QFT, Standard Model and EW Symmetry Breaking

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The Standard Model

= Quantum Field Theory

gauge invariant

with spontaneous symmetry breaking

gauge invariance

- powerful symmetry principle
- determines structure of interactions
- guarantees renormalizability $\rightarrow$ precision tests

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symmetry breaking

- massive leptons and quarks
- massive vector bosons of the weak interaction

Outline

1. Gauge Theories
2. Higgs mechanism
3. Electroweak interaction and Standard Model
4. Phenomenology of W and Z bosons, precision tests
5. Higgs bosons

1. Gauge theories

Notations and Conventions

$$\mu, \nu, .. = 0, 1, 2, 3; \qquad k, l, .. = 1, 2, 3$$

$$x = (x^\mu) = (x^0, \vec{x}), \quad x^0 = t \qquad (\hbar = c = 1)$$

$$p = (p^\mu) = (p^0, \vec{p}), \quad p^0 = E = \sqrt{\vec{p}^2 + m^2}$$

$$a_\mu = g_{\mu\nu}\, a^\nu, \qquad (g_{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$a^2 = a_\mu a^\mu, \quad a \cdot b = a_\mu b^\mu = a^0 b^0 - \vec{a} \cdot \vec{b}$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = g_{\mu\nu}\, \partial^\nu, \quad \partial^\nu = \frac{\partial}{\partial x_\nu} \qquad [\, \partial^0 = \partial_0, \;\; \partial^k = -\partial_k \,]$$

$$\Box = \partial_\mu \partial^\mu = \frac{\partial^2}{\partial t^2} - \Delta$$

Fields in the Standard Model

- spin 0 particles: scalar fields $\phi(x)$
- spin 1 particles: vector fields $A_\mu(x), \quad \mu = 0, \ldots 3$
- spin 1/2 fermions: spinor fields $\psi(x) = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$

Lagrangian: $\mathcal{L}(\phi, \partial_\mu \phi)$ Lorentz invariant

Action: $S = \int \mathrm{d}^4x \, \mathcal{L}(\phi(x), \cdots)$ Lorentz invariant

free fields: $\mathcal{L}$ is quadratic in the fields $\Rightarrow$ propagators

interacting fields: higher powers in the fields $\Rightarrow$ vertices

Constructing QED – main steps

- start with $\mathcal{L}_0 = \overline{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi$ for free fermion field ψ

  symmetric under global gauge transformations

  $\psi' = e^{i\alpha}\psi, \ \alpha \text{ real}$

- perform minimal substitution $\partial_\mu \ \rightarrow \ D_\mu = \partial_\mu - ieA_\mu$

  $\Rightarrow$ invariance under local gauge transformations

  $$\psi' = e^{i\alpha(x)}\,\psi, \quad A'_\mu = A_\mu + \tfrac{1}{e}\,\partial_\mu\alpha(x)$$

  - involves additional vector field A_μ
  - induces interaction between A_μ and ψ

  $$e\,(\overline{\psi}\gamma^\mu\psi)\,A_\mu \ \equiv \ e\,j^\mu A_\mu$$

- make A_μ a dynamical field by adding

  $$\mathcal{L}_A = -\tfrac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Non-Abelian gauge theories

Generalization: “phase” transformations that do not commute

$$\psi \to \psi' = U\psi \qquad \textit{with} \qquad U_1 U_2 \neq U_2 U_1$$

requires matrices, *i.e.* ψ is a multiplet

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{pmatrix}, \qquad U = n \times n \text{ -}\textit{matrix}$$

each $\psi_k = \psi_k(x)$ is a Dirac spinor

(i) global symmetry

starting point: $\mathcal{L}_0 = \overline{\psi}\,(i\gamma^\mu\,\partial_\mu - m)\,\psi$

where $\overline{\psi} = (\overline{\psi}_1, \cdots, \overline{\psi}_n)$

consider unitary matrices: $U^\dagger = U^{-1}$

$$\psi' = U\psi \quad \Rightarrow \quad \overline{\psi'} = \overline{\psi}\,U^\dagger = \overline{\psi}\,U^{-1}$$

