Neutrino Physics III (Additional Material)

Neutrino mass models

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Neutrino Physics III Outline

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1. Introduction
   - Effective Lagrangian Approach
2. Tree-level Majorana Masses
   - See-saw Types I,II,III
   - Variations (Inverse See-saw)

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   - The Zee Model
   - The Inert Doublet (Inert)

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   - Double W Exchange contributions
   - The Zee-Babu Model
5. Masses not Described by the Weinberg Operator

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6. Masses in extensions of the SM
   - Grand Unification (GUT)
   - Supersymmetry
   - Extra Dimensions

Masses of Neutrinos in Models BSM

In the SM the neutrinos do not have mass by several reasons:

- (a) There are not ν_R
- (b) It only contains a doublet of scalars
- (c) Renormalizable theory (interactions with dim ≤ 4)

(a)$\Longrightarrow$ Neutrinos can not have Dirac masses
(b)+(c) $\Longrightarrow$ L and B conserved exactly (perturbativelly)
$\Longrightarrow$ no Majorana masses
$m_\nu \neq 0$ requires to abandon (a) , (b) or (c)

We will begin by (c), the most radical but the most general.

Effective Lagrangian Approach

If the new particles are much heavier than m_W their effect can be parametrized at low energies by an effective Lagrangian

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \sum_{n=5}^{\infty} \sum_i \left(\frac{C_i^{(n)}}{\Lambda^{n-4}} \mathscr{O}_i^{(n)} + \text{h.c.} \right)$$

with $C_i^{(n)}$ coupling constants and Λ the new physics scale

If $E, m \ll \Lambda$, effects of $\mathscr{O}_i^{(n)}$ suppressed by powers of E/Λ or m/Λ

(They decouple and we recover the SM).

Effects of new physics dominated by the lowest dimension operators.

Weinberg Operator

Only a dimension 5 operator (Weinberg operator) which does not conserve LN

$$\mathcal{O}^{(5)} = (\overline{\tilde{L}_L}\Phi)(\tilde{\Phi}^\dagger L_L)$$

with $\tilde{L}_L = i\tau_2 L_L^c$ and $\tilde{\Phi} = i\tau_2\Phi^*$
(transform well under Lorentz and SU(2))
After SSB ($\langle\Phi^{(0)}\rangle = v/\sqrt{2}$) in the unitary gauge

$$\mathcal{O}^{(5)} \to -\frac{1}{2}v^2\overline{\nu_L^c}\nu_L$$

For three generations $C^{(5)}_{\alpha\beta}$ has family indices

$$(M_\nu)_{\alpha\beta} = C^{(5)}_{\alpha\beta}\frac{v^2}{\Lambda}$$

To obtain $m_\nu < 1\,\text{eV}$ one needs $\Lambda > 10^{14}$ GeV if $C^{(5)} \sim 1$

Most neutrino Majorana masses can be paremetrized by the Weinberg operator, but

Most neutrino Majorana masses can be paremetrized by the Weinberg operator, but

Not always!

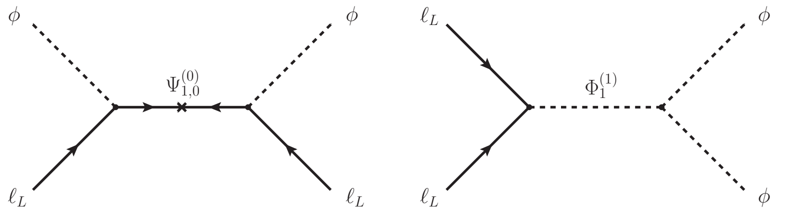
- Not, if the models contain particles lighter than the electroweak scale
  - Light Dirac neutrinos
  - Light sterile fermions
  - Majorons, Axions, ...
  - Additional light scalars
- Not, if the scalar discovered is not exactly the SM Higgs.

