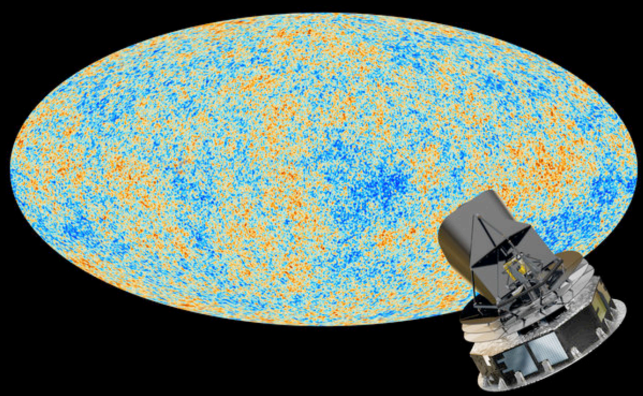
4TH IDPASC SCHOOL

Braga, 20 - 28 January 2014

CMB and Galaxy Clusters at Planck frequencies

António da Silva
DF-FCUL



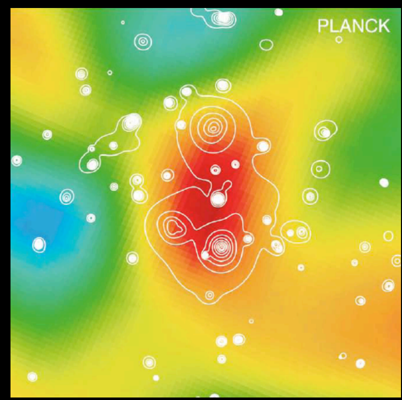


Fig. credits: ESA and the Planck collaboration

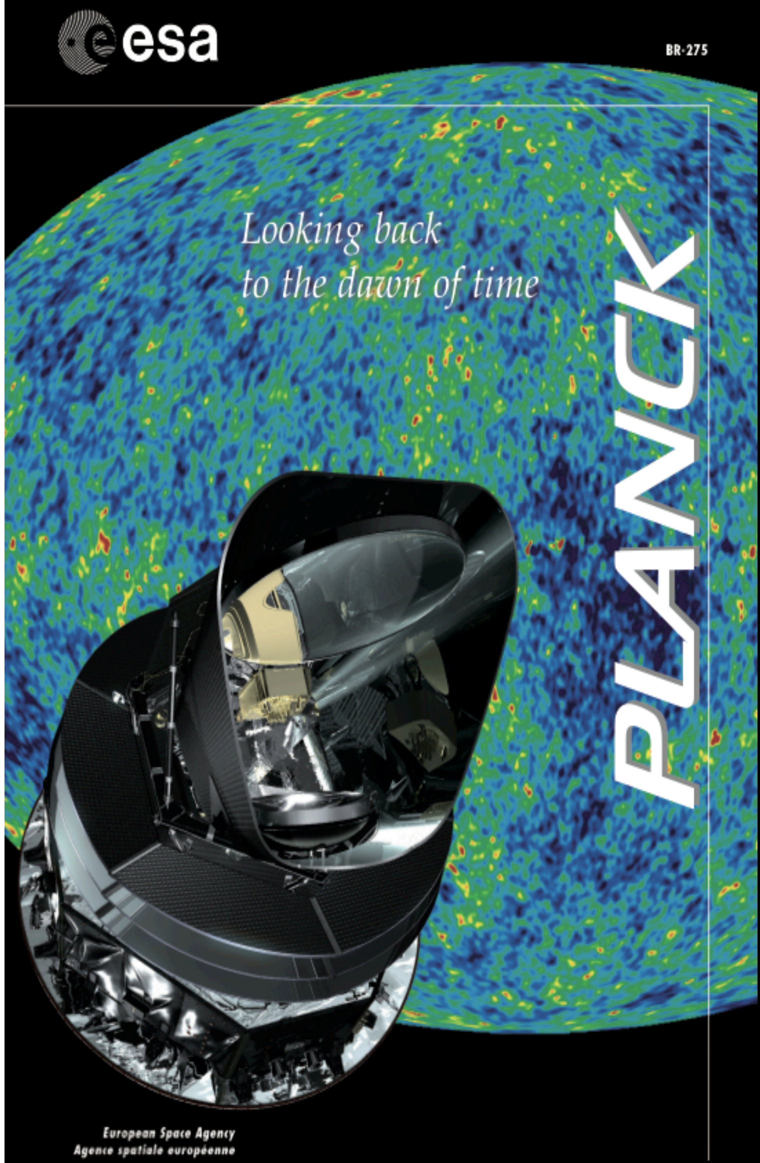
Syllabus:

Outline: This course provides a short introduction to the Cosmic Microwave Background (CMB) radiation and Galaxy clusters as probes of Cosmology and Large Scale Structure (LSS). After briefly reviewing the current standard model of cosmology, we will describe the main properties of the CMB and Sunyaev-Zeld'ovich (SZ) clusters and finalize with an overview of the latest findings obtained with the Planck satellite (ESA).

Contents:
Part 1: Review of the Standard Model of Cosmology (SMC):
Part 2: CMB and Sunyaev-Zel'dovich (SZ) Galaxy Clusters
Part 3: Latest results from the Planck Satellite

Bibliography:
- Primordial cosmology, P. Peter, J.-P. Uzan, Oxford University Press, 2009
- An Introduction to Modern Cosmology, A. Liddle, J. Wiley & Sons, 2003
- Course notes by the course's responsible

Planck Surveyor: looking back to the dawn of time



Project: ESA lead mission to observe the temperature and polarization anisotropies of the Cosmic Microwave Background (CMB) radiation with unprecedented precision.

Total Cost: about €700 million (€1 / person in EU)

Mission timeline:

- Launch: 14 May 2009
- Operational orbit at L2: July 2009
- Nominal science phase: end of January 2011
- Extended mission: Shut down date: 19 Oct. 2013

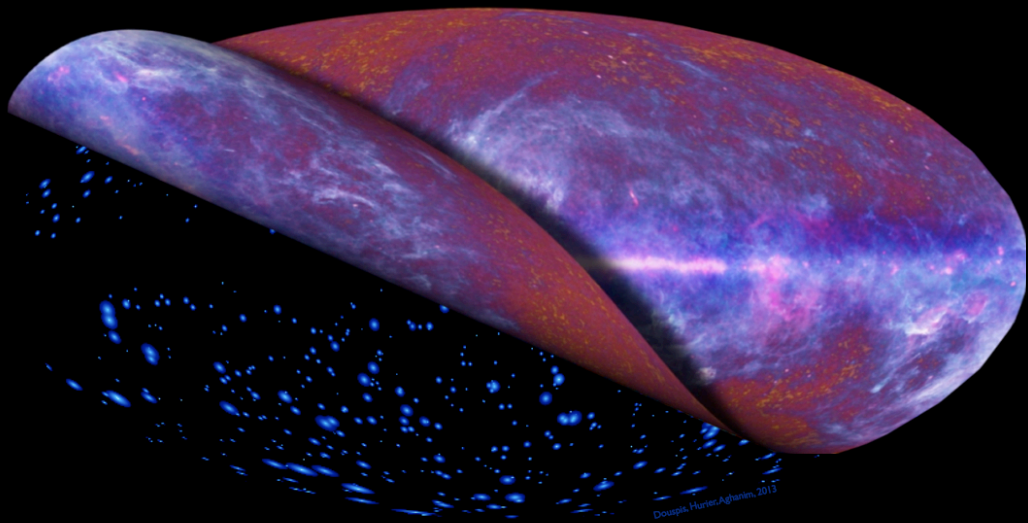
Payload:

- Telescope: 1.5 m projected apertures
- Low Frequency Instrument (LFI): array of 22 tuned radio receivers operating at 30, 44 and 70 GHz.
- High Frequency Instrument (HFI): array of 52 bolometers operating at 100, 143, 217, 353, 545, and 857 GHz.

Fig. credits: ESA

Planck CMB observations

2009-2013: Planck satellite observes the CMB sky with unprecedented angular resolution and sensitivity.



Animation credits: ESA and the Planck collaboration; Cluster map by Douspis, Hurier, Aghanim 2013

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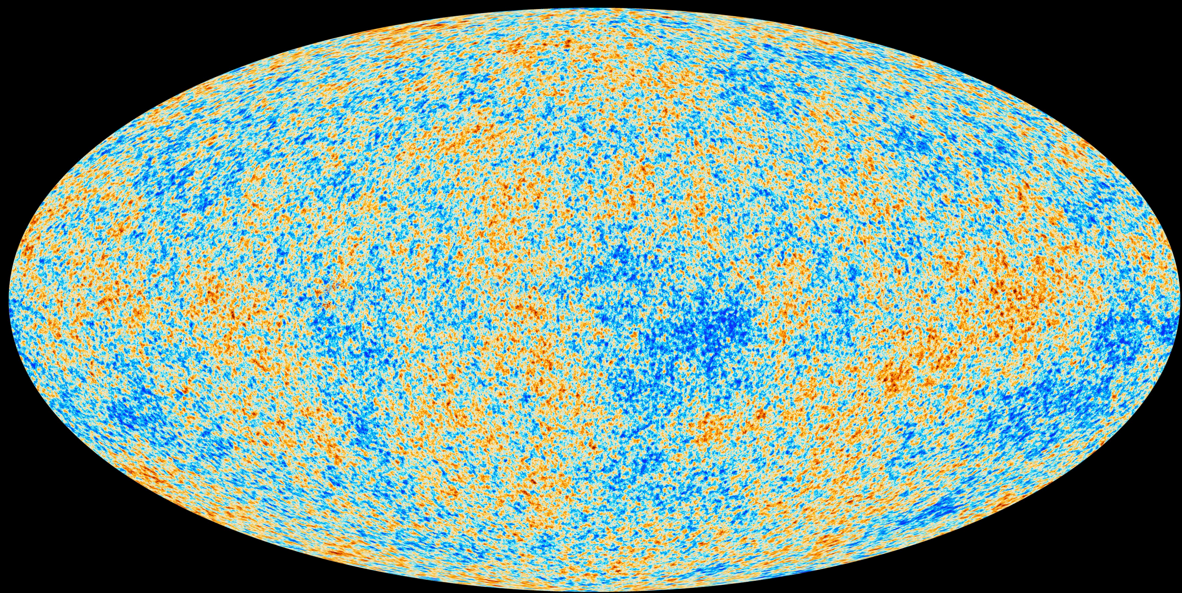


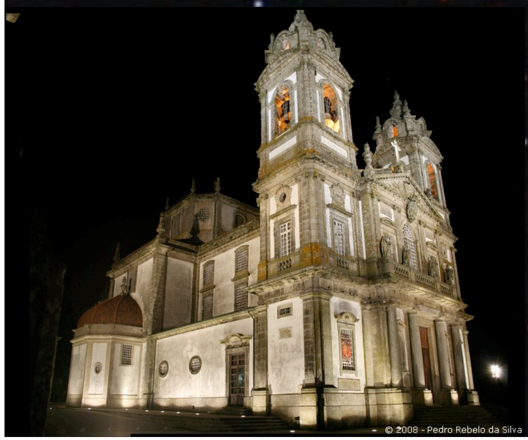
Animation & Fig. credits: ESA and the Planck collaboration

Part I: Standard Cosmology

Foreword: Why is the sky dark at night?

Heinrich Olbers

Olbers' paradox (1826) : argues that the **darkness of the sky** at night **conflicts with the concept of an eternal and static universe**, with stars distributed uniformly.



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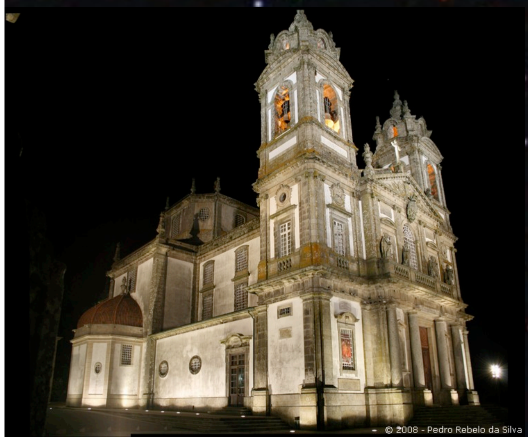
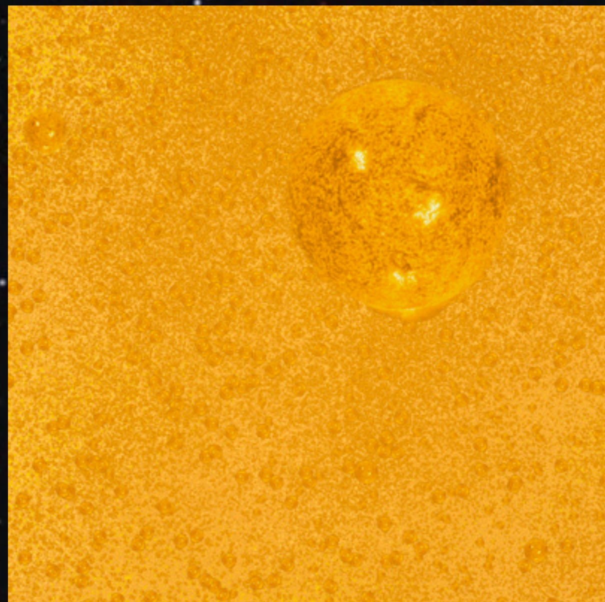


Oblers paradox in action.

Exercise: prove why this happens



Foreword: Why is the sky dark at night?

Heinrich Olbers

Some possible explanations:

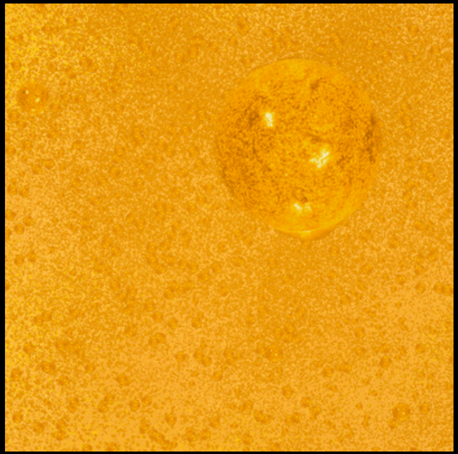
1. Too much dust absorbs light from distant stars.

2. The number of stars in the Universe is finite.

3. The distribution of stars is not uniform.

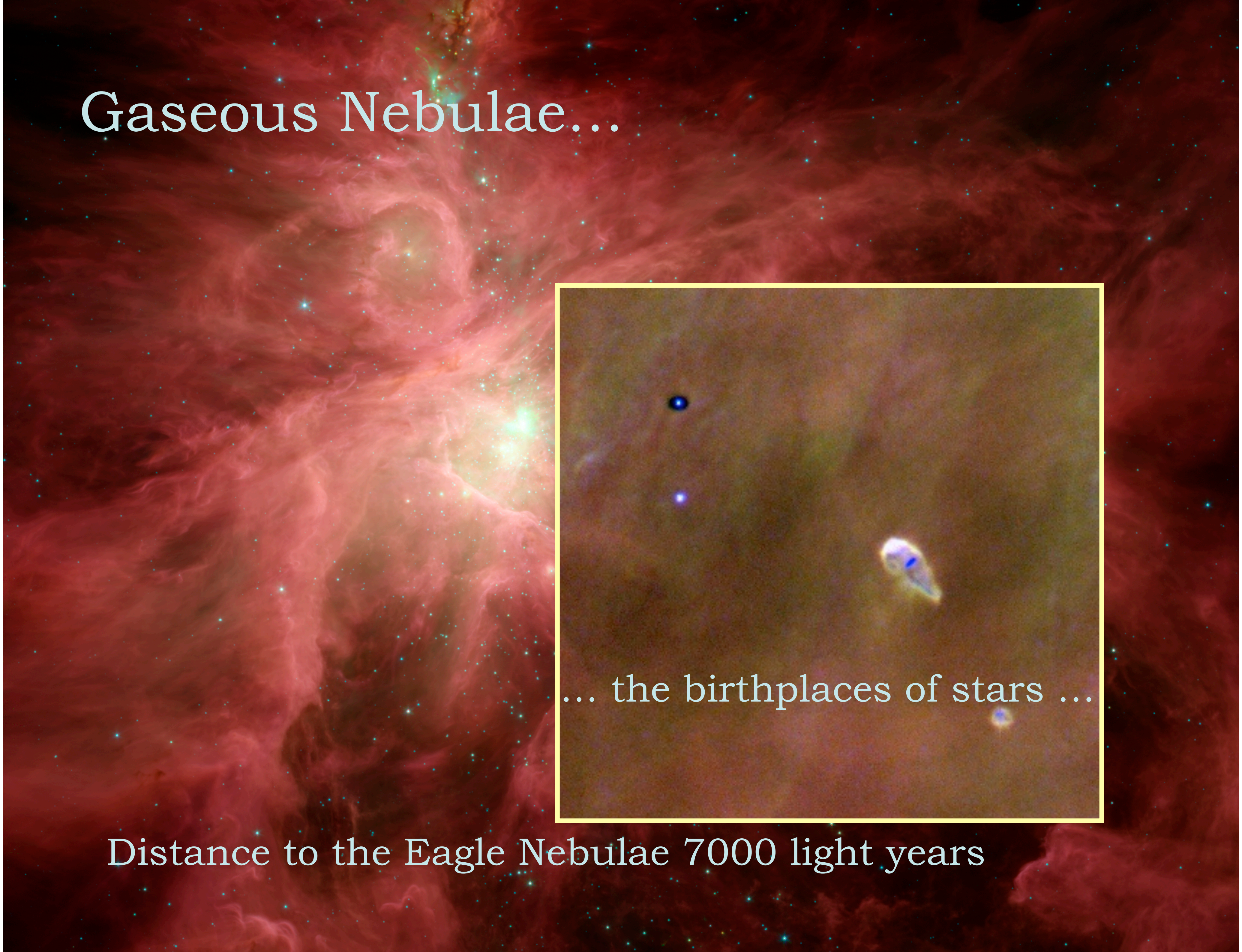
4. The Universe is expanding. Light from distant stars are redshifted into obscurity.

