3. HEFT

Motivation and goals

Power counting

Operator bases

Experimental constraints

Higgs physics

Motivation and goals. The use of an EFT approach to BSM physics and searches is well motivated from the fact that new physics seems to be heavier than the EW scale (negative LHC searches), $\Lambda > M_{EW}$. Besides the usual good properties discussed in Lect. 1, the EFT approach has three other advantages in the present context: 1) it offers a model-independent approach (complementary to the study of particular BSM scenarios well motivated for whatever reason), 2) it is very useful to guide us in what interactions to look for by telling us if a given operator is expected to be more or less suppressed (we will refine our power-counting below) and also if it is already constrained by existing data or relatively unconstrained (in which case, it might well be the place to look for a relatively large departure from the SM) and 3) it allows to probe (although indirectly) mass scales above the direct reach of the LHC. This latter point could be crucial (if a deviation is found!) e.g. to decide which next-generation collider to build.

With the Higgs discovery and all data LHC has accumulated so far, we are now in the position to look at the complete $\mathcal{L}_{d=6}$ and (making use of data from previous experiments, in particular LEP I & II + Tevatron) perform a global analysis to determine what are the most promising deviations from the SM we can expect. As mentioned at the end of the previous lecture, the Higgs plays a central role in any BSM model that addresses the hierarchy problem so that we will focus mostly on Higgs physics as a promising ground for deviations.

Power counting We saw in Lect. 2 that operators that violate B or L_i numbers require a very large suppresion scale while we are rather interested in new physics expected to appear not far from the TeV scale (if naturalness is a good guide). We will therefore consider those $d=6$ ops. that respect B and L_i. For their flavor structure, for the same reason we will assume MFV holds, up to possible violations for the top quark. Under these assumptions we expect that $d=6$ ops. will be suppressed by $1/\Lambda^2$ with $\Lambda \sim$ few TeV. Here we want to be more precise and include also an estimate for the effect of couplings in the irrelevant ops. The low-E impact of a given operator $\mathcal{O}_i$ will be very different depending on such effects, e.g.

$$\frac{1}{\Lambda^2} g_i^2 \mathcal{O}_i \quad \text{vs} \quad \frac{1}{\Lambda^2} g_i^4 \mathcal{O}_i \quad \text{vs} \quad \frac{1}{\Lambda^2} \frac{g_i^2}{16\pi^2} \mathcal{O}_i \qquad (\text{with } g_i \ll 1)$$

Such factors depend of course on the particular UV physics that generates the ops. but one can make rather generic estimates that are useful to guess the relative importance of different ops. in whole classes of BSM theories (eg if they are perturbative or if some particular sector couples strongly to the new physics, etc.).

First we have to agree on a good notation for couplings (eg, a quartic coupling $|H|^2S^2$ of the Higgs to some heavy scalar S I could call g_H^2 or g_H^4 or λ_H making the task of determining the coupling factors of irrel. ops. ill-defined). The sensible way to do this is to remember that the perturbative loop expansion in a QFT is an expansion in powers of $\hbar$.

For instance, take $S/\hbar = \frac{1}{\hbar}\int d^4x\left[\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}g^2\phi^4\right]$, the quantity that enters in the path integral as $\exp(-S/\hbar)$.

By redefining the field as $\phi = \phi'/g$, this takes the form

$$\frac{1}{g^2\hbar}\int d^4x\left[\frac{1}{2}(\partial\phi')^2 - \frac{1}{2}m^2\phi'^2 - \frac{1}{4}\phi'^4\right]$$

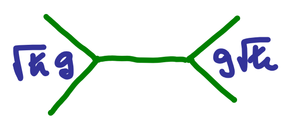
which depends on g^2 and $\hbar$ just in the combination $g^2\hbar$ and shows that the loop expansion (in $\hbar$) is the same as the perturbative expansion (in g). Alternatively, take any field and rescale it as $\phi = \sqrt{\hbar}\,\phi'$ to get

$$\int d^4x\left[\frac{1}{2}(\partial\phi')^2 - \frac{1}{2}m^2\phi'^2 - \frac{1}{4}g^2\hbar\,\phi'^4\right]$$

with the same conclusion. You can check in the SM that the perturbative expansion is an expansion in $g_i^2\hbar$ $(i=1,2,3)$, $y_i^2\hbar$ and $\lambda\hbar$, so that for this $\hbar$ counting $\lambda \sim g_i^2 \sim y_i^2$ (we should use g_h^2 instead of λ !). The general rule to assign a proper power of the couplings to a term in the Lagrangian with n fields is therefore $g^{n-2}\,\text{field}^n$, so that

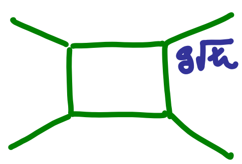
$$\frac{1}{\hbar}\int g^{n-2}\,\text{field}^n \xrightarrow[\text{field}\to\sqrt{\hbar}\,\text{field}']{} \int (g^2\hbar)^{n/2-1}\,\text{field}'^{\,n}$$

