EFFECTIVE QFT AND BSM

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1. EFT GENERALITIES

WHAT are EFTs? is its structure and ingredients?... Examples.

WHY are they useful? do they work?...

HOW to construct them? to use them?

Introduction

Generally speaking, an effective theory (EFT) is a theory derived (or derivable in principle!) from a more complete theory in order to study some physical sub-system, focusing only on the most relevant physics.

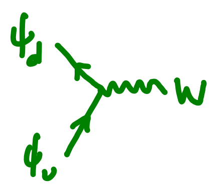
The EFTs we will be interested in are based on the fact that physics at a given energy scale can be studied ignoring physics at much higher scales. For example, the size of a hydrogen atom could be understood in terms of the electron mass, m_e, and the electromagnetic coupling strength, α, as $a_0 \sim \frac{1}{m_e \alpha}$, before knowing that the nucleus, a proton, is made of quarks and gluons, that electrons and quarks feel the electroweak force by interchanging W's and Z's or that, if string theory is right, an electron is a tiny string! At low enough E, it is an excellent approximation to treat the proton as a point-like charged particle, and so on. Atomic physics does not care about this short-distance structure, which cannot be probed without recourse to much higher energies.

In QFT we use EFT Lagrangians, $\mathcal{L}_{eff}$, to describe physics in some limited energy range, below some UV cutoff scale Λ, (typically associated with some heavy physics) ignoring short-distance physics above Λ (or $L \ll 1/\Lambda$). We can do that because the effects of particles heavier than Λ (the UV physics) can always be captured by local operators of the light dofs in the EFT. This follows ultimately from the uncertainty principle: heavy intermediate states must be virtual ⇒ short-lived ⇒ short-distance compared with typical $\lambda \sim 1/p \gg 1/\Lambda$.

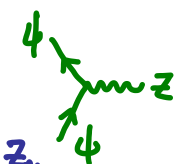
The paradigmatic example is Fermi's theory of weak interactions.

In $\mathcal{L}_{SM}$ fermion currents couple to EW gauge bosons as:

$$\mathcal{L}_{CC} = -g J^{\mu -} W_\mu^+ + \text{h.c.}$$
$$= -\underbrace{g}_{e/s_W} \frac{1}{\sqrt{2}} \sum_i (\bar{u}_{L_i} \gamma^\mu d_{L_i} + \bar{\nu}_{L_i} \gamma^\mu e_{L_i}) W_\mu^+ + \text{h.c.}$$



$$\mathcal{L}_{NC} = -e J^\mu_{em} A_\mu - \frac{e}{2 s_W c_W} J^\mu_Z Z_\mu$$
$$= -e \sum_i (\bar{\psi}_i \gamma^\mu Q_i \psi_i) - \frac{e}{2 s_W c_W} \sum_i \bar{\psi}_i \gamma^\mu (T_i^3 P_L - Q_i s_W^2) \psi_i Z_\mu$$



At energies too small to produce W/Z ($E \ll M_{W,Z}$) one can describe physics with an $\mathcal{L}_{eff}$ that does not contain $W_\mu^\pm$ or Z_μ.

$\sim J_\mu J^\mu$

SM non-local int. 4-Fermion local interaction

This results from the low-momentum expansion of the propagator

$$\frac{1}{p^2 - M^2} \underset{p^2 \ll M^2}{\simeq} \frac{-1}{M^2} + \underbrace{O\left(\frac{p^2}{M^4}\right)}_{\text{small correction}}$$

⇒ Interaction collapsed to point-like

We see that the strength of the 4-fermion interaction is determined by "UV physics". Making the p^2/M^2 expansion of the SM amplitude calculation one gets

$$\mathcal{L}_{eff} = \frac{8}{\sqrt{2}}\underbrace{\left(\frac{\sqrt{2}\,e^2}{8 s_W^2 M_W^2}\right)}_{G_F \simeq 1.166\times 10^{-5}\,\mathrm{GeV}^{-2}}\left(J_+^\mu J_{-\mu} + \frac{1}{2} J_Z^\mu J_{Z\mu}\right) \qquad (1)$$

With this simple Lagrangian one can compute $2\to 2$ fermion scatterings or $1\to 3$ decays (like neutron or muon decay). EX1.