5. The Universe has a finite age. Distant light hasn't even reached us yet.



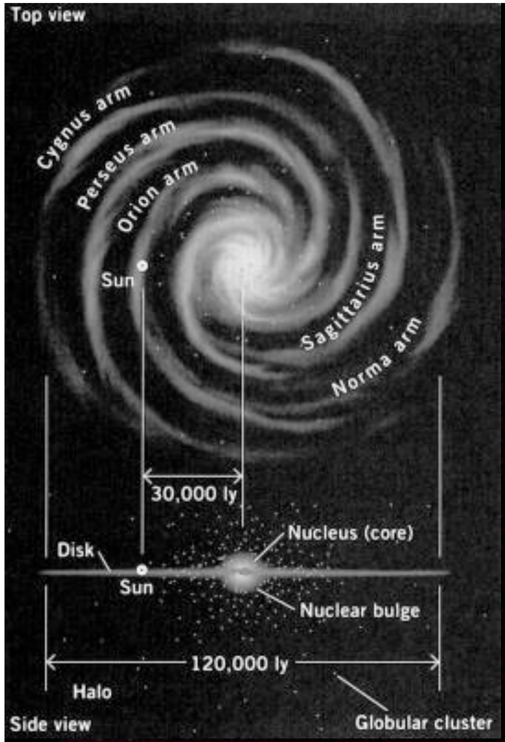
The Observable Universe


Stars...
Sun: 8 light minutes away
α -Centauro 4,25 light years


Gaseous Nebulae…
… the birthplaces of stars …
Distance to the Eagle Nebulae 7000 light years


The galaxy…
A home for billions of stars…

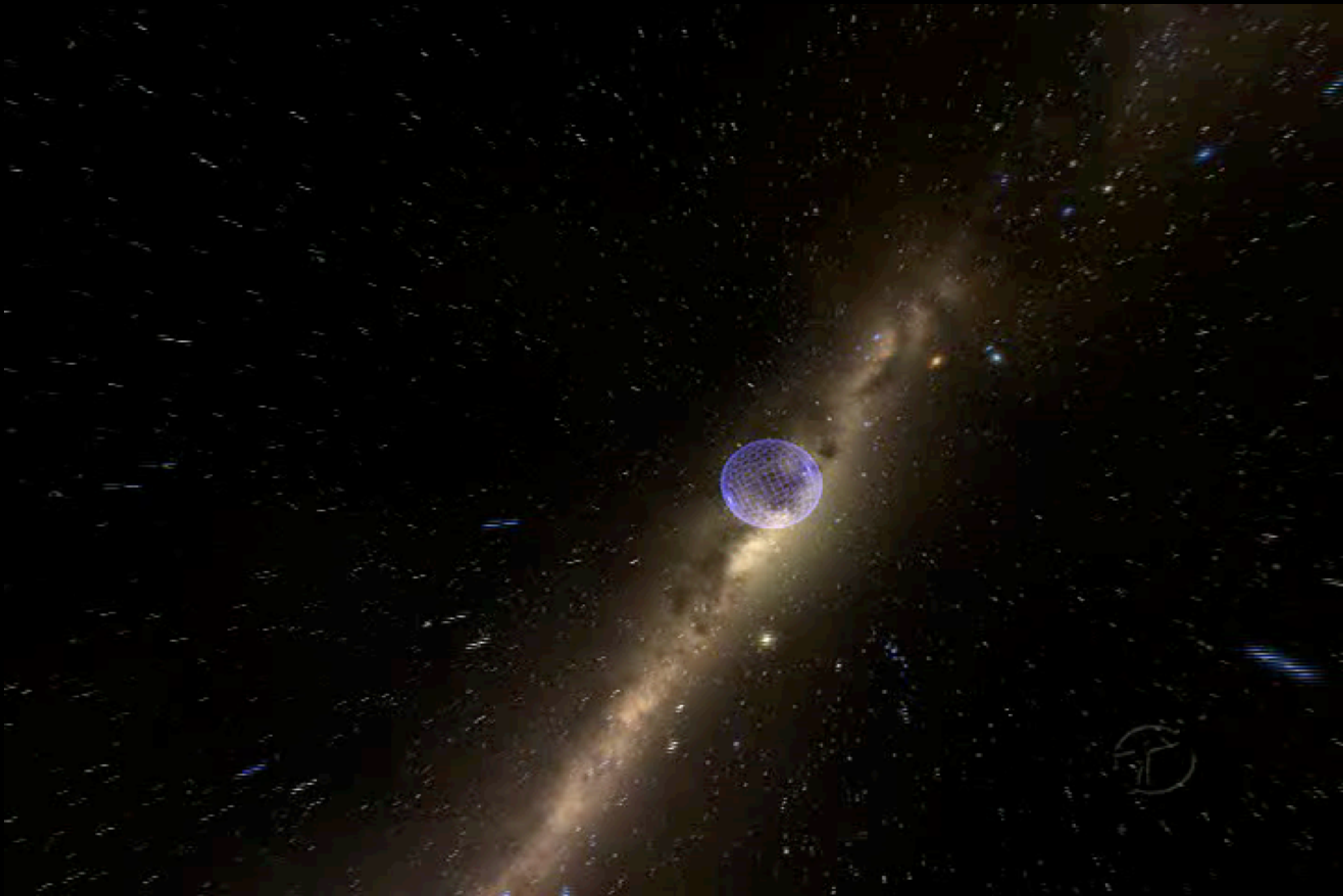
Milky Way



Top view
Cygnus arm
Perseus arm
Orion arm
Sun
Sagittarius arm
Norma arm
30,000 ly
Nucleus (core)
Disk
Sun
Nuclear bulge
120,000 ly
Halo
Side view
Globular cluster

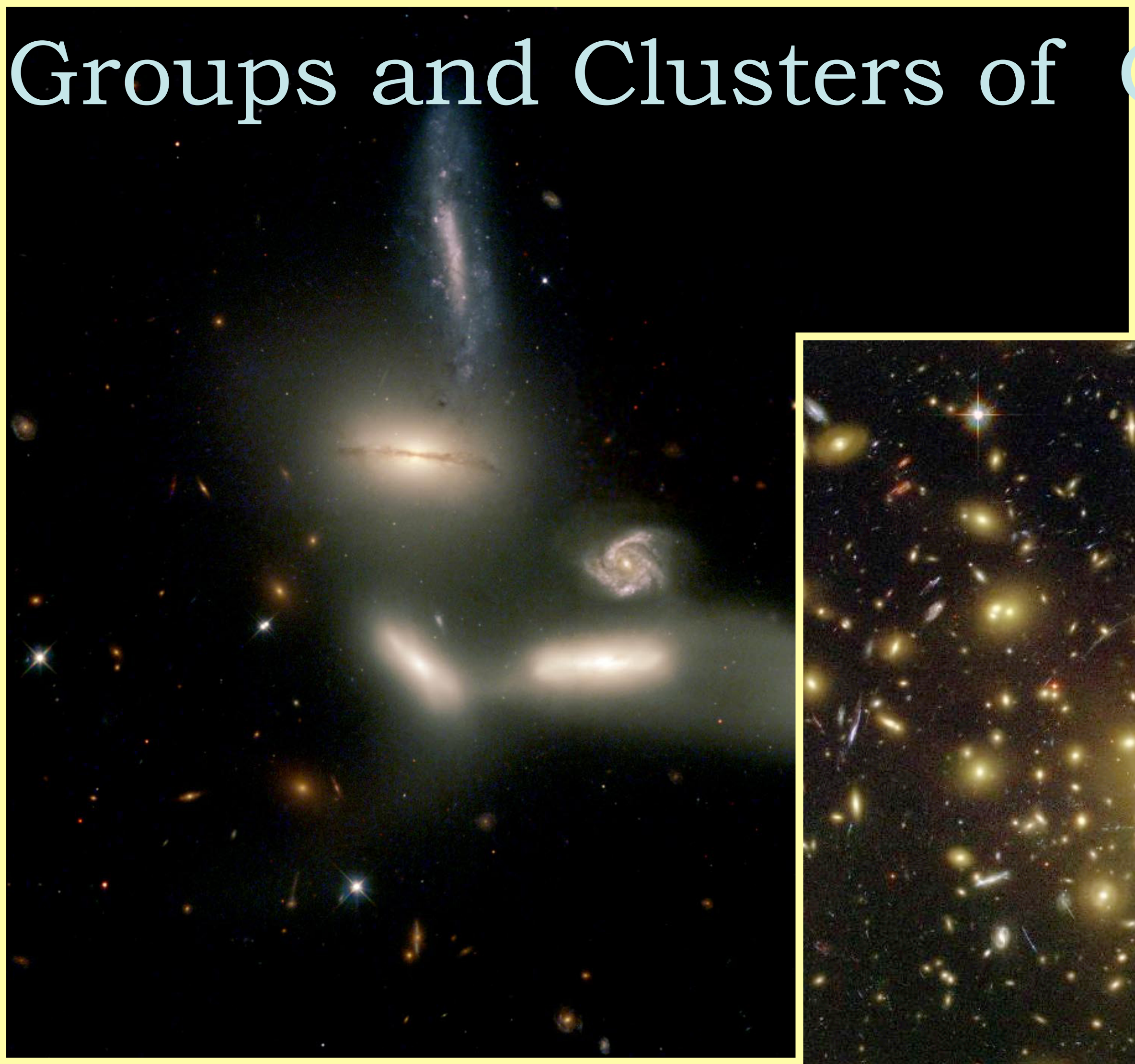
16 000 light years
120 000 light years

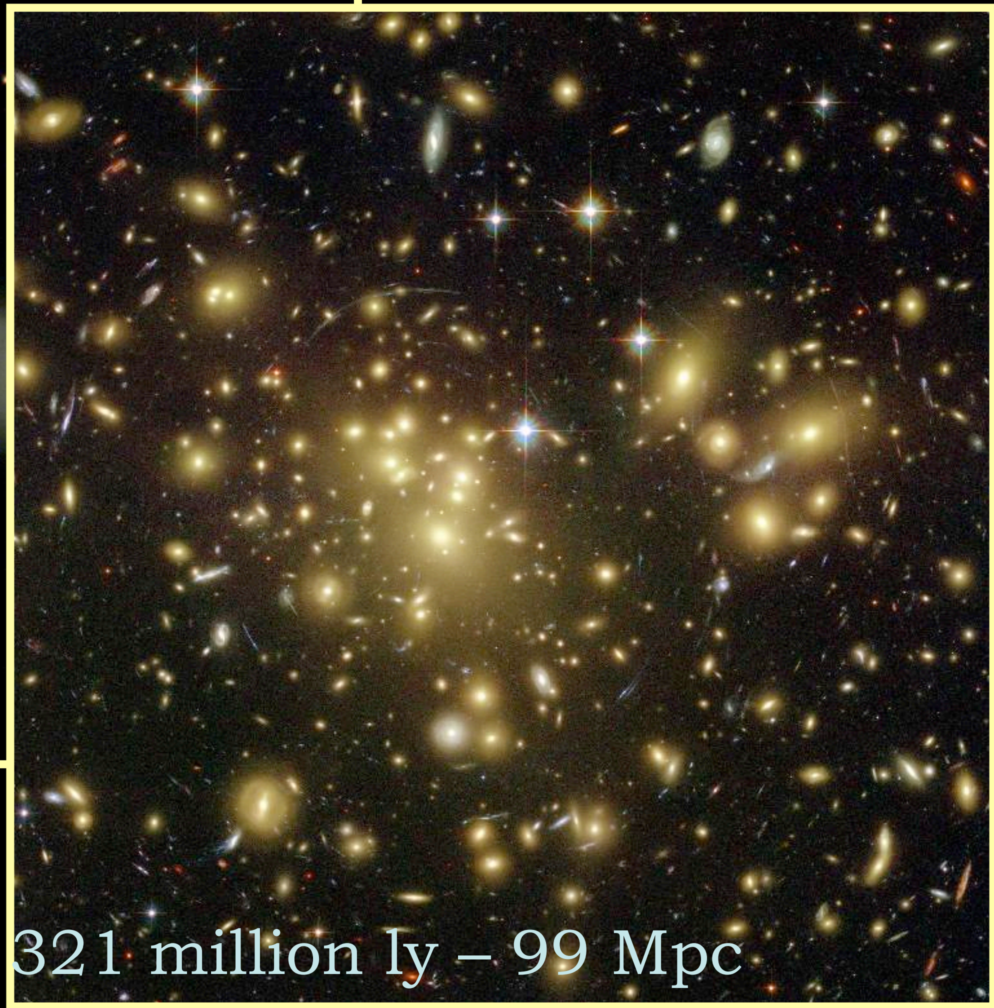
Our place in the Universe...



Credits: American Natural History Museum; gently provided by Miguel de Avillez U. Évora

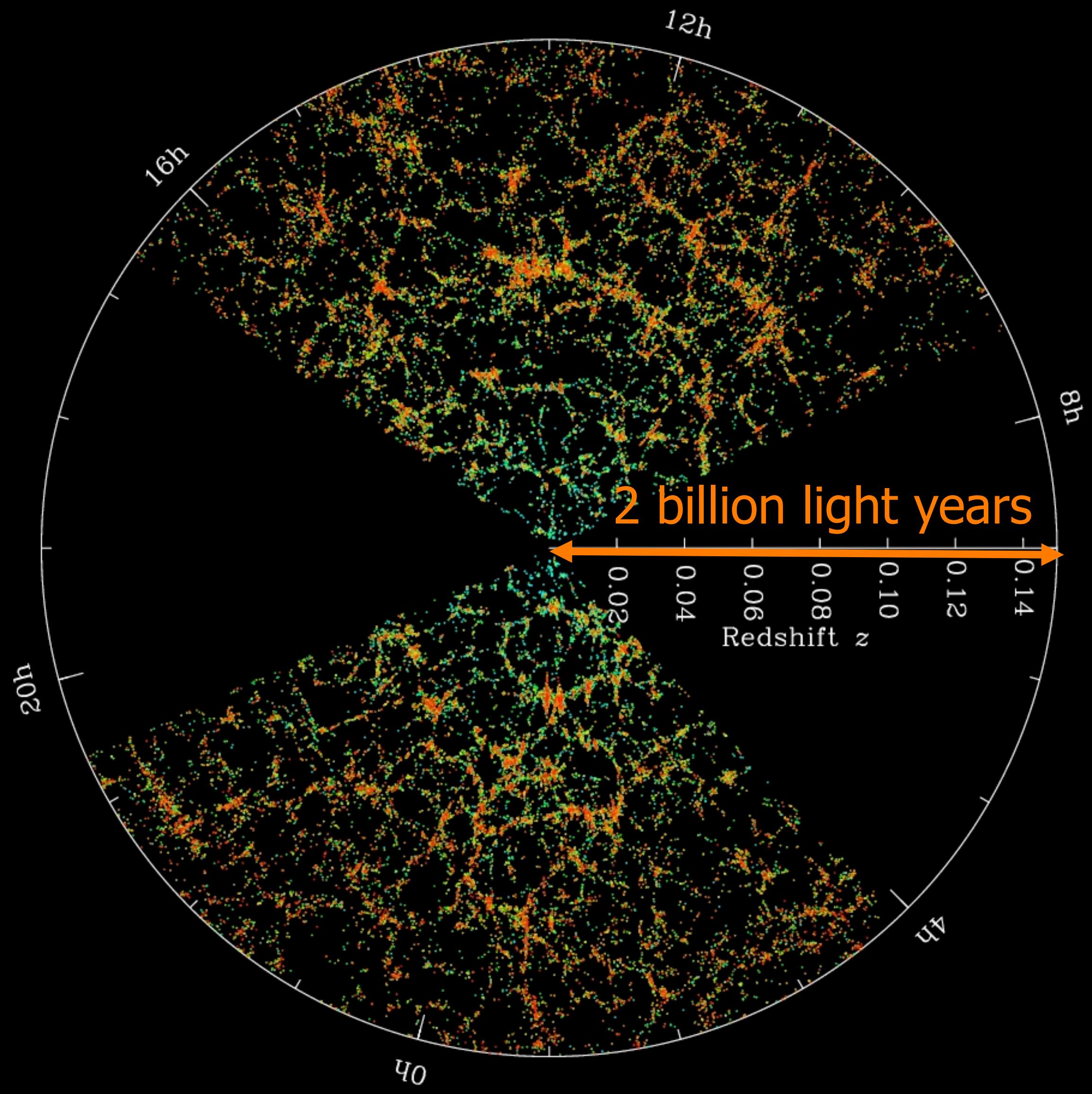
Groups and Clusters of Galaxies...



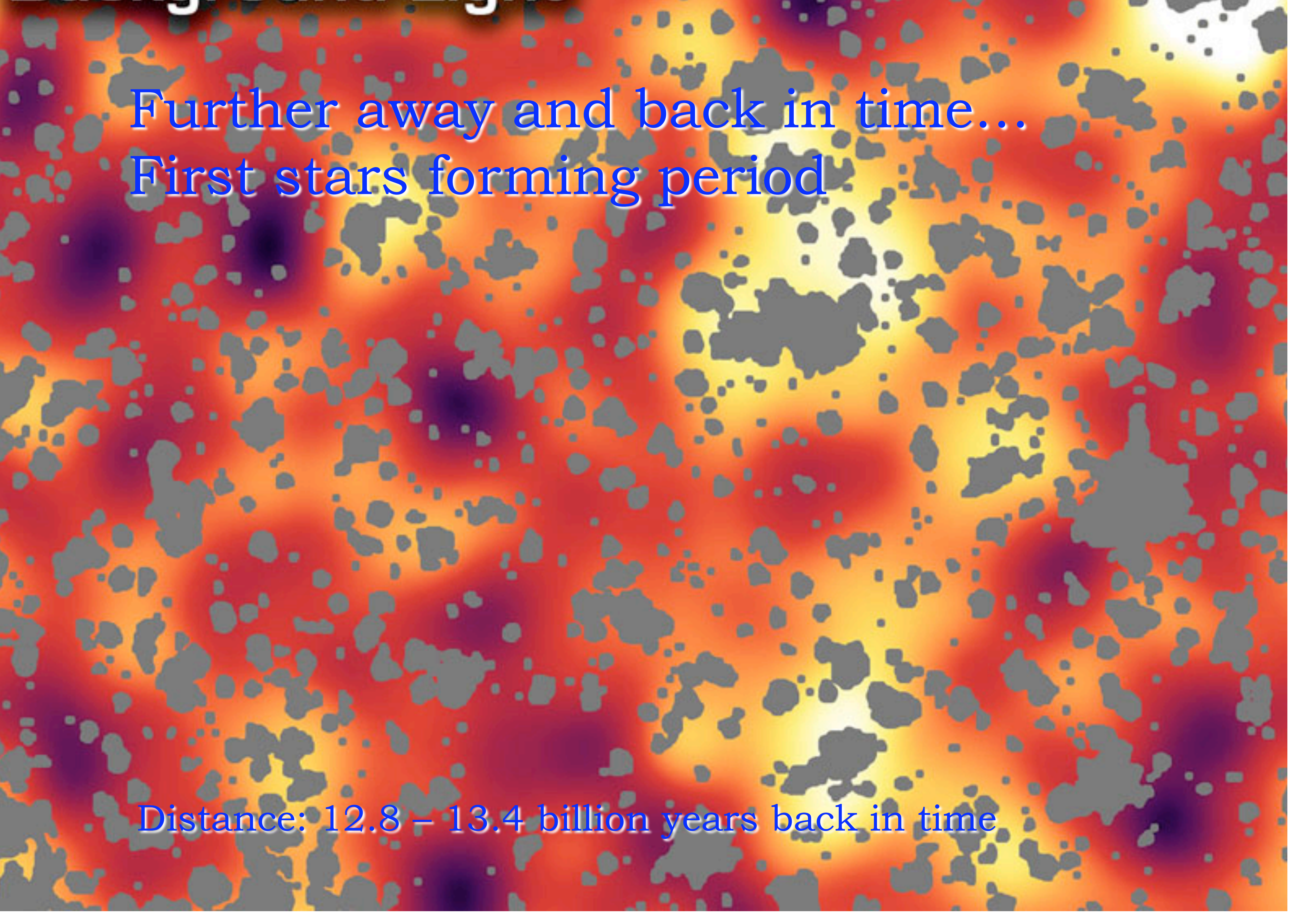


Distance to Coma cluster 321 million ly – 99 Mpc

LSS: "The fingers of God"...

