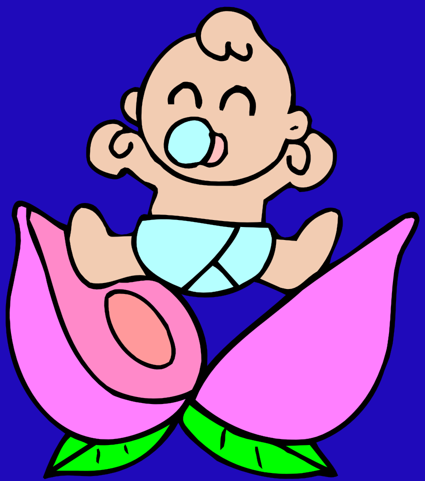
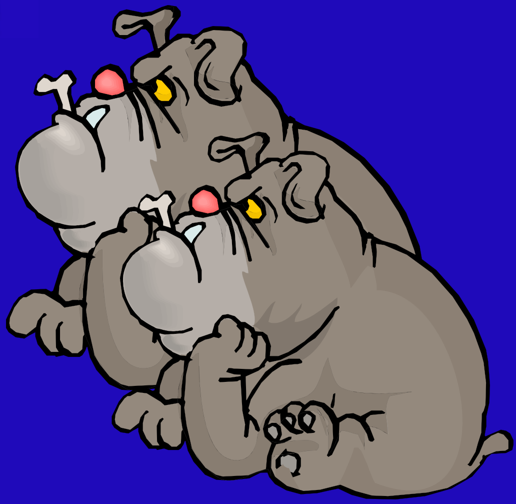
2. Symmetry Breaking

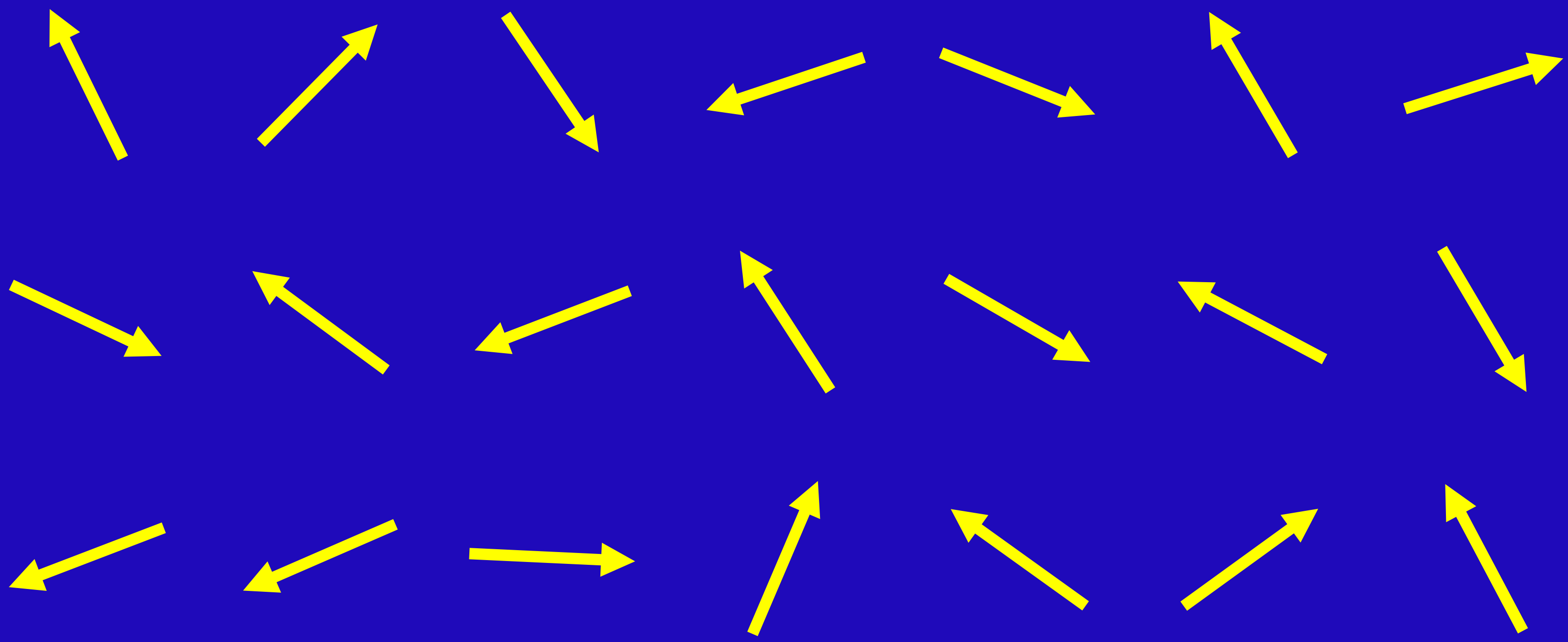


- Spontaneous Symmetry Breaking
- Scalar Potential
- Ground State Symmetry
- Higgs Mechanism
- The Higgs Boson
- Fermion Masses
- Fermion Mixings



