu_\mu) \propto L^2$ but $\Phi_\nu \propto 1/L^2$ Necessary a compromise



C) Averaged oscillations $\langle S_{ij} \rangle \approx 1/2$

$$P(\nu_e \to \nu_\mu) = \frac{1}{2} \sin^2 2\theta$$



B) Intermediate region. Due to the uncertainty ΔE on the energy (and possibly on the path-length), coherence gets lost when neutrinos of different energies have too different oscillation phases $\phi \sim \Delta m^2 L/E$, i.e. when

$$\Delta\phi \simeq \frac{\Delta E}{E}\phi \gtrsim 1$$

Therefore one can see n~E/ΔE oscillations before they average out

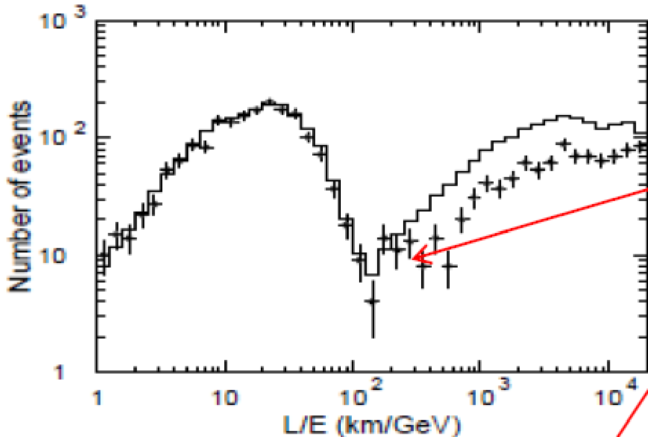


FIG. 3: Number of events as a function of L/E for the data (points) and the atmospheric neutrino MC events without oscillations (histogram). The MC is normalized by the detector live-time.

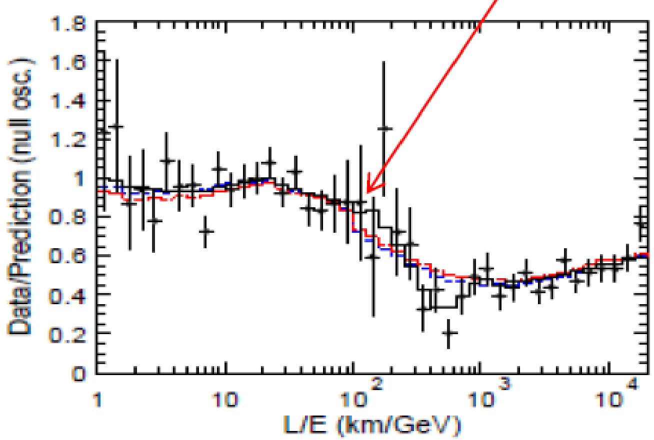


FIG. 4: Ratio of the data to the MC events without neutrino oscillation (points) as a function of the reconstructed L/E together with the best-fit expectation for 2-flavor $\nu_\mu \leftrightarrow \nu_\tau$ oscillations (solid line). The error bars are statistical only. Also shown are the best-fit expectation for neutrino decay (dashed line) and neutrino decoherence (dotted line).

Evidence of oscillation dip in atmospheric $\nu_\mu \rightarrow \nu_\tau$ oscillation (Super-Kamiokande)

Evidence for an oscillatory signature in atmospheric neutrino oscillation

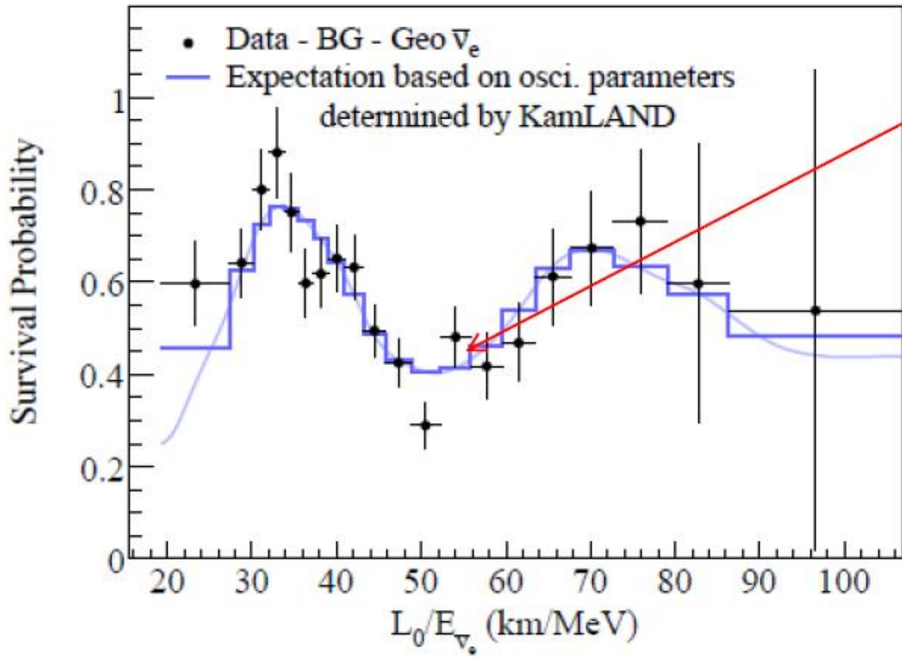
Y.Ashie,[1] J.Hosaka,[1] K.Ishihara,[1] Y.Itow,[1] J.Kameda,[1] Y.Koshio,[1] A.Minamino,[1] C.Mitsuda,[1] M.Miura,[1] S.Moriyama,[1] M.Nakahata,[1] T.Namba,[1] R.Nambu,[1] Y.Obayashi,[1] M.Shiozawa,[1] Y.Suzuki,[1] Y.Takeuchi,[1] K.Taki,[1] S.Yamada,[1] M.Ishitsuka,[2] T.Kajita,[2] K.Kaneyuki,[2] S.Nakayama,[2] A.Okada,[2] K.Okumura,[2] T.Ooyabu,[2] C.Saji,[2] Y.Takenaga,[2] S.Desai,[3] E.Kearns,[3] S.Likhoded,[3] J.L.Stone,[3] L.R.Sulak,[3] C.W.Walter,[3] W.Wang,[3] M.Goldhaber,[4] D.Casper,[5] J.P.Cravens,[5] W.Gajewski,[5] W.R.Kropp,[5] D.W.Liu,[5] S.Mine,[5] M.B.Smy,[5] H.W.Sobel,[5] C.W.Sterner,[5] M.R.Vagins,[5] K.S.Ganezer,[6] J.Hill,[6] W.E.Keig,[6] J.S.Jang,[7] J.Y.Kim,[7] I.T.Lim,[7] R.W.Ellsworth,[8] S.Tasaka,[9] C.Guillian,[10] A.Kibayashi,[10] J.G.Learned,[10] S.Matsuno,[10] D.Takemori,[10] M.D.Messier,[11] Y.Hayato,[12] A.K.Ichikawa,[12] T.Ishida,[12] T.Ishii,[12] T.Iwashita,[12] T.Kobayashi,[12] T.Maruyama,[12,*] K.Nakamura,[12] K.Nitta,[12] Y.Oyama,[12] M.Sakuda,[12] Y.Totsuka,[12] A.T.Suzuki,[13] M.Hasegawa,[14] K.Hayashi,[14] T.Inagaki,[14] I.Kato,[14] H.Maesaka,[14] T.Morita,[14] T.Nakaya,[14] K.Nishikawa,[14] T.Sasaki,[14] S.Ueda,[14] S.Yamamoto,[14] T.J.Haines,[15,5] S.Dazeley,[16] S.Hatakeyama,[16] R.Svoboda,[16] E.Blaufuss,[17] J.A.Goodman,[17] G.W.Sullivan,[17] D.Turcan,[17] K.Scholberg,[18] A.Habig,[19] Y.Fukuda,[20] C.K.Jung,[21] T.Kato,[21] K.Kobayashi,[21] M.Malek,[21] C.Mauger,[21] C.McGrew,[21] A.Sarrat,[21] E.Sharkey,[21] C.Yanagisawa,[21] T.Toshito,[22] K.Miyano,[23] N.Tamura,[23] J.Ishii,[24] Y.Kuno,[24] Y.Nagashima,[24] M.Takita,[24] M.Yoshida,[24] S.B.Kim,[25] J.Yoo,[25] H.Okazawa,[26] T.Ishizuka,[27] Y.Choi,[28] H.K.Seo,[28] Y.Gando,[29] T.Hasegawa,[29] K.Inoue,[29] J.Shirai,[29] A.Suzuki,[29] M.Koshiba,[30] Y.Nakajima,[31] K.Nishijima,[31] T.Harada,[32] H.Ishino,[32] R.Nishimura,[32] Y.Watanabe,[32] D.Kielczewska,[33,5] J.Zalipska,[33] H.G.Berns,[34] R.Gran,[34] K.K.Shiraishi,[34] A.Stachyra,[34] K.Washburn,[34] and R.J.Wilkes[34]

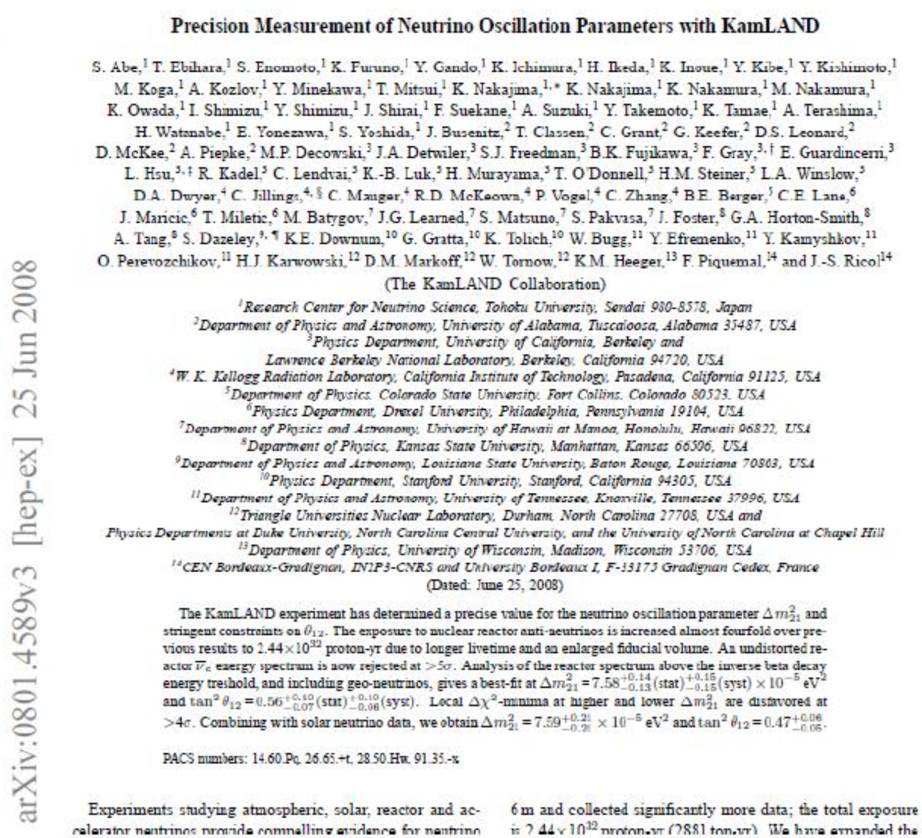
(The Super-Kamiokande Collaboration)

[1] Kamioka Observatory, Institute for Cosmic Ray Research, University of Tokyo, Kamioka, Gifu, 506-1205, Japan
[2] Research Center for Cosmic Neutrinos, Institute for Cosmic Ray Research, University of Tokyo, Kashiwa, Chiba 277-8582, Japan
[3] Department of Physics, Boston University, Boston, MA 02215, USA
[4] Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA
[5] Department of Physics and Astronomy, University of California, Irvine, Irvine, CA 92697-4575, USA
[6] Department of Physics, California State University, Dominguez Hills, Carson, CA 90747, USA
[7] Department of Physics, Chonnam National University, Kwangju 500-757, Korea
[8] Department of Physics, George Mason University, Fairfax, VA 22030, USA
[9] Department of Physics, Gifu University, Gifu, Gifu 501-1193, Japan
[10] Department of Physics and Astronomy, University of Hawaii, Honolulu, HI 96822, USA
[11] Department of Physics, Indiana University, Bloomington, IN 47405-7105, USA
[12] High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan
[13] Department of Physics, Kobe University, Kobe, Hyogo 657-8501, Japan
[14] Department of Physics, Kyoto University, Kyoto 606-8502, Japan
[15] Physics Division, P-23, Los Alamos National Laboratory, Los Alamos, NM 87544, USA
[16] Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA
[17] Department of Physics, University of Maryland, College Park, MD 20742, USA
[18] Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
[19] Department of Physics, University of Minnesota, Duluth, MN 55812-2496, USA
[20] Department of Physics, Miyagi University of Education, Sendai,Miyagi 980-0845, Japan
[21] Department of Physics and Astronomy, State University of New York, Stony Brook, NY 11794-3800, USA
[22] Department of Physics, Nagoya University, Nagoya, Aichi 464-8602, Japan
[23] Department of Physics, Niigata University, Niigata, Niigata 950-2181, Japan
[24] Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan
[25] Department of Physics, Seoul National University, Seoul 151-742, Korea
[26] International and Cultural Studies, Shizuoka Seika College, Yaizu, Shizuoka 425-8611, Japan
[27] Department of Systems Engineering, Shizuoka University, Hamamatsu, Shizuoka 432-8561, Japan
[28] Department of Physics, Sungkyunkwan University, Suwon 440-746, Korea
[29] Research Center for Neutrino Science, Tohoku University, Sendai, Miyagi 980-8578, Japan
[30] University of Tokyo, Tokyo 113-0033, Japan
[31] Department of Physics, Tokai University, Hiratsuka, Kanagawa 259-1292, Japan
[32] Department of Physics, Tokyo Institute for Technology, Meguro, Tokyo 152-8551, Japan
[33] Institute of Experimental Physics, Warsaw University, 00-681 Warsaw, Poland
[34] Department of Physics, University of Washington, Seattle, WA 98195-1560, USA