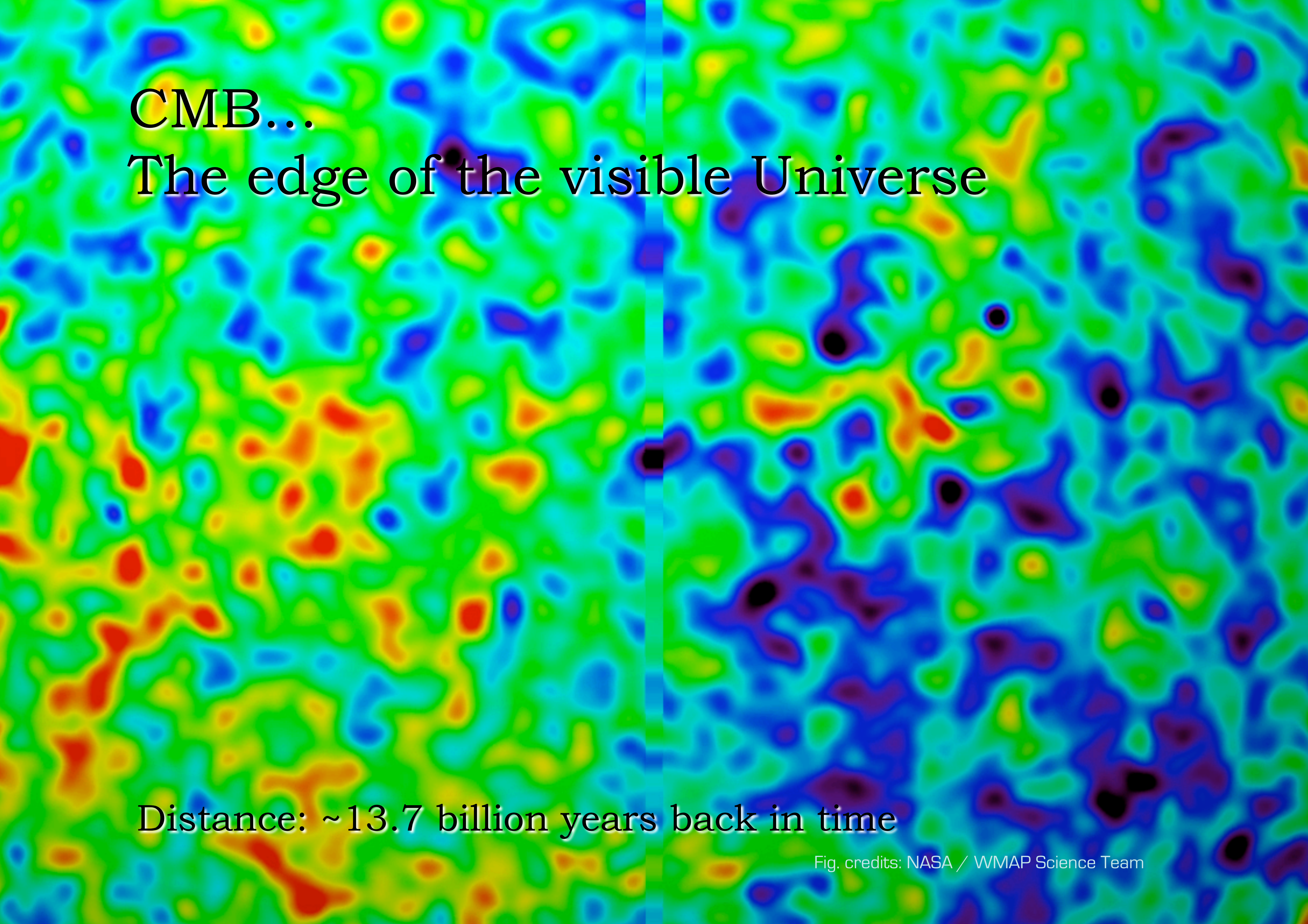


Credit: M. Blanton and the Sloan Digital Sky Survey



Further away and back in time…
First stars forming period

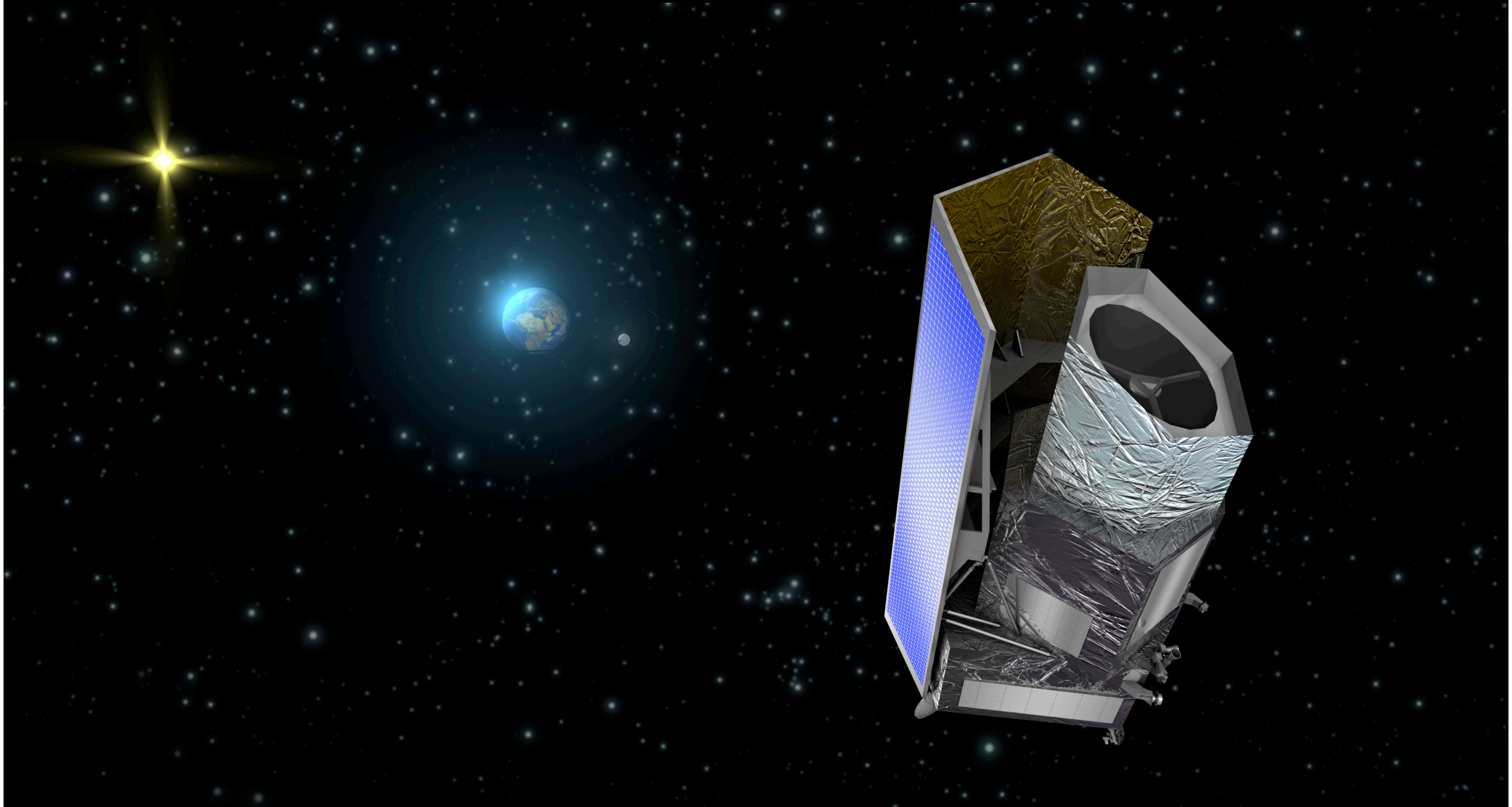
Distance: 12.8 – 13.4 billion years back in time


CMB…
The edge of the visible Universe
Distance: ~13.7 billion years back in time
Fig. credits: NASA / WMAP Science Team

Galaxy surveys: 3D mapping of the Universe…

SDSS: aims at ~25% of the sky; ~100 million objects

Euclid mission (ESA): Galaxy Surveys from space



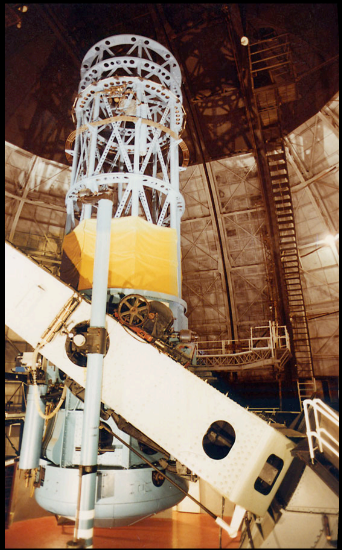
Euclid: ~ 2000 milions of galaxies

Fig. credits: ESA - C. Carreau.

Observational cosmology: main findings about the Universe

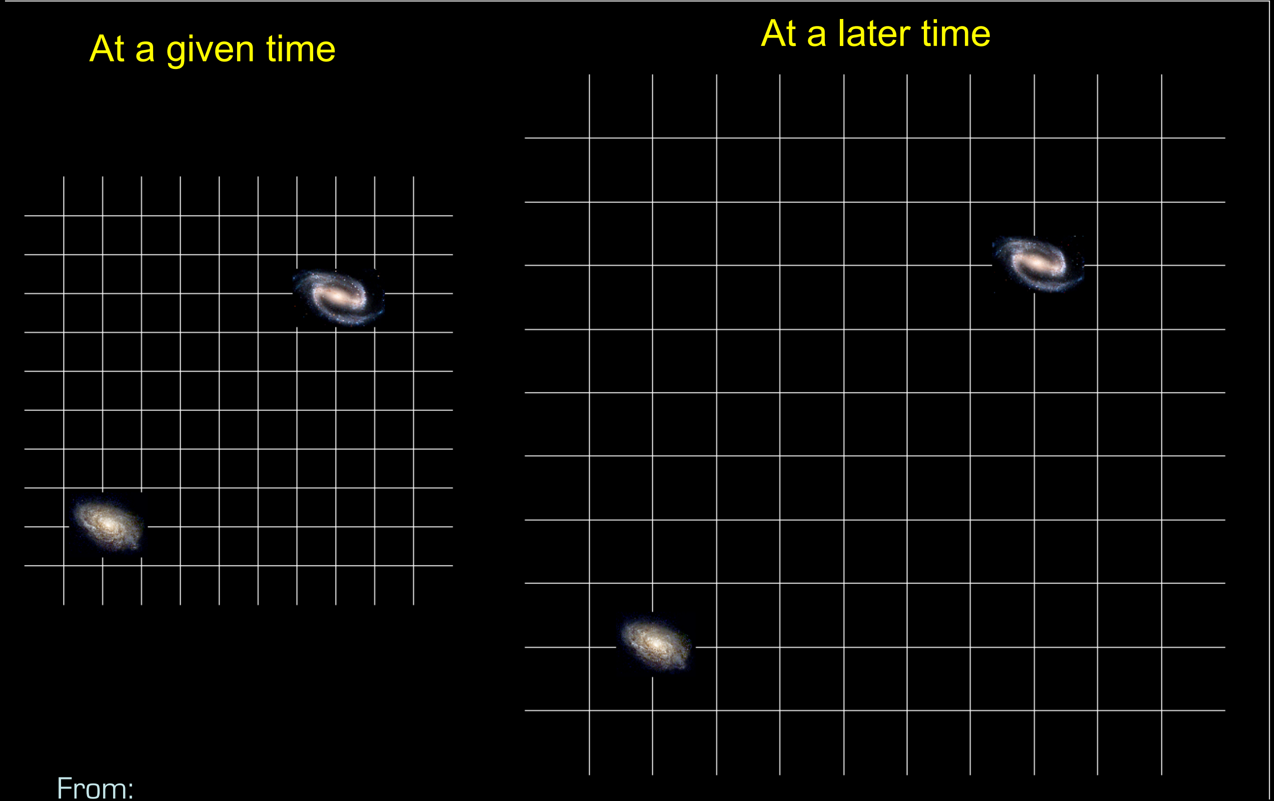
1. The Universe is expanding

Edwin Hubble



1924: Edwin Hubble ends debate on the nature of nebulae being galactic objects

1929: reports a linear relation between radial velocity and distance: $v = H\ d$



From:

1. The Universe is expanding

This leads to the basic idea behind the Big-Bang theory

- If the universe is expanding and matter-energy is conserved during the expansion then the universe had to be smaller, denser and hotter in the past!

- If so, the Universe must have evolved from a state where matter and radiation form a ultra dense and hot ionized plasma of fundamental particles

- As the universe expands and cools down:

    - interactions between the plasma components become less frequent;

    - different particle species should decouple from the plasma;

    - eventually the universe becomes neutral and transparent to radiation

1. The Universe is expanding

Exercise: Derive the Hubble's law, using the relation between physical and commoving coordinates in a homogeneous and isotropic Universe:

$$r_{phys} = a(t)\, x_{com}$$

Is H (Hubble proportionality parameter) a constant?

What's the expression for the peculiar velocity (relative to the global cosmic expansion)?

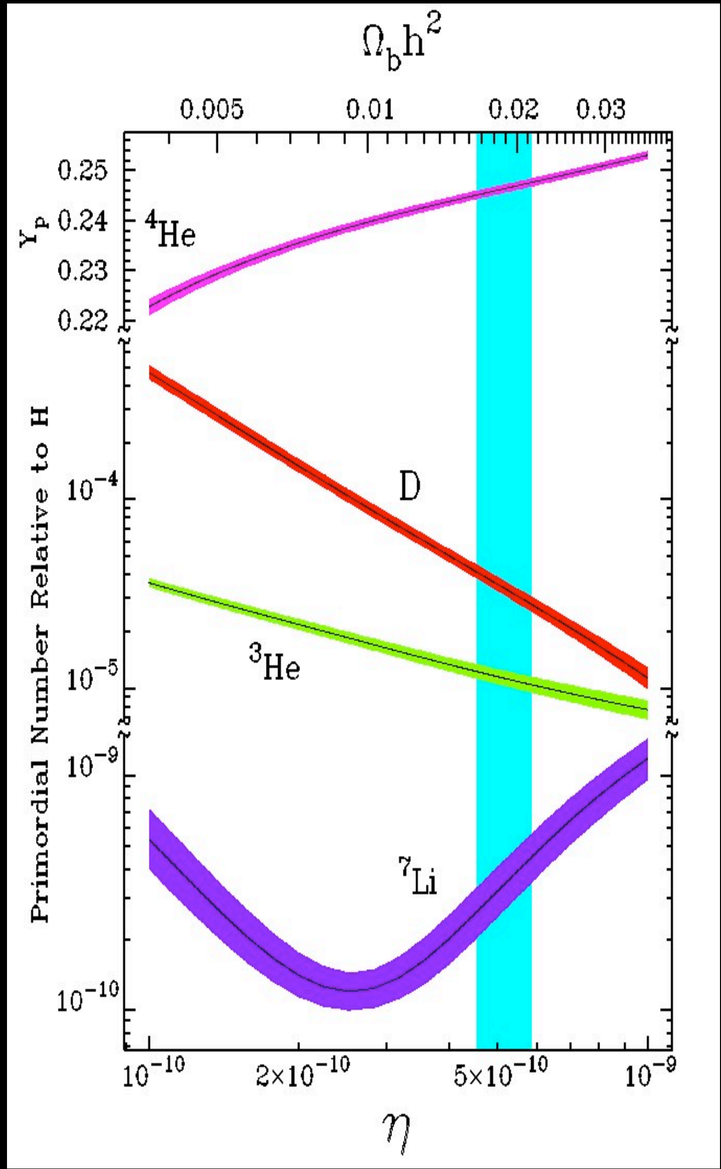
2. The abundance of light nuclei

Herman, Gamow, Alpher

The relative abundance of light elements can not be explained by stellar nucleosyntesis

1948: Alpher & Gamow computed the abundance of light elements in the context of the Big Bang theory

Light elements were produce at low temperatures (<1e9K and high densities) during several tens of minutes



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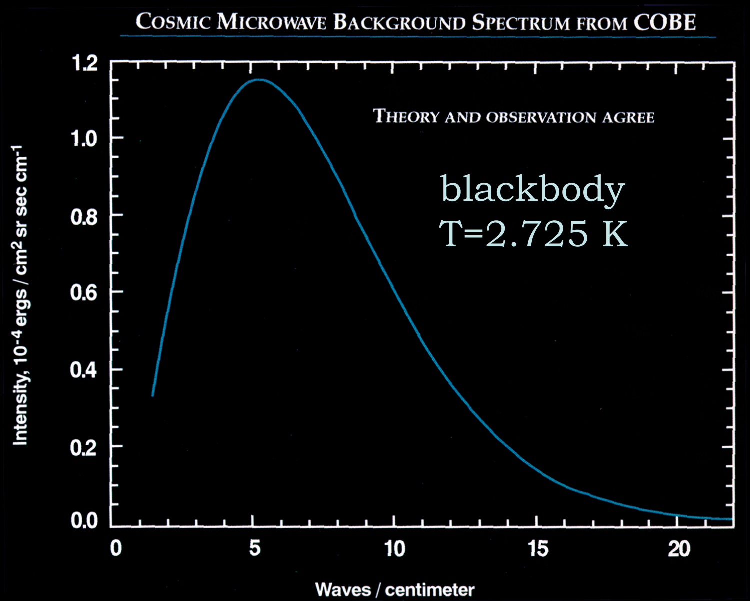
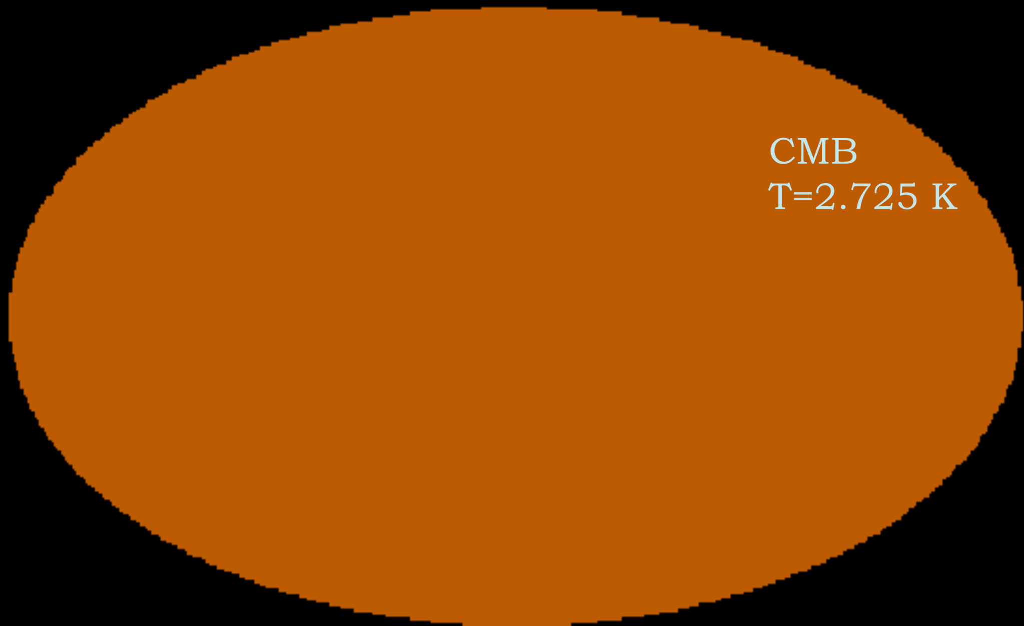
3. Cosmic Microwave Background

Penzias & Wilson

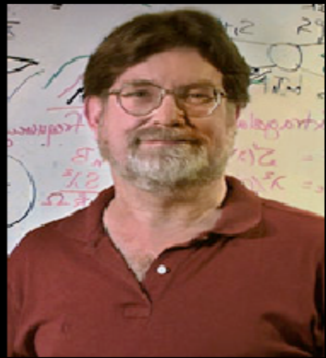
1965: Penzias & Wilson serendiputsly discovered a uniform radiation ("excess") across the sky.

