

Supersymmetry

- Lecture Benasque, Sept. 2014, 3 hours -

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- Plan:
- Aim: understand structure of SUSY theories: SUSY-QED, ..., MSSM, ...
 - fundamental motivation
 - "guess" SUSY-QED $\leadsto$ important lessons
 - derivation:
 - SUSY algebra
 - superspace, superfields
 - SUSY Lagrangian
 - applications

Special Relativity [Einstein, 1905]

- "Nature is Lorentz invariant"



QFT:

Lorentz transf. (rotation etc)

boson:	0	$\longleftrightarrow$	1	$\longleftrightarrow$	2	$\longleftrightarrow$	...
fermion:	$\frac{1}{2}$	$\longleftrightarrow$	$\frac{3}{2}$	$\longleftrightarrow$	$\frac{5}{2}$	$\longleftrightarrow$	...

Wouldn't it be more natural to have a symmetry with $\Delta J = \frac{1}{2}$?

$$0 \longleftrightarrow \frac{1}{2} \longleftrightarrow 1 \longleftrightarrow \frac{3}{2} \longleftrightarrow 2 \longleftrightarrow \frac{5}{2} \longleftrightarrow \dots$$

Yes, indeed

→ this is supersymmetry (SUSY) — a unique **F–B** symmetry

1. Introduction

1.1 Motivation

SUSY = fundamental new symmetry of relativistic QFT

- changes spin: $\Delta J = \pm \frac{1}{2}$

- Fermions $\leftrightarrow$ Bosons

- $Q \hat{=} \sqrt{\text{Poincaré}}$

↪ improves QFT, maybe Quantum Gravity

↪ relation to cosmological constant

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Phenomenological motivation for SUSY at TeV-scale :

- fine tuning problem of M_{Higgs}
 - radiative electroweak symmetry breaking
 - unification of gauge couplings
 - dark matter candidate
- } "understand scalars" !

1.2 "Guess" SUSY-QED — important aspects of SUSY

$$\mathcal{L}_{QED} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\Psi} (i \mathcal{D}^\mu \gamma_\mu - m) \Psi, \quad \mathcal{D}^\mu = \partial^\mu + ie A^\mu$$

1st guess: need selectron (charged scalar),
same mass / interaction as electron

→ scalar field ϕ and

$$\mathcal{L}_\phi = |\mathcal{D}^\mu \phi|^2 - m^2 |\phi|^2$$

→ not enough!

Ψ : 4 deg. of freedom ($e_{L,R}^\pm$)

ϕ : 2 deg. of freedom

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ϕ : 2 deg. of freedom

2nd guess: need same # deg. of freedom for selectrons

→ two scalar fields $A_L, A_R \hat{=} \tilde{e}_L^\pm, \tilde{e}_R^\pm$

$$\mathcal{L}_{A_L} + \mathcal{L}_{A_R}$$

→ not enough!

3rd guess: Need photino! $\tilde{\gamma} = \tilde{\gamma}^c$ (Majorana spinor), massless

$$\mathcal{L}_{\tilde{\gamma}} = \overline{\tilde{\gamma}} i \not{\partial} \tilde{\gamma}$$

→ not enough!

Interactions?

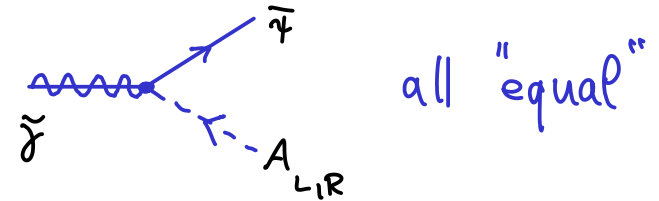
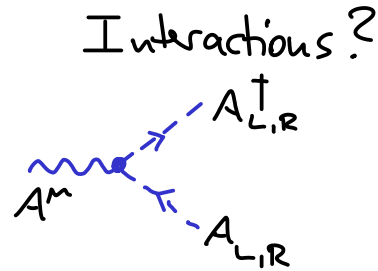
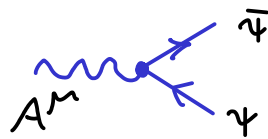
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→ not enough!

4th guess:



all "equal"

$$\mathcal{L}_{\tilde{\gamma}\text{-int}} = -\sqrt{2} e \left(\bar{\tilde{\gamma}} P_L \psi A_L^\dagger - \bar{\tilde{\gamma}} P_R \psi A_R^\dagger + \text{h.c.} \right)$$

→ still not enough! Need also

$$\mathcal{L}_{\phi^4} = -\frac{e^2}{2} \left(|A_L|^2 - |A_R|^2 \right)^2$$

(Then: "  =  ")

2. SUSY-Algebra

Symmetries in quantum theory $\rightarrow$ operators on Hilbert space of states

e.g. rotational invariance $\rightarrow$ angular momentum $\vec{J}$
 $[J_x, J_y] = i J_z$ etc.

rotation e.g. $|\Psi\rangle \rightarrow e^{i\alpha J_z} |\Psi\rangle$

translational ($\vec{x}, t$) invariance $\rightarrow$ momentum operator P^μ

translation e.g. $|\Psi\rangle \rightarrow e^{i a^\mu P_\mu} |\Psi\rangle$

Relativistic QFT : need P^μ (translations)
 $g_{\mu\nu}$ (rotations & boosts)

SUSY: need operator Q

- Fermions $\leftrightarrow$ Bosons $\Rightarrow Q$ is fermionic (anticommuting)
- $\Delta J = \pm \frac{1}{2} \Rightarrow Q$ must transform as a relativistic spinor
 $\leadsto$ simplest: Majorana-spinor

$$\Rightarrow \begin{aligned} [Q, P^\mu] &= 0, \\ [Q, J^{\mu\nu}] &= \text{fixed} \end{aligned}$$

- algebra $\{Q_\alpha, \bar{Q}_\beta\}$ must be
 - symmetry
 - transforms like $\psi_\alpha \bar{\psi}_\beta$
 - cannot vanish since positive definite

$$\Rightarrow = 2 \gamma_{\alpha\beta}^\mu P_\mu \quad (\text{up to normalization})$$

SUSY - algebra result:

$$[Q, P^\mu] = 0$$

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\gamma^\mu_{\alpha\beta} P_\mu$$

$$Q = Q^c = \text{Majorana spinor}$$

proof that this is most general possibility: Haag, Lopuszanski, Sohnius.

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Direct consequences:

- $[Q, P^2] = 0 \rightarrow$ equal rest mass of particle/supartner
- multiply algebra by $\gamma^0_{\beta\alpha} \Rightarrow \{Q_\alpha, Q^\dagger_\beta\} = 2 \text{Tr}(\gamma^\mu \gamma^0) P_\mu$

$$Q_1 Q_1^\dagger + \dots + Q_4 Q_4^\dagger = 8 \text{ Hamiltonian}$$

- " $Q \hat{=} \sqrt{H}$ " $\rightarrow H \geq 0$, relation to cosm. const.
- on states with $E \neq 0$: "Q invertible" $\Rightarrow \# F = \# B$

3. Superspace and superfields

3.1 Construction

Aim: SUSY QFT $\rightarrow$ made much simpler
by beautiful formalism

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So far: abstract operators : $\vec{J}$: rotations
 $P^\mu, g^{\mu\nu}$: translations, Lorentz
 Q : SUSY

Idea: translations, rotations etc described much simpler by

$$\left. \begin{array}{l} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow R(\alpha, \beta, \gamma) \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\ x^\mu \longrightarrow x^\mu + a^\mu \end{array} \right\} \begin{array}{l} \text{transformations} \\ \text{on suitable} \\ \text{coordinate space} \end{array}$$

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$\Rightarrow$ Hope: SUSY : $\left\{ \begin{array}{l} x^\mu \longrightarrow x^\mu + \dots \\ \theta \longrightarrow \theta + \xi \end{array} \right\} \leftarrow \text{new coordinate \& trans. parameter}$

$\Rightarrow$ Need to know relation between coordinate/operator representation

Operator $\leftrightarrow$ coordinate for translations:

(1) coordinate translation by a^μ : $x^\mu \longrightarrow x'^\mu = x^\mu + a^\mu$

(2) define fields on coordinate space: $\phi(x)$

$$\phi \longrightarrow \phi' \text{ with } \phi'(x') = \phi(x)$$

[math.: linear representation of translation group]

(3) define translation operators on space of functions: $e^{i a^\mu P_\mu}$

$$\phi' = e^{i a^\mu P_\mu} \phi$$

$$\Rightarrow \phi(x-a) = e^{i a^\mu P_\mu} \phi(x) \quad \forall \phi$$

$$\Rightarrow P_\mu = i \partial_\mu \quad (\text{Taylor operator})$$

$\Rightarrow$ the operators constructed in this way define the commutation rules of the abstract operators.