Opening the Weinberg Operator at Tree Level

If we want to generate the Weinberg operator at tree level with renormalizable interactions, need to open the effective vertex:

- Fermionic lines do not branch
- Only Yukawa interactions allowed

Only 2 topologies (Notation: $\Psi_T^{(Y)}$,Ψ fermion and Φ scalar)



- $L_L \Phi \to \Psi^{(0)}_{1,0}$
- $L_L L_L \to \Phi^{(1)}_{1,0}$, $\Phi^{(1)}_0$ does not have neutral component and does not give mass at tree level. Besides there is no $\Phi^{(1)}_0 \Phi\Phi$ coupling with only one doublet
- $L_L \Phi^* \to \Psi^{(1)}_{1,0}$, has hypercharge, cannot have Majorana mass
- $L_L \bar{L}_L \to \Phi^{(0)}_{1,0}$ by chirality does not couple to scalar. Besides it does not carry LN

Only three possibilities $\Psi^{(0)}_0, \Phi^{(1)}_1, \Psi^{(0)}_1$

- $\Psi^{(0)}_0$ Fermion singlet with $Y = 0$
- $\Phi^{(1)}_1$ Scalar triplet with $Y = 1$
- $\Psi^{(0)}_1$ Fermion triplet with $Y = 0$

See-saw Type I

As discussed already we consider the case $\nu_R = \Psi_0^{(0)}$

$$\mathscr{L}_Y = -\bar{L}_L Y_e \Phi e_R - \bar{L}_L Y_\nu \tilde{\Phi} \nu_R - \frac{1}{2}\overline{\nu_R^c} M \nu_R + \text{h.c.}$$

We will consider n ν_R fields and only three families of L_L. Then M is $n \times n$ symmetric matrix and Y_ν a $3 \times n$ matrix. After (SSB)

$$\mathscr{L}_{\nu\,\mathrm{mass}} = -\frac{1}{2}\begin{pmatrix}\overline{\nu_\mathrm{L}} & \overline{\nu_\mathrm{R}^\mathrm{c}}\end{pmatrix}\begin{pmatrix}0 & M_\mathrm{D} \\ M_\mathrm{D}^\mathrm{T} & M\end{pmatrix}\begin{pmatrix}\nu_\mathrm{L}^\mathrm{c} \\ \nu_\mathrm{R}\end{pmatrix} + \mathrm{H.c.}\ ,$$

With $M_D = Yv/\sqrt{2}$. If $M \gg M_\mathrm{D}$, diagonalized by blocks

$$M_\nu^\dagger \simeq -M_\mathrm{D}\, M^{-1}\, M_\mathrm{D}^\mathrm{T}$$

$$M_N \simeq M$$

and M_ν will be small. M_ν has, in general, rank $\min(3, n)$

Comparing with the Weinberg operator (identifying Λ with the smaller eigenvalue of M,$M_1 = \Lambda$)

$$C^{(5)\dagger} = -\frac{1}{2} Y_\nu \frac{M_1}{M} Y_\nu^T$$

Small neutrino masses with M very large and/or Y_ν very small
In any case the effects of the ν_R extremelly suppressed:
Difficult to check!
The case $M \lesssim m_Z$ cannot be described by the Weinberg operator, in particular

- $M = 0$ Dirac neutrinos
- $0 < M \ll m_Z$ sterile neutrinos (new mass scales: effects in oscillations)

See-saw Type II

Consider the case $\chi = \Phi_1^{(1)}$ (triplet scalar with $Y = 1$)

$$\chi = \begin{pmatrix} \chi^+/\sqrt{2} & \chi^{++} \\ \chi_0 & -\chi^+/\sqrt{2} \end{pmatrix}$$

