Naturalness, renormalisation, high energies puzzles

Many people are puzzled by two recent experimental observations:

The Higgs mass is low

We have not seen any new particles around the TeV scale. The SM seems to be fine at that scale, and perhaps it may be fine to much higher energies.

What is the origin of these puzzles?

The naturalness principle. In high E physics, we have a fairly clear way of formulating it, and it has to do with UV completion of theories, the hierarchy problem, the separation of scales….

General remarks

It is a good idea to reflect on the way we have gained information in Science over the last 2-3 milennia, before we get into the technicalities of our current (mis)understanding on naturalness.

It is useful to read P. Nelson (Am. Sci. 1985), and G. Giudice (Naturally Speaking, 2008) to reflect on these problems.

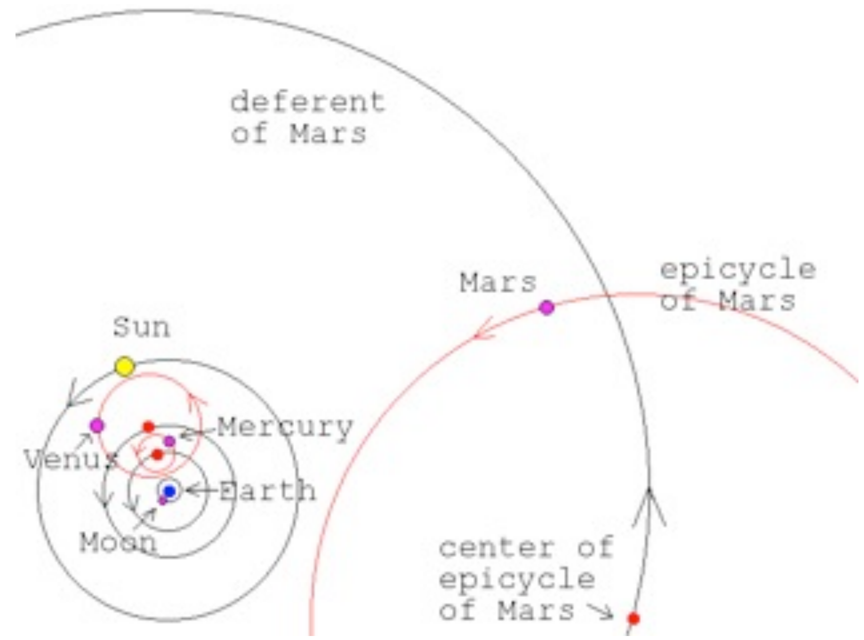
We will start with general arguments, then present technically how in QFT we can ask the questions sharply. The dependence on regulators, UV-completions, and why we should or should not be puzzled about the lightness of the Higgs particle.