You can also check that the rule holds when you combine different couplings. Eg, at tree level, like in



$$\sim g^2\hbar$$

or at loop level :



$$\sim \frac{\hbar^2}{16\pi^2}g^4 = \overbrace{(g^2\hbar)}^{\substack{\text{tree-level counting for}\\ \text{field}^4}}\underbrace{\left(\frac{\hbar g^2}{16\pi^2}\right)}_{\substack{\text{loop}\\ \text{correction}}}$$

Armed with this very simple counting we can formally write our general EFT Lagrangian (with BSM mass scale Λ) as

$$\mathcal{L}_{eff} = \frac{\Lambda^4}{g_*^2}\,\mathcal{L}\left(\frac{D_\mu}{\Lambda}, \frac{g_H H}{\Lambda}, \frac{g_{f_{L,R}} f_{L,R}}{\Lambda^{3/2}}, \frac{g F_{\mu\nu}}{\Lambda^2}\right) \qquad (1)$$

Here g_* is a generic coupling,

and $\left\{\begin{matrix} g_H \\ g_{f_L, f_R} \\ g \end{matrix}\right\}$ is a typical coupling of $\left\{\begin{matrix} \text{Higgs} \\ \text{fermions} \\ \text{gauge fields} \end{matrix}\right\}$ to the BSM sector

[This power counting is useful even when some of the couplings become strong. For instance, if one takes $g_* \sim g_H \sim g_{f_{L,R}} \sim 4\pi$ the power counting in (1) will reproduce the so-called naive-dimensional-analysis (NDA) counting, usually written in terms of Λ and $f = \Lambda/4\pi$, that gives the following scaling for a generic term of the effective Lagrangian:

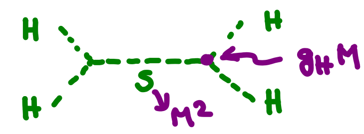
$$f^2\Lambda^2 \left(\frac{\phi}{f}\right)^A \left(\frac{\psi}{f\sqrt{\Lambda}}\right)^B \left(\frac{g F_{\mu\nu}}{\Lambda^2}\right)^C \left(\frac{D}{\Lambda}\right)^D \quad]$$

In discussing $d=6$ ops. we find useful to write them as products of currents (like in Fermi's theory) as this makes transparent the connection with potential examples of UV theories that can generate such operators by the tree-level exchange of some heavy particle (be it a scalar, a fermion, or a gauge boson). Obviously, the fact that such op. can be generated that way does not mean it must be, but it is good to know. It is interesting that some operators cannot be written as the product of two currents, although they can be easily generated via loops. We can then classify $d=6$ operators in these two classes: "current-current" (or "tree-level") ops. and "one-loop" ops. Combining this classification with the power counting discussed above, we find the following three types of $d=6$ operators:

Type 1: "current-current" with a potential g_*^2 enhancement

$$\Delta\mathcal{L} = \frac{g_*^2}{\Lambda^2} c_i \mathcal{O}_i$$

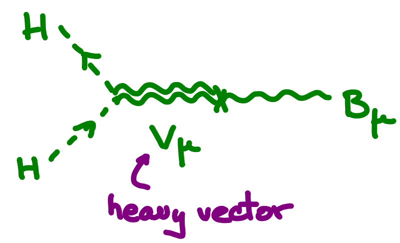
Ex: $\frac{g_H^2}{\Lambda^2}\underbrace{\frac{1}{2}(\partial_\mu|H|^2)^2}_{O_H} = -\frac{g_H^2}{2\Lambda^2}\underbrace{|H|^2}_{J_H}\partial^2\underbrace{|H|^2}_{J_H}$:



Type 2: "current-current" without g_*^2 enhancement

$$\Delta\mathcal{L} = \frac{1}{\Lambda^2} c_i O_i$$

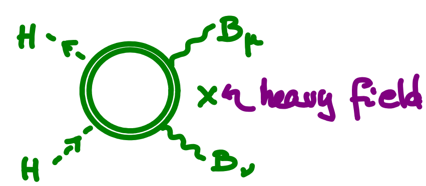
Ex: $O_B = \frac{ig'}{2}\underbrace{(H^\dagger \overleftrightarrow{D}^\mu H)}_{J_H^\mu}\underbrace{\partial^\nu B_{\mu\nu}}_{J_{B\mu}}$



Type 3: "one-loop"

$$\Delta\mathcal{L} = \underbrace{\frac{g_*^2}{16\pi^2}}_{\kappa_i} c_i O_i$$

Ex: $O_{BB} = g'^2|H|^2 B_{\mu\nu}B^{\mu\nu}$



Obviously, if g_* is strong, $g_* \sim 4\pi$, the expected suppresion (for weakly coupled theories) will not be present.