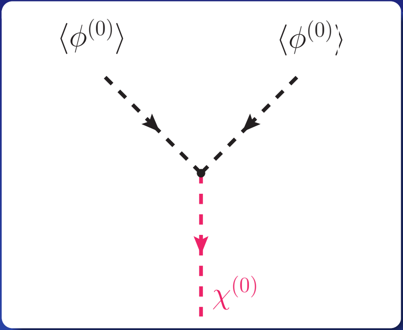
with Lagrangian

$$\mathscr{L}_\chi = -\left(\bar{\tilde{L}}_L Y_\chi \chi L_L + \mathrm{h.c.} \right) - V(\phi, \chi)$$

$$V(\Phi, \chi) = m_\chi^2 \mathrm{Tr}\{\chi \chi^\dagger\} + \left(\mu \tilde{\Phi}^\dagger \chi^\dagger \Phi + \mathrm{H.c.} \right) + \ldots$$

Y_χ is a symmetric matrix. Allow to assign LN -2 to the χ
μ breaks LN explicitly
If $\mu = 0$ and $m_\chi^2 < 0$, LN spontaneously broken
(Goldstone theorem $\Longrightarrow$ Majoron)

If $\mu \neq 0$ a VEV for the triplet is induced even if $m_\chi > 0$



In the limit $m_\chi \gg v$

$$\langle \chi \rangle \equiv v_\chi \simeq -\frac{\mu v^2}{2 m_\chi^2}$$

And therefore

$$M_\nu = -2 Y_\chi v_\chi = Y_\chi \frac{\mu v^2}{m_\chi^2}$$

Comparing with the expression obtained with the Weinberg operator (with $m_\chi = \Lambda$)

$$C^{(5)\dagger} = Y_\chi \frac{\mu}{m_\chi}$$

Triplet scalar VEV's contribute to the ρ parameter

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{\langle\Phi\rangle^2 + 2\langle\chi\rangle^2}{\langle\Phi\rangle^2 + 4\langle\chi\rangle^2} \approx 1 - \frac{2\langle\chi\rangle^2}{\langle\Phi\rangle^2} \approx 1 - \frac{2\mu^2\langle\Phi\rangle^2}{m_\chi^4}$$

$\rho = 1.0008^{+0.0017}_{-0.0010}$ and $\langle\chi\rangle \lesssim 3\,\mathrm{GeV}$.
Can be achieved easily by taking $\mu \ll m_\chi$ and/or $m_\chi \gg \langle\Phi\rangle$.
Possible to adjust the masses of the neutrinos with values of m_χ acccessible at the LHC and Y_χ not too small ($m_\nu \propto Y_\chi(\mu/m_\chi)(\langle\Phi\rangle/m_\chi)$ three-factor suppression)
In this case the model produces a very rich phenomenology tied to the masses of the neutrinos:

- Production at the LHC. Doubly charged scalar easy.
- Processes with LNV in the charged sector $\mu \to e\gamma, \tau \to \mu\gamma, \mu \to 3e, \tau \to 3e, \mu 2e, 2e\mu, 3\mu, \mu \leftrightarrow e$ in nuclei,...

See-saw Type III

Take zero hypercharge fermion triplets $\Sigma = \Psi_1^{(0)}$
(Will need at least 2 to have 2 light massive lneutrinos)
In cartesian components

$$\vec{\Sigma} = (\Sigma_1, \Sigma_2, \Sigma_3)$$

Σ has family indices. Under Lorentz Σ transform like two component right-handed fields. The Lagrangian is

$$\mathscr{L}_\Sigma = i\overline{\vec{\Sigma}}\not{D}\vec{\Sigma} - \left(\frac{1}{2}\overline{\vec{\Sigma}^c}M\vec{\Sigma} + \bar{\ell}Y(\vec{\Sigma}\cdot\vec{\tau})\tilde{\phi} + \mathrm{h.c.}\right)$$

and after SSB

$$\mathscr{L}_{\nu\,\mathrm{mass}} = -\frac{1}{2}\begin{pmatrix}\overline{\nu_L} & \overline{\Sigma_3^c}\end{pmatrix}\begin{pmatrix}0 & M_D \\ M_D^T & M\end{pmatrix}\begin{pmatrix}\nu_L^c \\ \Sigma_3\end{pmatrix} + \mathrm{h.c.}$$