For SUSY: go backwards!

Operator $\leftrightarrow$ coordinate for SUSY:

- (3) abstract algebra $\rightarrow$ differential operators on some space
- (2) how do they act on the arguments of fields
- (1) coordinate transformations

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(3): which space?

Q anticommuting, spinor $\Rightarrow$ fermionic spinor coordinate
 $\{Q, \bar{Q}\} = \text{translation} \Rightarrow$ describe susy + translations

$\rightarrow$ superspace coordinates

$$\begin{Bmatrix} x^\mu \\ \theta \end{Bmatrix}$$

$\leftarrow$

$\leftarrow$ fermionic
Majorana spinor

4 bosonic dimensions

4 fermionic dimensions

Diff. operators:

$$Q := i \left(\frac{\partial}{\partial \bar{\theta}} + i \gamma^\mu \theta \partial_\mu \right), \quad \bar{Q} := i \left(-\frac{\partial}{\partial \theta} - i \bar{\theta} \gamma^\mu \partial_\mu \right)$$

$$P^\mu := i \partial^\mu$$

$\rightarrow$ satisfy the susy algebra (check!)

(2) field transformations :

superfields $F(x, \theta)$ = fields on superspace

SUSY: $F \rightarrow F' := e^{i\bar{\theta}\xi} F$, ξ = fermionic Majorana spinor

evaluate:

$$F'(x, \theta) = e^{(-\frac{\partial}{\partial \theta} \xi - i\bar{\theta} \gamma^\mu \xi \partial_\mu)} F(x, \theta)$$
$$= F(x^\mu + i\bar{\theta} \gamma^\mu \xi, \theta - \xi)$$
$$F(x^\mu - \Delta x^\mu, \theta - \xi)$$

(1) identify with

→ SUSY coordinate transformation on superspace:

$$\begin{Bmatrix} x^\mu \\ \theta \end{Bmatrix} \rightarrow \begin{Bmatrix} x'^\mu = x^\mu + \Delta x^\mu \\ \theta' = \theta + \xi \end{Bmatrix}$$

units : $[x] = \frac{1}{\text{Mass}} = [\theta^2]$, $[\theta] = \frac{1}{\sqrt{\text{Mass}}}$

What is this good for?

- Nice and easier than $\{Q, \bar{Q}\} = \dots$
- construct susy theories : e.g.

define many superfields $F_1, \dots, F_n$
 $\mathcal{L} = \mathcal{L}(F_1 \dots F_n)$

$\Rightarrow \int d^4x \, d^4\theta \, \mathcal{L}$ is susy invariant
(provided $\int d^4\theta$ is transl. invariant)

$\int d^4\theta \, \mathcal{L}$ only depends on x
 $\leadsto$ ordinary Lagrangian

3.2 Types of superfields

Fields with special properties $\rightarrow$ special theories

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First: properties of $\mathcal{D}$ and functions of $\mathcal{D}$:

a) $\mathcal{D} = P_L \mathcal{D} + P_R \mathcal{D} =: \mathcal{D}_L + \mathcal{D}_R \quad \rightarrow \mathcal{D}_{L,R} \text{ don't mix under Lorentz/SUSY}$

$$\mathcal{D} = \mathcal{D}^c = i\gamma^0 \gamma^2 \bar{\mathcal{D}}^T$$

$$\bar{\mathcal{D}}_R = \bar{\mathcal{D}} P_L = \bar{\mathcal{D}}^c P_L = \mathcal{D}^T \overline{i\gamma^0 \gamma^2} P_L = (P_L \mathcal{D})^T \overline{i\gamma^0 \gamma^2} =: \mathcal{D}_L^T \in$$

b) $\mathcal{D}_L \sim \bar{\mathcal{D}}_R^T$ contains two indep. entries; each $(\text{entry})^2 = 0$

$$\Rightarrow \mathcal{D}_L \dots \mathcal{D}_L \dots \mathcal{D}_L = \mathcal{D}_L \dots \bar{\mathcal{D}}_R \dots \mathcal{D}_L = 0$$

$$\text{any } f(\mathcal{D}_L) = a + \bar{\mathcal{D}}_R \Psi_L + \underbrace{\bar{\mathcal{D}}_R \mathcal{D}_L}_= \mathcal{D}_L^T \in \mathcal{D}_L b$$

c) $\mathcal{D}$ contains four indep. entries: any $f(\mathcal{D}) = a + \dots + (\bar{\mathcal{D}} \mathcal{D})(\bar{\mathcal{D}} \mathcal{D}) b$

Interlude: Weyl spinors: SUSY $\Rightarrow$ spinors important

Weyl = minimal Lorentz covariant spinors

$$\gamma\text{-matrices} : \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad P_L = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$4\text{-spinor} \quad \underline{\psi} = \begin{pmatrix} \psi_L^\alpha \\ \bar{\psi}_R^{\dot{\alpha}} \end{pmatrix} \quad \underline{\psi}_L = \begin{pmatrix} \psi_L^\alpha \\ 0 \end{pmatrix} \quad \underline{\psi}_R = \begin{pmatrix} 0 \\ \bar{\psi}_R^{\dot{\alpha}} \end{pmatrix}$$

$$\underline{\psi}^c = \begin{pmatrix} \psi_R^\alpha \\ \bar{\psi}_L^{\dot{\alpha}} \end{pmatrix} \quad \partial = \begin{pmatrix} \partial_\alpha \\ \bar{\partial}^{\dot{\alpha}} \end{pmatrix}$$

$$\overline{\underline{\psi}} = (\psi_R^\alpha \quad \bar{\psi}_L^{\dot{\alpha}}), \quad \psi^\alpha = \epsilon^{\alpha\beta} \psi_\beta, \quad \epsilon^{\alpha\beta} = \text{antisymm.}$$

$$\begin{aligned} \text{mass term} \quad m \overline{\underline{\psi}} \underline{\psi} &= m (\overline{\underline{\psi}}_R \underline{\psi}_L + \overline{\underline{\psi}}_L \underline{\psi}_R) \\ &= m (\psi_R^\alpha \psi_{L\alpha} + \bar{\psi}_L^{\dot{\alpha}} \bar{\psi}_R^{\dot{\alpha}}) \end{aligned}$$

$$\text{kinetic term} \quad \overline{\underline{\psi}} \gamma^\mu \underline{\psi} = \overline{\underline{\psi}}_L \gamma^\mu \underline{\psi}_L + \overline{\underline{\psi}}_R \gamma^\mu \underline{\psi}_R$$

$$\overline{\underline{\psi}}_L \gamma^\mu \underline{\psi}_L = \bar{\psi}_L \bar{\sigma}^\mu \psi_L = \bar{\psi}_L (-\sigma^2 \sigma^\mu \sigma^2) \psi_L = -\psi_L \sigma^\mu \bar{\psi}_L$$

general superfields:

any function $F(x, \theta)$

"vector" superfields:

real function : $V(x, \theta) = V^\dagger(x, \theta)$

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expansion in θ : $V(x, \theta) = a(x) + \dots + \bar{\theta}_L \gamma^\mu \theta_L V_\mu(x) + \dots$

$\rightarrow$ contains vector field $V_\mu(x)$

unit: $[\bar{\theta}_L \gamma^\mu \theta_L V_\mu] = (\text{Mass})^0 \rightarrow \text{dimensionless}$

search for simpler superfields:

try $\mathbb{F}_{\text{try}} := \{ f(x, \theta_L) \}$

$f(x, \theta_L) \xrightarrow{\text{susy}} f(x + i \bar{\theta} \gamma^\mu \xi, \theta_L - \xi_L)$
 $\hookrightarrow$ depends also on θ_R

