3. Electroweak Phenomenology

- Inputs
- $Z \to f\,\bar{f}$, $W \to f_1\,\bar{f}_2$, Asymmetries
- Sensitivity to Higher Scales
- Gauge Self-interactions
- Higgs Search
- Flavour Dynamics
- CP Violation

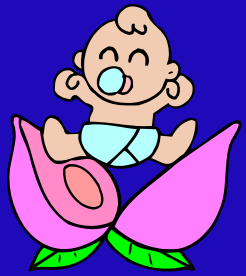
Standard Model Parameters

QCD: $\alpha_S(M_Z)$

1

EW Gauge / Scalar Sector: **4**

$$g\, ,\, g'\, ,\, \mu^2\, ,\, \lambda \quad \Longleftrightarrow \quad \alpha\, ,\, \theta_W\, ,\, M_W\, ,\, M_H \quad \Longleftrightarrow \quad \alpha\, ,\, G_F\, ,\, M_Z\, ,\, M_H$$



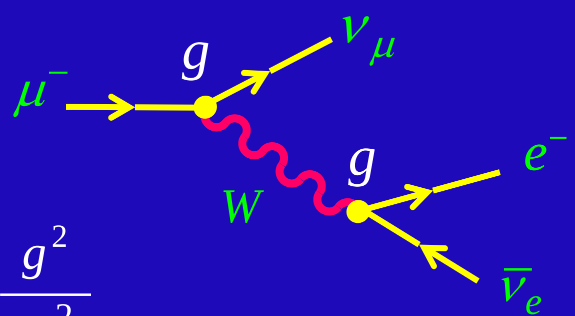
INPUTS

$$G_F = \left(1.166\,3788 \pm 0.000\,000\,7\right) \times 10^{-5}\ \text{GeV}^{-2}$$

$$\alpha^{-1} = 137.035\,999\,084 \pm 0.000\,000\,051$$

$$M_Z = \left(91.1875 \pm 0.0021\right)\ \text{GeV}$$

$$\alpha^{-1}\left(M_Z^2\right) = 128.95 \pm 0.05$$



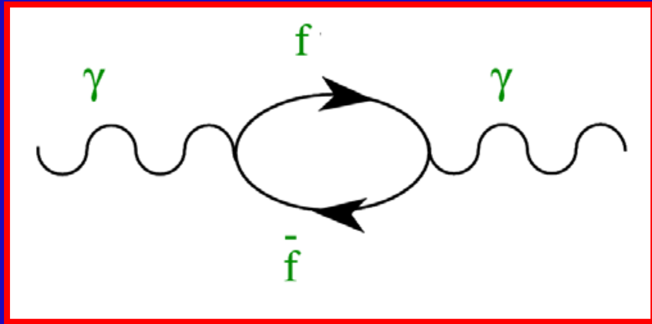
$$M_W^2 \sin^2\theta_W = \frac{\pi\alpha}{\sqrt{2}\,G_F}$$

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

$$G_F \sim \frac{g^2}{M_W^2}$$

$$M_W = 80.94\ \text{GeV} \qquad (79.96) \qquad \left[\text{Exp: } 80.399 \pm 0.023\right]$$

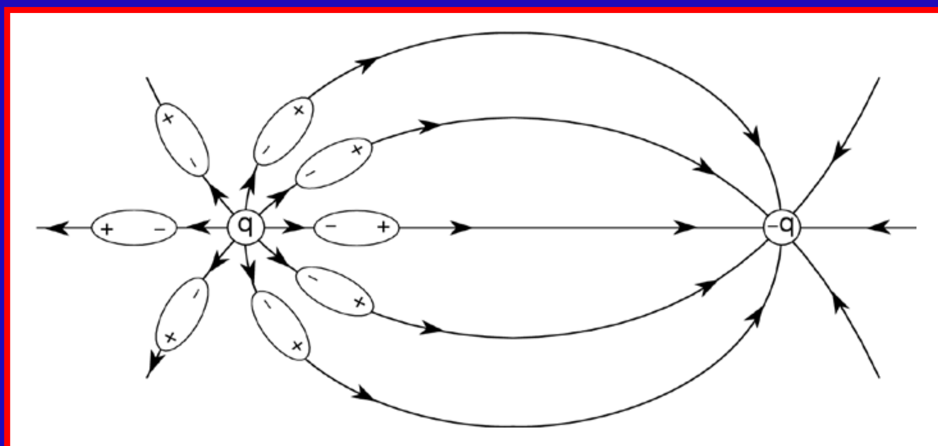
$$\sin^2\theta_W = 0.212 \qquad (0.231)$$



VACUUM POLARIZATION

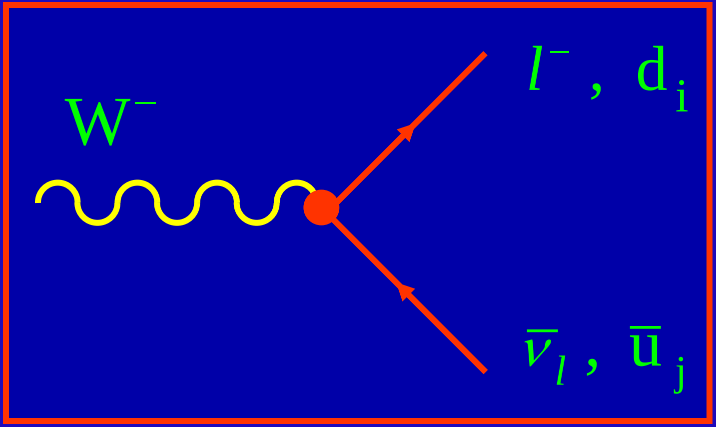
The Photon Couples to *Virtual* $f\,\bar{f}$ *Pairs*

Vacuum ⟷ **Polarized Dielectric Medium**



$$\alpha^{-1} = \alpha(m_e^2)^{-1} = 137.035999084\;(51) \quad ; \quad \alpha\,(M_Z^2)^{-1} = 128.95 \pm 0.05$$

($l^-\, l^+$ and $q\,\bar{q}$ contributions included)



$$W^- \to e^- \bar{\nu}_e \,,\; \mu^- \bar{\nu}_\mu \,,\; \tau^- \bar{\nu}_\tau \,,\; d' \,\bar{u} \,,\; s' \,\bar{c}$$

$$\bar{u}_j = \bar{u}\,,\bar{c} \qquad ; \qquad \begin{pmatrix} d' \\ s' \end{pmatrix} \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$\mathrm{Br}\left(W^- \to l^- \bar{\nu}_l\right) \equiv \frac{\Gamma\left(W^- \to l^- \bar{\nu}_l\right)}{\Gamma\left(W^- \to \mathrm{all}\right)} = \frac{1}{3 + 2\, N_C} = 11.1\%$$

QCD: $$N_C \left\{ 1 + \frac{\alpha_s(M_Z)}{\pi} \right\} \approx 3.115 \quad \Rightarrow \quad \mathrm{Br}\left(W^- \to l^- \bar{\nu}_l\right) \approx 10.8\%$$

Experiment:

$$\mathrm{Br}\left(W^- \to e^- \bar{\nu}_e\right) = \left(10.65 \pm 0.17\right)\%$$

$$\mathrm{Br}\left(W^- \to \mu^- \bar{\nu}_\mu\right) = \left(10.59 \pm 0.15\right)\%$$

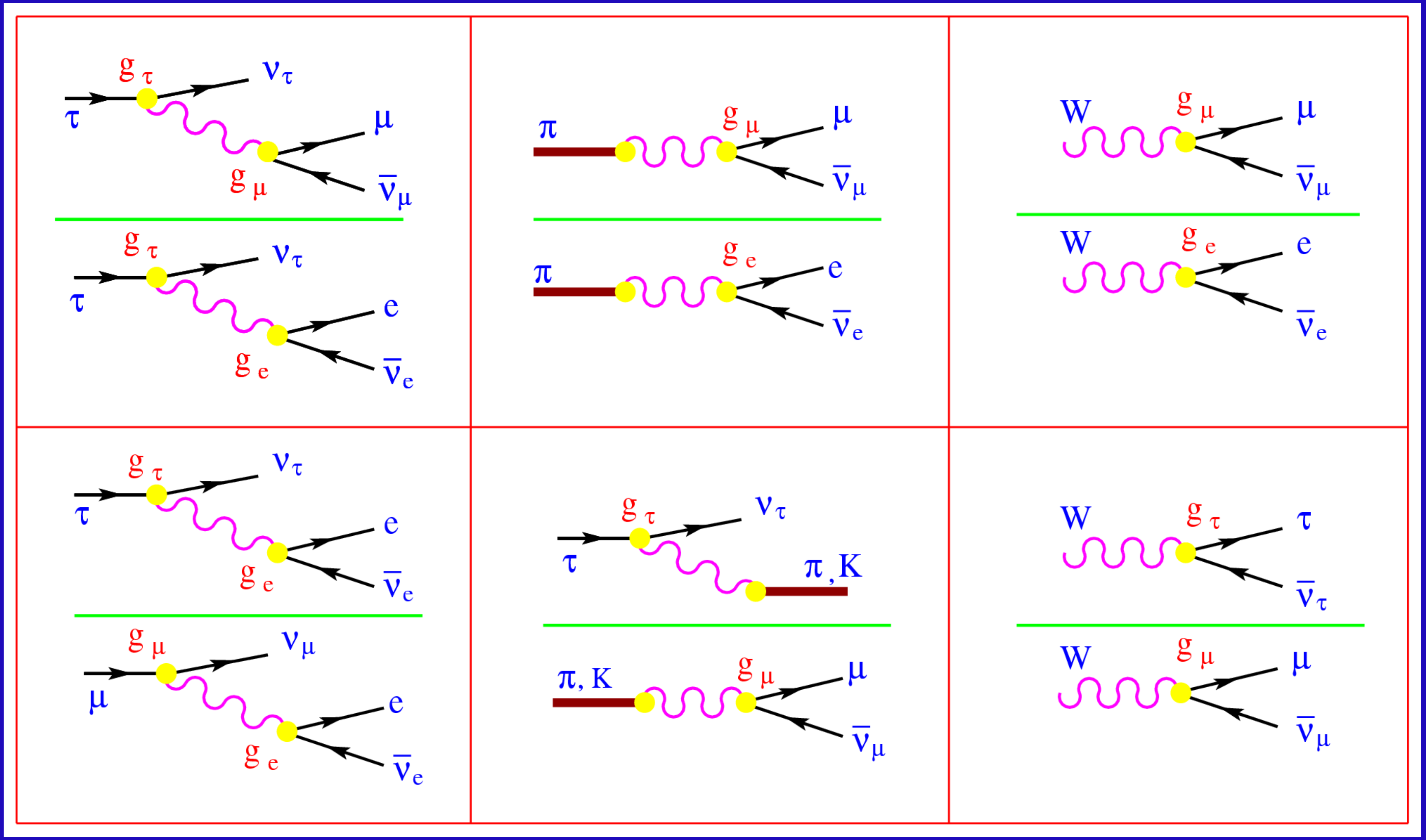
$$\mathrm{Br}\left(W^- \to \tau^- \bar{\nu}_\tau\right) = \left(11.44 \pm 0.22\right)\%$$