Some remarks of more general relevance:

- Higher orders in the p^2/M^2 expansion correspond to derivative operators in $\mathcal{L}_{eff} \sim (\bar\psi\psi)\partial^2(\bar\psi\psi)/M^4$. Depending on the precision needed one keeps more or less operators from an infinite series of them.

- That series converges only for $p^2 < M^2$. We can't use $\mathcal{L}_{eff}$ beyond that scale, which acts as a cutoff or limit of applicability of the EFT. Beyond that scale we should use a more fundamental theory that describes also the new physics dofs above Λ.

- Low-E neutrinos interact with matter only through (1). [No em interactions as they are neutral]. The amplitude for ν scattering $a_\nu \sim G_F$ and a ν Xsection $\sigma_\nu \sim |a_\nu|^2 \sim G_F^2$ $(\sim 1/M^4)$. By dim. analysis, as $\sigma \sim (\text{area})$, $\Rightarrow$ $\sigma_\nu \sim G_F^2\, s$ ← c.o.m. energy

  So, $\sigma_\nu \to 0$ at low energy $\Rightarrow$ E^2/M^2 suppression

- σ_ν violates unitarity for $s \gg M_{W,Z}^2$ $\Rightarrow$ beyond limit of validity of $\mathcal{L}_{eff}$.

Operator classification

To discuss further some other properties of EFTs, let us use a simple model with two scalars, one light ϕ and one heavy Φ with

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\Phi^2 - \frac{g}{2}M\phi^2\Phi \qquad (2)$$

This Lagrangian represents the more fundamental theory but we want to have a low-energy EFT Lagrangian to describe processes with $m \ll E \ll M$. Such $\mathcal{L}_{eff}$ should describe the light scalar only as the energy is too low to produce the heavy one.

Its generic form is a series expansion in ∂/Λ and φ/Λ (with $\Lambda \sim M$)

terms with more ∂'s

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}\mu^2\varphi^2 - \frac{\lambda}{4!}\varphi^4 - \sum_n \left[\frac{c_n}{\Lambda^{2n}}\varphi^{4+2n} + \frac{d_n}{\Lambda^{2n}}(\partial\varphi)^2\varphi^{2n} + \dots\right]$$

The key point for this to be useful is that, at low energy, the different Lagrangian terms can be classified in order of relevance and terms with higher and higher powers of φ or ∂ are less and less important.

More generally, defining the canonical scaling dimensions

$$[\phi] = 1 \quad [m^2] = 2 \quad [\lambda] = 0 \quad [c_n/\Lambda^{2n}] = [d_n/\Lambda^{2n}] = -2n$$

the operators are classified as

RELEVANT	$[\mathcal{O}] < 4$	more imp. in IR	$(m^2\varphi^2)$
MARGINAL	$[\mathcal{O}] = 4$		$(\lambda\varphi^4)$
IRRELEVANT	$[\mathcal{O}] > 4$	less imp. in IR	$(c\varphi^6/\Lambda^2)$

We could have guessed this naively by noting that the strength of a given interaction should be given by a dimensionless quantity. E.g., the strength of the quartic φ^4 interaction is set by the dimensionless λ coupling while the strength of the φ^6 cannot be determined without specifying the energy scale of the experiment E. Then, the strength is set by $c(E/\Lambda)^2$. This should be familiar to us from the example of the gravitational interaction: Newton's constant has $[G_N]=-2$ (remember $G_N = 1/M_{Pl}^2$, with $M_{Pl} = 1.2\times 10^{19}\,\mathrm{GeV}$) and therefore the strength of gravitational interactions is determined by the ratio $(E/M_{Pl})^2$ or $(m/M_{Pl})^2$. It is very small at the electroweak scale but becomes very strong at energies near the Planck scale.