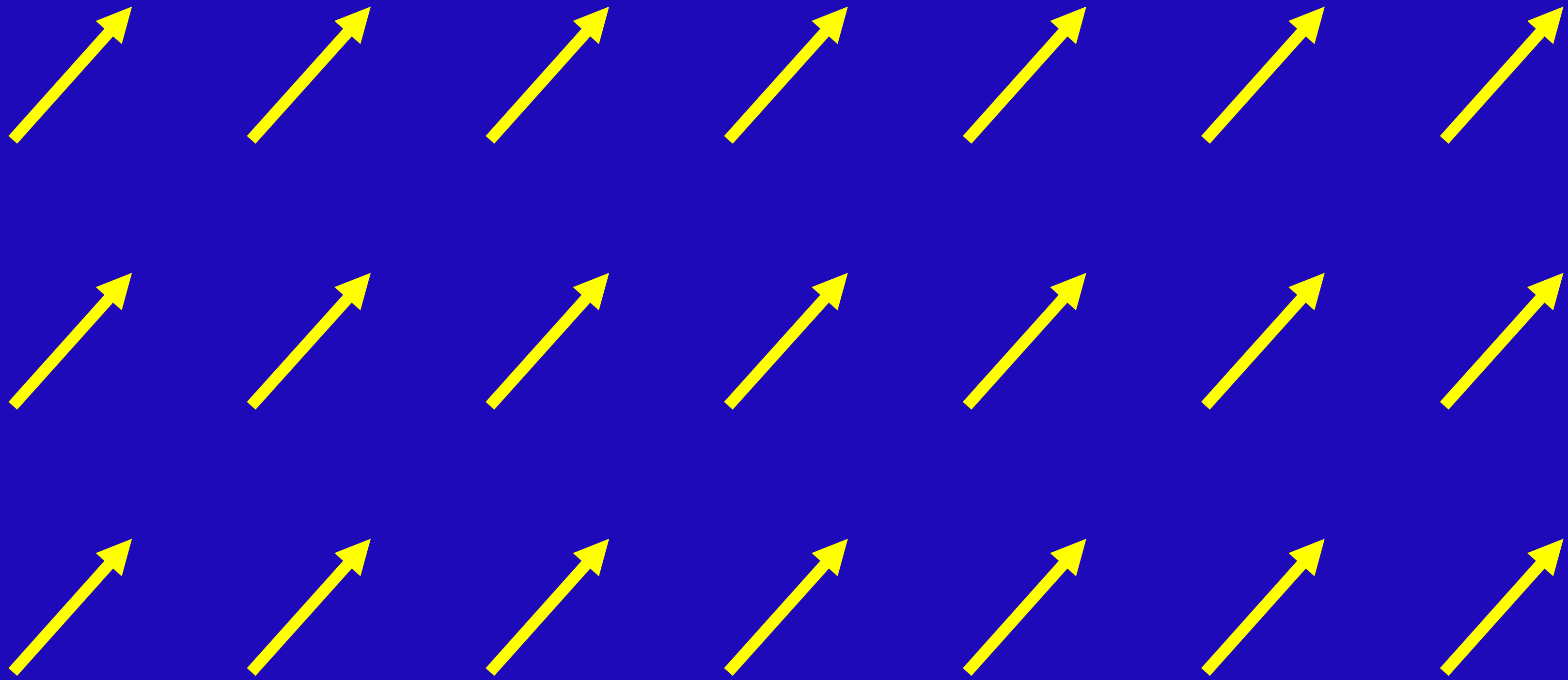
FERROMAGNET

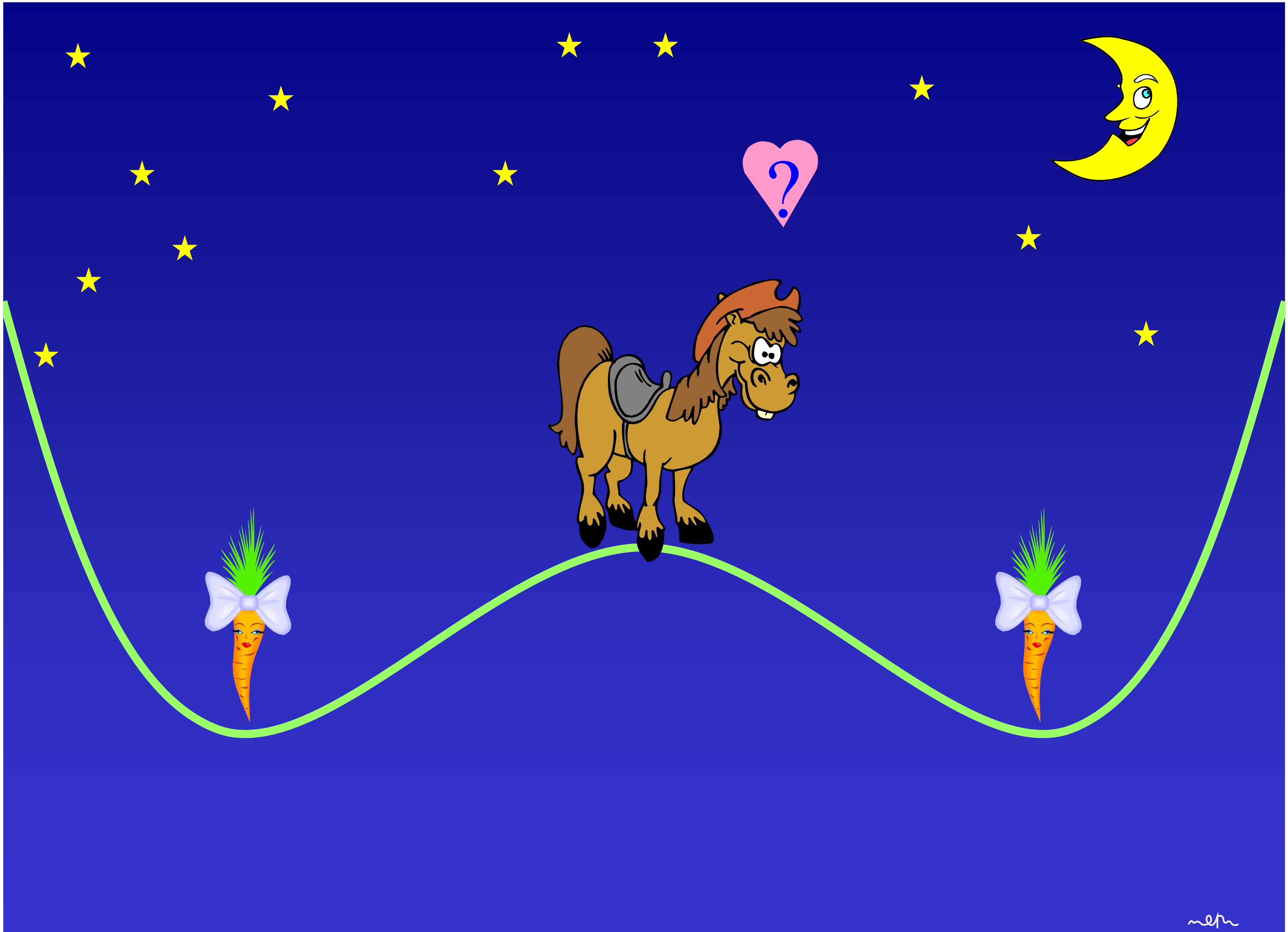
$T > T_C$

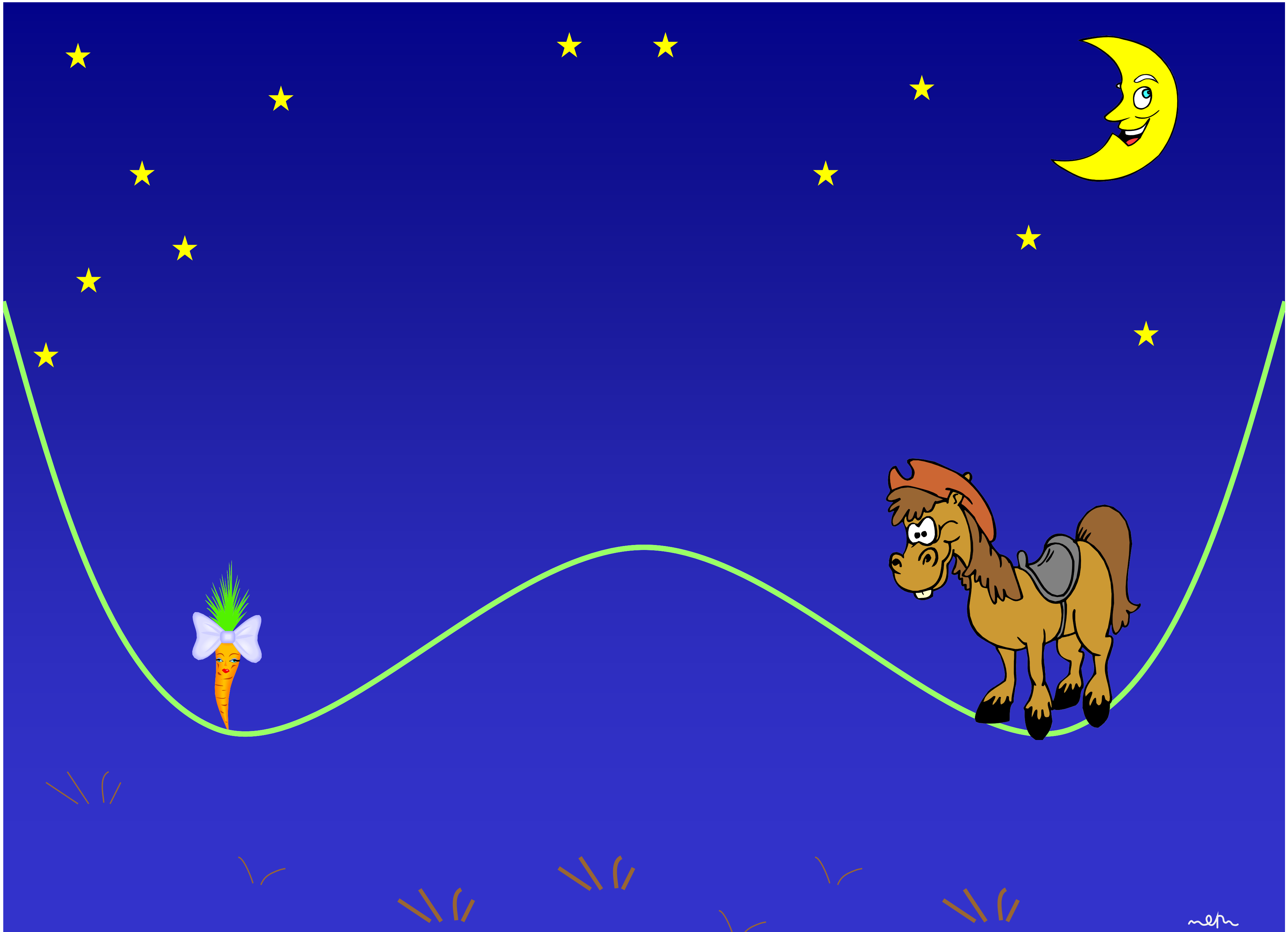


