

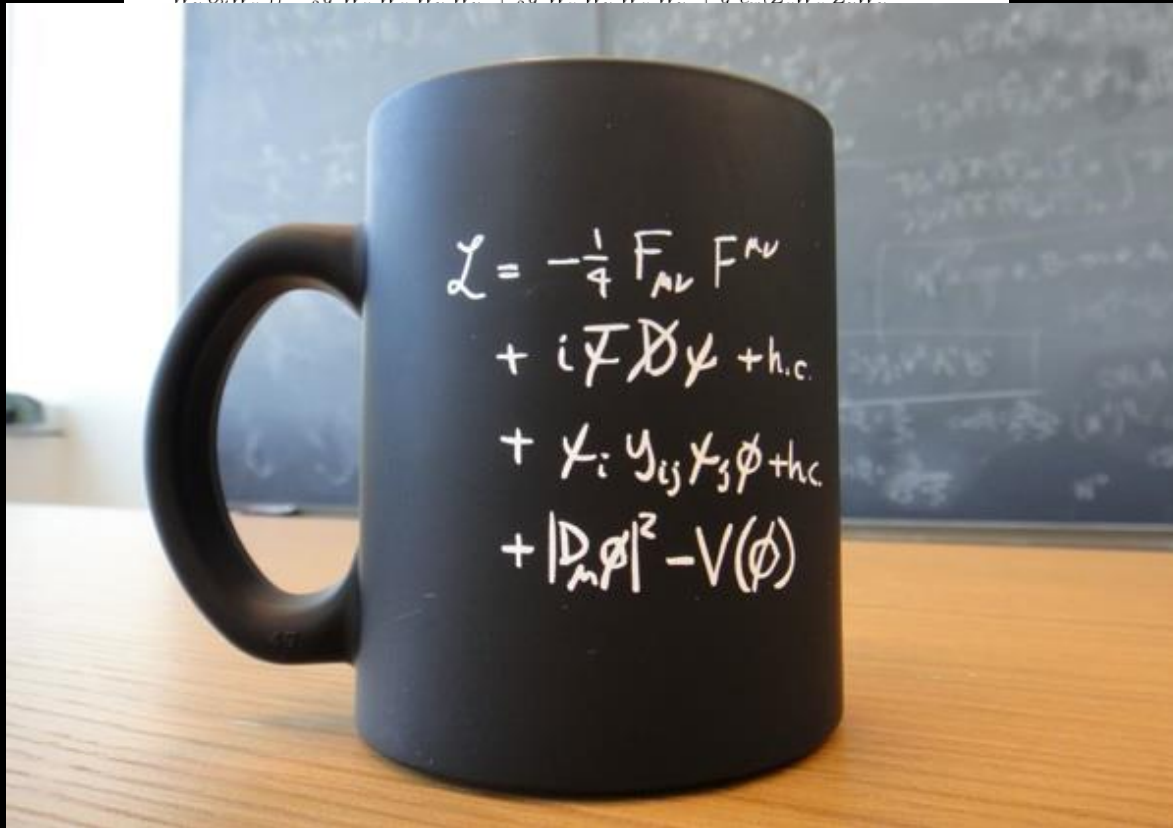
LHC physics: Interrogating the Standard Model of particle physics



See how in the
LHC experimental
lectures by Marco

The Standard Model Lagrangian:

$$\begin{aligned} \mathcal{L}_{SM} = & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_w^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\ & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - ig_{c_w}(\partial_\nu Z_\mu^0(W_\mu^+ W_\nu^- - \\ & W_\nu^+ W_\mu^-) - Z_\nu^0(W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\mu^0(W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)) - \\ & ig_{s_w}(\partial_\nu A_\mu(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu(W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + A_\mu(W_\nu^+ \partial_\nu W_\mu^- - \\ & W_\nu^- \partial_\nu W_\mu^+)) - \frac{1}{2}g^2 W^+ W^- W^+ W^- + \frac{1}{2}g^2 W^+ W^- W^+ W^- + g^2 c_w^2 (Z^0 W^+ Z^0 W^- - \end{aligned}$$



$$\begin{aligned} & \partial_\mu \bar{X}^+ X^0 + ig_{s_w} W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + ig_{c_w} W_\mu^- (\partial_\mu \bar{X}^- X^0 - \\ & \partial_\mu \bar{X}^0 X^+) + i g_{c_w} W_\mu^+ (\partial_\mu \bar{X}^- X^+ - \partial_\mu \bar{X}^+ X^0) + i g_{c_w} Z_\mu^0 (\partial_\mu \bar{X}^+ X^+ - \end{aligned}$$

accords (very) well with all measurements ever made in high-energy physics!

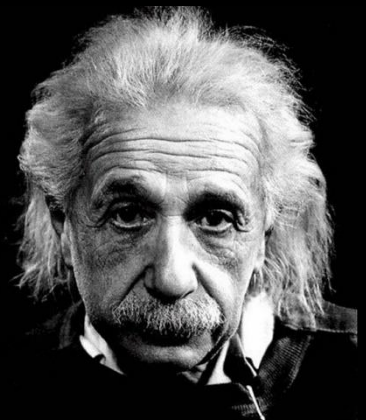
Outline:

- **Lecture 1 :**
 - *Why Nature needs a light Higgs boson?*
- **Lecture 2 :**
 - *Why a light Higgs boson is not the end?*

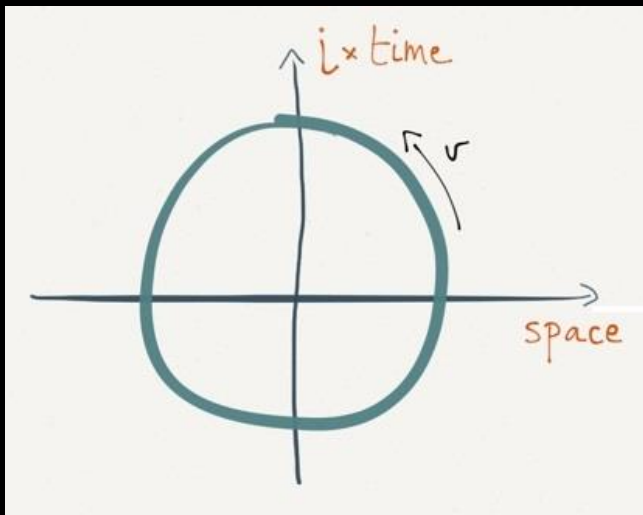
Lecture 0:

*XXth century physics
in a (hazel)nut-shell*

Lessons from relativity



- All inertial observers see the same physics:
 - light speed $c < \infty$
 - Lorentz symmetries = space-time “rotations”



$$x^\mu \equiv (ct, \vec{x})$$

$$x^2 = \eta_{\mu\nu} x^\mu x^\nu = x^\mu x_\mu = \text{invariant}$$

$$\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$$



- Mass is just another form of energy: $E = mc^2$

$$p^\mu = \left(\frac{E}{c}, \vec{p} \right) \rightarrow p^2 = \left(\frac{E^2}{c^2} - \vec{p}^2 \right) = m^2 c^4$$

Lessons from quantum mechanics

- Determinism is not fundamental: $\Delta x^\mu \times \Delta p_\nu \geq (\hbar/2)\delta_\nu^\mu$
 - Nature is random \rightarrow probability rules
 - The vacuum is not void, it fluctuates!



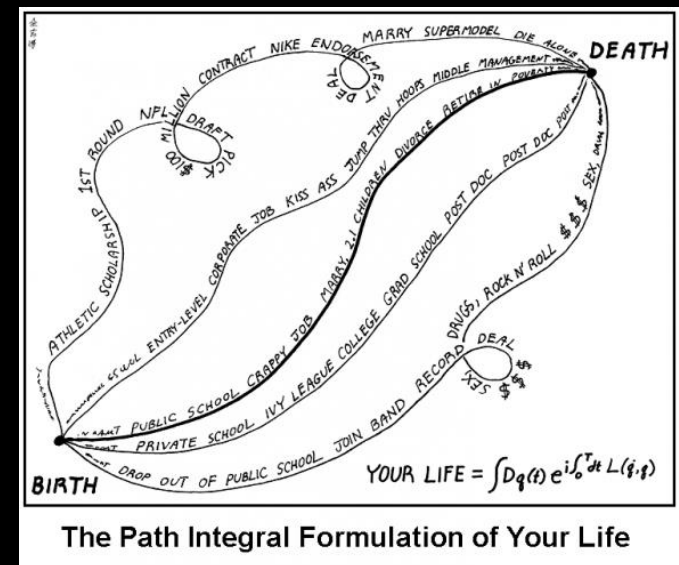
- Classical physics emerges from constructive interference of probability amplitudes:

Feynman's path integral:



$$A = \int [Dq] \exp \left[\frac{i}{\hbar} S[q(t), \dot{q}(t)] \right]$$

\rightarrow a rational for the least action principle!



Lessons from QM+relativity

- *number of particles in the system is no longer conserved:*

kinetic energy ↔ massive particles

→ particles = excitations (quanta) of fields

explaining why all electrons are the same

- *High-energy colliders are quantum “microscopes”.*

A word on units

- [m],[g],[s] are not well suited units for fundamental particles

$$\Delta x \sim 10^{-18} \text{m}, m_{\text{proton}} \sim 10^{-24} \text{g}, \tau_{\text{Higgs}} \sim 10^{-22} \text{s}, \dots$$

- most phenomena are set by the particle's mass

→ Can one measure length, time, energy, momentum in “units” of mass?

$$c = \hbar = 1 \quad [\text{natural units}]$$

$$E = mc^2 = m$$

$$L = \hbar/mc = 1/m$$

$$t = \hbar/E = 1/m$$

$$p = \hbar/L = m$$

$$(1 \text{ GeV})/c^2 = 1.783 \times 10^{-24} \text{ g}$$

$$(1 \text{ GeV})^{-1}(\hbar c) = 0.1973 \times 10^{-13} \text{ cm} = 0.1973 \text{ fm};$$

$$(1 \text{ GeV})^{-2}(\hbar c)^2 = 0.3894 \times 10^{-27} \text{ cm}^2 = 0.3894 \text{ mbarn}$$

$$1 \text{ barn} = 10^{-24} \text{ cm}^2$$

$$(1 \text{ volt/meter})(e\hbar c) = 1.973 \times 10^{-25} \text{ GeV}^2$$

$$(1 \text{ tesla})(e\hbar c^2) = 5.916 \times 10^{-17} \text{ GeV}^2$$

$$[\text{length}] = [\text{time}] = [\text{energy}]^{-1} = [\text{momentum}]^{-1} = [\text{mass}]^{-1}$$

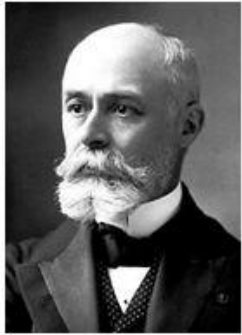
Lecture 1:

What is Higgs good for?

Discovery of radioactivity

- Earliest evidence by Becquerel in 1896 in Uranium salts:

The Nobel Prize in Physics 1903



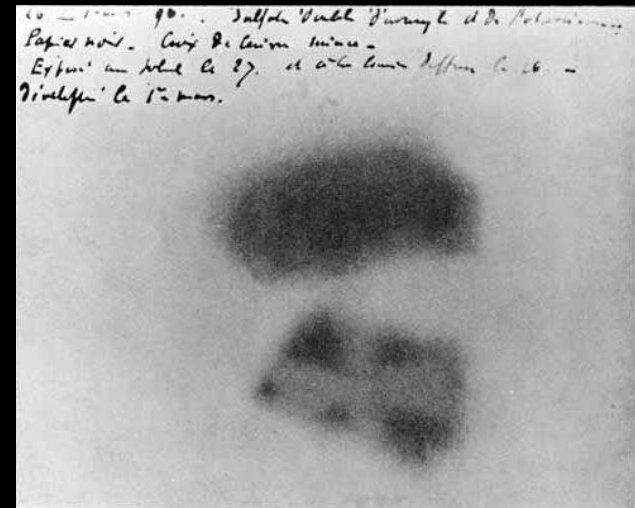
Antoine Henri
Becquerel
Prize share: 1/2



Pierre Curie
Prize share: 1/4

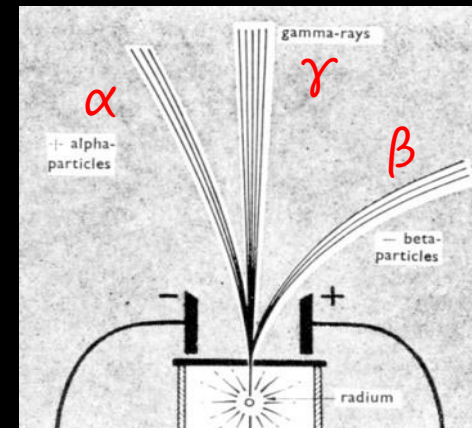


Marie Curie, née
Skłodowska
Prize share: 1/4



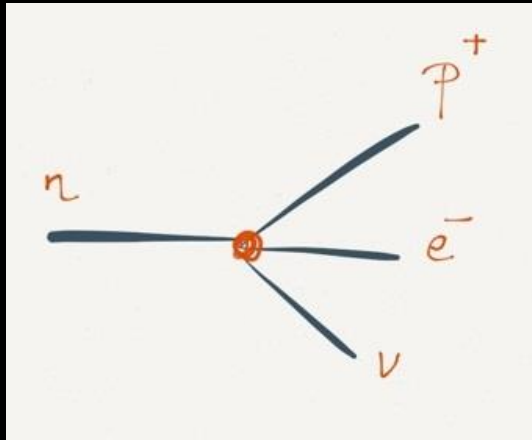
[Becquerel, 1896]

- Further studies by Rutherford/Villard quickly revealed 3 kinds of radioactivities: α, β, γ



Toward a theory of the weak interaction

- β -decay = nuclear transition+electron+antineutrino emission



Fermi “Weak” constant:

$$G_F \approx 1.166 \times 10^{-5} \text{ GeV}^{-2}$$

- 1934, Fermi proposed a first successful theory description, introducing a new fundamental (contact) interaction:

$$\frac{G_F}{\sqrt{2}} (\bar{n} \gamma^\mu p)_{V-A} (\bar{\nu} \gamma_\mu e)_{V-A} + h.c.$$

- 1940's, muon decay also described by Fermi theory with $(\bar{n} \gamma^\mu p) \rightarrow (\bar{\mu} \gamma^\mu \nu_\mu)$ suggesting **β -decay is universal!**
(similarly to QED)

Toward a theory of the weak interaction

- Fermi's "Weak theory" worked well in describing low-energy nuclear processes known at the time:
e.g. e^- nuclear capture, muon decay...