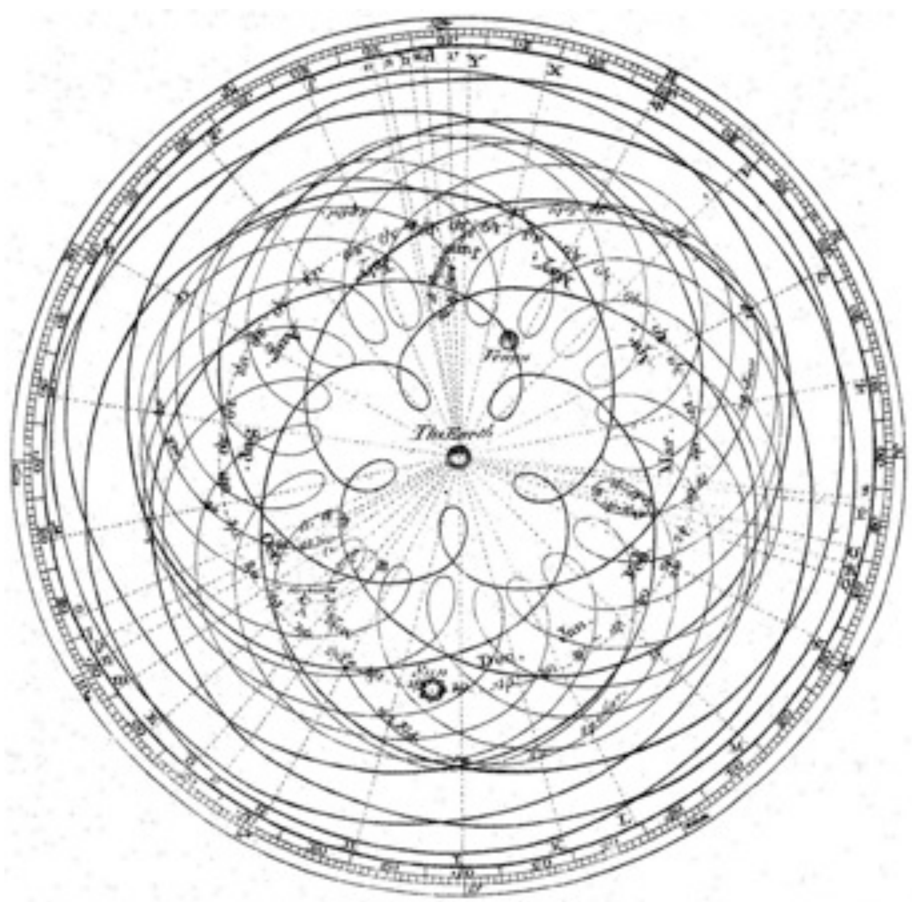
Structural Naturalness

Numerical Naturalness

Ptolemy is structurally unnatural, as seen from today's point of view

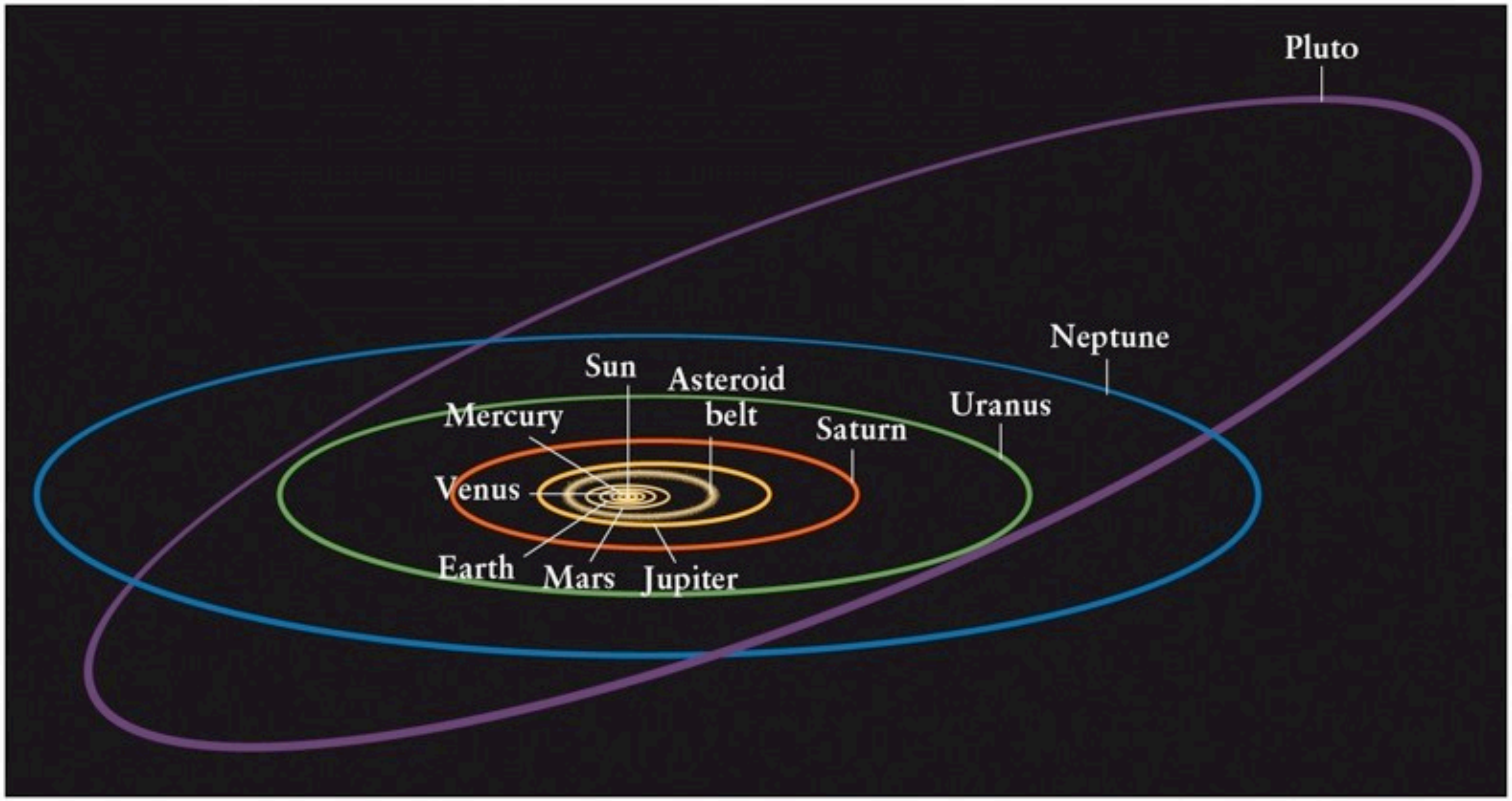
Numerical Naturalness: it is very unnatural that the velocity of the earth vanishes





Thursday, 23 January, 2014

There is no rationale for the distance of the planets to the Sun. It is all environmental. It depends on the initial conditions. Other planetary systems are vastly different. It is not a good question to ask for a law for the distances of planets to the Sun. Lanscapes, multiverses



Some general features of scientific progress seem to be related to our ability to accomplish a

SEPARATION OF SCALES

Reductionism, hierarchical structures. Insulation of scales

Copernican arguments: Landscapes

The role of symmetries, and their implementation (space-time, internal, unbroken, broken…