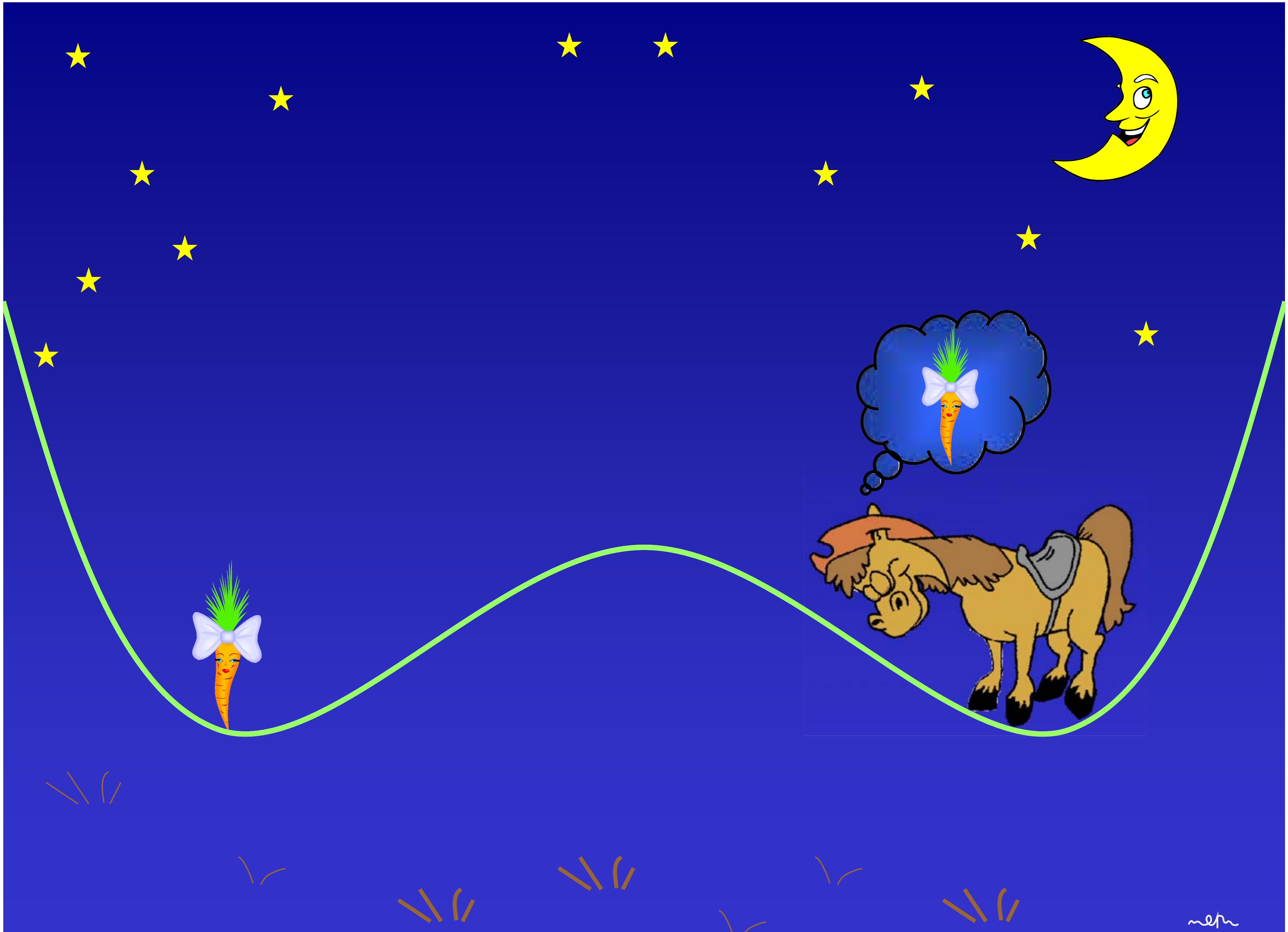
FERROMAGNET

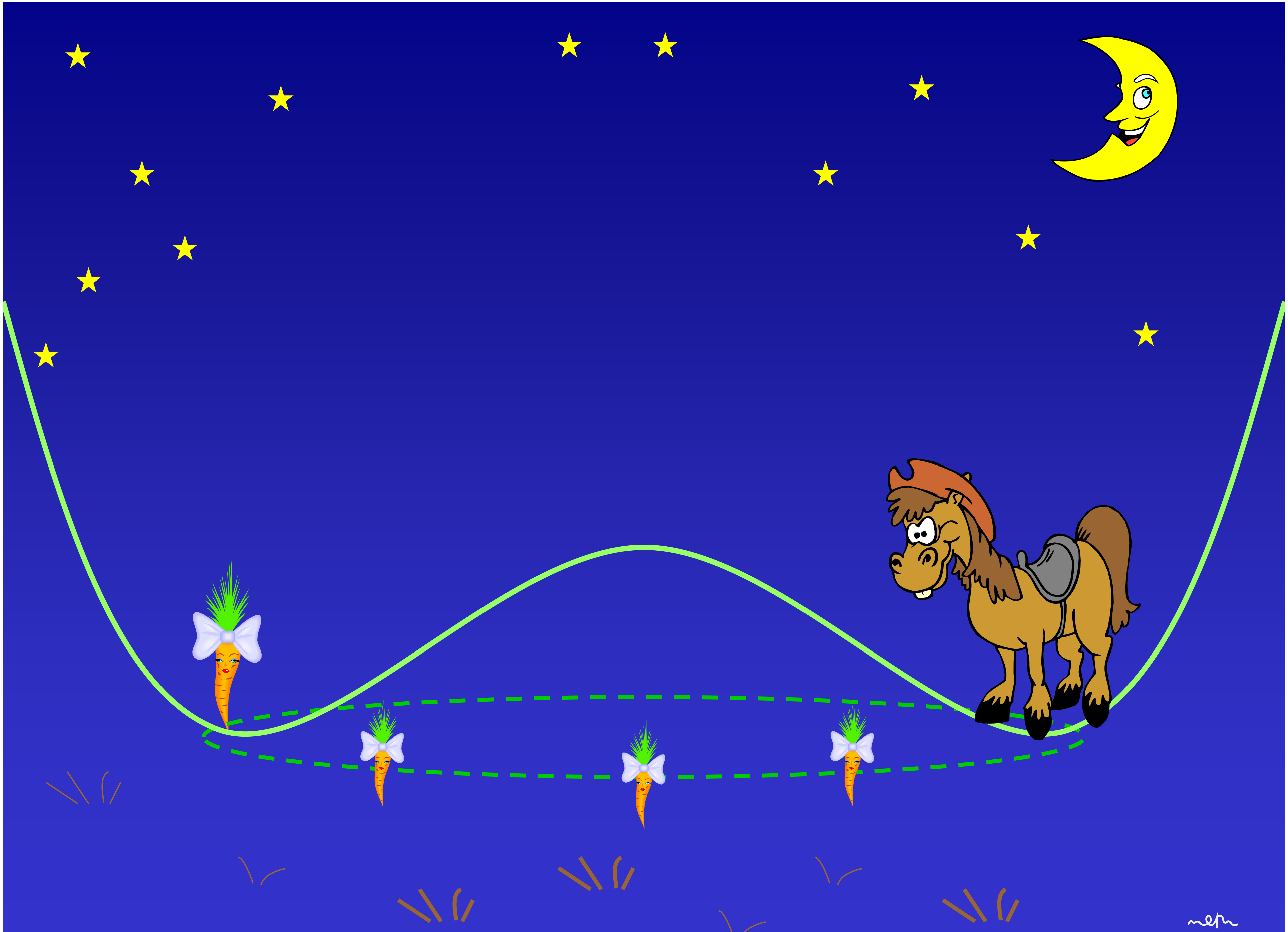
$T < T_C$

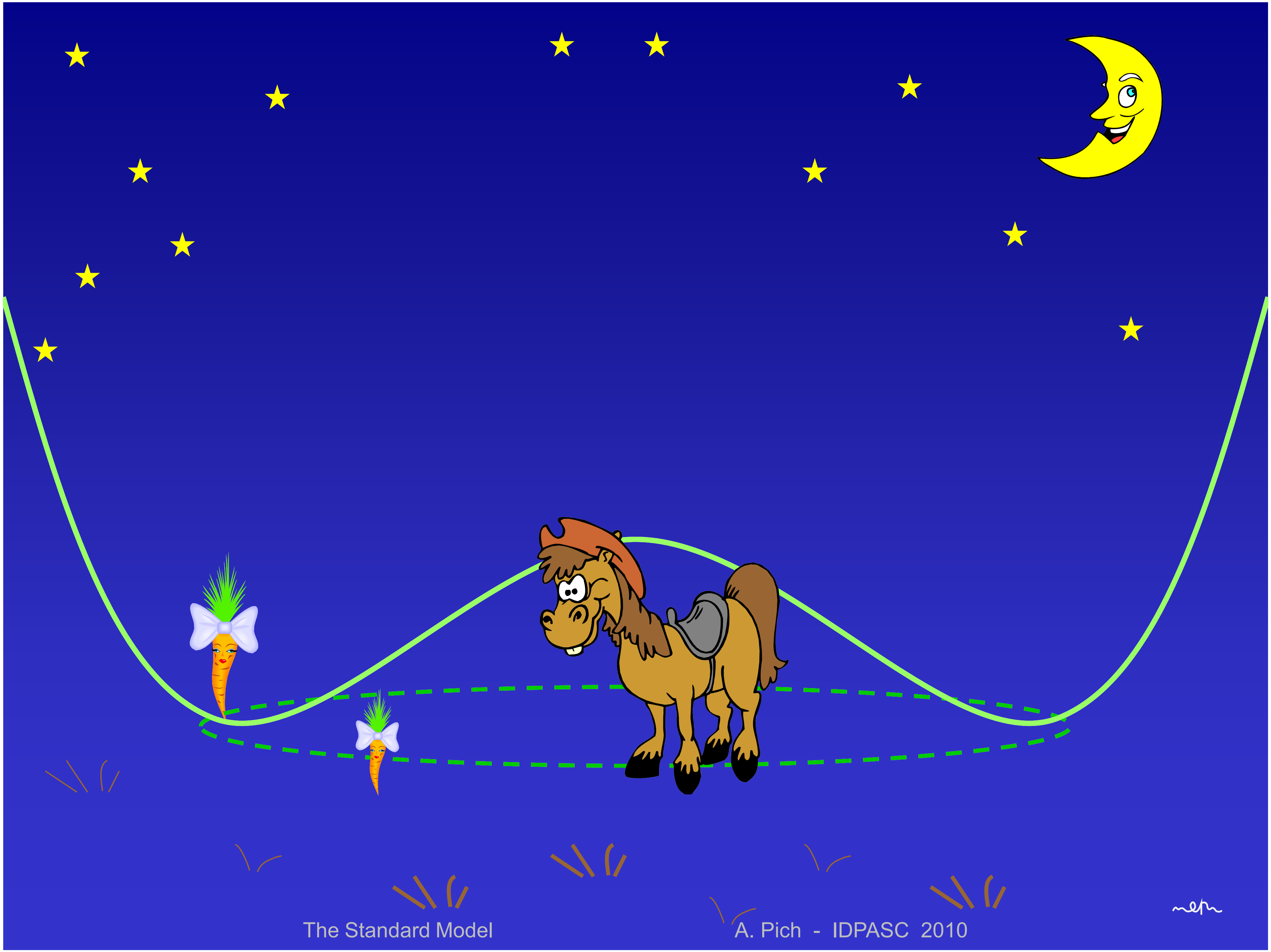











The Standard Model