- But the theory is problematic at higher energies:

$$\sigma_{e\nu_{\mu} \rightarrow \mu\nu} \approx \frac{G_F^2 s}{4\pi} \xrightarrow{\sqrt{s} \rightarrow \infty} \infty \quad \sqrt{s} = \text{center of mass } E$$

- Unitary evolution in QM \rightarrow cross-sections are bounded:

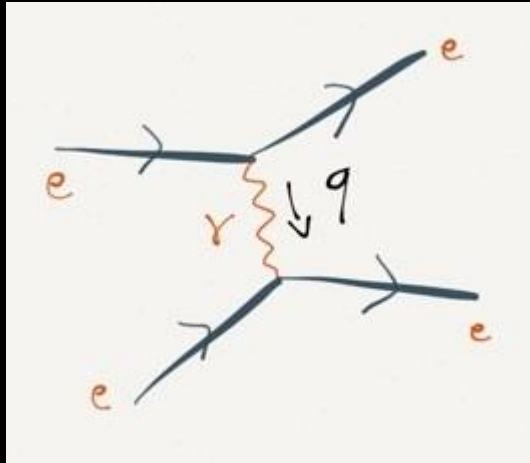
$$\text{any } \sigma \leq \sum_J \frac{2\pi(2J+1)}{s} \approx \frac{2\pi}{s} + \text{higher partial-wave}$$

- QM \rightarrow Fermi's theory **must** be modified below $E \sim 900 \text{ GeV}!!$

Toward a theory of the weak interaction

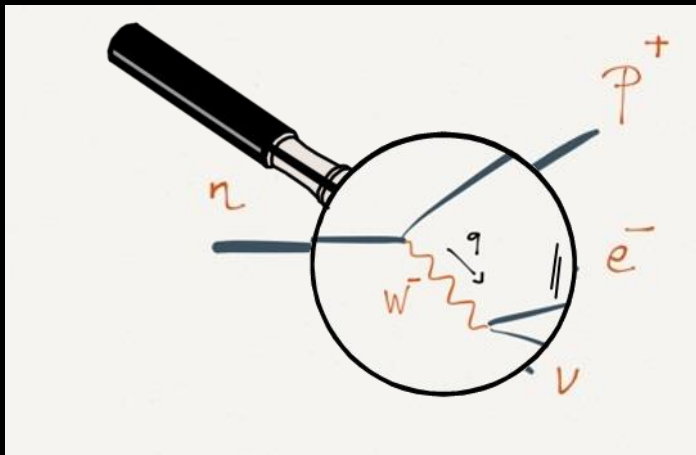
- Idea = mimic current-current interactions in QED:

[Schwinger, '57]



$$\approx i(\bar{e}\gamma^\mu e) \frac{4\pi\alpha}{q^2} (\bar{e}\gamma^\mu e) = i \frac{J_\mu^{em} J_{em}^\mu}{q^2}$$

massless photon propagator
→ long-range force



$$= i(\bar{\nu}\gamma^\mu \nu) \frac{G_F}{\sqrt{2}} \eta_{\mu\nu} (\bar{\nu}\gamma^\nu \nu)$$

$$\frac{(\pi/2)\alpha_W}{q^2 - m_W^2} (\eta_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2})$$

W longitudinal polarization
(see later)

Toward a theory of the weak interaction

- back to $\nu_\mu + e \rightarrow \mu + \nu$ scattering:

$$\sigma_{e\nu_\mu \rightarrow \mu\nu} = \frac{\pi\alpha_W^2 s}{8(s - m_W^2)^2}$$

low-energy regime, $s \ll m_W^2$:

$$\sigma_{e\nu_\mu \rightarrow \mu\nu} \approx \frac{\pi\alpha_W^2 s}{8m_W^4} \left[1 + \mathcal{O}\left(\frac{s}{m_W^2}\right) \right]$$

$$\frac{G_F}{\sqrt{2}} \approx \frac{\pi\alpha_W}{2m_W^2}$$

it “matches”
Fermi’s theory

pushing QED analogy further:

$$\alpha_W \approx \alpha \approx \frac{1}{137} \rightarrow m_W \sim 40 \text{ GeV}$$

high-energy regime, $s \gg m_W^2$:

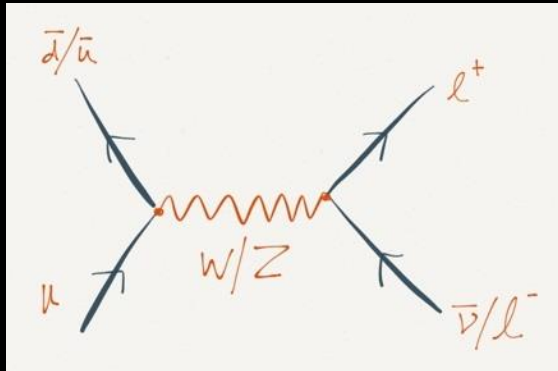
$$\sigma_{e\nu_\mu \rightarrow \mu\nu} \approx \frac{\pi\alpha_W^2}{8s} \left[1 + \mathcal{O}\left(\frac{m_W^2}{s}\right) \right]$$

falls like $\sim \frac{1}{s}$

safe from unitarity problem!

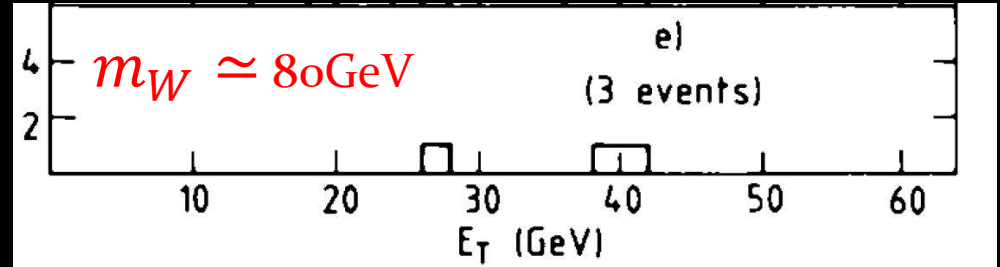
meanwhile at the CERN SPS*:

* $p\bar{p}$ collisions at $\sqrt{s}=540\text{GeV}$



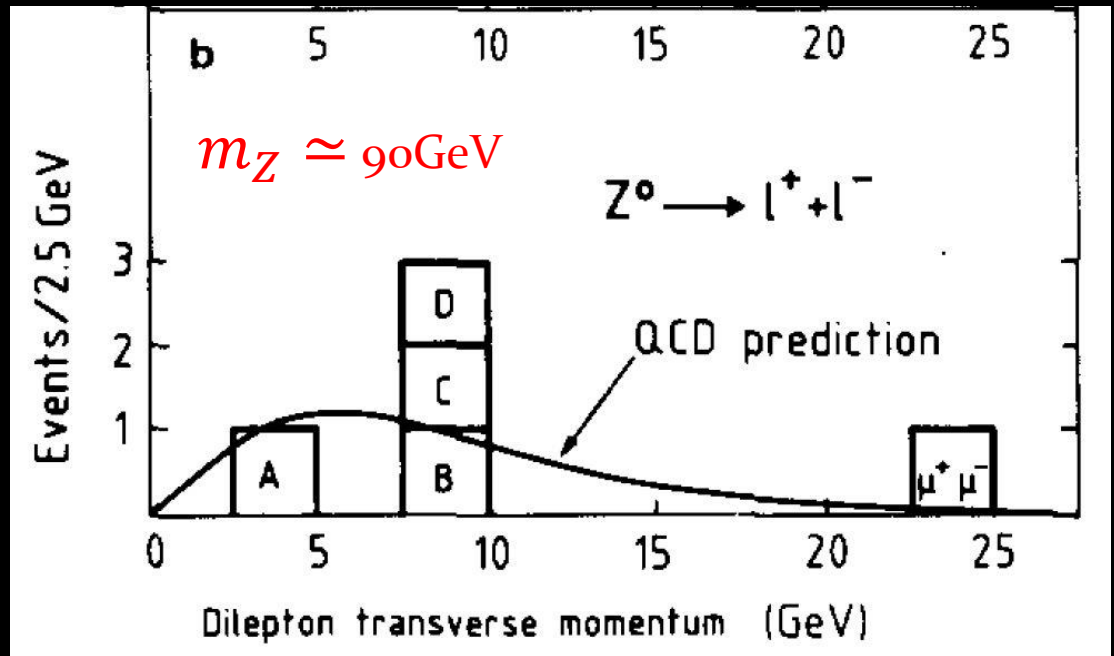
Rubbia, Van der Meer, early '80's

W discovery at UA2, '83



electrons recoiling against invisible ν 's

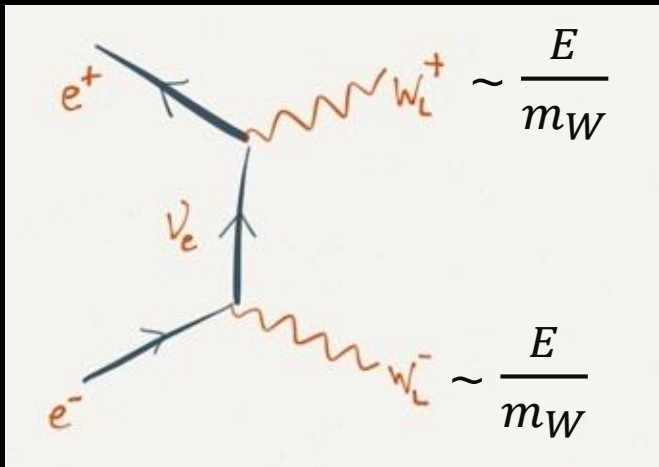
Z discovery at UA1, '83



$\rightarrow \alpha_W \approx \frac{1}{35} \approx 4\alpha$ weak force is not weak

Toward a theory of the weak interaction

- still problem with longitudinally polarized W : $e^+e^- \rightarrow W_L^+W_L^-$



$$E \gg m_W \quad \sim \quad A_0 \frac{E^2}{m_W^2} \quad \text{violate unitarity bound}$$

polarization vectors for plane waves:

W rest frame: $p^\mu = (m_W, 0, 0, 0)$

3 indep. vectors satisfying $\epsilon_i^\mu p_\mu = 0$

$$\epsilon_1^\mu = (0, 1, 0, 0)$$

$$\epsilon_2^\mu = (0, 0, 1, 0)$$

$$\epsilon_3^\mu = (0, 0, 0, 1)$$

\rightarrow
z-boost

$$p^\mu = (E, 0, 0, p) \quad (E^2 - p^2 = m_W^2)$$

$$\epsilon_1^\mu = (0, 1, 0, 0)$$

$$\epsilon_2^\mu = (0, 0, 1, 0)$$

$$\epsilon_3^\mu = \left(\frac{p}{m_W}, 0, 0, \frac{E}{m_W} \right) \xrightarrow{E \gg m_W} \frac{p^\mu}{m_W} + \mathcal{O}^\mu \left(\frac{m_W}{E} \right)$$