$$\Rightarrow \quad \overline{\psi'}\psi' = \overline{\psi}\psi, \quad \overline{\psi'}\gamma^\mu\partial_\mu\psi' = \overline{\psi}\gamma^\mu\partial_\mu\psi$$

if U does not depend on x

$\Rightarrow$ $\mathcal{L}_0$ is invariant under $\psi \to U\psi$

U: global gauge transformation

similar for **scalar fields:**

$$\phi \to \phi' = U\phi, \qquad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{pmatrix}$$

each $\psi_k = \phi_k(x)$ is a scalar field, $\phi^\dagger = (\phi_1^\dagger, \cdots, \phi_n^\dagger)$

terms $\phi^\dagger\phi$, $(\partial_\mu\phi)^\dagger(\partial^\mu\phi)$ are invariant

$\Rightarrow$ $\mathcal{L}_0 = (\partial_\mu\phi)^\dagger(\partial^\mu\phi) - m^2\,\phi^\dagger\phi$ is invariant relevant in physics:

the special unitary $n \times n$-matrices with det=1

group $SU(n)$

<u>examples:</u>

$$SU(2): \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad e.g. \quad \psi = \begin{pmatrix} \psi_\nu \\ \psi_e \end{pmatrix}$$ *isospin*

$$SU(3): \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \quad e.g. \quad \psi = \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$$ *colour*

$SU(n)$ matrices U can be written as exponentials

$$U(\theta_1, \cdots, \theta_N) = e^{i\theta_a T_a} \qquad \textit{sum over } a = 1, \cdots N$$

$\theta_1, \cdots \theta_N$: real parameters

$T_1, \cdots T_N$: $n \times n$-matrices, generators, $T_a^\dagger = T_a$

infinitesimal θ : $U = \mathbf{1} + i\theta_a T_a \quad (+O(\theta^2))$

N-dimensional Lie Group

det=1 and unitarity $\Rightarrow$ $N = n^2 - 1$

$n = 2: \quad N = 3, \qquad n = 3: \quad N = 8$

commutators $[T_a, T_b] \neq 0$ non-Abelian

$$\boxed{[T_a, T_b] = i\, f_{abc}\, T_c}$$

Lie Algebra

$f_{abc}:$ real numbers, structure constants

$f_{abc} = -f_{bac} = \cdots$ antisymmetric commutators $[T_a, T_b] \neq 0$ non-Abelian

$$\boxed{[T_a, T_b] = i\, f_{abc}\, T_c}$$

Lie Algebra

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$\boxed{SU(2)}$ $f_{abc} = \epsilon_{abc}$ *(like angular momentum)*

$T_a = \frac{1}{2}\,\sigma_a,$ σ_a : *Pauli matrices (a=1,2,3)*

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$\boxed{SU(3)}$ $T_a = \frac{1}{2}\, \lambda_a,$ λ_a : *Gell-Mann matrices (a=1,...8)*

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \ldots$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

(ii) local symmetry

now: $\theta_a = \theta_a(x)$ for $a = 1, \dots N$

covariant derivative $\partial_\mu \to D_\mu = \partial_\mu - ig\,\mathbf{W}_\mu$

vector field $\mathbf{W}_\mu$ is $n \times n$ matrix: $\mathbf{W}_\mu(x) = T_a\, W_\mu^a(x)$

induces interaction term $\mathcal{L}_0 \to \mathcal{L}_0 + \mathcal{L}_{\text{int}}$

with $\mathcal{L}_{\text{int}} = g\,\overline{\Psi}\gamma^\mu \mathbf{W}_\mu \Psi = g\,(\overline{\Psi}\gamma^\mu T_a \Psi)\, W_\mu^a \equiv j_a^\mu\, W_\mu^a$

For a multiplet of scalar fields Φ:

$$\mathcal{L}_0 = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) \quad \to \quad \mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi)$$