A. Pich - IDPASC 2010

SCALAR POTENCIAL

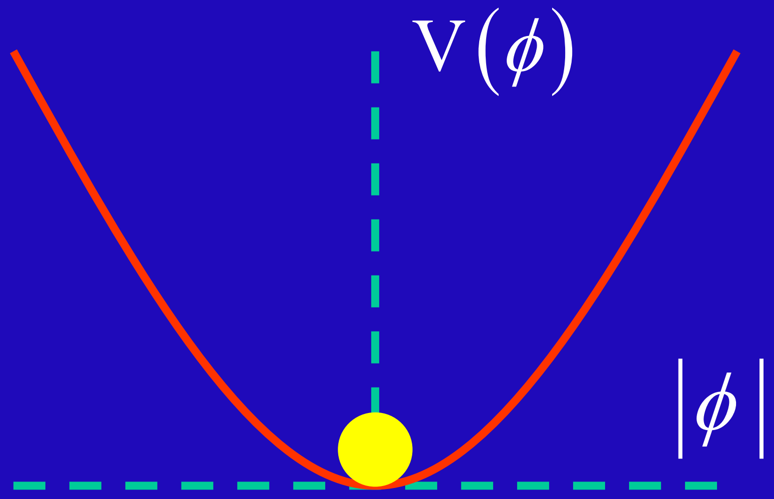
$$\mathcal{L}(\phi) = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi)$$