Operator bases The number of $d=6$ ops. (that preserve B and L_i numbers) is 59 (for a single fermion family): 59 ways to deviate from the SM at order $1/\Lambda$.

Naively one would write more but some of them are redundant: they can be eliminated from the Lagrangian by field redefinitions and have no impact on S-matrix elements. As an example, take $O_r = |H|^2|D_\mu H|^2$. It is straightforward to show that $\Delta\mathcal{L} = g_*^2 c_r O_r/\Lambda^2$ can be removed by the shift $H \to H - \frac{g_*^2}{2} c_r H|H|^2/\Lambda^2$, and it is known that such field redefinitions (field $\to$ field + F(fields)) do not change the physics. An equivalent way of removing redundant operators is by using ($d=4$) EoM on the $d=6$ terms. To see this is allowed consider a field redefinition $\varphi \to \varphi + \frac{1}{\Lambda^2} F(\varphi)$. It changes the action $S = \int d^4x \left[\mathcal{L}_{d\le 4} + \frac{1}{\Lambda^2} \mathcal{L}_{d=6} + \dots \right]$ by $\delta S = \int d^4x \left[\frac{\delta \mathcal{L}_{d\le 4}}{\delta \varphi} \cdot \frac{1}{\Lambda^2} F(\varphi) \right] + \dots$ so that the $d=6$ Lagrangian shifts as $\mathcal{L}_{d=6} \to \mathcal{L}_{d=6} + \frac{\delta \mathcal{L}_{d\le 4}}{\delta \varphi} F(\varphi)$ without changing the physics. Therefore we can drop such terms by using $\frac{\delta \mathcal{L}_{d\le 4}}{\delta \varphi} = 0$ which is nothing but the $d=4$ EoM for field φ.

This point is not just a technicality. It is very important because it implies there are different ways of removing redundant operators to arrive at a particular basis of 59 operators. Although physics will not depend on such basis choice, some bases prove more convenient than others (and the literature is plagued with errors due to "wrong" choices of basis) depending of course on the physical effect one is after.

The basis I will use appears in the literature under the name "SILH", from Strongly Interacting Light Higgs as it was chosen as ideally suited to study scenarios of pseudo-Goldstone composite Higgses. Needless to say, it is not restricted to such scenarios and it's as good as any other to parametrize $\mathcal{L}_{d=6}$.

It contains the following operators: there are 14 CP-even operators made of bosons only (table on the left, with the type of each op. as indicated) plus the following 6 CP-odd ones:

Type	
1	$\mathcal{O}_H = \frac{1}{2}(\partial^\mu \lvert H\rvert^2)^2$
1	$\mathcal{O}_T = \frac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$
1	$\mathcal{O}_6 = \lambda \lvert H\rvert^6$
2	$\mathcal{O}_W = \frac{ig}{2}\left(H^\dagger \sigma^a \overleftrightarrow{D^\mu} H\right) D^\nu W^a_{\mu\nu}$
2	$\mathcal{O}_B = \frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right) \partial^\nu B_{\mu\nu}$
2	$\mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W^a_{\mu\nu})^2$
2	$\mathcal{O}_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2$
2	$\mathcal{O}_{2G} = -\frac{1}{2}(D^\mu G^A_{\mu\nu})^2$
3	$\mathcal{O}_{BB} = g'^2 \lvert H\rvert^2 B_{\mu\nu} B^{\mu\nu}$
3	$\mathcal{O}_{GG} = g_s^2 \lvert H\rvert^2 G^A_{\mu\nu} G^{A\mu\nu}$
3	$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}$
3	$\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$
3	$\mathcal{O}_{3W} = \frac{1}{3!} g \epsilon_{abc} W^{a\nu}_\mu W^b_{\nu\rho} W^{c\rho\mu}$
3	$\mathcal{O}_{3G} = \frac{1}{3!} g_s f_{ABC} G^{A\nu}_\mu G^B_{\nu\rho} G^{C\rho\mu}$

$$O_{F\tilde{F}} = g_\alpha^2 \lvert H\rvert^2 F^\alpha_{\mu\nu} \tilde{F}^{\alpha\mu\nu}$$

$$O_{H\tilde{F}} = i g_\alpha (D^\mu H)^\dagger T^\alpha (D^\nu H) \tilde{F}^\alpha_{\mu\nu}$$

$$O_{3\tilde{F}} = \frac{1}{3!} g_\alpha f_{\alpha\beta\gamma} \tilde{F}^{\alpha\nu}_{\mu} F^\beta_{\nu\rho} F^{\gamma\rho\mu}$$

There are many more operators that involve fermions. I list the 44 of them (for a single family) in the table of the next page. The total number of operators in this SILH basis is then $14+6+44 = 64$, five more than the 59 advertised as the number of indep. ops. It is at times convenient to live with 5 redundant operators that can be removed from the basis by using EoMs at will depending on the physics one is interested in.