(Dated: February 4, 2008)

Evidence of oscillation dip in reactor $\overline{\nu}_e$ oscillations (KamLAND)





arXiv:0801.4589v3 [hep-ex] 25 Jun 2008

Precision Measurement of Neutrino Oscillation Parameters with KamLAND

S. Abe,[1] T. Ebihara,[1] S. Enomoto,[1] K. Furuno,[1] Y. Gando,[1] K. Ichimura,[1] H. Ikeda,[1] K. Inoue,[1] Y. Kibe,[1] Y. Kishimoto,[1] M. Koga,[1] A. Kozlov,[1] Y. Minekawa,[1] T. Mitsui,[1] K. Nakajima,[1,*] K. Nakajima,[1] K. Nakamura,[1] M. Nakamura,[1] K. Owada,[1] I. Shimizu,[1] Y. Shimizu,[1] J. Shirai,[1] F. Suekane,[1] A. Suzuki,[1] Y. Takemoto,[1] K. Tamae,[1] A. Terashima,[1] H. Watanabe,[1] E. Yonezawa,[1] S. Yoshida,[1] J. Busenitz,[2] T. Classen,[2] C. Grant,[2] G. Keefer,[2] D.S. Leonard,[2] D. McKee,[2] A. Piepke,[2] M.P. Decowski,[3] J.A. Detwiler,[3] S.J. Freedman,[3] B.K. Fujikawa,[3] F. Gray,[3,†] E. Guardincerri,[3] L. Hsu,[3,‡] R. Kadel,[3] C. Lendvai,[3] K.-B. Luk,[3] H. Murayama,[3] T. O'Donnell,[3] H.M. Steiner,[3] L.A. Winslow,[3] D.A. Dwyer,[4] C. Jillings,[4,§] C. Mauger,[4] R.D. McKeown,[4] P. Vogel,[4] C. Zhang,[4] B.E. Berger,[5] C.E. Lane,[6] J. Maricic,[6] T. Miletic,[6] M. Batygov,[7] J.G. Learned,[7] S. Matsuno,[7] S. Pakvasa,[7] J. Foster,[8] G.A. Horton-Smith,[8] A. Tang,[8] S. Dazeley,[9,¶] K.E. Downum,[10] G. Gratta,[10] K. Tolich,[10] W. Bugg,[11] Y. Efremenko,[11] Y. Kamyshkov,[11] O. Perevozchikov,[11] H.J. Karwowski,[12] D.M. Markoff,[12] W. Tornow,[12] K.M. Heeger,[13] F. Piquemal,[14] and J.-S. Ricol[14]

(The KamLAND Collaboration)

[1]*Research Center for Neutrino Science, Tohoku University, Sendai 980-8578, Japan*
[2]*Department of Physics and Astronomy, University of Alabama, Tuscaloosa, Alabama 35487, USA*
[3]*Physics Department, University of California, Berkeley and Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[4]*W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125, USA*
[5]*Department of Physics, Colorado State University, Fort Collins, Colorado 80523, USA*
[6]*Physics Department, Drexel University, Philadelphia, Pennsylvania 19104, USA*
[7]*Department of Physics and Astronomy, University of Hawaii at Manoa, Honolulu, Hawaii 96822, USA*
[8]*Department of Physics, Kansas State University, Manhattan, Kansas 66506, USA*
[9]*Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803, USA*
[10]*Physics Department, Stanford University, Stanford, California 94305, USA*
[11]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*
[12]*Triangle Universities Nuclear Laboratory, Durham, North Carolina 27708, USA and Physics Departments at Duke University, North Carolina Central University, and the University of North Carolina at Chapel Hill*
[13]*Department of Physics, University of Wisconsin, Madison, Wisconsin 53706, USA*
[14]*CEN Bordeaux-Gradignan, IN2P3-CNRS and University Bordeaux I, F-33175 Gradignan Cedex, France*

(Dated: June 25, 2008)

The KamLAND experiment has determined a precise value for the neutrino oscillation parameter Δm^2_{21} and stringent constraints on θ_{12}. The exposure to nuclear reactor anti-neutrinos is increased almost fourfold over previous results to 2.44×10^{32} proton-yr due to longer livetime and an enlarged fiducial volume. An undistorted reactor $\overline{\nu}_e$ energy spectrum is now rejected at $>5\sigma$. Analysis of the reactor spectrum above the inverse beta decay energy treshold, and including geo-neutrinos, gives a best-fit at $\Delta m^2_{21} = 7.58^{+0.14}_{-0.13}(\mathrm{stat})^{+0.15}_{-0.15}(\mathrm{syst}) \times 10^{-5}\ \mathrm{eV}^2$ and $\tan^2\theta_{12} = 0.56^{+0.10}_{-0.07}(\mathrm{stat})^{+0.10}_{-0.06}(\mathrm{syst})$. Local $\Delta\chi^2$-minima at higher and lower Δm^2_{21} are disfavored at $>4\sigma$. Combining with solar neutrino data, we obtain $\Delta m^2_{21} = 7.59^{+0.21}_{-0.21} \times 10^{-5}\ \mathrm{eV}^2$ and $\tan^2\theta_{12} = 0.47^{+0.06}_{-0.05}$.

PACS numbers: 14.60.Pq, 26.65.+t, 28.50.Hw, 91.35.-x

Experiments studying atmospheric, solar, reactor and accelerator neutrinos provide compelling evidence for neutrino

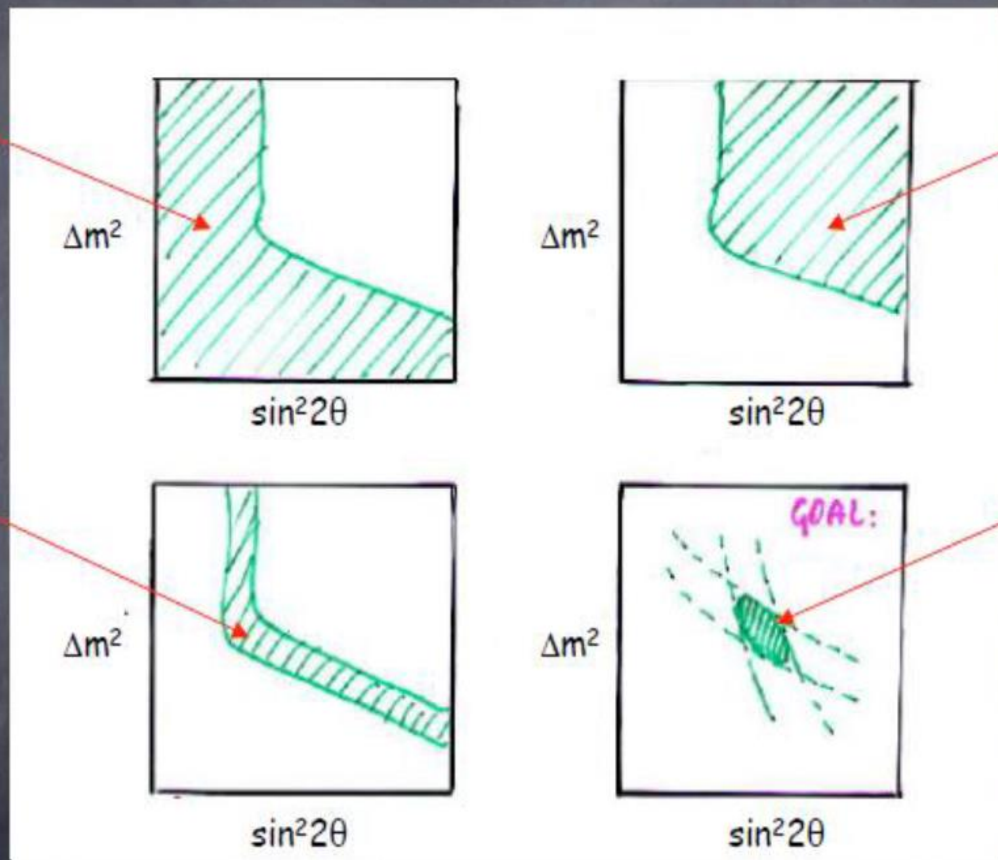
6 m and collected significantly more data; the total exposure is 2.44×10^{32} proton-yr (2881 ton-yr). We have expanded the

Typical experimental results

allowed

negative result: $P_{\alpha\beta} < c$

positive result: $P_{\alpha\beta} > c$



accurate positive result: $P_{\alpha\beta} = c \pm \Delta c$

several accurate results

(more exps. or 1 exp. with good spectral data)

THREE-FLAVOR NEUTRINO VACUUM OSCILLATIONS

PGD Parametrization

$$U = \mathcal{O}(\theta_{23})\Gamma\mathcal{O}(\theta_{13})\mathcal{O}(\theta_{12})$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{23} & S_{23} \\ 0 & -S_{23} & C_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\delta} & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix}$$

$$\times \begin{pmatrix} C_{13} & 0 & S_{13} \\ 0 & 1 & 0 \\ -S_{13} & 0 & C_{13} \end{pmatrix} \begin{pmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

δ =CP-violation phase

$$P(\nu_\alpha \to \nu_\beta) = |\sum_i U_{\beta i} \exp(-iE_i t) U^*_{\alpha i}|^2$$

$$= \sum_i |U_{\beta i}|^2 |U_{\alpha i}|^2 \longrightarrow \text{Classical probability}$$

$$+ Re \sum_{i \neq j} U_{\beta i} U^*{}_{\beta j} U^*{}_{\alpha i} U_{\alpha j}$$

$$\times \exp[-i\Delta m^2_{ij} L/2E] \longrightarrow \text{Phase information}$$

NEUTRINO REFRACTION IN MEDIUM

Interactions modify the effective mass that a particle exhibits while traveling through a medium. A well-known example is photon, which is massless in vacuum, but develops an effective mass in medium. One usually defines a refractive index n_{refr} which relates wavenumber and frequency of a particle by

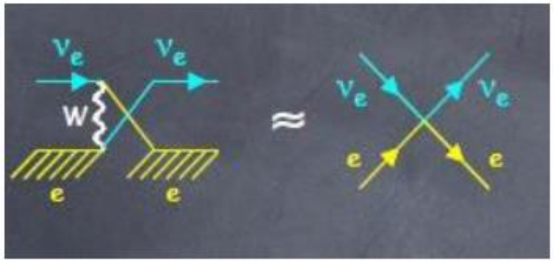
$$k = n_{refr}\,\omega$$

Refraction in a medium arises from the interference of the incoming wave with the scattered waves in the forward direction. Therefore, the refractive index is given in terms of the forward-scattering amplitude f_0 by

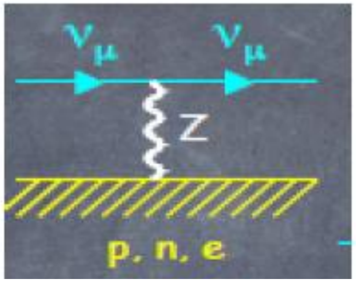
$$n_{\rm refr} = 1 + \frac{2\pi}{\omega^2} n f_0(\omega)$$

where *n* is the number density of the scattering targets. This formula applies to any particle propagating in a medium, except that f_0 must be calculated according to the interactions of that particle with the medium constituents.