$$\mathrm{V}(\phi) = \mu^2 \phi^\dagger \phi + h (\phi^\dagger \phi)^2$$

Phase Symmetry: $\phi(x) \to \mathrm{e}^{i\theta} \phi(x)$

$h > 0 \quad ; \quad \mu^2 > 0$

$M_\phi = \mu$



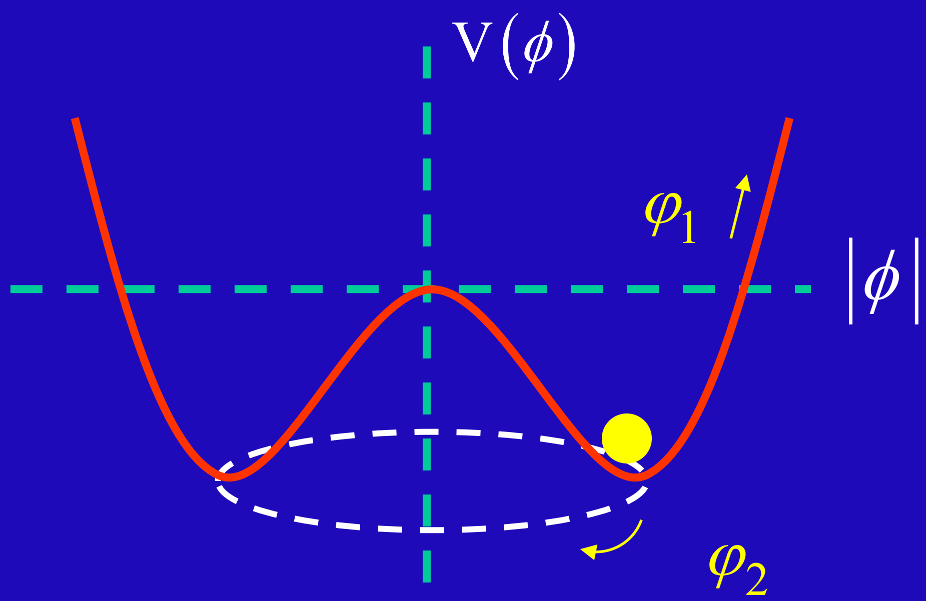
Trivial Minimum (Ground State / Vacuum): $\phi = \phi_0 = 0$

$$V(\phi) = \mu^2\, \phi^\dagger \phi + h\, (\phi^\dagger \phi)^2$$

Phase Symmetry:

$$\phi(x) \to e^{i\theta}\, \phi(x)$$

$$\mu^2 < 0$$



Degenerate Minima

(Ground State / Vacuum)

$$|\phi_0| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}} > 0 \quad ; \quad V(\phi_0) = -\frac{1}{4} h\, v^4$$

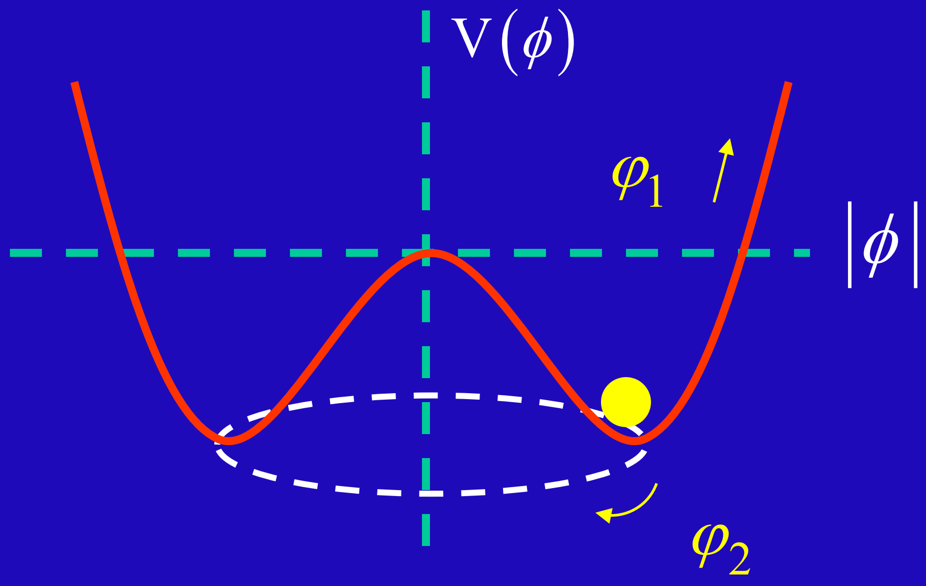
Spontaneous Symmetry Breaking:

$$\phi \equiv \frac{1}{\sqrt{2}} \left[v + \varphi_1(x) \right] e^{i\, \varphi_2(x)/v}$$

Vacuum Choice

Spontaneous Symmetry Breaking

$\mu^2 < 0$



$$\phi \equiv \frac{1}{\sqrt{2}} [\mathrm{v} + \varphi_1(x)]\; e^{i\varphi_2(x)/\mathrm{v}}$$

$$\mathcal{L}(\phi) = \frac{1}{2} \partial_\mu \varphi_1 \partial^\mu \varphi_1 + \frac{1}{2}\left(1 + \frac{\varphi_1}{\mathrm{v}}\right)^2 \partial_\mu \varphi_2 \partial^\mu \varphi_2 - V(\phi)$$

$$V(\phi) = V(\phi_0) + \frac{1}{2} M^2_{\varphi_1} \varphi_1^2 + h\,\mathrm{v}\,\varphi_1^3 + \frac{1}{4} h\, \varphi_1^4$$

$M^2_{\varphi_1} = -2\mu^2 > 0 \quad ; \quad M^2_{\varphi_2} = 0$

1 Massless Goldstone Boson

ELECTROWEAK SSB

New Scalar Doublet $\phi(x) \equiv \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix}$; $y_\phi = Q_\phi - T_3 = \frac{1}{2}$

$$\mathcal{L}(\phi) = (\mathbf{D}_\mu \phi)^\dagger \mathbf{D}^\mu \phi - \mu^2 \phi^\dagger \phi - h (\phi^\dagger \phi)^2$$

$$\mathbf{D}^\mu \phi = \left[\partial^\mu + i g \mathbf{W}^\mu + i g' y_\phi B^\mu\right]\phi \quad ; \quad \mathbf{W}^\mu = \frac{\vec{\tau}}{2} \cdot \vec{W}^\mu$$

$SU(2)_L \otimes U(1)_Y$ Symmetry

Degenerate Vacuum States:

$(\mu^2 < 0 \ , \ h > 0)$

$$\left|\langle 0 | \phi^{(0)} | 0 \rangle\right| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}}$$

Spontaneous Symmetry Breaking: $\phi(x) = \exp\left\{ i \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x) \right\} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$

HIGGS MECHANISM

$$\phi(x) = \exp\left\{ i \frac{\vec{\tau}}{2} \cdot \vec{\theta}(x) \right\} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mathrm{v} + H(x) \end{pmatrix}$$

$SU(2)_L$ Invariance ➡ $\vec{\theta}(x)$ Unphysical

Unitary Gauge: $\vec{\theta}(x) = 0$

$$(D_\mu \phi)^\dagger D^\mu \phi \quad \to \quad \frac{1}{2} \partial_\mu H \, \partial^\mu H + \frac{g^2}{4} (\mathrm{v} + H)^2 \left\{ W_\mu^\dagger W^\mu + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu \right\}$$

➡ $$M_Z \cos\theta_W = M_W = \frac{1}{2} \mathrm{v}\, g$$

Massive Gauge Bosons

Bosonic Degress of Freedom

Massless $W^{\pm}\,,\ Z$

3 x 2 polarizations = 6

+

3 Goldstones $\vec{\theta}$

SSB ↓

Massive $W^{\pm}\,,\ Z$

3 x 3 polarizations = 9

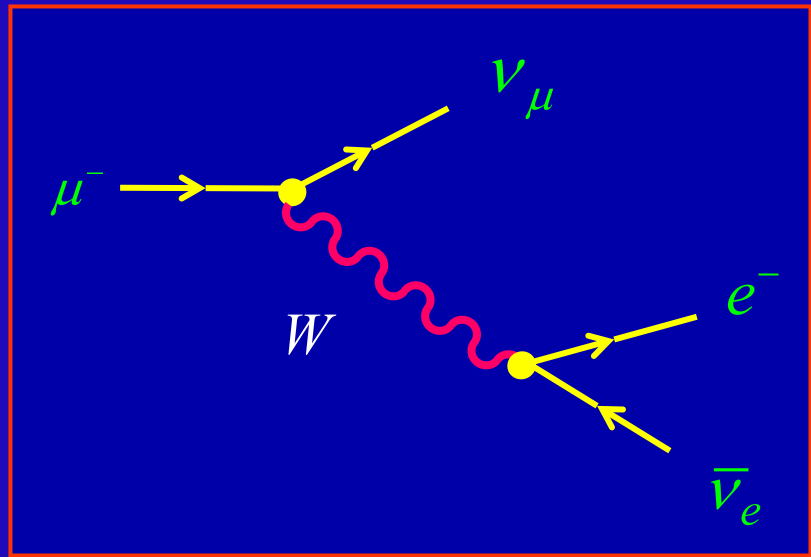
SAME PHYSICS

$$M_Z \cos\theta_W = M_W = \frac{1}{2}\, \mathrm{v}\, g$$

$$M_Z = 91.1875 \text{ GeV} > M_W = 80.399 \text{ GeV} \quad \Rightarrow \quad \sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.223$$