There are other bases often used in the literature, the most common being the "Hagiwara basis" and the "Polish basis". (see bibliography for refs.). The first maximizes the number of ops that involve SM bosons only without specifying fermion ops. The second tries to minimize the number of ops. containing

Type			
	$\mathcal{O}_{y_u} = y_u\vert H\vert^2 \bar{Q}_L \tilde{H} u_R$	$\mathcal{O}_{y_d} = y_d\vert H\vert^2 \bar{Q}_L H d_R$	$\mathcal{O}_{y_e} = y_e\vert H\vert^2 \bar{L}_L H e_R$
	$\mathcal{O}_R^u = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}_R\gamma^\mu u_R)$ $\mathcal{O}_L^q = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{Q}_L\gamma^\mu Q_L)$ $\mathcal{O}_L^{(3)\,q} = (iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H)(\bar{Q}_L\gamma^\mu \sigma^a Q_L)$	$\mathcal{O}_R^d = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{d}_R\gamma^\mu d_R)$	$\mathcal{O}_R^e = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}_R\gamma^\mu e_R)$ $\mathcal{O}_L^l = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{L}_L\gamma^\mu L_L)$ $\mathcal{O}_L^{(3)\,l} = (iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H)(\bar{L}_L\gamma^\mu \sigma^a L_L)$
1	$\mathcal{O}_{LR}^u = (\bar{Q}_L\gamma^\mu Q_L)(\bar{u}_R\gamma_\mu u_R)$ $\mathcal{O}_{LR}^{(8)\,u} = (\bar{Q}_L\gamma^\mu T^A Q_L)(\bar{u}_R\gamma_\mu T^A u_R)$ $\mathcal{O}_{RR}^u = (\bar{u}_R\gamma^\mu u_R)(\bar{u}_R\gamma_\mu u_R)$ $\mathcal{O}_{LL}^q = (\bar{Q}_L\gamma^\mu Q_L)(\bar{Q}_L\gamma_\mu Q_L)$ $\mathcal{O}_{LL}^{(8)\,q} = (\bar{Q}_L\gamma^\mu T^A Q_L)(\bar{Q}_L\gamma_\mu T^A Q_L)$	$\mathcal{O}_{LR}^d = (\bar{Q}_L\gamma^\mu Q_L)(\bar{d}_R\gamma_\mu d_R)$ $\mathcal{O}_{LR}^{(8)\,d} = (\bar{Q}_L\gamma^\mu T^A Q_L)(\bar{d}_R\gamma_\mu T^A d_R)$ $\mathcal{O}_{RR}^d = (\bar{d}_R\gamma^\mu d_R)(\bar{d}_R\gamma_\mu d_R)$	$\mathcal{O}_{LR}^e = (\bar{L}_L\gamma^\mu L_L)(\bar{e}_R\gamma_\mu e_R)$ $\mathcal{O}_{RR}^e = (\bar{e}_R\gamma^\mu e_R)(\bar{e}_R\gamma_\mu e_R)$ $\mathcal{O}_{LL}^l = (\bar{L}_L\gamma^\mu L_L)(\bar{L}_L\gamma_\mu L_L)$
	$\mathcal{O}_{LL}^{ql} = (\bar{Q}_L\gamma^\mu Q_L)(\bar{L}_L\gamma_\mu L_L)$ $\mathcal{O}_{LL}^{(3)\,ql} = (\bar{Q}_L\gamma^\mu \sigma^a Q_L)(\bar{L}_L\gamma_\mu \sigma^a L_L)$ $\mathcal{O}_{LR}^{qe} = (\bar{Q}_L\gamma^\mu Q_L)(\bar{e}_R\gamma_\mu e_R)$ $\mathcal{O}_{LR}^{lu} = (\bar{L}_L\gamma^\mu L_L)(\bar{u}_R\gamma_\mu u_R)$ $\mathcal{O}_{RR}^{ud} = (\bar{u}_R\gamma^\mu u_R)(\bar{d}_R\gamma_\mu d_R)$ $\mathcal{O}_{RR}^{(8)\,ud} = (\bar{u}_R\gamma^\mu T^A u_R)(\bar{d}_R\gamma_\mu T^A d_R)$ $\mathcal{O}_{RR}^{ue} = (\bar{u}_R\gamma^\mu u_R)(\bar{e}_R\gamma_\mu e_R)$	$\mathcal{O}_{LR}^{ld} = (\bar{L}_L\gamma^\mu L_L)(\bar{d}_R\gamma_\mu d_R)$ $\mathcal{O}_{RR}^{de} = (\bar{d}_R\gamma^\mu d_R)(\bar{e}_R\gamma_\mu e_R)$	
	$\mathcal{O}_R^{ud} = y_u^\dagger y_d (i\tilde{H}^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}_R\gamma^\mu d_R)$		
2	$\mathcal{O}_{y_u y_d} = y_u y_d (\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{Q}_L^s d_R)$ $\mathcal{O}_{y_u y_d}^{(8)} = y_u y_d (\bar{Q}_L^r T^A u_R)\epsilon_{rs}(\bar{Q}_L^s T^A d_R)$ $\mathcal{O}_{y_u y_e} = y_u y_e (\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{L}_L^s e_R)$ $\mathcal{O}'_{y_u y_e} = y_u y_e (\bar{Q}_L^{r\alpha} e_R)\epsilon_{rs}(\bar{L}_L^s u_R^\alpha)$ $\mathcal{O}_{y_e y_d} = y_e y_d^\dagger (\bar{L}_L e_R)(\bar{d}_R Q_L)$		
3	$\mathcal{O}_{DB}^u = y_u \bar{Q}_L \sigma^{\mu\nu} u_R \tilde{H} g' B_{\mu\nu}$ $\mathcal{O}_{DW}^u = y_u \bar{Q}_L \sigma^{\mu\nu} u_R \sigma^a \tilde{H} g W_{\mu\nu}^a$ $\mathcal{O}_{DG}^u = y_u \bar{Q}_L \sigma^{\mu\nu} T^A u_R \tilde{H} g_s G_{\mu\nu}^A$	$\mathcal{O}_{DB}^d = y_d \bar{Q}_L \sigma^{\mu\nu} d_R H g' B_{\mu\nu}$ $\mathcal{O}_{DW}^d = y_d \bar{Q}_L \sigma^{\mu\nu} d_R \sigma^a H g W_{\mu\nu}^a$ $\mathcal{O}_{DG}^d = y_d \bar{Q}_L \sigma^{\mu\nu} T^A d_R H g_s G_{\mu\nu}^A$	$\mathcal{O}_{DB}^e = y_e \bar{L}_L \sigma^{\mu\nu} e_R H g' B_{\mu\nu}$ $\mathcal{O}_{DW}^e = y_e \bar{L}_L \sigma^{\mu\nu} e_R \sigma^a H g W_{\mu\nu}^a$