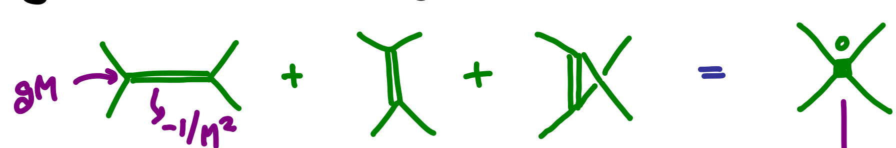
The same argument applies in determining the importance of a relevant operator like $m^2\varphi^2$: the dimensionless ratio is then $(m/E)^2$. And that's why mass terms dominate the long-distance (IR) behaviour of correlation functions.

As we find that the effect of irrelevant operators is suppressed by powers of E_{exp}/Λ, only if great precision is needed (or in special cases) one should worry about them. From the modern PoV, renormalizability just emerges in the IR and there is nothing fundamental about it.

Integrating out and matching

When, as in the example, we know the short-distance theory (and we can rely on perturbation theory) we can derive the effective theory from the more fundamental one (top-down approach) by requiring that both agree in their description of the low-energy physics. In order to do this we match scattering amplitudes (of light particles) or, equivalently, 1LPI (1 light-particle-irreducible) graphs in both theories.

E.g. in our example, $\mathcal{L}_{eff}$ should include a ϕ^4 interaction coming from Φ interchange:



(3)

✓ taylored to give right the low-E amplitudes
✓ fixed by the UV theory

← $c_0 g^2 \phi^4$

The matching can be done off-shell (provided $p^2/M^2 \ll 1$) as both EFT and full theory should agree completely in the IR.

Matching at non-zero external momentum, in a series in p^2/M^2, introduces higher-order ops. involving $\partial^2\phi$ too.

The most straightforward way of deriving the low-E EFT in a perturbative theory is by "integrating-out" the heavy physics using path-integral methods. The physical content of a QFT is encoded in its field correlators. For the low-E EFT these will be correlators of the light field ϕ

only. In the full theory these are given by :

$$\langle 0|T\phi(x_1)\ldots\phi(x_n)|0\rangle = \int \mathcal{D}\phi\,\mathcal{D}\Phi\;\phi(x_1)\ldots\phi(x_n)\,e^{iS[\phi,\Phi]/\hbar}$$

Performing the integral over Φ (integrating it out) we can rewrite the previous path integral as :

$$\int \mathcal{D}\phi\;\phi(x_1)\ldots\phi(x_n)\,\underbrace{e^{iS_{eff}[\phi]/\hbar}}_{\int \mathcal{D}\Phi\, e^{iS[\phi,\Phi]/\hbar}}$$

At tree-level, the integral is dominated by the classical solution $\quad \partial S/\partial\Phi\big|_{\Phi=\bar{\Phi}(\phi)} = 0$

which in our example leads to the EoM:

$$\partial^2\Phi + M^2\Phi + \frac{1}{2}gM\phi^2 = 0$$

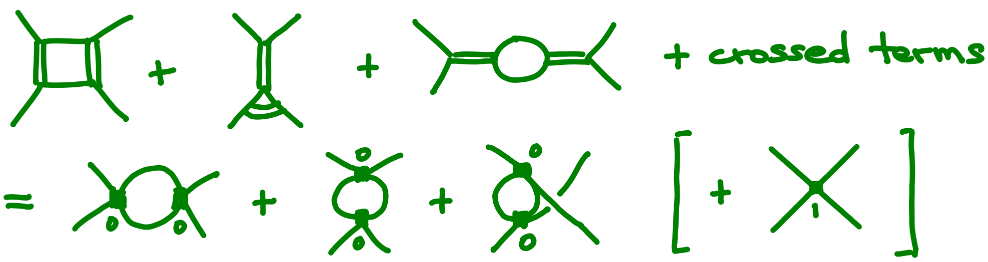
Solved by $\quad \bar{\Phi}(\phi) = -\frac{1}{2}gM\frac{1}{M^2+\partial^2}\phi^2$

Then $\quad S_{eff}[\phi] = S[\phi,\bar{\Phi}(\phi)] + O(\hbar)$

Expanding in powers of ∂^2/M^2 one obtains the series of operators in the Lagrangian of the low-E EFT.