Universal $W\, l\, \bar{\nu}_l$ **Couplings**

LEPTON UNIVERSALITY

$$\frac{g_\mu}{g_e}$$

$$\frac{g_\tau}{g_\mu}$$



CHARGED CURRENT UNIVERSALITY

$$\left| g_\mu / g_e \right|$$

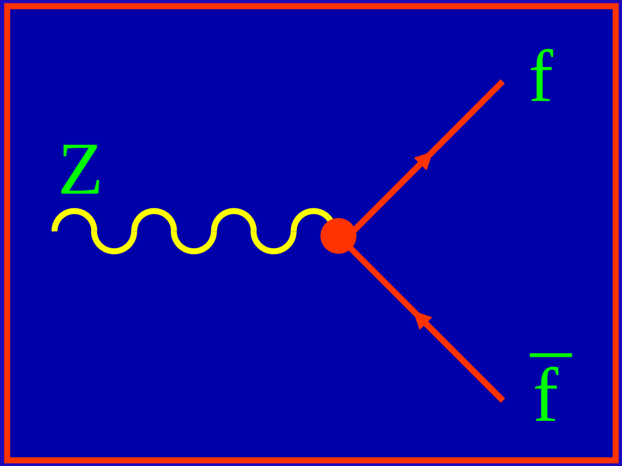
$B_{\tau\to\mu} / B_{\tau\to e}$	1.0018 ± 0.0014
$B_{\pi\to\mu} / B_{\pi\to e}$	1.0021 ± 0.0016
$B_{K\to\mu} / B_{K\to e}$	0.996 ± 0.006
$B_{K\to\pi\mu} / B_{K\to\pi e}$	1.001 ± 0.002
$B_{W\to\mu} / B_{W\to e}$	0.997 ± 0.010

$$\left| g_\tau / g_\mu \right|$$

$B_{\tau\to e} \;\; \tau_\mu / \tau_\tau$	1.0006 ± 0.0022
$\Gamma_{\tau\to\pi} / \Gamma_{\pi\to\mu}$	0.991 ± 0.004
$\Gamma_{\tau\to K} / \Gamma_{K\to\mu}$	0.982 ± 0.008
$B_{W\to\tau} / B_{W\to\mu}$	1.039 ± 0.013

$$\left| g_\tau / g_e \right|$$

$B_{\tau\to\mu} \;\; \tau_\mu / \tau_\tau$	1.0005 ± 0.0023
$B_{W\to\tau} / B_{W\to e}$	1.036 ± 0.014



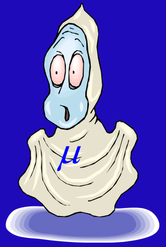
$$Z \to l^- l^+ , \; \nu_l \bar{\nu}_l$$

$$\Gamma(Z \to l \bar{l}) \propto \left(|v_l|^2 + |a_l|^2 \right)$$

$$\frac{\Gamma_{inv}}{\Gamma_{ll}} \equiv \frac{\Gamma(Z \to \text{invisible})}{Z \to l^+ l^-} = N_\nu \frac{\Gamma(Z \to \nu_l \bar{\nu}_l)}{\Gamma(Z \to l^+ l^-)} = N_\nu \frac{2}{(1 - 4\sin^2\theta_W)^2 + 1} = 1.955 \; N_\nu$$

(1.989)

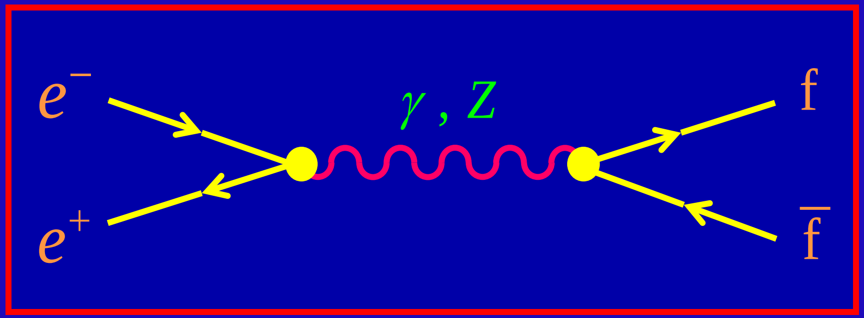
Experiment:

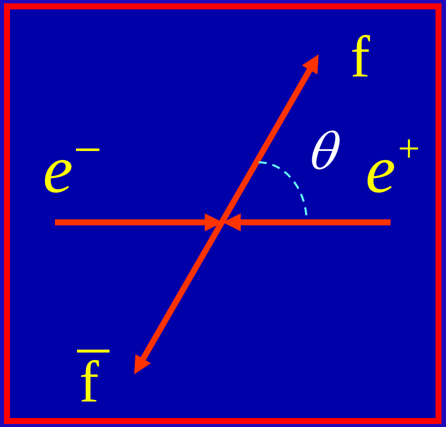


$$\frac{\Gamma_{inv}}{\Gamma_{ll}} = 5.942 \pm 0.016 \quad \Rightarrow \quad N_\nu = 3.04 \qquad (2.99)$$

$$N_\nu = 2.9840 \pm 0.0082$$

$e^+ e^- \to \gamma\,,\, Z \to f\,\bar{f}$





$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8s}\, N_f \left\{ A\,(1+\cos^2\theta) + B\cos\theta - h_f\left[C\,(1+\cos^2\theta) + D\cos\theta\right]\right\}$$

$$N_l = 1 \quad ; \quad N_q = N_C\left\{1 + \frac{\alpha_s(M_Z^2)}{\pi} + \cdots\right\} \quad ; \quad h_f = \pm 1$$

$$A = 1 + 2\, v_e v_f\, \mathrm{Re}(\chi) + (v_e^2 + a_e^2)(v_f^2 + a_f^2)\, |\chi|^2$$

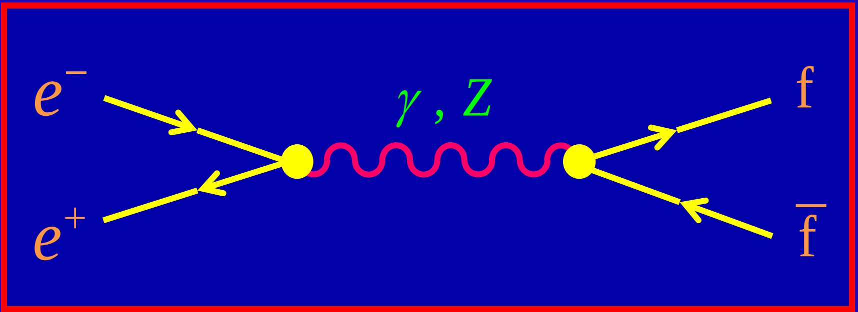
$$B = 4\, a_e a_f\, \mathrm{Re}(\chi) + 8\, v_e a_e v_f a_f\, |\chi|^2$$

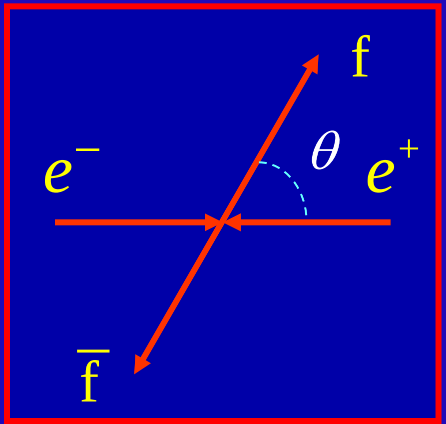
$$C = 2\, v_e a_f\, \mathrm{Re}(\chi) + 2\,(v_e^2 + a_e^2)\, v_f a_f\, |\chi|^2$$

$$D = 4\, a_e v_f\, \mathrm{Re}(\chi) + 4\, v_e a_e\, (v_f^2 + a_f^2)\, |\chi|^2$$

$$\chi = \frac{G_F M_Z^2}{2\sqrt{2}\,\pi\alpha}\, \frac{s}{s - M_Z^2 + i\, s\,\Gamma_Z / M_Z}$$

$$e^+ e^- \to \gamma\,,\, Z \to f\,\bar{f}$$





$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8s}\, N_f \left\{ A\,(1+\cos^2\theta) + B\cos\theta - h_f \left[C\,(1+\cos^2\theta) + D\cos\theta \right] \right\}$$

$$\mathcal{A}_{FB}(s) \equiv \frac{N_F - N_B}{N_F + N_B} = \frac{3}{8}\,\frac{B}{A}$$

$$\mathcal{A}_{Pol}(s) \equiv \frac{\sigma^{(h_f=+1)} - \sigma^{(h_f=-1)}}{\sigma^{(h_f=+1)} + \sigma^{(h_f=-1)}} = -\frac{C}{A} \qquad ; \qquad \sigma = \frac{4\pi\alpha^2}{3s}\, N_f\, A$$

$$\mathcal{A}_{FB}^{Pol}(s) \equiv \frac{N_F^{(+1)} - N_F^{(-1)} - N_B^{(+1)} + N_B^{(-1)}}{N_F^{(+1)} + N_F^{(-1)} + N_B^{(+1)} + N_B^{(-1)}} = -\frac{3}{8}\,\frac{D}{A}$$

Z Peak $\quad (s = M_Z^2)$

$$\sigma = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \, \Gamma_f}{\Gamma_Z^2} \quad ; \quad \Gamma_f \equiv \Gamma(Z \to f\,\bar{f})$$

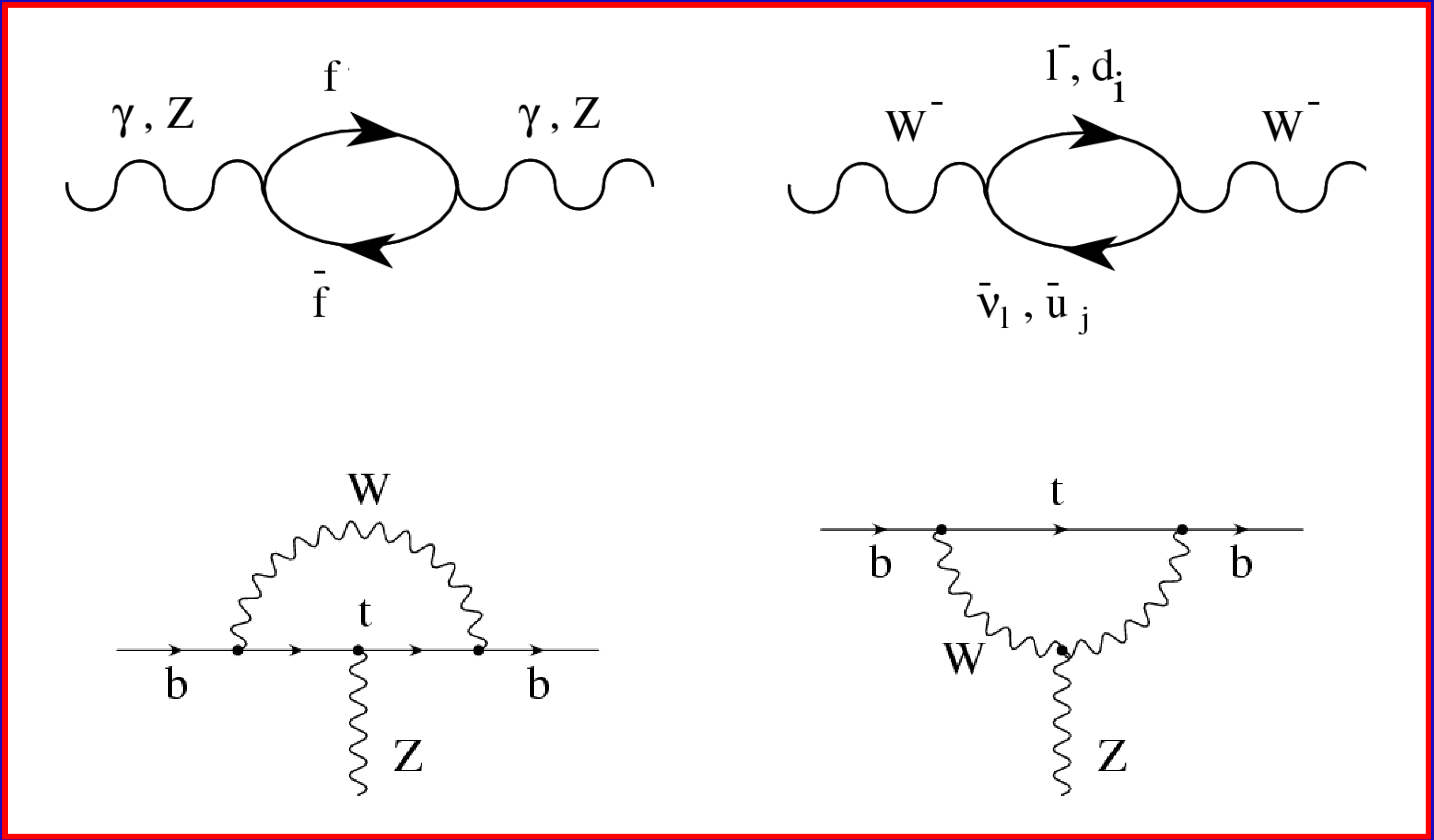
$$\mathcal{A}_{FB}(s) = \frac{3}{4} \mathcal{P}_e \, \mathcal{P}_f \quad ; \quad \mathcal{A}_{Pol}(s) = \mathcal{P}_f \quad ; \quad \mathcal{A}_{FB}^{Pol}(s) = \frac{3}{4} \mathcal{P}_e$$

$$\mathcal{A}_{LR}(s) \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\mathcal{P}_e \quad ; \quad \mathcal{A}_{FB}^{LR}(s) = -\frac{3}{4} \mathcal{P}_f$$