$\mathcal{L}$ is invariant under local gauge transformations

$$\Psi \rightarrow \Psi' = U\,\Psi\,,$$

$$\mathbf{W}_\mu \rightarrow \mathbf{W}'_\mu = U\,\mathbf{W}_\mu\,U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}$$

for infinitesimal transformations:

$$W^a_\mu \rightarrow {W'}^a_\mu = W^a_\mu + \frac{1}{g}\,\partial_\mu\theta^a + f_{abc}\,W^b_\mu\,\theta^c$$

crucial property of covariant derivative

$$\boxed{D'_\mu U = U\,D_\mu}$$

(iii) dynamics of W^a_μ fields

need: additional term $\mathcal{L}_W \quad \Rightarrow \quad$ e.o.m., propagators

naive: $\quad \sum_a (\partial_\mu W^a_\nu - \partial_\nu W^a_\mu)^2 \quad$ *not gauge invariant*

instead:
$$\mathbf{F}_{\mu\nu} = D_\mu \mathbf{W}_\nu - D_\nu \mathbf{W}_\mu \equiv F^a_{\mu\nu}\, T_a$$
$$= \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - ig\,[\mathbf{W}_\mu, \mathbf{W}_\nu]$$
$$= \frac{i}{g}\,[D_\mu, D_\nu]$$

gauge transformation: $\quad \mathbf{W}_\mu \to \mathbf{W}'_\mu, \quad D_\mu \to D'_\mu$
$$\Rightarrow \quad \mathbf{F}_{\mu\nu} \to \mathbf{F}'_{\mu\nu} = U\,\mathbf{F}_{\mu\nu}\,U^{-1}$$

$$\Rightarrow \mathrm{Tr}\,(\mathbf{F}'_{\mu\nu}\mathbf{F}'^{\,\mu\nu}) = \mathrm{Tr}\,(U\mathbf{F}_{\mu\nu}U^{-1}\,U\mathbf{F}^{\mu\nu}U^{-1}) = \mathrm{Tr}\,(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu})$$

gauge invariant

Lagrangian:

$$\mathcal{L}_W = -\frac{1}{2}\,\mathrm{Tr}\,(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}) = -\frac{1}{4}\sum_a F^a_{\mu\nu}F^{a,\mu\nu}$$

components of $\mathbf{F}_{\mu\nu}$ *[using normalization* $\mathrm{Tr}(T_a T_b) = \frac{1}{2}\,\delta_{ab}$ *]*

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\, f_{abc}\, W^b_\mu W^c_\nu$$

$F^a_{\mu\nu}$ = *Abelian* + *non-Abelian*

$$\mathcal{L}_W = \underbrace{\text{quadratic}}_{\textit{free part}} + \underbrace{\text{cubic} + \text{quartic}}_{\textit{tri- and quadri-linear interactions}}$$

$$
\begin{aligned}
\mathcal{L}_W &= -\frac{1}{4}\,(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)^2 && \Rightarrow \ \text{propagator} \\
&= -\frac{1}{2}\,g f_{abc}\,(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)\,W^{b,\mu}\,W^{c,\nu} \\
&= -\frac{1}{4}\,g^2 f_{abc}\,f_{ade}\,W_\mu^b\,W_\nu^c\,W^{d,\mu}\,W^{e,\nu}
\end{aligned}
$$

new type of couplings:

- self-couplings of vector fields (gauge couplings)
- universal coupling constant g for fermions and vector fields

propagator $D_{\rho\nu}$ for massless spin-1 particles

$$\left[\Box\, g^{\mu\rho} - \partial^\mu \partial^\rho\right] D_{\rho\nu}(x-y) = g^\mu{}_\nu\, \delta^4(x-y)$$

in momentum space

$$\left(-k^2\, g^{\mu\rho} + k^\mu k^\rho\right) D_{\rho\nu}(k) = g^\mu{}_\nu$$

has no solution (det = 0)

way out: add gauge fixing term $\mathcal{L}_{\rm fix} = -\frac{1}{2\xi}\left(\partial_\mu W^\mu\right)^2$

$$\left[-k^2\, g^{\mu\rho} + \left(1 - \frac{1}{\xi}\right) k^\mu k^\rho\right] D_{\rho\nu}(k) = g^\mu{}_\nu$$

which now has a solution:

$$D_{\rho\nu}(k) = \frac{1}{k^2 + i\epsilon}\left[-g_{\nu\rho} + (1-\xi)\,\frac{k_\nu k_\rho}{k^2}\right]$$