We turn now to the neutrino dispersion relation in media. Usually we are concerned with very low energies. Therefore, we may take the low-energy limit of the weak-interaction Hamiltonian, contracting the propagator for the massive gauge bosons to the 4-fermion approximation



Turning now to NC contributions, we find in an exactly similar way the following contributions to effective energies of both ν_e and ν_μ



The neutral current contribution is the same for all the flavors whereas the charged current contribution affects only ν_e

This difference of interaction energies produces an effective potential for the ν_e

$$\boxed{V = \sqrt{2} G_F n_e}$$

EQUATIONS OF MOTION IN MATTER

In flavor basis, where the neutrino potential is diagonal, the propagation equation for two flavors is

Wolfenstein equation

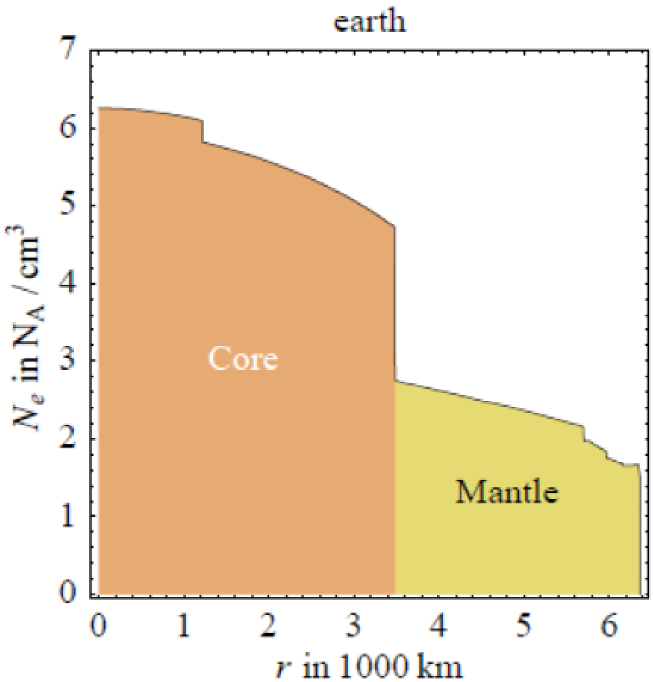
$$i\frac{d}{dx}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \frac{\mathcal{M}^2}{2E}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

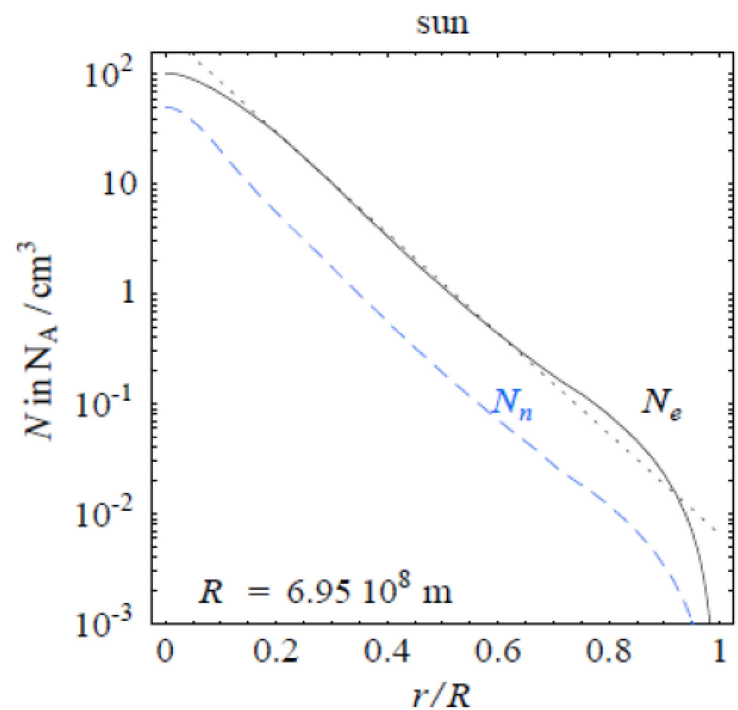
$$\mathcal{M}^2 = U\begin{pmatrix} m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}U^\dagger + \begin{pmatrix} A & 0\\ 0 & 0\end{pmatrix}$$

$$A = \frac{2\sqrt{2}G_F n_e E}{[\mathrm{eV}^2]} = 1.54\times 10^{-4}\frac{n_e}{[\mathrm{mol/cm^3}]}\frac{E}{[\mathrm{GeV}]}$$

For antineutrinos, $A \rightarrow -A$, $U \rightarrow U^*$

EXAMPLES OF DENSITY PROFILES





TWO-FLAVOR SOLUTIONS TO THE WAVE EQUATIONS

- Eigenvalues and eigenfunctions for fixed density

The effect of matter on neutrino propagation is contained in the parameter *A*, given by

$$A \equiv 2\sqrt{2} G_F n_e E = 2\sqrt{2} G_F (Y_e/m_n) \rho E$$

Y_e electron fraction

The ν propagation equations can be rewritten as:

$$i\frac{d}{dx}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{1}{4E}\left((\Sigma + A) + \begin{pmatrix} A - \Delta C_{2\theta} & \Delta S_{2\theta} \\ \Delta S_{2\theta} & -A + \Delta C_{2\theta} \end{pmatrix}\right)\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

where

$$\Sigma = m_2^2 + m_1^2, \;\; \Delta = m_2^2 - m_1^2, \;\; C_{2\theta} = \cos 2\theta, \;\; S_{2\theta} = \sin 2\theta$$

M^2 can be diagonalized to find instantaneous eigenvalues and eigenstates

$$U_m^\dagger \mathcal{M}^2 U_m \equiv \begin{pmatrix} M_1^2 & 0 \\ 0 & M_2^2 \end{pmatrix}$$

The eigenvalues of the mass matrix M^2 are

$$M^2_{2,1} = \{(\Sigma + A) \pm [(A - \Delta C_{2\theta})^2 + (\Delta S_{2\theta})^2]^{1/2}\}/2$$

The mixing matrix in medium, U_m, can be parametrized analogous to the vacuum mixing parametrization

$$\begin{pmatrix} \nu^m{}_1 \\ \nu^m{}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_m & -\sin\theta_m \\ \sin\theta_m & \cos\theta_m \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \equiv U^{m\dagger} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

ν_1^m , ν_2^m are the mass eigenstates in medium

Θ_m is the neutrino mixing angle in matter given by

$$\tan 2\theta_m = \frac{\Delta \sin 2\theta}{(-A + \Delta \cos 2\theta)}$$

or, equivalently,

$$\sin^2 2\theta_m = \frac{(\Delta \sin 2\theta)^2}{[(A - \Delta \cos 2\theta)^2 + (\Delta \sin 2\theta)^2]}$$

EFFECTIVE MIXING ANGLE IN MATTER

$$\sin^2 2\theta_m = \frac{(\Delta \sin 2\theta)^2}{[(A - \Delta \cos 2\theta)^2 + (\Delta \sin 2\theta)^2]}$$

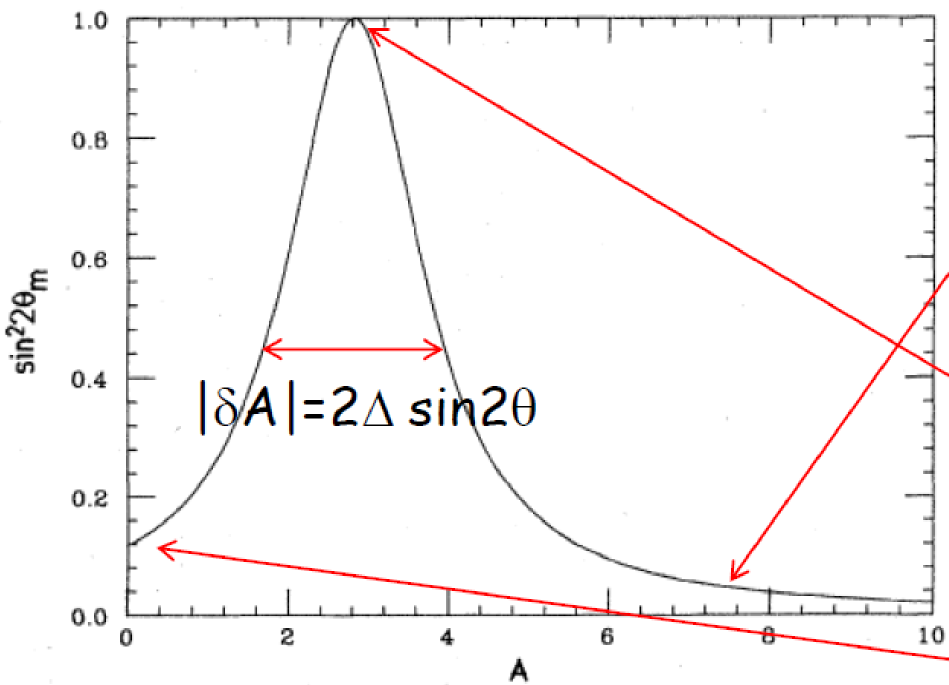


FIG. 2. Plot of $\sin^2 2\theta_m$, where θ_m is the effective neutrino mixing angle in matter, as a function of A, the induced electron-neutrino mass. Here we take $m_2^2 - m_1^2 = 3.0$ and $\sin^2\theta = 0.03$.

Matter dominance

$A \gg \Delta$, $\theta_m \longrightarrow \pi/2$

Resonance

$A = \Delta\cos 2\theta$, $\theta_m \longrightarrow \pi/4$

maximal mixing

Vacuum

$A \longrightarrow 0$, $\theta_m \longrightarrow \theta$

For constant matter density, the vacuum solution can be converted to the solution in a medium by replacing the vacuum quantities with their corresponding medium quantities.

The induced mass A can be converted into a characteristic length scale for the medium

$$\lambda_0 \equiv 4\pi E/A = \sqrt{2}\pi/(G_F n_e)$$

For a medium with ρ=1 g/cm^3 and $Y_e = \frac{1}{2}$, $\lambda_0 = 2 \times 10^9$ cm.

The ν wavelength in matter is given by

$$\begin{aligned}\lambda_m &= 4\pi E/(M_2^2 - M_1^2)\\ &= \lambda/[S_{2\theta}^2 + (A/\Delta - C_{2\theta})^2]^{1/2}\\ &= \lambda/[S_{2\theta}^2 + (\lambda/\lambda_0 - C_{2\theta})^2]^{1/2}\end{aligned}$$

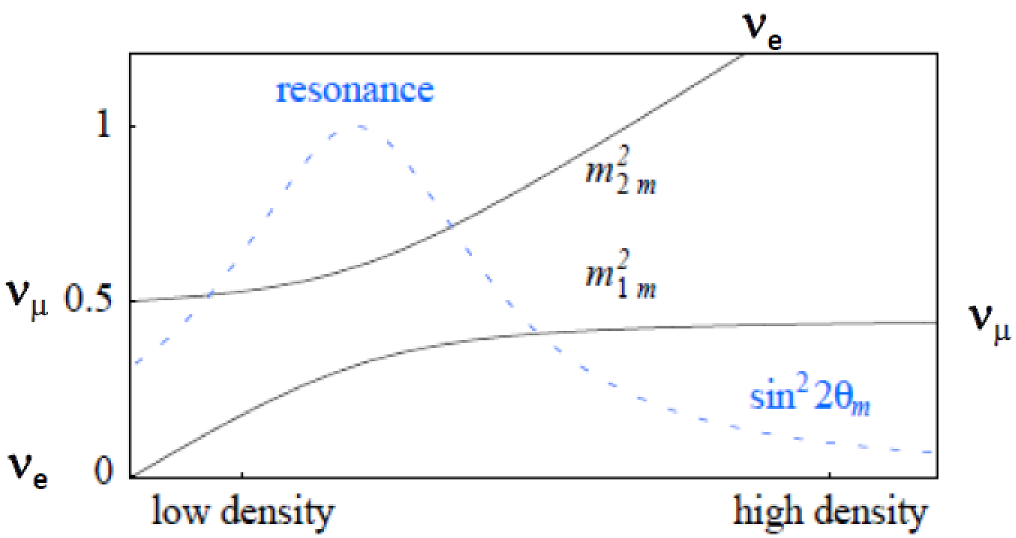
Making the appropriate replacements, $\theta \rightarrow \theta_m$, $\Delta \rightarrow (M_2^2 - M_1^2)$, the probability is given by

$$P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta_m (1 - \cos 2\pi x/\lambda_m)$$

As in the case of vacuum oscillations, one often averages over the phase information, thereby arriving at the classical probability

$$P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta_m$$

MIKHEYEV-SMIRNOV-WOLFENSTEIN (MSW) EFFECT



$$\nu_e = \cos\theta_m \nu_1 + \sin\theta_m \nu_2$$

Mixing angle in matter

$$\sin 2\theta_m = \frac{\sin 2\theta}{\sqrt{\left(\frac{2EV}{\Delta m^2} - \cos 2\theta\right)^2 + \sin^2 2\theta}}$$

ADIABATIC EVOLUTION → The ν state remains the same ν mass eingenstates for all the propagation

- $\theta << 1, \;\; V >> \frac{\Delta m^2}{2E} \longrightarrow \theta_m \approx \frac{\pi}{2}, \nu_e \approx \nu_{2m}$ **Matter dominance**
- $V \approx \frac{\Delta m^2}{2E} \longrightarrow \theta_m \approx \frac{\pi}{4}$ **Resonance**
- $V \approx 0 \longrightarrow \theta_m \approx \theta, \nu_2 \approx \nu_\mu$ **Resonant flavor conversions**

Resonances in matter occur either for ν or for anti-ν

In the adiabatic limit is easy to describe the MSW effect. The adiabaticity implies that ν is propagating for many oscillation wavelengths. The phase information is lost

Adiabatic probabilites = classical probabilities

$$P(\nu_e \to \nu_e) = \sum_i P_m(\nu_e \to \nu_i) P(\nu_i \to \nu_e)$$

Final ν_e — Back to flavor basis — No crossing — Rotate to mass basis — Initial ν_e

$$= (1 \;\; 0) \begin{pmatrix} \cos^2\theta & \sin^2\theta \\ \sin^2\theta & \cos^2\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos^2\theta_m & \sin^2\theta_m \\ \sin^2\theta_m & \cos^2\theta_m \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