$\notin \mathbb{F}_{\text{try}}$

chiral superfields: $\mathbb{F}_{\text{ch},L} := \{ f(x^n - i \bar{\partial}_R \gamma^n \partial_R, \partial_L) \}$

$$\begin{aligned}
 f &\xrightarrow{\text{susy}} f(x^n + i \bar{\partial} \gamma^n \xi - i \overline{\partial_R - \xi_R} \gamma^n (\partial_R - \xi_R), \partial_L - \xi_L) \\
 &= f(x^n - i \bar{\partial}_R \gamma^n \partial_R + 2i \bar{\partial}_R \gamma^n \xi_R, \partial_L - \xi_L) \\
 &= f(x^n - i \bar{\partial}_R \gamma^n \partial_R, \partial_L) \quad \uparrow \sim \partial_L \\
 &\in \mathbb{F}_{\text{ch},L}
 \end{aligned}$$

chiral superfields:

$$\mathbb{F}_{\text{ch},L} := \left\{ f \left(x^\mu - i \bar{\partial}_R \gamma^\mu \partial_R, \partial_L \right) \right\}$$

$$\begin{aligned} f &\xrightarrow{\text{susy}} f \left(x^\mu + i \bar{\partial} \gamma^\mu \xi - i \overline{\partial_R - \xi_R} \gamma^\mu (\partial_R - \xi_R), \partial_L - \xi_L \right) \\ &= f \left(x^\mu - i \bar{\partial}_R \gamma^\mu \partial_R + 2 i \bar{\partial}_R \gamma^\mu \xi_R, \partial_L - \xi_L \right) \\ &= g \left(x^\mu - i \bar{\partial}_R \gamma^\mu \partial_R, \partial_L \right) \quad \uparrow \sim \partial_L \end{aligned}$$

$$\in \overline{\mathbb{F}}_{\text{ch},L}$$

→ SUSY invariant set of simpler superfields
 → sums/products are again chiral, but $f^\dagger$ is not!

expansion in θ : $\Phi \in \overline{\mathbb{F}}_{\text{ch},L}$: $\Phi(x, \theta) = \phi(\underbrace{x^\mu - i \bar{\partial}_R \gamma^\mu \partial_R}_{=: y^\mu}, \partial_L)$

where

$$\phi(y, \partial_L) = \underbrace{A(y)}_{\text{Lorentz scalar}} + \sqrt{2} \bar{\partial}_R \underbrace{\psi_L(y)}_{\text{L-spinor}} + \bar{\partial}_R \partial_L \underbrace{F(y)}_{\text{scalar}}$$

Exercise 3a, 4a!

3.3 Lagrangians

Suppose we define integral $\int d^4 \theta = \int d^2 \theta_L \int d^2 \theta_R$
such that

$$\int d^4 \theta f(\theta) = \int d^4 \theta f(\theta + \xi) \text{ etc.}$$

$\Rightarrow$ easy to construct susy Lagrangians
out of vector and chiral superfields Φ_i

$$\text{general: } \int d^4 x \mathcal{L} = \int d^4 x \int d^4 \theta \underbrace{K}_{= \text{superfield}} \text{ [arbitrary combination]}$$

$$\text{chiral: } \int d^4 x \mathcal{L} = \int d^4 x \int d^2 \theta_L W \underbrace{\left[\begin{array}{c} \text{arbitrary polynomial in } \Phi_i \\ \text{(not } \Phi_i^\dagger) \end{array} \right]}_{= \text{"superpotential"} = \text{chiral}}$$

Summary and lessons:

- Susy transformations become simple on superspace $\{x^\mu, \theta\}$
- Superfields have simple transformations
chiral s.f. only depend on θ_L explicitly
- $\int d^4\theta$, $\int d^2\theta_L$ integrals yield susy Lagrangians

Summary and lessons:

- Susy transformations become simple on superspace $\{x^\mu, \theta\}$
- Superfields have simple transformations
chiral s.f. only depend on θ_L explicitly
- $\int d^4\theta$, $\int d^2\theta_L$ integrals yield susy Lagrangians

Tasks now:

- define integrals (translationally invariant!)
- evaluate integrals to obtain explicit form of $\mathcal{L}$

4. Detailed Lagrangians

4.1 Detailed evaluations (no vector superfields)

Integral definitions: [first only one θ -component]

we want $\int d\theta f(\theta + \xi) = f(\theta)$

but we know $f(\theta) = a + b\theta$ since $\theta^2 = 0$

therefore, $f(\theta + \xi) = a + b\xi + b\theta$

$\Rightarrow$ only possibility: $\int d\theta f(\theta) := b$ (times normalization)

$\int d\theta \sim \frac{\partial}{\partial \theta}$ for fermionic variables

Similarly :

$$\int d^2 \theta_L f(\theta_L) := f \Big|_{\substack{\text{coefficient of } \bar{\partial}_R \partial_L = \partial^\alpha \partial_\alpha \\ \text{with } \partial_R = 0}}$$

$$\int d^4 \theta f(\theta) := f \Big|_{\text{coefficient of } \frac{1}{2}(\bar{\theta}\theta)(\bar{\theta}\theta) = \partial^\alpha \partial_\alpha \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}}}$$

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Units and renormalizability :

(no couplings $\sim \frac{1}{\text{Mass}^n}$)

$$\mathcal{L} = \underbrace{\int d^4\theta}_{\sim \text{Mass}^2} \underbrace{K}_{\sim \text{Mass}^2}$$

$$\Rightarrow K \sim \underbrace{\Phi_i^\dagger \Phi_i}_{\sim \text{Mass}^2} * f(V)$$

$$\mathcal{L} = \underbrace{\int d^2\theta_L}_{\sim \text{Mass}} \underbrace{W}_{\sim \text{Mass}^3}$$

$$\Rightarrow W = c; \Phi_i + \frac{m_{ij}}{2} \Phi_i \Phi_j + \frac{\chi_{ijk}}{6} \Phi_i \Phi_j \Phi_k$$

Computations of $\Phi^\dagger \Phi$, $\bar{\Phi} \Phi$: (use Weyl spinors)

$$\begin{aligned}\bar{\Phi}(x, \theta) &= \phi(\overbrace{x^\mu \bar{\partial}_R y^\mu \partial_R}^{=y^\mu}, \theta_L) \\ &= A(y) + \sqrt{2} \bar{\partial}_R \psi_L(y) + \bar{\partial}_R \partial_L F(y)\end{aligned}$$

$\bar{\Phi}_1 \Phi_2$:

Computations of $\Phi^\dagger \Phi$, $\bar{\Phi} \Phi$: (use Weyl spinors)

$$\begin{aligned}\bar{\Phi}(x, \theta) &= \phi(\overbrace{x^\mu - i \bar{\theta}_R \gamma^\mu \theta_R}^{=y^\mu}, \theta_L) \\ &= A(y) + \sqrt{2} \bar{\theta}_R \psi_L(y) + \bar{\theta}_R \theta_L F(y)\end{aligned}$$

$\bar{\Phi}_1 \Phi_2$:

$$(y^\mu = x^\mu)$$

$$\begin{aligned}& (A_1 + \sqrt{2} \partial^\alpha \psi_{1L\alpha} + \partial^\alpha \theta_\alpha F_1) (A_2 + \sqrt{2} \partial^\alpha \psi_{2L\alpha} + \partial^\alpha \theta_\alpha F_2) \\ &= A_1 A_2 + \sqrt{2} \partial^\alpha (\psi_{1L\alpha} A_2 + \psi_{2L\alpha} A_1) \\ &+ \partial^\alpha \theta_\alpha (F_1 A_2 + A_1 F_2) + 2 \underbrace{\partial^\alpha \psi_{1L\alpha} \partial^\beta \psi_{2L\beta}}_{= -\frac{1}{2} \partial^\alpha \theta_\alpha \psi_{1L}^\beta \psi_{2L\beta}} \\ &= -\frac{1}{2} \partial^\alpha \theta_\alpha \psi_{1L}^\beta \psi_{2L\beta}\end{aligned}$$

$$\Rightarrow \int d^2\theta_L \bar{\Phi}_1 \Phi_2 = F_1 A_2 + A_1 F_2 - \psi_{1L}^\alpha \psi_{2L\alpha}$$

Spinor rearrangements:

$$\theta^1 = \varepsilon^{12} \theta_2 \quad \theta^1 \theta_1 = \varepsilon^{12} \theta_2 \theta_1 = -\varepsilon^{12} \theta_1 \theta_2 = +\theta^2 \theta_2$$

$$\theta^\alpha \theta_\beta = \delta^\alpha_\beta \quad \frac{1}{2} \theta^\gamma \theta_\gamma$$

$$\psi^\alpha \theta_\alpha = \varepsilon^{\alpha\beta} \psi_\beta \theta_\alpha = -\varepsilon^{\alpha\beta} \theta_\alpha \psi_\beta = +\theta^\beta \psi_\beta$$