44 d=6 ops. involving fermions (one family)

derivatives: it gets rid of many bosonic operators using their EoMs. A comparison of the bosonic operators in these three bases is presented in the table of the following page. Generically there are two sides to consider in choosing a good basis:

$$\text{Theory} \downarrow \text{Basis} \downarrow \text{Experiment}$$

Ideally, the basis should provide a transparent connection to

"POLISH"	"HAGIWARA"	SILH
$\mathcal{O}_W = \epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathcal{O}_{WWW} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\nu\rho}\hat{W}_\rho^\mu]$	$\mathcal{O}_{3W} = \frac{1}{3!} g \epsilon_{abc} W_\mu^{a\nu} W_{\nu\rho}^b W^{c\rho\mu}$
$\mathcal{O}_{\varphi W} = \varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{WW} = \Phi^\dagger \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Phi$	—
$\mathcal{O}_{\varphi B} = \varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{BB} = \Phi^\dagger \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\Phi$	$\mathcal{O}_{BB} = g'^2 \|H\|^2 B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{\varphi WB} = \varphi^\dagger \sigma^I \varphi W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{BW} = \Phi^\dagger \hat{B}_{\mu\nu}\hat{W}^{\mu\nu}\Phi$	—
—	$\mathcal{O}_W = (D_\mu\Phi)^\dagger \hat{W}^{\mu\nu}(D_\nu\Phi)$	$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W_{\mu\nu}^a$
—	$\mathcal{O}_B = (D_\mu\Phi)^\dagger \hat{B}^{\mu\nu}(D_\nu\Phi)$	$\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$
—	$\mathcal{O}_{DW} = \mathrm{Tr}\left([D_\mu, \hat{W}_{\nu\rho}][D^\mu, \hat{W}^{\nu\rho}]\right)$	$\mathcal{O}_{2W} = -\frac{1}{2}\left(D^\mu W_{\mu\nu}^a\right)^2$
—	$\mathcal{O}_{DB} = -\frac{g'^2}{2}(\partial_\mu B_{\nu\rho})(\partial^\mu B^{\nu\rho})$	$\mathcal{O}_{2B} = -\frac{1}{2}\left(\partial^\mu B_{\mu\nu}\right)^2$
—	—	$\mathcal{O}_W = \frac{ig}{2}\left(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H\right) D^\nu W_{\mu\nu}^a$
—	—	$\mathcal{O}_B = \frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right) \partial^\nu B_{\mu\nu}$
$\mathcal{O}_{\varphi D} = \left(\varphi^\dagger D^\mu \varphi\right)^* \left(\varphi^\dagger D_\mu \varphi\right)$	$\mathcal{O}_{\Phi,1} = (D_\mu\Phi)^\dagger \Phi\Phi^\dagger (D^\mu\Phi)$	$\mathcal{O}_T = \frac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$
$\mathcal{O}_{\varphi\Box} = (\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$\mathcal{O}_{\Phi,2} = \frac{1}{2}\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi)$	$\mathcal{O}_H = \frac{1}{2}(\partial^\mu \|H\|^2)^2$
$\mathcal{O}_\varphi = (\varphi^\dagger\varphi)^3$	$\mathcal{O}_{\Phi,3} = \frac{1}{3}\left(\Phi^\dagger\Phi\right)^3$	$\mathcal{O}_6 = \lambda \|H\|^6$
—	$\mathcal{O}_{\Phi,4} = (D_\mu\Phi)^\dagger (D^\mu\Phi)(\Phi^\dagger\Phi)$	—