Faddeev-Popov ghosts, BRS symmetry

[important for quantization and renormalization]

gauge group G, generators T_a, structure constants f_{abc}

for quantization: $\mathcal{L} = \mathcal{L}_{\rm sym} + \mathcal{L}_{\rm fix} + \mathcal{L}_{\rm ghost}$

$$\mathcal{L}_{\rm fix} = \tfrac{1}{2} \sum F_a^2 , \quad F_a = \partial_\mu W^{a,\mu}$$

requires ghost fields c_a and anti-ghosts $\bar{c}_a$

$$\mathcal{L}_{\rm ghost} = (\partial^\mu \bar{c}_a)\,(D_\mu^{\rm adj})_{ab}\, c_b , \quad D_\mu^{\rm adj} = \partial_\mu - ig\, W_\mu^r\, T_r^{\rm adj}$$

- $\mathcal{L}$ is symmetric under BRS transformations

$$\delta W_\mu^a = (D_\mu^{\rm adj})_{ab}\, c_b \qquad [W_\mu^a \to W_\mu^a + \delta W_\mu^a \quad \texttt{etc.}]$$

$$\delta \bar{c}_a = -\partial^\nu W_\nu^a , \quad \delta c_a = -\tfrac{1}{2} g\, f_{abc}\, c_b c_c$$

BRS *[Becchi, Rouet, Stora]* symmetry guarantees

– renormalizability

– gauge invariant and unitary S matrix

important: ST identities = symmetry relations between Green functions, valid to all orders

basic quantity: effective action $\Gamma(\mathcal{L})$

generating functional of vertex functions

$$\frac{\delta\Gamma}{\delta\varphi_i\delta\varphi_j\ldots}\Big|_{\varphi=0} = \Gamma_{\varphi_i\varphi_j\ldots}$$

classical action: $\Gamma_{\mathrm{cl}}(\mathcal{L}) = \int \mathrm{d}^4x\,\mathcal{L}$

$\Rightarrow$ tree level vertices

general: vertex functions with loop contributions, building blocks for renormalization

BRS symmetry: invariance of Γ under BRS transformations,

$$S(\Gamma) = \int \mathrm{d}^4x \left[\frac{\delta\Gamma}{\delta\varphi_i}\,\delta\varphi_i + \cdots\right] = 0$$

S: ST-operator

$$\Rightarrow \quad \frac{\delta S(\Gamma)}{\delta\varphi_j \cdots}\Big|_{\varphi=0} = 0$$

relations between vertex functions

Slavnov-Taylor (ST) identities

$\Rightarrow$ determines the structure of the counter terms for renormalization

all UV divergences in vertex functions can be removed by (multiplicative) renormalization of parameters and fields in the classical Lagrangian/action

2. Higgs mechanism

Vector field for massive spin-1 particles

generic vector field $A_\mu(x)$, $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

free Lagrangian $\quad \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu$

e.o.m. $\quad \left[(\Box + m^2)\, g^{\mu\nu} - \partial^\mu \partial^\nu\right] A_\nu = 0$

solutions $\quad \epsilon_\nu^{(\lambda)}\, e^{\pm ikx} \quad \text{with} \quad k^2 = m^2$