Toward a theory of the weak interaction

- part of the solution involves introducing a neutral current:

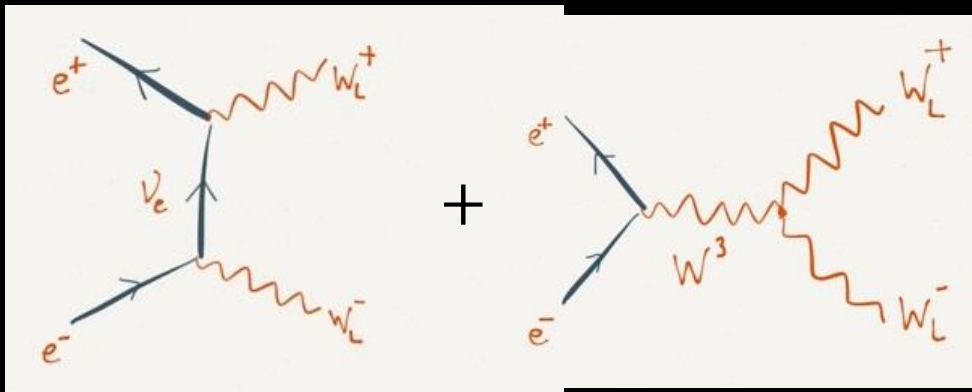
$$\mathcal{L}_{weak} \supset -\frac{g}{2\sqrt{2}} (J_+^\mu W_\mu^- + J_-^\mu W_\mu^+), \quad J_-^\mu = \bar{\nu}_e \gamma^\mu (1-\gamma_5) e + \dots$$

define “weak charges” $Q_\pm \equiv \int d^3x J_\pm^0(x) \rightarrow [Q_+, Q_-] = 2Q_3$

$$Q_3 = \int d^3x J_3^0(x),$$

$$J_3^\mu = \bar{e} \gamma^\mu (1-\gamma_5) e - \bar{\nu}_e \gamma^\mu (1-\gamma_5) \nu_e \neq J_{em}^\mu!$$

introduce associated mediator $\mathcal{L}_{weak} \rightarrow \mathcal{L}_{weak} - \frac{g}{2} J_3^\mu W_\mu^3$



$$\sim \frac{A'_0 E^2}{m_W^2}$$

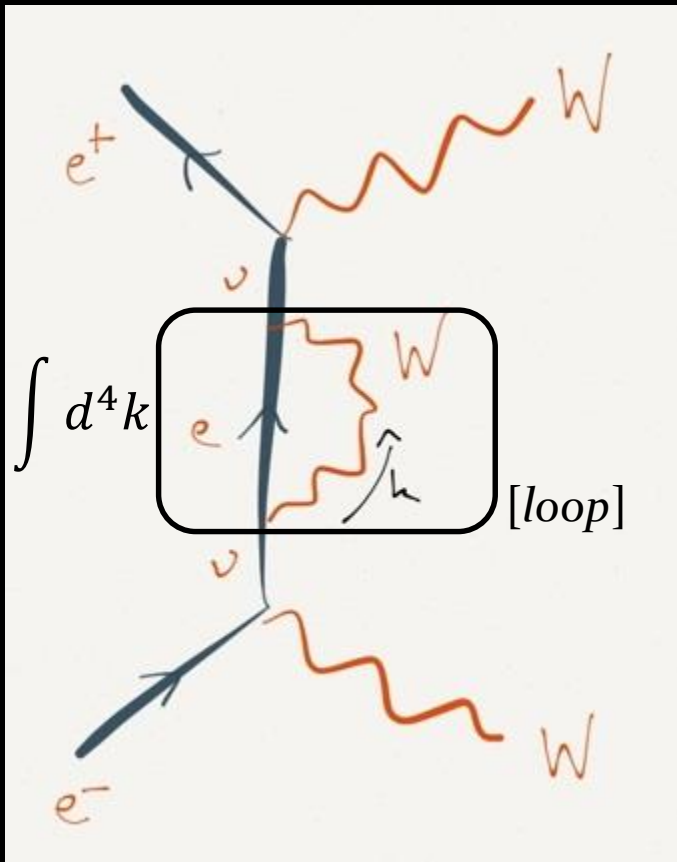
$$\frac{E^2}{m_W^2}$$

$SU(2)_L$ isospin

unitarity saved

Towards a theory of weak interactions

- only classical so far, theory badly behaves at quantum level:



$$W \text{ propagator} = \frac{1}{k^2 - m_W^2} \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_W^2} \right)$$

$$e \text{ propagator} = \frac{1}{k'^2 - m_e^2} (k'_\mu \gamma^\mu + m_e)$$

$k' = k - p$

$$\int d^4k \text{ [loop]} \sim \int^{\Lambda^2} k^2 dk^2 \frac{k^2}{k^4} \approx \Lambda^2$$

→ need gauge theories

QED

interlude

A gauge theory of electrodynamics

- **QED** = $U(1)$ locally invariant theory

$$\mathcal{L}_{Dirac} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \quad \bar{\psi} = \psi^\dagger \gamma^0$$

global symmetry: $\psi \rightarrow e^{i\vartheta} \psi$, $\mathcal{L}_{Dirac} \rightarrow \mathcal{L}_{Dirac}$



Noether's theorem \rightarrow conserved charge/current:

$$Q = \int d^3x J^0(x), \quad J^\mu(x) = \bar{\psi} \gamma^\mu \psi$$

declare local invariance: (*inspired by special relativity?*)

$\psi \rightarrow e^{i\vartheta(x)} \psi$ implies/dictates Q interactions through a long range force!!!

A gauge theory of electrodynamics

- **QED** = $U(1)$ locally invariant theory

$$\mathcal{L}_{Dirac} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

local symmetry:

$$\psi \rightarrow e^{i\vartheta(x)}\psi$$

$$A_\mu \rightarrow A_\mu + e^{-1}\partial_\mu\vartheta$$

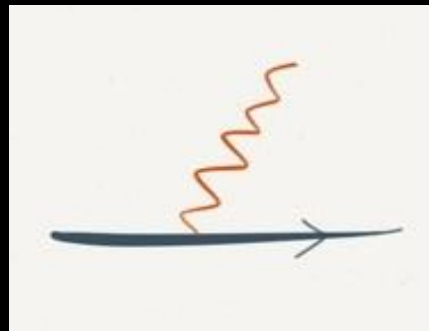
$$D_\mu \equiv \partial_\mu - ieA_\mu$$

$$\mathcal{L}_{QED} = \mathcal{L}_{Dirac} - eJ^\mu A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

current/photon
interaction

kinetic term
photon propagation



A gauge theory of electrodynamics

- **QED** = $U(1)$ locally invariant theory $\psi \rightarrow e^{i\vartheta(x)}\psi$

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - eJ^\mu A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

action:

least action \rightarrow *Euler-Lagrange equations:*

$$S = \int d^4x \mathcal{L}(\varphi_i, \partial_\mu \varphi_i) \quad \delta S = 0 \quad \rightarrow \quad \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_i)} \right) - \frac{\partial \mathcal{L}}{\partial \varphi_i} = 0$$

EL eq. for A_μ :

$$\partial_\mu F^{\mu\nu} = eJ^\nu$$

Maxwell's equations!

$$(\varepsilon^{\mu\nu\rho\sigma} \partial_\nu F_{\rho\sigma} = 0)$$

$$F^{\mu\nu} : \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 4\pi \rho_{\text{tot.}} \\ \nabla \wedge \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} &= 0 \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \wedge \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} &= \frac{4\pi}{c} \mathbf{J}_{\text{tot.}} \end{aligned}$$

A gauge theory of electrodynamics

- QED is impressively predictive at quantum level:

$$e^- \text{ magnetic moment: } \vec{\mu} = g_e(\alpha) \frac{e}{2m} \vec{S}, \quad \alpha = \frac{e^2}{4\pi}$$

experimentally: $\frac{g_e}{2} \approx 1.001159652180(76)$ [Gabrielse, '07]

theoretically:

$$\frac{g_e}{2} = \left[\text{Diagram 1} \right] + \left[\text{Diagram } o(\alpha) \right] + \dots + \left[\text{Diagram } o(\alpha^4) \right] + \dots$$

→ best determination of $\alpha^{-1} \approx 137.035999174(35)$

other independent measurements = testing QED at deep quantum level:

$$e.g. R_\infty = \frac{\alpha^2 m}{4\pi} \rightarrow \alpha^{-1} \approx 137.03599878(91) \quad \approx 10^{-9} \text{ agreement!!}$$

QED *lesson:*

$\psi \rightarrow e^{i\vartheta(x)}\psi$ local invariance \rightarrow extremely successful quantum theory of electrodynamics (tested at the 10^{-9} level!).

\rightarrow Could weak interactions inherit the powerful features of QED? Could one build a gauge theory of the weak interaction?

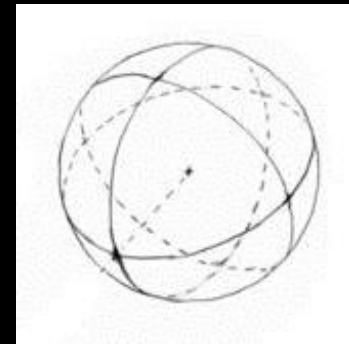
A gauge theory of weak interactions

what's needed?

[not $U(1)^3$]

- 3 conserved currents \rightarrow $SU(2)$ local symmetry

$$[\sigma_i, \sigma_j] = 2\varepsilon_{ijk}\sigma_k \text{ like spin } \rightarrow \text{“isospin”}$$



lepton doublet: $L = \begin{pmatrix} \nu_l \\ l^- \end{pmatrix} \begin{matrix} \uparrow \text{ up} \\ \downarrow \text{ down} \end{matrix} \sim \text{“isospinor”}$

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

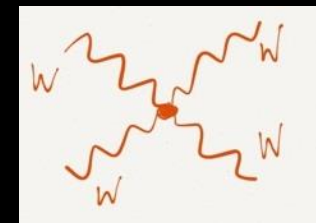
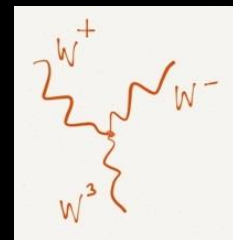
impose local $SU(2)$ invariance: $L \rightarrow e^{i\sigma_a \vartheta^a(x)/2} L$

$$\mathcal{L}_{weak} = i\bar{L}\gamma^\mu \left(\partial_\mu - ig \frac{\sigma_a}{2} W_\mu^a \right) L - \frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu}$$

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\varepsilon_{ijk} W_\mu^j W_\nu^k$$

charged ($\bar{L}\sigma_\pm L$)
& neutral ($\bar{L}\sigma_3 L$) current
Interactions with $W_\mu^{\pm,3}$

$W_\mu^{\pm,3}$ propagation



A gauge theory of weak interactions

- *what's also needed?*

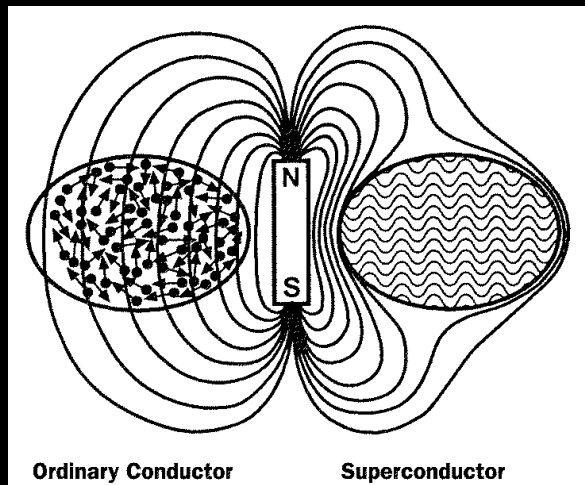
- $m_W \approx 0(100\text{GeV}) \leftarrow \text{Problem!!}$ gauge theories only describe massless force carriers

e.g. U(1) : $m^2 A_\mu A^\mu \rightarrow m^2 (A_\mu - \frac{1}{e} \partial_\mu \vartheta) (A^\mu - \frac{1}{e} \partial^\mu \vartheta) \neq m^2 A_\mu A^\mu$

Solution: *break the symmetry in the vacuum!*

[Y. Nambu, '60]

introduce a vacuum condensate $\langle 0|\varphi|0\rangle \equiv v \neq 0$



*dynamics (\mathcal{L}_{weak}) invariant,
field configurations are not!*

relativistic analogue of
Cooper pair condensate

$\langle e^- e^- \rangle \neq 0 \rightarrow m_{photon} \neq 0$

→ Meissner effect



A gauge theory of weak interactions

- what's also needed?