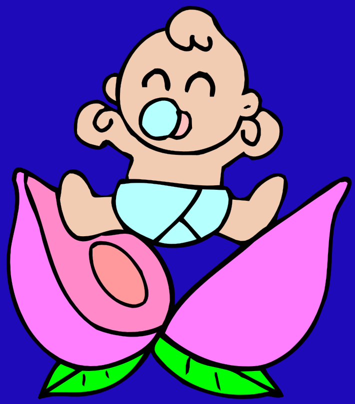
$$\frac{g^2}{M_W^2 - q^2} \approx \frac{g^2}{M_W^2} \equiv 4\sqrt{2}\, G_F$$

$$\frac{1}{\tau_\mu} \equiv \Gamma = \frac{G_F^2\, m_\mu^5}{192\,\pi^3}$$



$$\left. \begin{array}{l} G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2} \\ g = \dfrac{e}{\sin\theta_W} \quad , \quad M_W \end{array} \right\} \quad \Rightarrow \quad \begin{array}{l} \sin^2\theta_W = 0.215 \\ \mathrm{v} = \left(\sqrt{2}\, G_F\right)^{-1/2} = 246 \text{ GeV} \end{array}$$

THE HIGGS BOSON



$$\mathcal{L}_S = \frac{h\,\mathrm{v}^4}{4} + \mathcal{L}_H + \mathcal{L}_{HG^2}$$

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H\,\partial^\mu H - \frac{1}{2}M_H^2\,H^2 - \frac{M_H^2}{2\,\mathrm{v}}\,H^3 - \frac{M_H^2}{8\,\mathrm{v}^2}\,H^4$$

$$\mathcal{L}_{HG^2} = \left[M_W^2\,W_\mu^\dagger W^\mu + \frac{1}{2}M_Z^2\,Z_\mu Z^\mu\right]\left\{1 + \frac{2}{\mathrm{v}}H + \frac{H^2}{\mathrm{v}^2}\right\}$$

1 Scalar Particle H^0 to be Discovered

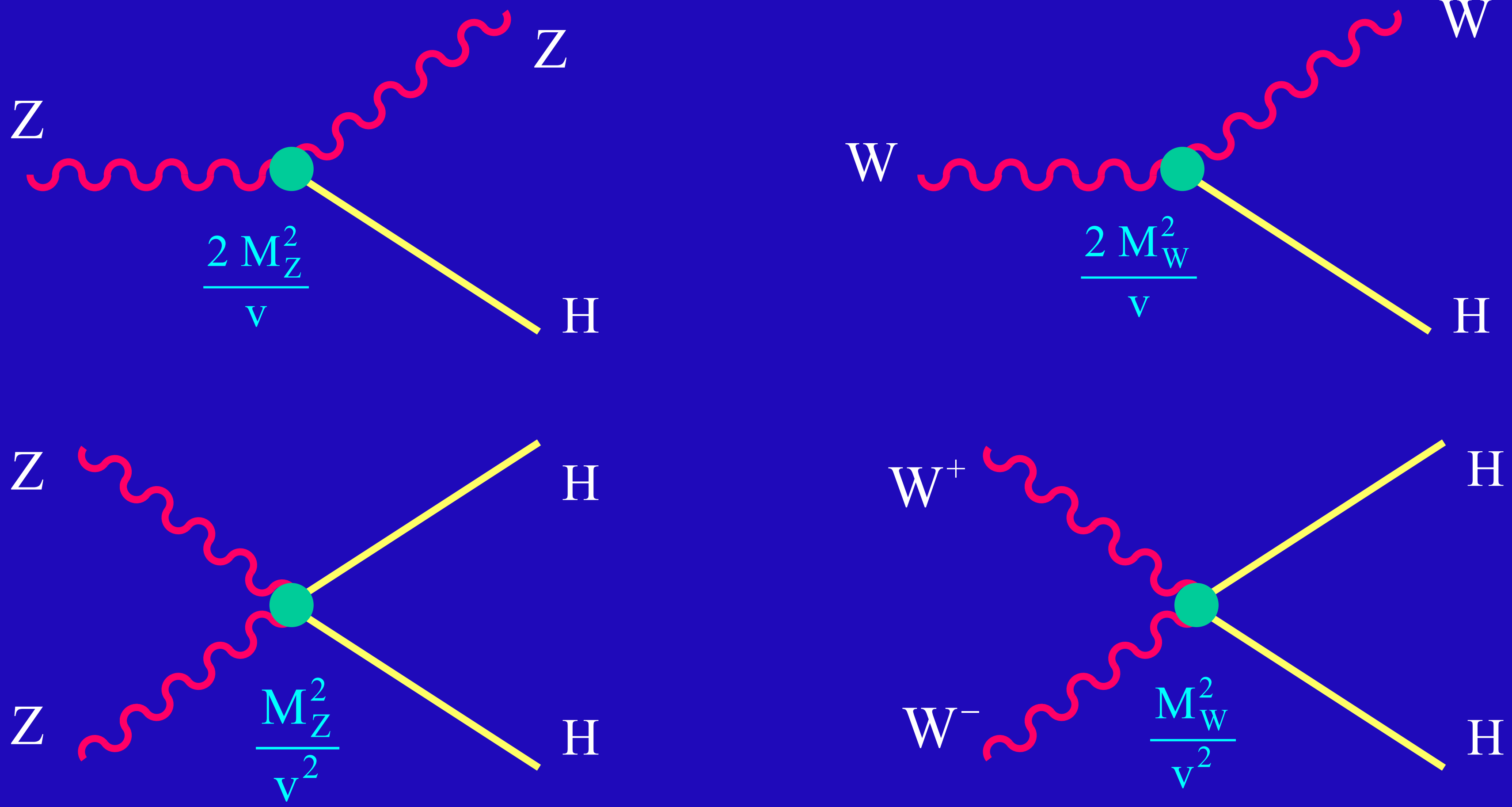
$$M_H = \sqrt{-2\,\mu^2} = \sqrt{2\,h}\;\mathrm{v}$$

Free Parameter

LEP: $114.4\ \mathrm{GeV} < M_H < 185\ \mathrm{GeV}$ (95% CL)

(Direct) (Indirect)

Higgs Couplings $\propto$ Masses



$$v = \left(\sqrt{2}\, G_F\right)^{-1/2} = 246 \quad \text{GeV}$$

FERMION MASSES

Scalar – Fermion Couplings allowed by Gauge Symmetry

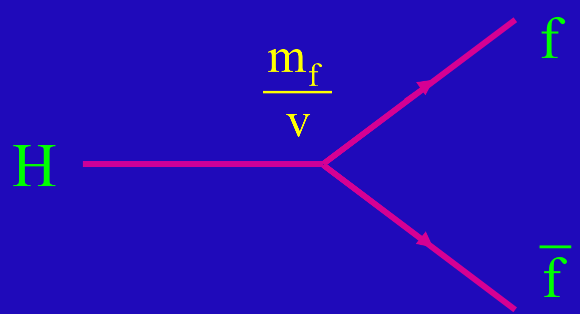
$$\mathcal{L}_Y = (\bar{q}_u, \bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right] + (\bar{\nu}_l, \bar{l})_L \; c^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l_R + \text{h.c.}$$

