$N_\nu^{\rm eff}$ beyond the instantaneous approximation

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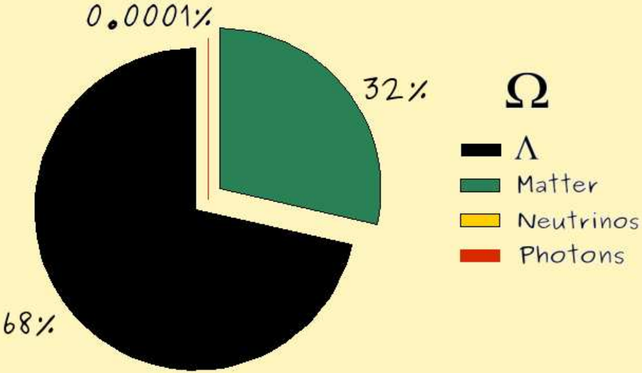
Components of the Universe

(Current time)

- $T_\gamma = 2.73\,\mathrm{K}$

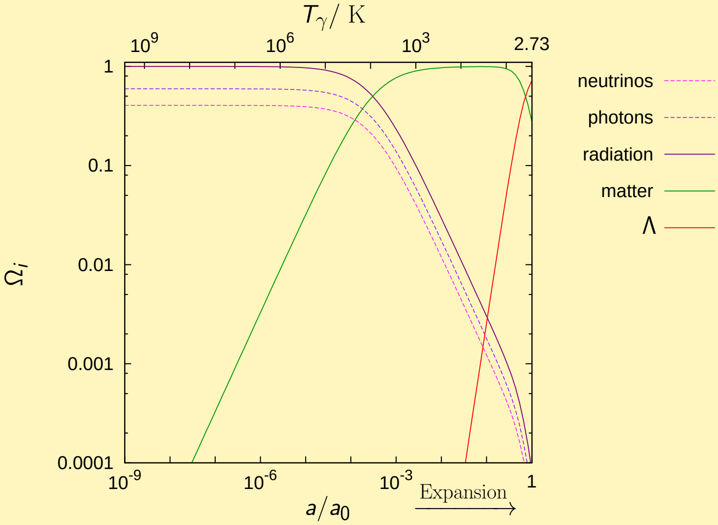
$\Omega_i = \rho_i/\rho_{\rm cr}$

Flat Universe: $\sum_i \Omega_i = 1$



Components of the Universe

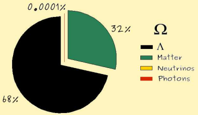
(Evolution)

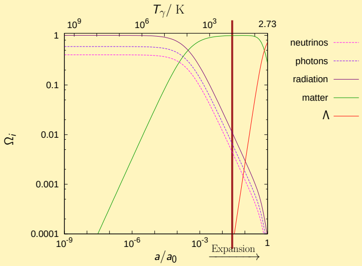


Components of the Universe

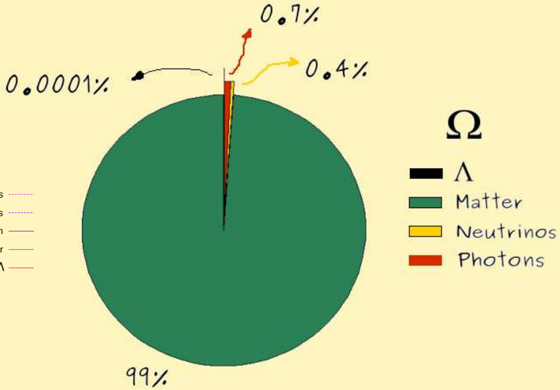
(Matter domination)

- $T_\gamma = 2.73\,\mathrm{K}$





- $T_\gamma = 10^2\,\mathrm{K}$



Components of the Universe

(Radiation domination)