- $m_W \approx O(100\text{GeV})$

introduce:

$$\Sigma(x) \equiv v \times \exp \left[\frac{i\sigma_a \chi^a(x)}{v} \right]$$

$$\mathcal{L}_{\text{weak}} \rightarrow \mathcal{L}_{\text{weak}} + \frac{1}{4} \text{Tr}[D_\mu \Sigma^\dagger D^\mu \Sigma]$$

$$\Sigma^\dagger \Sigma = 1, \quad \langle \Sigma \rangle = v$$

in unitary gauge

$$\vartheta^a = 2\chi^a/v$$

$$\Sigma = 1$$

$$= \frac{1}{2} m_W^2 W_\mu^a W_a^\mu, \quad m_W = g v / 2$$

$$\Sigma \rightarrow e^{i\sigma_a \vartheta^a(x)/2} \Sigma$$

$$D_\mu \Sigma = \partial_\mu \Sigma - ig \frac{\sigma_a}{2} W_\mu^a \Sigma$$

$$\chi^a \rightarrow \chi^a + \frac{v}{2} \vartheta^a$$

The vacuum condensate v breaks spontaneously $SU(2)$ gauge invariance

$$\rightarrow m_W \sim O(v) \quad \& \quad \chi^a = \text{longitudinal } W_\mu^a$$

A gauge theory of weak interactions

- This theory explains pages of particle physics data, but is still not consistent with unitarity at high E 's:

$$\mathcal{A}_{W_L^+ W_L^- \rightarrow W_L^+ W_L^-} = \text{[Three Feynman diagrams for } W_L^+ W_L^- \rightarrow W_L^+ W_L^- \text{]} \approx \frac{g^2 E^2}{m_W^2}$$

$$\sim \mathcal{O}\left(\frac{E^4}{m_W^4}\right) + \mathcal{O}\left(\frac{E^2}{m_W^2}\right) + \dots \quad \sim \mathcal{O}\left(\frac{E^4}{m_W^4}\right) + \mathcal{O}\left(\frac{E^2}{m_W^2}\right) + \dots$$

The Feynman diagrams in the box show three tree-level processes for $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$: s-channel, t-channel, and u-channel exchange of a W_L boson. Red arrows point from the $\frac{E^4}{m_W^4}$ terms in the asymptotic expansions below to the corresponding diagrams in the box.

→ something new must happen before $E \approx \mathcal{O}(4\pi v) \approx \text{TeV} !!$
in order to restore unitarity

A gauge theory of weak interactions

- This theory explains pages of particle physics data, but is still not consistent with unitarity at high E 's:

$$\mathcal{A}_{W_L^+ W_L^- \rightarrow W_L^+ W_L^-} = \text{[Three diagrams showing } W_L \text{ exchange]} \approx \frac{g^2 E^2}{m_W^2}$$

simplest (weakly coupled) **solution:**

add a scalar particle h

$$m_h < \mathcal{O}(\text{TeV})$$

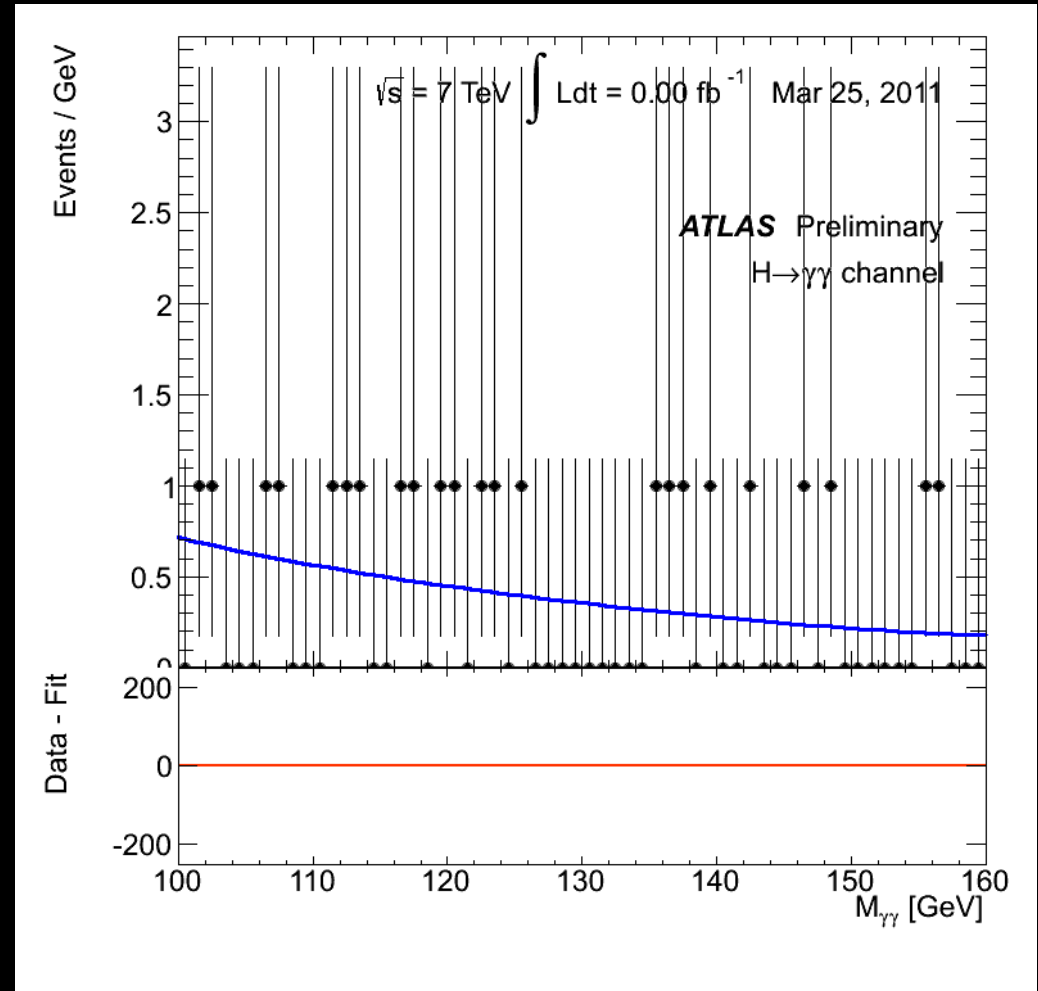
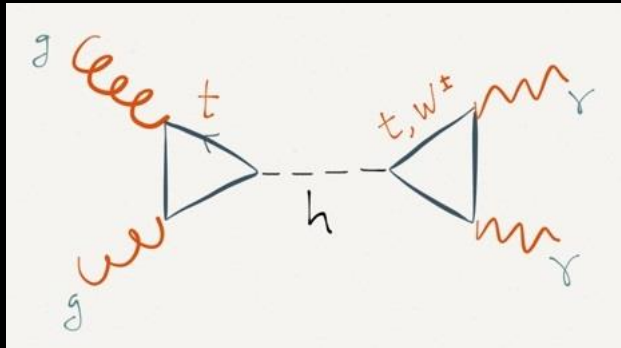
$$g_{hWW} = \frac{gm_W}{2} a$$

$$\text{[Two diagrams showing } h \text{ exchange]} \approx -\frac{g^2 E^2}{m_W^2} a^2$$

if $a \approx 1$, consistent theory up to $E \approx \frac{4\pi v}{\sqrt{|1-a^2|}} \gg \text{TeV}$

$\rightarrow h = \text{“a” Higgs boson}$

meanwhile at the CERN LHC:



On July 4th, 2012 at CERN:
ATLAS/CMS announces to the world
they've found a Higgs-like particle

$$m_h \approx 125\text{GeV}$$

$h \rightarrow WW^*, ZZ^*$ also observed
 $|a - 1| \lesssim 20\%$

The Higgs mechanism

- What if $a = b = 1$ exactly? Theory consistent up to $E \rightarrow \infty$

$$\mathcal{L} = \mathcal{L}_{weak} + \frac{1}{4} Tr[D_\mu \Sigma^\dagger D^\mu \Sigma] \left(1 + 2a \frac{h}{v} + b \frac{h^2}{v^2} + \dots \right) + \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{m_h^2}{2} h^2 + \dots$$

$$\rightarrow \mathcal{L}_{weak} + D_\mu H^\dagger D^\mu H - V(H^\dagger H), \quad H = e^{\frac{i\sigma_a \chi^a(x)}{v}} \left(\frac{0}{v + h(x)} \right) / \sqrt{2}$$

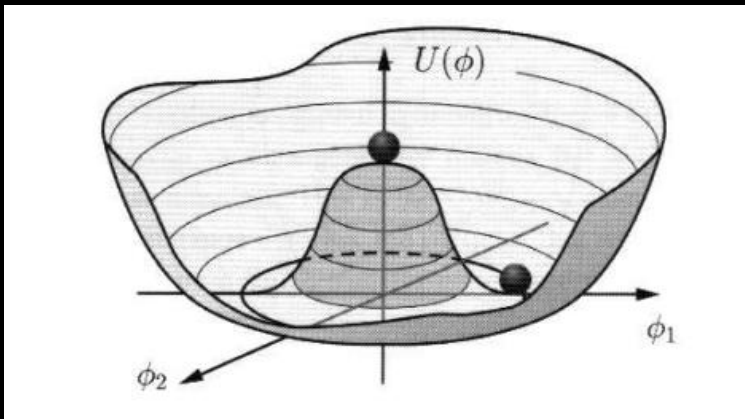
$$V(H^\dagger H) = m^2 H^\dagger H + \lambda (H^\dagger H)^2$$

if $m^2 < 0, \lambda > 0$
 SU(2) broken by
 the vacuum:

$$v = \sqrt{-m^2 / \lambda}$$

This is the so called
 Higgs mechanism

$$m_h^2 = 2\lambda v^2$$



Higgs, Englert, Brout '64

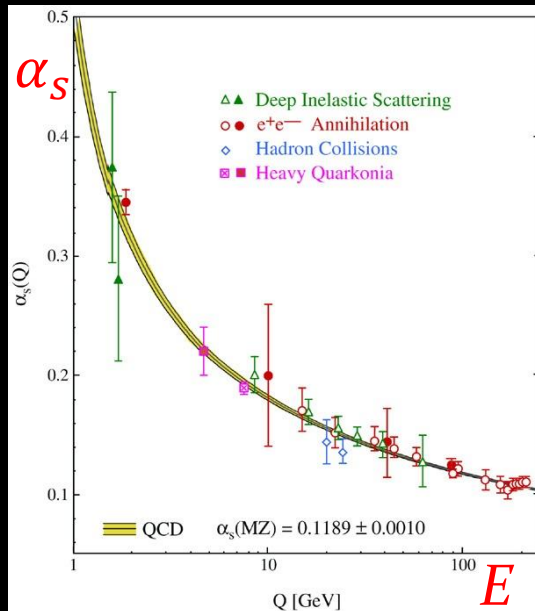
QCD *interlude*

QCD crash course

- Quantum **Chromo**Dynamics = SU(3) gauge theory
 - quarks carry color charge, force carrier = *massless gluons*

$$\mathcal{L}_{QCD} = i\bar{q}\gamma^\mu(\partial_\mu - ig_s T^A G_\mu^A)q - \frac{1}{4}G_{\mu\nu}^A G_A^{\mu\nu}$$

- unlike EW, QCD is weakening at high energy



→ asymptotic freedom



Gross, Politzer, Wilczek '73

- quarks bound in hadrons $\sim \Lambda_{QCD} \sim 200\text{MeV}$
- rational for the parton model for hadron collisions at $E \gg \Lambda_{QCD}$

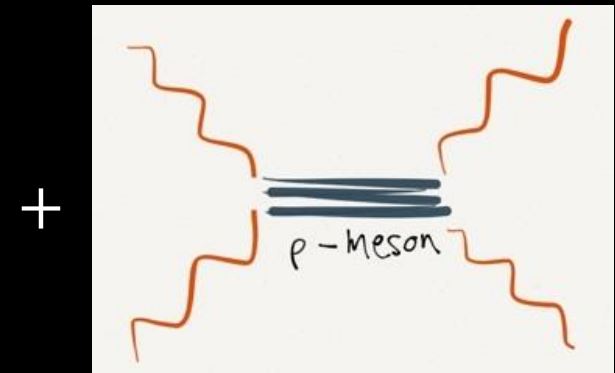
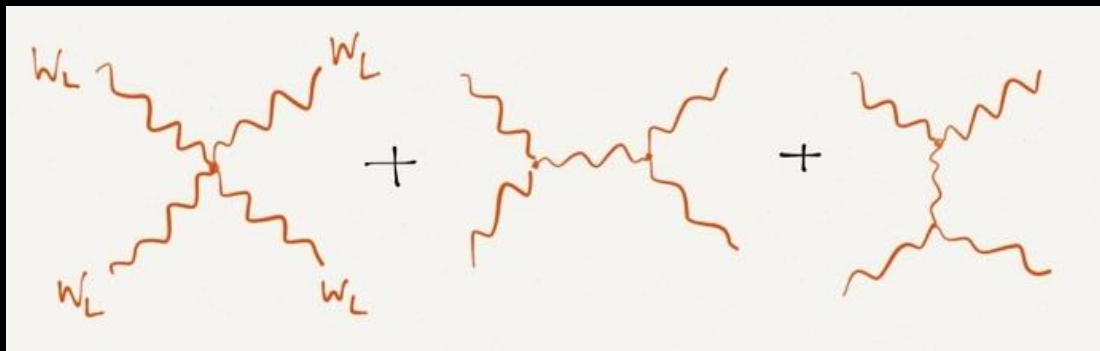
EW symmetry breaking from QCD

- Quantum **Chromo**Dynamics = SU(3) gauge theory
 - quarks carry color charge, force carrier = *massless gluons*

$$\mathcal{L}_{QCD} = i\bar{q}\gamma^\mu(\partial_\mu - ig_s T^A G_\mu^A)q - \frac{1}{4}G_{\mu\nu}^A G_A^{\mu\nu}$$

- for $E \lesssim \text{GeV}$, quark fluctuations “freeze” in the vacuum:

$\langle \bar{q}_L q_R \rangle \sim \Lambda_{QCD}^3$ carries weak charge \rightarrow breaks SU(2)
 even if $v \rightarrow 0$, $m_W \approx \Lambda_{QCD}$



what restores unitarity above Λ_{QCD} ?

spin-1 QCD resonances!