⬇ **SSB**

$$\mathcal{L}_Y = -\left(1 + \frac{H}{\mathrm{v}}\right) \left\{ m_{q_d} \, \bar{q}_d \, q_d + m_{q_u} \, \bar{q}_u \, q_u + m_l \, \bar{l} \, l \right\}$$

Fermion Masses are New Free Parameters

$$\left[m_{q_d}, \, m_{q_u}, \, m_l \right] = -\left[c^{(d)}, \, c^{(u)}, \, c^{(l)} \right] \frac{\mathrm{v}}{\sqrt{2}}$$



Couplings Fixed: $g_{Hf\bar{f}} = \frac{m_f}{\mathrm{v}}$

FERMION GENERATIONS

$N_G = 3$ Identical Copies

Masses are the only difference

$$\begin{matrix} Q=0 \\ Q=-1 \end{matrix} \quad \begin{pmatrix} \nu'_j & u'_j \\ l'_j & d'_j \end{pmatrix} \quad \begin{matrix} Q=+2/3 \\ Q=-1/3 \end{matrix} \qquad (j = 1, \cdots, N_G)$$

WHY ?

$$\mathcal{L}_Y = \sum_{jk} \left\{ \left(\bar{u}'_j, \bar{d}'_j\right)_L \left[c_{jk}^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c_{jk}^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] + \left(\bar{\nu}'_j, \bar{l}'_j\right)_L c_{jk}^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l'_{kR} \right\} + \text{h.c.}$$

⬇ **SSB**

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \left\{ \bar{d}'_L \cdot \mathbf{M}'_d \cdot d'_R + \bar{u}'_L \cdot \mathbf{M}'_u \cdot u'_R + \bar{l}'_L \cdot \mathbf{M}'_l \cdot l'_R + \text{h.c.} \right\}$$

Arbitrary Non-Diagonal Complex Mass Matrices

$$\left[\mathbf{M}'_d , \mathbf{M}'_u , \mathbf{M}'_l \right]_{jk} = - \left[c_{jk}^{(d)} , c_{jk}^{(u)} , c_{jk}^{(l)} \right] \frac{v}{\sqrt{2}}$$

DIAGONALIZATION OF MASS MATRICES

$$\mathbf{M}'_d = \mathbf{H}_d \cdot \mathbf{U}_d = \mathbf{S}_d^\dagger \cdot \mathcal{M}_d \cdot \mathbf{S}_d \cdot \mathbf{U}_d$$
$$\mathbf{M}'_u = \mathbf{H}_u \cdot \mathbf{U}_u = \mathbf{S}_u^\dagger \cdot \mathcal{M}_u \cdot \mathbf{S}_u \cdot \mathbf{U}_u$$
$$\mathbf{M}'_l = \mathbf{H}_l \cdot \mathbf{U}_l = \mathbf{S}_l^\dagger \cdot \mathcal{M}_l \cdot \mathbf{S}_l \cdot \mathbf{U}_l$$

$$\mathbf{H}_\mathrm{f} = \mathbf{H}_\mathrm{f}^\dagger$$
$$\mathbf{U}_\mathrm{f} \cdot \mathbf{U}_\mathrm{f}^\dagger = \mathbf{U}_\mathrm{f}^\dagger \cdot \mathbf{U}_\mathrm{f} = 1$$
$$\mathbf{S}_\mathrm{f} \cdot \mathbf{S}_\mathrm{f}^\dagger = \mathbf{S}_\mathrm{f}^\dagger \cdot \mathbf{S}_\mathrm{f} = 1$$

$$\mathcal{L}_Y = -\left(1 + \frac{H}{\mathrm{v}}\right) \left\{ \bar{\mathrm{d}} \cdot \mathcal{M}_d \cdot \mathrm{d} + \bar{\mathrm{u}} \cdot \mathcal{M}_u \cdot \mathrm{u} + \bar{l} \cdot \mathcal{M}_l \cdot l \right\}$$

$$\mathcal{M}_u = \mathrm{diag}\,(m_u, m_c, m_t) \quad ; \quad \mathcal{M}_d = \mathrm{diag}\,(m_d, m_s, m_b) \quad ; \quad \mathcal{M}_l = \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$

$$\mathrm{d}_L \equiv \mathbf{S}_d \cdot \mathrm{d}'_L \quad ; \quad \mathrm{u}_L \equiv \mathbf{S}_u \cdot \mathrm{u}'_L \quad ; \quad l_L \equiv \mathbf{S}_l \cdot l'_L$$
$$\mathrm{d}_R \equiv \mathbf{S}_d \cdot \mathbf{U}_d \cdot \mathrm{d}'_R \quad ; \quad \mathrm{u}_R \equiv \mathbf{S}_u \cdot \mathbf{U}_u \cdot \mathrm{u}'_R \quad ; \quad l_R \equiv \mathbf{S}_l \cdot \mathbf{U}_l \cdot l'_R$$

Mass Eigenstates ≠ Weak Eigenstates

$$\bar{\mathrm{f}}'_L \, \mathrm{f}'_L = \bar{\mathrm{f}}_L \, \mathrm{f}_L \quad ; \quad \bar{\mathrm{f}}'_R \, \mathrm{f}'_R = \bar{\mathrm{f}}_R \, \mathrm{f}_R \quad \Longrightarrow \quad \mathcal{L}'_{\mathrm{NC}} = \mathcal{L}_{\mathrm{NC}}$$

$$\bar{\mathrm{u}}'_L \, \mathrm{d}'_L = \bar{\mathrm{u}}_L \cdot \mathbf{V} \cdot \mathrm{d}_L \quad ; \quad \mathbf{V} \equiv \mathbf{S}_u \cdot \mathbf{S}_d^\dagger \quad \Longrightarrow \quad \mathcal{L}'_{\mathrm{CC}} \neq \mathcal{L}_{\mathrm{CC}}$$

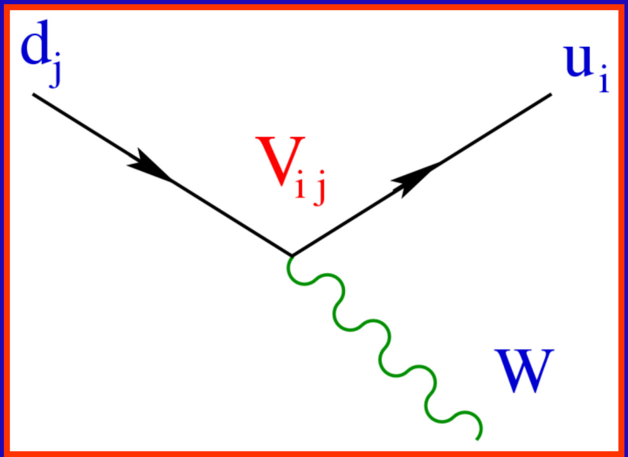
QUARK MIXING

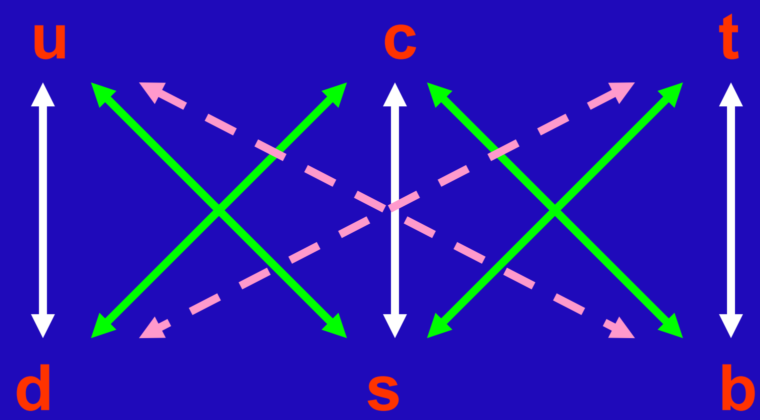
$$\mathcal{L}_{NC}^{Z} = \frac{e}{2\sin\theta_W \cos\theta_W}\, Z_\mu \sum_f \bar{f}\,\gamma^\mu \left[v_f - a_f\,\gamma_5\right] f$$