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{8}g^2\phi^4 + \frac{1}{2}\frac{g^2}{M^2}\phi^2(\partial\phi)^2 + \ldots$$

So far we have done this matching to reproduce with the effective theory the IR physics and it is obvious that the UV behaviour is completely different. However, we are matching QFTs and we should be allowed to consider loop corrections in our effective theory. Should we worry about having the wrong UV behaviour? After all, loops are sensitive to large momenta beyond the effective theory cutoff and moreover, the full theory includes heavy particles (not in the effective theory) that can also affect loop corrections as virtual states entering loops... Suppose we want to do the same matching in (3) but now at 1-loop level of precision:



(4)

The matching fails: as long as p^2 is small in the loop, heavy particle propagators can be collapsed to a point and both sides of the matching agree in such IR contributions. But the UV contributions for large loop momentum disagree (even the UV divergences differ). However, the key point is that unknown UV physics can be mimicked by local ops, so the missmatch in (4) can be simply fixed by adding a 1-loop shift to , the tree-level quartic. Technically, we regulate UV divs on both sides using dimensional regularization and a mass-independent renormalization scheme (this point is crucial to keep the power counting simple). This procedure does introduce an scale, the renormalization scale μ through logs only. Formally the amplitude matching looks like

$$a_{low}\left(\underset{\uparrow}{\lambda_i}, m^2, \log \underset{\uparrow}{m^2/\mu^2}\right) = a_{full}\left(\underset{\uparrow}{g_i}, m^2, M^2, \log \frac{m^2}{\mu^2}, \log \frac{M^2}{\mu^2}\right) \quad (5)$$

low-E couplings & masses. high E couplings & masses

To perform this matching it is convenient to choose $\mu = M$ the heavy threshold scale, so that matching corrections are determined at the boundary between EFTs (low-E one and short-distance one). This choice has the advantage that potentially large corrections from the terms $\sim \log M^2/\mu^2$ on the RHS of (5) are minimized while the $\log m^2/\mu^2$ which are of IR origin should cancel between both sides. Matching enough amplitudes we can get the values of the low-E EFT parameters as determined by short-distance physics at $\mu = M$

$$\begin{aligned} m^2(\mu=M) &= f_{m^2}(g_i, m^2, M^2) \\ \lambda_i(\mu=M) &= f_{\lambda_i}(g_i, m^2, M^2) \end{aligned} \quad (6)$$

This cancelation of logs is more general and occurs for all IR non-analytic terms which come from loop integration in low p^2 regions, where $1/(p^2-m^2)$ propagators get singular (for $m^2 \to 0$). As both sides of the matching eqns. have the same IR behaviors, those IR singular terms cancel. As a result, threshold corrections like f_{m^2} and f_{λ_i} are analytic functions of low-energy masses, a useful check of EFT calculations. In particular, threshold corrections are "finite" → they involve no logs. Logs can always be understood as coming from RG evolution between two separate mass scales. Eg, m^2 and λ_i fixed at $\mu = M$, need to be evolved down to the energy scales of interest, say $E \sim m$. This will give $\log M^2/m^2$ effects. EFTs are very useful to understand theoretically the structure of such corrections and to organize radiative corrections in a form suitable for resummations (of large logs).

One does this matching at the level of the Lagrangian, once and for all, so that low-E processes can then be described by such $\mathcal{L}_{eff}$. In other cases one does not know the short-distance theory (or it is impossible to derive perturbatively the low-E effective theory, as in QCD) and the parameters in $\mathcal{L}_{eff}$ cannot be computed but have to be extracted from experiment. Predictivity will not be lost because to a given accuracy only a finite numbers of terms need to be considered. In such cases, symmetries are all important to restrict the form of the possible terms in $\mathcal{L}_{eff}$.