Final Polarization $\mathcal{P}_f \equiv -A_f = \dfrac{-2\, v_f \, a_f}{|v_f|^2 + |a_f|^2}$ **Only Available for** $f = \tau$

$|v_l| = \frac{1}{2} \left| -1 + 4\sin^2\theta \right| \ll 1$ ➡ $\mathcal{P}_l$ **Sensitive to Higher Order Corrections**

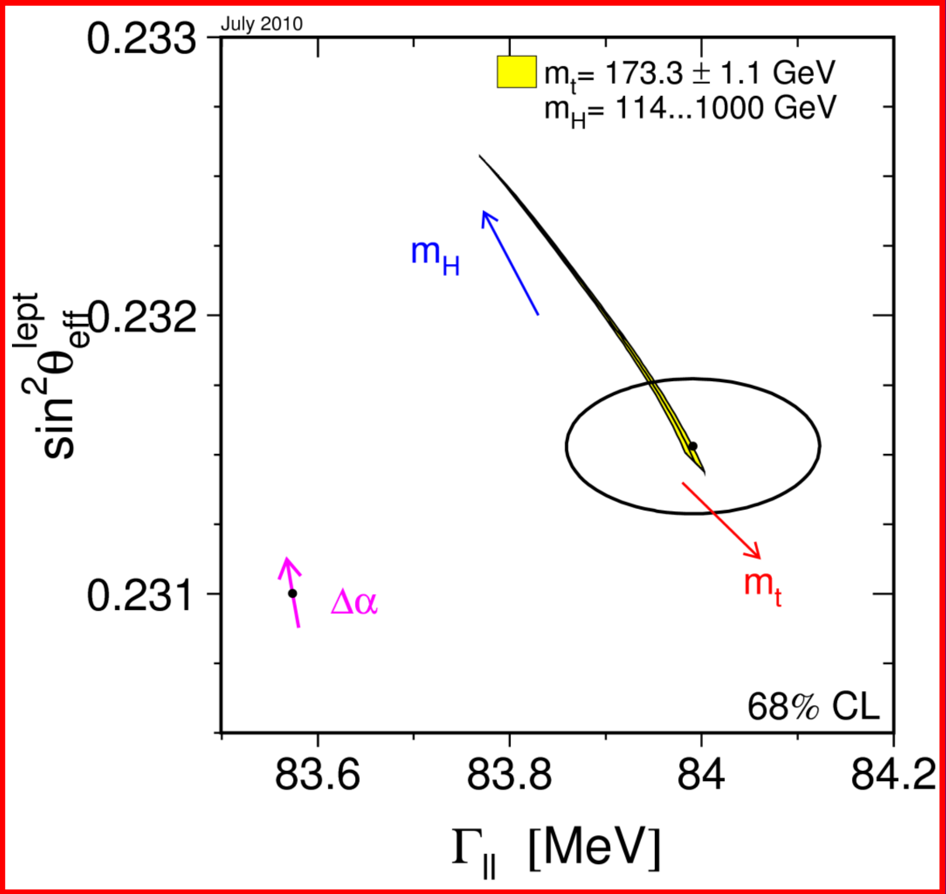
Higher Order Corrections

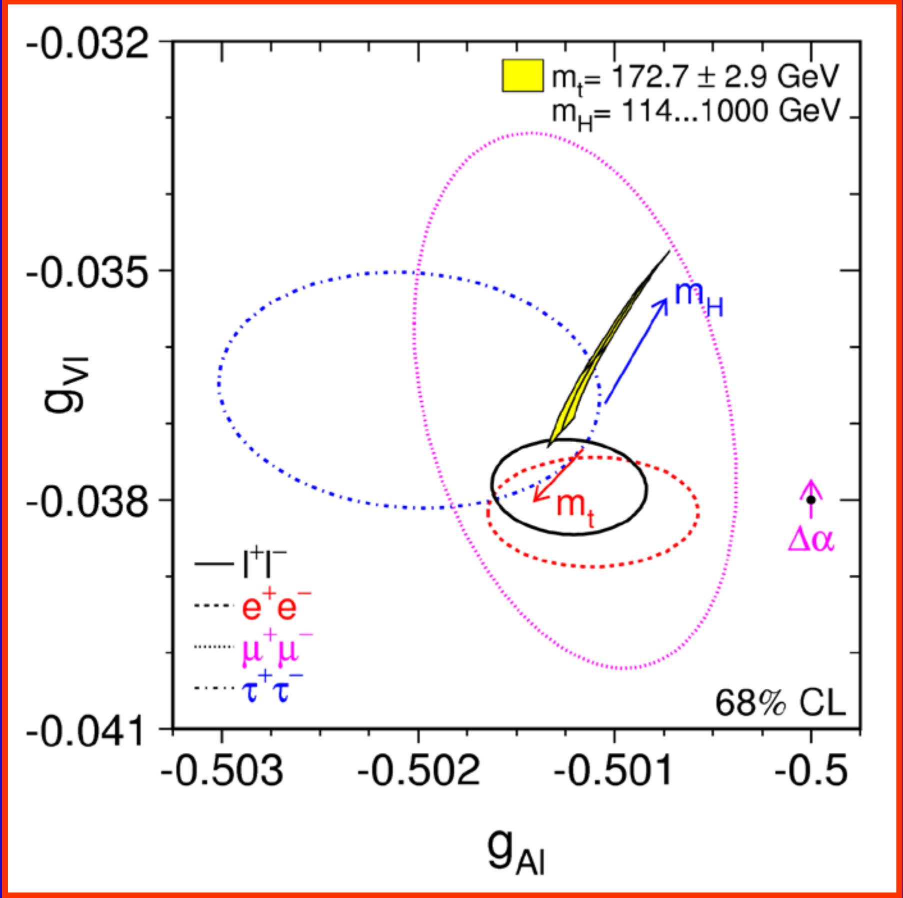


Sensitive to Heavier Particles: TOP , HIGGS

Evidence of Electroweak Corrections

July 2010 LEPEWWG September 2005



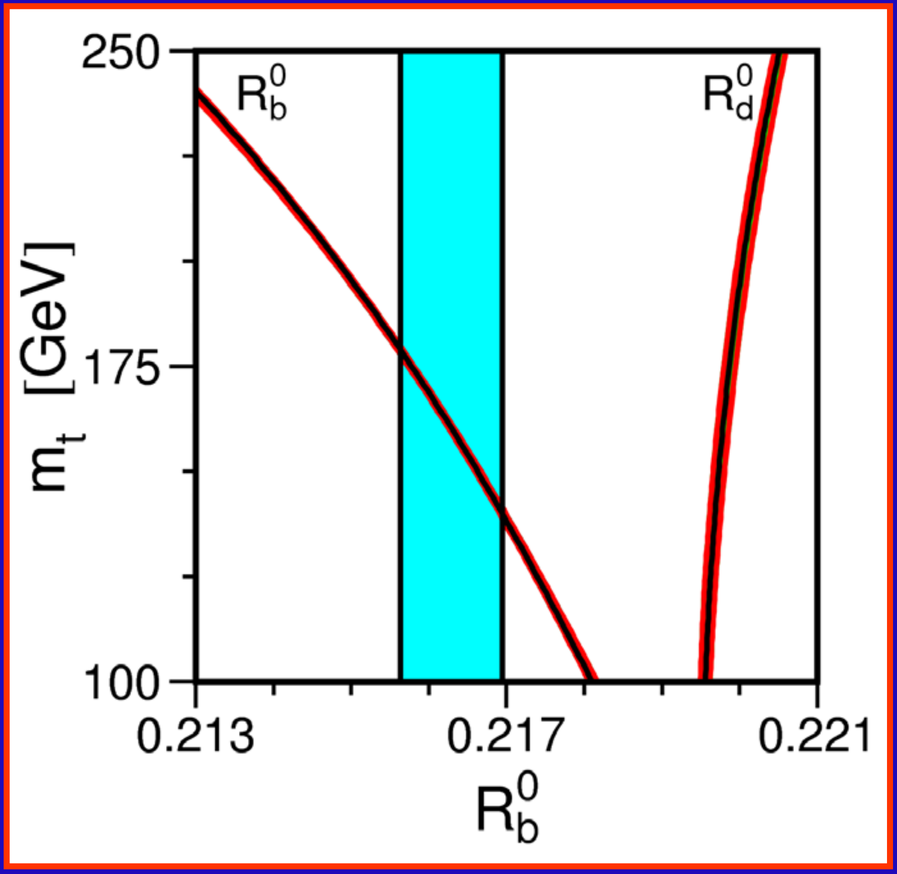


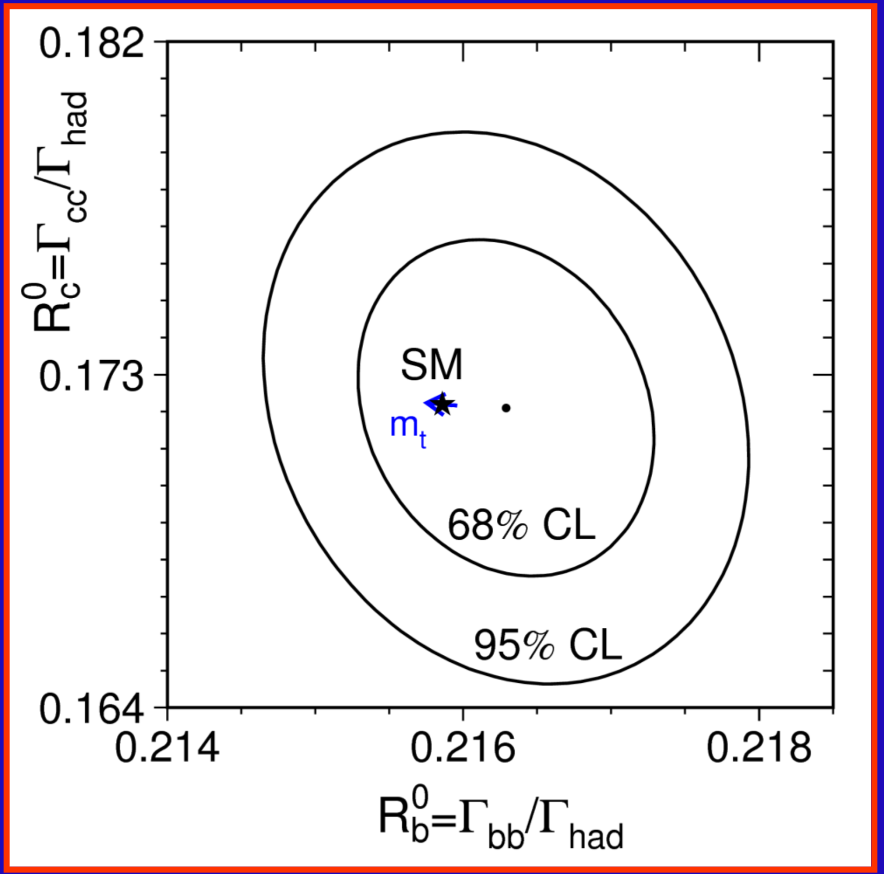
$\alpha(M_Z^2)^{-1} = 128.95 \pm 0.05$

Low Values of M_H Preferred

$R_b \equiv \Gamma(Z \to b\bar{b}) / \Gamma(Z \to \text{hadrons})$