Same results for the mass as in see-saw type I

$$M_\nu^\dagger \simeq -M_\mathrm{D}\, M^{-1}\, M_\mathrm{D}^\mathrm{T}\,.$$

Charged components are combined into a Dirac field

$$E^- = \frac{1}{\sqrt{2}}(\Sigma_1 + i\Sigma_2 + \Sigma_1^c + i\Sigma_2^c)$$

with mass M (present limits give $M \gtrsim 100\,\mathrm{GeV}$)
The E mix with charged leptons giving rise to tree-level FCNC (proportional to $M_D/M \ll 1$, therefore supressed)
$\vec{\Sigma}$ have gauge interactions: much easily produced than ν_R .

- Masses of neutrinos like see-saw type I
- New charged particles, more restricted ($M > 100\,\mathrm{GeV}$)
- Gauge Interactions
- Much richer phenomenology

Variations (Inverse See-saw)

See-saw I very general: adding more sterile fermions one obtains qualitativelly different pictures.

A peculiarity of see-saw I,III: active-sterile mixing is order M_D/M while mass of active is M_D^2/M

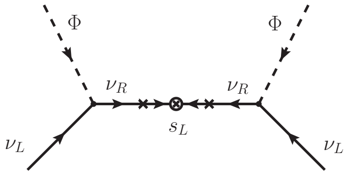
$\Longrightarrow$ Small masses require small mixings!

Interesting variation: $3\nu_L$, $3\nu_R$ And $3s_L$ with mass matrix (in the basis ν_L, ν_R^c, s_L)

$$\begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix}$$

$M_D = Y_\nu \langle \Phi \rangle$ and Y_ν standard Yukawas

M and μ mass matrices of singlets (μ symmetric)



If $\mu = 0$ LN conserved

Spectrum contains 3 massles ν's and 3 heavy Dirac.

If $\mu \ll M_D \ll M$: the 3 heavy split into 2×3 pseudo-Dirac

the 3 light obtain a mass

$$M_\nu^\dagger \approx M_D M^{-1} \mu (M^T)^{-1} M_D^T$$

while the active-sterile mixing remains $\sim M_D/M$:

masses and active-sterile mixing decoupled!

The active neutrino masses can be small and the active-sterile mixing big!

Interesting phenomenology

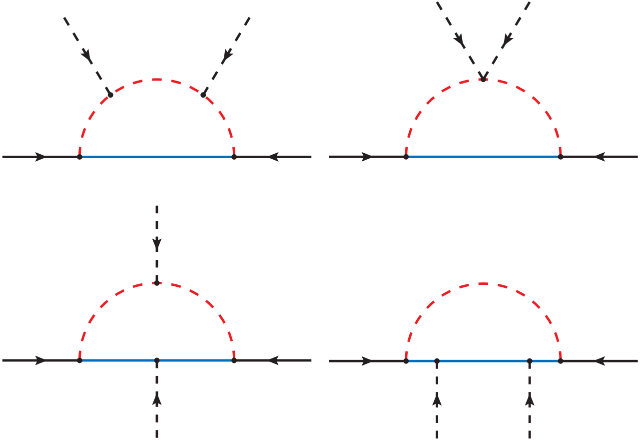
One-loop Majorana Masses

To have small masses from the Weinberg operator one needs $\Lambda \gg \langle\Phi\rangle$ or $C^{(5)} \ll 1$:

- If $\Lambda \gg \langle\Phi\rangle$ difficult to distinguish the different models, the only effect is the generation of the Weinberg operator (higher dimension operators are supressed)
- If $C^{(5)} \ll 1$ only in the Weinberg operator (which does not conserve LN),
  $\Longrightarrow$ New accessible physics not tied to neutrino masses.