Flavour Conserving Neutral Currents

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}}\, W_\mu^\dagger \left[\sum_{ij} \bar{u}_i\,\gamma^\mu (1-\gamma_5)\, V_{ij}\, d_j + \sum_l \bar{\nu}_l\,\gamma^\mu (1-\gamma_5)\, l \right] + \text{h.c.}$$

Flavour Changing Charged Currents





LEPTON MIXING

$$\mathcal{L}_{CC}^{(l)} = \frac{g}{2\sqrt{2}} W_\mu^\dagger \sum_{ij} \bar{\nu}_i \gamma^\mu (1-\gamma_5) \mathbf{V}_{ij}^{(l)} l_j + \text{h.c.}$$

- **IF** $m_{\nu_i} = 0$ ⟶ $\mathcal{L}_{CC}^{(l)} = \frac{g}{2\sqrt{2}} W_\mu^\dagger \sum_l \bar{\nu}_l \gamma^\mu (1-\gamma_5) l + \text{h.c}$

  $\bar{\nu}_{l_j} \equiv \bar{\nu}_i \mathbf{V}_{ij}^{(l)}$

Separate Lepton Number Conservation (Minimal SM without ν_R)

- **IF** ν_R^i exist and $m_{\nu_i} \neq 0$

  $\not{L}_e , \not{L}_\mu , \not{L}_\tau$ $\quad (L_e + L_\mu + L_\tau \;\; \text{Conserved})$

BUT $\text{Br}(\mu \to e\gamma) < 1.2\times 10^{-11}$; $\text{Br}(\tau \to \mu\gamma) < 4.4\times 10^{-8}$

(90 % CL)

QUARK MIXING MATRIX

- **Unitary** $N_G \times N_G$ **Matrix:** N_G^2 **parameters**

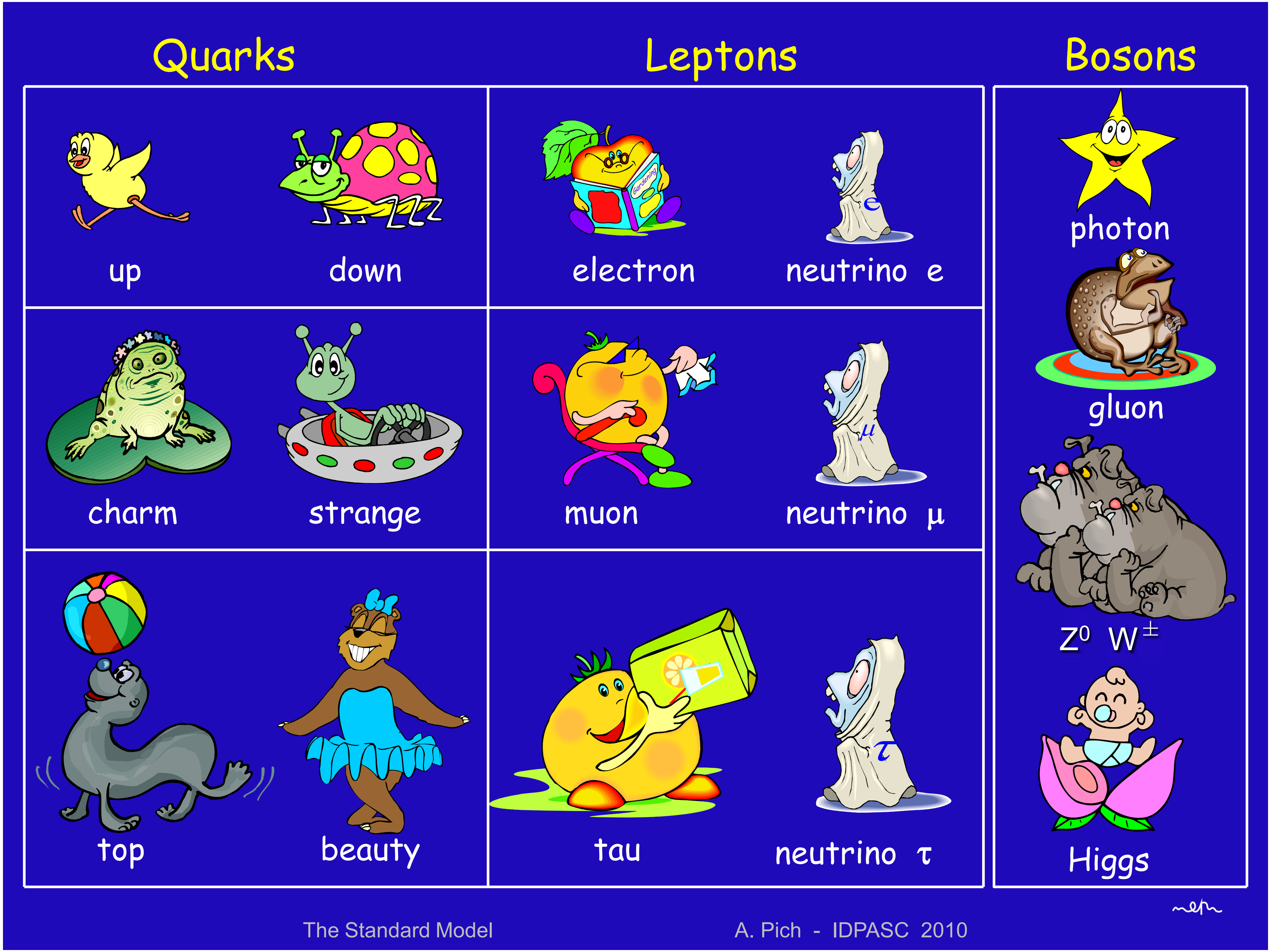
$$\mathbf{V} \cdot \mathbf{V}^\dagger = \mathbf{V}^\dagger \cdot \mathbf{V} = \mathbf{1}$$

- $2 N_G - 1$ **arbitrary phases:**

$$u_i \to e^{i\phi_i} u_i \quad ; \quad d_j \to e^{i\theta_j} d_j \quad \Longrightarrow \quad \mathbf{V}_{ij} \to e^{i(\theta_j - \phi_i)} \mathbf{V}_{ij}$$

$\mathbf{V}_{ij}$ **Physical Parameters:**

$\frac{1}{2} N_G (N_G - 1)$ **Moduli** ; $\frac{1}{2} (N_G - 1) (N_G - 2)$ **phases**



Quarks
Leptons
Bosons
up
down
electron
neutrino e
photon
charm
strange
muon
neutrino μ
gluon
top
beauty
tau
neutrino τ
Z^0 $W^\pm$
Higgs
The Standard Model
A. Pich - IDPASC 2010