3 orthogonal polarization vectors $\epsilon_\nu^{(\lambda)}$ with polarization sum

$$\sum_{\lambda=1}^{3} \epsilon_\mu^{(\lambda)*} \epsilon_\nu^{(\lambda)} = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}$$

longitudinal polarization $\quad \epsilon_\nu \simeq k_\nu / m \quad$ for high momentum propagator $D_{\rho\nu}(x-y)$

solution of $\left[(\Box + m^2)\, g^{\mu\rho} - \partial^\mu \partial^\rho\right] D_{\rho\nu}(x-y) = g^\mu{}_\nu\, \delta^4(x-y)$

in momentum space

$$\left[(-k^2 + m^2)\, g^{\mu\rho} + k^\mu k^\rho\right] D_{\rho\nu}(k) = g^\mu{}_\nu$$

solution

$$D_{\rho\nu}(k) = \frac{1}{k^2 - m^2 + i\epsilon}\left(-g_{\nu\rho} + \frac{k_\nu k_\rho}{m^2}\right)$$

problem: weak interaction, gauge bosons are massive

mass terms $\sim M^2 W_\mu^a W^{a,\mu}$ spoil local gauge invariance

- bad high energy behaviour of amplitudes and cross sections, conflict with unitarity
  reason: longitudinal polarization $\epsilon^\mu \simeq \frac{k^\mu}{M} \sim k^\mu$
- bad divergence of higher orders with loop diagrams
  reason: propagators contain $-g_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}$
  $\Rightarrow$ additional powers of momenta in loop integration
  $\Rightarrow$ spoil renormalizability

renormalizable theories: divergences can be removed by a finite number of counter terms

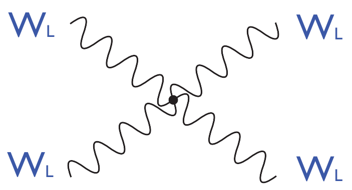
gauge invariant theories: counter terms for parameters (and fields)

W and Z are massive

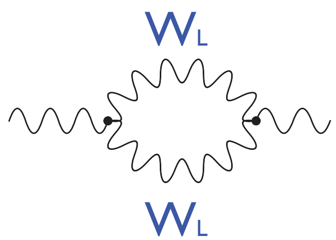
- W, Z have longitudinal polarization states

  polarization vectors of W (Z) $\epsilon_{\mathrm{L}} \sim k/M_{\mathrm{W}}$

  for large momentum k

- 

  bad high energy behaviour of WW scattering

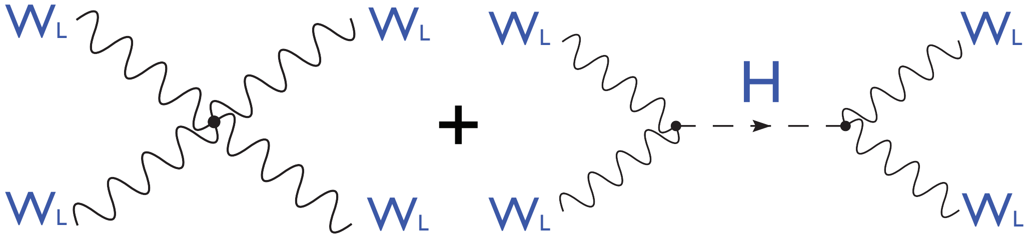
- 

  bad divergence of loop integrals

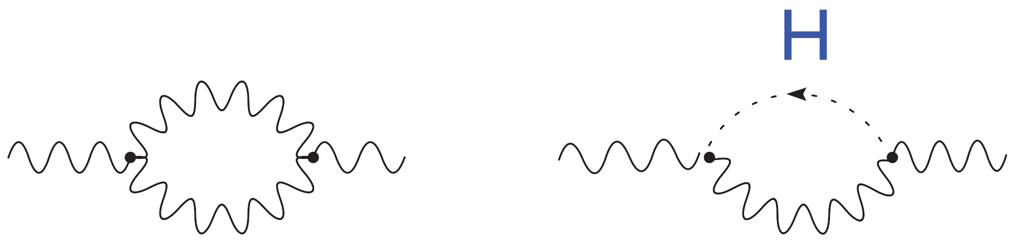
way out:

new scalar with appropriate couplings to W,Z

- restoration of unitarity



- restoration UV finiteness ⇒ renormalizability

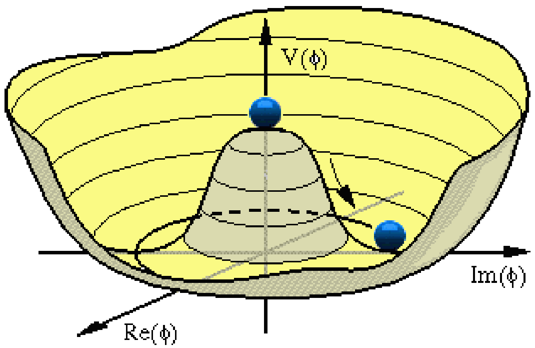


consistent way: Higgs mechanism
= scalar with gauge invariant interactions
and non-invariant ground state

example: complex scalar field $\phi \neq \phi^\dagger$

Lagrangian with interaction V (potential), minimum at $\phi_0 = v$

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi)$$



$V = V(|\phi|)$: *$\mathcal{L}$ symmetric under* $\phi \to e^{i\alpha}\,\phi, \quad U(1)$

$v \neq 0$: $\phi_0' = e^{i\alpha}\,v \neq \phi_0$ *not symmetric*

$V = V(|\phi_0'|) = V(|\phi_0|)$: *vacuum is degenerate*

spontaneous symmetry breaking (SSB)

write $\phi(x) = \eta(x) e^{i\theta(x)}$, η and θ real

$V(|\phi|) = V(\eta)$, minimum at $\eta = v$: $V'(v) = 0$, $V''(v) > 0$

expand around minimum: $\eta(x) = v + \frac{1}{\sqrt{2}} H(x)$

$$V(\eta) = V(v) + \tfrac{1}{2} V''(v) \cdot \tfrac{1}{2} H^2 + \cdots$$

$$\mathcal{L} = \tfrac{1}{2} |\partial_\mu H|^2 - \underbrace{\frac{1}{2} V''(v)}_{m_H^2 > 0} \cdot \tfrac{1}{2} H^2 + v^2 |\partial_\mu \theta|^2 + \cdots$$

$m_H^2 > 0$ *mass of H*

- H field is massive
- θ field is massless, no θ^2 term: Goldstone field
- special case of Goldstone theorem:

  for each broken generator T_a with $T_a \phi_0 \neq 0$

  there is a massless Goldstone field $\theta(x)$

SSB in gauge theories

$$\mathcal{L} = \left(D_\mu \phi\right)^+ \left(D^\mu \phi\right) - V(|\phi|) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad D_\mu = \partial_\mu - ieA_\mu$$

invariant under local U(1) transformations:

$$\phi'(x) = e^{i\alpha(x)}\, \phi(x) = e^{i\alpha(x)}\, e^{i\theta(x)} \eta(x)$$

$$A'_\mu(x) = A_\mu(x) + \frac{1}{e} \partial_\mu \alpha(x)$$

choose $\alpha(x) = -\theta(x)$: $\quad \phi'(x) = \eta(x)$

$$\mathcal{L} = |(\partial_\mu - ieA'_\mu)\eta|^2 - V(\eta) - \frac{1}{4} F'_{\mu\nu} F'^{\,\mu\nu}$$

massless θ field removed (unphysical)

$$\mathcal{L} = |(\partial_\mu - ieA'_\mu)(v + \tfrac{1}{\sqrt{2}}H)|^2 - \tfrac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - V$$

$$= \underbrace{-\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + v^2 e^2\, A'_\mu A'^\mu}_{\text{massive A-field, } m_A \sim ev} + \underbrace{\frac{1}{2}[(\partial_\mu H)^2 - m_H^2 H^2]}_{\text{neutral scalar, } m_H \neq 0} + \cdots$$

in this special gauge: no Goldstone field **unitary gauge**

A_μ-field propagator: $$\frac{i}{k^2 - m_A^2 + i\varepsilon} \underbrace{\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_A^2}\right)}_{\text{polarization sum of 3 pol. states}}$$

massive vector field without spoiling gauge symmetry of $\mathcal{L}$

two different gauges

properties	ϕ field	A_μ field
symmetry manifest	H, θ	2 polarizations (transverse)
physics manifest	H	3 polarizations (2 transverse + 1 longitudinal)

$\theta \quad \rightarrow \quad$ *longitudinal polarization of* A_μ