But, what is the reason for $C^{(5)} \ll 1$?
If there are no fields with the quantum numbers of the see-saw $\Psi_0^{(0)}, \Phi_1^{(1)}, \Psi_1^{(0)}$ masses cannot be generated at tree level
But, if LN is not conserved the Weinberg operator will be generated at one loop (or more).
This gives a natural justification for $C^{(5)} \ll 1$

Topologies 1PI



Topologies that are not 1PI can be reduced to variations of the see-saw I,II,III.

The Zee Model

Adds to the SM a scalar singlet h^+ and a new doublet Φ'

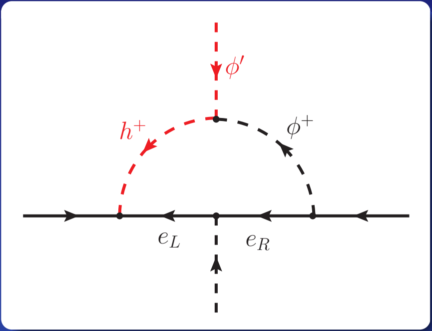
$$h^+ \sim (0,1), \qquad \Phi' \sim (\frac{1}{2},\frac{1}{2})$$

$\mathscr{L} = \mathscr{L}_{\rm SM} + \mathscr{L}_{\rm Zee}$. The relevant terms terms are

$$\mathscr{L}_{\rm Zee} = \overline{\tilde{L}_L} f L_L h^+ + \mu h^+ \Phi^\dagger \tilde{\Phi}' + \cdots$$

- f_{ab} antisymmetric 3×3
- μ can be taken real

The μ coupling only exists for two different doublets ($i\tau_2$ is antisymmetric).
Variations depending on the Yukawa couplings of the new doublet Φ'



A finite M_ν is generated at one loop.
Simplest versions (with only Φ coupling to leptons) give

$$M_\nu \sim \frac{\langle\Phi\rangle\langle\Phi'\rangle\mu}{(4\pi)^2 m_h^2}\left(f\,Y\,Y^\dagger + Y^*\,Y^T f\right)$$

Y charged lepton Yukawas (can be taken diagonal) $m_\ell = Y_{\ell\ell} v$.

$$(M_\nu)_{\alpha\beta} \propto f_{\alpha\beta}(m_\alpha^2 - m_\beta^2)$$

Zero entries in the diagonal because the $f_{\alpha\beta}$ antisymmetry! This structure predicts maximal mixing for the solar angle (in fact too maximal).

- The simplest version "too predictive" and cannot explain the spectrum of masses and mixings
- More complicated versions of the model can fit all the data
- Small m_ν by taking μ small and/or f small and/or m_h large

If μ small (natural):

- h could be seen at the LHC
- Simplest version has no $0\nu\beta\beta$ ($(M_\nu)_{ee} = 0$)
  $0\nu\beta\beta$ linked the departure of maximal mixing in the solar angle.
- Rich LFV phenomenology ($\mu \to e\gamma, \tau \to e\gamma, \ldots$)

The Inert Doublet

Add a fermion singlet (like the ν_R) s_R, and scalar doublet η

$$s_R \sim (0,0) \qquad \eta \sim (\frac{1}{2},\frac{1}{2})$$

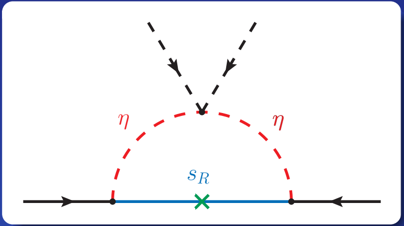
with (in addition to the SM couplings of Φ)

$$\mathscr{L}_Y = -\bar{L}_L Y_s \tilde{\eta}\, s_R - \frac{1}{2}\overline{s_R^c} M s_R - \lambda_5 \left(\Phi^\dagger \eta\right)^2 + \cdots + \text{h.c.}$$

Like see-saw type I if $\eta \leftrightarrow \Phi$ and $s_R \leftrightarrow \nu_R$, but