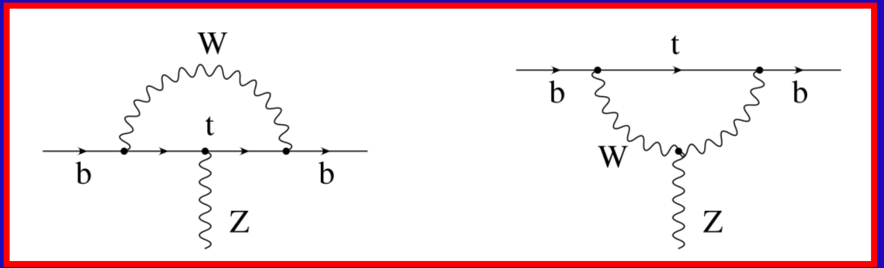
LEPEWWG September 2005

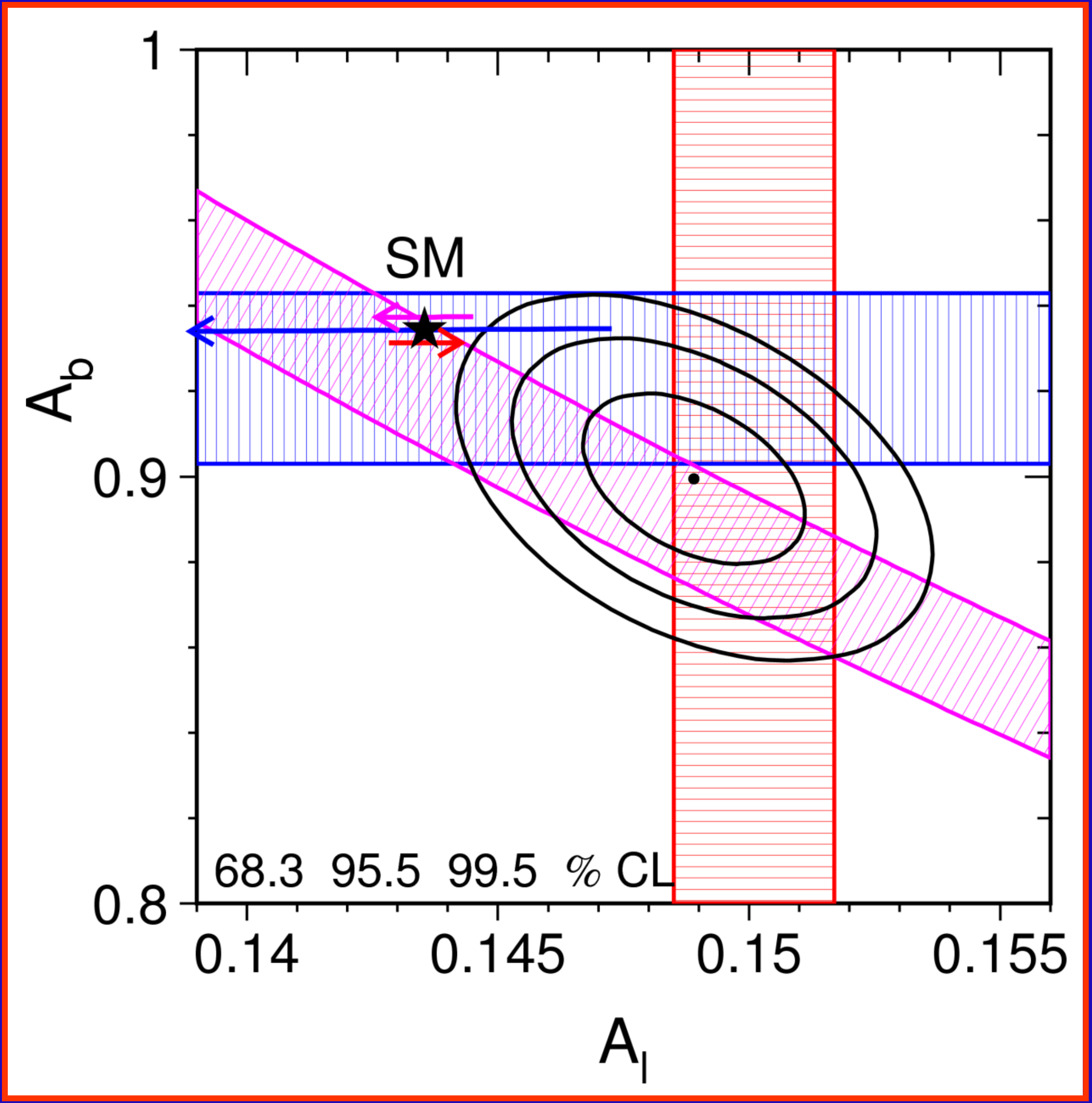




Bernabéu-Pich-Santamaría 1988

- Measurement
- $\Delta\alpha^{(5)}_{had} = 0.02758 \pm 0.00035$
- $\alpha_s = 0.118 \pm 0.003$
- $m_H = 114...1000$ GeV





LEPEWWG

September 2005

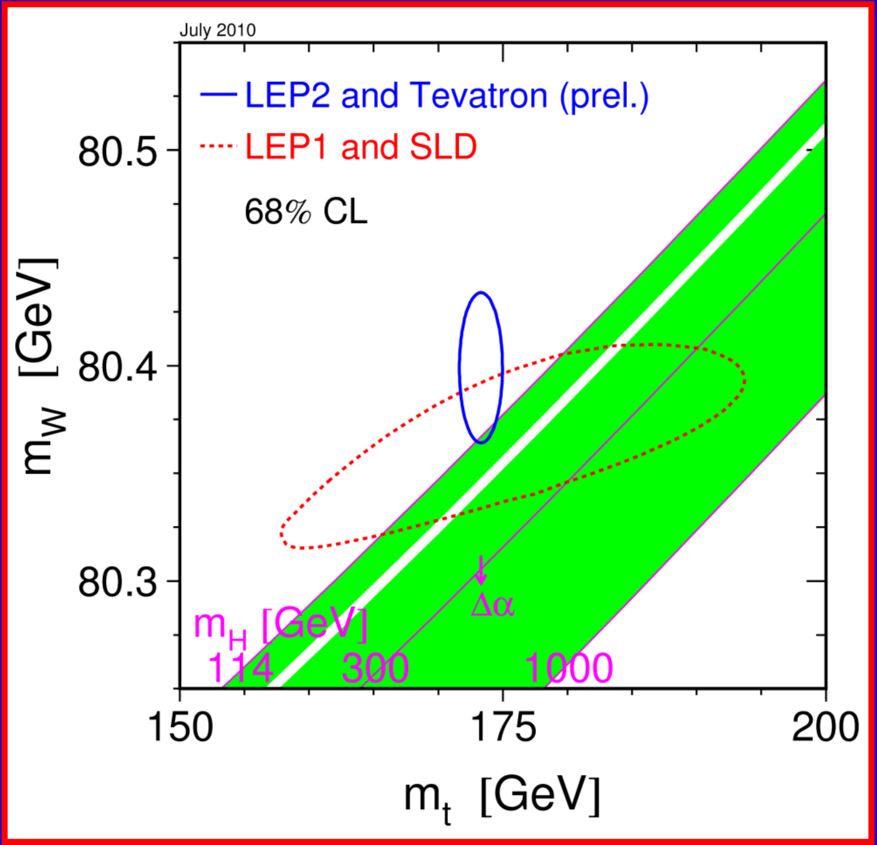
$m_t = (172.7 \pm 2.9)$ GeV

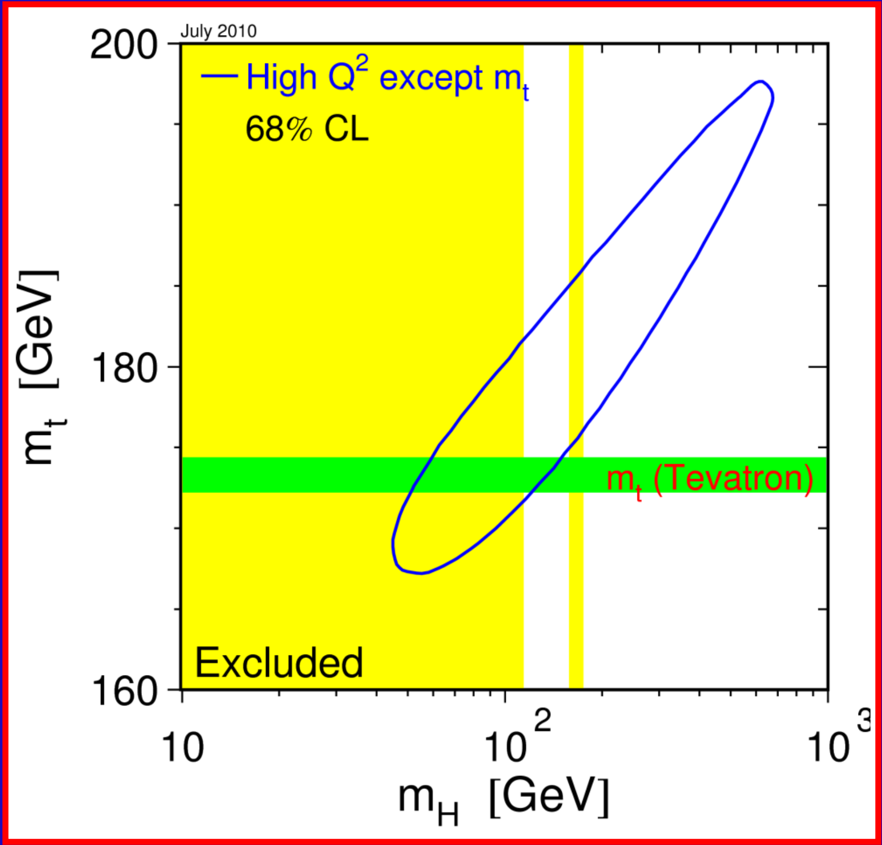
$M_H = (300\,^{+700}_{-186})$ GeV

$\alpha(M_Z^2)^{-1} = 128.95 \pm 0.05$

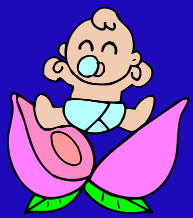
Heavy Quarks (Leptons) Favour High (Low) M_H

LEPEWWG July 2010

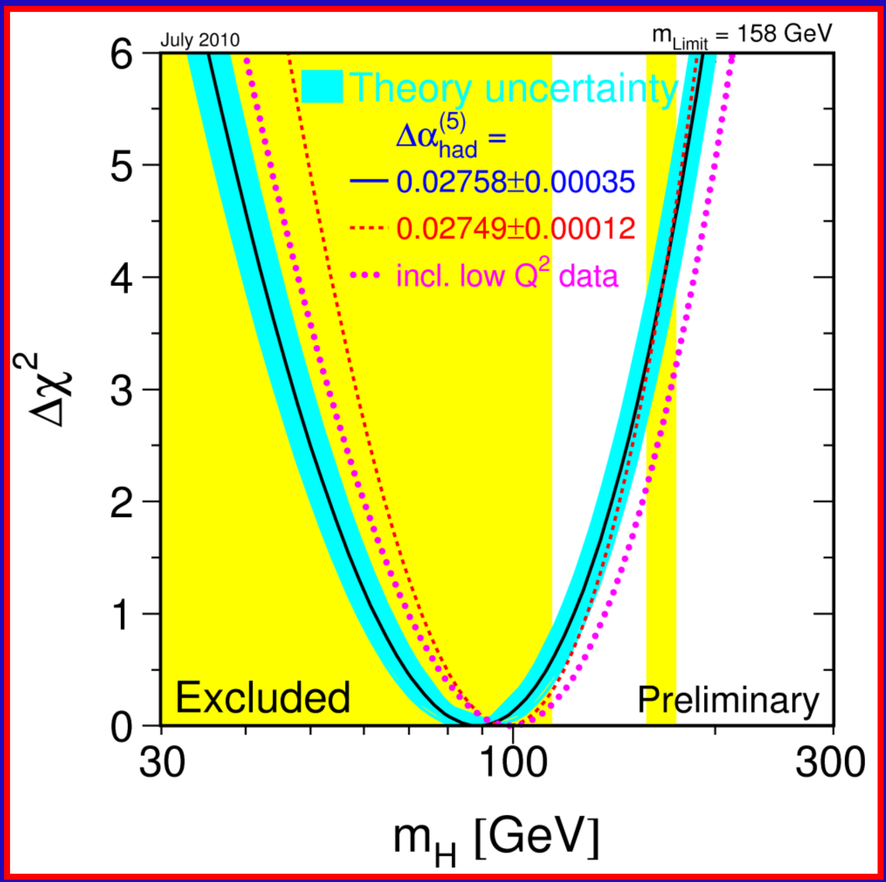




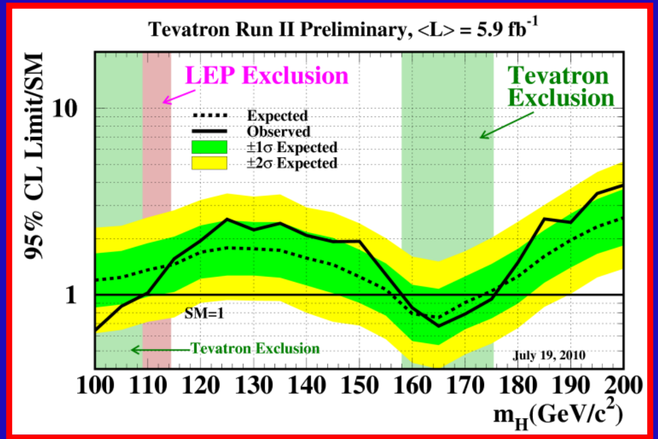
$m_t = (173.3 \pm 1.1)$ GeV **(CDF + D0)**