Ingredients and usefulness

Let us briefly summarize the ingredients of an EFT:

1) Use only the relevant d.o.f.s (eg remove heavy stuff $M \gg E$)

In some cases the d.o.f. in the EFT are totally different from those of the full theory, the most famous example being QCD: quarks and gluons are the fundamental dofs but the low-E description is in terms of pions.

2) Identify the symmetries (useful to restrict $\mathcal{L}_{eff}$)

3) Identify the power counting that organizes the hierarchy of the infinite series of operators, controls the precision of the approximations and sets the limit of validity of the EFT. E.g. $\epsilon = E/M$

When one knows the short-distance theory why would one go through the trouble of deriving an EFT? In general, EFTs are extremely useful tools. An incomplete list of reasons for that follows:

- One avoids wasting time calculating negligible effects (eg from heavy physics) by focusing on the relevant physics.
- EFTs allow a better understanding of multi-scale problems using simple dimensional analysis and making the separation of hard/soft effects much easier.
- It can be simpler to calculate in the EFT (especially when the UV theory is too complicated or non-perturbative, like QCD).
- EFTs are the most natural way to resum potentially

large logarithmic corrections $\sim (g^2 \log E/M)^n$ (which can be large even for $g \ll 1$ if $E \ll M$) in a systematic way, using renormalization group (RG) techniques. Note that the relevant β-functions are those of the EFT, not those of the full theory. Ex2.

- EFTs (truncated to some level of approximation) might posses accidental/emergent symmetries not present in the fundamental theory.
- The EFT language is the natural language to discuss renormalization (as understood from a modern PoV) as well as naturalness or finetuning issues.

Given this long list of virtues it is not surprising to find so many EFTs being used to attack all sort of problems. More or less familiar examples include HQET, NRQCD, NRQED, Chiral $\mathcal{L}$, SCET, (for a long list see e.g.[†]) with their different symmetries, d.o.fs, expansion parameters for power counting, fundamental theory from which they follow and appropiate physical system to describe. Even General Relativity or the Standard Model can be thought of as low-energy EFT approximations of more fundamental UV theories.

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† http://ocw.mit.edu/courses/physics/8-851-effective-field-theory-spring-2013/projects/

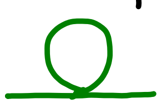
UV divergences and renormalization

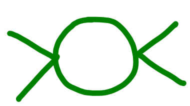
From the modern point of view UV divergences are not mysterious at all but just due to our ignorance of the UV physics (that we probe when letting loop momenta go to infinity). It is also intuitively clear, by thinking in position space, that short-distance, UV, effects can be captured by local operators. Renormalization just absorbs in relevant and marginal operators the UV physics effects. We also understand how renormalizabity is just an emergent property of QFTs in the IR.

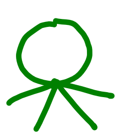
Let us see in a bit more detail how radiative corrections impact a low-E EFT. Take the following interaction Lagrangian

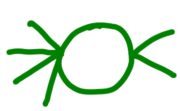
$$\mathcal{L}_{eff} \supset \lambda \phi^4 + c_6 g^4 \frac{\phi^6}{\Lambda^2} + \dots$$

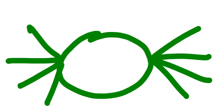
At one-loop we get corrections like these $\left(\kappa \equiv \frac{1}{16\pi^2}\right)$

 $\sim \kappa \lambda \Lambda^2 \phi^2$ $\qquad \delta m^2 \sim \kappa \Lambda^2$ (threshold)

 $\sim \kappa \lambda^2 \log\Lambda \cdot \phi^4$ $\qquad \lambda$ renormalizes itself ($\beta_\lambda \sim \kappa \lambda^2$)

 $\sim \kappa c_6 g^4 \frac{1}{\Lambda^2} \Lambda^2 \cdot \phi^4$ $\qquad \delta\lambda \sim \kappa c g^4$ (threshold)

 $\sim \kappa \frac{c_6 g^4}{\Lambda^2} \lambda \log\Lambda \cdot \phi^6$ $\qquad \lambda$ renormalizes sixtic ($\beta_{cg^4} \sim \kappa \lambda c g^4$)