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$$\theta^\alpha \theta_\beta = \delta^\alpha_\beta \quad \frac{1}{2} \theta^\gamma \theta_\gamma$$

$$\psi^\alpha \theta_\alpha = \varepsilon^{\alpha\beta} \psi_\beta \theta_\alpha = -\varepsilon^{\alpha\beta} \theta_\alpha \psi_\beta = +\theta^\beta \psi_\beta$$

$$\begin{aligned} \theta^\alpha \psi_{1L\alpha} \theta^\beta \psi_{2L\beta} &= \psi_{1L}^\alpha \theta_\alpha \theta^\beta \psi_{2L\beta} = \psi_{1L}^\alpha \left(-\frac{1}{2} \delta^\beta_\alpha \theta^\gamma \theta_\gamma \right) \psi_{2L\beta} \\ &= -\frac{1}{2} \theta^\gamma \theta_\gamma \psi_{1L}^\alpha \psi_{2L\alpha} \end{aligned}$$

$$\begin{aligned} \theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \theta^\beta \sigma_{\beta\dot{\beta}}^\nu \bar{\theta}^{\dot{\beta}} &= -\varepsilon^{\beta\gamma} \frac{1}{2} (\theta\theta) \sigma_{\gamma\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \sigma_{\beta\dot{\beta}}^\nu \bar{\theta}^{\dot{\beta}} \\ &= -\varepsilon^{\beta\gamma} \frac{1}{2} (\theta\theta) \varepsilon^{\dot{\alpha}\dot{\beta}} \frac{1}{2} (\bar{\theta}\bar{\theta}) \sigma_{\gamma\dot{\alpha}}^\mu \sigma_{\beta\dot{\beta}}^\nu \\ &= \frac{1}{4} \theta\theta \bar{\theta}\bar{\theta} \text{Tr}(\sigma^2 \sigma^\mu \sigma^2 \sigma^{\nu T}) \\ &= \frac{1}{2} \theta\theta \bar{\theta}\bar{\theta} g^{\mu\nu} \end{aligned}$$

$$\underbrace{\bar{\Phi}^\dagger \Phi}_{\text{wavy}} \Big|_{\theta^4} :$$

$$\begin{aligned} \bar{\Phi} &= \phi(x - i \bar{\theta}_R \gamma^\mu \theta_R, \theta_L) \\ &= e^{-i \bar{\theta}_R \gamma^\mu \theta_R \partial_\mu} \phi(x, \theta_L) \end{aligned}$$

explicitly, Weyl notation:

$$\begin{aligned} \bar{\Phi} &= \left[1 - i \theta \sigma^\mu \bar{\theta} \partial_\mu - \underbrace{\frac{1}{2} (\theta \sigma^\mu \bar{\theta}) (\theta \sigma^\nu \bar{\theta}) \partial_\mu \partial_\nu}_{= -\frac{1}{4} \theta \theta \bar{\theta} \bar{\theta}} \right] [A + \sqrt{2} \theta \psi_L + \theta \theta F] \\ &= -\frac{1}{4} \theta \theta \bar{\theta} \bar{\theta} \quad \square \end{aligned}$$

$$\underbrace{\bar{\Phi}^\dagger \Phi}_{\text{wavy}} \Big|_{\theta^4} : \quad \bar{\Phi} = \phi(x - i \bar{\partial}_R \gamma^\mu \partial_R, \theta_L)$$

$$= e^{-i \bar{\partial}_R \gamma^\mu \partial_R \partial_\mu} \phi(x, \theta_L)$$

explicitly, Weyl notation:

$$\bar{\Phi} = \left[1 - i \partial \sigma^\mu \bar{\partial} \partial_\mu - \underbrace{\frac{1}{2} (\partial \sigma^\mu \bar{\partial}) (\partial \sigma^\nu \bar{\partial}) \partial_\mu \partial_\nu}_{= -\frac{1}{4} \partial \partial \bar{\partial} \bar{\partial} \quad \square} \right] [A + \sqrt{2} \partial \psi_L + \partial \partial F]$$

$$\begin{aligned} \bar{\Phi}^\dagger \Phi \Big|_{\theta^4} &= A^\dagger \left(-\frac{1}{4} \theta^4 \square A \right) + (-i \partial \sigma^\mu \bar{\partial} \partial_\mu A)^\dagger (-i \partial \sigma^\nu \bar{\partial} \partial_\nu A) - \frac{1}{4} \theta^4 (\square A^\dagger) A \\ &\quad + \sqrt{2} \bar{\partial} \bar{\psi}_L (-i \partial \sigma^\mu \bar{\partial} \partial_\mu \sqrt{2} \partial \psi_L) + \text{h.c.} \\ &\quad + \bar{\partial} \bar{\partial} F^\dagger \partial \partial F \end{aligned}$$

$$= \theta^4 \left\{ (\partial^\mu A^\dagger) (\partial_\mu A) + \bar{\psi}_L \bar{\sigma}^\mu i \partial_\mu \psi_L + F^\dagger F \right\}$$

Result: general $\mathcal{L}$ for chiral superfields

$$(1) = \int d^4 \theta \quad \bar{\Phi}_i^\dagger \Phi_i = (\partial^\mu A_i)^\dagger (\partial_\mu A_i) + \bar{\psi}_{Li} \bar{\sigma}^\mu_i \partial_\mu \psi_{Li} + F_i^\dagger F_i$$

$$\int d^2 \theta_L \quad \frac{1}{2} m_{ij} \Phi_i \Phi_j = \frac{1}{2} m_{ij} (A_i F_j + A_j F_i - \psi_{Li} \psi_{Lj})$$

$$(2) = \int d^2 \theta_L \quad W(\Phi) = \sum_i F_i \frac{\partial W(A)}{\partial A_i} - \frac{1}{2} \sum_{ij} \psi_{Li} \psi_{Lj} \frac{\partial^2 W(A)}{\partial A_i \partial A_j}$$

Result: general $\mathcal{L}$ for chiral superfields

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$\Rightarrow$ no kinetic term for $F \Rightarrow$ no deg. of freedom $\Rightarrow$ eliminate

eq.o. motion: $0 = \frac{\partial \mathcal{L}}{\partial F_i} = F_i^\dagger + \frac{\partial W(A)}{\partial A_i} \Rightarrow F_i^\dagger = - \frac{\partial W(A)}{\partial A_i}$

$$\Rightarrow (1) + (2) = \mathcal{L} = \text{kinetic terms for } A_i, \psi_{Li} - \sum_i \left| \frac{\partial W(A)}{\partial A_i} \right|^2 - \frac{1}{2} \sum_{ij} \psi_{Li} \psi_{Lj} \frac{\partial^2 W(A)}{\partial A_i \partial A_j}$$

4.2 Example: SUSY-QED matter

$$\Phi = A(y) + \sqrt{2} \bar{\partial}_R \psi_L(y) + \bar{\partial}_R \partial_L F(y)$$

$\uparrow$ complex scalar $\uparrow$ left-handed spinor $\uparrow$ no deg. of freedom

Describe electron: $\psi = \psi_L + \psi_R = \begin{pmatrix} \psi_L^\alpha \\ \bar{\psi}_R^{\dot{\alpha}} \end{pmatrix}$

$\psi_L \rightarrow$ chiral superfield Φ_L , $Q_L = -1$

$\psi_R = \begin{pmatrix} 0 \\ \bar{\psi}_R^{\dot{\alpha}} \end{pmatrix} \rightarrow ?$

$\bar{\psi}_R = (\psi_R^\alpha \ 0) \rightarrow$ chiral superfield Φ_R , $Q_R = +1$
(physics: anti-electron/positron-R)

Electron mass: $m \bar{\psi} \psi = m (\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R) = m (\psi_R^\alpha \psi_{L\alpha} + \bar{\psi}_{L\dot{\alpha}} \bar{\psi}_R^{\dot{\alpha}})$
 $\rightarrow$ superpotential term $m \Phi_L \Phi_R$

Hence choose

$$\Phi_L = A_L + \sqrt{2} \partial^\alpha \psi_{L\alpha} + \partial^\alpha \partial_\alpha F_L$$

$\uparrow$
 $\tilde{e}_L, \text{charge } -1$

$$\Phi_R = A_R + \sqrt{2} \partial^\alpha \psi_{R\alpha} + \partial^\alpha \partial_\alpha F_R$$

$\uparrow$
 $\tilde{e}_R^\dagger, \text{charge } +1$

$$\mathcal{L} = \int d^4\theta \left(\Phi_L^\dagger \Phi_L + \Phi_R^\dagger \Phi_R \right) + \int d^2\theta W + \text{h.c.}, \quad W = m \Phi_L \Phi_R$$

$$= \mathcal{L}_{\text{kinetic}, A_L, A_R, \psi_L, \psi_R} - m \left(\psi_L^\alpha \psi_{R\alpha} + \bar{\psi}_{L\dot{\alpha}} \bar{\psi}_R^{\dot{\alpha}} \right) - m^2 \left(|A_L|^2 + |A_R|^2 \right)$$

- scalar superpartners $\tilde{e}_L, \tilde{e}_R$
- kinetic terms for all fields ; mass terms with equal masses

4.3 Vector superfields

contain $V^\mu \rightsquigarrow$ only consistent in gauge theory !