- η does not take VEV ($m_\eta^2 > 0$), cannot generate masses.
- Discrete symmetry $\eta \to -\eta$ and $s_R \to -s_R$ forbids the couplings s_R- Φ
- LN assigned as ($L(\eta) = -1$, $L(s_R) = 0$) broken explicitly in the potential by the λ_5 term

Φ Weinberg op. cannot be generated at tree level
W. op. generated for the η, but $\langle\eta\rangle = 0$
$\Longrightarrow$ no tree-level m_ν
LN is broken (by λ_5): Majorana masses will be generated



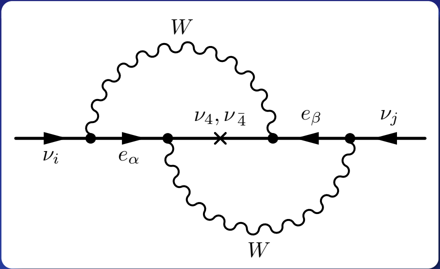
$$M_\nu^\dagger \sim \frac{\lambda_5 \langle\Phi\rangle^2}{(4\pi)^2} Y_s M^{-1} Y_s^T \,, \quad m_\eta \ll M$$

If $m_\eta \gg M$ (change $M^{-1} \to M/m_\eta^2$)
M_ν like in see-saw but with a supression $\lambda_5/(4\pi)^2$

Φ Weinberg op. induced by the η Weinberg op. at one loop!

- Charged particles could be seen in the LHC
- The discrete symmetry makes the lightest of s_R and η's stable: it could be a good dark matter candidate

If one of the neutrinos is Majorana and mixes with the others it will necessarily generate two-loop masses for the others



If ν_4 mass is generated by see-saw I

$$(M_\nu)_{ij}^{(2)} = -\frac{g^4}{m_W^4} m_{\rm R} m_{\rm D}^2 \sum_\alpha V_{\alpha i} V_{\alpha 4} m_\alpha^2 \sum_\beta V_{\beta j} V_{\beta 4} m_\beta^2 I_{\alpha\beta} \,,$$

$I_{\alpha\beta}$ two-loop integral $m_{\alpha,\beta}$ masses charged leptons.
If neutrino masses are Majorana $m_{\rm lightest}$ cannot be zero!

The Zee-Babu Model

Minimal model: add only two scalar singlets to the SM

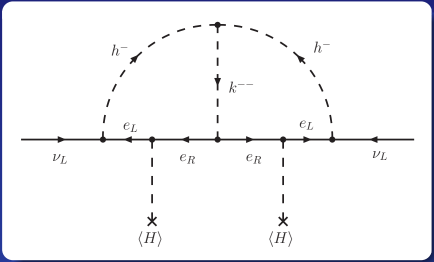
$$h^+ \sim (0,1), \qquad k^{\pm\pm} \sim (0,2)$$

$\mathscr{L} = \mathscr{L}_{\rm SM} + \mathscr{L}_{\rm ZB}$. The relevant terms are

$$\mathscr{L}_{ZB} = \overline{\tilde{L}_L} f L_L h^+ + \overline{e_R^c} g e_R k^{++} + \mu (h^-)^2 k^{++} + \cdots$$

- f_{ab} Antisymmetric 3×3
- g_{ab} Symmetric 3×3
- μ Can take real

f coupling allows to assign LN 2 to the h^-
g coupling allows to assign LN 2 to the k^{--}
μ coupling breaks LN in 2 units $\Longrightarrow$ Majorana neutrino masses



Mass arises at two loops, calculable and Finite!

$$M_\nu = \frac{\langle \Phi \rangle^2 \mu}{48\pi^2 M^2} \tilde{I} f\, Y g^\dagger Y^T f^T \,, \quad M \equiv \max(m_h, m_k)\,, \quad \tilde{I} \approx 1$$

Y charged lepton Yukawa coupling.$m_\ell = Y_{\ell\ell} v$.