comparison of CP-even bosonic ops. between 3 popular bases (taken from Willenbrock & Zhang)

a theory (or class of theories) so as to minimize the number of ops. required to capture the physical effects. In addition it is convenient that there is a direct connection between ops. and the heavy states that produce them and some handle on the size expected for the ops.[†] As an example, consider BSM theories in which heavy states only couple to SM bosons (the so-called universal

[†] These kind of arguments are called "theory biased" in some quarters. A better name would be "theory informed".

theories). The leading $d=6$ physics effects are most conveniently captured by operators containing bosons only. These are the 14 ops (respecting CP) listed in the table of page 3.8. This is the number of independent parameters that map the physical effects at $d=6$ level of precision. Now, in some bases several of these ops. are removed in favor of other ops. using the gauge boson EoMs, e.g.

$$D^\nu W^a_{\mu\nu} = igH^\dagger \frac{\sigma^a}{2} \overleftrightarrow{D}_\mu H + g \sum_f \bar{f}_L \frac{\sigma^a}{2} \gamma_\mu f_L$$

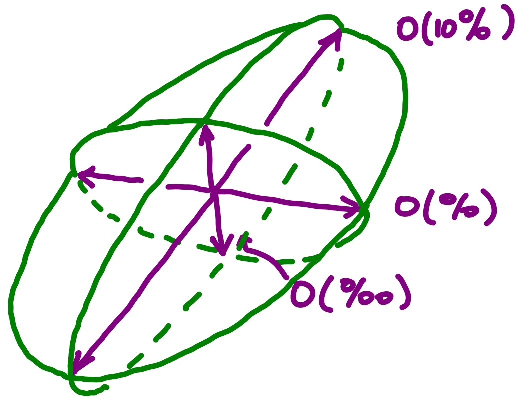
Clearly, such bases are not the best suited to study this kind of scenarios: a single physical effect captured by one Wilson coefficient, say $c_W O_W / \Lambda^2$, in a "good" basis, is spread among many different operators (involving all fermions!) in bases that remove O_W by using the EoM above. Moreover, the physical effects controlled by the single parameter c_W will appear in the "bad" bases through many Wilson coefficients that are correlated in a very precise way. More precisely, one has

$$c_W O_W \to \frac{g^2}{g_*^2} c_W \left[-\frac{3}{2} O_H + 2 O_6 + \frac{1}{2} \left(O_{y_u} + O_{y_d} + O_{y_e} + h.c. \right) + \frac{1}{4} \sum_f O_L^{(3)f} \right]$$

Such potential correlations can be rather misleading on the other front that determines the good properties of a particular basis: its connection to experimental data.

Experimental constraints

A good basis should have a clean connection between operators and observables, ideally in a $1\to 1$ correspondence, which is however not possible in practice: a given experimental constraint generically involves some linear combination of Wilson coefs. Extracting model-independent bounds on the Wilson coefs. of $d=6$ operators requires using a complete basis and to pay special attention to possible hidden correlations. The common practice of setting bounds for a single coefficient at a time ignores correlations (that can be due to theory or simply basis choice!) overestimates the experimental constraints and should be avoided. On the other hand, concrete BSM scenarios predict particular patterns of correlations between Wilson coefficients, allowing for more stringent experimental constraints on them. Finally, it is clear that in order to set constraints from experiment on the c_i's one doesn't care about their possible enhancement (by g_*^2) or suppression (by $g_*^2/16\pi^2$) which only plays a role on the theory $\leftrightarrow$ ops connection. One could proceed by making a global complete fit of all data for the 59 Wilson coefficients, or the relevant subset for Higgs physics, which is the sector where significant deviations are well motivated. Alternatively, one can save effort by noticing the hierarchical pattern of experimental constraints (on $c_i M_W^2/\Lambda^2$) that range from the very precise per-mille level (eg. from EW precision data) to percent level (eg. from TGC data) to loose constraints of order 10% or less (eg. for some Higgs couplings). We can imagine pictorially such bounds as an ellipsoid in the space of Wilson coefs:



(where each axis represents several dimensions in the c_i space). A good basis should be well aligned with such ellipsoid so that a well defined subset of operators can be constrained at the per-mille level (and therefore can be dropped safely from the discussion of the constraints on other operators) and so on towards less constrained ops. until one can determine the most promising ones to expect possible large deviations.