Effective field theories between different scales

Consistent matching

Large numbers, large ratios

We know at least two fundamental scales in our understanding of the fundamental forces in Nature

$$\frac{G_F\,\hbar^2}{G_N\,c^2} = 1.73859(15)\,10^{33}$$

$$\frac{M_{\rm NP}}{M_{\rm SM}} \sim 10^{13}\text{-}10^{16}$$

What keeps these scales stably separated? What explains large numbers. Dirac, Dicke arguments, back to Landscapes and multiverses

Large numbers, large ratios

In QFT the problem of large numbers, or naturalness, appears in a very dramatic way in terms of the so-called hierarchy problem

What protects low energy scales from high energy. Naturalness criterion. In a nutshell, the problem is summarised in the next two formula. They contain several hidden structures: UV structures, renormalisation, effective field theories, separation of scales…

We are used to deconstructing reality and scaling

$$\delta m^2 \sim \frac{g^2}{16\pi^2}\Lambda^2$$
$$m_{\rm H}^2 = m_0^2 + \delta m^2 \ll \Lambda^2$$

Reviewing regularisation, renormalisation and effective field theories

In QFT when computing quantum corrections, we need to regulate the theory to tame the infinities originating from the relativistic invariance and the locality principle.

It is precisely the qualitative behaviour of the theory as we change the scale at which we look at it that allows us to ask questions on separation of scales in a precise way.

The naive believe is that when some “unnatural” dependence on an intermediate scale appears, some new “physics” particles will show up to tame the unnatural behaviour. There are several nice examples

A few examples perhaps illustrate the naturalness reasoning:

Electron self-energy

Charge pion mass difference

Kaons mass difference

The Higgs mass correction, the failed example

The electron as a sphere of radius r

$$\frac{\alpha}{r} < m_e c^2 \qquad r > 3 \times 10^{-15}\mathrm{m}$$

In QED:

$$\alpha\, m_e \log(m_e r)$$

For the pion mass difference:

$$M^2_{\pi^+} - M^2_{\pi^0} = \frac{3\alpha}{4\pi}\Lambda^2 \qquad \Lambda < 800\mathrm{MeV}$$

For the kaon mass difference:

$$\frac{M_{K^0_L} - M_{K^0_S}}{M_{K^0_L}} = \frac{G_F^2 f_K^2}{6\pi^2}\sin^2\theta_C\,\Lambda^2 \qquad \Lambda < 2\mathrm{GeV}$$

$$\delta m_H^2 = \frac{3G_F}{4\sqrt{2}\pi^2}\left(4m_t^2 - 2m_W^2 - m_Z^2 - m_H^2\right)\Lambda^2 \qquad \Lambda < 1\mathrm{TeV}$$

Practical computational tools
The mathematical coding of the high energy dependence

DR a useful computational tool

Multiloop integrals are integrals over rational functions. In the early 70's a beautiful regularisation system was introduced: Analytically continue in the number of dimensions. In principle to complex dimension d, and then the divergences are recovered as we go back to four-dimensions in terms of poles in (d-4) of different order.

Many advantages to this procedure:

❖ It automatically preserves the symmetries that can be extended to d-dimensions. This is the case of vector-like theories like QCD, QED…

❖ It easily permits a simple renormalisation in terms of a mass independent subtraction scheme, more difficult to implement in other regularisation prescriptions. The RGE are particularly simple.

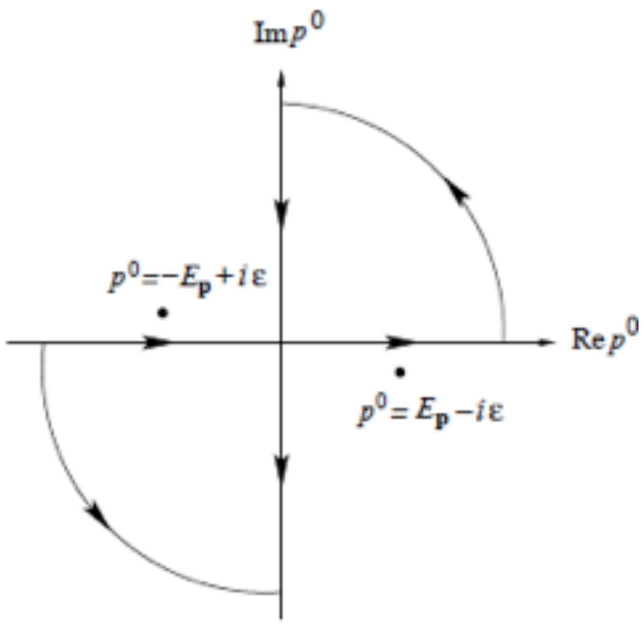
❖ Absence of polynomial divergences

❖ Subtle for chiral theories

❖ So far, nobody has found a non-perturbative extension of DR

$$I_n(d,m^2) = \int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2 - m^2 + i\varepsilon)^n}.$$

$$\frac{1}{(p^2 - m^2 + i\varepsilon)^n} = \frac{1}{[(p^0)^2 - E_{\vec{p}}^2 + i\varepsilon]^n}$$