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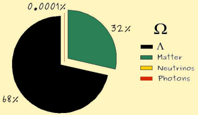
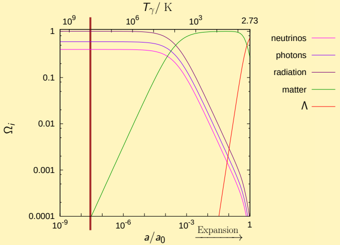
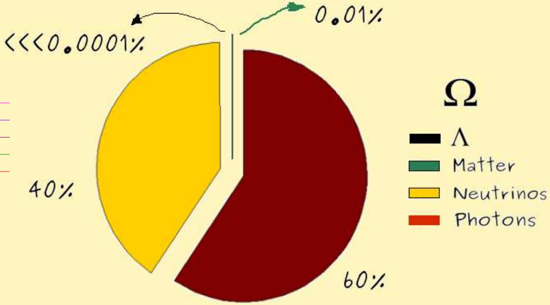
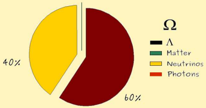




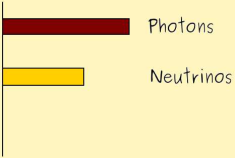
- $T_\gamma = 10^8\,\mathrm{K}\;\;(8.6\,\mathrm{keV})$



Radiation energy density in the radiation era



$$\rho_R = \rho_\gamma + \rho_\nu$$



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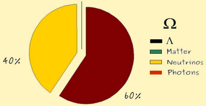
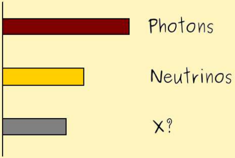
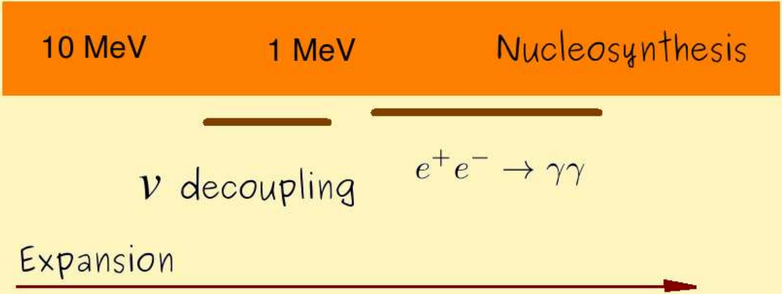


$$\rho_R = \rho_\gamma + \rho_\nu + \rho_X$$



ν decoupling and $e^{\pm}$ annihilations



Effect of $e^{\pm}$ annihilations on T_γ

- Relativistic energy densities

$$\rho_\gamma = \frac{\pi^2}{15} T_\gamma^4 \qquad\qquad \rho_\nu^0 = \frac{7}{8}\frac{\pi^2}{15} T_\nu^4$$

$$\longrightarrow \quad \rho_\nu^0 = \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 \rho_\gamma$$

- Temperature difference after $e^{\pm}$ annihilations

$$\frac{T_\gamma^f}{T_\nu^f} = \left(\frac{11}{4}\right)^{1/3} \simeq 1.40102$$

Effective number of neutrinos

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$$\rho_R = \left(1 + N_\nu^{\mathrm{eff}}\,\frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right)\rho_\gamma$$

N_ν^{eff} accounts for all contributions to ρ_R different from ρ_γ

$$N_\nu^{\mathrm{eff}} \equiv \left(\frac{\rho_R - \rho_\gamma}{\rho_{\nu_{\mathrm{eq}}}}\right)\left(\frac{\rho_\gamma^0}{\rho_\gamma}\right)$$

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Not to be confused with the number of neutrino generations

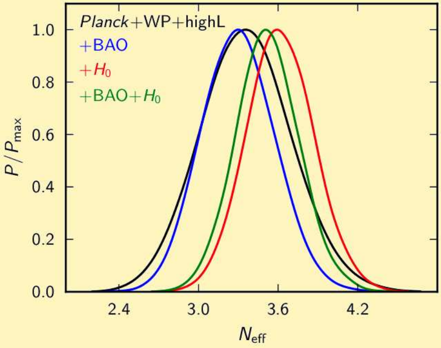
Experimental value

- (CMB alone) Planck+WP+high-l

$$N_\nu^{\rm eff} = 3.36^{+0.68}_{-0.64} \quad (95\,\%\ \mathrm{C.L.})$$

- Planck+WP+high-l $+H_0$+BAO

$$N_\nu^{\rm eff} = 3.52^{+0.48}_{-0.45} \quad (95\,\%\ \mathrm{C.L.})$$



(P.A.R. Ade et al. Planck 2013 results)