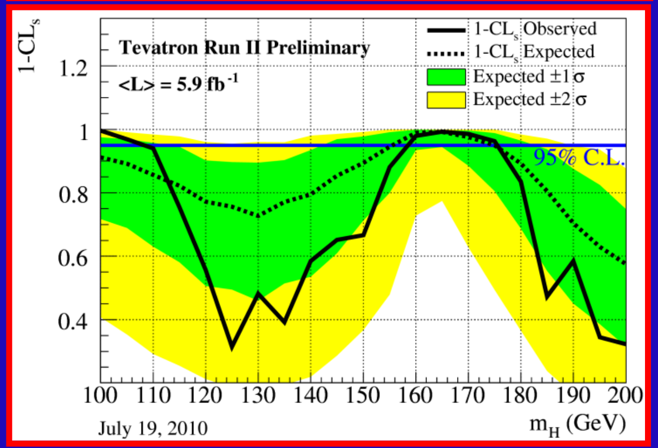


LEPEWWG (July 2010)



$H \to W^+W^-$ CDF / D0 (July 2010)

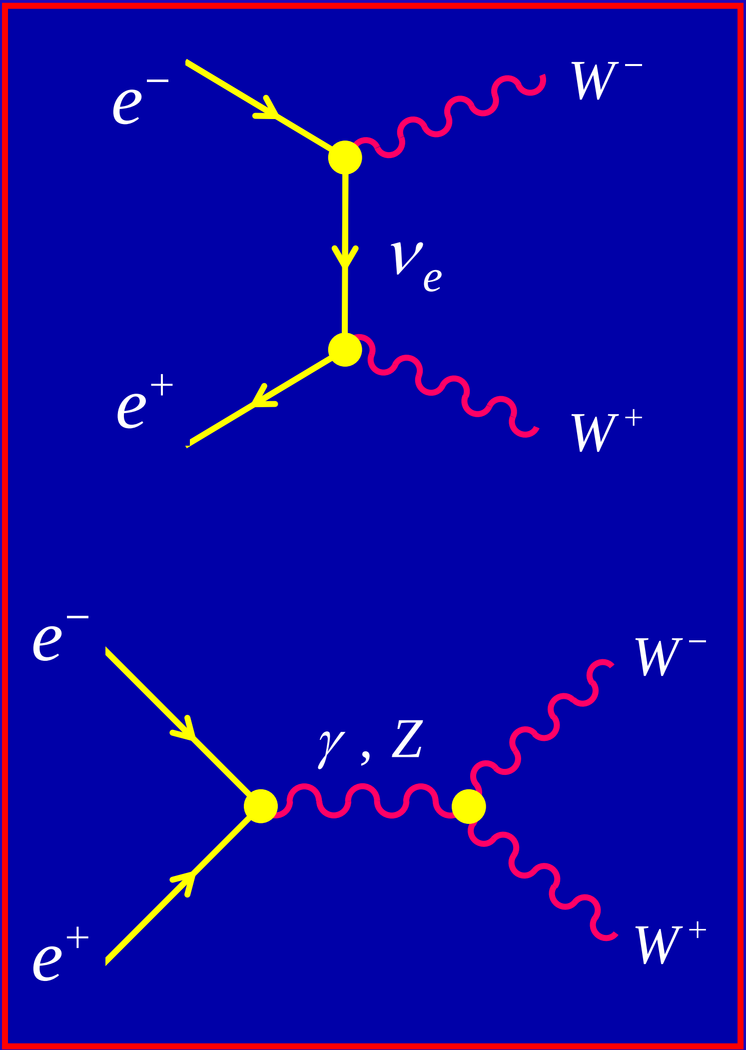


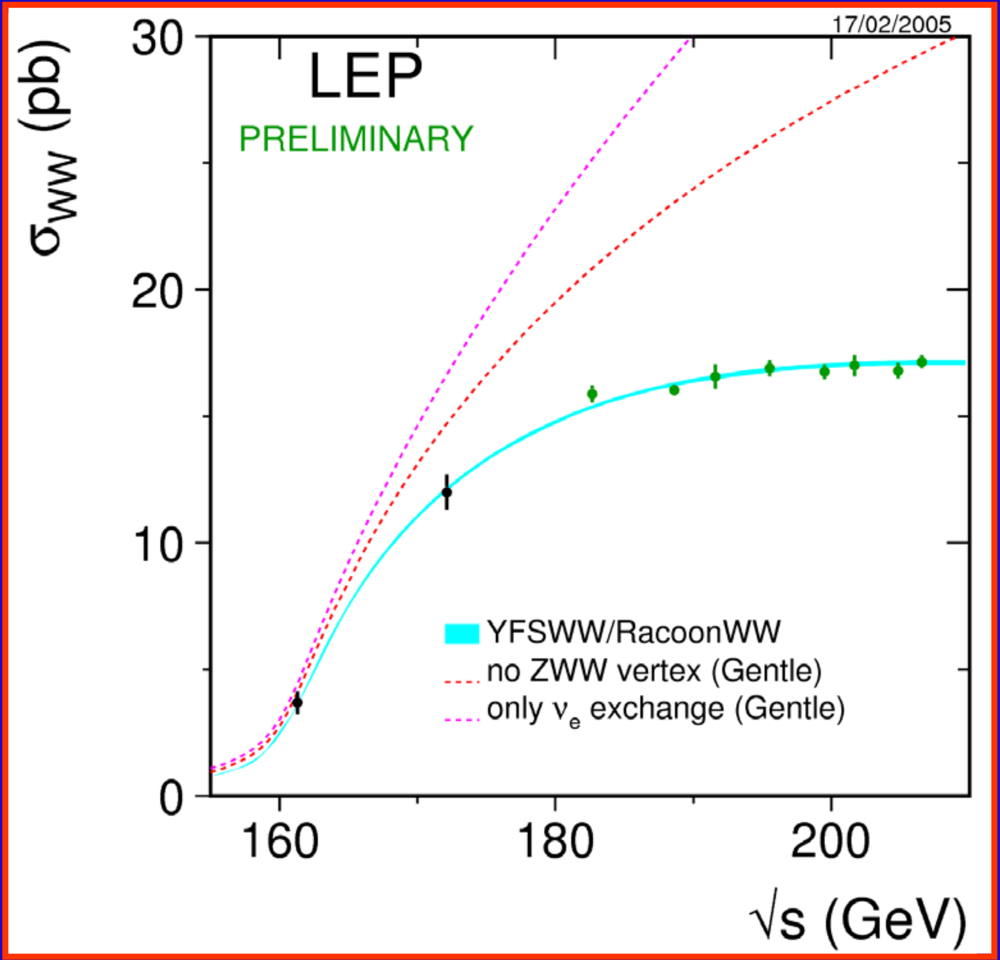


$M_H \in [158,175]$ excluded (95% CL)

$$114.4 \text{ GeV} < M_H < 158 \quad (185) \quad \text{GeV} \qquad (95\% \text{ CL})$$