Higgs Physics

Such program has been carried through with a focus on Higgs physics in the last 2 papers of the bibliography (where you can find a detailed discussion). Here I will sketch the procedure and main points. The starting point (under the assumptions of MFV) is the subset of 20 operators below, which are the only ones directly relevant to Higgs physics.†

Rescales H prop. ←	$\mathcal{O}_H = \frac{1}{2}(\partial^\mu \lvert H\rvert^2)^2$
	$\mathcal{O}_T = \frac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$
h→hh ←	$\mathcal{O}_6 = \lambda \lvert H\rvert^6$
	$\mathcal{O}_W = \frac{ig}{2}\left(H^\dagger \sigma^a \overleftrightarrow{D^\mu} H\right) D^\nu W^a_{\mu\nu}$
	$\mathcal{O}_B = \frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right) \partial^\nu B_{\mu\nu}$

$\mathcal{O}_{BB} = g'^2 \lvert H\rvert^2 B_{\mu\nu}B^{\mu\nu}$	→ h→γγ
$\mathcal{O}_{GG} = g_s^2 \lvert H\rvert^2 G^A_{\mu\nu}G^{A\mu\nu}$	→ gg→h
$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}$	} h→Zγ
$\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$	}
$\mathcal{O}_{3W} = \frac{1}{3!} g \epsilon_{abc} W^{a\,\nu}_{\mu} W^b_{\nu\rho} W^{c\,\rho\mu}$	

$\mathcal{O}_{y_u} = y_u \lvert H\rvert^2 \bar{Q}_L \tilde{H} u_R$	$\mathcal{O}_{y_d} = y_d \lvert H\rvert^2 \bar{Q}_L H d_R$	$\mathcal{O}_{y_e} = y_e \lvert H\rvert^2 \bar{L}_L H e_R$	} h→f$\bar{f}$
$\mathcal{O}^u_R = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}_R \gamma^\mu u_R)$ $\mathcal{O}^q_L = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{Q}_L \gamma^\mu Q_L)$ $\mathcal{O}^{(3)\,q}_L = (iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H)(\bar{Q}_L \sigma^a \gamma^\mu Q_L)$	$\mathcal{O}^d_R = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{d}_R \gamma^\mu d_R)$	$\mathcal{O}^e_R = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}_R \gamma^\mu e_R)$	} h→Wf$\bar{f}$, Zf$\bar{f}$
$\mathcal{O}^{(3)\,ql}_{LL} = (\bar{Q}_L \sigma^a \gamma_\mu Q_L)(\bar{L}_L \sigma^a \gamma^\mu L_L)$		$\mathcal{O}^{(3)\,l}_{LL} = (\bar{L}_L \sigma^a \gamma^\mu L_L)(\bar{L}_L \sigma^a \gamma_\mu L_L)$	†

If these operators are present, they will modify the couplings of the Higgs wrt SM values, offering potential windows to new physics. I have indicated above several such effects, some of which have already started to be probed (for the first time!) by the LHC, others will be in the future.

Which ones can still show some large deviation from the SM?

First one should realize that many of these operators have an impact on other sectors of the theory not

† $\mathcal{O}^{(3)l}_{LL}$ affects indirectly through its modification of G_F. $\mathcal{O}^{(3)ql}_{LL}$ is important for combined exp. constraints (through G_F^q).

involving explicitly Higgses being produced, that is, with $H \to \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$. We've seen this already with $\mathcal{O}_T$, which changes the $M_W \leftrightarrow M_Z$ relationship. Other operators will modify the couplings of fermions to gauge bosons (eg $\mathcal{O}_R^f = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{f}\gamma^\mu f)$) and others triple gauge-boson couplings (TGCs), like $\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$. Ex 5. One can then use experimental data from LEP + Tevatron to set constraints in many of these ops. even before looking at Higgs data.

As we mentioned already, working at order $1/\Lambda^2$, the expected deviations in observables of interest will be some linear combinations of Wilson coefficients of the $d=6$ ops. in the table above. We can now be more explicit about what it means in practice to have an operator basis well aligned with experimental observables. The linear system relating observables q_i and Wilson coefficients c_i in the SILH (sub)basis above takes the schematic form:

$$\begin{bmatrix} q_i^L \\ q_i^q \\ q_i^{TGC} \end{bmatrix} = \begin{bmatrix} T_1 & 0 & 0 \\ \times & T_2 & 0 \\ \times & \times & T_3 \end{bmatrix} \cdot \begin{bmatrix} c_i^a \\ c_i^b \\ c_i^c \end{bmatrix}$$