$\det(\mathscr{M}_\nu) = 0$: one of the ν's is massless

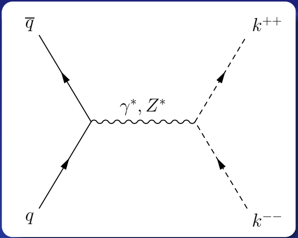
The corresponding eigenvector is fixed:

$f_{e\tau}/f_{\mu\tau}, f_{e\mu}/f_{\mu\tau}$ fixed in terms of mixings

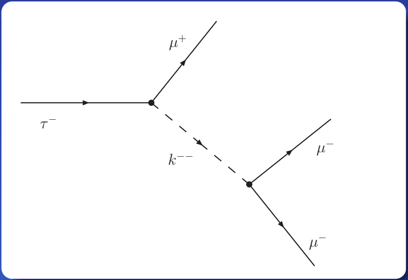
Main features

k^{++} easily produced by Drell-Yan

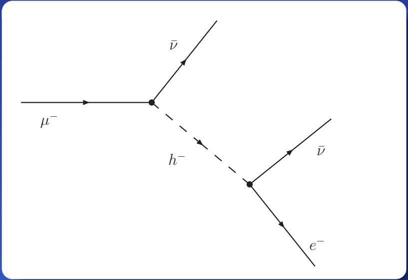
Decays mainly to leptons (if $m_k > 2m_h$ also to $2h^+$)



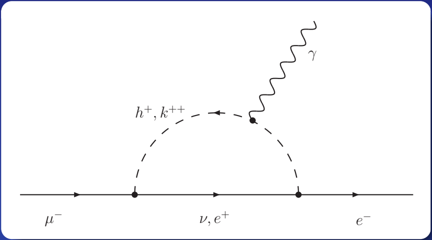
$\ell_a^- \to \ell_b^+ \ell_c^- \ell_d^-$: limits on g_{ab}



$\ell_a \to \ell_b \nu \bar{\nu}$: limits on f_{ab}



$\ell_a^- \to \ell_b^- \gamma$: constraints g_{ab} and f_{ab}



Testable Model!

- Predictions on ν masses and mixings
- LHC (now $m_k > 200$–$400\ \mathrm{GeV}$ at 95% CL)
- Many LFV process which should be large if $m_{k,h}$ are discovered

Models with Light Sterile Neutrinos

The Weinberg Op. parametrizes a large class of models but!
Does not parametrize Dirac neutrinos!
Does not parametrize light sterile neutrinos
Minimum model $3\nu_L$ and $2\nu_R$:
If $\Delta L = 0$ (no ν_R Majorana mass):

- 1 massles ν_L
- 2 massive Dirac neutrinos

if $\Delta L \neq 0$ (ν_R with Majorana masses)

- 1 massless ν_L (will get a two-loop Majorana mass)
- 4 massive Majorana neutrinos (4 mass differences)

Scenario not excluded at present, could explain LNSD-MiniB, reactor and Gallium anomalies and N_s in cosmology!

Majoron Models

In models with Majorana masses there is always a dimensionful parameter associated with the breaking of LN
I can be promoted to a complex field:
LN spontaneously broken . Example:

$$\overline{\nu_R^c} M \nu_R \to h\sigma \overline{\nu_R^c} \nu_R \quad \sigma \to \frac{1}{\sqrt{2}}(v_\sigma + \rho) e^{i\theta/v_\sigma}$$

θ Goldstone boson "Majoron". It survives at low energies
Majoron-neutrino couplings (Goldstone theorem)

$$\frac{m_\nu}{v_\sigma} \overline{\nu} \gamma_5 \nu \theta$$

Possibility of decays and annihilations

$$\nu \to \nu' \theta \,, \quad \nu\nu \to \theta\theta$$

Could avoid cosmological bounds (modification of primordial abundances, CMB, LSS, BBN)