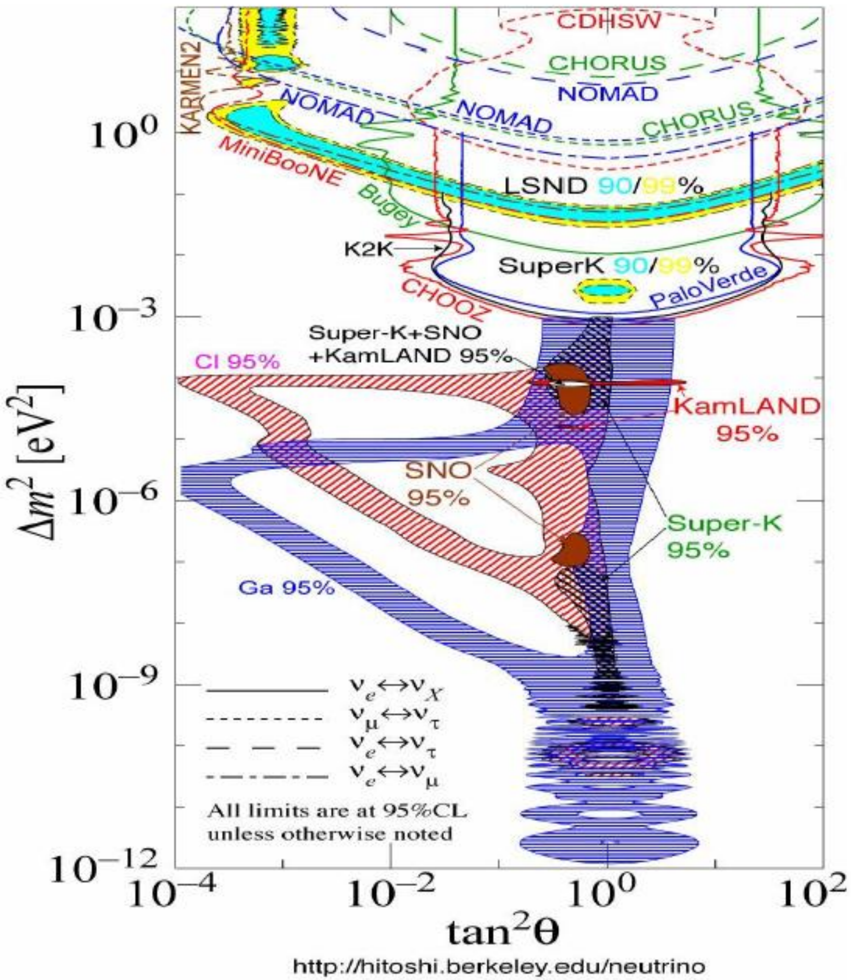
$$= \sin^2\theta \sin^2\theta_m + \cos^2\theta \cos^2\theta_m = \frac{1}{2}(1 + \cos 2\theta \cos 2\theta_m)$$

If initial n_e is large $\to$ $\cos 2\theta_m \sim -1$

And if θ small $\;\to\;$ $P(\nu_e \to \nu_e) \simeq \frac{1}{2}(1 - \cos 2\theta) \ll 1$

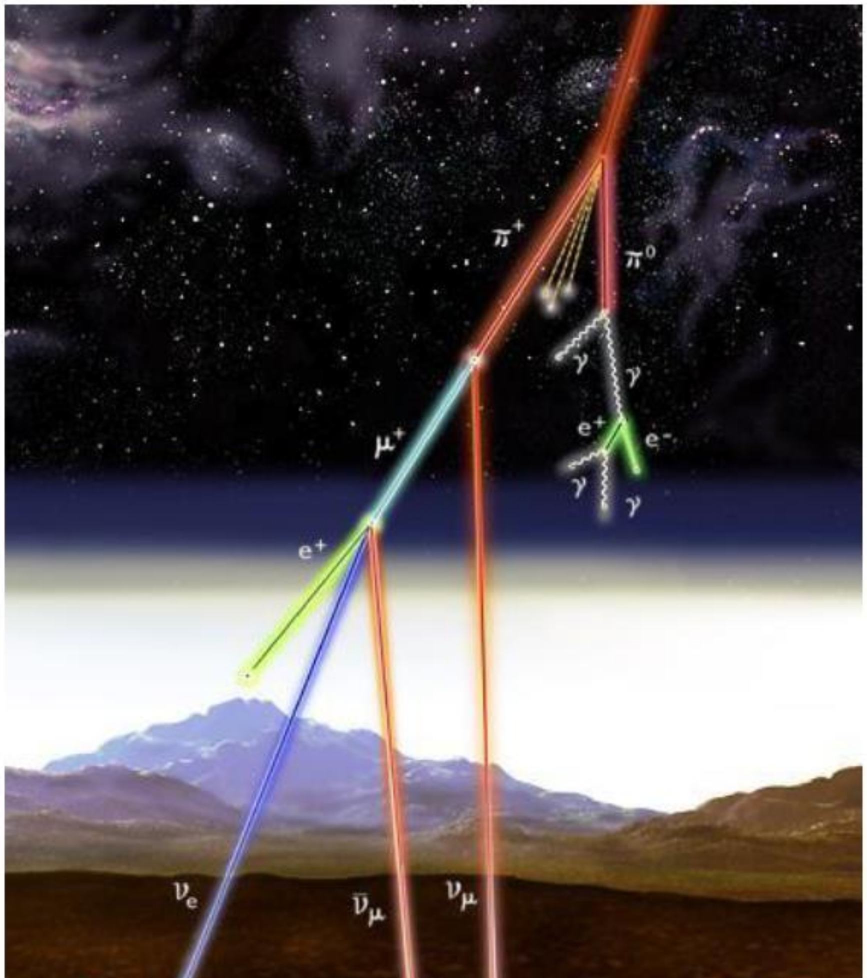
Strong conversion probabilities

NEUTRINO OSCILLATIONS: PHENOMENOLOGY



ATMOSPHERIC NEUTRINOS

When cosmic rays, i.e. protons and heavier nuclei interact with the Earth's atmosphere they produce kaons and pions, which in turn decay into muons, electrons and neutrinos.

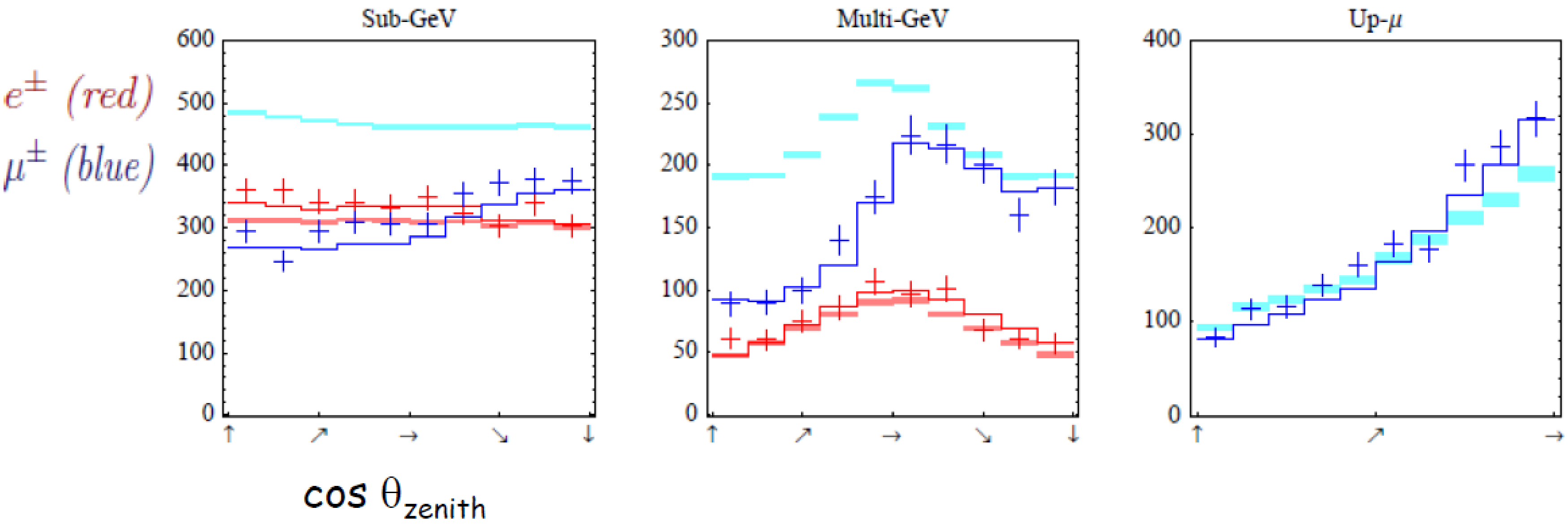


The flavor can be very well measured; from the simple production mechanism one expects

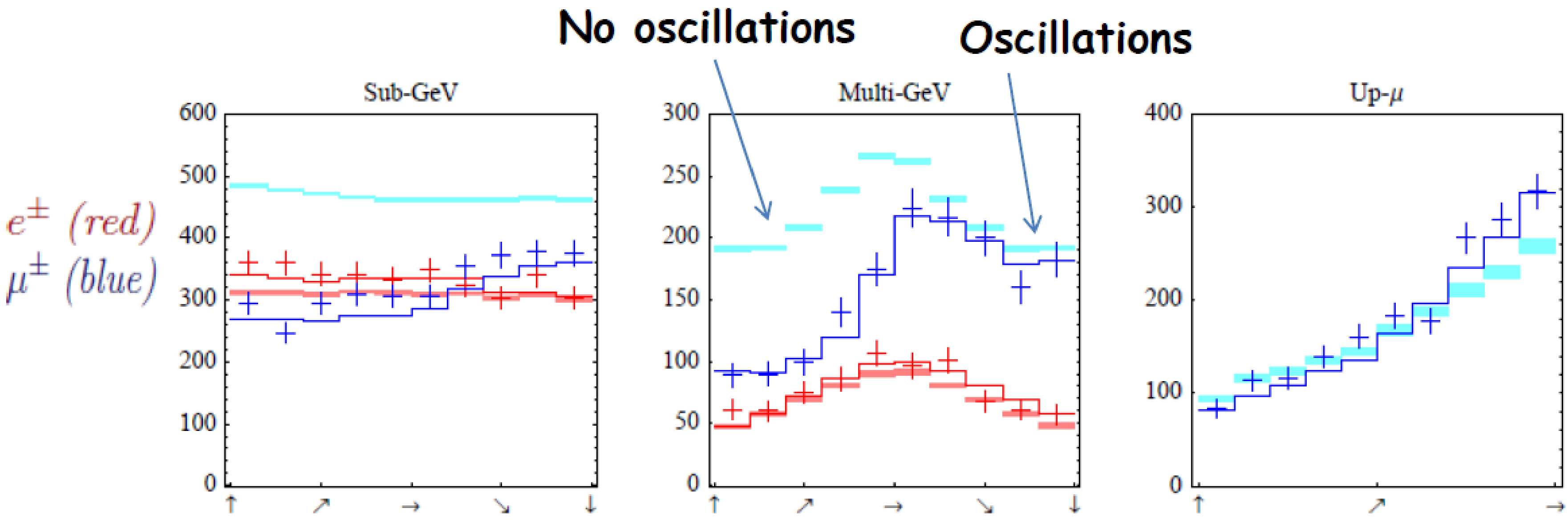
$$N(\nu_\mu) : N(\nu_e) = 2 : 1.$$

This ratio depends on the energy of the measured neutrinos since the lifetime of high energy muons is increased by their Lorentz factor so that they may hit the ground before decaying (at energies > few GeV).

SUPER-KAMIOKANDE DATA



- Sub-GeV events: $E_l < 1.4$ GeV
- Multi-GeV events: : $E_l > 1.4$ GeV
- Up-going stopping muons: the μ is produced in the rock below the detector (energy cannot be measured) and stops inside the detector. Typical energy of the parent ν is ~ 10 GeV



Histogram shows the prediction without oscillations and the best $\nu_\mu \longrightarrow \nu_\tau$ oscillation fit, for Δm^2_{atm} = 2.5 x 10^{-3} eV^2 and $\sin^2 2\theta_{atm}$=1.

Data in the multi-GeV show a neutrino anomaly (clearly asymmetric distribution). Electron samples are compatible with no oscillations

The flux of up-ward going muons (↑) is about two times lower than the flux of down-ward muons (↓).

Up-μ: there are fewer μ coming from straight up than from near the horizon - they've had to travel further, so have had more of a chance to oscillate to something else, such as a ν_τ, which SK cannot detect.

Therefore the data can be interpreted assuming that nothing happens to ν_e and ν_μ oscillates into ν_τ

$$
\begin{aligned}
P(\nu_e \to \nu_e) &= 1 \\
P(\nu_e \to \nu_\mu) &= 0 \\
P(\nu_\mu \to \nu_\mu) &= 1 - \sin^2 2\theta_{\rm atm} \sin^2 \frac{\Delta m^2_{\rm atm} L}{4E_\nu}
\end{aligned}
$$

Looking at the zenith-angle dependence (multi-GeV) we notice that down-ward going neutrinos are almost unaffected by oscillations, while up-ward going neutrinos feel almost averaged oscillations, and therefore their flux is reduced by a factor 1 -1/2 $\sin^2 2\theta_{atm}$. This must be equal to the up/down ratio N(up)/N(down) = 0.5±0.05, so that $\sin^2 2\theta_{atm} = 1 \pm 0.1$. Furthermore, multi-GeV neutrinos have energy E ~ 3 GeV, and they begin to oscillate around the horizontal direction ($\cos\theta \approx 0$) i.e. at a pathlength of about $L \approx 1000$ km. Therefore $\Delta m^2_{atm} \sim E/L \sim 3 \times 10^{-3}$ eV^2.