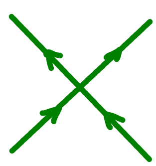
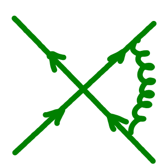
 $\sim \kappa \frac{c_6^2 g^8}{\Lambda^4} \log\Lambda \cdot \phi^8$ $\qquad$ sixtic generates ϕ^8

I have regulated divergent integrals using a cutoff, identified with the scale that suppresses the operators, to mimick in a simple way the effect of heavy states at Λ. We would get the same behaviour working in dimensional regularization (eg. quadratic divergences reappear from threshold corrections $\sim M^2 \sim \Lambda^2$).

Besides the typical contributions to β-functions of the parameters of the original Lagrangian we find a contribution $\delta m^2 \sim k \Lambda^2$ due to the fact that $m^2 \varphi^2$ is a relevant operator. This can be dangerous to maintain $m^2 \ll \Lambda^2$ and is at the root of the hierarchy problem, see below. On the other hand we see that even if we start with operators not higher than φ^6, they will be generated radiatively anyway (eg φ^8 as shown), and to be consistent the infinite tower of them should be considered. This was the reason traditionally given against non-renormalizable field theories. We know better now: if we work at some level of precision, we can ignore operators of dimension higher than some given number and work consistently within such approximation.

We mentioned that power-law corrections $\sim \Lambda^n$ are problematic theoretically. However, they can be renormalized away to agree with experimental data (possibly hiding some naturalness problems). On the other hand, $\log \Lambda$ divergences, which affect almost all ops. give real physical effects and are related to RG effects: they appear as $\log(\Lambda/\mu_{IR})$, where μ_{IR} is some IR scale. If one compares these corrections at two different IR scales μ_{IR} and μ'_{IR}, the difference goes like $\log(\mu'_{IR}/\mu_{IR})$ which cannot be renormalized away and is a physical effect, associated with the running of couplings with scale and captured by the appropriate RG equations.

Such corrections can also be interpreted as quantum modifications of the scaling dimension of ops. For instance, a 4-quark operator can get renormalized by gluons as

$\frac{c}{\Lambda^2}(\bar{q}q)^2$  $+$  $\Rightarrow \Delta c \sim \frac{\alpha_s}{4\pi} c \log\frac{\Lambda}{\mu}$

$$\Downarrow$$

$$\delta\mathcal{L}_{eff} = \frac{c}{\Lambda^2}\underbrace{\left[1 - \frac{\alpha_s}{4\pi}\log\frac{\mu}{\Lambda}\right]}_{= \left(\frac{\mu}{\Lambda}\right)^{-\alpha_s/4\pi}}(\bar{q}q)^2$$

So that the suppresion of the operator is not by 2 powers of Λ but by $2 - \frac{\alpha_s}{4\pi}$ making the op. a little bit more important in the IR. Such effects can be important to determine precisely the rates of rare processes, although for perturbative theories they will not change the nature of irrelevant ops. However, these quantum effects are crucial for marginal ops, as they can turn them into relevant ones (like it happens for α_s which increases/decreases in the IR/UV) or into irrelevant ones (like it happens for α_{em} which decreases/increases in the IR/UV) due to such quantum "running".

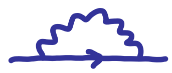
Naturalness and tuning

We consider the parameters of a given theory, with typical mass scale Λ to be natural if $\lambda_i \sim O(1)$, $m_i \sim O(\Lambda)$. where λ_i are dimensionless couplings and m_i are masses. Cases with $\lambda_i \ll 1$ or $m_i \ll \Lambda$ can still be natural provided there is a symmetry reason that explains their smallness. Typically the theory recovers some symmetry in the limit $\lambda_i \to 0$ or $m_i \to 0$.