Extensions of the SM

Apart from the neutrino mass problem, the SM has a series of unsatisfactory aspects:

- Only 3 generations? Why?
- Hierarchy problem
- Several gauge coupling constants
- Flavour problem (hierarchies of masses and mixings)
- It does not includes gravity
- It can not explain the baryon asymmetry of the universe
- It does not have a good dark matter candidate

Several theories have been proposed to solve these problems. They have to incorporate neutrino masses!
The mechanisms will be generalizations of the mechanisms discussed above (after all these theories should behave like the SM at low energies)

Grand Unification (GUT)

Unification based in SU(5) is like the SM
(ν_R singlet under SU(5))
Unification based in SO(10) very interesting:

- ν_R member of the 16 representation which fits a complete family of the SM

$$\overbrace{3}^{N_c}\left(\overbrace{2}^{Q_L}+\overbrace{1}^{d_R}+\overbrace{1}^{u_R}\right)+\overbrace{2}^{L_L}+\overbrace{1}^{e_R}+\overbrace{1}^{\nu_R}=16$$

- Automatic cancellation of anomalies
- $B-L$ is a generator of the group, necessarily broken spontaneously (Majorana neutrino masses)
- GUT relations between the Yukawas of the top and of the neutrino

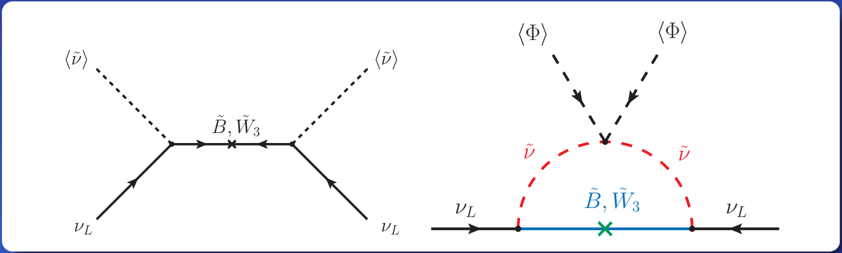
Neutrinos in SO(10) were the inspiration of the see-saw!

Supersymmetry

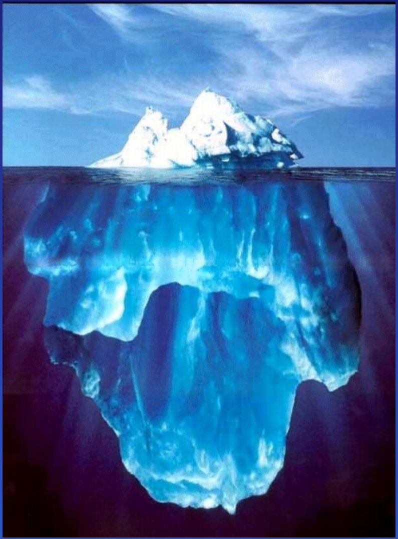
In the Minimal Supersymmetric Standard Model (MSSM) LN conservation imposed “by hand” when imposing the conservation of the parity R, $(-)^{3B+L+2j}$.
Many terms in the superpotential could break explicitly L

$$Le^c\Phi_1, \quad LLe^c \quad , L\Phi_2 \quad , LQd^c$$

Masses of neutrinos even without ν_R!
SUSY contains $\tilde{B}, \tilde{W}$ which just have the quantum numbers of the see-saw I,III and and also Majorana masses!



Extra Dimensions



In extra dims couplings can be small because we only see "The tip of the iceberg". In 5 dims Yukawa couplings are not dimensionless $[Y_{d=5}] = M^{-1/2}$. (M Natural scale of interactions in 5 dims). Then, size of Yukawas in 4 dims

$$Y_{d=4} \sim \sqrt{M_c} Y_{d=5} \sim \sqrt{\frac{M_c}{M}} \ll 1$$

In 5 dims γ_5 is γ_5, (no chirality). Neutrinos are naturally Dirac with small masses.