QCD lesson:

Strong interactions can solve the apparent unitarity violation in longitudinal W scattering

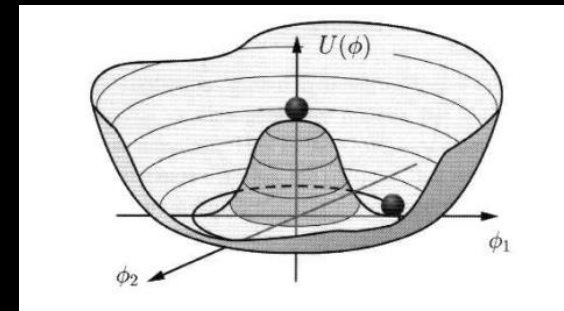
→ *Higgs-less theories like technicolor (i.e. more energetic version of QCD) was also a possibility, as good as a light Higgs scalar,*

but Nature did not choose this...

Lecture 2:

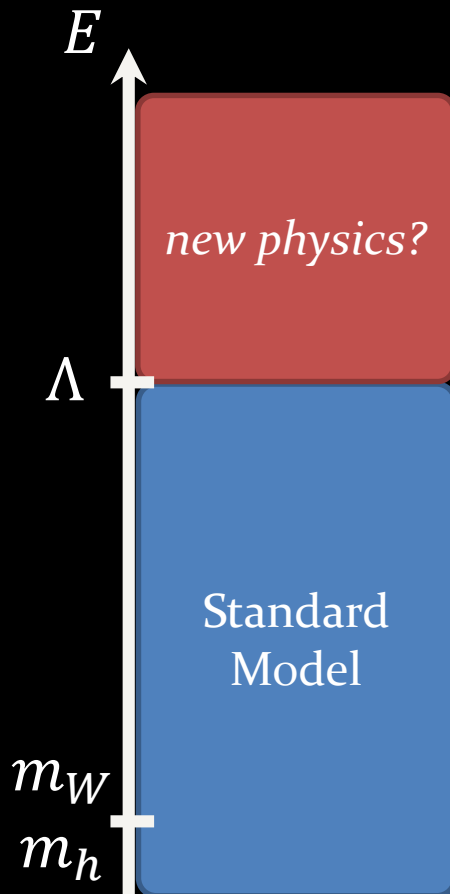
*What is lying behind/beyond
the Higgs?*

The Higgs mechanism



- **Virtues** of the Higgs mechanism:
 - Simple description of weak boson masses:
it provides 3 longitudinal W + unitarization scalar
 - Theory of Weak interactions consistent at all energies
- **Short-comings** of the Higgs mechanism:
 - Mere description of the breaking, not an explanation
what makes $m^2 < 0$?
 - Higgs scalar is very sensitive to unknown physics in the UV
→ severe “hierarchy problem”

The Hierarchy problem



$$m_{h, \text{measured}}^2 = \text{---} \text{---} + \text{---} \text{---}$$

classical mass



quantum correction
new physics at $E \approx \Lambda$

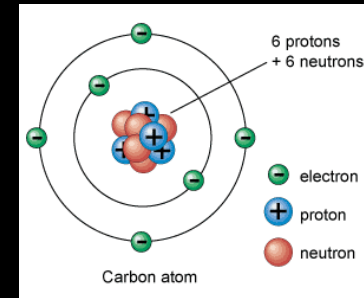
$$(125\text{GeV})^2 = \sim -\mathcal{O}(\Lambda^2) \quad \sim \frac{\alpha_{\text{new}}}{4\pi} \Lambda^2$$

if there is any new dynamics at energies $\Lambda \gg m_h$ which couples to h , its quantum fluctuations will destabilize m_h , unless the (unobservable) classical mass is chosen so to almost exactly cancel this large correction.

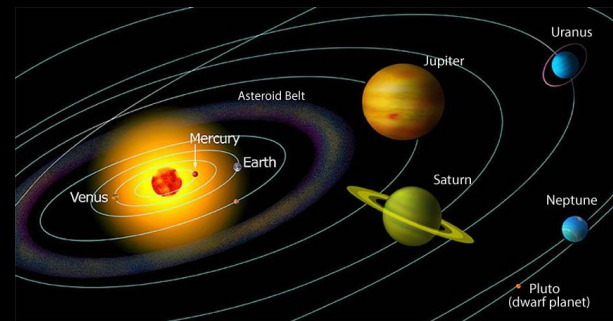
Hierarchy problem \rightarrow short/long distance fine-tuning!

UV physics is irrelevant for IR physics

One does not need to tweak atomic physics:



in order to understand Kepler's laws:



Short distance dynamics “factors out” from long distance one in physical observables → theories are only effective descriptions

$$\mathcal{O}_{full}(E \ll \Lambda) = \mathcal{O}_{long}(E) \times \left[1 + o\left(\frac{E}{\Lambda}\right)^{n>0} \right]$$

Only scalar masses are UV sensitive

| | # of physical polarizations for massless excitations | | # of physical polarizations for massive excitations |
|-------------------|---|---|--|
| vector (spin 1) | 2 | ≠ | 3 |
| spinor (spin 1/2) | 2 | ≠ | 4* |
| scalar (spin 0) | 1 | = | 1 |

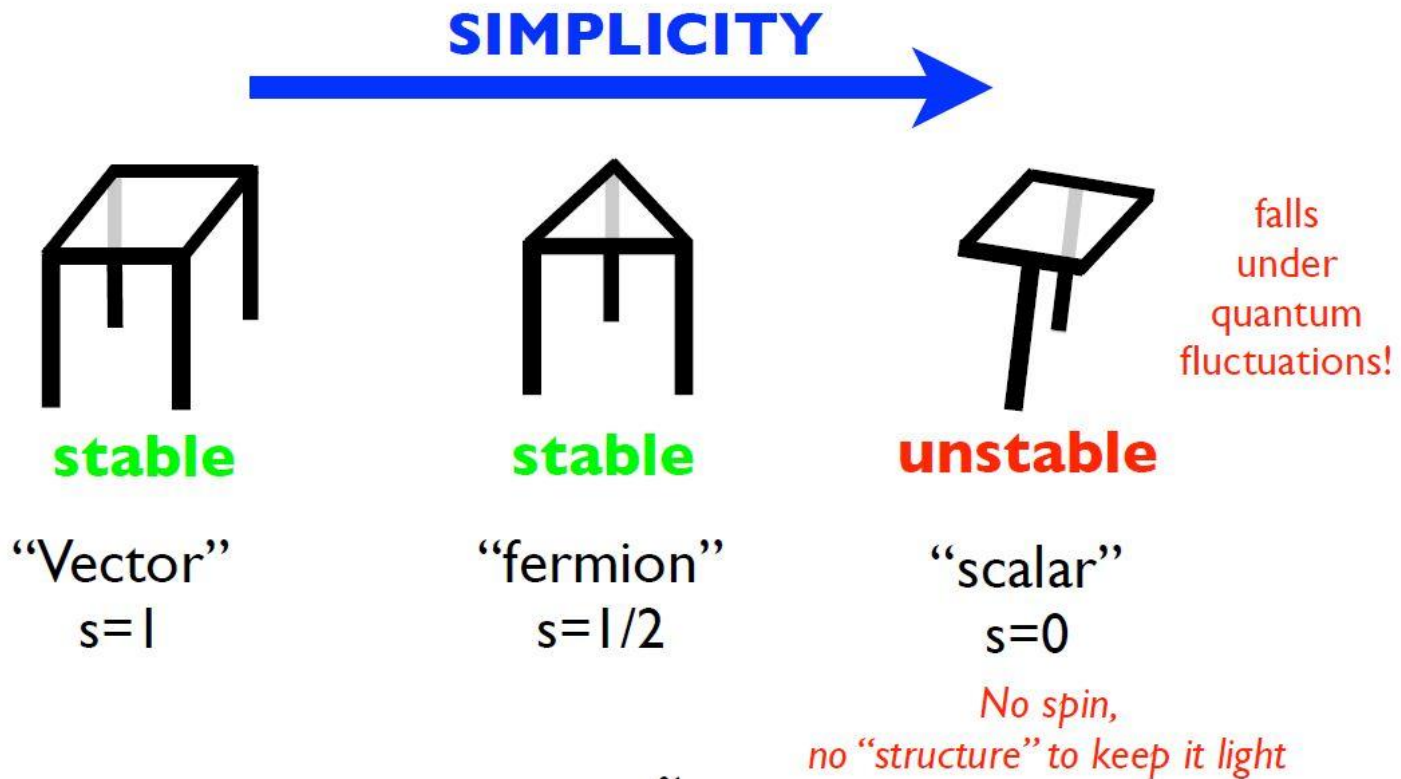
Gauge boson and fermion* masses are stable under quantum fluctuations at the shortest distance because massless and massive states propagate different degrees of freedom.

Scalar particles are too simple to enjoy this property.

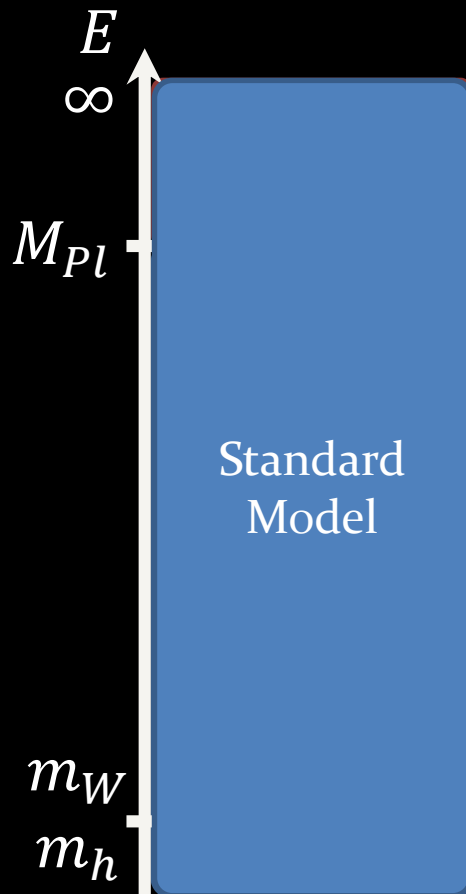
* Majorana fermions are the exception, but they can't carry any charge.

Here is where the **simplicity** of the Higgs mechanism puts it into **trouble**

Since not always simplicity is good:



Is there another scale above m_W ?