EVIDENCE OF OSCILLATIONS FROM ATMOSPHERIC NEUTRINOS

[SK Collaboration, hep-ex/0404034]

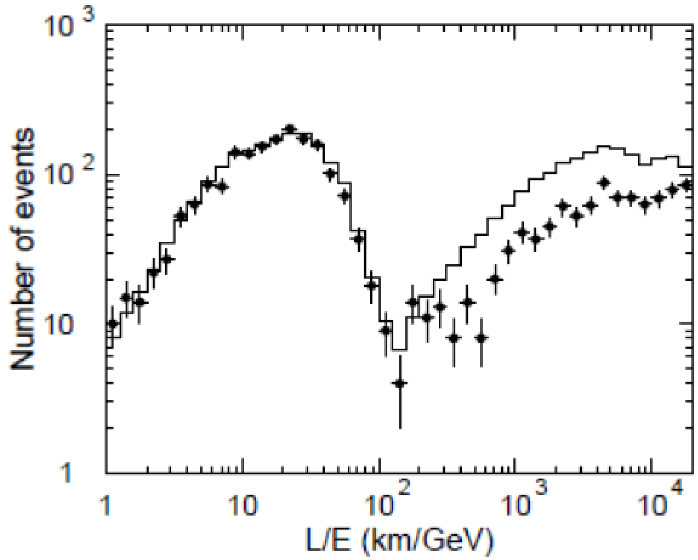


FIG. 3: Number of events as a function of L/E for the data (points) and the atmospheric neutrino MC events without oscillations (histogram). The MC is normalized by the detector live-time.

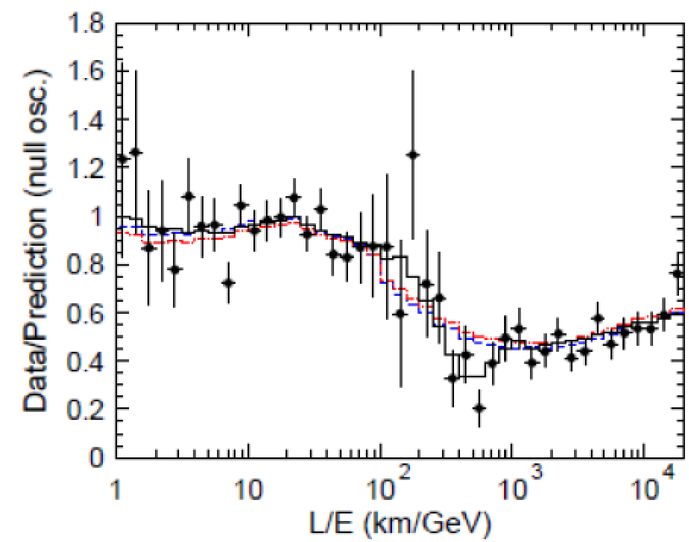


FIG. 4: Ratio of the data to the MC events without neutrino oscillation (points) as a function of the reconstructed L/E together with the best-fit expectation for 2-flavor $\nu_\mu \leftrightarrow \nu_\tau$ oscillations (solid line). The error bars are statistical only. Also shown are the best-fit expectation for neutrino decay (dashed line) and neutrino decoherence (dotted line).

Dip in L/E distribution observed in the data, as predicted by the oscillation probability

Other possible interpretations (nu decoherence, nu decay) disfavoured by data

ATMOSPHERIC NEUTRINO MASS AND MIXING PARAMETERS

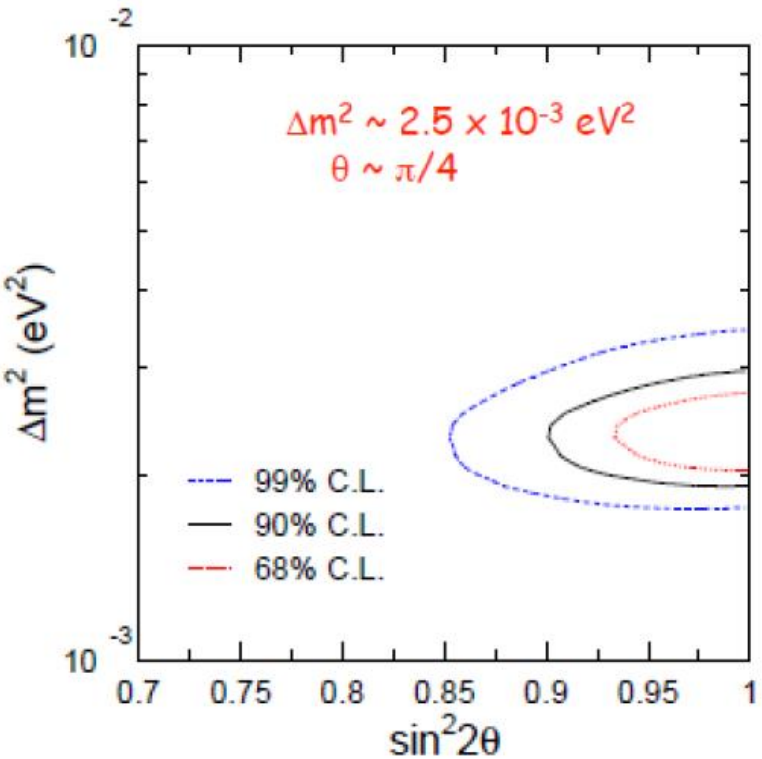
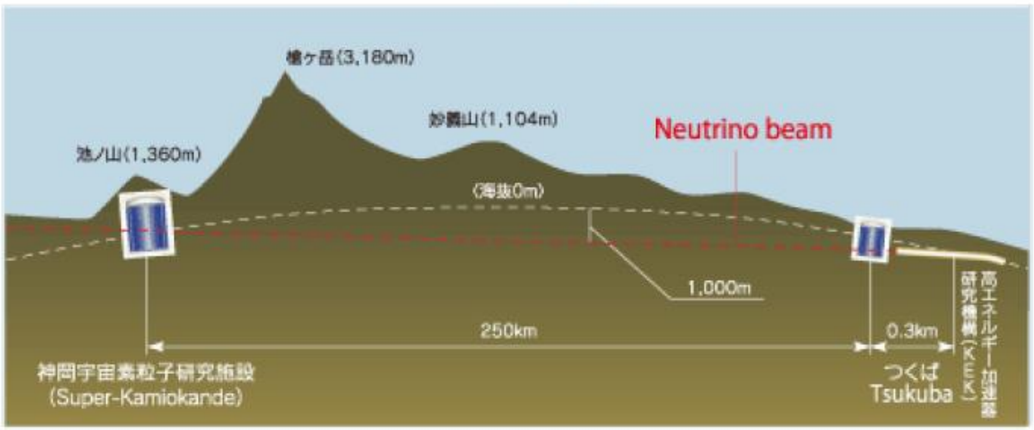


FIG. 5: 68, 90 and 99% C.L. allowed oscillation parameter regions for 2-flavor $\nu_\mu \leftrightarrow \nu_\tau$ oscillations obtained by the present analysis.

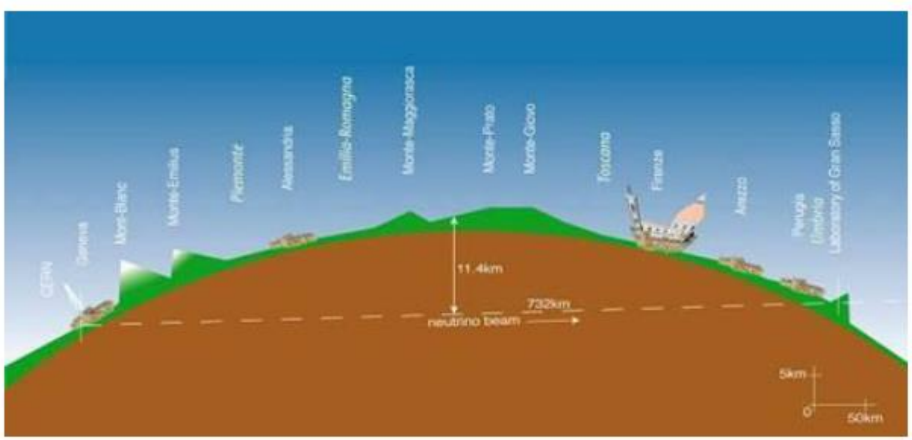
LONG-BASELINE EXPERIMENTS

Reproducing atmospheric ν_μ physics in controlled conditions

K2K (Japan)



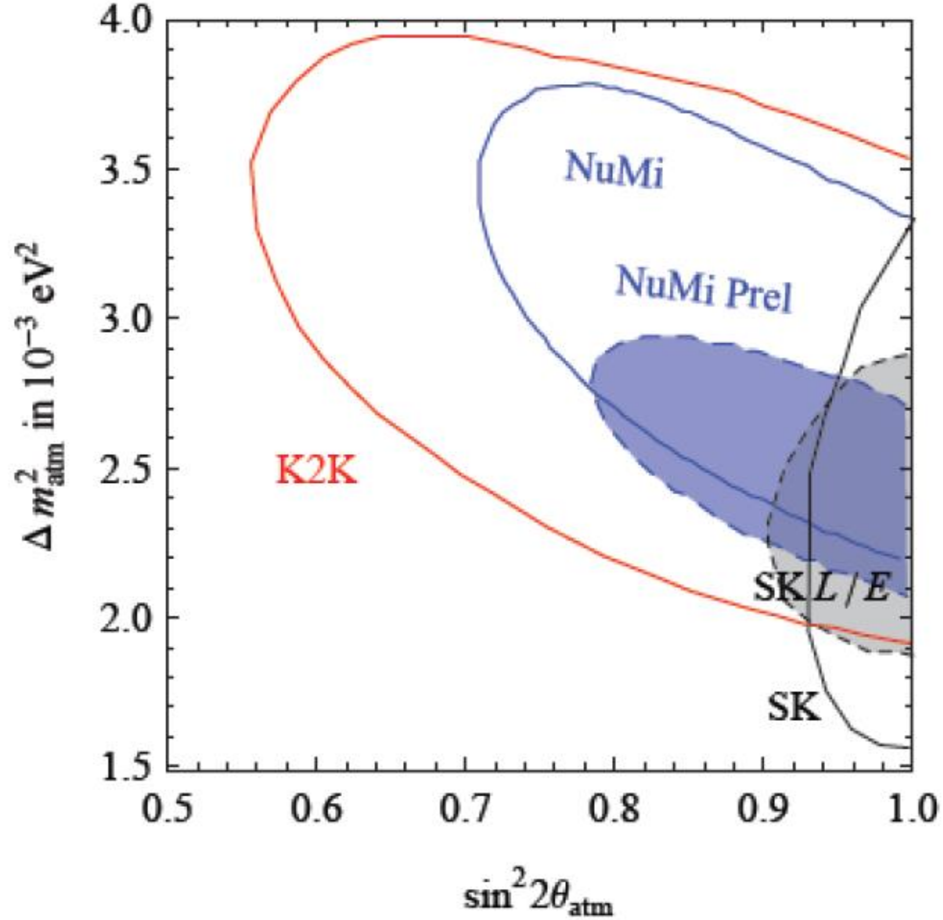
MINOS (USA)