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contain $V^\mu \rightsquigarrow$ only consistent in gauge theory!

non-susy: (scalar) matter $\phi := \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix} \xrightarrow{\text{gauge transf.}} e^{i T^a \alpha_a} \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$
 $\alpha_a = \text{real}$

covariant deriv.

$$\mathcal{D}^\mu = \partial^\mu + ig T^a V_a^\mu$$

such that $(\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) = \text{gauge invariant}$

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covariant deriv.

$$\mathcal{D}^\mu = \partial^\mu + i g T^a V_a^\mu$$

such that $(\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) = \text{gauge invariant}$

susy: chiral matter $\underline{\Phi} \xrightarrow{\text{gauge transf.}} e^{i T^a \Lambda_a} \underline{\Phi}$
 $\Lambda_a = \text{chiral}$

postulate term $e^{2 g T^a V_a}$

such that $\underline{\Phi}^\dagger e^{2 g T^a V_a} \underline{\Phi}$ is gauge-invariant

requires:

$$e^{2 g T^a V_a} \xrightarrow{\text{gauge transf.}} e^{i T^a \Lambda_a^\dagger} e^{2 g T^a V_a} e^{-i T^a \Lambda_a}$$

Result: consistent theory with vector fields

$$\mathcal{L} = \int d^4\theta \quad \bar{\Phi}^\dagger e^{2g T^a V_a} \Phi$$

susy and gauge invariant

- multiplet of chiral matter fields Φ
- multiplet of vector superfields V_a (adjoint rep.)

Result: consistent theory with vector fields

$$\mathcal{L} = \int d^4\theta \quad \Phi^\dagger e^{2g T^a V_a} \Phi$$

susy and gauge invariant

- multiplet of chiral matter fields Φ
- multiplet of vector superfields V_a (adjoint rep.)

Immediate simplification: Wess-Zumino gauge:

use gauge transformation $\Lambda \rightarrow$ achieve $V_a|_{\theta, \bar{\theta}} = 0$

$$\begin{aligned} V(x, \theta) &= \bar{\theta}_L \gamma^\mu \theta_L V_\mu(x) + (\bar{\theta} \theta) \bar{\theta} \lambda(x) + \frac{1}{4} (\bar{\theta} \theta) (\bar{\theta} \theta) D(x) \\ &= \theta^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} V_\mu(x) + \theta^\alpha \partial_\alpha \bar{\theta}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}}(x) + \frac{1}{2} \theta^\alpha \partial_\alpha \bar{\theta}_{\dot{\alpha}} \partial^{\dot{\alpha}} D(x) \\ &\quad + \bar{\theta}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial^\alpha \lambda_\alpha(x) \end{aligned}$$

Final ingredient: field strength superfield

- one more possible susy, gauge invariant term:
- take certain derivative of vector superfield
- result = chiral, gauge covariant superfield:

$$W_\alpha = \lambda_\alpha - \frac{i}{2} (\sigma^\mu \bar{\sigma}^\nu \partial)_\alpha F_{\mu\nu} + \partial_\alpha \mathbb{D} + \dots$$

$$\rightarrow \frac{1}{4} \int d^2 \theta_L W_a^\alpha W_{a\alpha} + h.c. = -\frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} + \bar{\lambda}_a \bar{\sigma}^\mu i D_\mu \lambda_a + \frac{1}{2} \mathbb{D}_a^2$$

$$= \text{kinetic terms for } V_a^\mu, \lambda_a + \frac{1}{2} \mathbb{D}_a^2$$

4.4 General renormalizable $\mathcal{L}_{\text{susy}}$

multiplet of chiral superfields + suitable vector superfields

$$\begin{aligned}\mathcal{L} = & \int d^4\theta \left(\Phi^\dagger e^{2gT^a V_a} \Phi \right) \\ & + \int d^2\theta_L \left(\frac{1}{4} W_a^\alpha W_{a\alpha} + W(\Phi) \right) + \text{h.c.}\end{aligned}$$

4.4 General renormalizable $\mathcal{L}_{\text{susy}}$

multiplet of chiral superfields + suitable vector superfields

$$\mathcal{L} = \int d^4\theta \left(\Phi^\dagger e^{2gT^a V_a} \Phi \right) \\ + \int d^2\theta_L \left(\frac{1}{4} W_a^\alpha W_{a\alpha} + W(\Phi) \right) + \text{h.c.}$$

evaluate $\rightarrow F_i, D_a$ have no kinetic terms $\rightarrow$ eliminate

$$F_i^\dagger = - \frac{\partial W(A)}{\partial A_i}, \quad D_a = -g A^\dagger T^a A$$

4.4 General renormalizable $\mathcal{L}_{\text{susy}}$

multiplet of chiral superfields + suitable vector superfields

$$\mathcal{L} = \int d^4\theta \left(\Phi^\dagger e^{2gT^a V_a} \Phi \right) + \int d^2\theta_L \left(\frac{1}{4} W_a^\alpha W_{a\alpha} + W(\Phi) \right) + \text{h.c.}$$

evaluate $\rightarrow F_i, D_a$ have no kinetic terms $\rightarrow$ eliminate

$$F_i^\dagger = - \frac{\partial W(A)}{\partial A_i}, \quad D_a = -g A^\dagger T^a A$$

Result: $\mathcal{L} = \mathcal{L}_{\text{kinetic}}, A, \psi, V^a, \lambda \mid \partial^n \rightarrow D^n$

+ $\mathcal{L}_{\text{gaugino}}$ + $\mathcal{L}_{\text{superpot.}}$ + $\mathcal{L}_F$ + $\mathcal{L}_D$

$$\mathcal{L}_{\text{gaugino}} = -\sqrt{2} g \left(\bar{\psi}_{\dot{\alpha}} T^a \bar{\lambda}_a^{\dot{\alpha}} A + A^\dagger T^a \lambda_a^\alpha \psi_\alpha \right)$$

$$\mathcal{L}_{\text{superpot}} = -\frac{1}{2} \psi_i^\alpha \psi_{j\alpha} \frac{\partial^2 W(A)}{\partial A_i \partial A_j} + \text{h.c.}$$

$$\mathcal{L}_F = -\sum_i \left| \frac{\partial W(A)}{\partial A_i} \right|^2 \quad \mathcal{L}_D = -\frac{1}{2} g^2 (A^\dagger T^a A)(A^\dagger T^a A)$$

4.5 Full SUSY-QED

two chiral superfields

$$\Phi_L : A_L, \psi_{L\alpha} \quad Q_L = -1$$

$$\Phi_R : A_R, \psi_{R\alpha} \quad Q_R = +1$$

$$\text{electron} \quad \psi = \begin{pmatrix} \psi_{L\alpha} \\ \bar{\psi}_{R\dot{\alpha}} \end{pmatrix}$$

$$\text{selectrons} \quad \tilde{e}_L = A_L, \quad \tilde{e}_R = A_R^\dagger$$

$$\left. \begin{array}{l} 4 \text{ fermionic} \\ 4 \text{ bosonic} \end{array} \right\} \text{deg.o.f.}$$

one vector superfield

$$V : A^\mu, \lambda_\alpha, \bar{\lambda}_{\dot{\alpha}}$$

$$\text{photon} \quad A^\mu$$

$$\text{photino} \quad \tilde{\gamma} = \begin{pmatrix} \lambda_\alpha \\ \bar{\lambda}_{\dot{\alpha}} \end{pmatrix} = \tilde{\gamma}^c \quad (\text{Majorana}) \quad \left. \begin{array}{l} 2 \text{ bos.} \\ 2 \text{ ferm.} \end{array} \right\} \text{deg.o.f.}$$