Wick rotation

$$\int_{-\infty}^{\infty} \frac{dp^0}{2\pi} \frac{1}{[(p^0)^2 - E_{\vec{p}}^2 + i\varepsilon]^n} = i(-1)^n \int_{-\infty}^{\infty} \frac{dp_E^0}{2\pi} \frac{1}{[(p_E^0)^2 + E_{\vec{p}}^2]^n}$$

continued...

$$\frac{1}{a^n} = \frac{1}{\Gamma(n)} \int_0^\infty dt\, t^{n-1} e^{-az} \qquad (a > 0)$$

$$I_n(d, m^2) = \frac{i(-1)^n}{(4\pi)^{2+\frac{d-4}{2}}} \frac{\Gamma(n-2-\frac{d-4}{2})}{\Gamma(n)(m^2)^{n-2-\frac{d-4}{2}}}$$

$$\Gamma(-k+\varepsilon) = \frac{(-1)^k}{k!}\left[\frac{1}{\varepsilon} + \psi(k+1) + \mathcal{O}(\varepsilon)\right] \qquad k \in \mathbb{N},$$

$$\gamma = -\psi(1) = 0.5772\ldots$$

$$\psi(z) = \frac{d}{dz}\log\Gamma(z), \qquad \psi(k+1) = -\gamma + \sum_{n=1}^{k}\frac{1}{n}$$

$$I_n(d, m^2) \xrightarrow{d\to 4} -\frac{i(m^2)^{2-n}}{16\pi^2}\frac{2}{d-4} + \text{finite part}, \qquad n = 1, 2.$$

$$I_n(d, m^2, q) = \int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2 + 2p \cdot q - m^2 + i\epsilon)^n}$$

$$I_n(d, m^2, q) = I_n(d, m^2 + q^2)$$

If the number of loops is L, generically we get a Laurent expansion

$$\frac{1}{(d-4)^L}, \quad \frac{1}{(d-4)^{L-1}}, \; \cdots, \frac{1}{d-4},$$

General principles

The analytic continuation satisfies some simple axioms

$$\begin{aligned}\int d^dp\left[f(p)+g(p)\right] &= \int d^dp\,f(p)+\int d^dp\,g(p),\\ \int d^dp\,f(\lambda\,p) &= \lambda^{-d}\int d^dp\,f(p) \qquad (\text{with }\lambda\in\mathbb{C}),\\ \int d^dp\,f(p+k) &= \int d^dp\,f(p).\end{aligned}$$

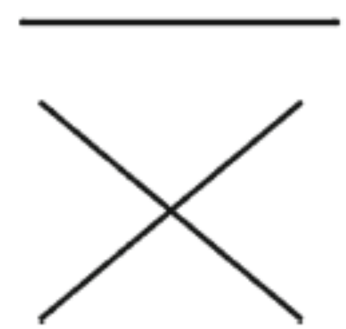
By carefully analysing the previous integrals we obtain from analytic continuation that

$$\int d^dp\,(p^2)^n = 0.$$

The fact that DR eliminates quadratic divergences might seem surprising in the light of the previous discussion of the hierarchy problem. Indeed, since DR regularizes the quadratic divergences to zero it seems that the whole hierarchy problem results from using a clumsy regulator and that by using DR we could shield the Higgs mass from the scale of new physics. This is not the case, but for interesting reasons. In spite of DR the Higgs mass is still sensitive to high energy scales. If it is ever found with a low mass, we will also get relevant information on what shields its mass from the higher scales. Before explaining these interesting reasons, we need to develop more theory.

A case study

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4.$$



$$\Longrightarrow \quad \frac{i}{p^2 - m^2 + i\varepsilon}$$

$$\Longrightarrow \quad -i\lambda$$

d-dimensional action and vertices

$$S = \int d^d x\; \mathcal{L}(\phi, \partial_\mu\phi)$$

d-dimensions of fields and couplings

$$D_\phi = \frac{d-2}{2} \qquad D_\psi = \frac{d-1}{2}, \qquad D_A = \frac{d-2}{2} D_m = 1, \qquad D_\lambda = 4-d$$

$$\lambda'\phi^3 \qquad \Longrightarrow \qquad D_{\lambda'} = 1 + \frac{4-d}{2}$$

$$g\phi\overline{\psi}\psi \qquad \Longrightarrow \qquad D_g = \frac{4-d}{2}$$

introducing a scale by making the coupling dimensionless

$$\lambda \longrightarrow \mu^{4-d}\lambda$$

DR one-loop renormalisation

the renormalised Lagrangian depends on the bare couplings and fields

$$\mathcal{L}_{\rm ren} \;=\; \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 - \frac{m_0^2}{2}\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4 = \mathcal{L} + \mathcal{L}_{\rm ct}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\mu^{4-d}\phi^4$$