This was the cosmic microwave background radition predicted by Gamow and Alpher in 1948





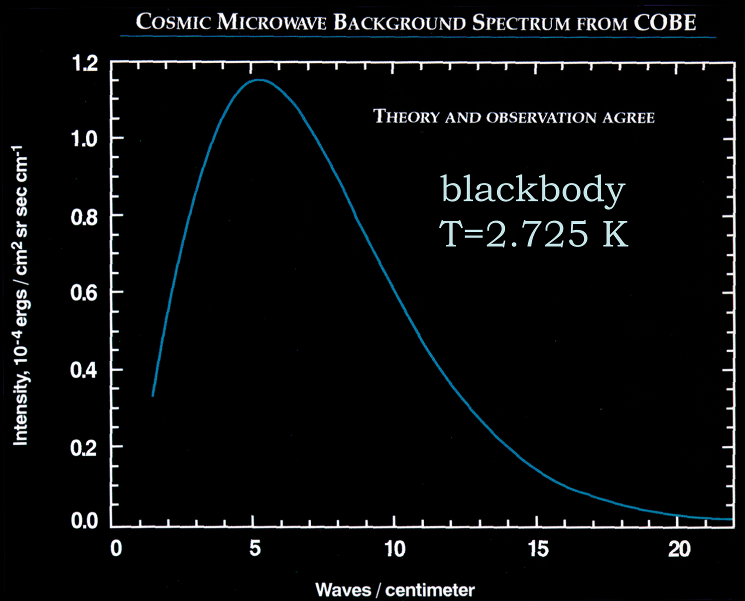
3. Cosmic Microwave Background



John Matter &
Geroge Smooth

1991: High precision measurement of CMB temperature by COBE and 1st detection of temperature fluctuations (Mather & Smoot)

2001: State of the art measurements of dT/T~1e-5 temperature fluctuations by WMAP



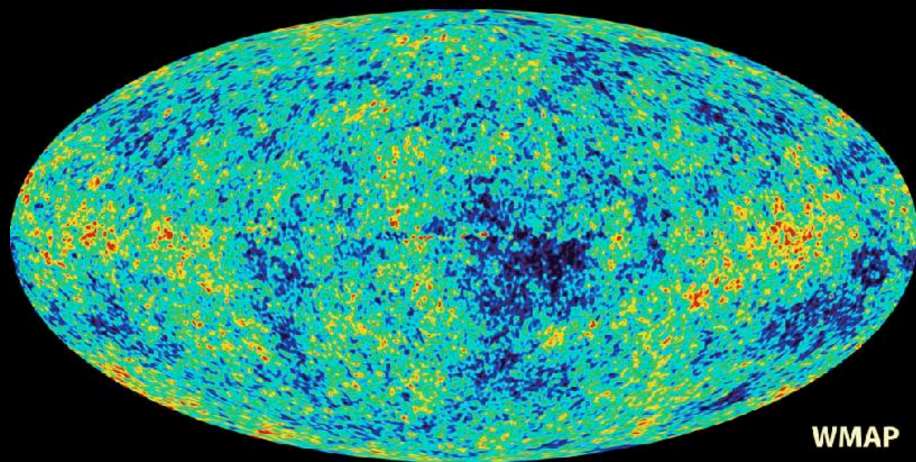
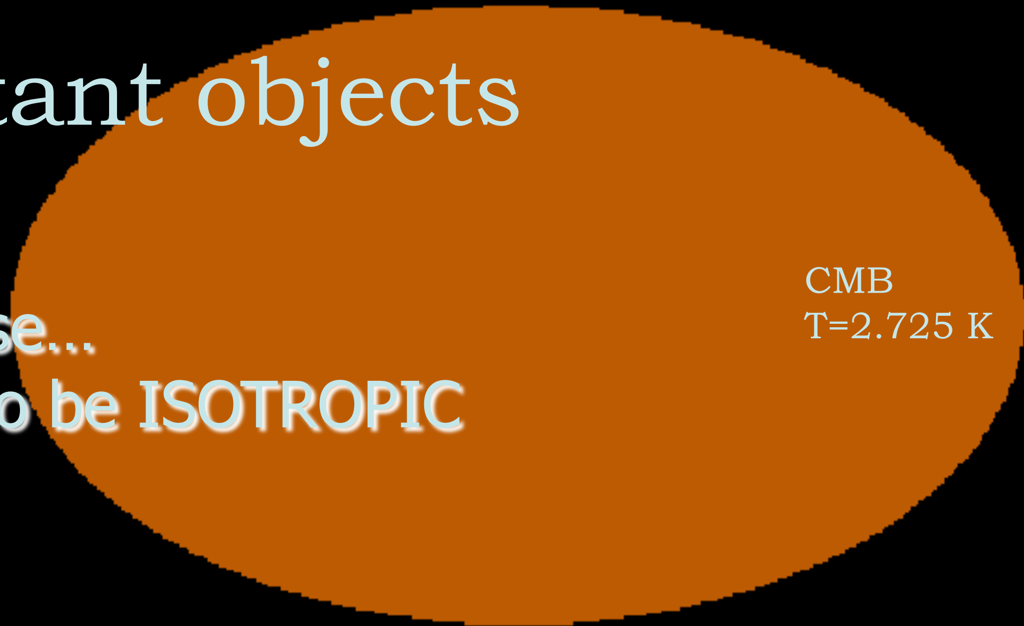


Fig. credits: NASA / WMAP Science Team

4. Isotropy of distant objects



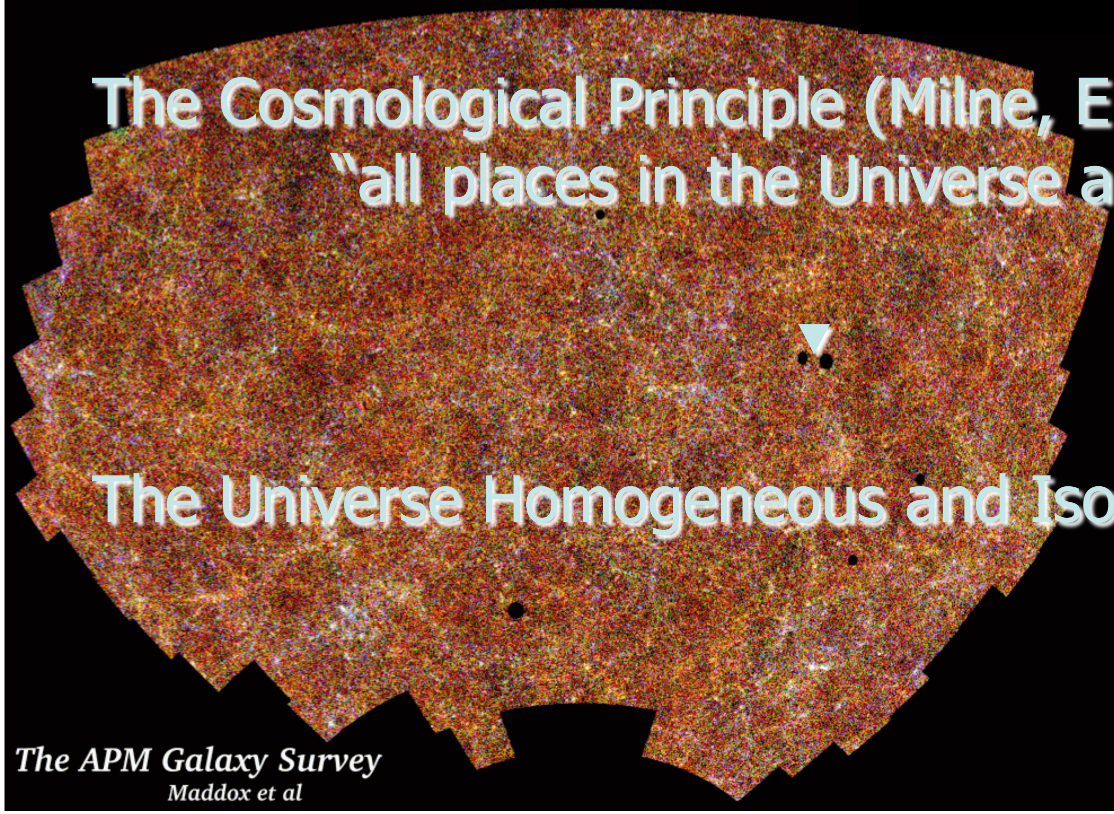
On Large Scales the Universe…
… appears to be ISOTROPIC

+

The Cosmological Principle (Milne, Einstein):
"all places in the Universe are alike" – Einstein

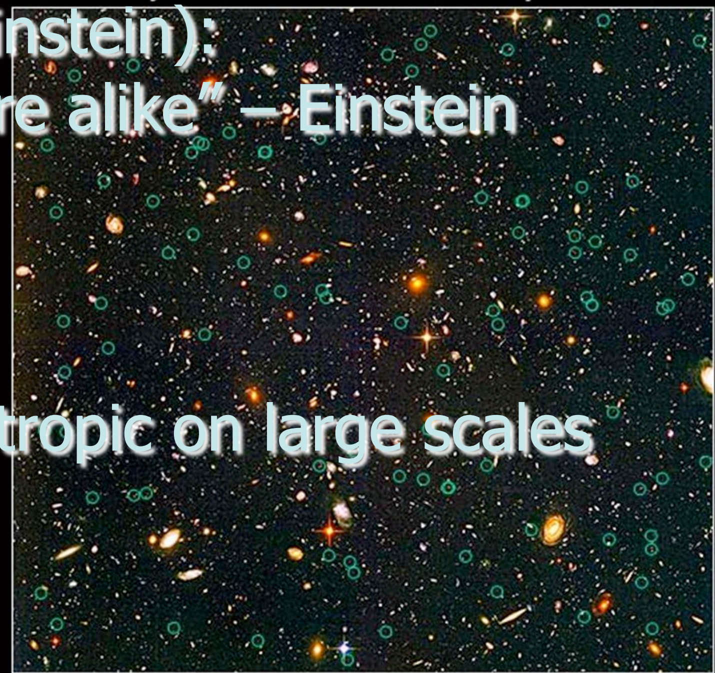
▼

The Universe Homogeneous and Isotropic on large scales



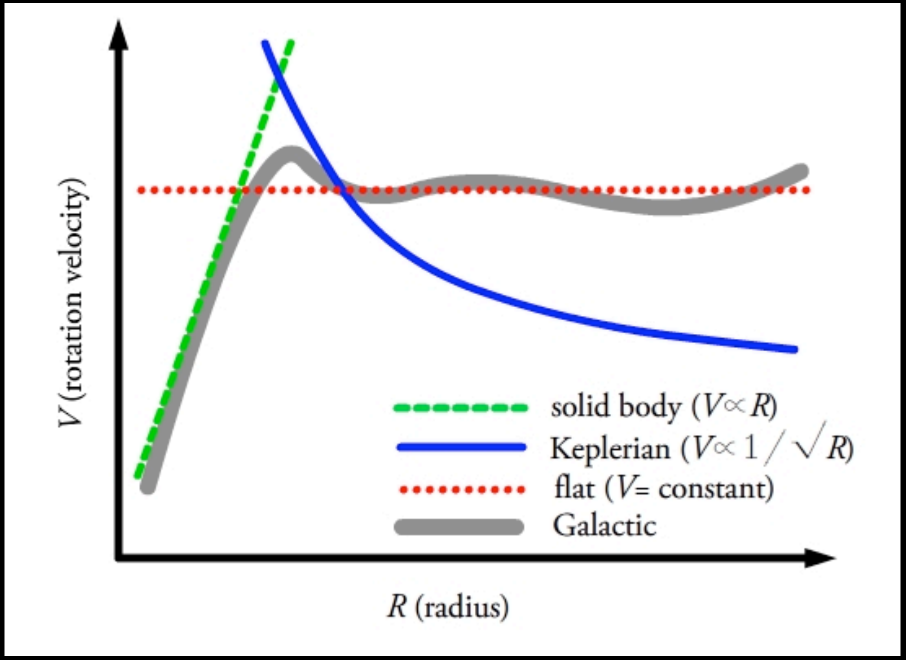
The APM Galaxy Survey
Maddox et al

Distant Objects in the Hubble Ultra Deep Field



NASA, ESA, R. Windhorst (Arizona State University) and H. Yan (Spitzer Science Center, Caltech)

5. The existence of Dark Matter



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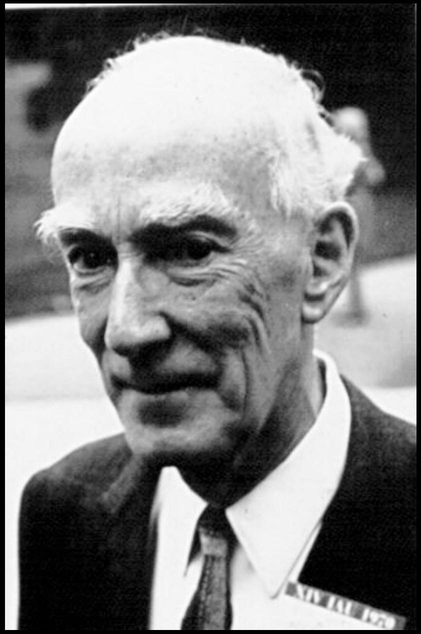


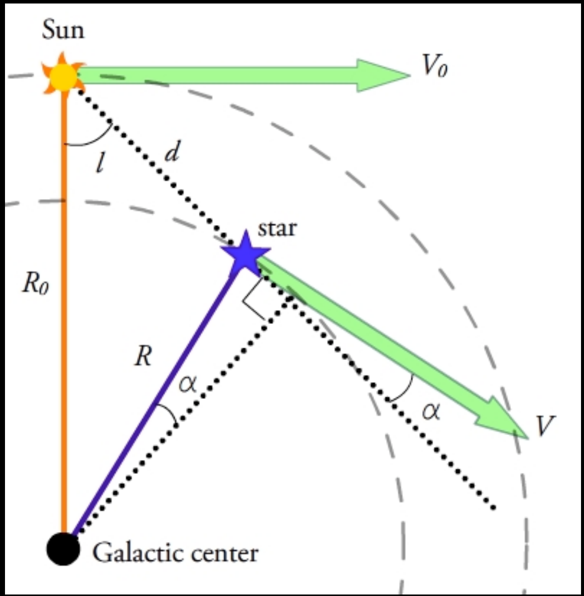
Jan Oort

1927: Jan Oort studies the rotation of stars in our galaxy and infers that their rotation is not consistent with Keplerian motion.

$$v_{circ} = \sqrt{\frac{GM(R)}{r}}$$

5. The existence of Dark Matter

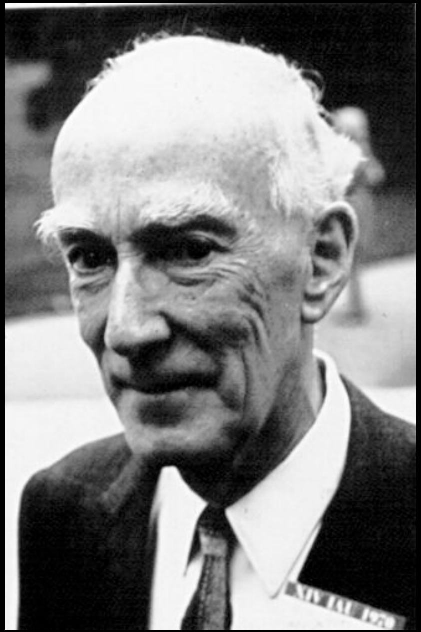
From:



Oorts constants:

$$A \equiv -\frac{1}{2}\left[\frac{dV_c}{dR}|_{R_0} - \frac{V_{c,0}}{R_0}\right]$$

$$B \equiv -\frac{1}{2}\left[\frac{dV_c}{dR}|_{R_0} + \frac{V_{c,0}}{R_0}\right]$$



Circular motion:

$$v_{circ} = \sqrt{\frac{GM(R)}{r}}$$

If the whole mass is mostly at the centre: $v_{cir}{}^{\wedge}2 \sim 1/r$

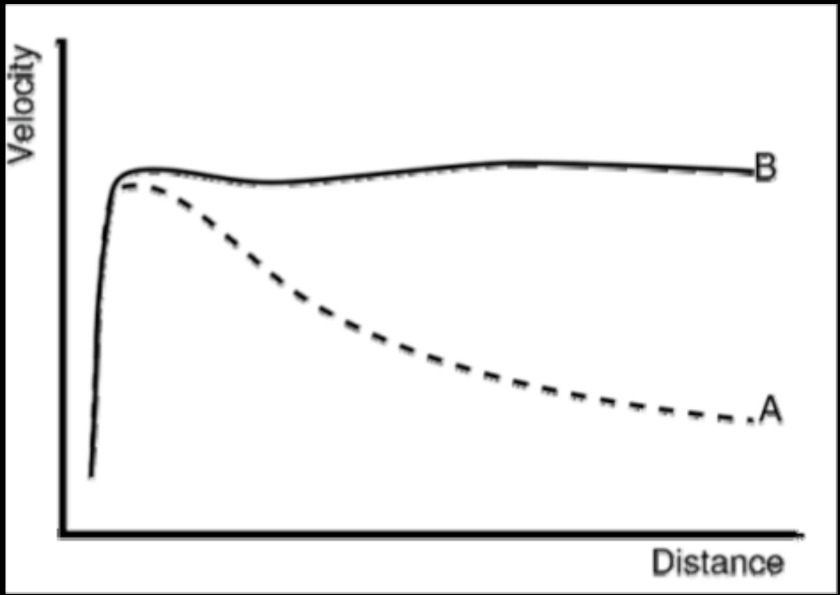
Observations vs Keplerian motion:

- Kepler. motion: (A-B)/(A+B) = 2
- Observations : (A-B)/(A+B) = 5

-Mass is not concentrated at the centre
-Non-luminous mass is required

http://icc.dur.ac.uk/~tt/Lectures/Galaxies/TeX/lec/node42.html

5. The existence of Dark Matter



B: Observations

A: theoretical expectations

1980: Vera Rubin and others also find that stars rotate too fast in the outskirts of spiral galaxies to remain bound assuming that gravity is produced only by visible matter.

5. The existence of Dark Matter

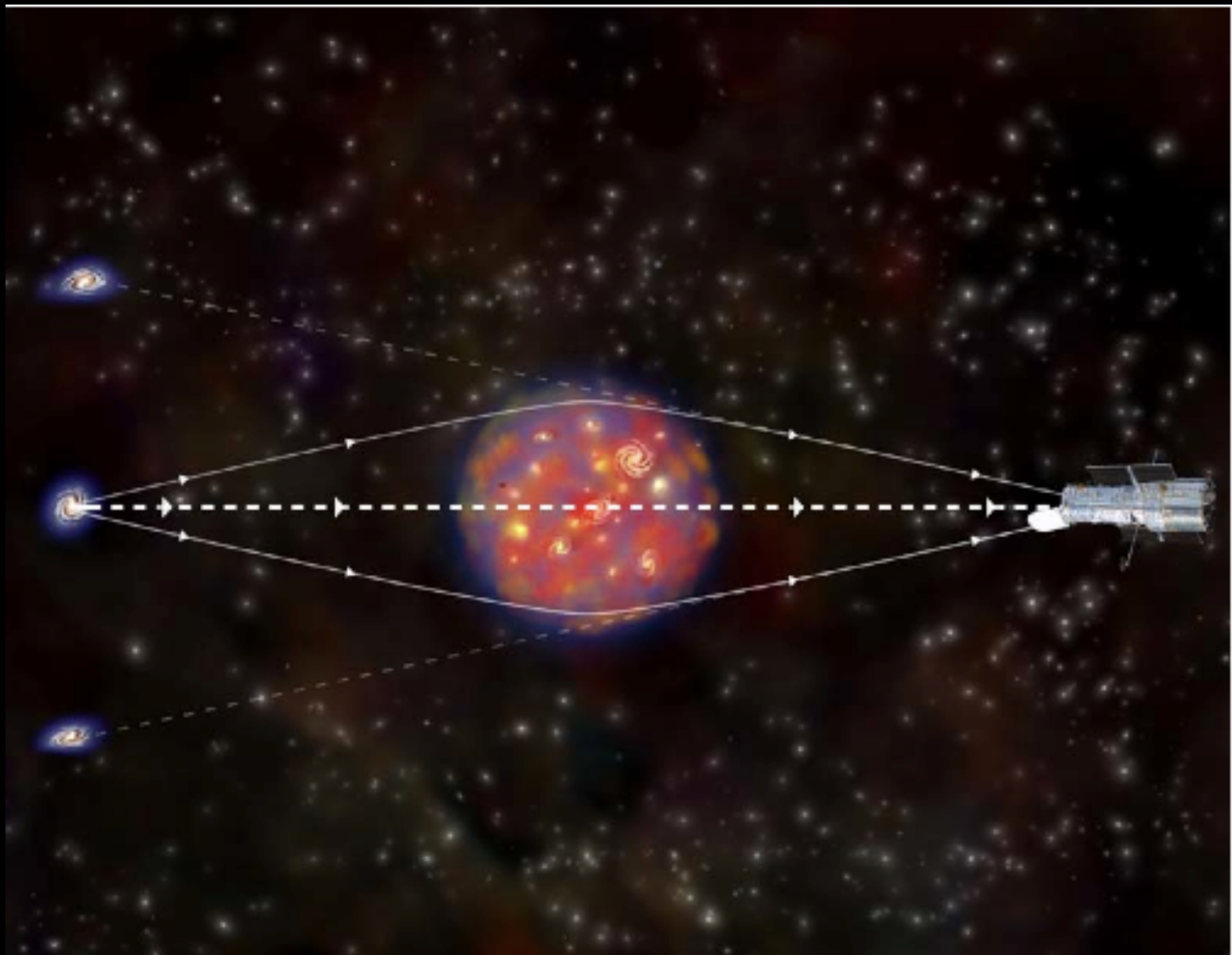
Fritz Zwicky

1936: Fritz Zwicky applied the virial theorem to the velocities of galaxies in the Coma cluster and finds M/L~500 for them to remain bound