Lagrangian:

$$\mathcal{L} = \int d^4\theta \left(\Phi_L^\dagger e^{2eQ_L V} \Phi_L + L \rightarrow R \right) \\ + \int d^2\theta_L \left(\frac{1}{4} W^\alpha W_\alpha + m \Phi_L \Phi_R \right) + \text{h.c.}$$

Lagrangian: $\mathcal{L} = \int d^4\theta \left(\Phi_L^\dagger e^{2eQ_L V} \Phi_L + L \rightarrow R \right) + \int d^2\theta_L \left(\frac{1}{4} \omega^\alpha \omega_\alpha + m \Phi_L \Phi_R \right) + h.c.$

evaluate: kinetic terms + gaugino + superpot. + F + D -terms

$$\mathcal{L}_{\text{kinetic}} = |\mathcal{D}^\mu A_L|^2 + |\mathcal{D}^\mu A_R|^2 + \bar{\Psi} i\gamma^\mu \mathcal{D}_\mu \Psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \tilde{\gamma} i\gamma^\mu \partial_\mu \tilde{\gamma}$$

$$\mathcal{L}_{\text{gaugino}} = -\sqrt{2} e Q_L \left(\bar{\Psi}_{L\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} A_L - \bar{\Psi}_{R\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} A_R + A_L^\dagger \lambda^\alpha \Psi_{L\alpha} - A_R^\dagger \lambda^\alpha \Psi_{R\alpha} \right)$$

$$\mathcal{L}_{\text{superpot}} = -m \Psi_L^\alpha \Psi_{R\alpha} + h.c. = -m \bar{\Psi} \Psi$$

$$\mathcal{L}_F = -m^2 |A_L|^2 - m^2 |A_R|^2$$

$$\mathcal{L}_D = -\frac{1}{2} e^2 Q_L^2 \left(|A_L|^2 - |A_R|^2 \right)^2$$

compare with "guess":

- equal # deg. of freedom : implemented in superfields
- equal masses $\leftrightarrow$ superpot. & F-terms
- equal interactions with γ and $\tilde{\gamma}$ and (scalar)⁴-terms

Summary:

- SUSY algebra $\leftarrow \Delta J = \pm \frac{1}{2}, F \leftrightarrow B$
- implement on superspace/fields
- $\int d^4\theta, \int d^2\theta_L$ give susy Lagrangians
- evaluation needs lots of spinor algebra
- chiral fields appear as $\Phi^\dagger \Phi \rightarrow$ kinetic terms
and in superpotential
- vector superfields appear in $\Phi^\dagger e^2 \partial^V \Phi \rightarrow$ gauge/
gaugino interactions
and in $W^\alpha W_\alpha \rightarrow$ kinetic terms
- implements a lot of physics:
 - equal # d.o.f.
 - equal interactions, masses
 - (scalar)⁴ interactions always fixed by F-, D-terms

5. Supersymmetric Standard Model

Strategy:

gauge field $\rightarrow$ vector s.f. $\rightarrow V^{\mu}, \lambda_{\alpha}, \bar{\lambda}_{\dot{\alpha}}$

scalar field $\rightarrow$ chiral s.f. $\rightarrow$ scalar, ψ_{α}

fermion $\Psi = \begin{pmatrix} \psi_{L\alpha} \\ \bar{\psi}_{R\dot{\alpha}} \end{pmatrix}$ (charge Q)

$\psi_{L\alpha} \rightarrow$ chiral s.f. $\rightarrow \tilde{f}_L, \psi_{L\alpha} \quad (Q)$

$\psi_{R\alpha} \rightarrow$ chiral s.f. $\rightarrow \tilde{f}_R^{\dagger}, \psi_{R\alpha} \quad (-Q)$

(anti-(s)fermion- $\bar{R}$)

5.1 MSSM fields

			$Q_{e.m.} = T_3 + Y/2$	$SU(3)_c$
$\hat{Q}$	$\tilde{q}_L = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$	$q_{L\alpha} = \begin{pmatrix} u_{L\alpha} \\ d_{L\alpha} \end{pmatrix}$	$+2/3$ $-1/3$	3
$\hat{U}$	$\tilde{u}_R^+$	$u_{R\alpha}$	$-2/3$	3^*
$\hat{D}$	$\tilde{d}_R^+$	$d_{R\alpha}$	$+1/3$	3^*
$\hat{L}$	$\tilde{l}_L = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}$	$l_{L\alpha} = \begin{pmatrix} \nu_{L\alpha} \\ e_{L\alpha} \end{pmatrix}$	0 -1	1
$\hat{E}$	$\tilde{e}_R^+$	$e_{R\alpha}$	$+1$	1
$\hat{H}_u$	$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$\begin{pmatrix} \tilde{H}_u^+ \\ H_u^0 \end{pmatrix}$	$+1$ 0	1
$\hat{H}_d$	$\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	0 -1	1
$\hat{V}_s^a$	$G^{\mu a}$	$\lambda_{s\alpha}^a, \bar{\lambda}_{s\dot{\alpha}}^a$	0	8
$\hat{V}_w^a$	$W^{\mu a}$	$\lambda_{\alpha}^a, \bar{\lambda}_{\dot{\alpha}}^a$	$0, \pm 1$	1
$\hat{V}'$	B^{μ}	$\lambda'_{\alpha}, \bar{\lambda}'_{\dot{\alpha}}$	0	1

5.2 MSSM Lagrangian

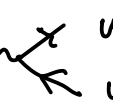
$$\mathcal{L} = \mathcal{L}_{\text{susy}} + \mathcal{L}_{\text{susy-breaking}} \longrightarrow \text{later}$$

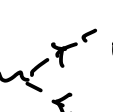
$$\mathcal{L}_{\text{susy}} = \int d^4\theta \left[\hat{Q}^\dagger e^{2g_s \hat{V}_s + 2g \hat{V} + 2g' Y_Q \hat{V}'} \hat{Q} + \dots + \hat{H}_d^\dagger e^{2g \hat{V} + 2g' Y_{H_d} \hat{V}'} \hat{H}_d \right] \\ + \int d^2\theta_L \left[\frac{1}{4} W_s^\alpha W_{s\alpha} + \frac{1}{4} W^\alpha W_\alpha + \frac{1}{4} W'^\alpha W'_\alpha + W \right] + \text{h.c.}$$

- kinetic terms, gauge interactions, gaugino interactions
- superpotential, F-, D-terms

$$W = -\gamma_u \underbrace{\hat{H}_u \hat{Q}}_{\hat{Q}^2} \hat{U} + \gamma_d \hat{H}_d \hat{Q} \hat{D} + \gamma_e \hat{H}_d \hat{L} \hat{E} + \mu \hat{H}_u \hat{H}_d \\ = \hat{H}_u^1 \hat{Q}^2 - \hat{H}_u^2 \hat{Q}^1 \quad \left(\text{SU(2) - invariant, without } \hat{H}_u^\dagger \right)$$

Obvious Feynman rules:

SM gauge interactions: γ, Z  $u, d, \dots$

stop squark gauge: γ, Z  $\tilde{u}, \tilde{d}, \dots$

gaugino λ, λ'  $u, \dots$

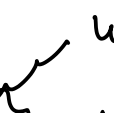
SM $F^{\mu\nu} F_{\mu\nu}$ - int.: γ, Z  W^+

gaugino: 

W^+  u_L, ν_L

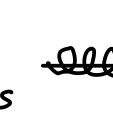
W^+  d_L, e_L

Z^+  u_L

W  $W \dots$

G^r  u, d

G^r  $\tilde{u}, \tilde{d}$

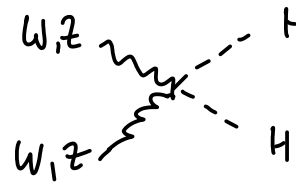
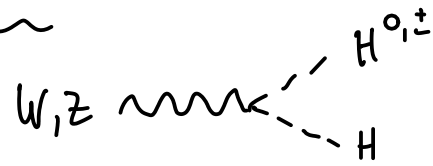
λ_s  $u, \dots$



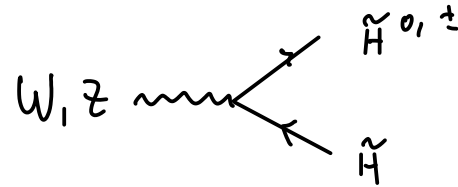
Feynman rules for $H, \tilde{H}$:

SM Higgs:

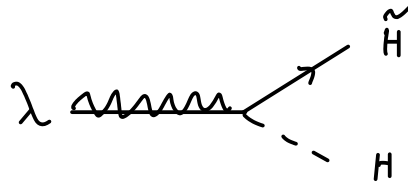


$\rightarrow WW v^2$
mass term...

gauge-Higgsino:

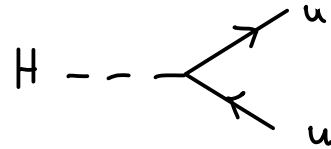


gaugino-Higgsino-Higgs:



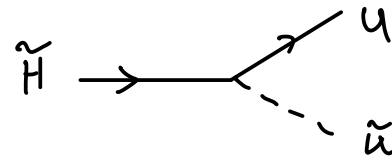
$\rightarrow \lambda \tilde{H} v$
mass/mixing
term

SM Yukawa:



$\rightarrow \bar{u} u v$
mass term

Higgsino/sfermions:



Sample: gluino-squark interactions:

$\gamma_u = \gamma_d = 0$, massless (s)quarks
neglect $su(2) \times u(1)$

$$D^\mu \tilde{u}_L = (\partial^\mu + i g_s G_a^\mu T^a) \tilde{u}_L$$

$$D^\mu \tilde{u}_R^\dagger = (\partial^\mu + i g_s G_a^\mu (-T^{a*})) \tilde{u}_R^\dagger$$

$$D^\mu \tilde{u}_R = (\partial^\mu + i g_s G_a^\mu T^a) \tilde{u}_R$$

$$\mathcal{L} = |D^\mu \tilde{u}_L|^2 + |D^\mu \tilde{u}_R|^2$$

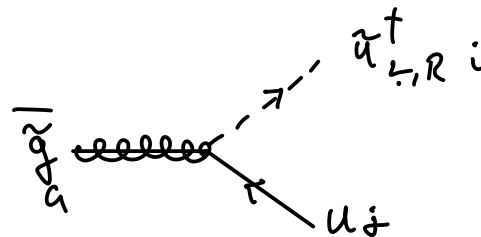
$$\begin{aligned} & \rightarrow G_a^\mu \text{ (wavy line)} \begin{array}{l} \xrightarrow{p'} \tilde{u}_{LRi}^\dagger \\ \xleftarrow{p} \tilde{u}_{L/Rj} \end{array} = -i g_s (p+p')^\mu T_{ij}^a \\ & \quad \begin{array}{l} G_a^\mu \text{ (wavy line)} \xrightarrow{\quad} \tilde{u}_L^\dagger \\ G_b^\mu \text{ (wavy line)} \xleftarrow{\quad} \tilde{u}_L \end{array} = i g_s^2 g^{\mu\nu} \{T^a, T^b\} \end{aligned}$$

$$\mathcal{L}_{\text{gaugino}} = -\sqrt{2} g_s \left(\begin{aligned} & \bar{u}_L \dot{\alpha} T^a \bar{\lambda}_{sa}^{\dot{\alpha}} \tilde{u}_L + \tilde{u}_L^\dagger T^a \lambda_{sa}^\alpha u_{L\alpha} \\ & + \bar{u}_R \dot{\alpha} (-T^{a*}) \bar{\lambda}_{sa}^{\dot{\alpha}} \tilde{u}_R^\dagger + \tilde{u}_R (-T^{a*}) \lambda_{sa}^\alpha u_{R\alpha} \end{aligned} \right)$$

Combine to 4-spinors:

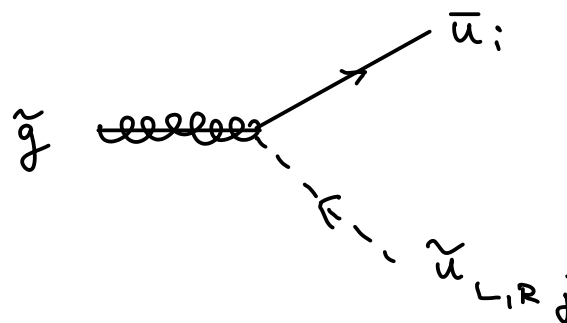
$$u = \begin{pmatrix} u_L^\alpha \\ \bar{u}_R^{\dot{\alpha}} \end{pmatrix}, \quad \tilde{g}_a = \begin{pmatrix} \lambda_{sa}^\alpha \\ \bar{\lambda}_{sa}^{\dot{\alpha}} \end{pmatrix}$$

$$(2) + (3) = \tilde{u}_L^\dagger \tilde{g}_a T^a P_L u - \tilde{u}_{Rj}^\dagger \tilde{g}_a \underbrace{T_{ij}^{a*}}_{=T_{ji}^a} P_R u_i$$



$$= -\sqrt{2} i g_s T_{ij}^a \begin{cases} + P_L \\ - P_R \end{cases}$$

$$(1) + (4) = \bar{u} P_R T^a \tilde{g}_a \tilde{u}_L - \bar{u}_j P_L \underbrace{T_{ij}^{a*}}_{=T_{ji}^a} \tilde{g}_a \tilde{u}_{Ri}$$



$$= -\sqrt{2} i g_s T_{ij}^a \begin{cases} + P_L \\ - P_R \end{cases}$$

5.3 Subtleties

oh no,
(not again !!!)

Why two Higgs doublets?

- SUSY Yukawa couplings
for up- and down-type

→ exercise !

- cancellation of gauge
anomalies (Higgsinos
are chiral fermions!)

Pheno: μ -problem motivates non-minimal susy

R-Parity:

- was already imposed on Lagrangian:

$R(SM) = +1$, $R(SUSY\text{-partner}) = -1$
 $\Rightarrow LSP$ stable, dark matter candidate
- otherwise extra terms possible:

$$\hat{H}_d \text{ and } \hat{L} \text{ have same quantum numbers}$$
$$\Rightarrow \hat{H}_d \hat{Q} \hat{D} + \hat{L} \hat{Q} \hat{D} + \hat{U} \hat{D} \hat{D} + \dots$$

Allows proton decay

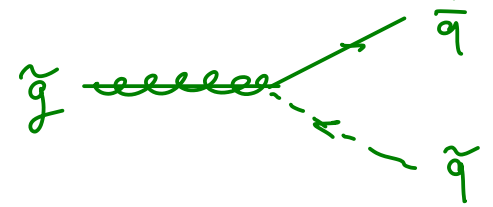
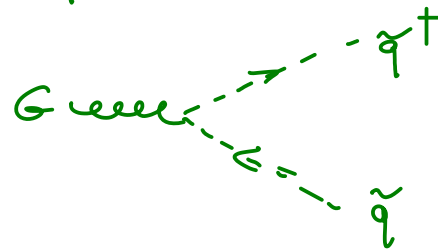
5.4 Summary of definite predictions

(not modified by susy breaking)

- Susy breaking does not affect dimension-4 terms in $\mathcal{L}$
(= "marginal terms" = terms with dim.-less couplings)

- existence of all susy particles

- interaction strength of SM / SUSY particles are equal



- also quartic scalar couplings predicted

$\Rightarrow$ prediction of m_{Higgs} , $\begin{array}{c} q \\ \diagup \\ \bar{q} \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} \tilde{g} \\ \diagdown \\ \tilde{q} \end{array}$ etc possible

5.5 SUSY breaking

"Hard breaking"

$:=$ change all terms in $\mathcal{L}$
 $\Rightarrow$ completely non-susy theory
 $\rightarrow$ not what we want

"Soft breaking"

$:=$ SUSY breaking terms which

- could originate from spontaneous susy breaking in some fundamental theory (Eff. QFT !)
- do not introduce quadratic divergences
- affect only terms in $\mathcal{L}$ with a dimensionful prefactor ("mass terms")

Soft breaking terms in the MSSM:

- all gauge invariant quadratic terms in scalar fields

$$m_Q^2 |\tilde{q}_L|^2 \quad m_U^2 |\tilde{u}_R|^2 \quad m_D^2 |\tilde{d}_R|^2 \quad \dots$$

(could even be 3×3 matrices in family space)

- all gauge invariant gaugino masses

$$M_3 \lambda_{sa}^\alpha \lambda_{sa\alpha} \quad M_2 \lambda_a^\alpha \lambda_{a\alpha} \quad M_1 \lambda'^\alpha \lambda'_\alpha$$

- certain triple-scalar interactions (only A^3 , not $A^\dagger A^2$)
(neglected in this lecture / exercises !)