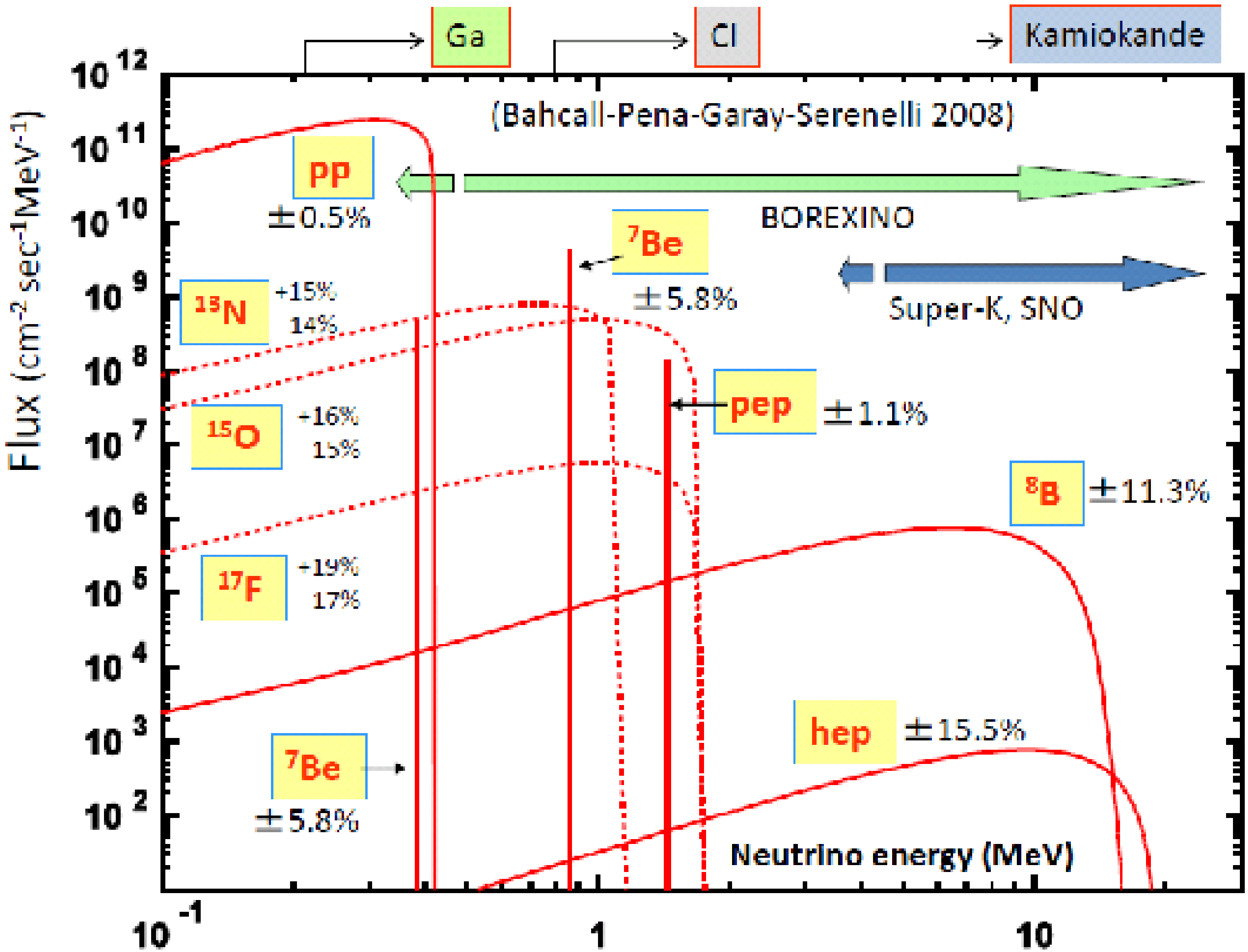
OPERA (Europe)

Once more… dominant $P_{\mu\tau} = \sin^2(2\theta_{23})\,\sin^2(\Delta m^2 L/4E_\nu)$

Oscillation parameters consistent among experiments

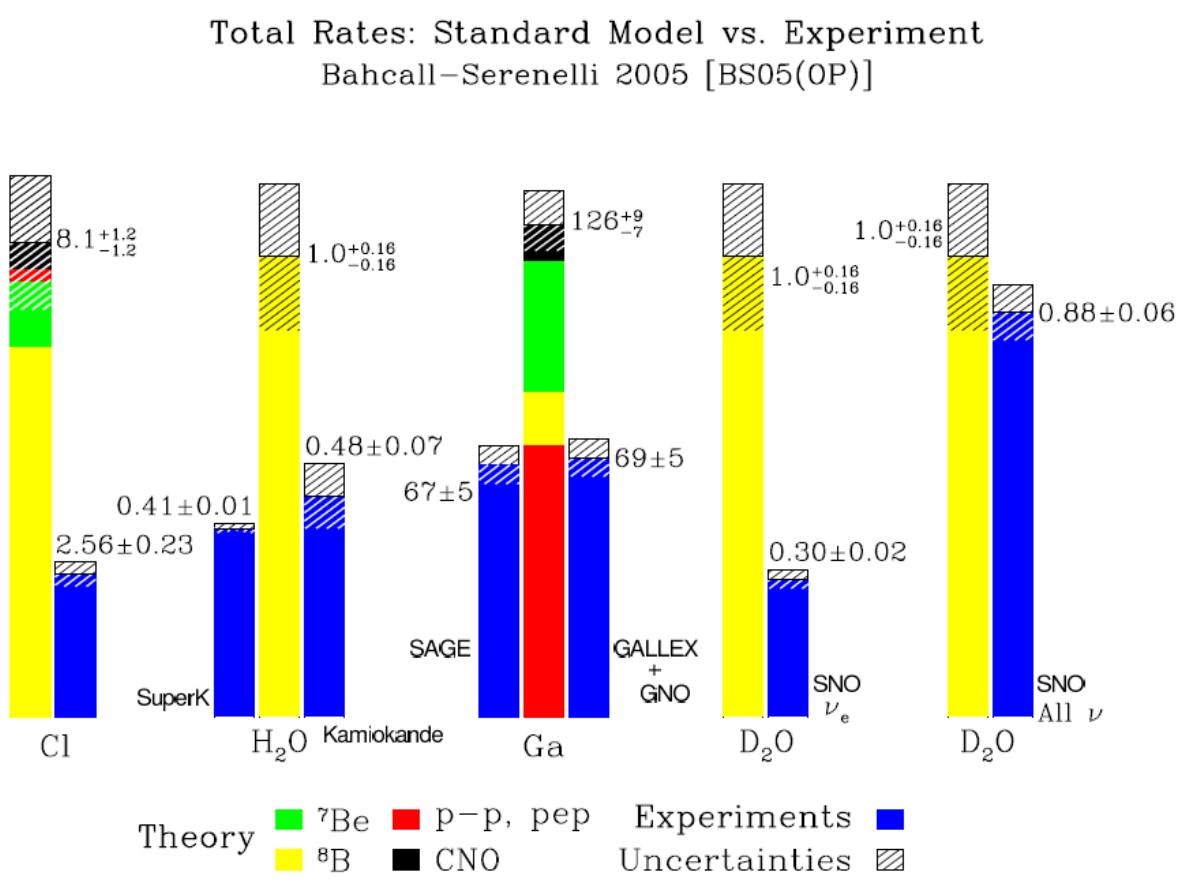


SOLAR NEUTRINO SPECTRUM



MISSING NEUTRINOS

All CC sensitive results indicate a ν_e deficit...



as compared to solar model expectations.

NEUTRINO OSCILLATIONS IN THE SUN

In the limit of small θ_{13} MSW equations reduce to only 2 generations of neutrinos

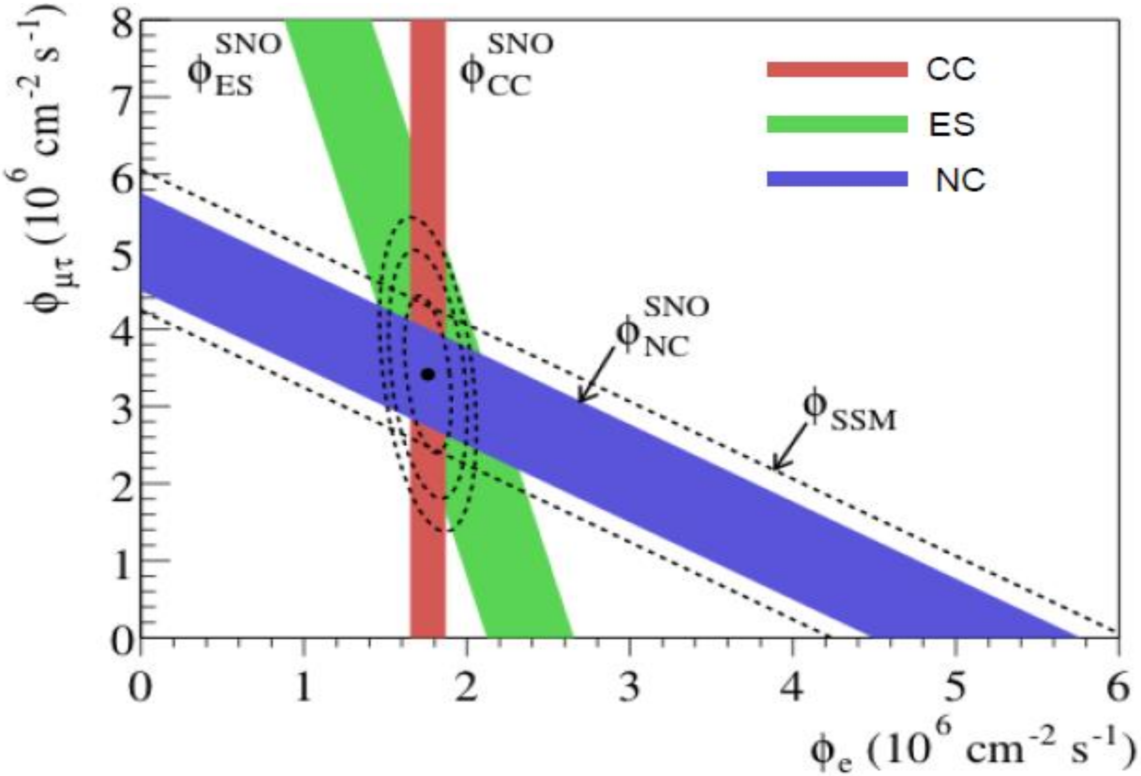
$$i\frac{d}{dx}\begin{pmatrix}\nu_e\\ \nu_a\end{pmatrix}=\begin{bmatrix}\frac{G_F n_e(x)}{\sqrt{2}}-\frac{k_\nu}{2}\cos 2\theta_{12} & \frac{k_\nu}{2}\sin 2\theta_{12}\\ \frac{k_\nu}{2}\sin 2\theta_{12} & -\frac{G_F n_e(x)}{\sqrt{2}}+\frac{k_\nu}{2}\cos 2\theta_{12}\end{bmatrix}\cdot\begin{pmatrix}\nu_e\\ \nu_a\end{pmatrix}$$

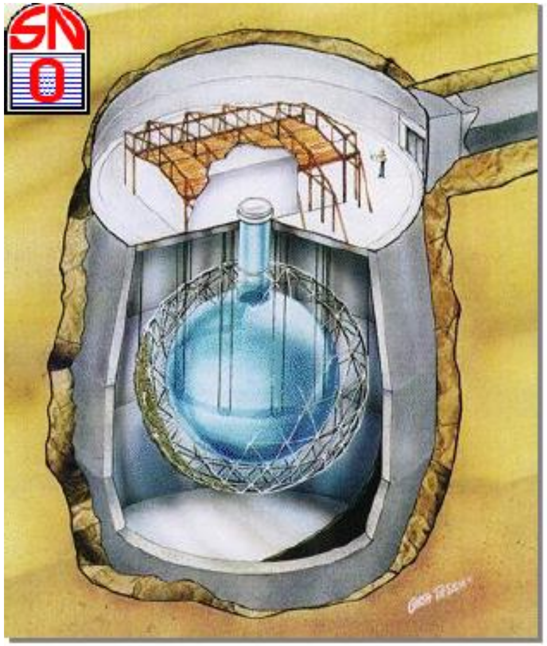
with $k_\nu=\delta m^2/2E=2\pi/\lambda_\nu$ and $\nu_a=\cos\theta_{23}\nu_\mu+\sin\theta_{23}\nu_\tau$

FLAVOR OSCILLATIONS FOR SOLAR NEUTRINOS

In 2002 the breakthrough with SNO: in deuterium one can separate CC events (induced by ν_e only) from NC events (induced by ν_e, ν_μ, ν_τ), and double check via elastic scattering events (due to both NC and CC)

[SNO collaboration, nucl-ex/0204008]





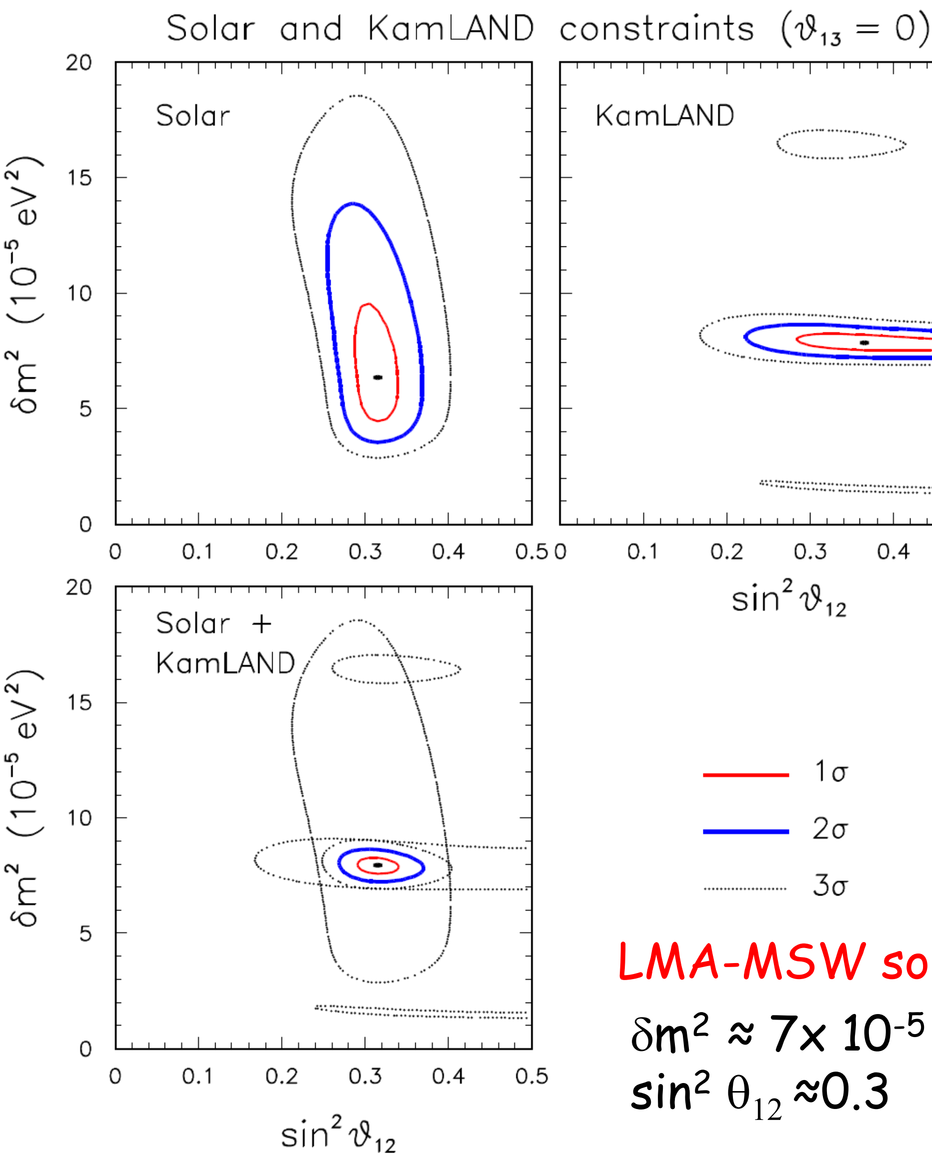
$$\frac{CC}{NC} \sim \frac{\phi(\nu_e)}{\phi(\nu_e)+\phi(\nu_{\mu,\tau})}$$ thus $\frac{CC}{NC} \sim \frac{1}{3} < 1 \Rightarrow \phi(\nu_{\mu,\tau}) > 1 \Rightarrow \nu_e \to \nu_{\mu,\tau}$ Flavor oscillations