$$\mathcal{L}_{\rm ct} = \frac{1}{2}A(d-4)\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2B(d-4)\phi^2 - \frac{\lambda}{4!}\mu^{4-d}C(d-4)\phi^4$$

the A, B, C functions contain all the dependence of the renormalised Lagrangian on the regulator and the bare quantities. The renormalisation conditions are used to express m and the coupling in terms of observable quantities

$$\phi_0(x) \;\equiv\; \sqrt{Z_\phi(d-4)}\phi(x) = \sqrt{1+A(d-4)}\phi(x)$$

$$m_0^2(d-4) \;=\; m^2\,\frac{1+B(d-4)}{1+A(d-4)}$$

$$\lambda_0(d-4) \;=\; \lambda\mu^{4-d}\frac{1+C(d-4)}{[1+A(d-4)]^2}$$

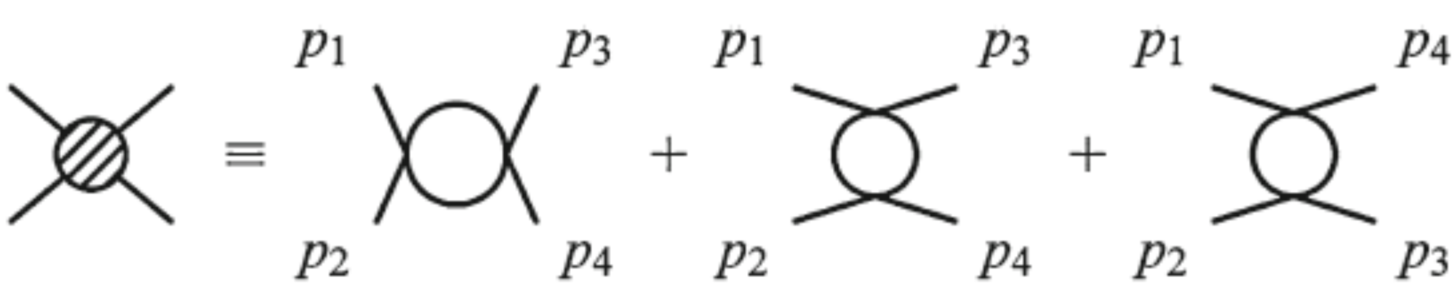
Examples

$$= \frac{1}{2}\lambda\mu^{4-d}\int\frac{d^dp}{(2\pi)^d}\frac{1}{p^2-m^2+i\varepsilon}$$

$$= -i\frac{\lambda m^2}{16\pi^2}\frac{1}{d-4} + \text{finite part.}$$

$$\delta m^2 = -\frac{\lambda m^2}{16\pi^2}\frac{1}{d-4}.$$

$$= -i\delta m^2(2\pi)^4\delta^{(4)}(p_1+p_2).$$



$$\frac{\lambda^2}{2}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2+i\varepsilon}\left[\frac{1}{(k+p_1+p_2)^2-m^2+i\varepsilon}\right.$$
$$\left. + \frac{1}{(k+p_1+p_3)^2-m^2+i\varepsilon} + \frac{1}{(k+p_1+p_4)^2-m^2+i\varepsilon}\right]$$

$$\frac{1}{a_1 a_2} = \int_0^1 \frac{dx}{[x a_1 + (1-x) a_2]^2} \qquad \delta\lambda = -\frac{3\lambda^2}{16\pi^2}\frac{1}{d-4}$$

$$-i\delta\lambda(2\pi)^4\delta^{(4)}(p_1+p_2+p_3+p_4)$$

$$\begin{aligned} A(d-4)_{1-\mathrm{loop}} &= 0, \\ B(d-4)_{1-\mathrm{loop}} &= -\frac{\lambda}{16\pi^2}\frac{1}{d-4}, \\ C(d-4)_{1-\mathrm{loop}} &= -\frac{3\lambda}{16\pi^2}\frac{1}{d-4}. \end{aligned}$$

We notice that the construction of counterterms is intrinsically ambiguous because together with the divergent part we can subtract a finite contribution. In our analysis we just removed the pole parts without imposing any renormalization condition at a particular value of the momenta. This is called minimal subtraction. Subtracting also some numerical factor is the modified minimal subtraction. These are mass-independent subtraction schemes, the simplest to work with. Let's see the effect on the 1PI correlation functions

$$\overline{\mathrm{MS}} \qquad \gamma - \log(4\pi)$$

1PI correlation functions, the RGE

$$\Gamma_n(p_i; m_0, \lambda_0, d-4)_0 = Z_\phi(d-4)^{-\frac{n}{2}}\Gamma_n(p_i; m, \lambda, \mu, d-4),$$

$$\Big[\mu\frac{\partial}{\partial\mu} + \beta\left(\lambda, \frac{m}{\mu}, d-4\right)\frac{\partial}{\partial\lambda} + \gamma_m\left(\lambda, \frac{m}{\mu}, d-4\right) m\frac{\partial}{\partial m} - n\gamma\left(\lambda, \frac{m}{\mu}, d-4\right)\Big]\Gamma_n(p_i; m, \lambda, \mu, d-4) = 0$$