→ *no hierarchy, no problem*

but we actually do know
of two scales beyond the SM:

- Gravity: $G_N = M_{Pl}^{-2}$, $M_{Pl} \approx 10^{19} \text{ GeV}$

- QED* Landau pole:

$$\Lambda_Y \sim m_W e^{\frac{2\pi\alpha_Y^{-1}(m_W)}{b_Y}} \approx 10^{41} \text{ GeV}$$

if $\Lambda \approx M_{Pl}$, humongous fine-tuning needed,
as precise as $1/10^{32}!!$

* actually hypercharge

A layman fine-tuning analogue



There are two possibilities:

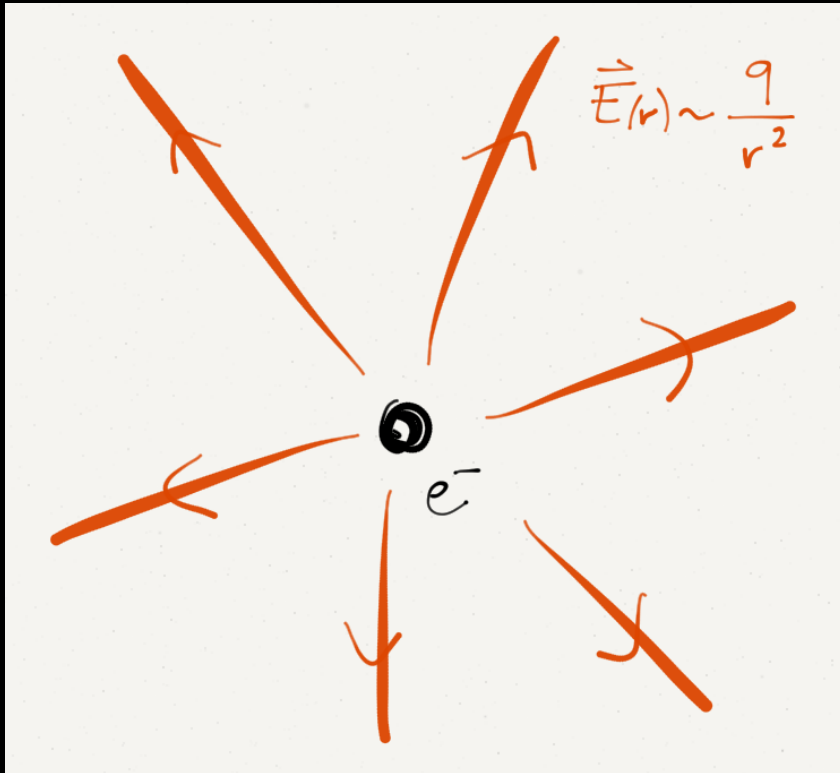
- 1) A few Avogadro numbers $\sim 10^{23}$ of air molecules conspire to all move upwards in order to balance the Earth's gravitational pull...
- 2) There is a trick! Some mechanical structure is hidden and explain the stability

Which would you think is right?

→ What is the structure stabilizing the Higgs mass?

A less pedestrian fine-tuning analogue

→ electron mass in classical physics: $m_e = m_0 + \Delta$



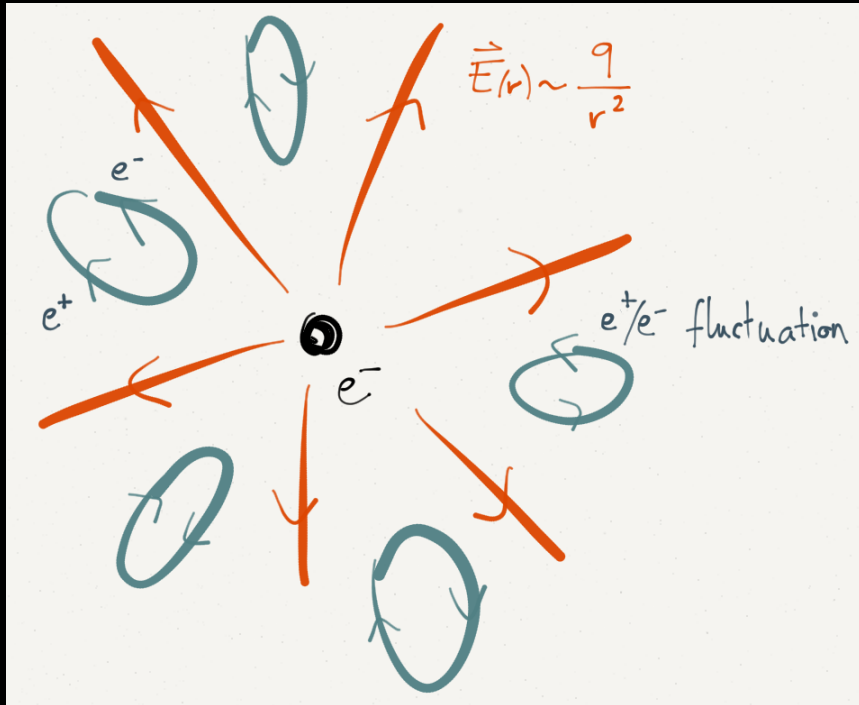
$$\Delta \approx q \int d^3\vec{r} \vec{E}(r)^2$$

$$\approx 4\pi q \int_{\frac{1}{\Lambda}}^{\infty} \frac{r^2 dr}{r^4} \propto \Lambda$$

→ there should be something new around $E \sim \mathcal{O}(m_e)$ to avoid large tuning

A less pedestrian fine-tuning analogue

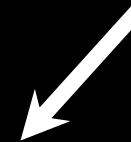
→ electron mass in classical physics: $m_e = m_0 + \Delta$



At $E \approx 2m_e$ or $r \approx 10^{-13}\text{m}$
two new phenomena emerge:
quantum fluctuations + positron



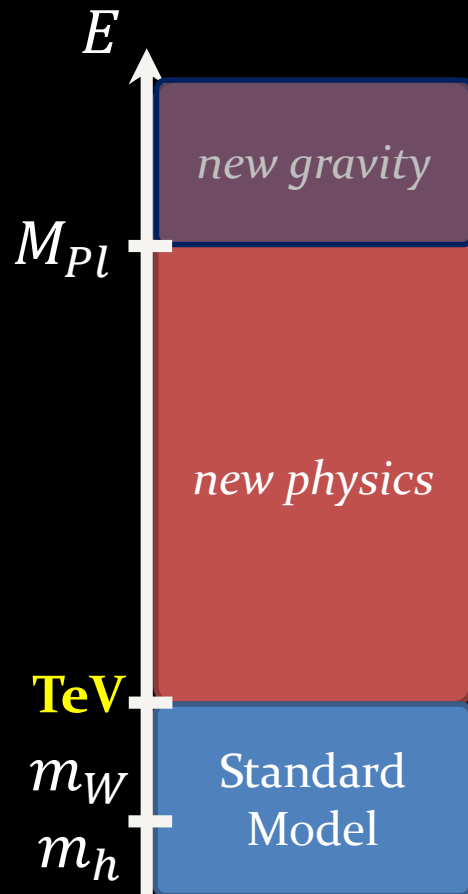
electron/positron pair fluctuations
screen out the (“valence”) electron
from its electric field for $r \lesssim m_e^{-1}$



thus stabilizing the electron mass:

$$\Delta \approx m_e \log \Lambda / m_e$$

TeV scale new physics from Naturalness



At what energy this structure should emerge?

No fine-tuning if $\frac{\alpha_{new}}{4\pi} \Lambda^2 \sim \mathcal{O}(m_h^2)$

$\rightarrow \Lambda \sim \text{TeV}$ within LHC reach!

what kind of structure?

technical naturalness:

m_h^2 is stable under quantum corrections if the theory enjoys a new symmetry when $m_h \rightarrow 0$.



't hooft, '79

Which symmetry to protect m_h ?

- how to forbid $m^2 H^\dagger H$?
 - can't be a new “charge” : $H \rightarrow e^{iX} H$
 - shift symmetry: $H \rightarrow H + c \rightarrow H = \text{Goldstone boson}$
 - “spin trick”: link H to $s \neq 0$ field whose mass is protected

spin = good quantum number under Lorentz symmetries
relating fields of different spin \rightarrow *extend space-time*

S=1:
extradimensions

$$A_M = \begin{pmatrix} A_\mu \\ H \end{pmatrix}$$

S=1/2:
supersymmetry

$$\Phi = \begin{pmatrix} H \\ \psi_H \end{pmatrix}$$

Extradimensions

- we only experience 3+1 dimensions, thus:
 - extradimensions are small
 - they are large, but we are confined on a 3+1 subspace

→ *how can we see them?*

- Gravity cannot be confined:

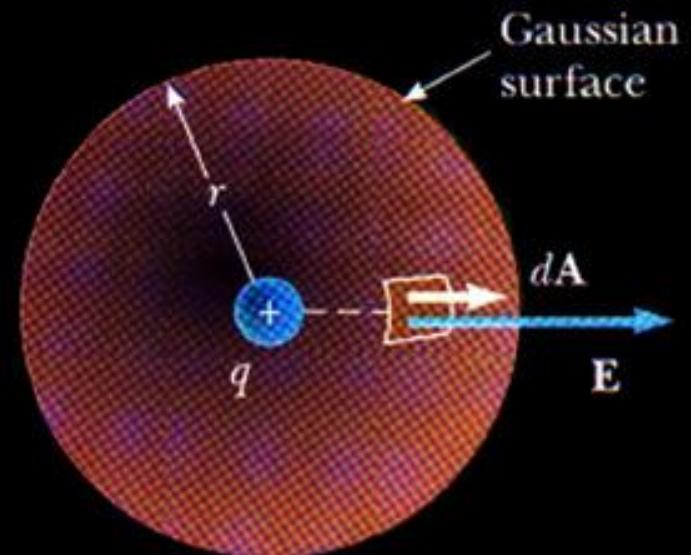
Gauss' theorem:

$$\int \vec{g}(x) \cdot d\vec{S} = -4\pi G_N M$$

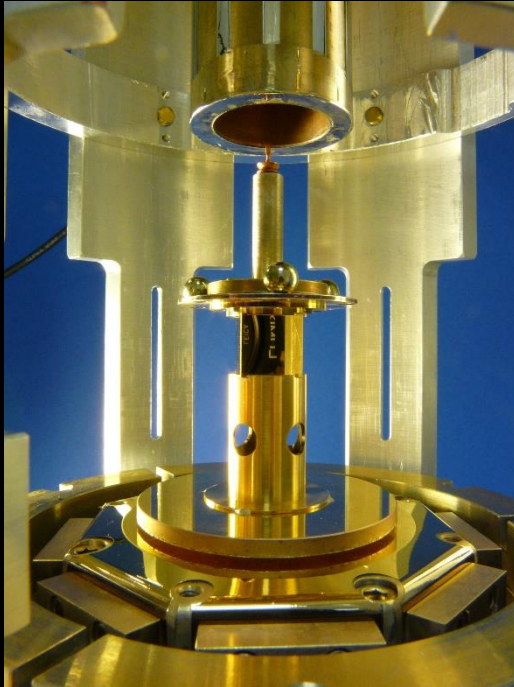
$$\vec{g}(r < \boxed{L}) \sim \frac{G_N M}{r^{2+n}}$$

size of extra dimensions

of extra space dimensions



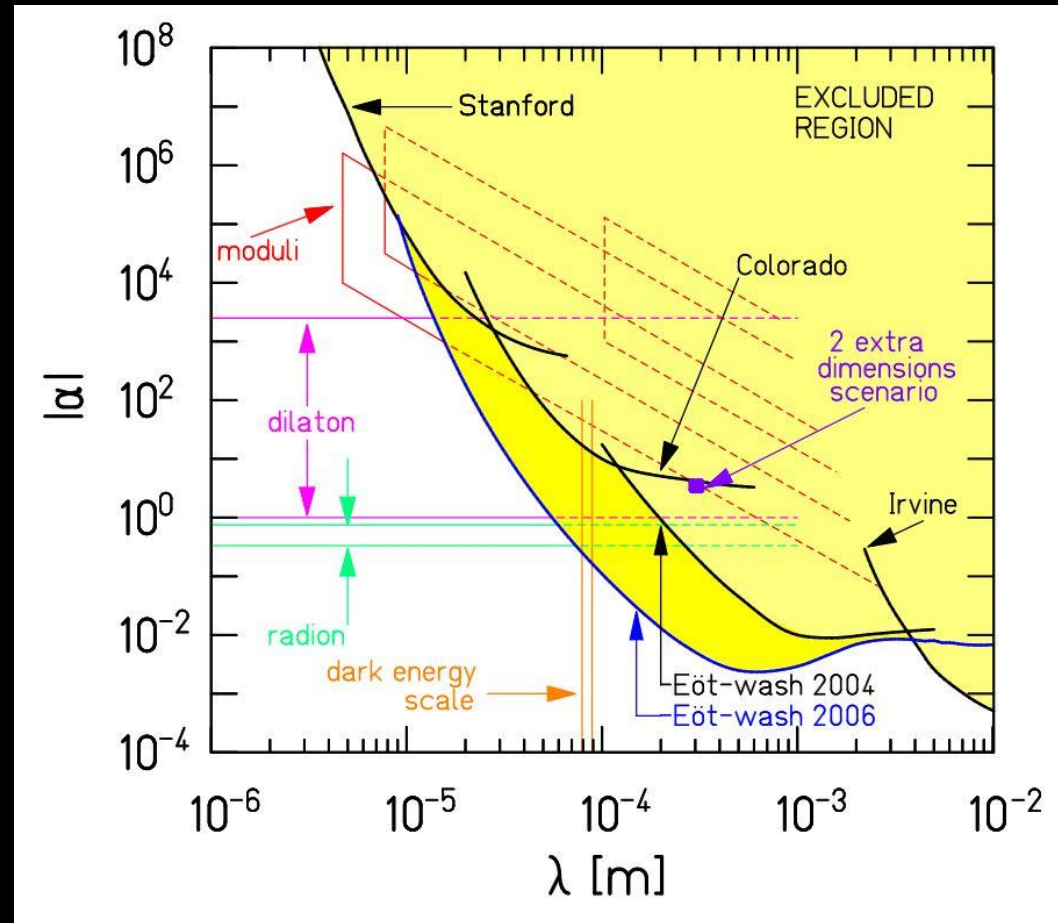
Short range test of Newton's Law



$$V(r) = \frac{G_N m_1 m_2}{r} (1 + \alpha e^{-r/\lambda})$$

No deviation to inverse square law observed down to $\lambda \approx 56\mu\text{m}$