Also **CC/NC ~ P_{ee} ~ $\sin^2\theta_{12}$(LMA) ~1/3 < 1/2**

Evidence of mixing in the first octant + matter effects

COMBINED ANALYSIS OF SOLAR NU + KAMLAND

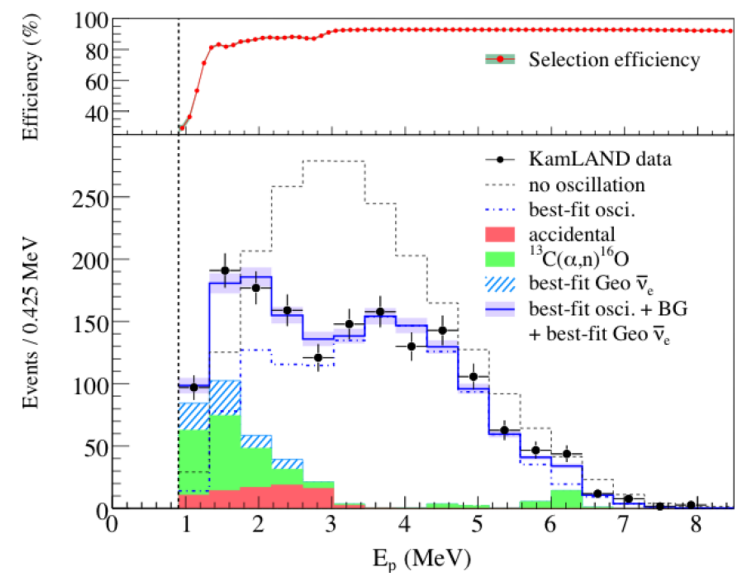
[Fogli et al., hep-ph/0506083]



Also in 2002…KamLAND:
1000 ton mineral oil detector, surrounded by nuclear reactors producing anti-ν_e. With solar (δm^2, θ) **LMA-MSW solution**, reactor neutrinos oscillate with large amplitude.
Definitive confirmation of solar neutrino oscillations

LMA-MSW solution

$\delta m^2 \approx 7\times 10^{-5}$ eV^2
$\sin^2 \theta_{12} \approx 0.3$



[KamLAND Collaboration, hep-ex/0212021]

2007: PMNS mixing after about 40 years of research ...

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

"Atmospheric" sector	"Interference" sector	"Solar" sector
Large rotation (~ maximal)	Small rotation (maybe null ?)	Large rotation (< maximal)
$\sin^2\theta_{23} \sim 1/2$	**$\sin^2\theta_{13} \sim 0$?**	**$\sin^2\theta_{12} \sim 1/3$**
$\Delta m^2 \sim 3\times10^{-3}$ eV^2		**$\delta m^2 \sim 8\times10^{-5}$ eV^2**

Open questions ...

How can we measure θ_{13}? And δ afterwards?
And finally the hierarchy, i.e. **sign($\pm\Delta m^2$)**?

Neutrino oscillations at nuclear reactors

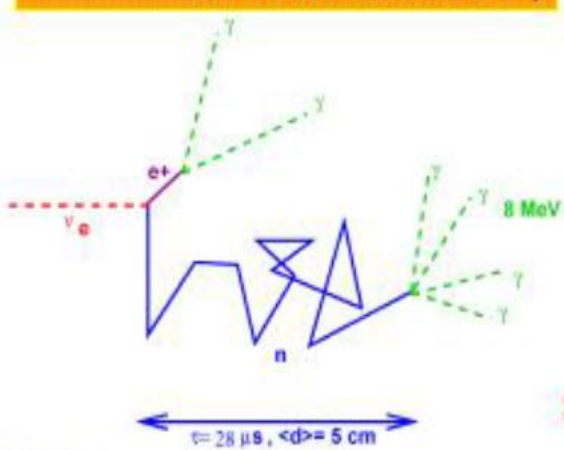
$\bar{\nu}_e$ disappearance searches

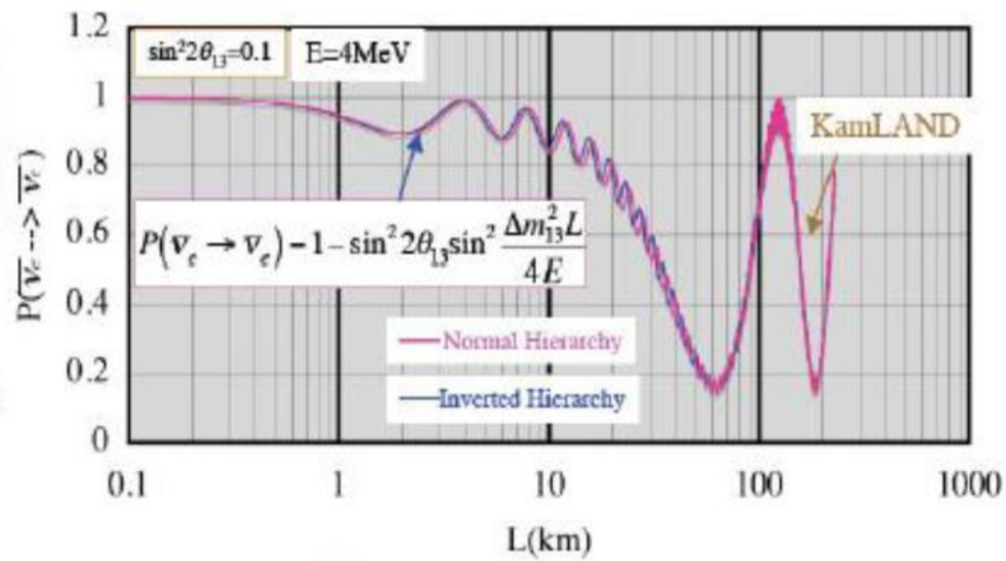
$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{13}{}^2 L}{4E_\nu}\right) - \cos^4\theta_{13} \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m_{21}{}^2 L}{4E_\nu}\right)$$

Reactor Neutrino Oscillation

$\bar{\nu}_e$ detection: inverse beta decay

$$\bar{\nu}_e + p \rightarrow n + e^+$$
$$n + Gd \rightarrow 8\ \mathrm{MeV}\ \gamma$$





Small deficit (= $\sin^2 2\theta_{13}$) ➲ high precision is necessary

6. The hunt to θ_{13}, the last mixing angle

Indeed, the hunt to θ_{13} is crucial in neutrino research, in order to plan future CP violation searches!

In 2006 the upper bound still comes from

CHOOZ expt. → **$\sin^2\theta_{13}$ < few %**

But, in the meantime, some weak hints of lower bounds have appeared ...

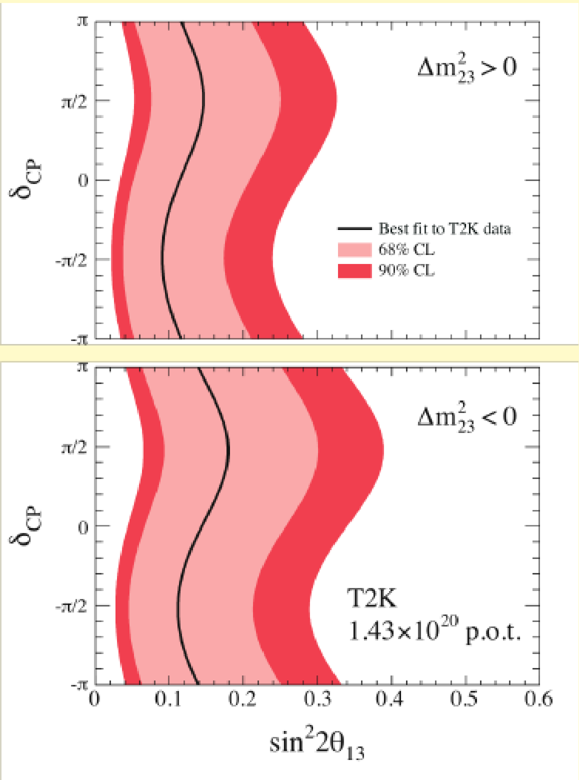
1. From a 3ν analysis of atmospheric data (+ long-baseline accelerator experiments + CHOOZ) by considering **subleading "solar term" effects**.

2. From an accurate comparison within a 3ν approach of **solar** (SNO dominated) and **KamLAND** data.

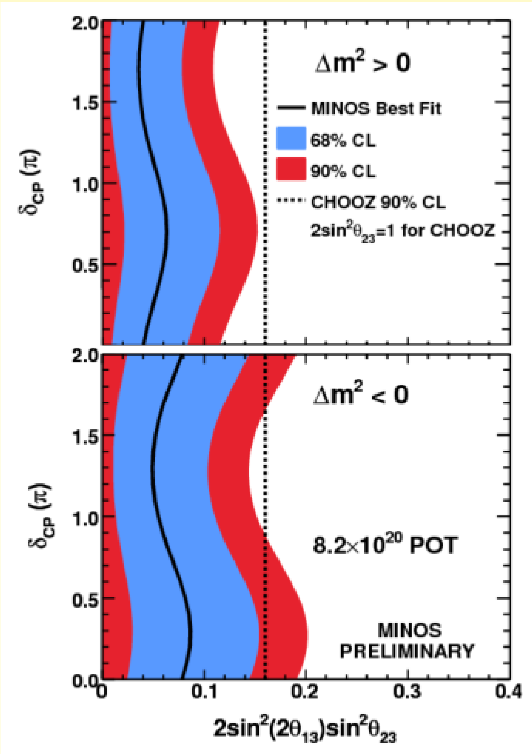
Once again, new experimental results come to rescue! In 2011, T2K and MINOS found some electron event excess when running in appearance mode ...

$\nu_\mu \rightarrow \nu_e$ oscillations, sensitive to θ_{13}

T2K

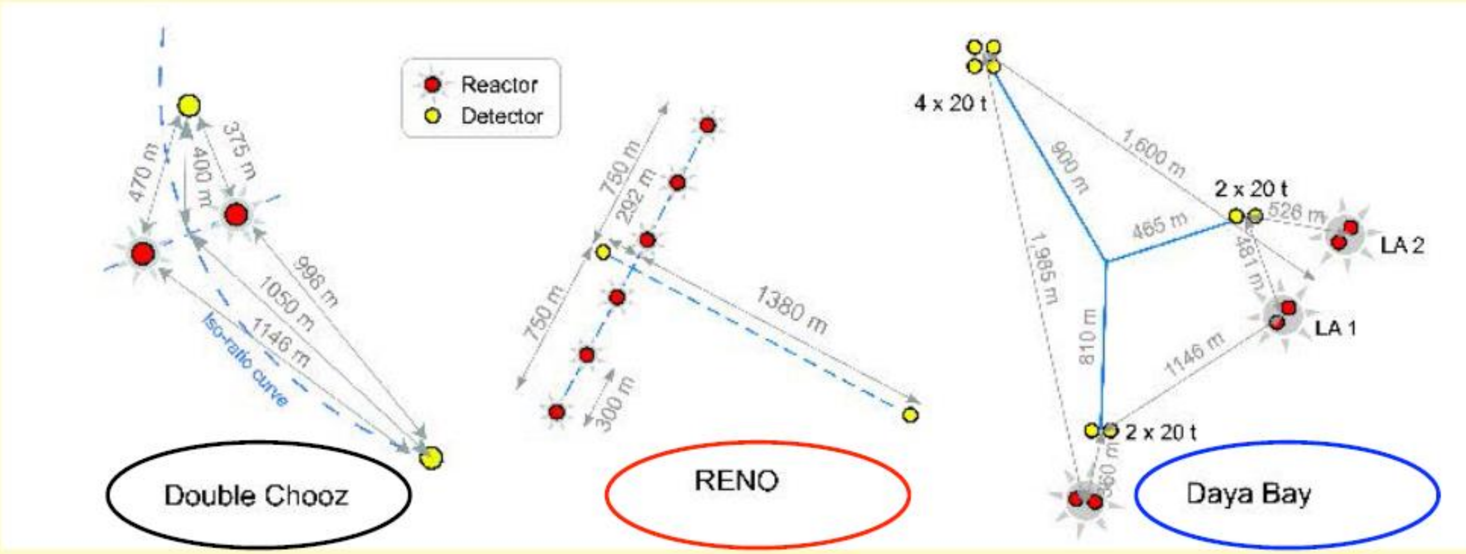


MINOS



Both experiments favor $\sin^2\theta_{13}$ ~ few % ! It makes sense to combine these with all the other oscillation data ...