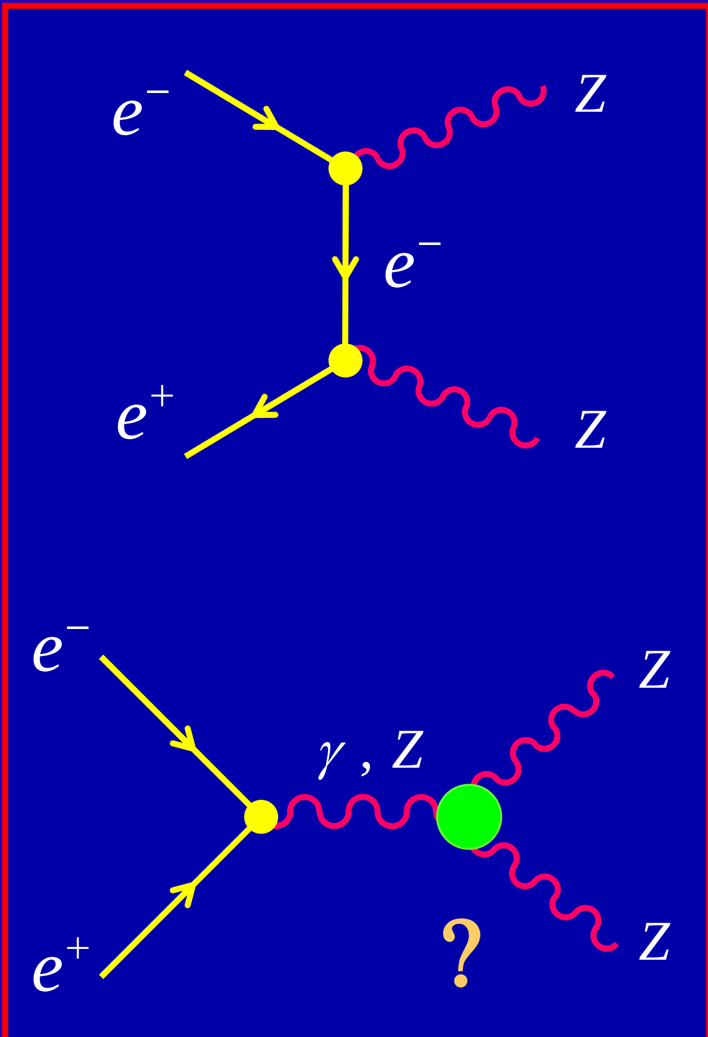
$$e^+ e^- \rightarrow W^+ W^-$$

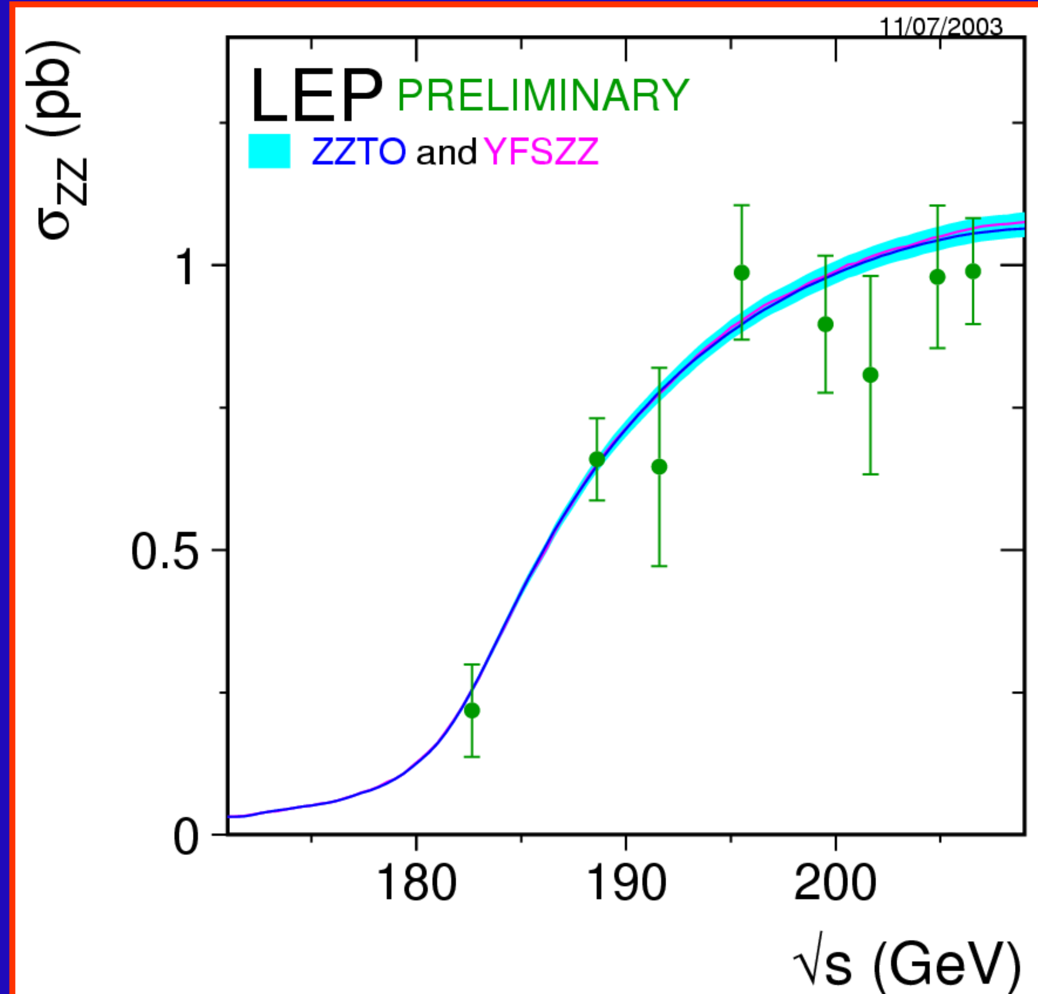




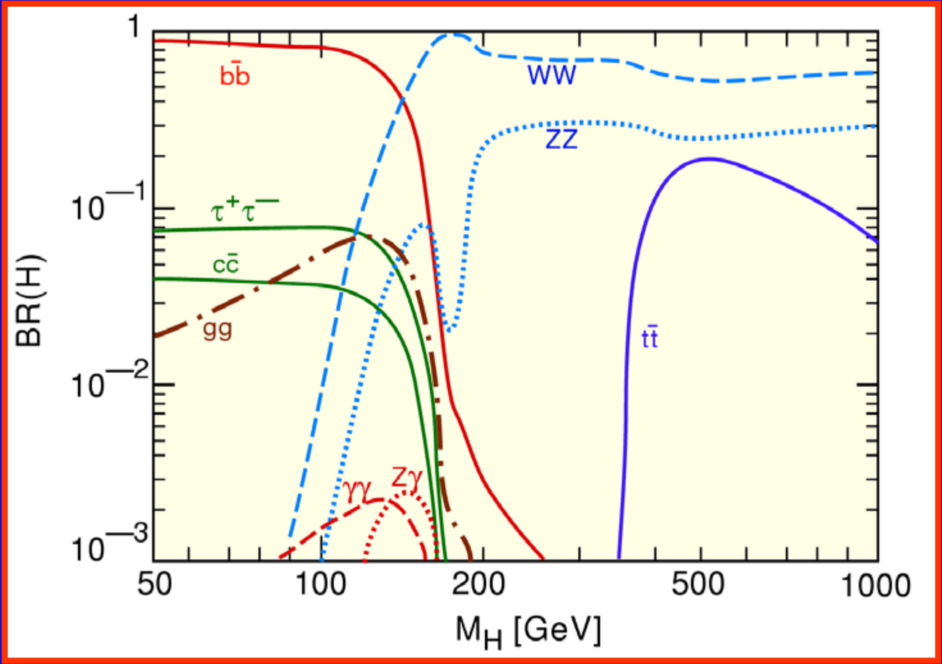
Evidence of Gauge Self-Interactions

$e^+ e^- \to Z\, Z$



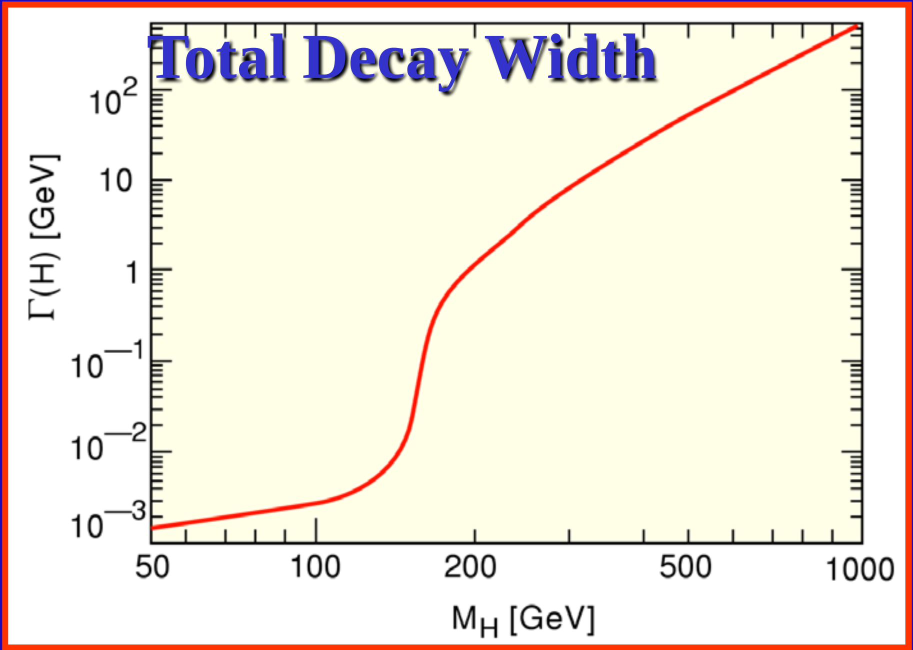


No Evidence of $\gamma Z Z$ or $Z Z Z$ couplings



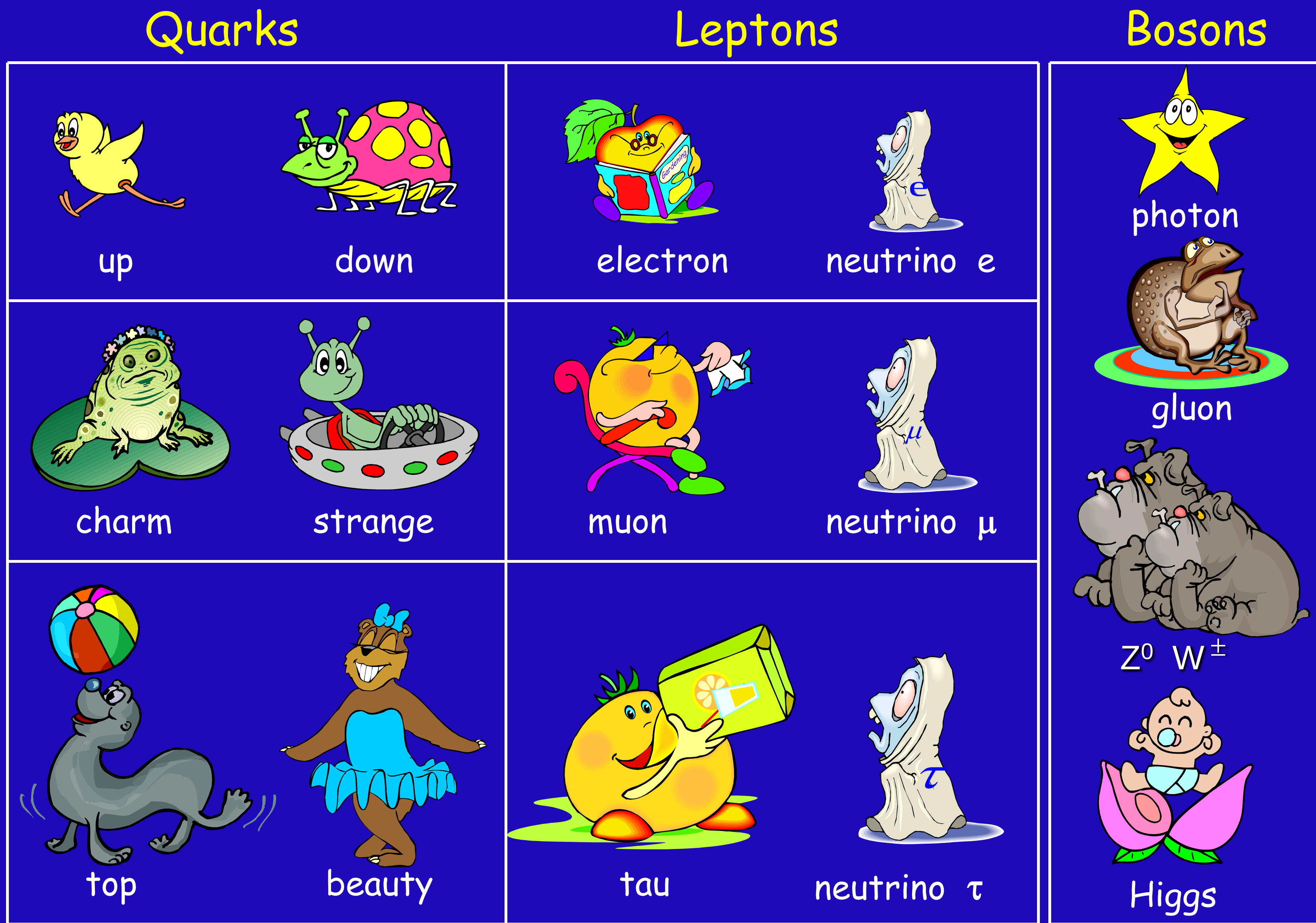
D. Denegri

Branching Ratios



Interaction proportional to mass (M_W^2, M_Z^2, m_f)

The Higgs decays into the heaviest possible particles


Quarks
Leptons
Bosons
up
down
electron
neutrino e
photon
gluon
charm
strange
muon
neutrino μ
Z0 W±
top
beauty
tau
neutrino τ
Higgs

Standard Model Parameters

QCD: $\alpha_s\,(M_Z)$ **1**

EW Gauge / Scalar Sector: **4**

$$g\,,\,g'\,,\,\mu^2\,,\,h \;\Leftrightarrow\; \alpha\,,\theta_W\,,\,M_W\,,\,M_H \;\Leftrightarrow\; \alpha\,,\,G_F\,,\,M_Z\,,\,M_H$$

Yukawa Sector: **13**

$$m_e\;,\;m_\mu\;,\;m_\tau$$

$$m_d\;,\;m_s\;,\;m_b$$

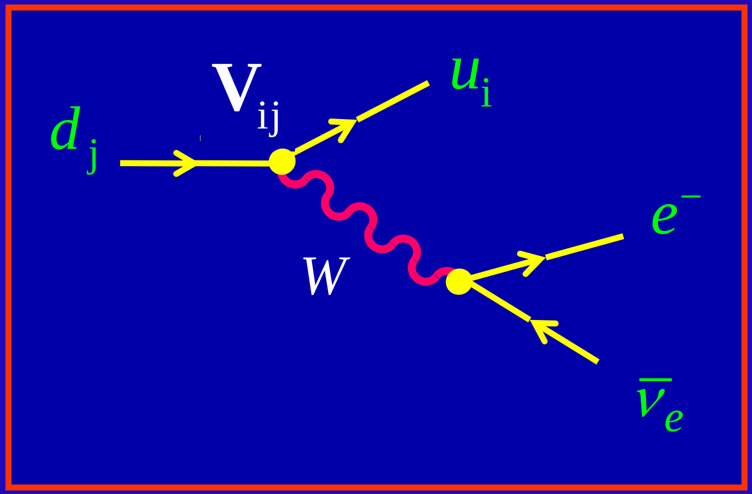
$$m_u\;,\;m_c\;,\;m_t$$

$$\theta_1\,,\,\theta_2\,,\,\theta_3\,,\,\delta$$

➡ **18 Free Parameters** (+ Neutrino Masses / Mixings ?)