$$\begin{aligned}
\beta\left(\lambda, \frac{m}{\mu}, d-4\right) &= \mu\frac{\partial\lambda}{\partial\mu}, \\
\gamma_m\left(\lambda, \frac{m}{\mu}, d-4\right) &= \frac{\mu}{m}\frac{\partial m}{\partial\mu}, \\
\gamma\left(\lambda, \frac{m}{\mu}, d-4\right) &= \frac{1}{2}\mu\frac{\partial}{\partial\mu}\log Z_\phi.
\end{aligned}$$

we are getting there, we want to study the energy dependence of the different correlation functions, masses, couplings etcetera. All that is needed now is to use Euler's theorem for homogeneous functions. It is a simple exercise to check that the dimension of the 1PI is given by:

$$D_n = 4 - n - \frac{d-4}{2}(n-2)$$

Energy scale dependence

$$\Gamma_n(\xi s p_i; \lambda, \xi m, \xi\mu, d-4) = \xi^{D_n} \Gamma_n(s p_i; \lambda, m, \mu, d-4)$$

$$\left(\mu \frac{\partial}{\partial \mu} + s \frac{\partial}{\partial s} + m \frac{\partial}{\partial m} - D_n\right) \Gamma_n(s p_i; \lambda, m, \mu, d-4) = 0$$

trading the mu derivative for the s-derivative yields the Callan-Symanzik equation:

$$\left\{ -s \frac{\partial}{\partial s} + \beta\left(\lambda, \frac{m}{\mu}\right) \frac{\partial}{\partial \lambda} + \left[\gamma_m\left(\lambda, \frac{m}{\mu}\right) - 1\right] m \frac{\partial}{\partial m} \right.$$
$$\left. +4 - n\left[1 + \gamma\left(\lambda, \frac{m}{\mu}\right)\right]\right\} \Gamma_n(s p_i; m, \lambda, \mu) = 0.$$

we know understand the advantages of using a mass independent scheme, the renormalisation functions do not depend on masses only on the coupling and hence we can integrate the equation immediately, by introducing scale dependent couplings and masses:

$$s \frac{\partial}{\partial s} \overline{\lambda}(s) = \beta\left(\overline{\lambda}(s)\right), \qquad \frac{s}{\overline{m}(s)} \frac{\partial}{\partial s} \overline{m}(s) = \gamma_m\left(\overline{\lambda}(s)\right) - 1$$

$$\log s = \int_\lambda^{\overline{\lambda}(s)} \frac{dt}{\beta(t)}, \qquad \overline{m}(s) = m \exp\left[\int_\lambda^{\overline{\lambda}(s)} dt \frac{\gamma_m(t)-1}{\beta(t)}\right].$$

$$\Gamma_n(sp_i; m, \lambda, \mu) = s^{4-n}\Gamma_n\Big(p_i, \overline{m}(s), \overline{\lambda}(s), \mu\Big) \exp\left[-n \int_1^s \frac{ds'}{s'}\gamma\Big(\bar{\lambda}(s')\Big)\right]$$

the fixed points determined the asymptotic behaviour. For a beta function fixed point we obtain: $\beta(\lambda^*) = 0$

$$\Gamma_n(sp_i; \lambda^*, \mu) = s^{4-n(1+\gamma^*)}\Gamma_n(p_i; \lambda^*, \mu)$$

$D_\phi = 1 + \gamma^*$ is the anomalous dimension of the field at the fixed point. For our simple example, we can compute the RG functions at the one-loop level, in general they are determined by the firs order pole in d-4. A remarkable result

$$m_0^2 = m^2\left(1 - \frac{\lambda}{16\pi^2}\frac{1}{d-4}\right), \qquad \beta(\lambda) \equiv \mu\frac{\partial\lambda}{\partial\mu} = \frac{3\lambda^2}{16\pi^2}$$

$$\lambda_0 = \mu^{4-d}\left(\lambda - \frac{3\lambda^2}{16\pi^2}\frac{1}{d-4}\right). \qquad \gamma_{m^2}(\lambda) \equiv \frac{\mu}{m^2}\frac{\partial m^2}{\partial\mu} = \frac{\lambda}{16\pi^2}$$

The issue of quadratic divergences

We can now go back to the beginning remarks before this long technical digression, and imagine that instead of using DR in our computation of the simple scalar theory, we use a momentum space cut-off to evaluate the integrals. This means:

$$\int_{|p_E|<\Lambda} \frac{d^4 p_E}{(2\pi)^4} \frac{1}{(p_E^2+m^2)^n} \sim \begin{cases} \frac{m^2}{8\pi^2}\left[\frac{\Lambda^2}{m^2} - \log\left(\frac{\Lambda^2}{m^2}\right)\right] & n=1 \\ \frac{1}{8\pi^2}\left[\log\left(\frac{\Lambda^2}{m^2}\right) - \frac{1}{2}\right] & n=2 \\ \frac{m^{4-2n}}{8\pi^2(n-1)(n-2)} & n>2 \end{cases}$$

$$\begin{aligned} m_0(\Lambda)^2 &= m^2\left\{1 - \frac{\lambda}{16\pi^2}\left[\frac{\Lambda^2}{m^2} - \log\left(\frac{\Lambda^2}{m^2}\right)\right]\right\}, \\ \lambda(\Lambda) &= \lambda\left[1 - \frac{3\lambda}{16\pi^2}\log\left(\frac{\Lambda^2}{m^2}\right)\right]. \end{aligned}$$