- Mass from the virial theorem: $M_V = \langle v^2 \rangle \langle R \rangle / G$
- Visible (luminous) Mass: $M_L = N_g R_{ML} L$ (R_{ML} – mass to light ratio)

5. The existence of Dark Matter

lensing effects:



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5. The existence of Dark Matter

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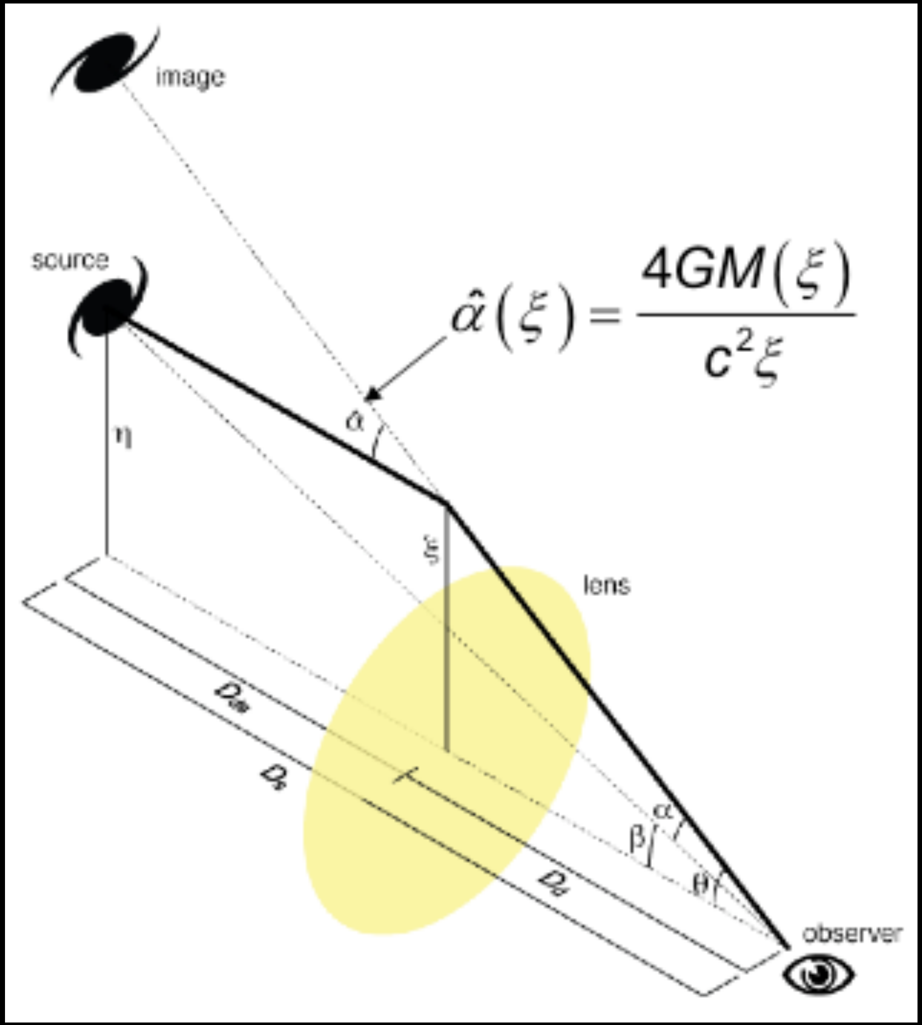

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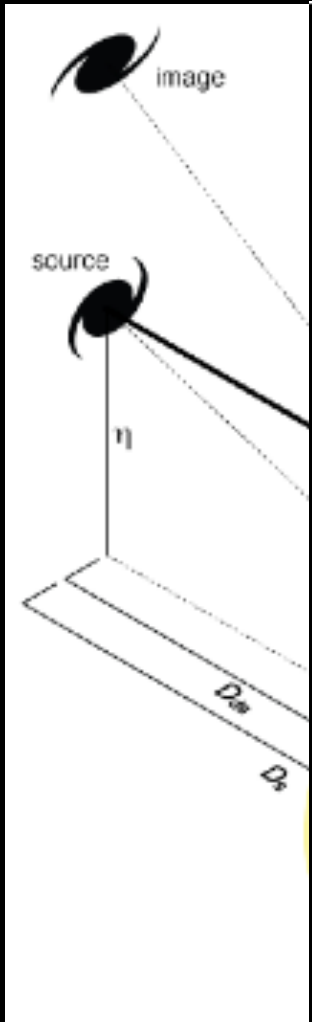
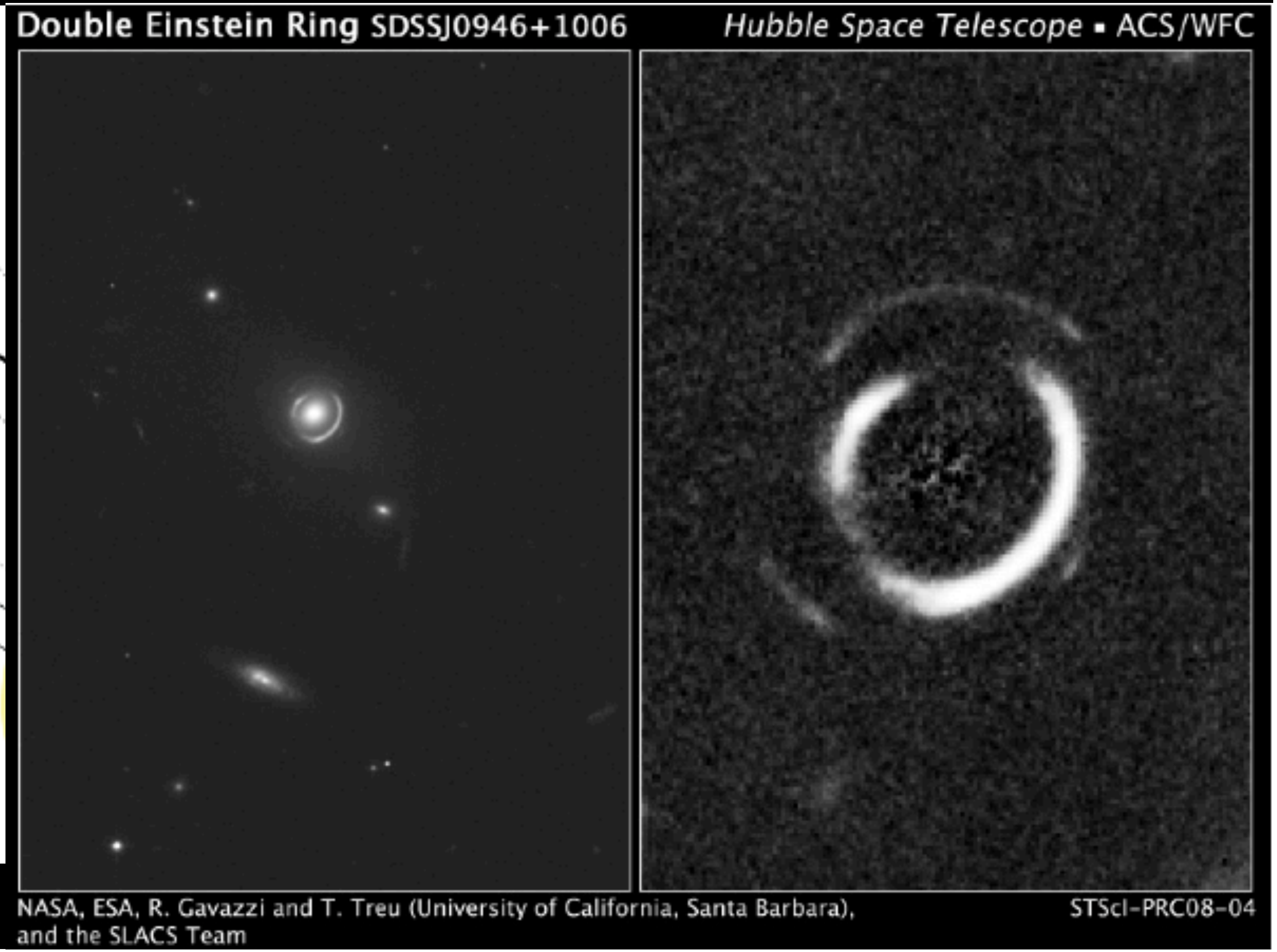
Einstein Rings

5. The existence of Dark Matter

lensing effects:

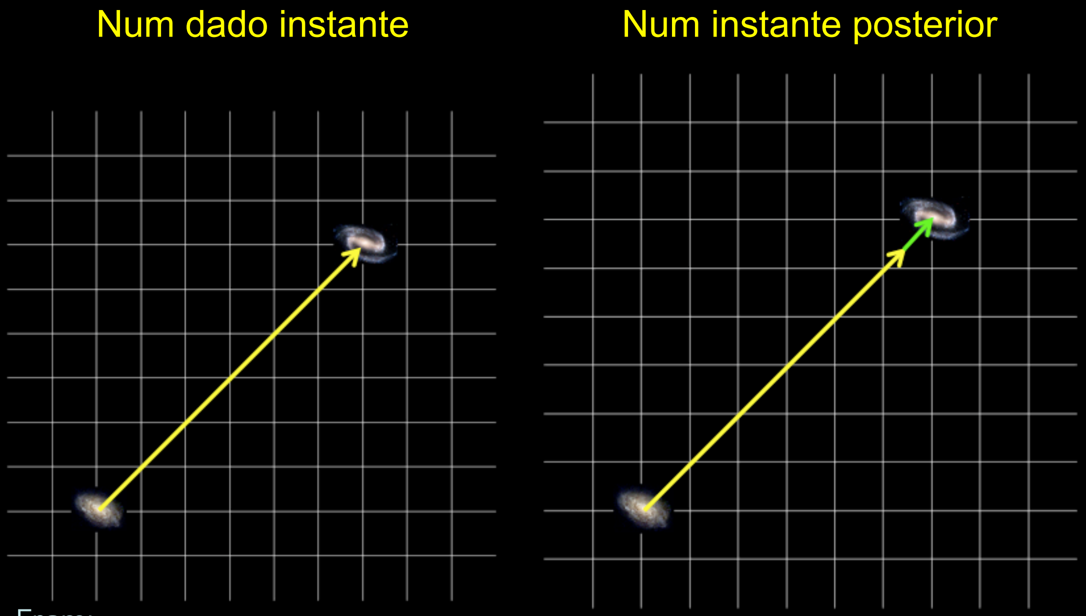




6. Cosmic expansion is accelerating

1998: S. Perlmutter and the supernova Cosmology project found first evidence for the accelerated expansion of the Universe.

assuming supernovae are standard candles, they appear further away then predicted by non-accelerating expansion models



From:

Standard Model of Cosmology

SMC: Mathematical framework

Fundamental assumptions:

- The Universe is homogeneous and isotropic when observed on large scales and expands uniformly with respect to any position

- The dynamics of space-time is described by Einstein's theory of general relativity (GR).

$$G_{ab} = R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{c^4} T_{ab} + \Lambda g_{ab}$$

for the Universe to be homogeneous and isotropic the stress-energy tensor has to be that of a perfect fluid

$$T_{ab} = (\rho + \frac{p}{c^2}) U_a U_b - \frac{p}{c^2} g_{ab}$$

SMC: Mathematical framework

Reprinted from the lecture notes “The Cosmic Microwave Background”, 2006, by Carlo Baccigalupi, available from:

http://people.sissa.it/~bacci/work/lectures/

1.3 notation and micro-elements of general relativity

Spacetime is described by three spatial dimensions plus the time coordinate. Greek indeces run from 0 to 3, while latin indeces are used for spatial directions, from 1 to 3. We use x to indicate a generic spacetime point, $\vec{x}$ and $\hat{x}$ for its spatial component and versor, respectively. The fundamental constants we use are the Boltzmann and Gravitational ones, indicated with k_B and G, respectively. We work with unitary light speed velocity, $c = 1$, never using the Planck constant $\hbar$.

In general relativity, the infinitesimal distance from two spacetime points is defined as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \ , \tag{1.1}$$

where $g_{\mu\nu}(x)$ is the metric tensor. By an appropriate change on reference frame, it is always possible to reduce the metric tensor to the Minkowski one, meaning that the system changes to the one which in free fall locally in x. The signature of the metric tensor we adopt is the following:

$$(-,+,+,+) \ . \tag{1.2}$$

The inverse of the metric tensor is represented with the indeces up:

$$g_{\mu\rho} g^{\rho\nu} = \delta^\nu_\mu \ , \tag{1.3}$$

where δ^ν_μ is the Kronecker delta. We shall use the Kronecker delta in arbitrary index configuration:

$$\delta^\nu_\mu = \delta_{\mu\nu} = \delta^{\mu\nu} = 1 \text{ if } \mu = \nu, \ 0 \text{ otherwise.} \tag{1.4}$$

The Christoffel symbols are defined as usual as

$$\Gamma_{\alpha\beta\gamma} = \frac{1}{2}\left(\frac{\partial g_{\alpha\gamma}}{\partial x^\beta} + \frac{\partial g_{\gamma\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\gamma}\right) \ , \ \Gamma^\alpha_{\beta\gamma} = \frac{1}{2} g^{\alpha\alpha'}\left(\frac{\partial g_{\alpha'\beta}}{\partial x^\gamma} + \frac{\partial g_{\alpha'\gamma}}{\partial x^\beta} - \frac{\partial g_{\beta\gamma}}{\partial x^{\alpha'}}\right) \ . \tag{1.5}$$

The Riemann, Ricci and Einstein tensors are given by

$$R^\alpha_{\beta\mu\nu} = \frac{\Gamma^\alpha_{\beta\nu}}{\partial x^\mu} - \frac{\Gamma^\alpha_{\beta\mu}}{\partial x^\nu} + \Gamma^\alpha_{\lambda\mu}\Gamma^\lambda_{\beta\nu} - \Gamma^\alpha_{\lambda\nu}\Gamma^\lambda_{\beta\mu} \ , \tag{1.6}$$

$$R_{\mu\nu} = R^\alpha_{\alpha\mu\nu} \ , \tag{1.7}$$

and

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \ , \tag{1.8}$$

where $R = R^\mu_\mu$ is the Ricci scalar and the repeated indeces are summed. To simplify the notation, let us introduce the following conventions for derivation in general relativity:

- $_{;\mu} \equiv \nabla_\mu$ means covariant derivative with respect to x^μ,
- $_{|a} \equiv {}^s\nabla_a$ means covariant derivative with respect to the spatial metric, i.e. the 3×3 array obtained removing the time column and row from the metric tensor in (1.1),
- $_{,\mu}$ means ordinary derivative with respect to x^μ .

A uni-dimensional tensor, or vector, v_μ can be obtained via covariant derivation of a scalar quantity s as

$$v_\mu = s_{;\mu} = s_{,\mu} \ , \tag{1.9}$$

where the last equality holds for scalars only. A tensor can be obtained via covariant derivation of a vector as

$$t_{\mu\nu} = v_{\mu;\nu} = v_{\mu,\nu} - v_\alpha \Gamma^\alpha_{\mu\nu} \ . \tag{1.10}$$

Further covariant derivative raises the rank of tensors:

$$\begin{aligned} u_{\mu\nu\rho} &= t_{\mu\nu;\rho} = t_{\mu\nu,\rho} - t_{\alpha\nu}\Gamma^\alpha_{\mu\rho} - t_{\mu\alpha}\Gamma^\alpha_{\rho\nu} \\ u^\nu_{\mu\rho} &= t^\nu_{\mu;\rho} = t^\nu_{\mu,\rho} - t^\nu_\alpha \Gamma^\alpha_{\mu\rho} + t^\alpha_\mu \Gamma^\nu_{\rho\alpha} \ . \end{aligned} \tag{1.11}$$

SMC: Mathematical framework

Fundamental assumptions:

- The Universe is homogeneous and isotropic when observed on large scales and expands uniformly with respect to any position

- The dynamics of space-time is described by Einstein's theory of general relativity (GR).

$$G_{ab} = R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{c^4} T_{ab} + \Lambda g_{ab}$$

$$T_{ab} = (\rho + \frac{p}{c^2}) U_a U_b - \frac{p}{c^2} g_{ab}$$

Solution is the FLRW metric:

$$ds^2 = c^2 dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right]$$

SMC: Mathematical framework

- Energy-momentum conservation $T^{ab}{}_{;b} = 0$

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) \quad \Rightarrow \quad d\left(\rho c^2 a^3\right) = -p d\left(a^3\right)$$

$$p = w\rho c^2 \qquad -1 \leq w \leq 1$$

Equation of state

$$\rho(t) = \rho_i \left(\frac{a(t)}{a_i}\right)^{-3(1+w)}$$

- Dynamical equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

Friedman equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

Raychaudhuri
(or acceleration) equation

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SMC: FLRW models

- Cosmological parameters

$$\frac{8\pi G}{3H^2}\rho + \frac{\Lambda c^2}{3H^2} - \frac{kc^2}{a^2H^2} = 1 \quad \Leftrightarrow \quad \sum_i \Omega_i + \Omega_\Lambda + \Omega_k = 1$$

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

Hubble parameter

$$\Omega = \frac{\rho}{\rho_c}, \qquad \Omega_\Lambda = \frac{\Lambda c^2}{3H^2}, \qquad \Omega_k = -\frac{kc^2}{a^2H^2}, \qquad \rho_c = \frac{3H^2}{8\pi G}$$

Critical energy density

total mater energy density parameter

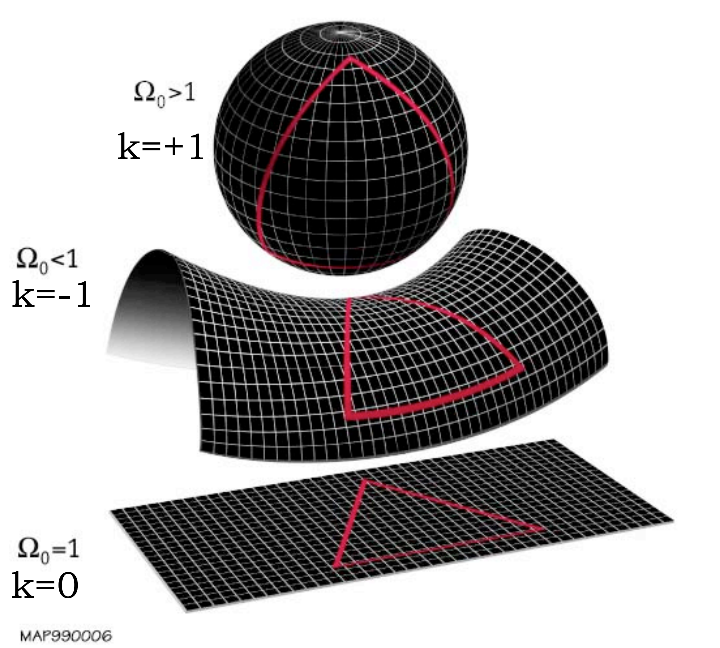
Vacuum or dark energy density parameter

Curvature density parameter

$$\rho \equiv \sum_i \rho_i$$

$$\Omega_i \equiv \frac{\rho_i}{\rho_{\text{crit}}}$$

includes all matter and radiation components (baryons, dark mater, radiation, …)