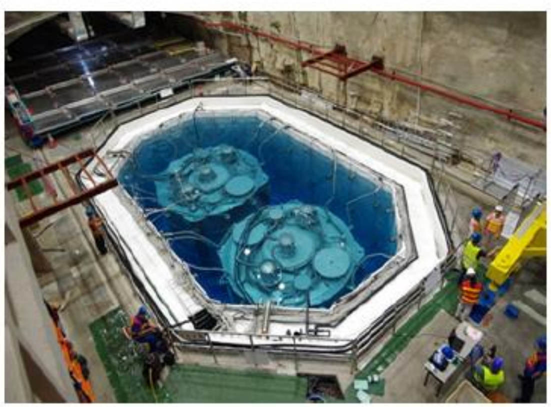
7. The Short Baseline Reactor experiments



Running with FD; ND in construction

Running with ND & FD

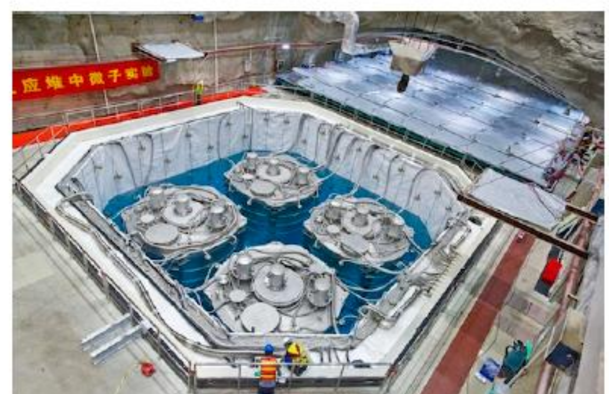
Running with ND & FD



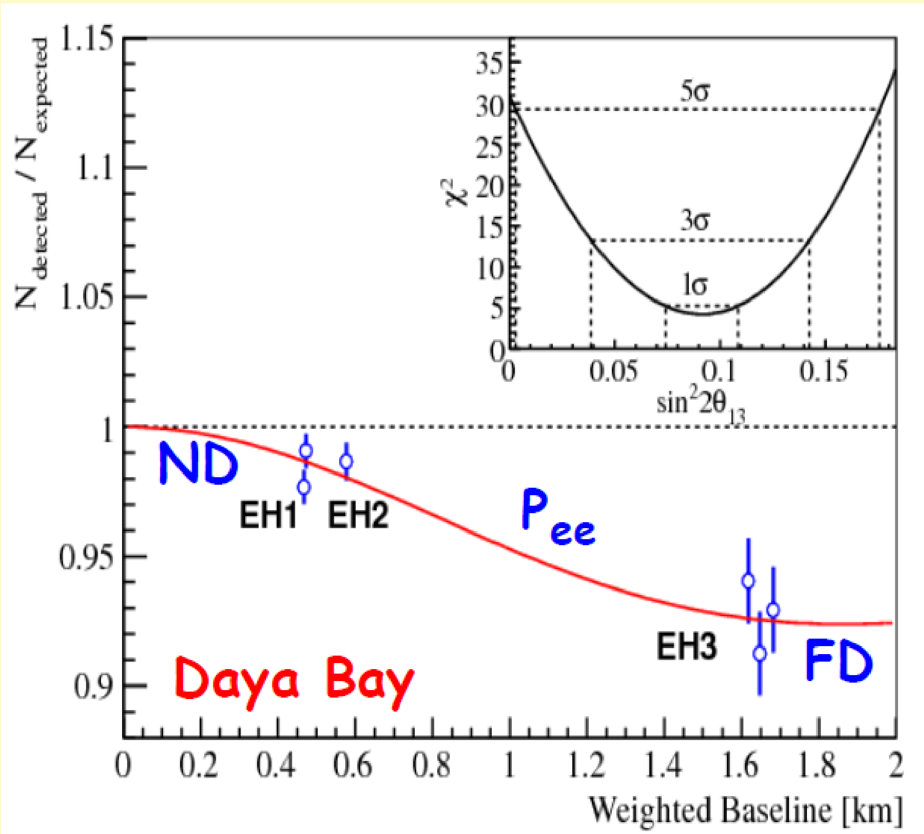
E.g, for Daya Bay:

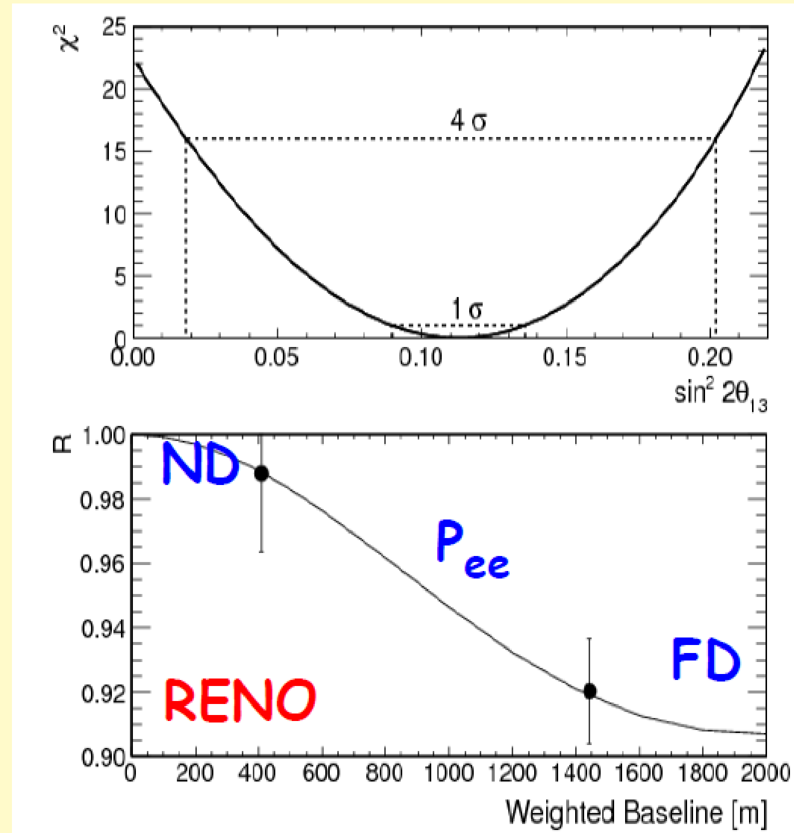
← ND

FD →



2012: experimental discovery of $\theta_{13}>0$! (value obtained at ~ fixed Δm^2)





Clear disappearance at FD with respect to ~ unoscillated signal at ND. Double Chooz results (FD only) also consistent with Daya Bay & RENO.

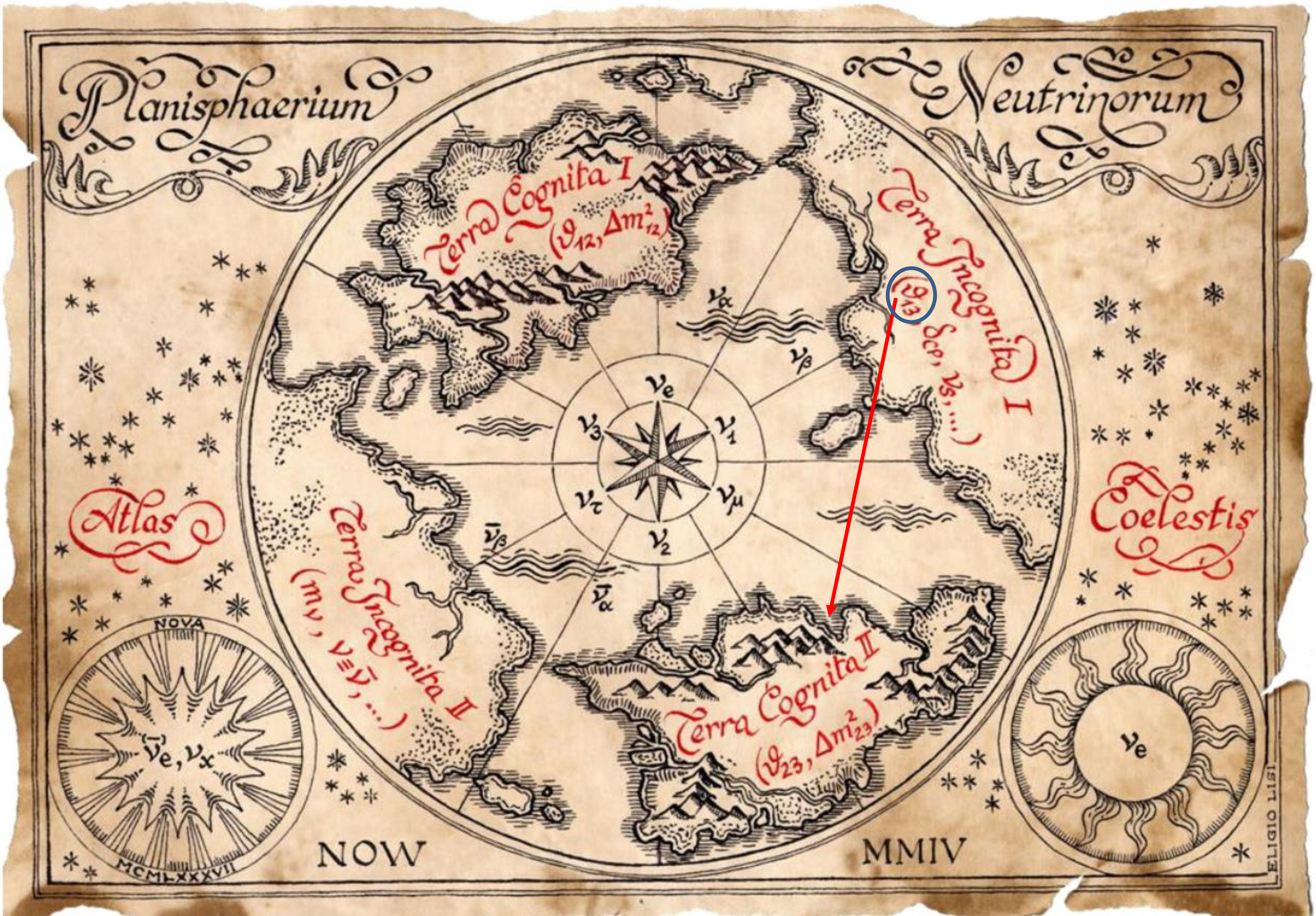
ENTERING THE PRECISION ERA OF NEUTRINO OSCILLATIONS

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.54	0.307	0.014	0.42	2.36
1σ range	7.32 – 7.79	0.291 – 0.325	0.006 – 0.023	0.38 – 0.51	2.26 – 2.48
2σ range	7.14 – 7.99	0.275 – 0.342	< 0.033	0.36 – 0.59	2.17 – 2.57
3σ range	6.98 – 8.17	0.259 – 0.360	< 0.042	0.33 – 0.64	2.07 – 2.67

Fractional 1σ accuracy [defined as 1/6 of $\pm 3\sigma$ range]

δm^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	Δm^2
2.6%	5.4%	~ 0.008	~12%	4.2%

[Capozzi et al., 1312.2878]



Planisphaerium
Neutrinorum
Terra Cognita I
($\vartheta_{12}, \Delta m^2_{12}$)
Terra Incognita I
(ϑ_{13}, δ_{CP}, ν_S, ...)
Terra Incognita II
(m_ν, $\nu \equiv \bar{\nu}$, ...)
Terra Cognita II
($\vartheta_{23}, \Delta m^2_{23}$)
Atlas
Coelestis
ν_e
ν_1
ν_μ
ν_2
ν_τ
ν_3
ν_α
ν_β
$\bar{\nu}_\beta$
$\bar{\nu}_\alpha$
NOVA
$\bar{\nu}_e, \nu_x$
MCMLXXXVII
NOW
MMIV
ν_e
ELIGIO LISI

HOPING TO DISCOVER MOST DISTANT LANDS!

FURTHER READINGS

- T.K. Kuo and J.T. Pantaleone, "Neutrino Oscillations in Matter," Rev. Mod. Phys. **61**, 937 (1989).
- A. Strumia and F. Vissani, "Neutrino masses and mixings and...," hep-ph/0606054.