The one-loop renormalisation can be done in the same way, the cancelling divergent parts. Inverting the first equation to write the renormalised mass in terms of the bare parameters to 1st order

$$m^2 = m_0(\Lambda)^2 + \frac{\lambda_0(\Lambda)}{16\pi^2}\left[\Lambda^2 - \log\frac{\Lambda^2}{m_0(\Lambda)^2}\right].$$

Should the scalar theory be valid at arbitrarily high energies, this would be the end of the story. The cut-off would be an artifact of the computation and should disappear at the end of the computation. Physical quantities would only depends on the renormalised values of m and \lambda. If we can only believe our theory up to a certain scale at which new physics plays a role. The the latter equation has to be interpreted in a Wilsonian way, by regarding Λ as the energy above which our theory is replaced by some unknown new dynamics. Just below this scale we keep our original Lagrangian, and the effect of the high energy d.o.f. is codified in the cut-off dependence of the bare parameters. The mass and coupling paramters characterise the theory well below the cut-off scale. There is a strong dependence of the value of the mass on the high energy scale. This is the hierarchy problem.

Any theory with fundamental scalars in afflicted with this problem, including the standard model due to the presence of the Higgs field. The only exception are theories with Nambu-Goldstone bosons.

In DR there are no quadratic divergences. The higher order divergences are signalled by poles in dimensions lower than 4. At one loop, we have a pole at d=2. For L-loops the additional loops happen at fractional dimensions:

$$d = 4 - \frac{2}{L}$$

The previous discussion might lead us to believe that the hierarchy problem is regularization artifact that can be disposed of by a smart choice of the regulator. The whole thing, however, is more complicated. Integrate one of the RG equations involving mass:

$$m(\mu)^2 = m(\mu_0)^2 \exp\left[\int_{\lambda(\mu_0)}^{\lambda(\mu)} \frac{dx}{\beta(x)}\gamma_{m^2}(x)\right].$$

There is clear dependence on the initial condition, but we do not know in principle how this one is sensitive to the initial value mu_0 at high energy.

We need to understand the dependence on high scales in DR in a Wilsonian framework. We need to have a description of effective field theories in this context. How separation of scales can be achieved.

It is clear that in a single scale theory there is no hierarchy problem. In fact if the high scale is the Planck scale, it is not clear at all whether the quadratic dependence will exist. We know too little about what happens to space-time at that scale to make any sound statement on quadratic dependences….

Fermions masses are protected by some symmetries

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - m)\psi - \frac{a}{\Lambda^2}(\overline{\psi}\psi)^2 + \dots$$

$$\delta m = -\frac{3am}{8\pi^2}\left(\frac{m}{\Lambda}\right)^2 \log\left(\frac{m^2}{\mu^2}\right).$$

when the mass vanishes, there is a discrete chiral symmetry

$$\psi \longrightarrow \gamma_5\psi, \qquad \overline{\psi} \longrightarrow -\overline{\psi}\gamma_5.$$

Consider now the following toy models of scalars coupled, or a scalar coupled to a massive fermion

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{M^2}{2}\Phi^2 - \frac{g}{2}\phi^2\Phi.$$

We can compute the mass renormalisation, and check that indeed there is quadratic dependence on the scale g

$$= -g^2\mu^{4-d}\int \frac{d^dq}{(2\pi)^d}\frac{i}{q^2-m^2+i\varepsilon}\frac{i}{(q+p)^2-M^2+i\varepsilon}$$

$$= g^2\mu^{4-d}\int_0^1 dx\; I_2\Big(d, x\,m^2+(1-x)M^2-x(1-x)p^2\Big).$$

$$\delta m^2 = \frac{g^2}{16\pi^2}\log\left(\frac{M^2}{\mu^2}\right)$$

The same computation for a massive fermion coupling, and we verify again the quadratic dependence on the high scale

$$\mathcal{L}_{\rm int} = g'\phi\overline{\psi}\psi.$$

$$\delta m^2 \sim g'^2 M^2\log\left(\frac{M^2}{\mu^2}\right)$$

The naturalness criterion

At any energy scale μ, a physical parameter or a set of physical parameters $\alpha_i(\mu)$ is allowed to be very small only if the replacement $\alpha_i(\mu) = 0$ would increase the symmetry of the system.