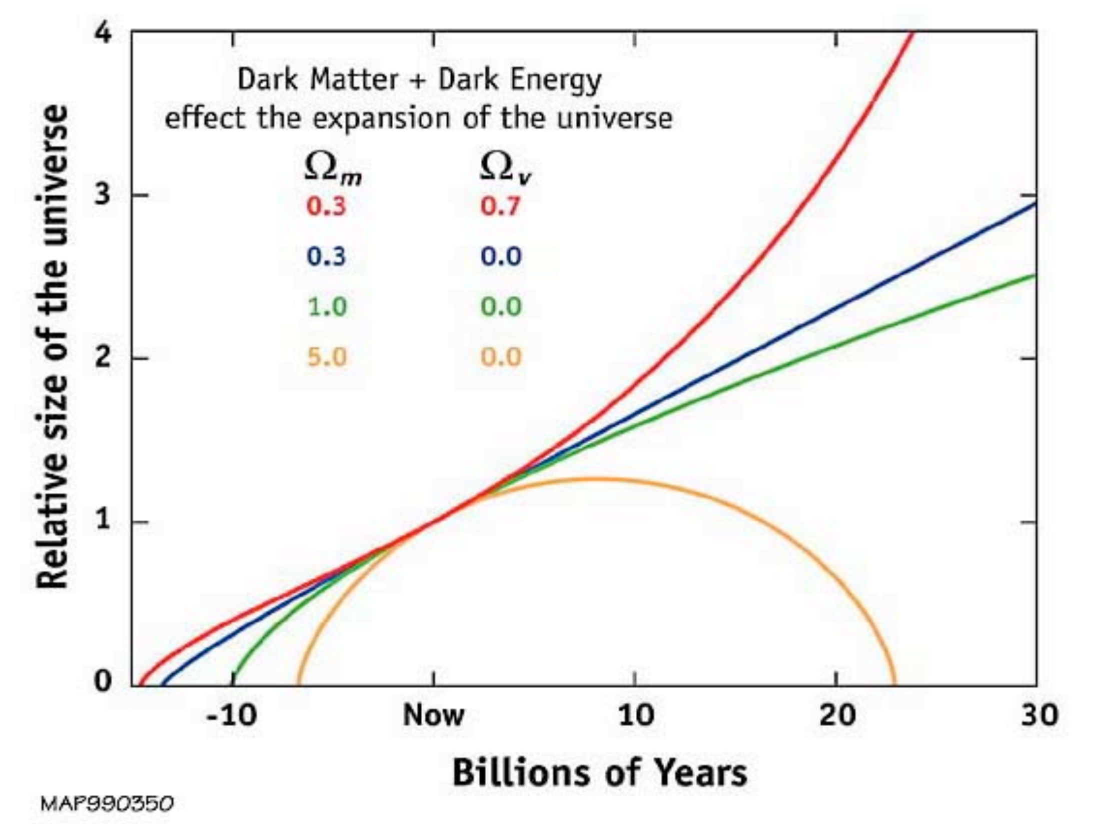


SMC: FLRW models

- Friedman equation revisited

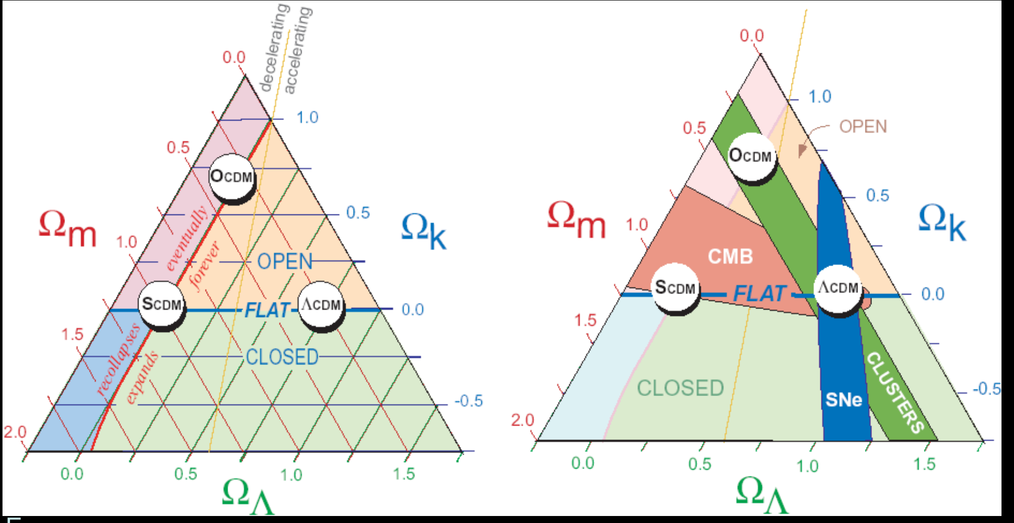
$$H^2(t) = \frac{8\pi G}{3}(\rho_r + \rho_m) - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$
$$= H_0^2 \left[\Omega_{r0} \left(\frac{a_0}{a} \right)^4 + \Omega_{m0} \left(\frac{a_0}{a} \right)^3 + \Omega_{k0} \left(\frac{a_0}{a} \right)^2 + \Omega_{\Lambda 0} \right]$$

The evolutionary fate of the Universe is determined by cosmological parameters



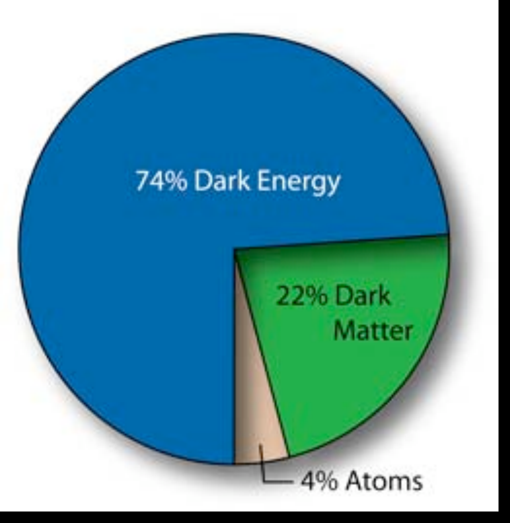
SMC: Concordance Cosmology

Combination of different observational datasets…



From:

… allow us to impose constraints on cosmological parameters



WMAP3 parameters

Parameter	Value	Description
	Basic parameters	
H_0	$70.9^{+2.4}_{-3.2}$ km s^{-1} Mpc^{-1}	Hubble parameter
Ω_b	$0.0444^{+0.0042}_{-0.0035}$	Baryon density
Ω_m	$0.266^{+0.025}_{-0.040}$	Total matter density (baryons + dark matter)
τ	$0.079^{+0.029}_{-0.032}$	Optical depth to reionization
A_s	$0.813^{+0.042}_{-0.052}$	Scalar fluctuation amplitude
n_s	$0.948^{+0.015}_{-0.018}$	Scalar spectral index
	Derived parameters	
ρ_0	$0.94^{+0.06}_{-0.09} \times 10^{-26}$ kg/m^3	Critical density
Ω_Λ	$0.732^{+0.040}_{-0.025}$	Dark energy density
z_{ion}	$10.5^{+2.6}_{-2.9}$	Reionization red-shift
σ_8	$0.772^{+0.036}_{-0.048}$	Galaxy fluctuation amplitude
t_0	$13.73^{+0.13}_{-0.17} \times 10^9$ years	Age of the universe

SMC: Exact solutions of the Friedman equation

- Scale factor:

$$\frac{d}{dt}\frac{a(t)}{a_0} = H_0\sqrt{1-\Omega_0+\Omega_{m0}\left(\frac{a}{a_0}\right)^{-1}+\Omega_{r0}\left(\frac{a}{a_0}\right)^{-2}-\Omega_{\Lambda 0}\left[1-\left(\frac{a}{a_0}\right)^{2}\right]}$$

for a critical density universe Ω=1 gives:

$$\begin{aligned}\frac{a(t)}{a_0} &= \left(\frac{3(1+w)}{2}H_0 t\right)^{2/(3(1+w))}\\ H(t) &= \frac{\dot{a}}{a} = \frac{2}{3(w+1)t}\end{aligned}$$

- Age of the Universe:

$$t = H_0^{-1}\int_0^{\frac{a(t)}{a_0}=(1+z)^{-1}} \frac{1}{\sqrt{1-\Omega_0+\Omega_{m0}x^{-1}+\Omega_{r0}x^{-2}-\Omega_\Lambda\left(1-x^2\right)}}\,dx$$

SMC: distances, horizons and volumes

- Coordinate distance:

$$\int_0^{r_e} \frac{dr}{\sqrt{1-kr^2}} = c\int_0^t \frac{dt'}{a(t')} = c\,\eta(t)$$

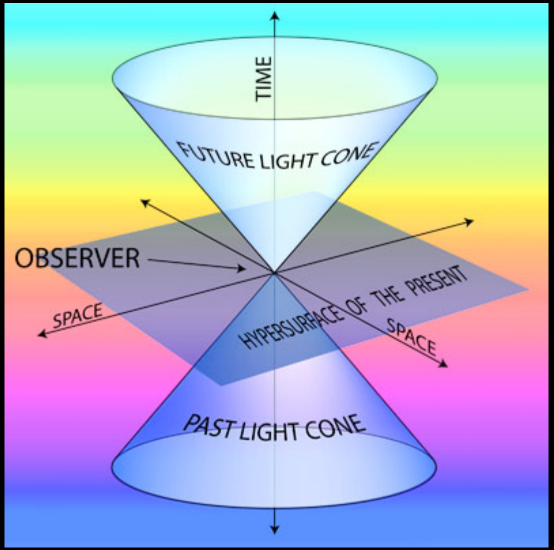
- Proper (physical) distance / particle horizon:

$$d_H(t) = \int_0^{r_e} \sqrt{|g_{rr}|}dr = a(t)\int_0^{r_e} \frac{dr}{\sqrt{1-kr^2}} = c\,a(t)\eta(t)$$

for a Λ=0 universe gives:

$$d_H(t) \simeq \frac{2}{3w+1}\frac{c}{H_0}\Omega_{w0}^{1/2}\left(\frac{a}{a_0}\right)^{3(1+w)/2} = 3\frac{1+w}{1+3w}ct$$

From:



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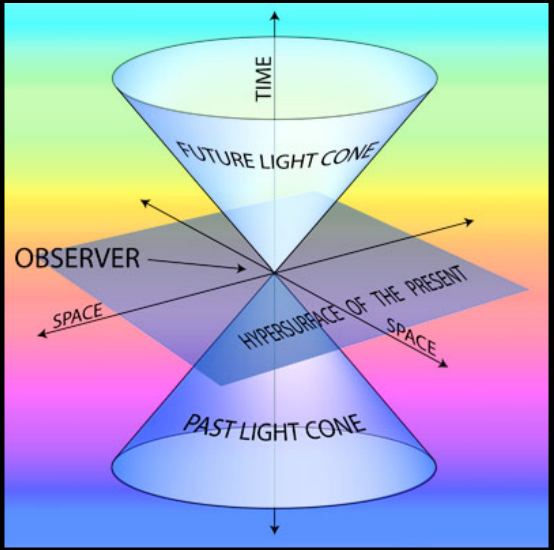
Angular size of the particle horizon for Λ=0:

$$\boxed{\theta_H \simeq 2\tan\frac{\theta_H}{2} = \frac{\Omega_0^{3/2}\sqrt{1+z}}{\Omega_0 z + (\Omega_0 - 2)\left(\sqrt{1+\Omega_0 z} - 1\right)}}$$

From:



SMC: distances, horizons and volumes

- Hubble length:

$$R_H(t) = \frac{c}{H(t)} = \frac{3(w+1)}{2}ct$$

where the last equality holds for a critical density universe Ω=1

- Physical volume element:

$$dV = \sqrt{|g|}\, dr\, d\theta\, d\phi = (ar)^2 \frac{a\, dr}{\sqrt{1-kr^2}}\, d\Omega$$

$$\frac{dV}{d\Omega\, dz} = \frac{c}{H(z)} \frac{(a_0 r)^2}{(1+z)^3} = \frac{c}{H_0} \frac{d_A^2}{\mathscr{H}(z)(1+z)}$$

where:

$$\mathscr{H}(z) = H(z)/H_0$$

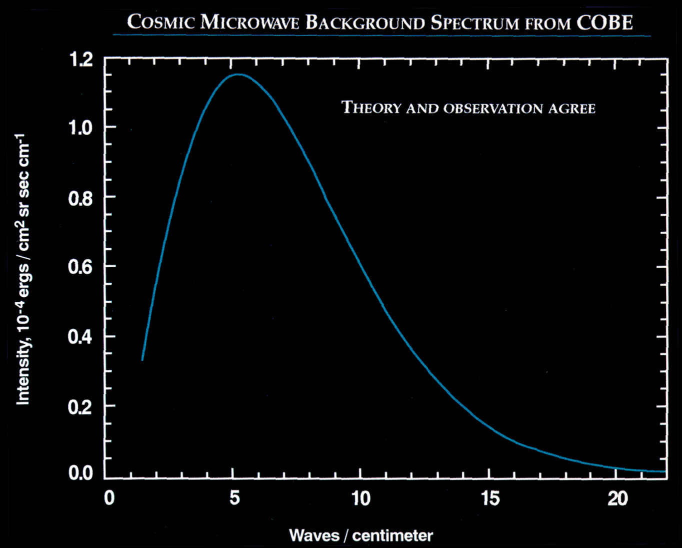
Initial Conditions and Inflation

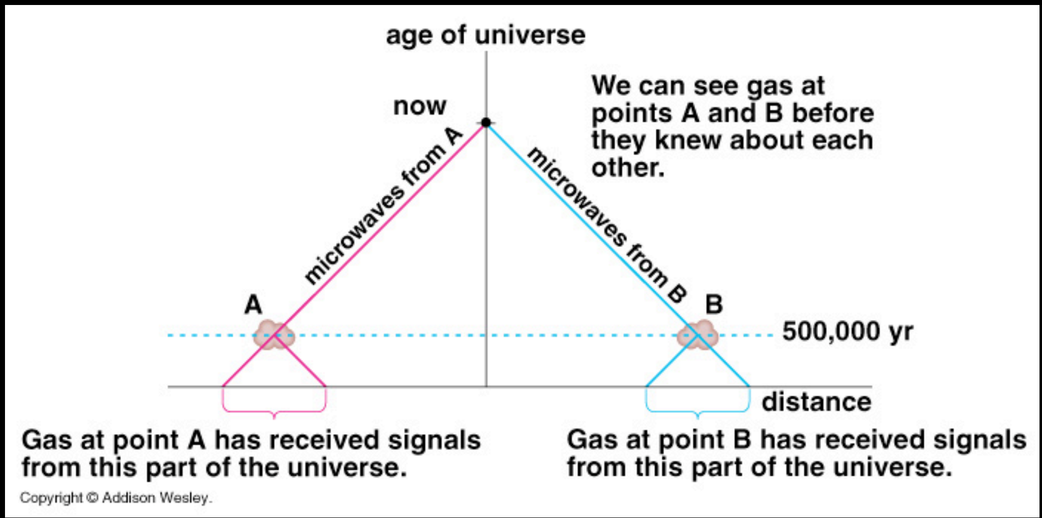
The Horizon Problem

At high redshift

$$\theta_H \simeq \frac{180}{\pi}\sqrt{\frac{\Omega_0}{z}}\,\mathrm{deg}$$

there are ~54000 causal disconnected areas in the CMB sky. Why the CMB has a thermal spectrum with a so uniform temperature in all directions (2.725 °K)





The Flatness Problem

From the Friedmann Equation, written at early times:

$$|\Omega(t) - 1| = \frac{|k|}{a^2(t)H^2(t)} = \frac{|k|}{\dot{a}^2(t)}$$

is a decreasing function of time

decreases as time approaches the big bang instant.

This means that as we go back in time the energy density of universe has to be extremely close to critical density. For t=1e-43 s (Planck time) Ω should deviate no more than 1e-60 from the unity.

Why has the universe to start with Ω(t) so close to 1?

The Monopoles & other relics Problem

Particle physics predicts that a variety of “exotic” stable particles, such as the magnetic monopoles, should be produced in the early phase of the Universe and remain in measurable amounts until the present.

No such particles have yet been observed. Why?

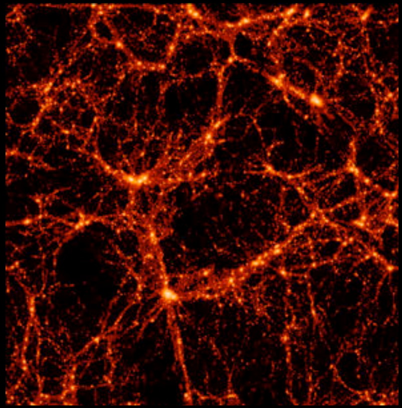
This either implies that the predictions from particle physics are wrong, or their densities are very small and therefore there's something missing from this evolutionary picture of the Big Bang.

The Origin of Perturbations Problem

Locally the universe is not homogeneous. It displays a complex hierarchical pattern of galaxies, clusters and super clusters.

What's the origin of cosmological structure?
Does it grew from gravitational instability?
What is the origin of the initial perturbations?

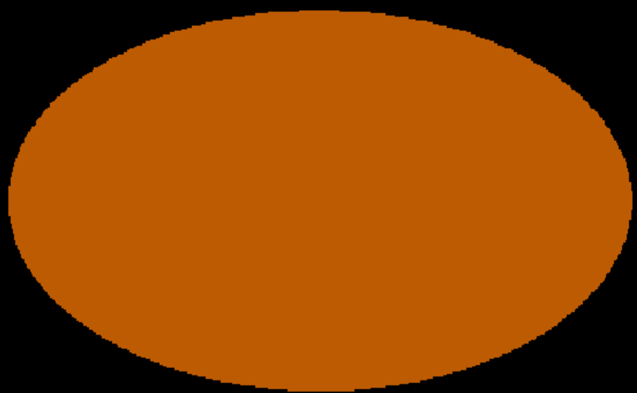
Without a mechanism to explain their existence one has to assume that they ``were born'' with the universe already showing the correct amplitudes on all scales, so that gravitaty can correctly reproduce the present-day structures?



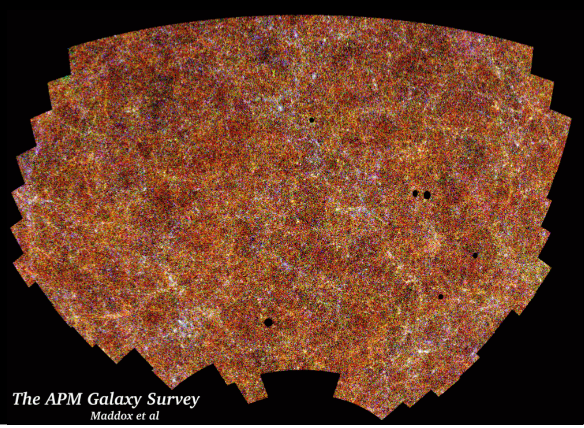
The homogeneity and isotropy Problem

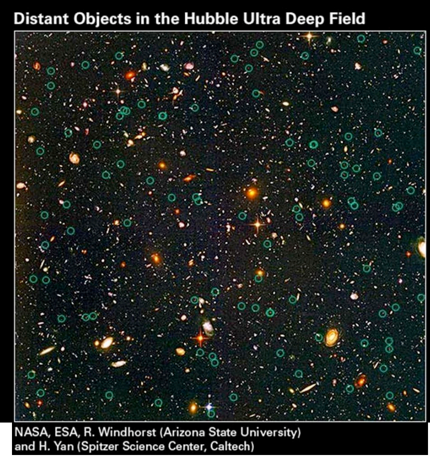
Why is the universe homogeneous on large scales? At early times homogeneity had to be even more “perfect”.