$\rightarrow R \lesssim 40\mu\text{m}$



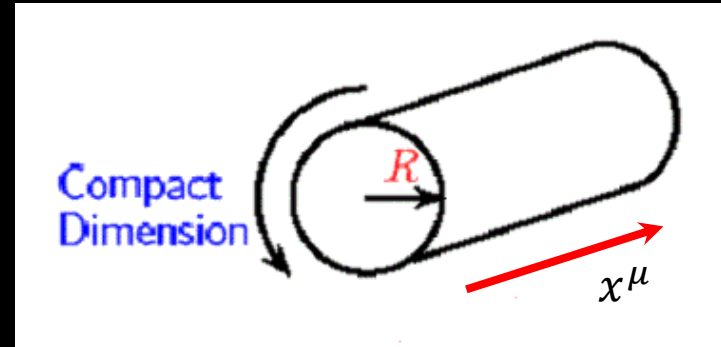
Eöt-Wash experiment, '07

Fields in extradimensions

- 5d example:

$$x^M = (x^\mu, y),$$

$$p^M = (p^\mu, p_5)$$



- if $0 \leq y \leq L \rightarrow p_5 = n/L$ quantized \sim 4d mass

5d momentum conservation: $p^M p_M = p^2 - p_5^2 = 0 \rightarrow p^2 = 0 + n^2/L^2 \equiv m_n^2$

- for $r \gg L$ ($E \ll 1/L$), massless 5d field describes a tower of massive 4d Kaluza-Klein fields:

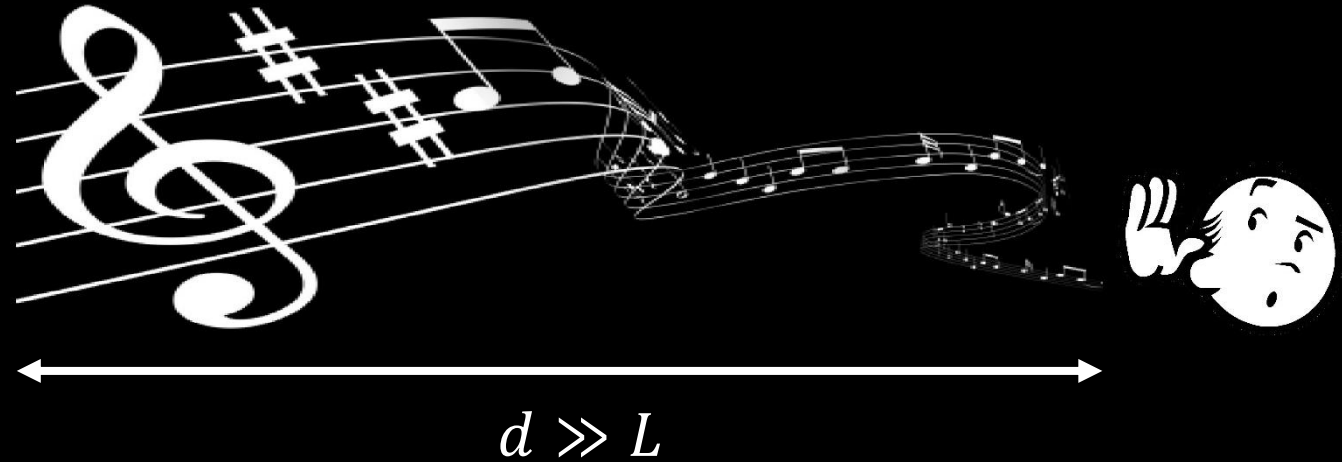
$$\phi(x, y) = \sum_{n=0}^{\infty} \phi_n(x) f_n(y)$$

5d "wave-function"



Kaluza, Klein, 1920's

KK states = signature of extradimensions



How to tell from afar that sounds originate from an extended object?

check the spectrum!

→ only $f \approx n/L$ harmonics propagate over long distances

Fields in extradimensions

- compact y breaks 5d space-time symmetries \rightarrow 4d:

- y -translation breaking \rightarrow 5d mass is not “conserved”

massless 5d field = Σ massive 4d fields

- $x^\mu y$ -rotation breaking \rightarrow 5d spin is not “conserved”

$A_M = (A_\mu, A_5)$ 5d spin 1 = 4d spin 1 + spin 0

- For $E \gg 1/L$, 5d symmetries are restored
 \rightarrow scalar mass protected by 5d gauge symmetry

$$m_h \sim 1/L \rightarrow L \sim \mathcal{O}(\text{TeV}^{-1})$$

Supersymmetry

- Extended space-time w/ “fermionic dimensions”:

Superspace: $X = (x^\mu, \theta^\alpha)$ $\alpha = 1, \dots, 4$

4d coordinates

fermionic coordinate
 $\theta^\alpha = (\text{Majorana}) \text{ spinor}$

$$\{\theta, \theta\} = 0 \rightarrow \theta^2 = 0$$

Only one step in θ is allowed

$$\{Q_\alpha, Q_\beta\} = 2(\gamma^\mu \gamma_0 \gamma_2)_{\alpha\beta} \mathcal{P}_\mu$$

two steps in $\theta = \text{translation}$

Superfields in Superspace

- Susy[**scalar**]~**spinor**, $Q_\alpha \phi \sim \psi_\alpha \rightarrow$ spin is not “conserved”:

$$\Phi(X) = \Phi(x, \theta) = \phi(x) + \bar{\theta}\psi(x)$$

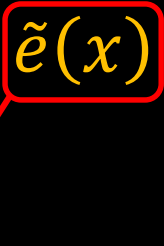
- $[Q_\alpha, \mathcal{P}_\mu] = 0 \rightarrow$ mass is still conserved $m_\phi = m_\psi$!

\rightarrow scalars inherit protection from fermions



Supersymmetry is broken

- By Susy, all fermions have a degenerate scalar partners:

$$\Phi_{electron}(X) = \tilde{e}(x) + \bar{\theta}\psi_e(x)$$


NOT observed, although it carries EW charge

- Supersymmetry has to be broken at some scale Λ_{SUSY} :

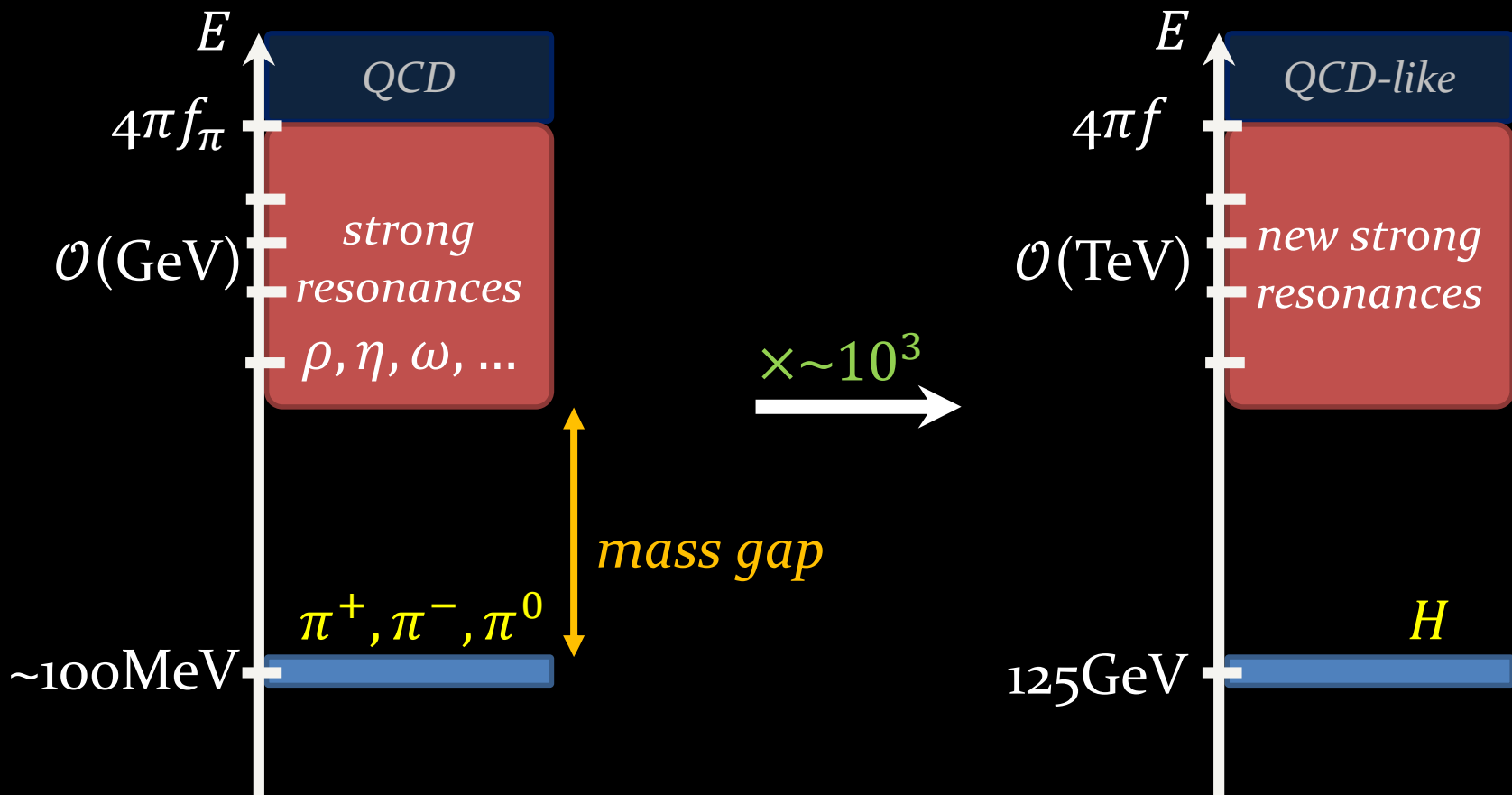
$$m_{\tilde{e}} \sim \Lambda_{\text{SUSY}} \gg m_e$$

- Higgs mass is no longer fully protected:

$$m_h \sim \Lambda_{\text{SUSY}} \rightarrow \Lambda_{\text{SUSY}} \sim \mathcal{O}(\text{TeV})$$

Higgs as a pseudo Goldstone boson

- more mundane option: stay in 4d, once again mimick QCD



Higgs as a pseudo Goldstone boson

- Light π 's thanks to chiral symmetry breaking: $q = u, d$

$$\mathcal{L}_{QCD} = i\bar{q}_L \gamma^\mu D_\mu q_L + i\bar{q}_R \gamma^\mu D_\mu q_R - \frac{1}{4} G_{\mu\nu}^A G_A^{\mu\nu} - \bar{q}_L m_q q_R + h.c.$$

global symmetry: $SU(2)_L \times SU(2)_R$, $q_{L,R} \rightarrow \mathcal{U}_{L,R} q_{L,R}$

- $g_s \rightarrow o(4\pi) \rightarrow \langle \bar{q}_L q_R \rangle \sim \Lambda_{QCD}^3$

symmetry breaks: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$

$$q_{L,R} \rightarrow \mathcal{U} q_{L,R}$$

- Goldstone theorem:

“global continuous symmetry

broken in the vacuum \rightarrow massless scalar”