Finally we come to the (in)famous cosmological constant, or dark energy, its contribution to the energy density of the Universe is:

$$\rho_\Lambda = \frac{\Lambda_c}{8\pi G_N}$$

The measured value is:

$$\rho_\Lambda \simeq (10^{-3}\mathrm{eV})^4 = 10^{-48}\mathrm{GeV}^4.$$

Compared with the critical density

$$\rho_c = \frac{3H_0^2}{8\pi G_N} \qquad \rho_\Lambda \simeq 0.74\rho_c$$

Should gravity be excluded from naturalness arguments?

Let's then be conservative. Assume that there is nothing beyond the SM until the Planck scale, and since we have no clue about quantum gravity, we can hope for a miracle that there will be no quadratic dependence on the Planck mass.

The problem is that then we are hit with….

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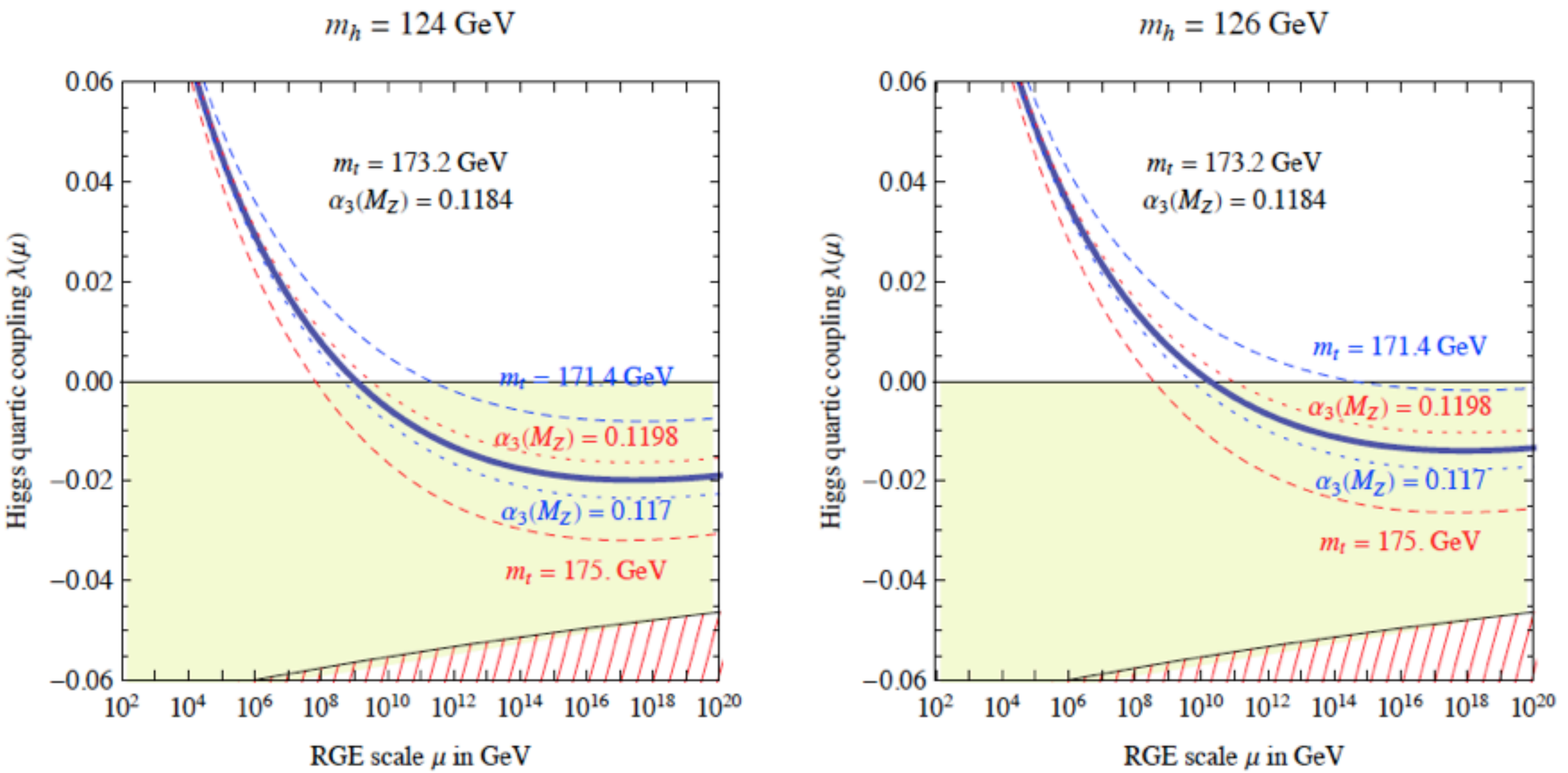
The problem is that then we are hit with….

The Planck chimney


m_h = 124 GeV
m_t = 173.2 GeV
$\alpha_3(M_Z)$ = 0.1184
m_t = 171.4 GeV
$\alpha_3(M_Z)$ = 0.1198
$\alpha_3(M_Z)$ = 0.117
m_t = 175. GeV
Higgs quartic coupling $\lambda(\mu)$
RGE scale μ in GeV
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Conclusions

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No Conclusions!!

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No Conclusions!!

Thank you for your attention