The FLRW universes form a very special subset of solutions of the GR equations. So *why nature “prefers” homogeneity and isotropy from the beginning as opposed to having evolved into that stage?*



CMB T=2.725 K





The Theory of Inflation

Inflation can be defined as

$$\text{Inflation} \Leftrightarrow \ddot{a} > 0 \Leftrightarrow \frac{d}{dt}\left(cH^{-1}/a\right) < 0.$$

This happens when

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right)$$

$\Rightarrow$ $p < -\rho c/3$

The Theory of Inflation

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If the universe experiences a phase of inflation prior to radiation domination we avoid…

… the flatness problem because the derivative of the scale factor increases in

$$|\Omega(t) - 1| = \frac{|k|}{a^2(t)H^2(t)} = \frac{|k|}{\dot{a}^2(t)}$$

Is an increasing function of time

The Theory of Inflation

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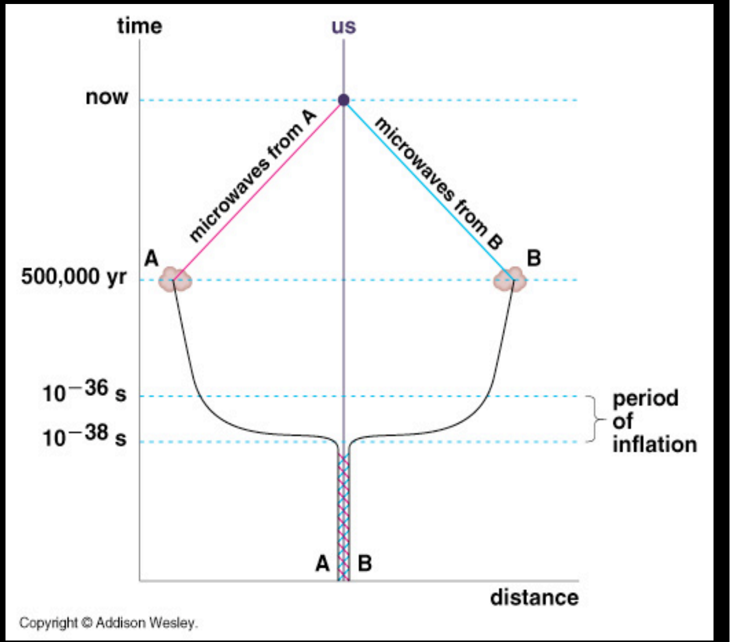
If the universe experiences a phase of inflation prior to radiation domination we avoid…

… the horizon problem because if the accelerated expansion is long enough all causally disconnected sky patches could have been in causal contact



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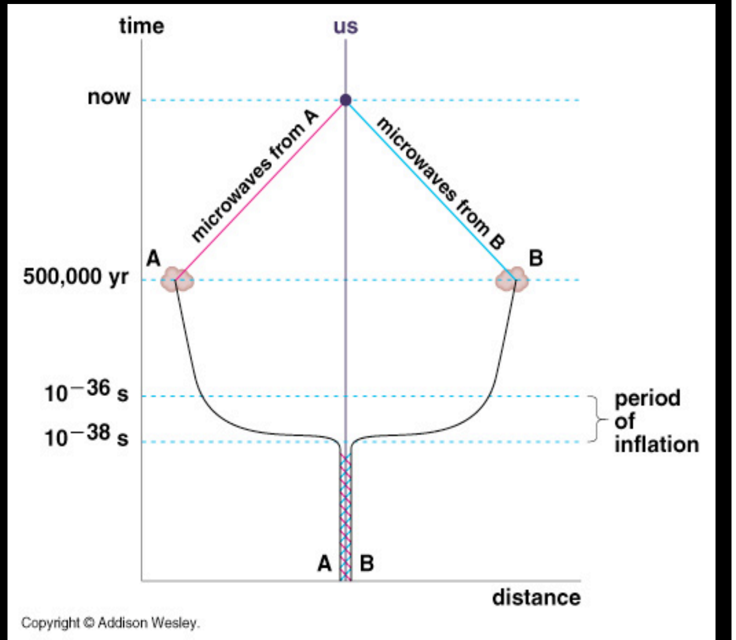
If the universe experiences a phase of inflation prior to radiation domination we avoid…

… the monopoles problem because their density is very much diluted to be observed.



The Theory of Inflation

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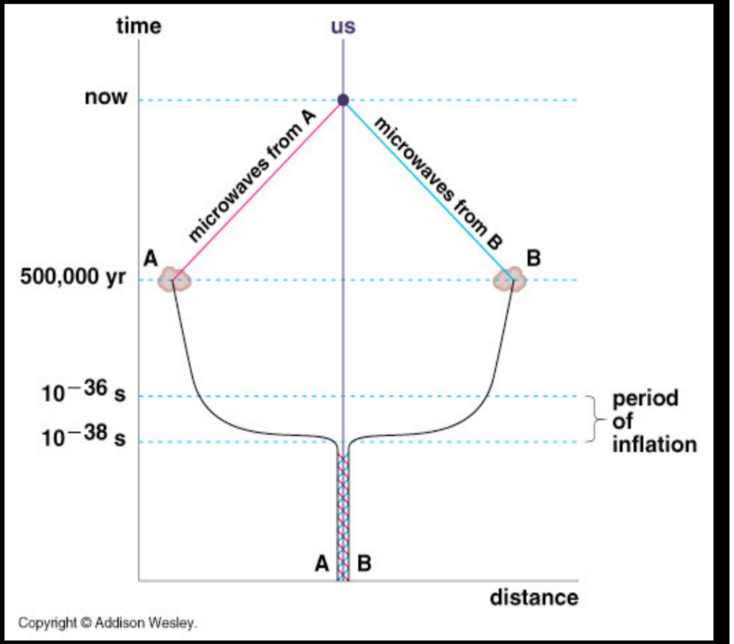
If the universe experiences a phase of inflation prior to radiation domination we avoid…

… the homogeneity problem because the visible universe expanded a lot and had time to undergo causal interactions



The Theory of Inflation

The simplest model of inflation is described by, a slow-rolling, scalar field with:

$$T_{ab} = \varphi_{;a}\ \varphi_{;b} - g_{ab}\mathscr{L}(\varphi)\ ; \qquad \mathscr{L}(\varphi) = \tfrac{1}{2}\varphi_{;a}\ \varphi_{;b}g^{ab} - V(\varphi)$$

with

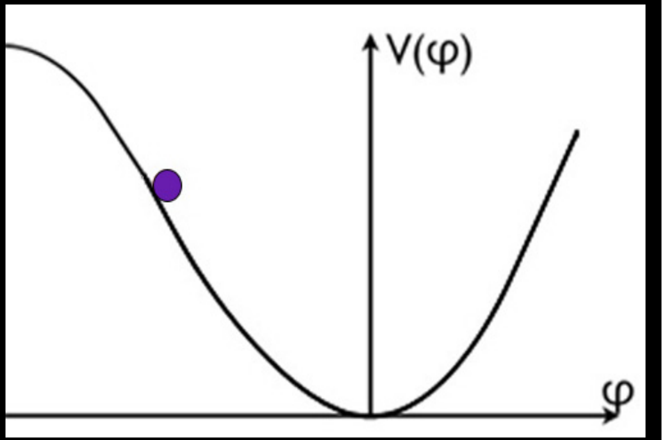
$$\rho_\varphi = \dot\varphi^2/2 + V(\varphi)$$
$$p_\varphi = \dot\varphi^2/2 - V(\varphi)$$

The conservation of the energy stress tensor gives:

$$\ddot\varphi + 3\frac{\dot a}{a}\dot\varphi + \frac{dV}{d\varphi} = 0$$

whenever φ "slow-rolls", $\ddot\varphi\,,\,\dot\varphi \simeq 0$

$$p_\varphi = -\rho_\varphi \simeq V = \text{const}$$



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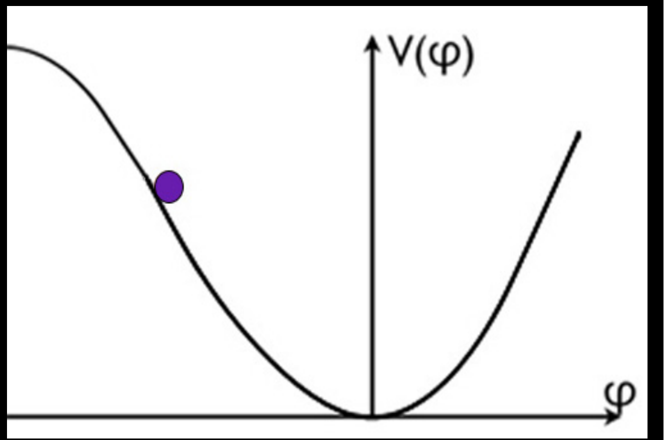
with

$$\rho_\varphi = \dot\varphi^2/2 + V(\varphi)$$
$$p_\varphi = \dot\varphi^2/2 - V(\varphi)$$

For $p_\varphi = -\rho_\varphi \simeq V = \text{const}$ one gets:

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho$$
$$\frac{a(t)}{a_i} = \exp\left[H(t - t_i)\right]$$

the universe expands exponentially



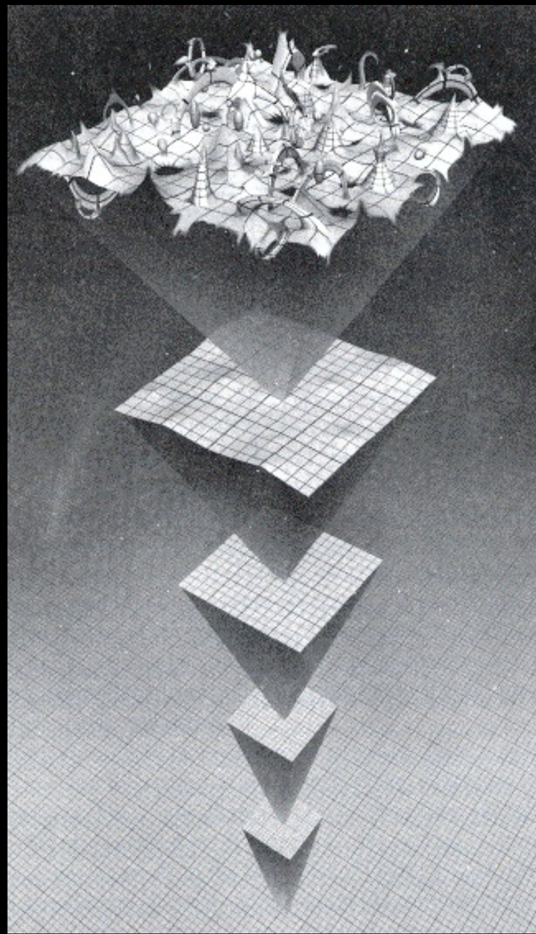
The Theory of Inflation

Inflation also provides a mechanism for the origin of fluctuations…

… fluctuations (density and grav. waves) are due to quantum fluctuations about the vacuum state of the inflationary potential.

During inflation fluctuations are "inflated" to macroscopic scales

From:



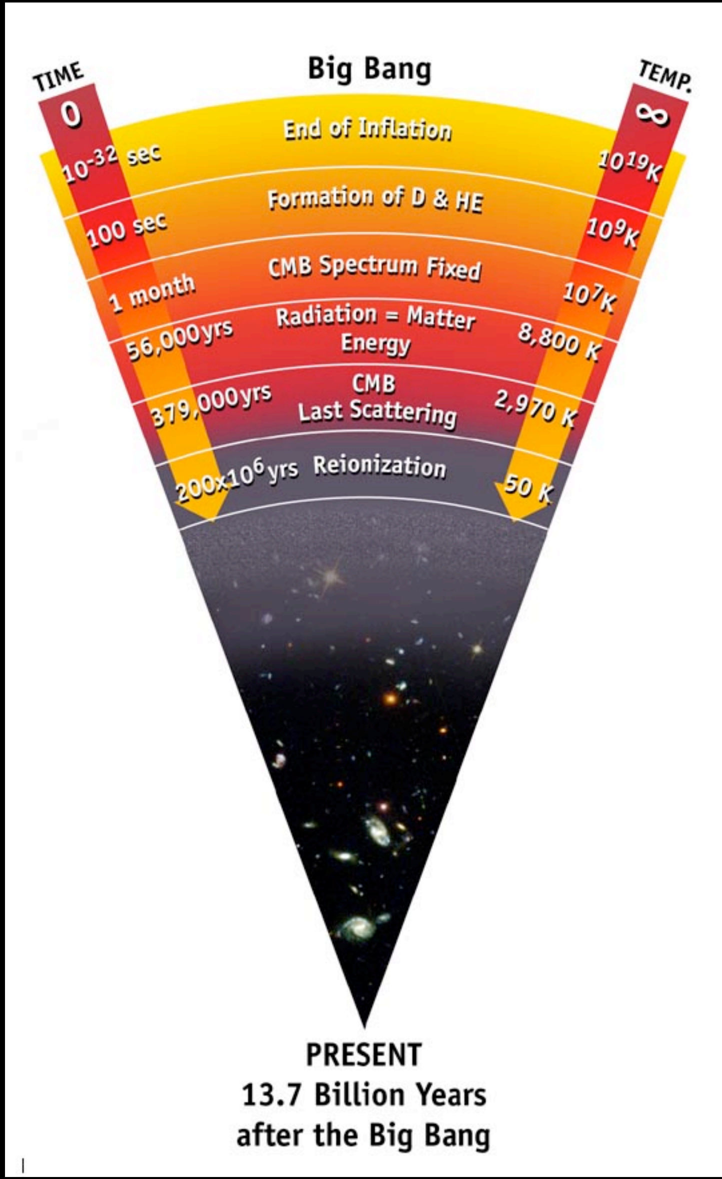
Standard Model of Cosmology: the origin of fluctuations

SMC = Hot Big Bang + Inflation

FLRW models provide a description for the evolution of the “background” Universe

provides a mechanism for the origin of perturbations to the “background Universe”

fluctuations (density and grav. waves) are due to quantum fluctuations about the vacuum state of the inflationary potential.



From:

Standard Model of Cosmology: the origin of fluctuations

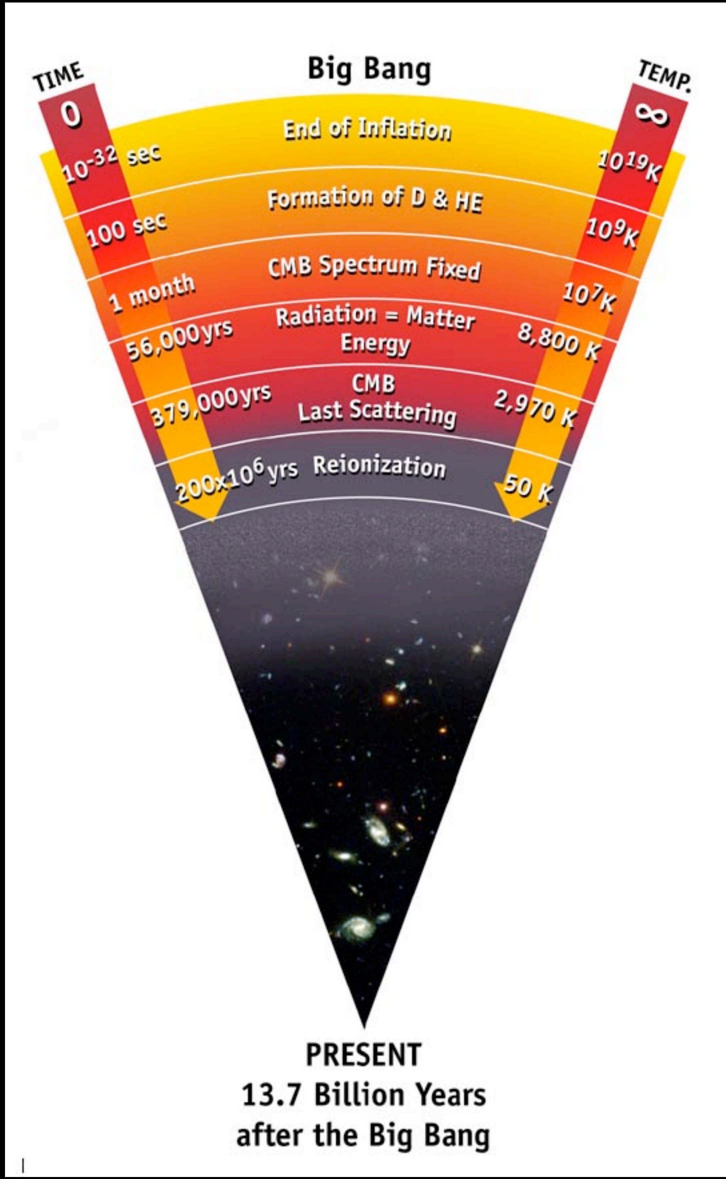
SMC = Hot Big Bang + Inflation

After the end of inflation

$$H^2(t) = \frac{8\pi G}{3}(\rho_r + \rho_m) - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$
$$= H_0^2 \left[\Omega_{r0} \left(\frac{a_0}{a}\right)^4 + \Omega_{m0} \left(\frac{a_0}{a}\right)^3 + \Omega_{k0} \left(\frac{a_0}{a}\right)^2 + \Omega_{\Lambda 0} \right]$$

• Background evolution is dominated by:

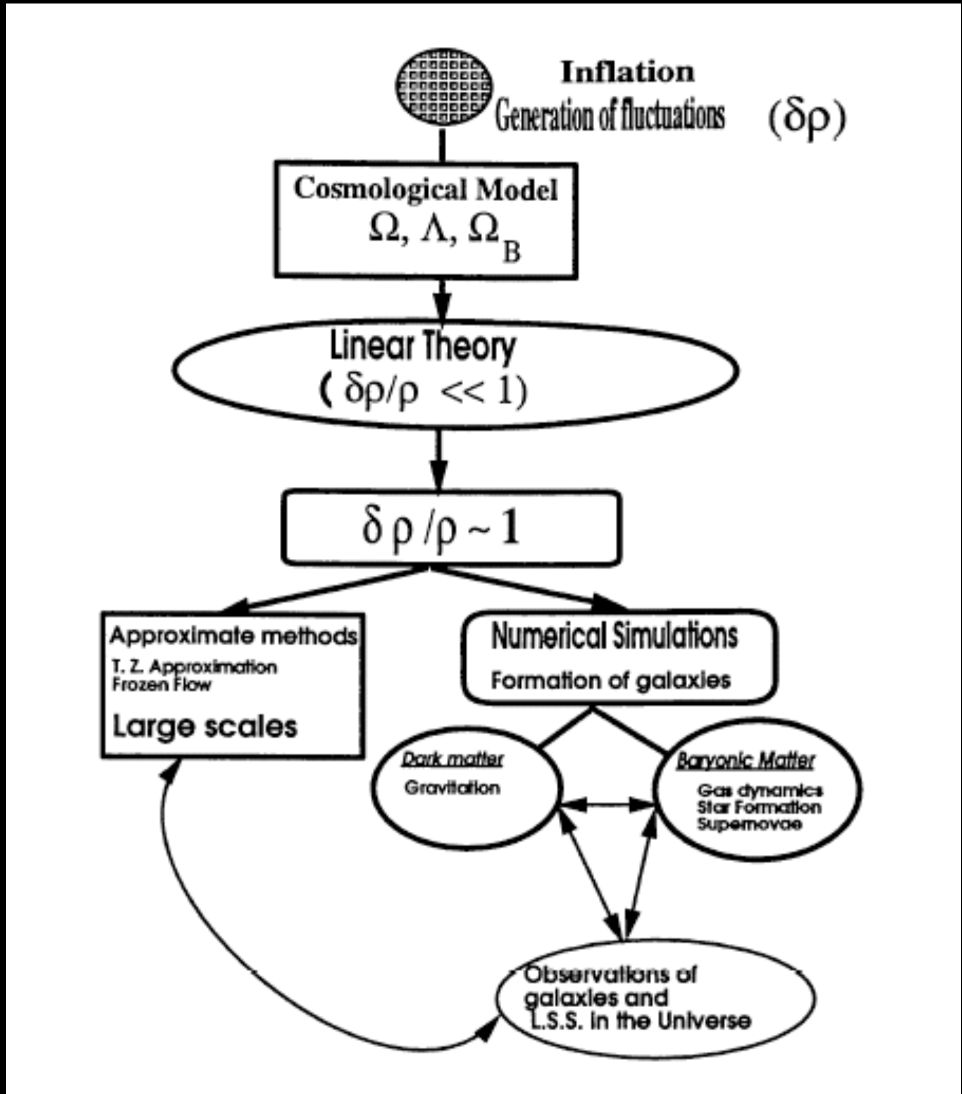
- Radiation
- Matter
- Dark Energy



From:

Formation of Cosmological Structure

Formation of Cosmological Structure: « overall picture »



From: G.Yepes, Cosmological Numerical Simulations, Astronomical Society of the Pacific Conference Series, Vol. 126 (1997)

Formation of Cosmological Structure

Cosmological random fields

- specified by an infinite set of joint probability distribution functions (pdfs):

$$\langle A(\mathbf{x})\rangle,\ \langle A(\mathbf{x})A(\mathbf{x}')\rangle, \ldots\ ,\ \langle A(\mathbf{x})A(\mathbf{x}')\ldots A(\mathbf{x}^{(\mathbf{n})})\rangle$$

- usually assumed **Ergodic** (ensemble average $\leftrightarrow$ spatial average)

- invariant under rotations and translations:
  - one-point pdf independent of **x**
  - two-point pdf depends only on **r**=|**x**-**x**'|

Formation of Cosmological Structure

Density contrast (overdensity field/excess function)

- $$\boxed{\delta(\mathbf{x},t) \equiv \frac{\rho(\mathbf{x},t)-\rho_0(t)}{\rho_0(t)}}$$ where $\rho_0(t) = \langle \rho(\mathbf{x},t) \rangle$

- Gaussian distributed, ergodic, invariant under rot. and trans.
  - one-point pdf: $\langle \delta(\mathbf{x}) \rangle = 0$
  - two-point pdf: $\xi(r) \equiv \langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r}) \rangle$ (correlation function)
- Fourier decomposition:

$$\delta(\mathbf{x},t) = \sum \delta_k(\mathbf{k},t) e^{-i\mathbf{k}\cdot\mathbf{x}} \qquad \delta_k(\mathbf{k},t) = \frac{1}{V}\int \delta(\mathbf{x},t) e^{i\mathbf{k}\cdot\mathbf{x}} d^3x$$

- Correlation function & power spectrum

$$\xi(r,t) = \frac{V}{(2\pi)^3}\int \langle |\delta_k(\mathbf{k},t)|^2 \rangle e^{-i\mathbf{k}\cdot\mathbf{r}} d^3k$$

$$P(k,t) = \langle |\delta_k(\mathbf{k},t)|^2 \rangle = \frac{1}{V}\int \xi(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^3r$$

Formation of Cosmological Structure

Linear perturbation theory

- linear regime: $\delta \ll 1$
- expanding universe: $R_H \propto t$ grows faster then $\lambda_{\rm phy} = a(t)\lambda_{\rm com} \propto t^\alpha$
- general treatment: GR multi-fluid component analysis

Sub-horizon perturbations in non-relativistic components

- Growth of perturbations **<u>collisional matter</u>** (experiences short-lived accelarations due to shocks, e.g. baryons)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\left(\phi + \frac{p}{\rho}\right)$$

$$\nabla^2 \phi = 4\pi G \rho$$

Continuity equation

Euler equation

Poisson equation

where

- physical coord.: $\mathbf{r} = a(t)\mathbf{x}$; $\mathbf{v} = \dot{a}\mathbf{x} + a\mathbf{u}$

Hubble expansion **peculiar velocity (** $\mathbf{u} = \dot{\mathbf{x}}$ **)**

Formation of Cosmological Structure

Linear perturbation theory

Sub-horizon perturbations in non-relativistic components

- Growth of perturbations **collisional matter**

$$\boxed{\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} + \left(\frac{c_S^2 k^2}{a^2} - 4\pi G\rho_0\right)\delta = 0}$$

where $c_S^2 \equiv (\partial p/\partial \rho)_S$ is the adiabatic sound speed.