Instantaneous approximation

- Neutrinos decouple before $e^{\pm}$ annihilation

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- They don't participate in the annihilation

Instantaneous approximation

$$\rho_\nu^0 = \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 \rho_\gamma$$

$$\rho_R = \left(1 + N_\nu^{\rm eff}\frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\right)\rho_\gamma$$

$$\rho_\nu = N_\nu \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \rho_\gamma$$

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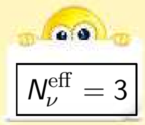
$$\rho_\nu = 3\frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \rho_\gamma$$



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- f_ν deviation from equilibrium

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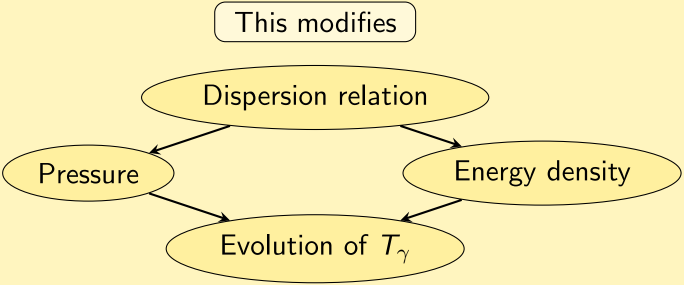

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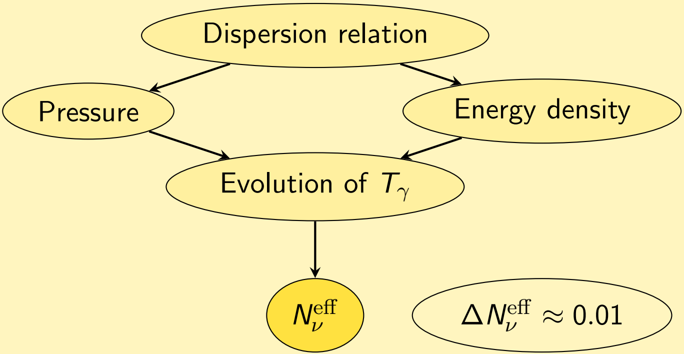

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Annihilation	$e^+ + e^- \to \nu + \bar{\nu}$
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- $\frac{T_\gamma^f}{T_{\gamma_0}^f} < \left(\frac{11}{4}\right)^{1/3}, \qquad f_\nu \neq f_{\rm eq}$

Effect of contributions

Contribution	$T_\gamma^f / T_{\gamma_0}^f$	$\delta\rho_{\nu_e}$	$\delta\rho_{\nu_{\mu,\tau}}$	$N_\nu^{\rm eff}$
Finite temperature QED	1.3998	0	0	3.011
Annihilation	1.3993	0.933 %	0.305 %	3.030
Scattering	1.4006	0.196 %	0.080 %	3.007
ν self-interaction	1.40098	0.0005 %	0.0005 %	3.00037
None (instantaneous)	1.40102	0	0	3

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Processes (+ QED)	$T_\gamma^f / T_{\gamma_0}^f$	$\delta\rho_{\nu_e}$	$\delta\rho_{\nu_{\mu,\tau}}$	$N_\nu^{\rm eff}$
Annihilation	1.3981	0.925 %	0.302 %	3.041
+ scattering	1.3980	0.996 %	0.331 %	3.043
+ ν self-interaction	**1.3979**	**0.915 %**	**0.390 %**	**3.044**

Conclusions

- Deviation from equilibrium of ρ_ν

$\delta\rho_{\nu_e} \approx 1\,\%$, $\delta\rho_{\nu_{\mu,\tau}} \approx 0.4\,\%$ $(\delta\rho{=}(\rho{-}\rho_{\rm eq})/\rho_{\rm eq})$

- Deviation from the instantaneous ν decoupling approx.

$$\Delta N_\nu^{\rm eff} = 0.044 \ \text{(without oscillations)}$$

in agreement with *G. Mangano et al. 2005 Nucl.Phys. B, 729, 221* ($\Delta N_\nu^{\rm eff} = 0.046$)

- Agreement with $N_\nu^{\rm eff}|_{\rm exp}$ (only CMB) within the 2σ region (some tension if CMB+H_0+BAO)

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