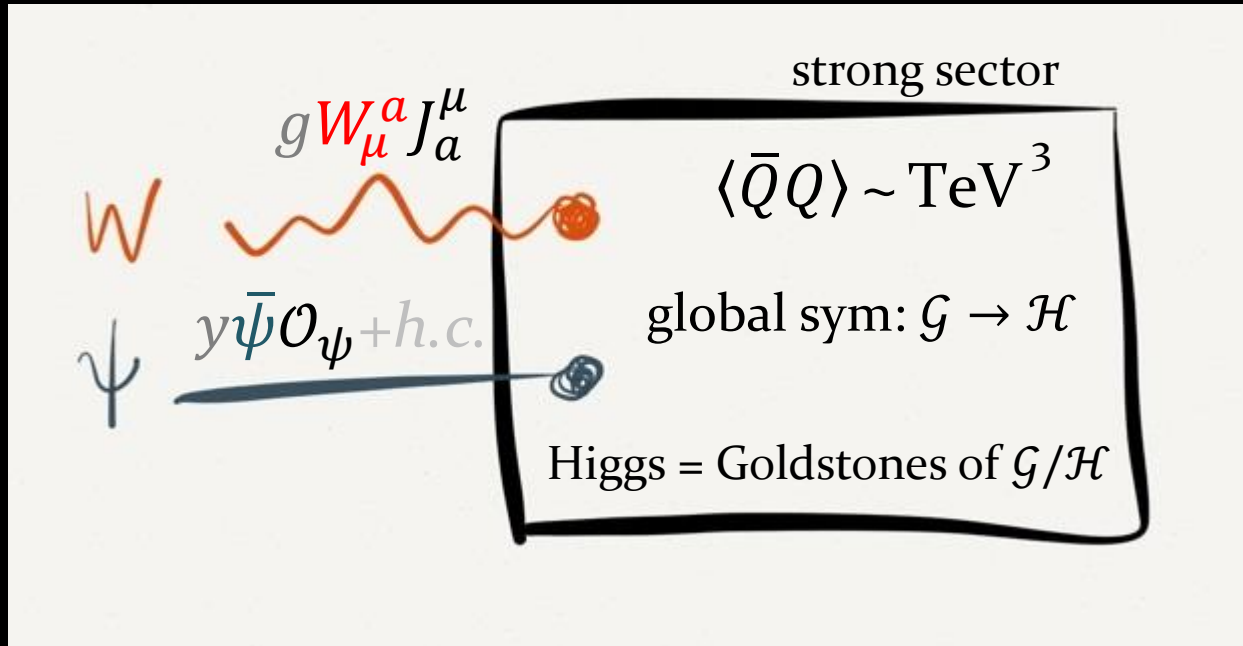
$\pi^{\pm,0} = 3$ massless Goldstone bosons
of $SU(2)^2/SU(2)$

$$\rightarrow m_\pi^2 \approx \frac{(m_u + m_d) \Lambda_{QCD}^3}{f_\pi^2}$$



Higgs as a pseudo Goldstone boson

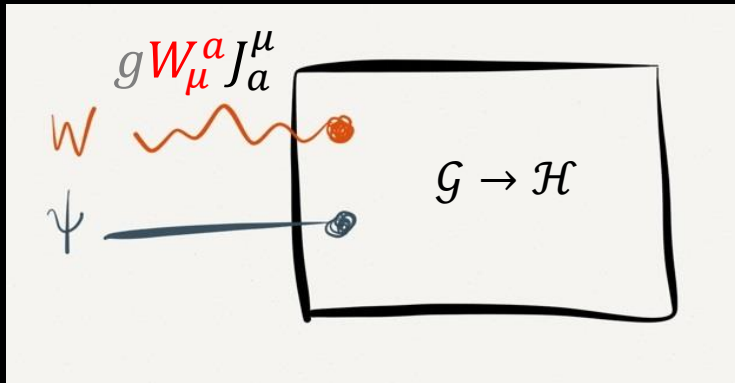
- Add a new strong force (SU(N)?) with “techni-quarks” Q



- *global symmetry breaking $\mathcal{G} \rightarrow \mathcal{H}$ at scale $f \sim \text{TeV}$*
- *unbroken \mathcal{H} must contain SU(2)*
- *g, y breaks \mathcal{G} explicitly $\rightarrow m_h = m_h(g, y)$ and naturally small!*

Higgs as a pseudo Goldstone boson

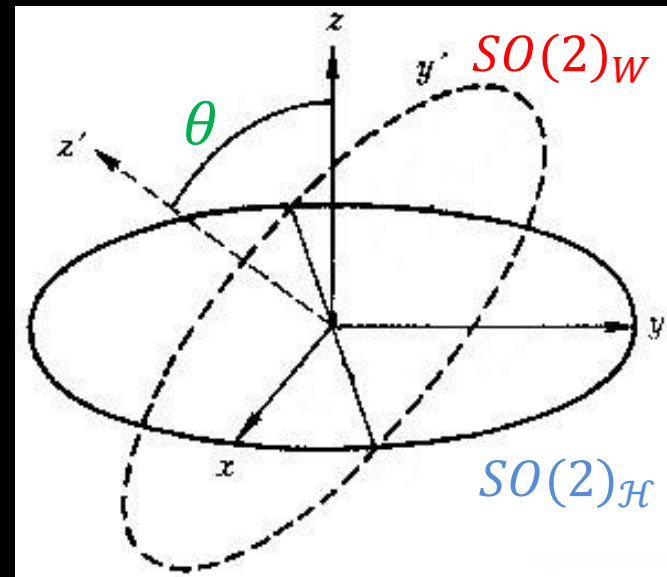
- EW symmetry breaking/ m_W from vacuum misalignment:



if the $SU(2)$ associated with J_μ^a is not aligned with the $SU(2)$ in \mathcal{H} , then $\mathcal{G} \rightarrow \mathcal{H}$ breaking induces EW gauge symmetry breaking.

$SO(3) \rightarrow SO(2)$ analogue:

$$v = f \sin \theta$$



Time to wrap up!

Conclusions 1

- Despite ~100yrs of progress, we still do not know the complete mechanism driving the weak force.
- Since Fermi, unitarity clearly indicated a scale below which the theory must be modified (in the form of introducing new particles)
- After the discovery of the Higgs boson, this is no longer the case. The Standard Model is consistent potentially up to the Planck scale, where gravity has to be modified.

Conclusions 2

- Yet, in the SM, the Higgs boson is much lighter only at the price of a $1/10^{32}$ fine-tuning miracle.
- A naturally light Higgs boson requires to extend the theory beyond the SM at $E \sim \text{TeV}$.
- Two well motivated avenues:
 1. Extend space-time |
 2. Add new forces
- All solutions predict new particles w/in LHC reach

Do not hesitate:



LHsea

more

Space-time symmetries crash course

- Poincaré group = Lorentz $SO(3,1)$ symmetry + translations

Symmetries → Generators

| | | | |
|--------------|---|-----------------------------------|--|
| Rotations | → | angular momentum \mathcal{J}_i | $[\mathcal{J}_i, \mathcal{J}_j] = i\varepsilon_{ijk} \mathcal{J}_k$ |
| Boosts | → | \mathcal{K}_i | $[\mathcal{K}_i, \mathcal{K}_j] = -i\varepsilon_{ijk} \mathcal{J}_k$ $[\mathcal{J}_i, \mathcal{K}_j] = i\varepsilon_{ijk} \mathcal{K}_k$ |
| Translations | → | energy/momentum \mathcal{P}_μ | $[\mathcal{J}_i, \mathcal{P}_j] = i\varepsilon_{ijk} \mathcal{P}_k$ $[\mathcal{K}_i, \mathcal{P}_j] = -i\delta_{ij} \mathcal{P}_0$ $[\mathcal{P}_0, \mathcal{J}_i] = 0, [\mathcal{P}_0, \mathcal{K}_i] = i\mathcal{P}_i$ |

Representations characterized by two invariants: *mass, spin*

- Physical particles are representations of Poincaré group:

e.g. ϕ =scalar, V^μ =vector, $T^{\mu\nu}$ =tensor,... + ψ =spinor

$S=0$

$S=1$

$S=2$

$S=1/2$

Spinor crash course

- $[J_m + iK_m, J_n - iK_n] = 0 \rightarrow SO(3,1) \approx SU(2)_L \times SU(2)_R$

$SU(2)_{L,R}$ representations labelled by $j_{L,R} = 0, 1/2, 1, 3/2, \dots$

$$\begin{aligned} (j_L, j_R) = & \quad (0,0) \quad \text{scalar} \\ & \quad (1/2,0) \quad \text{left-handed spinor} \\ & \quad (0,1/2) \quad \text{right-handed spinor} \\ & \quad (1/2,1/2) \quad \text{vector} \\ & \quad \dots \end{aligned}$$

$$\text{Dirac spinor} = (1/2,0) + (0,1/2)$$

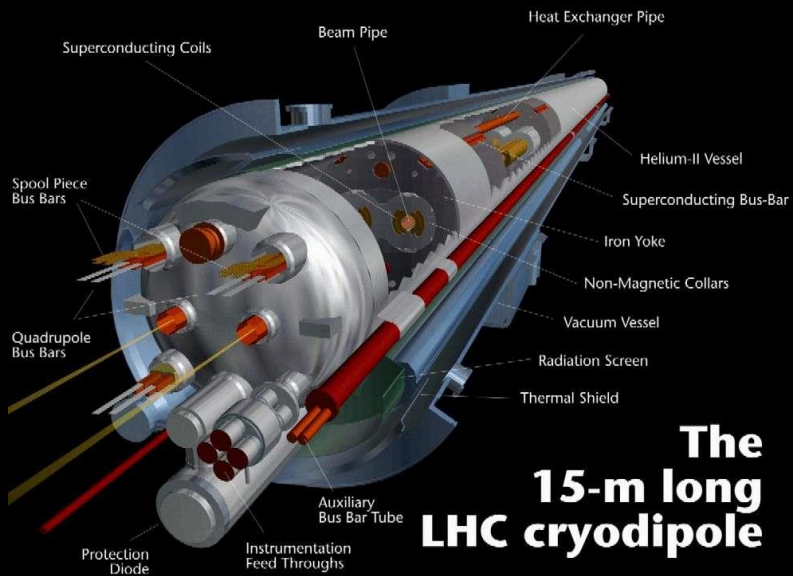
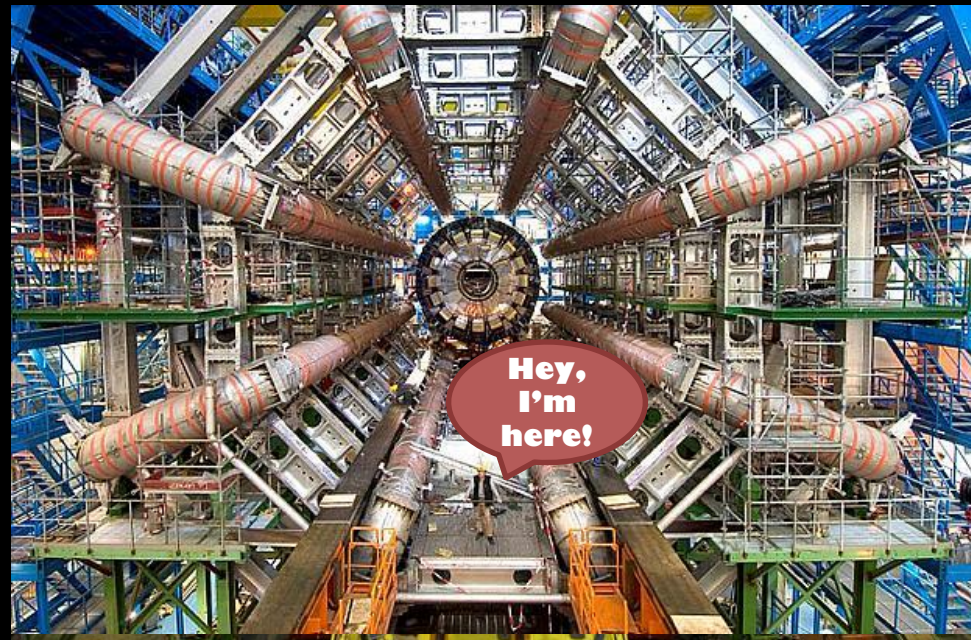
it is not “fundamental”, but reducible

$(1/2,0)$ and $(0,1/2)$ can a priori have different interactions

$$\text{Majorana spinor} = (1/2,0) + (1/2,0)^c \quad \text{for neutral fermions only}$$

LHC

Large Hadron Collider



The 15-m long LHC cryodipole

How large is large?

- **total cost** ~ 9 billion € (~1/5 of French public debt yearly interest)
- **size**: → ring ~ 27km | -z ~ 100m
→ detectors ~ $25 \times 50 \pi \text{m}^3$ [ATLAS]
- **beam pipe**:
 - T ~ 1.9K (~30% colder than the Universe sparsest regions)
 - B field ~ 8.4T (~300 000 Earth's magnetic field)
 - current ~ 12kA (~40 000 common light bulb current)
- **beam**:
 - proton/proton (occasionally lead)
 - quasi-luminal speed: $\gamma \sim 7000$ (~ 10^4 laps/♥-beat!!!)
 - kinetic energy ~ $7 \times 2 = 14 \text{TeV}$ (TeV = 10^{12} eV)
- **collisions**:
 - $6 \times 10^8/\text{s}$
 - ~GB/s of data, of which only ~100MB/s recorded

Why is LHC so large?

quantum mechanics: $\Delta x \times \Delta p \geq \hbar/2$

We want to probe $\Delta x \sim 10^{-18/-19} \text{m} \sim L_{\text{proton}}/10^3$,
where weak force separates from QED,

$$\rightarrow E \sim pc > 100 \text{GeV}/1 \text{TeV}$$

(~10TeV needed because protons are composite.)

High energies \rightarrow long accelerators (magnet limited)
+ big calorimeter to stop outgoing particles.