This defines the **Jeans** scale

$$\lambda_J = \frac{2\pi}{k_J} \quad ; \qquad k_J = \frac{2a}{c_S}\sqrt{\pi G\rho_0}$$

Solutions:

- **k < k**$_J$: 2 solutions with increasing, and decreasing amplitudes
- **k > k**$_J$: amplitude does not grow (oscillates)

Formation of Cosmological Structure

Linear perturbation theory

Sub-horizon perturbations in non-relativistic components

- Growth of perturbations **collisionless matter** (can develop multi-stream flows, where there isn't a unique velocity at a given r)

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}} - \nabla\phi\cdot\frac{\partial f}{\partial \mathbf{v}} = 0$$

$$\nabla^2\phi = 4\pi G\rho\,,$$

Boltzmann equation

Poisson equation

where f - phase space density, and $\rho = m\int f(\mathbf{r},\mathbf{v},t)d^3v$

Perturbing we find again two regimes of evolution around the characteristic scale (free streaming):

$$k_{\rm FS} = \frac{2a}{v_*}\sqrt{\pi G\rho_0} \quad ; \qquad v_*^{-2} = \frac{\int \frac{\mathbf{k}\cdot\partial f_0/\partial\mathbf{v}}{\mathbf{k}\cdot\mathbf{v}}\,d^3v}{\int f_0\,d^3v}$$

- **k < k**$_{FS}$: 2 solutions with increasing, and decreasing amplitudes
- **k > k**$_{FS}$: perturbations are dumped due to free streaming

Formation of Cosmological Structure

Linear perturbation theory

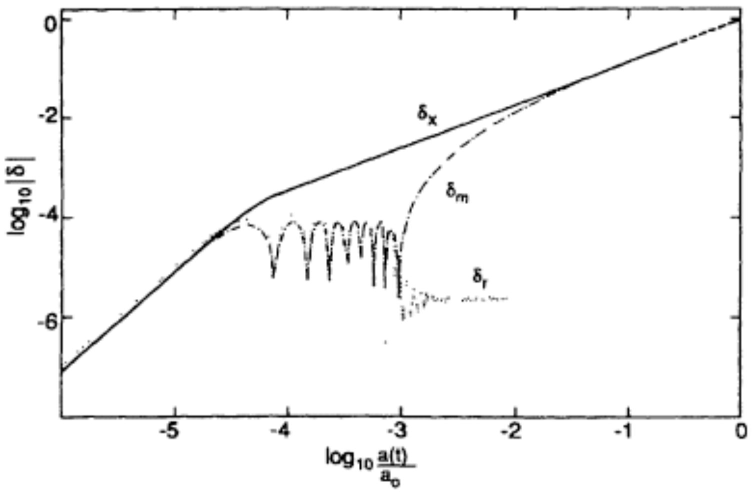
Sub-horizon perturbations in non-relativistic components

•Matter domination era: in both **collisional** and **collisionless** systems (e.g. baryons and dark matter) perturbations evolve according:

$$\boxed{\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\rho_0\delta = 0}$$

for scales larger than the Jeans (**k<k$_J$**) or free-stream (**k<k$_{FS}$**) scale.

The growing solution in both cases is: $\boxed{\delta_k \propto t^{2/3}}$



From: Coles & Lucchin, 2002

Figure 3. Evolution of perturbations in the baryonic, δ_m, cold dark

Formation of Cosmological Structure

Linear perturbation theory

Multi component fluid case

- Cosmic fluid is made of relativistic and non-relativistic components characterized by individual equations of state w_j

  - Radiation
  - Neutrinos
  - Baryons
  - Dark matter
  - Dark energy

Energy density components:

$$\rho(\mathbf{r},t) = \rho_b(\mathbf{r},t) + \rho_{DM}(\mathbf{r},t) + \rho_{rad}(\mathbf{r},t) + \rho_v(\mathbf{r},t)$$

Component perturbations:

$$\rho_u(t)\delta(\mathbf{r},t) = \rho_{b,u}\delta_b + \rho_{DM,u}\delta_{DM} + \rho_{rad,u}\delta_{rad} + \rho_{v,u}\delta_v$$

- all subjected to gravity + interactions between collisional species

- The evolution of the system requires integration of the Boltzmann transport equations for both collisional and colissionaless species

Formation of Cosmological Structure

Linear perturbation theory

Multi component fluid case

- The solution of the Boltzmann transport equations is numerical (public codes: CMBFAST, CAMB, etc)
  - Central quantity is the computation of the linearly extrapolated matter power spectrum:

$$\mathcal{P}(k,t) = \frac{g^2(\Omega,\Omega_\Lambda)}{g^2(\Omega_0,\Omega_{\Lambda 0})}\left(\frac{k}{aH}\right)^4 T^2(k,t)\,\delta_H^2(k)$$

(for t after during matter domination)

Linear growth factor of density perturbations

Transfer Function (cmbfast output)

Primordial power spectrum of density perturbations

where,

$$\delta_H^2(k) = \delta_H^2(k_0)\left(\frac{k}{k_0}\right)^{n-1}$$

(n=1 is the scale invariant Harrison-Zel'dovich power spectrum)

Formation of Cosmological Structure

Linear perturbation theory

Multi component fluid case

- Linear extrapolated matter power spectrum

$$\mathcal{P}(k,t) = \frac{g^2(\Omega,\Omega_\Lambda)}{g^2(\Omega_0,\Omega_{\Lambda 0})}\left(\frac{k}{aH}\right)^4 T^2(k,t)\,\delta_H^2(k)$$

T(k,t) fitting formula by BBKS (LCDM)

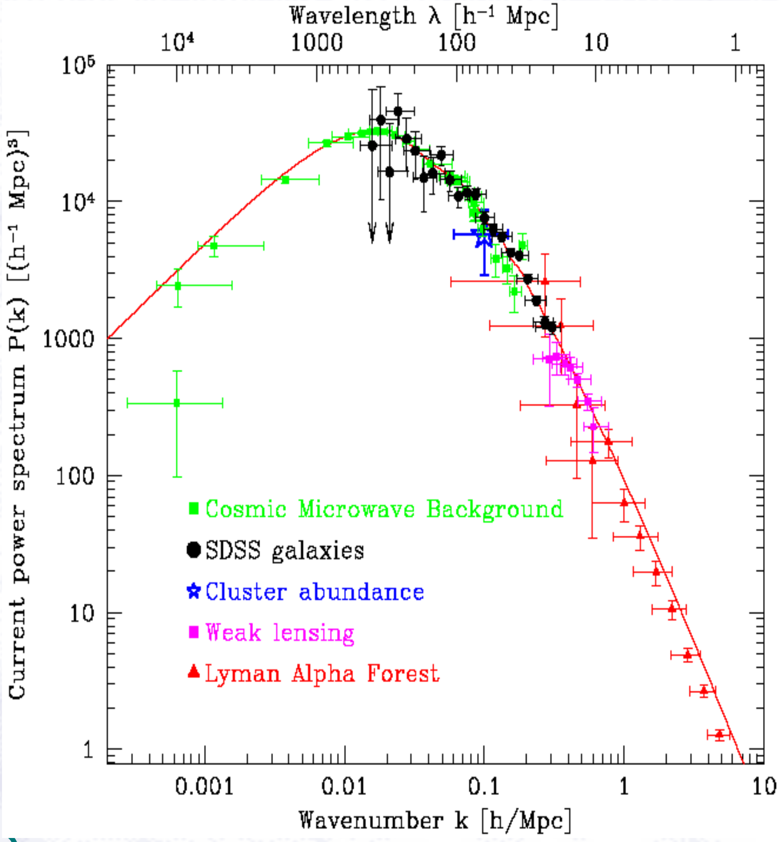
$$T(q) = \frac{\ln(1+2.34q)}{2.34q}[1+3.89q+(16.1q)^2+(5.46q)^3+(6.71q)^4]^{-1/4}$$

with:

$$q \equiv (k\theta/h\,\mathrm{Mpc}^{-1})/\Gamma,\ \theta = T_{CMB}/2.73$$

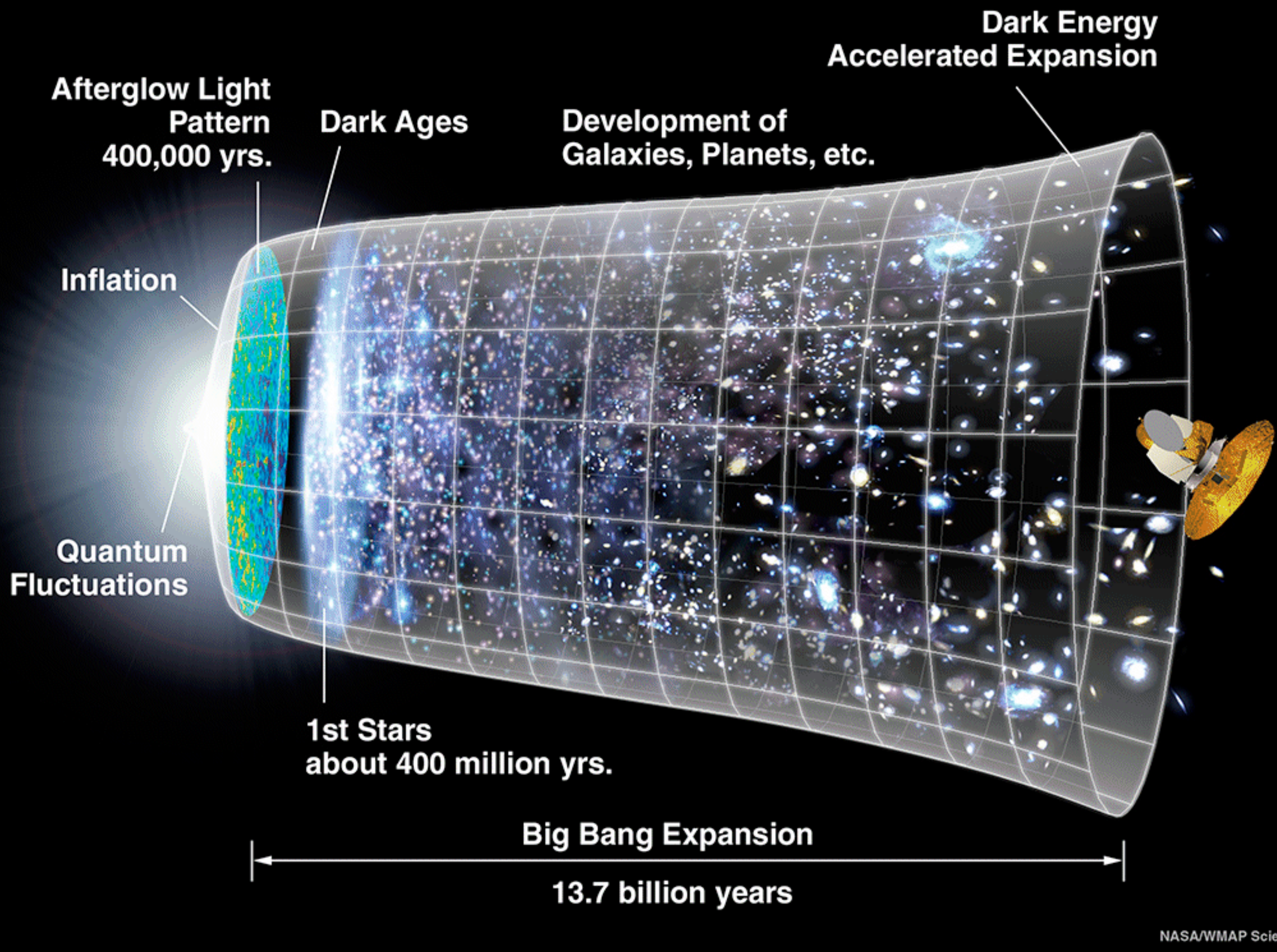
$$\Gamma = \Omega_0 h \exp[-\Omega_{B0}(1+\sqrt{2h}/\Omega_0)]$$ (shape paremeter)

$g(\Omega, \Omega_\Lambda)$ is the linear growth factor (computed using linear perturbation theory – see last 3 slides of the lecture)



From:

Structure formation history:



Appendix

Computation of the linear growth factor

Linear perturbation theory: matter perturbations

Linearized field equations in commoving coordinates (expressed in terms of perturbed quantities):

$$\begin{aligned} \frac{\partial \delta}{\partial t} + \frac{1}{a}\vec{\nabla}_x \cdot \vec{v} &= 0 \\ \frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} &= -\frac{1}{a}\vec{\nabla}\phi \\ \nabla^2\phi &= 4\pi G\, a^2\, \rho_u\, \delta \end{aligned}$$

$$\Rightarrow \quad \frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = \frac{3}{2}\Omega_0 H_0^2 \frac{1}{a^3}\,\delta$$

$$\Downarrow$$

$$\delta(\mathbf{x}, t) = \underset{\uparrow}{D_1(t)}\, \Delta_1(\mathbf{x}) + \underset{\uparrow}{D_2(t)}\, \Delta_2(\mathbf{x})$$

growing mode solution

decaying mode solution

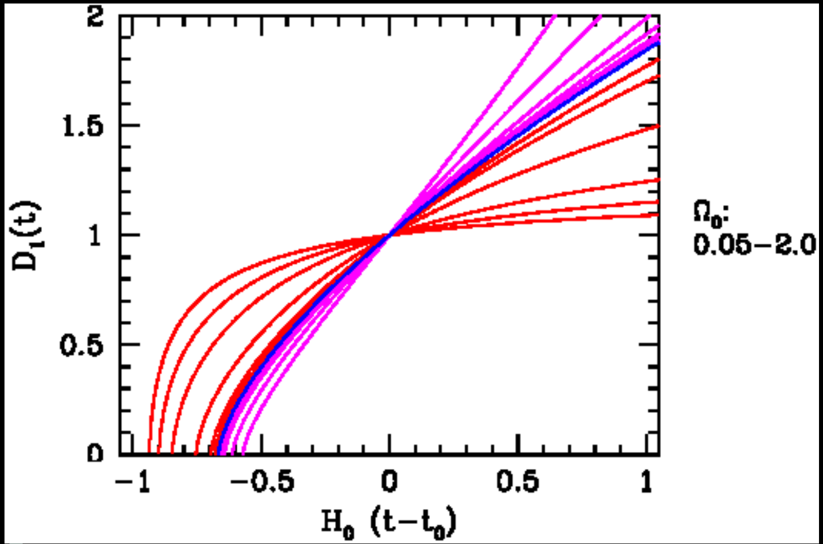
Which lead to the following equation

$$\frac{d^2 D}{dt^2} + 2\,\frac{\dot{a}}{a}\,\frac{dD}{dt} = \frac{3}{2}\Omega_0 H_0^2\, \frac{1}{a^3}\, D$$

Computation of the linear growth factor

Linear perturbation theory: matter perturbations

Linear growth factor is analytical for various FLRW models



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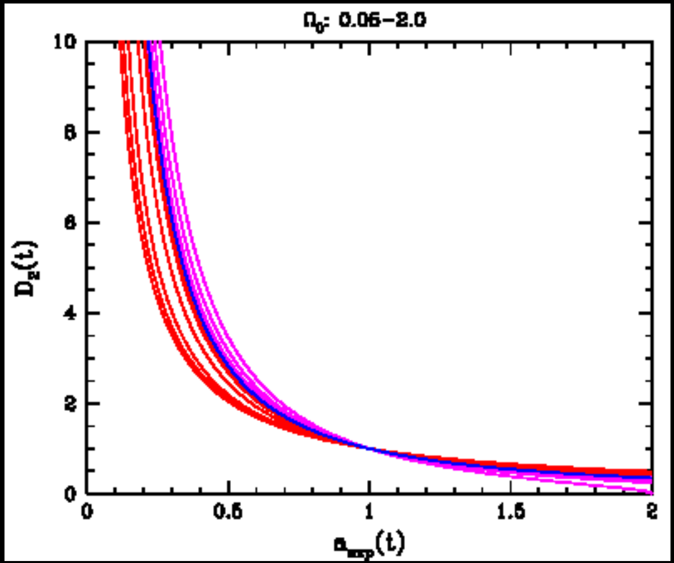

For a LCDM (concordance) model the solution is numerical:

$$\frac{d}{dt}\left\{a^2 H^2 \frac{d}{dt}\left(\frac{D}{H}\right)\right\} = 0 \quad \Rightarrow \quad D(z) = \frac{5\,\Omega_{m,0} H_0^2}{2} H(z) \int_z^\infty \frac{1+z'}{H^3(z')}\,dz'$$

Computation of the linear growth factor

Linear perturbation theory: matter perturbations

For a LCDM (concordance) model the solution is numerical:

$$D(z) = \frac{5\,\Omega_{m,0} H_0^2}{2} H(z) \int_z^{\infty} \frac{1+z'}{H^3(z')}\, dz'$$

Fitting formula:

$$g(z) \equiv (1+z)\, D(z) \approx \frac{5\Omega(z)}{2} \frac{1}{\Omega^{4/7}(z) - \Omega_\Lambda(z) + [1+\Omega_m(z)/2][1+\Omega_\Lambda(z)/70]}$$

with,

$$\begin{aligned}
\Omega_m(z) &= \Omega_{m,0}\,(1+z)^3 \left[\frac{H_0}{H(z)}\right]^2 \\
\Omega_\Lambda(z) &= \Omega_{\Lambda,0} \left[\frac{H_0}{H(z)}\right]^2
\end{aligned}$$