TOO MANY !

Measurements of V_{ij}



$$\Gamma(d_j \to u_i\, e^-\, \bar{\nu}_e) \propto |\mathbf{V}_{ij}|^2$$

We measure decays of hadrons (no free quarks)

Important QCD Uncertainties

V_{ij} DETERMINATIONS

CKM entry	Value	Source
$\|V_{ud}\|$	0.97425 ± 0.00022 0.9746 ± 0.0019 0.9741 ± 0.0026	Nuclear β decay $n \to p\, e^- \bar{\nu}_e$ $\pi^+ \to \pi^0 e^+ \nu_e$
$\|V_{us}\|$	0.2246 ± 0.0012 0.2164 ± 0.0031 0.2259 ± 0.0015 0.2244 ± 0.0012	$K \to \pi\, e^- \bar{\nu}_e$ τ decays $K/\pi \to \mu\nu$, Lattice
$\|V_{cd}\|$	0.230 ± 0.011 0.229 ± 0.026	$\nu\, d \to c\, X$ $D \to \pi l \nu$, Lattice
$\|V_{cs}\|$	0.985 ± 0.104	$D \to K l \nu$, Lattice
$\|V_{cb}\|$	0.0387 ± 0.0011 0.0415 ± 0.0007 0.0406 ± 0.0013	$B \to D^* / D\, l\, \bar{\nu}_l$ $b \to c\, l\, \bar{\nu}_l$
$\|V_{ub}\|$	0.0034 ± 0.0004 0.0043 ± 0.0004 0.0039 ± 0.0004	$B \to \pi\, l\, \bar{\nu}_l$ $b \to u\, l\, \bar{\nu}_l$
$\|V_{tb}\| \big/ \sqrt{\sum_q \|V_{tq}\|^2}$ $\|V_{tb}\|$	> 0.89 > 0.74 ; < 1	$t \to bW / qW$ $p\bar{p} \to tb + X$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9995 \pm 0.0010$$

$$\sum_j \left(|V_{uj}|^2 + |V_{cj}|^2 \right) = 2.002 \pm 0.027 \quad \text{(LEP)}$$

QUARK MIXING MATRIX

- **Unitary** $N_G \times N_G$ **Matrix:** N_G^2 **parameters**

$$\mathbf{V} \cdot \mathbf{V}^\dagger = \mathbf{V}^\dagger \cdot \mathbf{V} = \mathbf{1}$$

- $2\, N_G - 1$ **arbitrary phases:**

$$u_i \to e^{i\phi_i}\, u_i \quad ; \quad d_j \to e^{i\theta_j}\, d_j \quad \Rightarrow \quad \mathbf{V}_{ij} \to e^{i(\theta_j - \phi_i)}\, \mathbf{V}_{ij}$$

$\Downarrow$

$\mathbf{V}_{ij}$ **Physical Parameters:**

$\frac{1}{2} N_G\, (N_G - 1)$ **Moduli** ; $\frac{1}{2} (N_G - 1)\, (N_G - 2)$ **phases**

- $\mathbf{N_f = 2:}$ **1 angle, 0 phases** (Cabibbo)

$$\mathbf{V} = \begin{bmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{bmatrix} \Rightarrow \text{No } \not{CP}$$

- $\mathbf{N_f = 3:}$ **3 angles, 1 phase** (CKM)

$c_{ij} \equiv \cos\theta_{ij} \quad ; \quad s_{ij} \equiv \sin\theta_{ij}$

$$\mathbf{V} = \begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & c_{23}\, c_{13} \end{bmatrix}$$

$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$\lambda \approx \sin\theta_C \approx 0.225 \quad ; \quad A \approx 0.81 \quad ; \quad \sqrt{\rho^2+\eta^2} \approx 0.37$

$\delta_{13} \neq 0 \quad (\eta \neq 0) \Rightarrow \not{CP}$

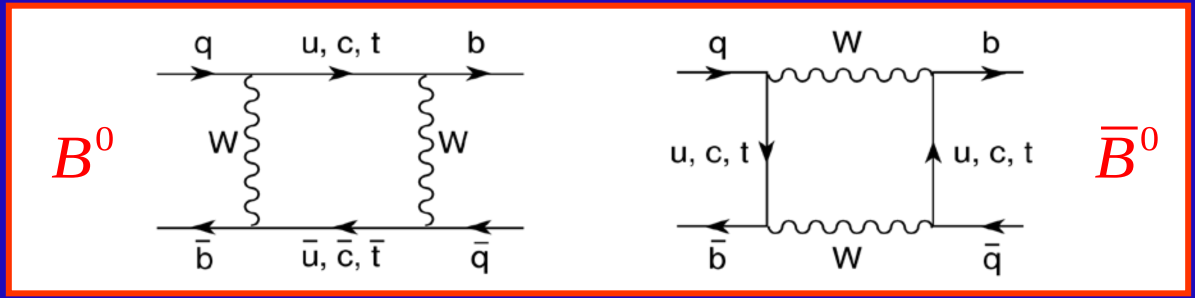
- $\mathcal{C}$, $\mathcal{P}$: Violated maximally in weak interactions
- $\mathcal{CP}$: Symmetry of nearly all observed phenomena
- Slight (~ 0.2 %) $\not{\mathcal{CP}}$ in K^0 decays (1964)
- Sizeable $\not{\mathcal{CP}}$ in B^0 decays (2001)
- Huge Matter — Antimatter Asymmetry in our Universe → Baryogenesis

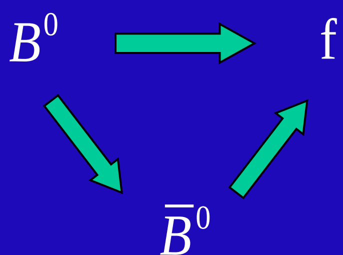
$\mathcal{CPT}$ Theorem: $\not{\mathcal{CP}} \Longleftrightarrow \not{\mathcal{T}}$

Thus, $\not{\mathcal{CP}}$ requires:

- Complex Phases
- Interferences

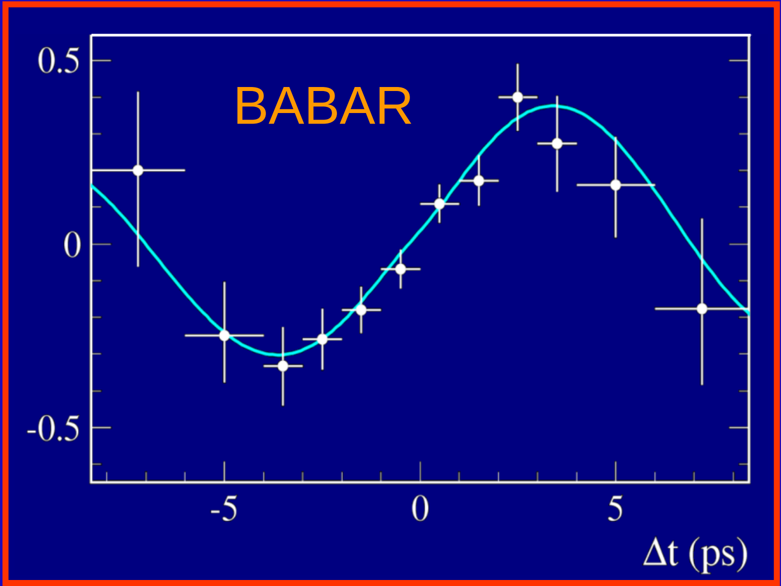
Meson – Antimeson Mixing





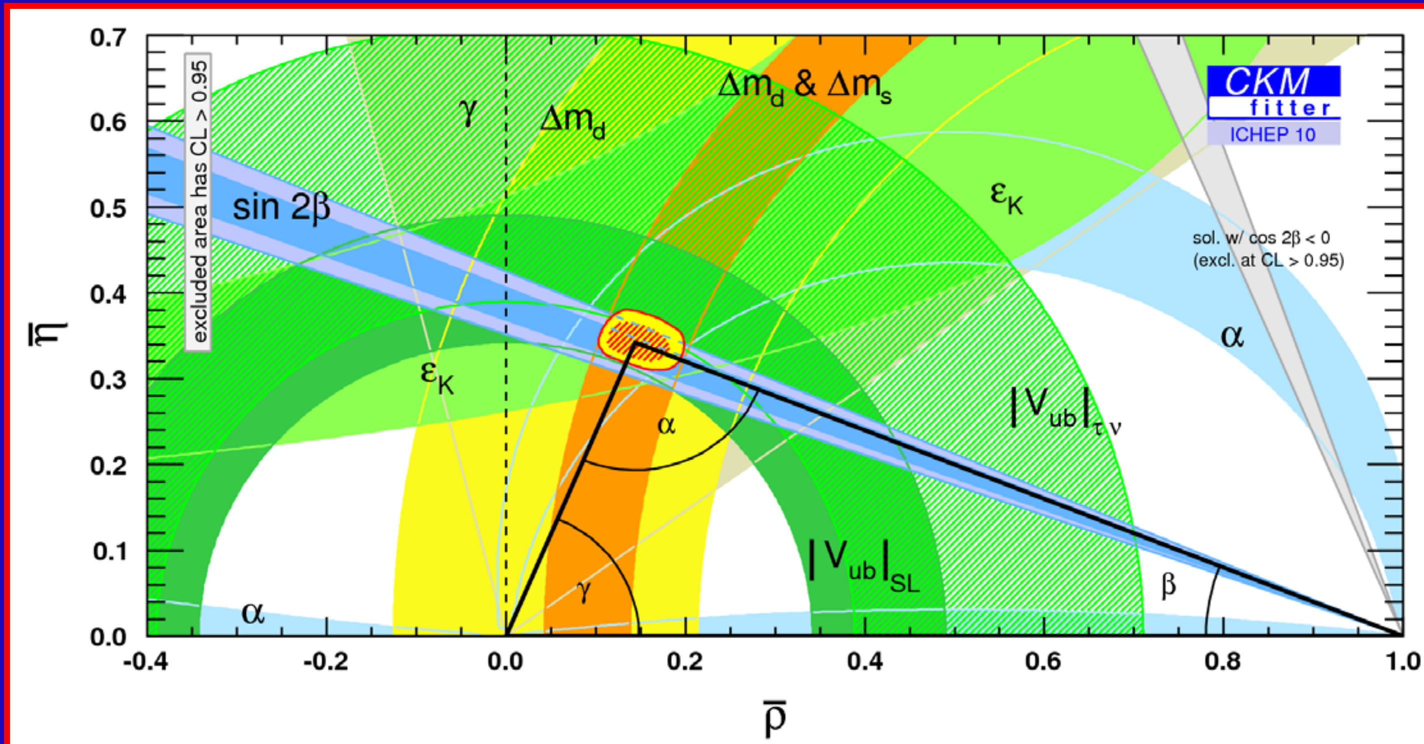
2 Interfering Amplitudes

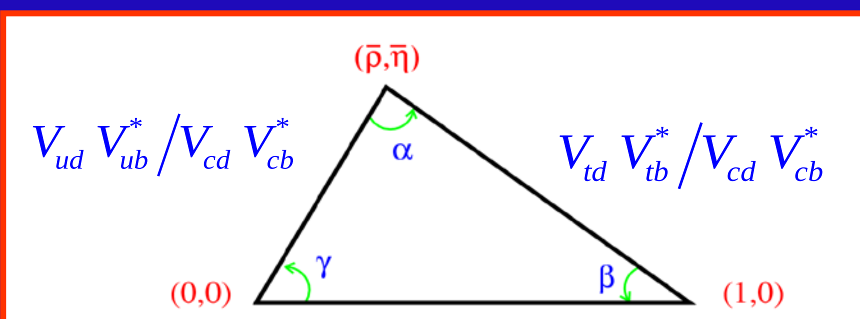
$\not{CP}$ Signal



$$\frac{\Gamma(B^0 \to J/\psi\, K_S) - \Gamma(\bar{B}^0 \to J/\psi\, K_S)}{\Gamma(B^0 \to J/\psi\, K_S) + \Gamma(\bar{B}^0 \to J/\psi\, K_S)} \neq 0$$