$\Rightarrow$ essentially all masses / mixings of superpartners
are free parameters

Note: either : soft parameters treated as ~ 126 free parameters
or : fixed by some ad-hoc / "well-motivated" assumptions
or : predicted by specific theory of susy breaking $\leftrightarrow$ test such theories

5.6 Masses, Mixings, Mass eigenstates

Masses from:

- dimensionful parameters in $\mathcal{L}_{\text{susy}}$
- soft susy breaking
- Higgs mechanism, $\langle H_{u,d} \rangle = v_{u,d} \neq 0$

One explicit example: selectrons: $\hat{L} \rightarrow \tilde{\ell}_L = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}$
 $\hat{E} \rightarrow \tilde{e}_R^+$

* relevant soft breaking terms: $\mathcal{L}_{\text{soft}} = \dots - m_L^2 |\tilde{\ell}_L|^2 - m_E^2 |\tilde{e}_R^+|^2$

* terms in $\mathcal{L}_{\text{susy}}$ without derivatives and two powers of selectrons;
only in $\mathcal{L}_F = - \sum_i \left| \frac{\partial W(A)}{\partial A_i} \right|^2$

$$W(\Phi) = \mu \hat{H}_u \hat{H}_d + \gamma_e \hat{H}_d \hat{L} \hat{E} + \dots$$

$$\leadsto \text{need } \frac{\partial W(A)}{\partial H_d}, \quad \frac{\partial W(A)}{\partial \tilde{e}_L}, \quad \frac{\partial W(A)}{\partial \tilde{e}_R^+}$$

$$\frac{\partial W(A)}{\partial \tilde{e}_R^\dagger} = \gamma_e H_d \tilde{\ell}_L \longrightarrow \gamma_e v_d \tilde{e}_L$$

$$\frac{\partial W(A)}{\partial \tilde{e}_L} = \gamma_e H_d^0 \tilde{e}_R^\dagger \longrightarrow \gamma_e v_d \tilde{e}_R^\dagger$$

$$\begin{aligned} \frac{\partial W(A)}{\partial H_d^0} &= \frac{\partial}{\partial H_d^0} \left[\mu H_u^\dagger H_d^- - \mu H_u^0 H_d^0 + \gamma_e H_d^0 \tilde{e}_L \tilde{e}_R^\dagger - \gamma_e H_d^- \tilde{\nu}_L \tilde{e}_R^\dagger \right] \\ &= -\mu H_u^0 + \gamma_e \tilde{e}_L \tilde{e}_R^\dagger \longrightarrow -\mu v_u + \gamma_e \tilde{e}_L \tilde{e}_R^\dagger \end{aligned}$$

$$\begin{aligned} \Rightarrow \sum_i \left| \frac{\partial W(A)}{\partial A_i} \right|^2 &= \dots + (\gamma_e v_d)^2 (|\tilde{e}_L|^2 + |\tilde{e}_R|^2) \\ &\quad - 2 \operatorname{Re} \left(\mu^* v_u \gamma_e \tilde{e}_L \tilde{e}_R^\dagger \right) \end{aligned}$$

$$\begin{aligned} \Rightarrow \text{obtain mass matrix} \quad & \left((\gamma_e v_d)^2 + m_L^2 \right) |\tilde{e}_L|^2 + \left((\gamma_e v_d)^2 + m_E^2 \right) |\tilde{e}_R|^2 \\ & - 2 \operatorname{Re} \left(\mu^* v_u \gamma_e \tilde{e}_L \tilde{e}_R^\dagger \right) \end{aligned}$$

Summary of results:

$\tilde{f}_L$	and	$\tilde{f}_R$	$\longrightarrow$	$\tilde{f}_{1,2}$
gauginos	and Higgsinos		$\longrightarrow$	$\tilde{\chi}^{\pm,0}$
almost all masses/mixings are free parameters				

5.7 Aspects of SUSY phenomenology

① sparticle production/decay



② dark matter

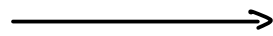
③ precision observables

$(g-2)_\mu$, M_W , M_h , ...

④ flavour/~~CP~~ observables

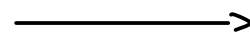
① Production / decay

production



robust (couplings fixed,
masses unknown)

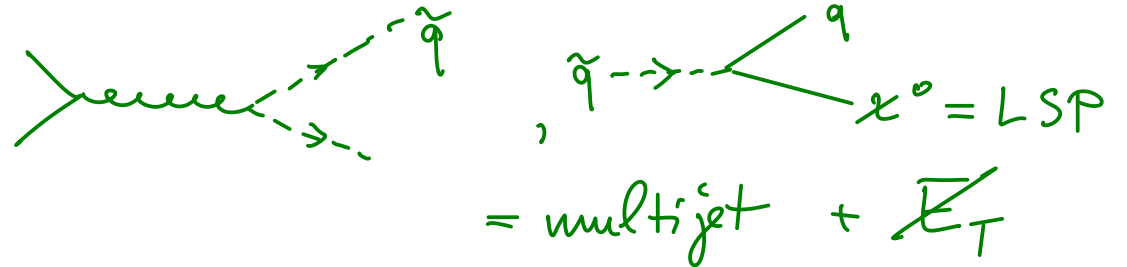
decay



- mass hierarchies/orderings
mixing important

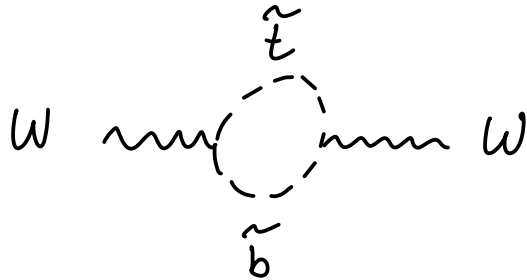
- stable or unstable LSP?

in simplest scenarios :



③

Precision observables



loops !

all masses / mixings matter

→ tests of susy spectra

→ $M_W, (g-2)_\mu$ agree better with
MSSM ($m_{\text{susy}} \lesssim \text{TeV}$)
than with SM

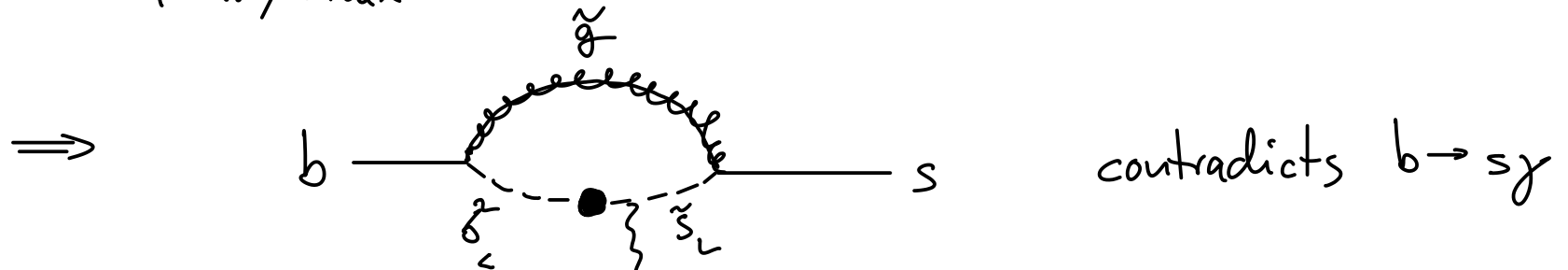
④ Flavour, ~~CP~~:

big problem for TeV-scale SUSY

(unless we assume there exists an explanation for very specific patterns of susy breaking)

e.g. $(m_Q^2)_{23}$ $\left(\begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix} \right)^\dagger \left(\begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix} \right)$

↑
family index



Many potential solutions (very heavy Staus, no flavour mixing, ...)

Summary: SUSY = very fundamental new symmetry of QFT

- could be relevant for TeV-scale physics
- $d=4$ - terms unambiguously predicted
(gauge, gaugino, Yukawa, (scalar)⁴ - couplings)
- masses / mixings influenced by susy breaking
- problems (flavour, CP)
- potential solutions (just don't know which!)
- improvements:
dark matter, gauge coupling unification, M_W , $(g-2)_\mu$
Symmetry involves scalars !!
Improved understanding of EWSB !

Only need discovery and measurement of soft breaking parameters!