The observables q_i^L are the very precise Z-pole observables measured at LEP-I (the leptonic Z widths into $\nu\nu$, $l_L l_L$, $l_R l_R$) and the W mass (Tevatron). Deviations in these 4 obs. depend through the matrix T_1 on just 4 Wilson coefs (c_T, c_R^e, $c_{LL}^{(3)l}$ and $c_W + c_B$) which can then be bounded at ‰ level. Next come quark observables g_i^q (δI_Z^{had}, R_b, A_c, A_b) also well measured by LEP plus G_F^q/G_F^L (KLOE + β decays) which depend on just 5 additional c_i's ($c_L^q, c_L^{(3)q}, c_R^u, c_R^d, c_{LL}^{(3)q}$) which can in turn be bound at a similar level of precision ($c_i m_W^2/\Lambda^2 \lesssim O(10^{-3})$). Finally, LEP-II data on TGCs, gives information on the 3 parameters g_1^Z, κ_γ, λ_γ (see Ex 5) and this, through T_3, can be used to bound κ_{3W}, and two linear combinations of c_W, κ_{HW}, κ_{HB}, at the per-cent level.

One concludes that previous data closes $4+5+3=12$ of the 20 possible windows for new physics from which we started. Let us focus then on the 8 operators that remain. They are

$$O_H = \frac{1}{2}(\partial_\mu |H|^2)^2 \qquad O_6 = \lambda |H|^6 \qquad O_{y_f} = y_f |H|^2 \bar{f}_L H f_R$$

$(f = t, b, e)$

$$O_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu} \qquad O_{GG} = g_s^2 |H|^2 G^A_{\mu\nu} G^{A\mu\nu}$$

and a linear combination of O_W, O_B, O_{HW} and O_{HB} that can be written as

$$O_{WW} = g^2 |H|^2 W^a_{\mu\nu} W^{a\mu\nu}$$

(an op. not in the basis used). Using the previous bounds on c_i's, it turns out that the unconstrained combination associated with this operator is $\kappa_{HW} - \kappa_{HB}$.

What these 8 operators have in common is that they involve $|H|^2$, in such a way that $H \to \langle H \rangle$ just gives operators already in $\mathcal{L}_{d=4}$ and their impact is not observable. This means that only Higgs data can constrain these operators.

O_H modifies the Higgs propagator and therefore changes in a universal way all Higgs couplings (leaving then all BRs the same)

O_6 will impact $h \to hh$, not accesible in the near future.

O_{y_b, y_τ} affect $BR(h \to b\bar{b})$ and $BR(h \to \tau\tau)$ respectively.

O_{y_t} impacts $BR(h \to \gamma\gamma)$ by changing the SM top-loop contribution. In addition, it changes the associated production $ht\bar{t}$ rate.

O_{BB} modifies directly $BR(h \to \gamma\gamma)$

O_{GG} changes $\sigma(GG \to h)$, the main Higgs production mechanism.

$\kappa_{HW} - \kappa_{HB}$ affects $BR(h \to Z\gamma)$.

Performing a global fit to all Higgs data from ATLAS and CMS the only significant bounds apply to the Wilson coefs. that contribute directly to SM loop-suppressed processes:

$$\frac{m_W^2}{\Lambda^2}\kappa_{GG} \in [-0.8, 0.8] \times 10^{-3}$$
$$\frac{m_W^2}{\Lambda^2}\kappa_{BB} \in [-1.3, 1.8] \times 10^{-3} \qquad 95\% \text{ C.L.}$$
$$\frac{m_W^2}{\Lambda^2}\kappa_{Z\gamma} \in [-6, 12] \times 10^{-3}$$

where $\kappa_{Z\gamma} \equiv -\frac{1}{4}(\kappa_{HW} - \kappa_{HB}) - 2 s_W^2 \kappa_{BB}$. Notice that the last bound is possible even though $h \to Z\gamma$ has not been seen yet but thanks to the exp. limit on its rate at about $10 \times \sigma_{SM}$.

The analysis reviewed above is useful to bound possible deviations from new physics in all sorts of Higgs measurements. As an example, consider the test of custodial symmetry based on measuring

$$\lambda_{WZ} - 1 = \frac{\Gamma(h\to WW)}{\Gamma^{SM}(h\to WW)} \cdot \frac{\Gamma^{SM}(h\to ZZ)}{\Gamma(h\to ZZ)} - 1$$

which is constrained experimentally to the range $(-0.45, 0.15)$. Using the $d=6$ EFT approach one gets

$$\lambda_{WZ} - 1 \simeq s_W^2\left[0.9c_W - 2.6c_B + 3\kappa_{HW} - 3.9\kappa_{HB}\right]\frac{m_W^2}{\Lambda^2}$$

$$\simeq \left(0.8\,\delta g_1^Z - 0.1\,\delta\kappa_\gamma - 1.6\,\kappa_{Z\gamma}\right)\frac{m_W^2}{\Lambda^2}$$

and using the bounds on these deviations from TGC and $h\to Z\gamma$ data, one gets the stronger bound

$$\lambda_{WZ} - 1 \in [-5, 6] \times 10^{-2}$$

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