$$V_{ud}\,V_{ub}^{*} + V_{cd}\,V_{cb}^{*} + V_{td}\,V_{tb}^{*} = 0$$



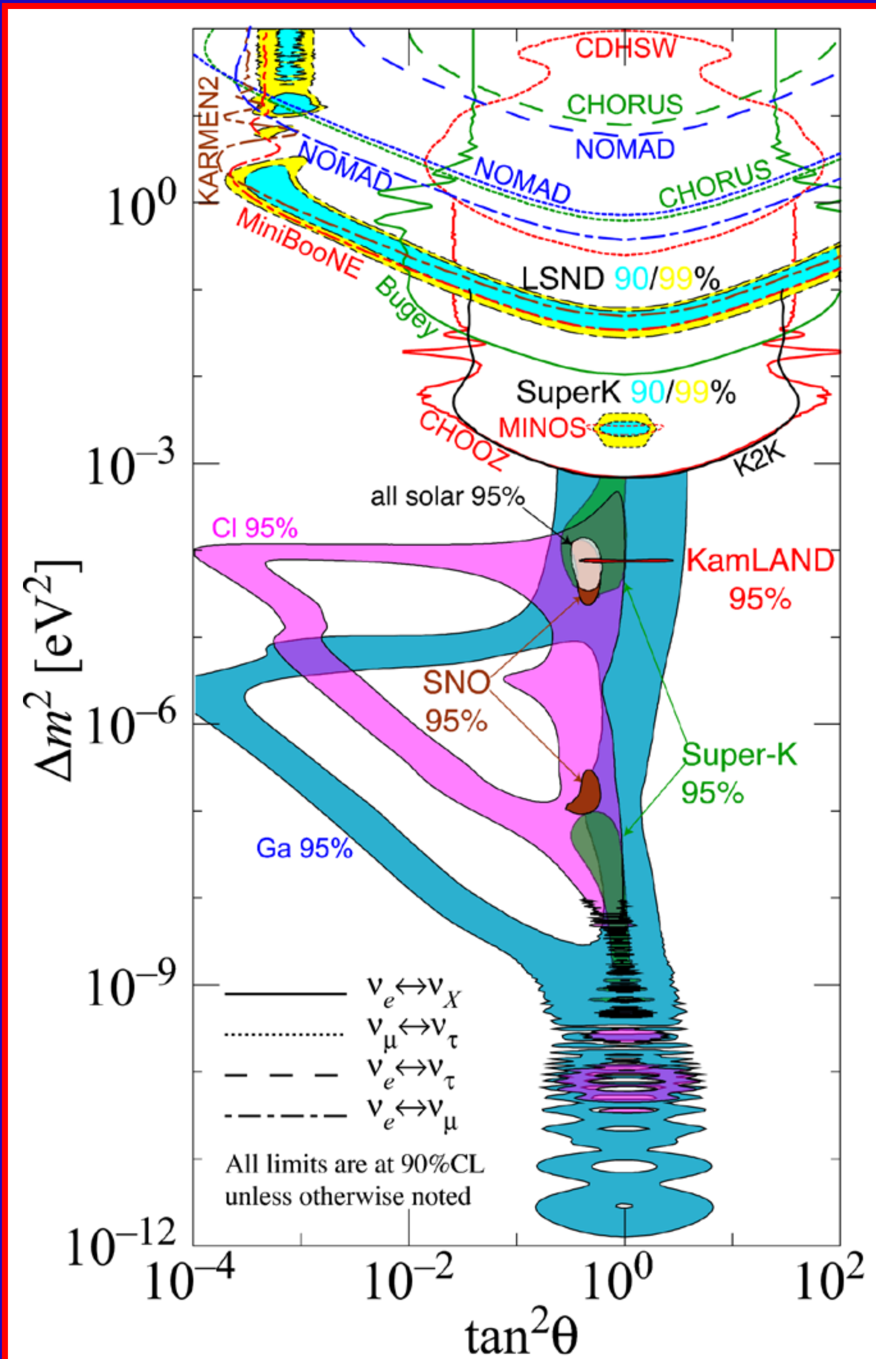


UT*fit*

$$\bar\eta \equiv \eta\left(1-\frac{1}{2}\lambda^2\right) = 0.358\pm0.012$$

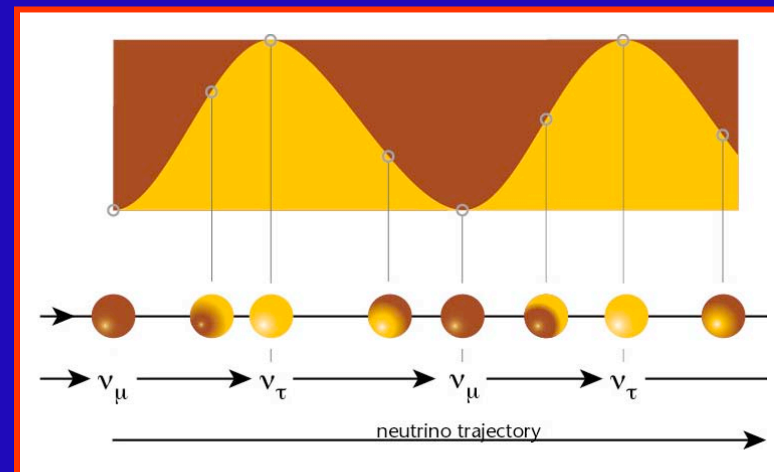
$$\bar\rho \equiv \rho\left(1-\frac{1}{2}\lambda^2\right) = 0.132\pm0.022$$

$$\alpha = 87.8\pm3.0^\circ \;;\; \beta = 22.42\pm0.74^\circ \;;\; \gamma = 69.8\pm3.0^\circ$$



Neutrino Oscillations

http://hitoshi.berkeley.edu/neutrino

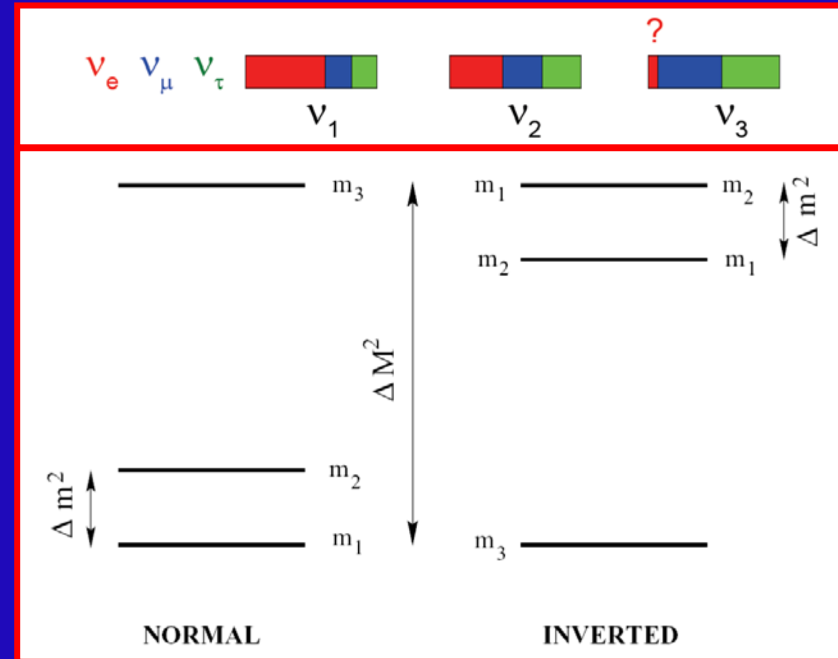


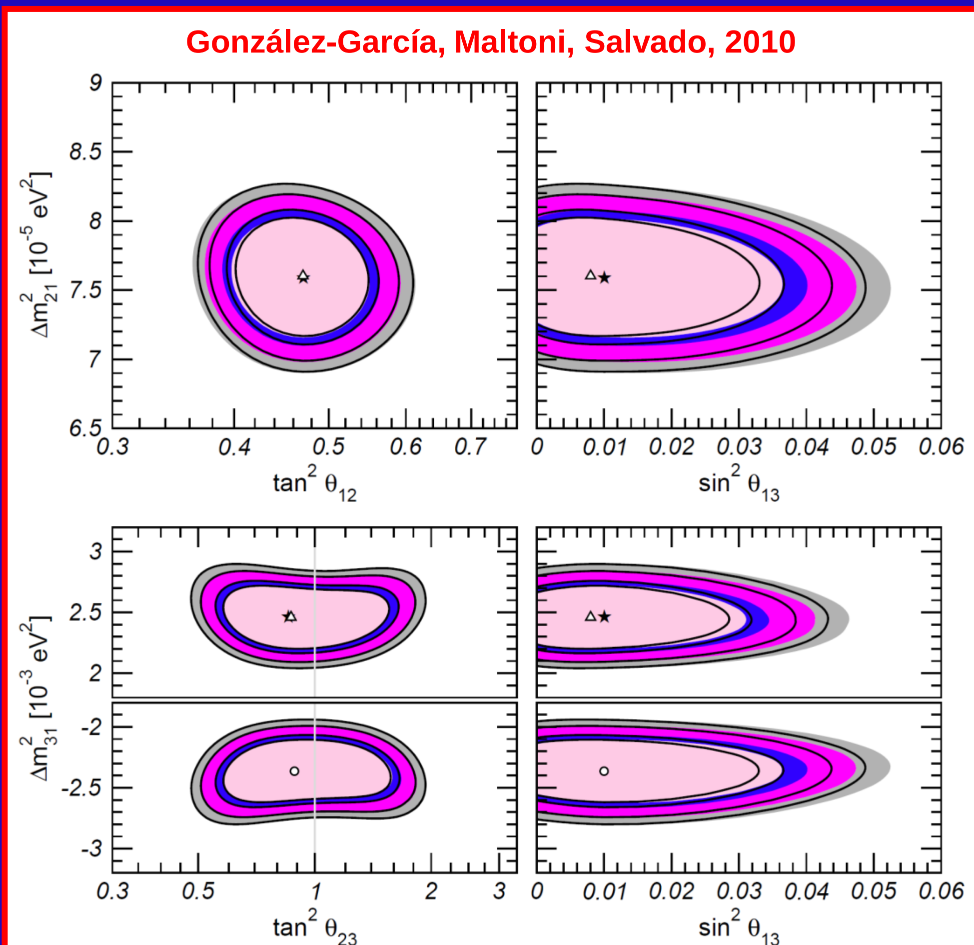
Lepton Mixing

ν_R , $\not{CP}$?

NEW PHYSICS

Neutrino Oscillations





$$\Delta m^2_{21} \;=\; (7.59 \pm 0.20)\cdot 10^{-5}\ \mathrm{eV}^2$$

$$\left|\Delta m^2_{32}\right| \;=\; (2.43 \pm 0.13)\cdot 10^{-3}\ \mathrm{eV}^2$$

$$\sin^2(2\theta_{12}) \;=\; 0.87 \pm 0.03$$

$$\sin^2(2\theta_{23}) \;>\; 0.92 \quad (90\%\ \mathrm{CL})$$

$$\sin^2(2\theta_{13}) \;<\; 0.15 \quad (90\%\ \mathrm{CL})$$

THE STANDARD THEORY OF FUNDAMENTAL INTERACTIONS

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

Electroweak + Strong Forces

- Gauge Symmetry → Dynamics
- 3 Gauge Parameters: $\alpha_s(M_Z^2)$, α , θ_W
- All Known Experimental Facts Explained
- Problem with Mass Scales / Mixings:

- 15 Additional Parameters
- Why 3 Families ?
- Why Left $\neq$ Right ?
- Why $m_t > M_Z$?
- Does the Higgs Exist ?
- Flavour Mixing
- $\mathcal{CP}$ Violation
- Neutrino Masses / Oscillations