In the absence of such symmetries one can still arrange for some particular coupling or mass to be very small compared to their natural values, but in such cases radiative corrections tend to spoil such small values. (While in the presence of some symmetry, radiative corrections will respect that symmetry too and the smallness will be protected.) In fact we can estimate what are the natural values for different quantities by looking at the size expected from radiative corrections to them. If we look at the list of one-loop diagrams above we find that it would not be natural to expect the following hierarchies: $m^2 \ll \kappa\lambda\Lambda^2$, $\lambda \ll \kappa c_8 g^4$, $c_8 \ll \kappa c_6^2 g^2$, where we have normalized the φ^8 op. as $c_8 g^6 \varphi^8/\Lambda^4$. (In addition, perturbativity requires $\lambda \ll 16\pi^2$).

Symmetries

How about fermion masses $m_f \bar{f}_L f_R$ which are also relevant operators? Do we have

 $\sim \Delta m_f \sim \kappa g^2 \Lambda$?

This does not happen due to chiral symmetry $f \to e^{i\alpha\gamma_5} f$ (or $f_L \to e^{i\alpha} f_L$, $f_R \to e^{-i\alpha} f_R$. "Chiral" means L and R are treated dif.) which is a symmetry of the fermionic Lagrangian if we set $m_f = 0$. Loop corrections from such $m_f = 0$ Lagrangian respect this symmetry. Using the full $m_f \neq 0$ Lagrangian any loop correction contributing to m_f should be proportional to the symmetry breaking parameter, m_f itself, and one gets

$$\Delta m_f \sim \kappa m_f g^2 \log \Lambda$$

at most. A huge difference compared with $\Delta m^2 \sim \kappa \Lambda^2$ for scalars. In this respect, therefore, fermion mass ops. behave as marginal rather than as relevant ops.

Symmetries are also important to determine what ops. appear in $\mathcal{L}_{eff}$ or even what are the light d.o.fs (eg. through Goldstone's theorem).

Gauge symmetries cannot be broken, only hidden, so that in principle the EFT inherits the gauge symmetries of the parent theory. The same will happen with exact global symmetries. When they are only approximate (i.e. broken by some small terms in $\mathcal{L}$)

this can happen in basically three different forms, depending on the type of op. breaking the symmetry:

a) Soft breaking (by relevant ops.)

E.g. chiral symmetry in QCD, broken by fermion masses. The breaking becomes more important in the IR.

b) Hard breaking (by marginal ops.)

Like isospin in QCD, broken by electromagnetism. In general, this breaking will extend radiatively to all kinds of other ops.

c) Accidental breaking (by irrelevant operators)

Unlike "broken" gauge symmetries, the symmetry in this case is better at low-E (a better name for such symmetry is "emergent") and is not a symmetry of the short-distance $\mathcal{L}$ (eg. B, L in SM)

BIBLIOGRAPHY

There are many excellent introductions to the use of EFTs. An incomplete list of some of my favorites is:

- What is renormalization? G.P. Lepage '89
  www.arxiv.org/pdf/hep-ph/0506330.pdf
- Effective Field Theory, H. Georgi '93
  www.people.fas.harvard.edu/~hgeorgi/review.pdf
- Selected topics in EFTs for particle physics A.G. Cohen '93
- Effective Field Theories, D.B. Kaplan www.arxiv.org/pdf/nucl-
- Five lectures on effective field theory, D.B. Kaplan
  www.arxiv.org/pdf/nucl-th/05100023.pdf
- TASI Lectures on EFTS, I.Z. Rothstein www.arxiv.org/pdf/hep-ph/0308266.pdf
- Effective Field Theories, A.V. Manohar, www.arxiv.org/pdf/hep-ph/9606222.pdf
- Introduction to EFT, C.P. Burgess, http://arxiv.org/pdf/hep-th/0701053.pdf
- On Effective Field Theories, and NDA, A. Wulzer
  http://www.pd.infn.it/~wulzer/Teaching/Strongly_coupled_2013/Chapter_4/
- Effective Field Theory, I. Stewart, MIT online course
  http://ocw.mit.edu/courses/physics/8-851-effective-field-theory-spring-2013/